Introduction to Biostatistics
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Lecture – 16
Counting Principle, Permutations, and Combination

Dear students, welcome to today’s lecture. I hope you have gotten the opportunity to go through your concepts of probability that we discussed in last lecture. So, today we will continue our discussion on probability. So, we begin with a brief recap of what we had discussed in last class.

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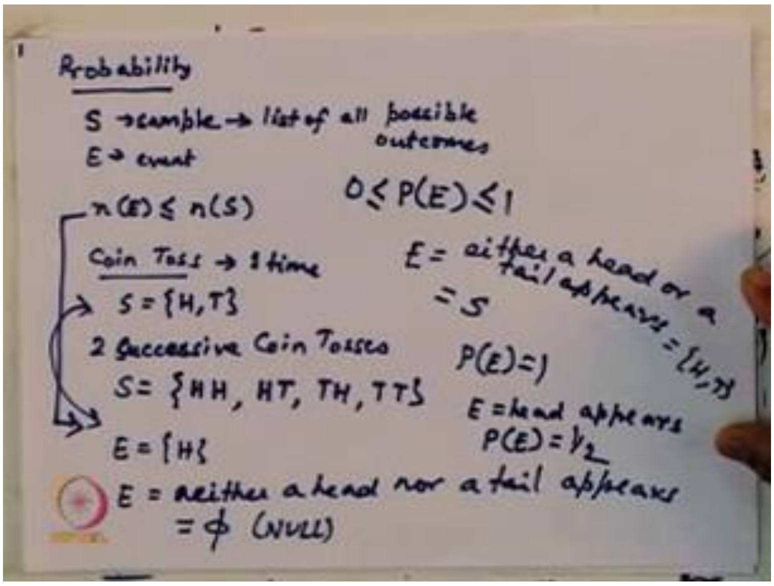

So, we were discussing the concept of probability. So, in probability you have there are two things; one is a sample and one is an event. So, of these the sample is defined as the list of all possible outcomes and of these event will depend on how you define the event. So, in the general case number of possibilities in as event is less or equal to the number of possibilities in the total sample space.

So, we discussed few cases; so in case of a coin toss. So, for a coin toss experiment, my sample space is H comma T. Now this is the sample space for a single coin toss. So, this is one time. So, if I have two coins, two successive coin tosses. So, my sample space becomes. So, in the first case let us say first I get a head, I can get the second time also I can head; this is one possibility. I can get first time head and second time tail, I can get the event the first time is tail the second time is a head or I can get two tails. So, for a single coin toss, I have two possibilities for a two time coin toss I have four possibilities. So, as you increase the number of coin tosses you get more and more you know number of possibilities in your sample space. So, in this case, so I can define an event as a head in this case. So, clearly you see. So, this is satisfied. So, n of e in this case is less than number in sample space.

So, there are three axioms; first is probability of an event E is bounded between 0 and 1. So, if I define an E which is, so if my event E was neither a head nor a tail occurs, appears; this event does not have any entity. So, this is nothing, but a null event. So, null is represented by the number phi. So, in this case for a coin toss, single coin tossing it is impossible that you will have neither a head nor a tail because the head and tail are the two (Refer Time: 03:30). In this case, my probability of E is going to return me a value of 0. So, if I change my definition of E to the E as either a head or a tail appears. So, in this case my E is nothing, but the sample space itself right. So, you are happy with either a head or a tail. So, in this case the event is simply the sample size. So, in this case my probability of E is equal to 1, but for the case of head appears, E is head appears. So, probability of E is going to be half.

So, these events can be you know, Venn diagrams can be conveniently used to describe events and sample space. So, sample space is typically drawn as a box and events are drawn a circles.

