Biomathematics
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Lecture No. # 07
Differentiation and its applications

Hello. Welcome to this lecture of Biomathematics. As we have been, in the last few lectures, we have been discussing differentiation, and in the last lecture, we discussed its application in Biology. We discussed, how certain filaments grow and the knowledge of derivative, how does it help us to understand about the rates, etcetera, etcetera. So, we, for example, we, knowing the rate of growth of actin, we could get some idea about the filament growth and then, the process of growth of filament. So, in this lecture, we will continue learning some more applications of Biology and how we can apply the idea of derivative, to understand some more problems in Biology; one more or two more examples, we will take and then, we will go to the next topic.

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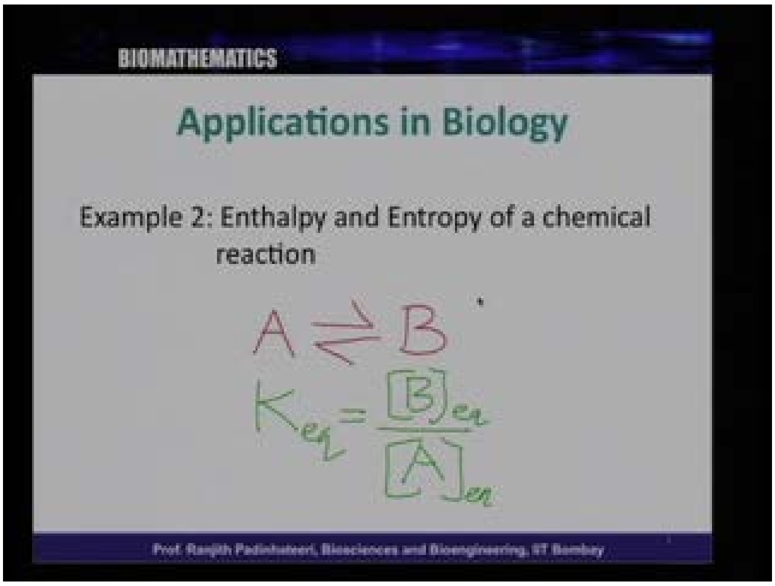

So, today's, as you can see, today's lecture again, is differentiation and its applications and we will discuss example 2 in the application of Biology. So, the example 2 is enthalpy and entropy of a chemical reaction. So, as you all know, one of the important thing in Biology is chemical processes. So, any chemical reaction, understanding of chemical reactions and things related to that, is very important in understanding various processes in Biology. So, let us take here, a simplest possible chemical reaction and let us try and understand a bit, thermodynamics of it and how does this knowledge of mathematics will help us there.

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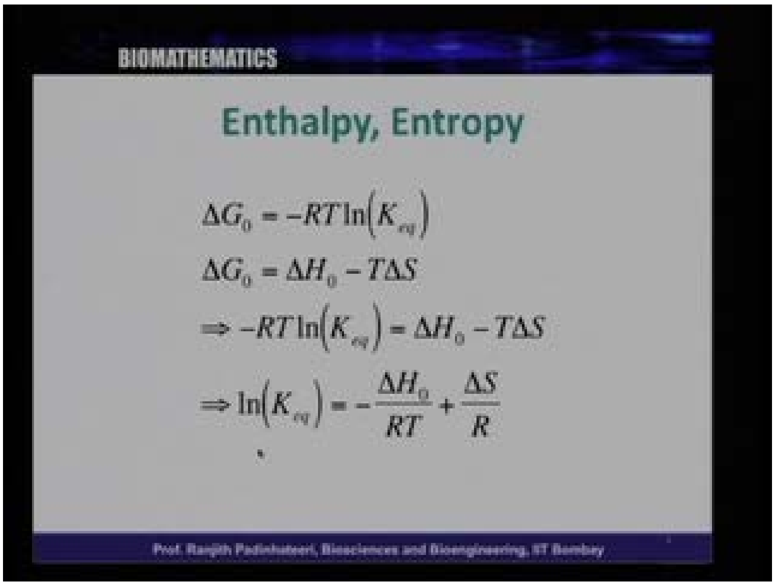

