Introduction to Biostatistics
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Lecture - 23
Probability Distribution: Poisson distribution and Uniform distribution Part-I

Hello and welcome to today's lecture. So, in the last lecture we had discussed about binomial random variables right.

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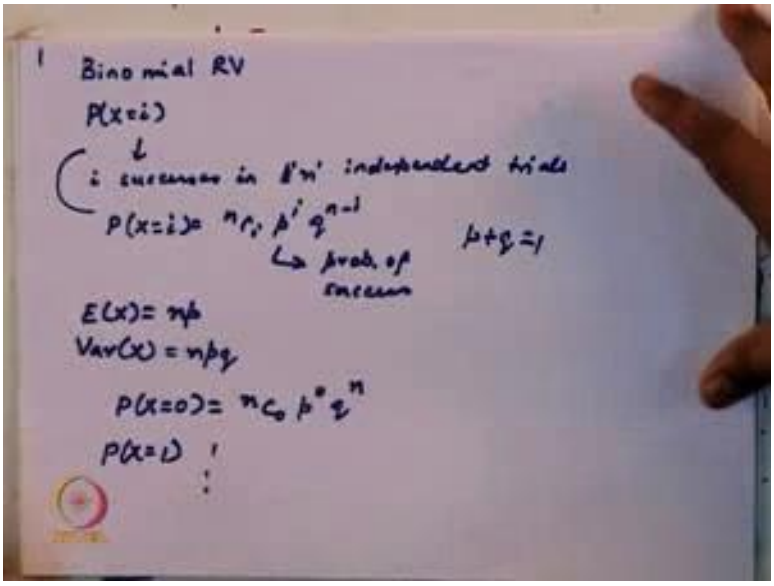

So, in case of a binomial random variable, so you have probability of x equal to i that is i successes in n independent trials is defined as p of x equal to i is equal to n c i p to the power i q to the power n minus I, where p is the probability of success, and q is the probability of failure so p plus q is equal to 1. And we showed that for this, but in distribution your expectation of x is given by n p, and variance of x is given by n p q. Also you can use recursion to actually compute the binomial probabilities as you go forward.

So, today in other words, we know that p of x equal to 0, p of x equal to 0 is n c 0, p to the power 0 q to the power n and so, and then accordingly you can know x equal to 1 and so on and so forth; you can compute in terms of probability of x equal to 0 and so on. So, today we will discuss about another very important probability distribution, just call the poisson probability distribution or Poisson distribution.

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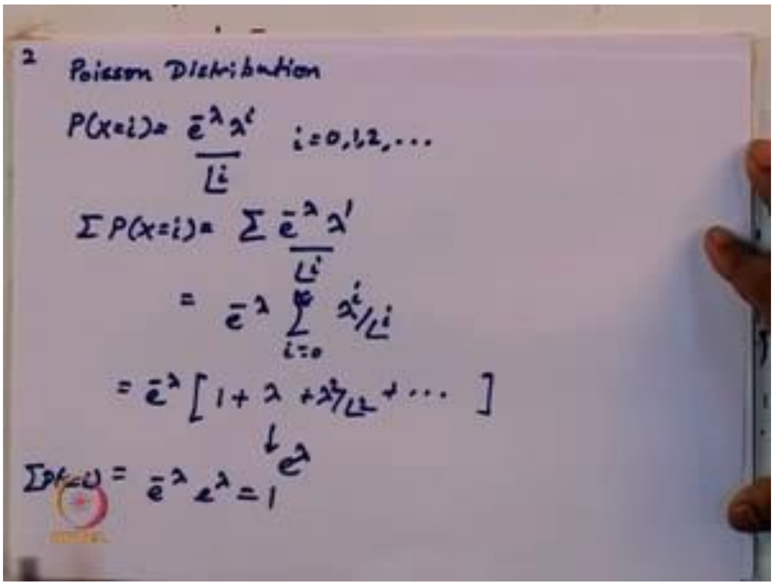

In Poisson distribution the probability mass function for probability x equal to i is given by e to the power minus lambda, lambda to the power i by factorial i where I can take values 0, 1, 2 dot dot dot. So, i is a discrete random variable. So, in this case let us first compute. So, what you can easily see is summation of probability of x equal to i, is equal to e to the summation. So, you can take out e to the power minus lambda, and you are left with (Refer Time: 03:08). So, this is nothing but e to the power lambda. So, this is nothing, but e to the power lambda. So, we have summation probability x equal to i is e to the power minus lambda into the lambda equal to 1 which fulfills our summation of probabilities must be equal to 1.