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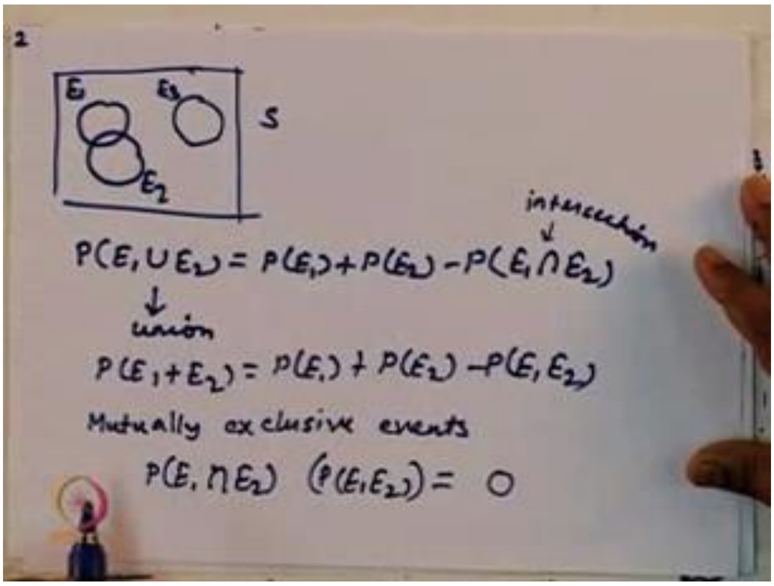

So, this is an event E 1, this is an event E 2, let us say this is an event E 3. So, I can write, down we had derived these expressions probability of E 1 union E 2. So, this is union is equal to P E 1 plus P E 2 minus P of E 1 intersection E 2. Now many places instead of writing like this you can also you will see, E 1 plus E 2 will be written as P 1 plus P E 2 minus probability of E 1, E 2. So, this is an intersection. So, many places instead of a union you will see a plus input and instead of E 1 intersection E 2, you will get the expression as E 1 into E 2.

So, what I have also drawn here, I have drawn an event E 3 which does not overlap with either E 1 or E 2. So, these are examples of mutually exclusive events. So, I can write P of E 1, E 2 intersection E 2 or P of E 1, E 2 is equal to 0 because there is nothing in commonality between the two events; P E 1 E 2 is phi. So, I can for any two events.

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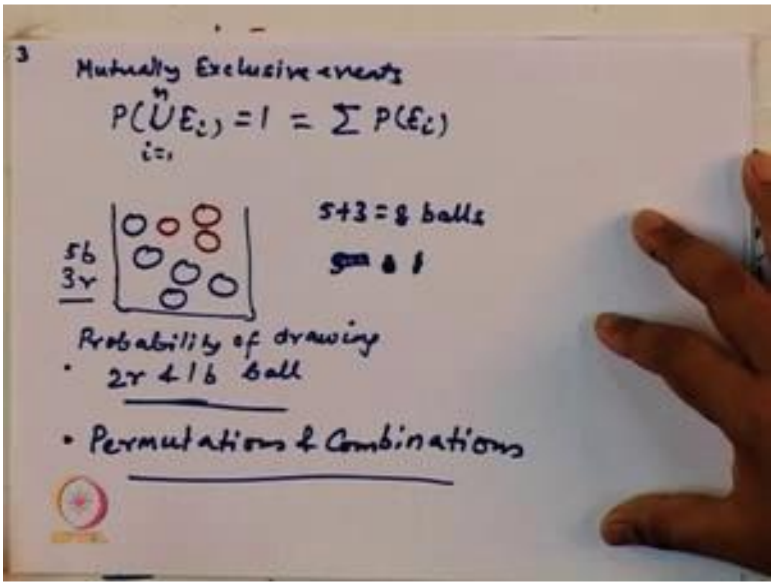

So, for mutually exclusive events, probability of union of E I; i equal to 1 to n is nothing, but equal to 1 and this is equal to summation probability of E i because each of the intersections. So, in this case in the most generic case, probability of E i intersection E j is going to be 0. So, in every edition is, simply these individual elements which are getting handed out. So, this is the second thing.

Now, let us take an example. Let us have, consider a case. You have a box which has 5 blue balls and 3 red balls. So, you have 5 blue and three red balls. This is a container having 5 blue and 3 red. So, I can ask a question, what is the chance probability of drawing 2 red and 1 blue ball from this box? So, what is the probability of drawing 2 red and 1 blue ball from this box? So, how do I do it? So, let us just say, if I am collecting at 1, I am basically taking out 3 balls. So, I have to take out three balls, I have a total of 5 plus 3 equal to 8 balls. So, in order to calculate to answer this question, we need to come up with ways of counting. So, that brings us to two ideas important in persons of permutations and combinations. To answer this question, I need to briefly discuss permutations and combinations.