So, one example, one thing here is…So, let us take the simplest case, as you see in this slide here. Let us take the K is A going to B and B goes to A, back. So, this is A goes to B and back. So, this is a reaction, A arrow, forward arrow B and backward arrow, this way. So, what does it mean is that, as you all know, A is converting to B and B is converting back to A. So, this A and B could be two states of a protein, proteins having state A and state B. It could be an enzyme moving, changing, something from A to B, it is could be anything, you can imagine. And, if you have any such, such a reaction, or A goes to B, then, the, you all know that, the equilibrant constant, the K equilibrium for such reaction. As you see here, is, K equilibrium is equal to the concentration of B at equilibrium divided by the concentration of A at equilibrium. So, when you say, when I say equilibrium, let me briefly tell you, equilibrium means, the rate of A changing to B is equal to the rate of B changing to A; that means, some amount of A is getting converted into B; some amount of B is getting converted into A; but the concentration of A and B, on an average, does not change; that is what we call equilibrium. So, at equilibrium, when the concentration of A and the concentration of B, on an average, do not change, the equilibrium constant is basically, the ratio of the concentrations of B divided by the concentration of A. So, this is, this bracket means, as you know, it is concentration. So, the concentration of B at equilibrium divided by the concentration of A at equilibrium and this constant is known as equilibrium constant.

So, now, another thing which we all know from thermodynamics is that, delta G. So, the delta g of a chemical reaction basically, is the change in energy or the change in free energy actually, for the chemical reaction and we all know, you might have, you might have studied this that, the formula the delta G 0 is basically, minus R T log K equilibrium. This is the formula for delta G 0. Now, you also know that, delta G 0 can be also written as delta H 0 minus T delta S. So, free energy is enthalpy minus temperature into entropy; this is also you know. So, if you combine this two, you can substitute, instead of this delta G 0 here, you can substitute this minus R T ln K equilibrium from the top equation. So, combining this equation and this equation, I substitute this here and I get R T ln K equilibrium is delta H 0 minus T delta S. And, if I rearrange this, that is, I take minus R T to the right hand side, so, I divide it throughout by minus R T, I will get ln K equilibrium is equal to delta H 0 divided by minus R T and minus T delta S divided by minus R T. So, essentially, we will get, ln K equilibrium is equal to minus delta H 0 divided by R T plus delta S by R. So, these three equations, combining, we can rewrite such an equation, which is very simple. Just simple algebra. One can rewrite an equation ln K equilibrium is equal to minus delta H 0 divided by R T plus delta S divided by R.

Now, for a chemical reaction, if you think of, like, typically, the chemical reaction happens at a fixed temperature. So, in Biology, like, it is like 310 Kelvin, 37 degree Celsius, the temperature is a constant. R is a universal constant. So, R is a constant, T is a constant, we can calculate ln K equilibrium. So, let us, for any reaction, you can calculate ln K equilibrium. Now, if we vary the temperature, if you imagine the experiments in a laboratory, one can actually control the temperature. So, let us say, you vary the temperature from 25 degree to 37 degrees, 25, 26, 27, 28. So, as you vary the temperature, you can imagine that, K equilibrium, ln K equilibrium also varies. So, as we vary the temperature, the equilibrium constant also varies; that means, ln K equilibrium changes. So, essentially, one can imagine doing an experiment, where you keep the temperature 25 degree Celsius and do the reaction A going to B, and when it reaches equilibrium, calculate the ratio of B and A concentrations, and you will get ln K equilibrium. So, I would write it down. So, let us think of a table. So, let us think of…So, if you do this experiment, you can draw a table. So, temperature versus ln K equilibrium. So, let us say, you do this temperature, this temperature at, let us say, 283 Kelvin.

So, so, 283 Kelvin, you do this, and you start the experiment. A will go to B and back and when it reaches equilibrium, you can calculate the constant, concentration B by A and that value and log of this and as there is, this is basically, K equilibrium and the log of this is basically, you can write it here, log of K equilibrium. Now, let us say, you do it at 285 Kelvin. You calculate ratio, again do the experiment, know the concentration at equilibrium of B and A, write down this number, here; like, you can vary 287, 289, 291, so on and so forth, upto 310, you can just do and calculate this value. Then, once you get such a table, you can plot a graph. The graph is...So, let us say, you plot a graph which is 1 over temperature, versus ln K equilibrium. So, let us see, what does this graph give us. So, 1 over T is, as we know that, T here, as we know, we did T is, 283 is the first temperature we did. So, here we will have, 1 over to 283. Here, you will have 1 over 285. So, all these are numbers. So, you just, for all this number, you will have a corresponding K equilibrium value. So, for 1 over 283, 1 over 285, 1 over 287… So, as you vary 1 over T, you will have ln K equilibrium varying. So, let us see, how this ln K equilibrium varies. So, let us look at the next slide.