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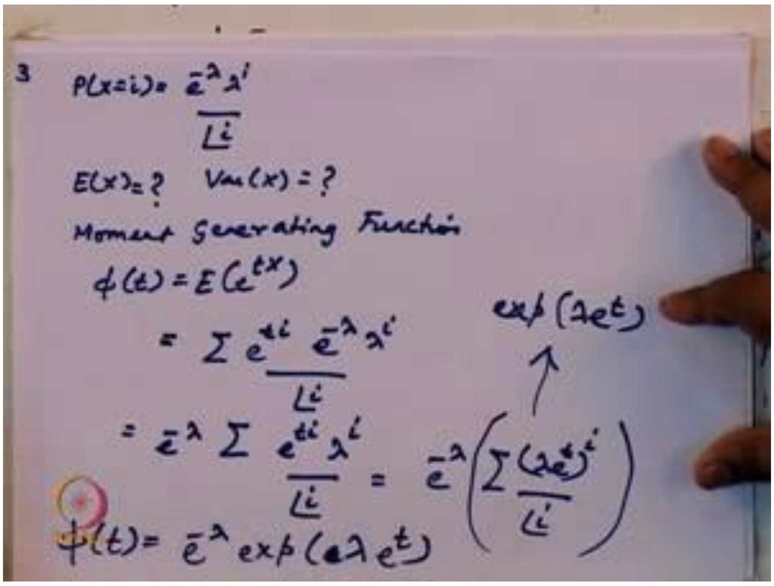

So, now, so again we write down the probability mass function. So, let us try to find out expectation and variance; we want to calculate what is expectation, and variance of x is how much. So, what we can do is we can use moment generating function, and if you remember so phi of t is given by expectation of e to the power t x right. So, in this case we will have this is nothing, but summation e to the power t i, e to the power minus lambda, lambda to the power i by factorial i. So, I can take out to the power minus lambda, and you have summation e to the power t i, lambda to the power i by factorial i; lambda t equal to the power i.

So, I can write I can combine these 2 parameters, and write lambda e to the power t whole to the power i. So, now, this; so this whole term can be written as this whole term can be written as exponential of lambda e to the power t. So, then my moment generating function phi t becomes e to the power minus lambda, into exponential of e to lambda e to the power t. So, phi of t once more is e to the power minus lambda exponential lambda e to the power t. So, this I can write as exponential lambda into e to the power t minus 1.

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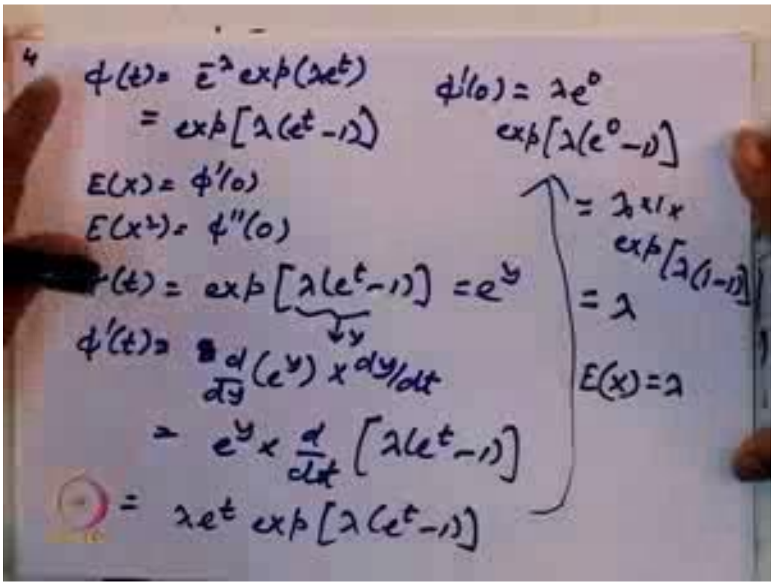

So, how do I calculate expectation? So I know that expectation of x is equal to phi prime of 0, and expectation of x square equal to phi double prime of 0. So, phi of t is given by exponential lambda e to the power t minus 1. So, phi prime of t will be. So, if you consider this as y. So, this is e to the power y. So, phi prime of t is equal to d d y of e to the power y, into d y d t.