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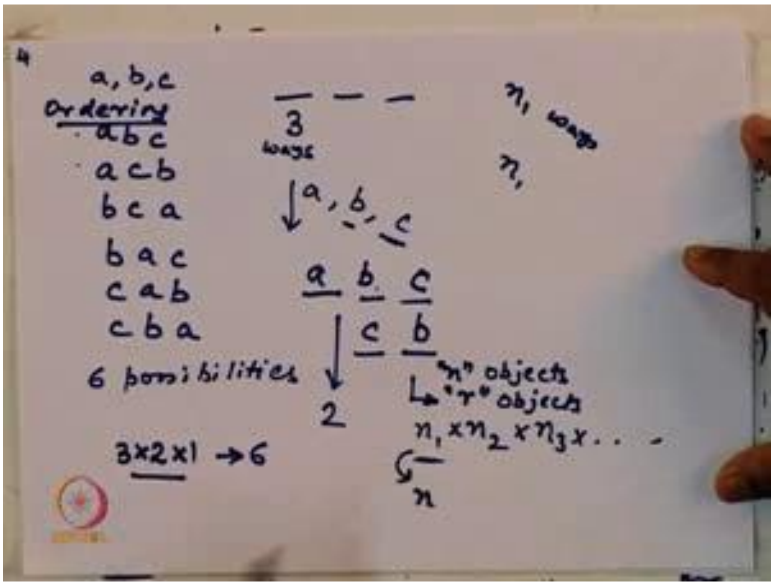

So, imagine let us take a very simple case, I have these 3 numbers a, b, c and let us think about in what ways can I arrange a, b, c in a linear order? So, I can write them as. So, ordering of a, b, c. So, I can write them as a, b, c. I can write them as a, c, b. I can write them as b, c, a. I can write it as b, a, c. I can write it as c, a, b and c, b, a. So, there are 6 possibilities. So, that the way to find out find this out is ask. So, you have three positions and you have three numbers a, b and c. You want to put them here, now imagine in how many ways? So, imagine you have three slots which were empty, in how many ways can you fill up the first three, the first slot?

So, I can either put a, I can either put or put b or put c. So, there are three ways of putting or filling up the first slot. So, I have this condition, I have three ways of filling of the first slot. After I fill up the first slot, let us just hypothetical example my first slot is a, in how many ways can I fill up the next slot? So, I am left once since I have put a here, I am left with b and c only. So, here I can either put b or I can put c. So, there are two possibilities here.

And once I have put b here, I am left with no alternative, but only I have to put c here because that is the only letter which is still available to me or vice versa. Similarly, if I put c here, the only other way is to put b here. So, for each of these three ways then there are two ways, so the way to do it. So this is basically three ways, for the first step into two ways, for the second step into one way, for the first third step. So, this is how you get a total of six possibilities. In the general case, if there are n 1 ways of putting in the first slot, I write down the expression n 1 then, you have n 2ways the fitting of the second slot. So, this is in the general case when you have n objects and then you want to find out r objects from that. So, you want to order take r objects from n objects and ordered them.

So, I can, there might be n 1 times n 1 possibilities for the first object, n 2 possibilities for the first of second object, second slot and so on and so forth. So, if there are n objects and I know n 1 is nothing, but is equal to n. So, I can do for n objects.

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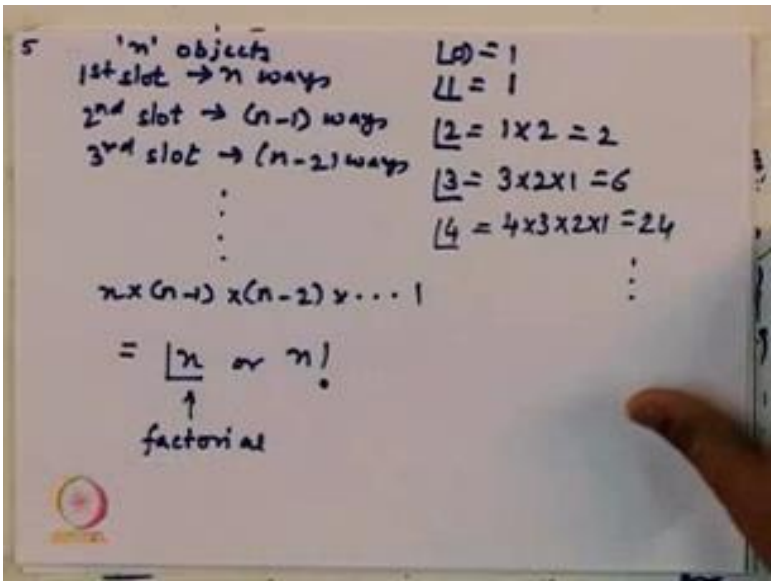