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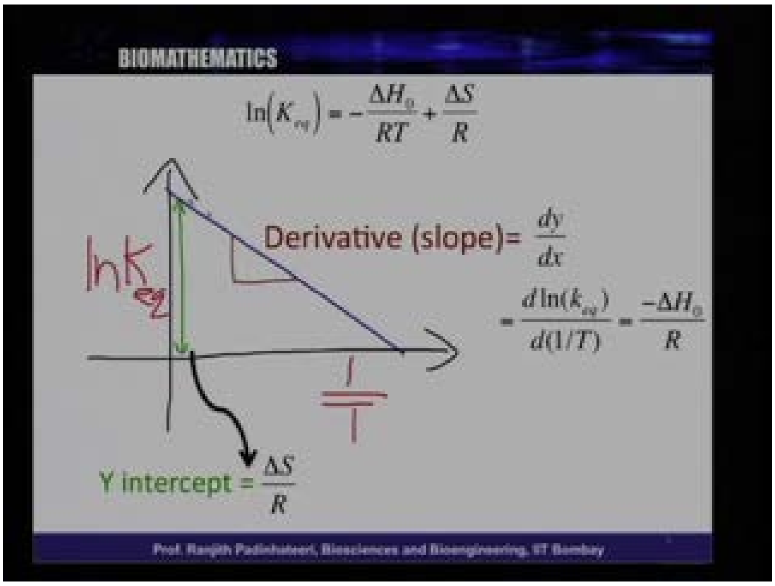

So, the typical graph will look like this. It is like a, this blue line. So, for, as the 1 over T increases, that is, as the temperature decreases, the ln K equilibrium will decrease. So, if you look at this equation, so, let us look at this equation. So, the equation we had was, ln K equilibrium is equal to minus delta H 0 by R into 1 over T. So, as you can see here, delta H 0 by R T can be written as delta H 0 by R into 1 over T. So, let us write it, delta H 0 by R into 1 over T plus delta S by R. So, this way, you can rewrite this equation and then, you can plot. So, now, ln K equilibrium versus 1 over T. So, this, if you look at this equation, this is the y. So, this equation looks like, y is equal to some constant, minus sign with some constant m, 1 over T is your x axis. So, this is x, plus some other constant c. So, this is a straight line with a minus m. So, this is a straight line; this is, as you all know, this is the slope. So, this equation essentially, gives a straight line with a negative slope. So, that is what you have here. So, if you look at this graph, straight line with a negative slope is essentially, this. So, this graph, as you see here, this graph is basically, straight line with a negative slope.

So, the derivative, in other words, slope is nothing, but...So, we learnt this slope is nothing, but derivative. So, the derivative, if we calculate, is basically, d y by d x. How does the y, that is ln k equilibrium, changes, when things in the x axis changes? So, it turns out that, if you calculate this d y by d x, as you can see from this equation, if you take this y is equal to m x plus c, and calculate the slope, derivative, you will see that, the derivative is minus delta H 0 by R. In other words, the slope gives you enthalpy divided by R. So, if, since we know the R, R is the number which we all know, we can easily calculate the enthalpy. So, by calculating the derivative, we can calculate delta H 0. So, that is the message here. So, from this (( )) experiment, by calculating the derivative of this curve, we can calculate the enthalpy, delta H 0. And, as you can see here, the y intercept, that is the value of y at, for this particular x axis, for, the value of y, when 1 over T is 0, basically, is the, is basically, delta S by R. So, that will give you some idea of entropy also. So, you can, since this is a straight line, you can extend this straight line upto the value, where 1 over T is 0 and at that point, you will get the entropy, delta S by R. So, from this particular experiment, by knowing this simple idea of that derivative is nothing, but the slope and by calculating the derivative of this curve, experimental curve, we can get the enthalpy, in a enthalpy delta H 0 and from the y intercept, we can also calculate the entropy.