So, first term will return it to the power y, into d y d t is d d t of lambda into e to the power t minus 1. So, what we will get is lambda e to the power t into e to the power y. So, it remains as exponential lambda e to the power t minus 1. So, from here I can write phi prime of 0. So, if I put t equal to 0, then this value is 1, e to the power 0 1 minus 1 is 0, so let me write it down; lambda e to the power 0, into exponential, lambda e to the power 0 minus 1. So, lambda e to the power 0 is 1 and exponential. So, it will be lambda exponential into 1 into exponential lambda into 1 minus 1. So, this will also be e to the power 0 is 1 so this gives me value of lambda. So, I can write e of x is equal to lambda ok.

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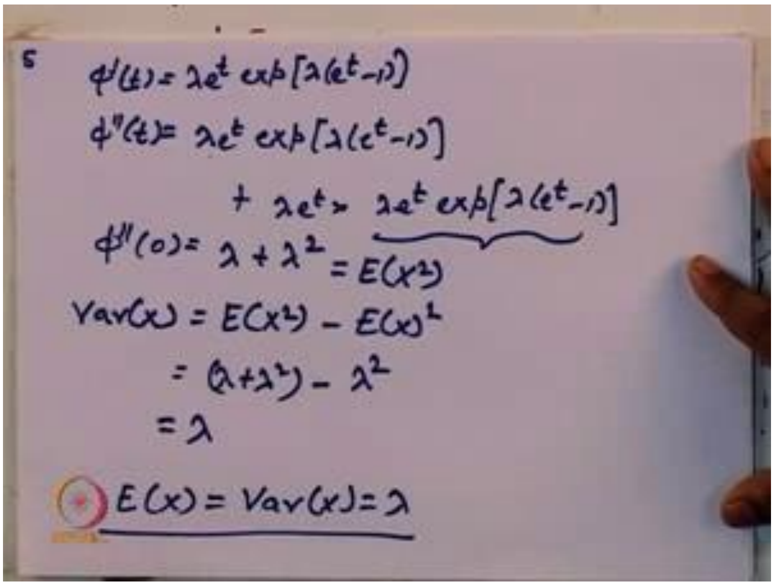

So, phi prime t is given by lambda e to the power t into exponential, lambda into e to the power t minus 1. So, I can write phi double prime t is equal to lambda e to the power t into exponential lambda into e to the power t minus 1 plus the derivative of; that means, lambda e to the power t into, lambda e to the power t exponential of lambda e to the power t minus 1. So, you have. So, this this component comes from taking a derivative. So, this component comes from taking a derivative of this part. So, first part derivative of first part remains as is. So, that is why we have lambda e to the power t into this, and then lambda e to the power t and derivative of this component will give you this result.

So, phi double prime of 0 then becomes lambda, and this just becomes lambda square. So, this is nothing, but E of x square. So, variance of x we know is expectation of x square minus E of x whole square. So, this becomes lambda plus lambda square minus lambda square is equal to lambda. So, for a poisson distribution you have expectation, for a poisson distribution your expectation and variance both return a value of lambda.

Now, let us solve few examples of Poisson distribution. Now what are the kind of you know variables which follow a poisson distribution let us say the number of visits to a clinic on a given day, or number of cases handled by insurance company on a given day so on and so forth. So, let us solve a sample example few some. So, let us assume that.

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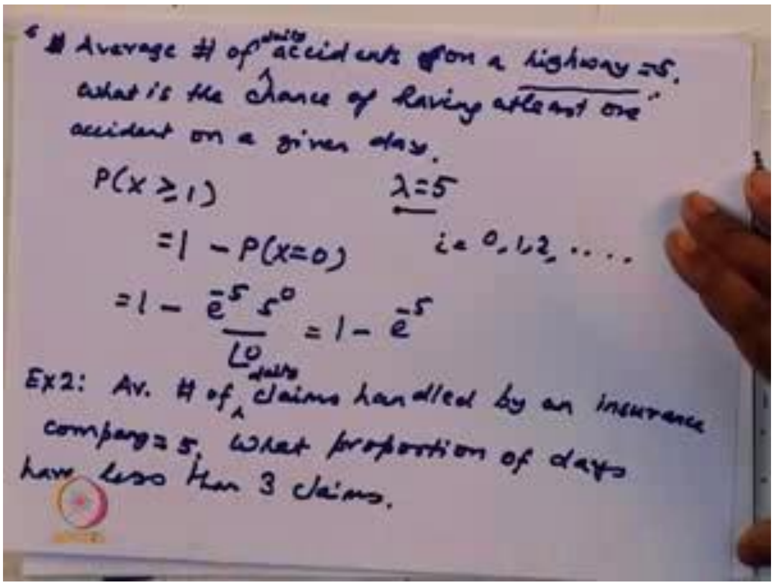