The first slot can be, for first slot there are n ways so for n objects. Second slot there are n minus 1 ways, third slot n minus 2 ways. So, you see it is decreasing by 1. So, the total number of possibilities is nothing, but n into n minus 1 into n minus 2 up to 1. This number is nothing, but the factorials of the number n, this is you right the fact, so this is the factorial of the number n. This is also or n exclamation is also used for describing the factorial of the number n. So, factorial of 1 is simply is equal to 1, factorial of 2 is equal to 1 into 2, so factorial of 0 is also defined as 1. Factorial of 3 is 3 into 2 into 1 is equal to 6, 4 into 3 into 2 into 1 is 24 so on and so forth.

So, there are n factorial ways of arranging n objects in a linear order. So, imagine, you have a bookcase imagine you consider the case you have four maths, four biology and two English books.

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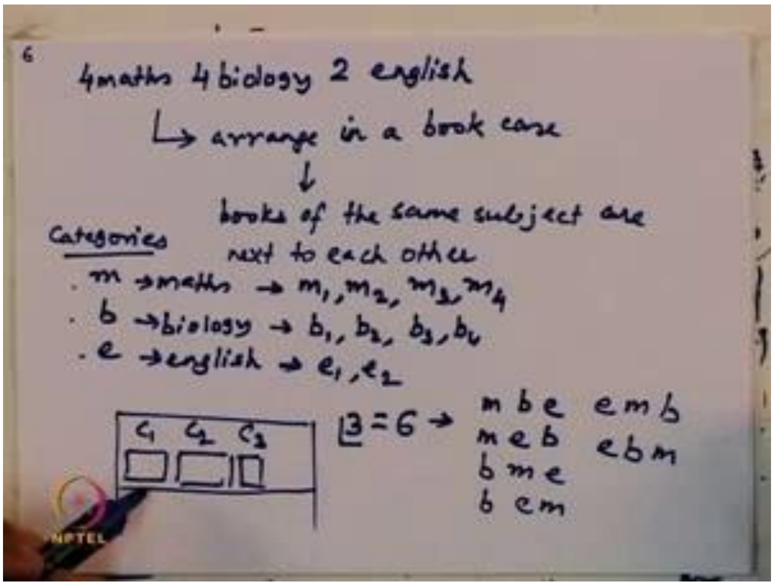

And you want to arrange them in a bookcase such that books of the same subject are next to each other. So, how will I do it? So, let us say, I named my maths book, so I have three categories. Categories are maths, biology and English; this is for maths, this is for biology and this is for English.

Maths, biology and English and in maths you have four books, you can label them as m 1, m 2, m 3 and m 4. Your biology you have b 1, b 2, b 3 and b 4 and you have English, e 1 and e 2. How will you arrange? So, there are three categories right; first is you think about it, if this is your book rack, if this is your book rack I can have of three categories; category 1, category 2, category 3. So, in three categories there are three categories of books maths, biology and English. I can order them three categories in how many different ways? We just saw which is going to be factorial 3 is equal to 6 possible way. Why? Because what are the six combination? I could have put m, b, e, I could have put maths, English, biology, I could have put b m e, b e m, I could have put e m b, e b m.

So, you have six combinations which is factorial 3 and three categories you can be arranged in a book rack in 3 factorial ways. Now for each of these categories, so imagine you have a one particular setup which is m b e.