So, enthalpy and entropy, from the simple experiment, where we vary the temperature and measure the concentration of A and B. So, if you just sit back and think, what did you do, is nothing, but vary the temperature and calculate the concentrations of A and B at equilibrium. From this simple experiment and by plotting this, and using this idea of derivative or slope in mathematics, we can get, what is the enthalpy delta H 0. We can also calculate delta S. So, let us...So, this is another example, where the idea of mathematics, the idea of derivative, the mathematics, the mathematical idea of derivative is used in Biology, to get some information. Now, we will go to the one more example. So, few, when we were discussing the idea of function, we were discussing many function. We actually, discussed an example of population growth. So, we said, an example of fish growth. So, the example was this. Let me just remind you, the example was this. Let us say, you are doing some farming, fish farming. So, you have a big pond and you add a lot fish and the fish grows. So, it takes few days for the fishes to grow. So, I get many, many fishes. So, the number, if you plot the number of fish versus the type or the density of fishes, how many fishes are there in a unit area or unit volume; that is, you can call it density, the population density of fish.

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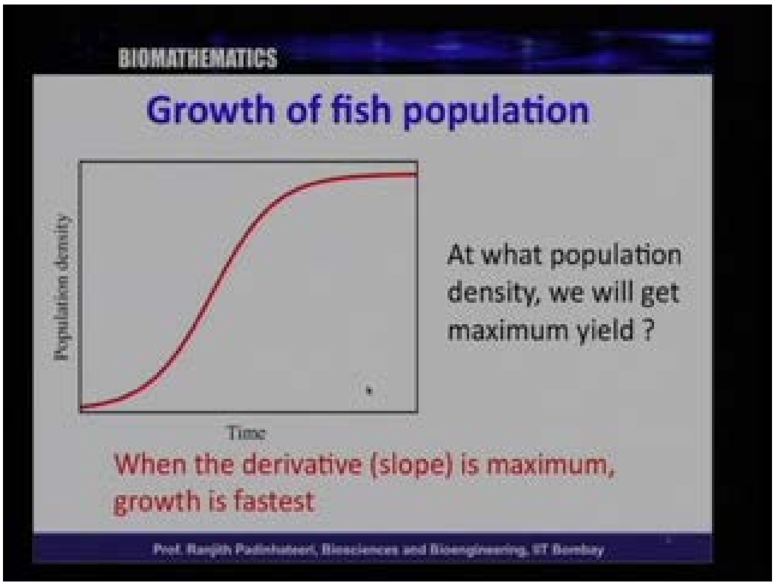

So, if you plot the population density versus time, we had got the following graph. So, let us look at this graph, which we had got many, few, few lectures ago. When we are discussing functions, we had shown this graph, which is basically, population density versus time. So, what does this mean? So, this mean is that, first, when you grow the fish, first the time is, let us take the time in days. So, the time is this X axis, in days. So, first, the fish population is very small and increasing very slowly. And, the population density increases here, and then, it reaches a constant value. So, this is the typical thing, which you see in any growth curve. And, we wanted to understand this question that, at what population density, we will get the maximum yield. So, what do you mean by maximum yield? When we say maximum yield, what we want? We want the number of fishes to increase very fast. So, the growth, the fish growth has to be fastest. So, that is where you will get the maximum fish. Like, if you had, like 10000 fish in the whole pond today, tomorrow you want it to be 20000, 30000. The more, the better. So, in one day, how many fish will it increase? So, now, change in number of fish in one day, that is what is yield, right; when the, increase in number of fish in one day, that is the yield you get.

So, and, so, this idea of change in number of fish, when you change the time, is basically, can saw, this is like derivative. So, let us look at, let us think about this curve a bit more. So, the curve basically, y is number of fish and this is time. Now, as you can see here, this has a shape, which is something like this. Now, if you change this, let us say, write the days here, 0, 1, 2, 3, 4, 5, 6. So, these are days. So, in a week, this gets saturated. Let us say, this is the case. So, now, if you just look at the first day, the 0, the…So, this is 1, this is 2. So, if you just look here, the increase is actually very small. If you just write this one day, the number increases only a little. On the other hand, if you look here, somewhere here, between second and third day, the increase was a lot. The number increased from…Second day the number was this; the third day the number is this. So, the number increased a lot in one day here. So, the yield was maximum in this particular region. So, how did...So, if you call this change in time, so, we can call delta t, that is, this delta t and this is delta N, change in number, delta N by delta t is basically, the yield. So, this is basically, the derivative; in other words, when the delta t is very small, I can call it d N by d t, the yield.