So, the accidents; so the number of accidents or average, number of accidents on a highway, number of delay accidents is equal to 5. So, what is the chance of having at least 1 accident on a given day? So, what we want to do is we want to calculate the probability of x greater or equal to 1. Now your average number of accidents of given highway is 5, which means that this is the parameter lambda. So, lambda is equal to 5. So, we want to calculate probability of x greater equal to 1. So, this can be written as nothing, but 1 minus p of x equal to 0; why because for a poisson random variable your, i can be values from 0 1 2 so on and so forth.

So, that is why so your p greater x equal to 1 is 1 minus p x equal to 0. So, I can write 1 minus e to the power minus 5, 5 to the power 0, by factorial of 0 this is 1 minus e to the power minus 5. So, this is your final answer. So, from the problem you are supposed to identify what is the value of 5 or lambda. So, when you say when you are given a language like average number of delay accidents is 5, then you know that lambda is equal to 5. Let us take another example 2. So, let us say the average number of claims handled by average number of daily claims handled by an insurance company is equal to 5.

So, what proportion of days have less than 3 claims? So, in order to solve this problem I will show this. So, average number of daily claims handled by the insurance company is

5, and what proportions of days have less than 3 claims. So, what we want to do is to calculate probability of x less or equal to 3.

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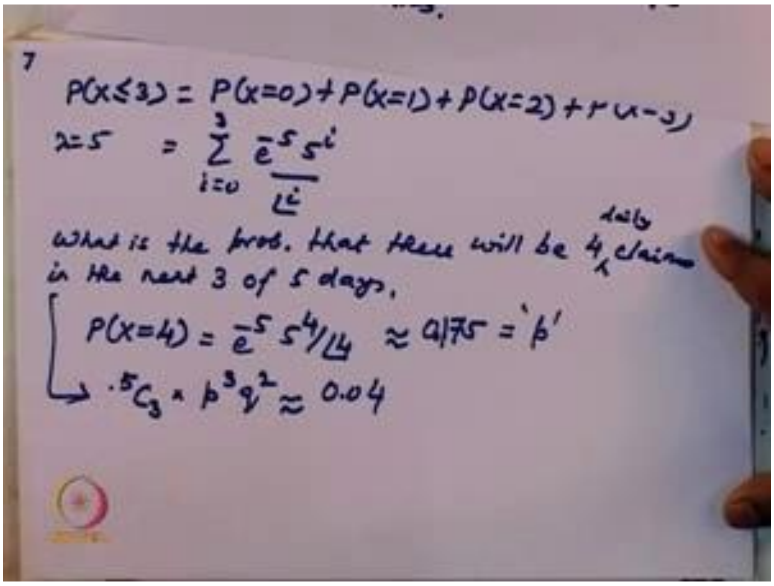

So, then this is nothing, but probability of x equal to 0, plus probability of x equal to 1, plus probability of x equal to 2, plus probability of x equal to 3 and then you can plug in the values. So, you are also given the average number of claims is 5 that means, lambda is 5. So, you can just expand the summation i is equal to 0 to 3 e to the power minus 5, 5 to the power i by factorial i this will be your final answer.

Now, let us say I can ask the next question. So, what is the probability that there will be 4 claims in the next 3 of 5 days? So, this is an example. So, what you calculate this probability p of x is equal to 3 is some given value right. So, now, you want to calculate what is the probability that there will be 4 claims? So, what is the probability of calculating 4 claims, we just need to calculate x equal to 4 so this is nothing, but e to the power minus 5, 5 to the power 4 by factorial 4, this will give you a value of roughly 0.175.

Now, the question if you look at the second part of the question that there will be 4 claims in next 3 days. So, in a sense you get the idea that you have to combine. So, this probability is you get is let us say probability of success in a binomial random variable. So, your answer to this case is basically 5 C 3. So, in any 3 of the 5 days you will get 4 claims. So, this is this p cubed q square; this is what we have been asked to determine.