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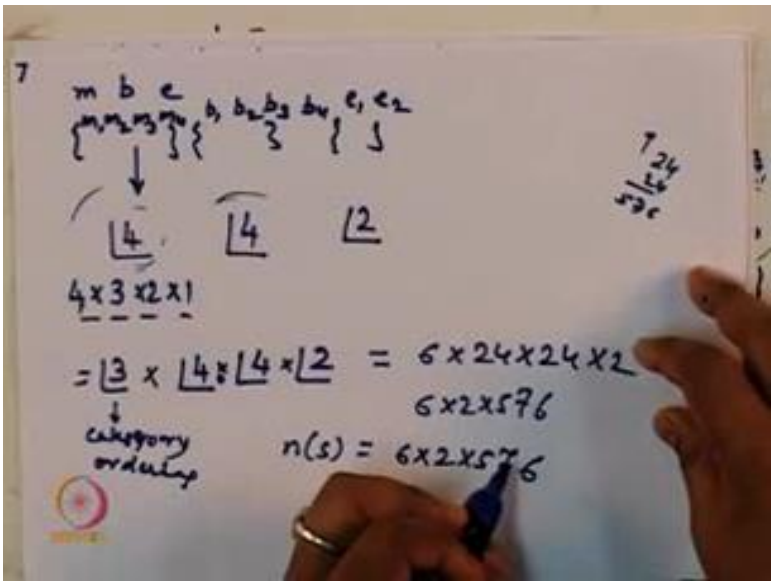

So, one particular setup you have chosen let us say m b e. Within this m, you actually have four combinations. You have to arrange m 1, m 2, m 3, m 4. Similarly, within the b you have to arrange b 1, b 2, b 3, b 4 and within the e you have to arrange e 1 and e 2. So, four sets of books within a block can be arranged in factorial 4 ways. Why? Because the first slot, so again you have four slots; the first slot can be filled it in four different ways. Once you have put a given number, let us just say m 1 you have put here then, in the second slot there are one of three different numbers can be chosen here. So, you can have three ways then, the next one is two ways then, next one is one way. So, factorial 4 ways is the number of arranging here.

Similarly factorial 4 ways is the number of possible arrangements here and factorial of 2 is the arrangement here. So, the total number of possible arrangements comes out. So, this is once you have decided on the category 4 factorial into 2 factorial. So, these are the number of possibilities, you see there are products of 4 into 4 into 2 is the number of categories and for each category for a given choose of a category order within this you have 4 factorial. For each of these ordering in maths, you can have another 4 factorial within biology which is why you multiply by 4 and this is by 2.

And the final answer is by you multiplied by factorial 3 because these are the categories, so category ordering. So, that is your answer. So, your sample space has factorial 3 into 4 into 4 into 2 which is 6 into 24 into 24 into 2. So, 24 square is 96 (Refer Time: 18:15).

So, 576 you have 6 into 2 into 576, you can calculate the finite. So, you have these many numbers of possibilities. So, this represents. So, n of sample space represents 6 into 2 into 576.

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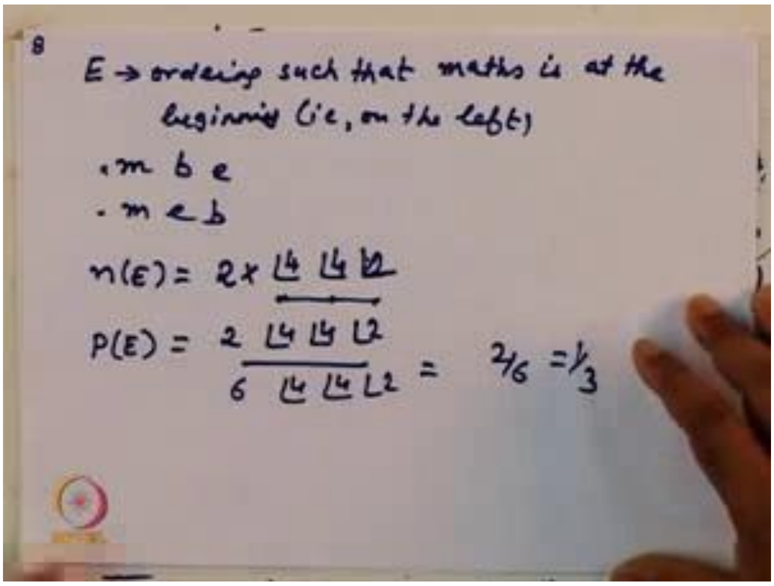

Now, let us define the event E as the ordering such that maths is at the beginning that is on the left.

So, this is the possibility of event of E. So, what are the categories that are possible? The numbers of categories which are possible are basically, you have m b e or m e b, but for each of these two categories for each of, so I have. So, the total number of possibilities of n of E is going to be 2 times the remaining things as before within each of these orderings you can again rearrange as per the maths or the bio or the English as before. So, if n of E is simply 2 into 4 factorial into 4 factorial into 2 factorial. So, then your probability of the event E is nothing by 2 into factorial 4 factorial 4 factorial 2 by 6 into factorial 4 factorial 4 factorial 2 is nothing by 2 by 6 is equal to one third.

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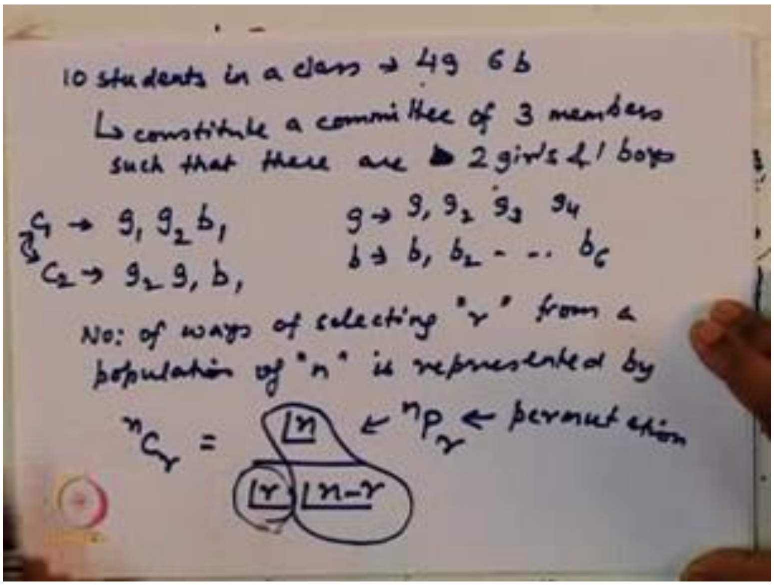

So, this is how you order, there is a slightly different you know thing (Refer Time: 20:19). So, this is ordering is called permutation or arrangement. Now imagine you want to you have 10 students in a class and you want to constitute a committee. So, of these 10, let us assume that there are 4 girls and 6 boys. You have 4 girls in the class and 6 boys in the class and you want to consider a committee of 3 members; such that there are boys, such that there are 2 girls and 1 boy. So, in a committee when you constitute a committee, what matters is not the order. So, let us say, I have a committee. I have let us say g 1, g 2, b 1. So, my girls are g 1, g 2, g 3, g 4 and my boys are b 1, b 2 up to b 6. So, in one particular set, let us say I have committee 1 is g 1, g 2, b 1; committee 2 is g 2, g 1, b 1.

See the ordering is different, but for a committee the order does not matter. So, this is the same event, I have the same event. So, the number of possibilities, so here you are not concerned about the ordering, but you are considering of the selection. So, in general the ways, the number of ways of selecting r from a population of n is represented by n C r and n C r is defined by factorial n by factorial r factorial of n minus r. So, this is different. So, this is defined as n P r.

So, this is permutation, permutation n P r is defined by factorial n by factorial n minus r and n C r is defined by n factorial n factorial r by factorial n minus r. So, you see there is this defined, division by this factorial r. So, this is an example. So, the ordering can be more, but in terms of combination it is still the same combination. So, that is why you divide by factorial r.

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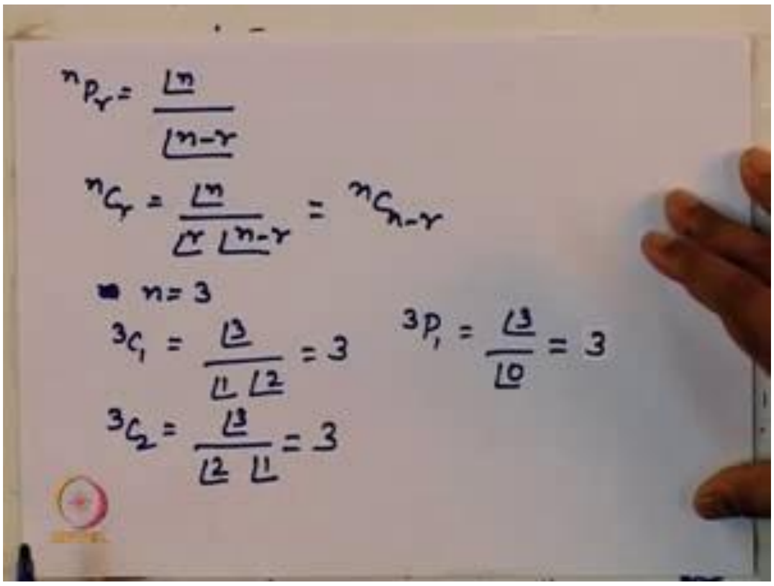