Here p is nothing, but the probability of x equal to 4 claims. So, 4 I guess daily claims. So, here you plug in the value of p here and then you will get the exact value here. So, in 3 of out of 5 days, so your n is 5, and your you want to. So, this is a binomial random variable with n 5, and p is as follows p is the probability that there will be 4 claims in a given day. Accordingly you can calculate this particular value, and this will be you will get a value of roughly 0.04 or something; just cross check put in the proper values and cross checked.

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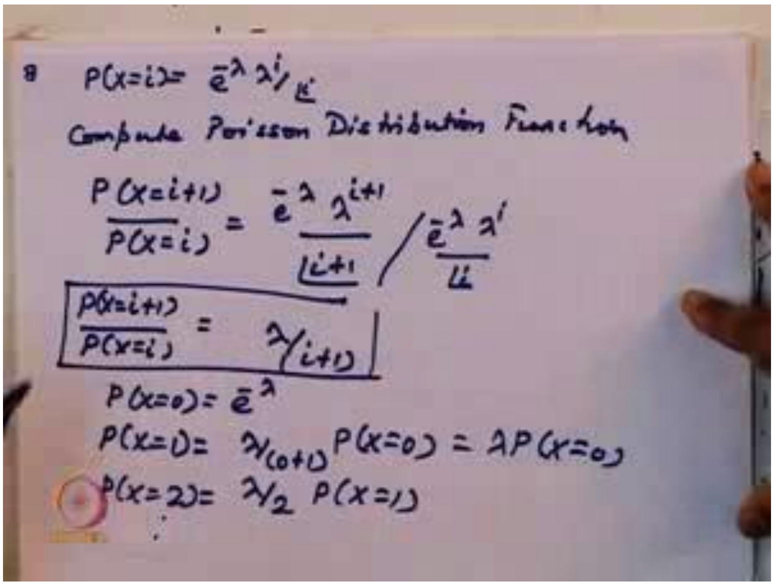

So, let us again write down the probability of x equal to i in this case of a poisson variable right. So, we have e to the power minus lambda, lambda to the power i by, e to the power minus lambda, lambda to the power i by factorial i. So, how can we compute the Poisson distribution function, in other words how can you write a program in order to find out each of the probabilities of the Poisson distribution? For doing that what you can do is you can write down probability x equal to i plus 1 by probability of x equal to i, is equal to nothing, but lambda by i plus 1; because factorial i plus 1 can be written as factorial i into i plus 1.

And these 2 will cancel each other out, these 2 cancel each other out, lambda to the power i plus 1 by lambda total for i is simply lambda. So, it would mean that so probability of x equal to 0 is nothing but e to the power minus lambda, probability of x equal to 1 will give you lambda by x equal to 1. So, i is 0 so 0 plus 1 into p of x equal to

0; this is nothing but lambda into p of x equal to 0. P of x equal to 2 accordingly we plug in the values will be lambda by 2 into probability of x equal to 1.

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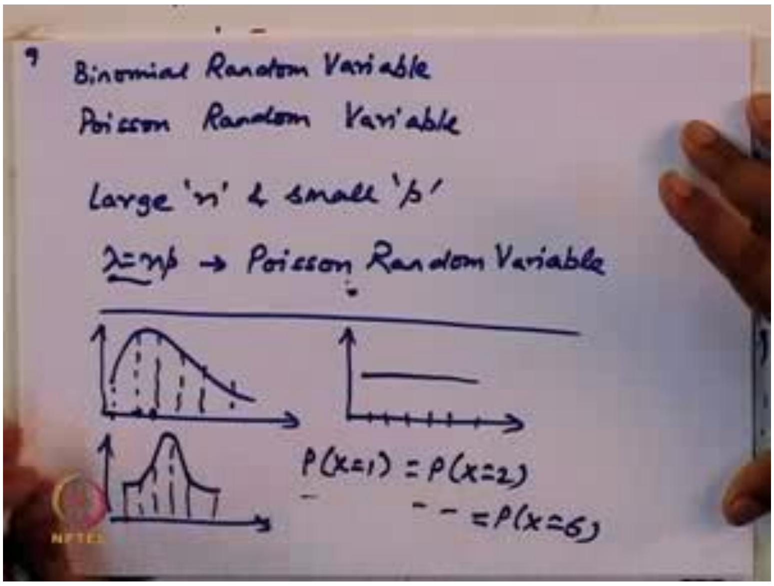

So, using this particular expression you can write down the recursive form of this calculating this probability is so on and so forth. So, that. So, another thing I wanted to mention is. So, you have a binomial random variable, and you have a poisson random variable. So, it can be shown that for large 'n' and small 'p'. So, lambda is equal to n p and we will have the poisson random variable. In other words if you choose lambda is equal to n p then the probability you calculate from poisson random variable will be very similar to the probability you calculate using binomial random variable. So, there are similarities when you use binomial and poisson random variable, for certain you know values for n and p. So, we are done with poisson random variable we want to discuss about uniform distribution.