So, n P r is factorial n by factorial of n minus r, n C r is equal to factorial n by factorial r factorial n minus r. So, ns, so let us say if n is 3, 3 C 1 is going to be factorial 3 by factorial 1 factorial 2 simply equal to 3 versus 3 P 1 is equal to factorial 3 by factorial 0 is equal to 3, 3 C 2 is same as. So, you see the definition of n C r, if I put instead of r if I put n minus r, if I put n minus r by r you get the same thing. So, n C r is same as n C n minus r. So, as is the case of 3 c 1 and 3 c 2. So, factorial 2 and factorial 1 you still get the value of 3. So, n C r is same as n C n minus r.

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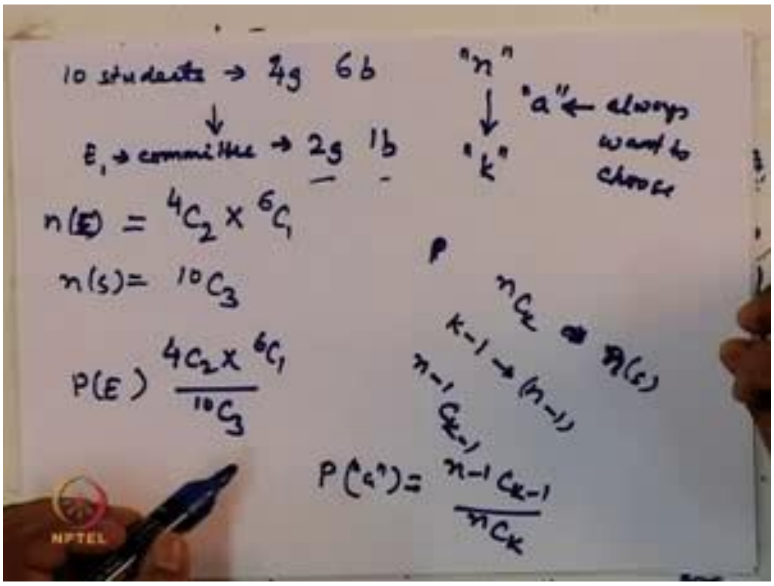

So, the way of choosing let us say you have 10 students where, you have (Refer Time: 25:07) 4 girls and + boys and you want to constitute a committee of 2 girls and 1 boy. So, the number of possibilities, so you want to choose 4 out of 10, you want to do 4 C 2 and for each you want to do 6 C 1. So, this is your n of sample space; 4 C 2 into 6 C 1 is the total number of ways in which you can constitute a committee of 2 girls and 1 boy from this. Now, so this is event E 1.

So, I can say the event is such that this, but in how many different ways can you choose 3 out of 10? So, your n of sample space is 10 C 3, so 10 C 3. So, I can ask the question, what is the chance that you have come up with a committee which has 2 girls and 1 boy? So, the answer is then. So, probability of constitute a committee which has 2 girls and 1 boy is simply 4 C 2 into 6 C 1 by 10 C 3. So, this is your probability of this event is the possibility that you have constituted committee of 2 girls and 1 boy.

Let us take another example. So, let us say, you have n objects from which you want to draw k objects, but you always. So, let us say there is an element a, which you always want to select. So, what is the probability that you choose k out of n such that a is always selected? So, to answer these question my probability; so in how many ways can I choose k from n? The answer is n C k. So, this is the probability of your sample associated with your sample n of s. So, if there is an element a, which is always selected the probability of selecting k out of n is equivalent of saying that I want to select k minus 1 from n minus 1.

So, which is nothing, but n minus 1 C k minus 1. So, probability of selecting this a, let us say that a is always selected is nothing, but n minus 1 C k minus 1 by n C k. So, with that I stop for today, is to briefly summarize we discussed about the basics of probability once more and we choose. So, one of the critical steps of calculating probability is to do a counting and then there are two ways of counting, which we call a permutation or a combination. A permutation of is done when you have ordering or an arrangement kind of setup. A combination is done when you just want to select and the order does not make a may have any importance as in case of constituting a committee or something like that.

With that I thank you for your attention and I look forward for our next lecture.