What is the uniform distribution? So, whenever you calculate these you always get some probabilities right. So, you might get a probability distribution like this, you might get a probability distribution like this. So, as you can see in these cases the probability of different values are not the same. So, in this particular example the probability of these intermediate values is higher or in this case there is a greater chance that these values in the center have greater chance of actually occurred. But what is an uniform random distribution is when there is no bias and all the values are equally likely to occur.

So, in other words if you had different values, all your variables will are equally probable to occur and this would be the case in case of rolling of a dice. So, your probability of x equal to 1 is equal to probability of x equal to 2 so on and so forth, equal to probability of x equal to 6 so this is an example of a uniform distribution. So, in general you define the probability density function for a uniform distribution as f x is equal to.

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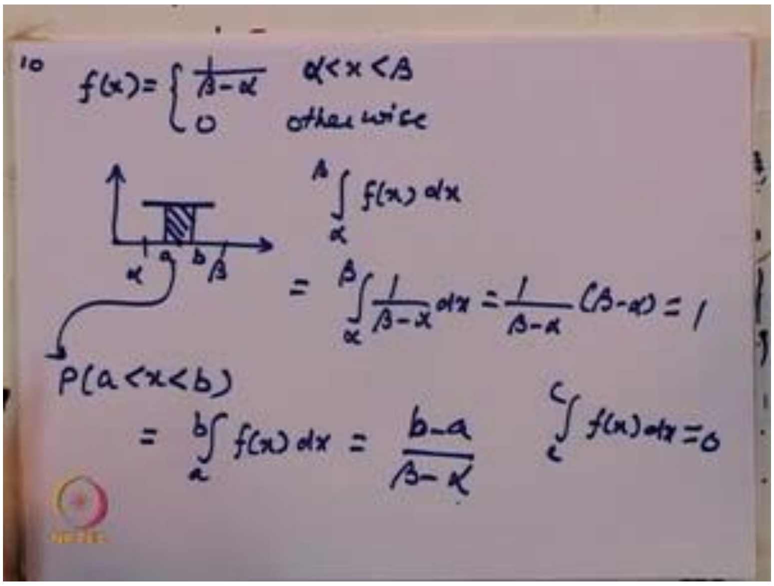

So, for a uniform distribution you write f x is uniform, 1 by beta minus alpha in the range 0 sorry. So, if you have a domain alpha to beta, then your probability is uniform between that region. Alpha and this is beta between that this poverty is uniform, and you understand why it has to be 1 by beta minus alpha; why because integration of let us say alpha to beta f x, d x is nothing, but integral alpha to beta this is a constant to 1. So, just for a uniform distribution you have a constant probability. So, then what is the probability of a less than x less than b, 4 given a uniform distribution like this. So, I can calculate probability of a less than x less than b as nothing, but integral of a to b, f x d x and this will give me the value of b minus a by beta minus alpha. So, as you can see if your b was equal to beta and equal to alpha then this is the entire regime and your probabilities returns you a value of unity.

So, as we will see so we will stop here for today and you can see that for a uniform distribution, the probability is equal for all of those values as again for a continuous random variable which follows the universal (Refer Time: 22:06) the probability associated with a single point is always 0, because integration of C 2 C f x d x is always equal to 0. So, you have a probability of a finite range which is this area under the curve a to b is this probability and this is finite. With that I you know I will end my class here and i look forward to meet you again in the next lecture.

So, we I would like to summarize today's lecture by saying we have this we have discussed 2 distributions the Poisson distribution and the uniform random distribution. So, we have shown that for a poisson distribution both your expectation and variance is equal to lambda, which is the parameter of the poisson distribution, and you can identify the what lambda is from the jargon of your problem let us say average number of deaths, or average number of cases, in a given day is this and then you know that is what your lambda is. And in the end we discussed about the uniform distribution, where you show that probability of each value is the same. So, your priority is simply a flat line over the domain alpha to beta over which that uniform distribution is defined.

With that I thank you for your attention and I look forward to next class.