So, this is the d N by d t. So, this slope or the derivative gives you the yield. So, think about it, think about it once more. Yield means, you want the maximum number. The delta N, the number should increase, when the time changes by one day, by, time changes by one small factor. If you look here, if the time changes from, by one day, here, the yield was very small. The number only increased from here to here. The number only increased this much. On the other hand, here to here, the number increased this much. So, as you can see here, the, as you can see in this graph here, if you keep calculating the slope from here to here, you will find them in the middle, somewhere here, where the slope is maximum; slope is maximum somewhere in the middle.

So, that is the place, where you get maximum yield; that is, in one day, in unit time, how much more fish you get, that is the yield. So, this idea of derivative, essentially, helped us to understand, where do we get the maximum yield, like, at what time. So, at this particular time, after the here, you can see, here, you can say, after that second day, in this, in this, if you look at this, after this second day, if, you get the maximum yield here in this graph. Here also, somewhere here, is you will get the maximum yield. So, if you could, by calculating the derivative of the curve at various times and looking at where the derivative is maximum, where is the derivative is large, bigger number, where the slope is maximum, you will get some idea about, where the growth is maximum; and, when the growth is maximum, when the number is, growth means, increase in number.

Where the number of fish and the number of any organism increases very fast, you can say, the yield is maximum, or the large yield. So, here again, the idea of derivative helps us to understand something about growth processes. Now, talking about the growth process, one more thing I want to discuss today is that, typically, when you plot a growth curve, so, the growth…When I said growth is a very fast, typically, what I mean is that, the growth is exponential. So, when you say fast, very fast growth, what does it mean? So, let us have a look at it. Let us just revisit the bacterial growth, what we had discussed and look at what does it mean fast growth. And, you also might have used this log phase some time, the word log phase; why log? So, those kind of things, we will briefly discuss in this context. And so, let us, let us go back and revisit this bacterial growth a bit, because this is one important area in Biology. We will also come back to this, because growth and decay processes, is something, which we, which is very important thing in Biology. We will discuss more and more, more about this, but for the moment, let us…Since we now understand the idea of derivative, let us revisit this bacterial growth.

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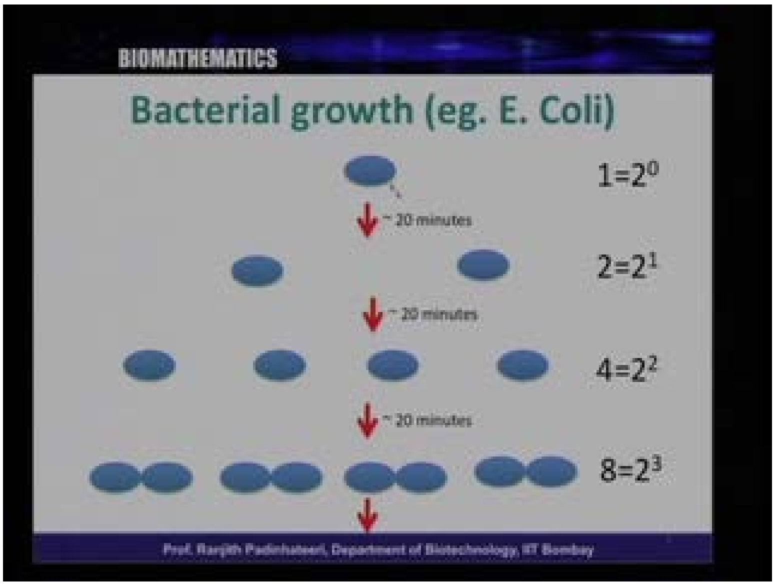

So, let us go to look at this slide here; we had seen this slide. So, we are discussing about bacterial growth. Let us take the example e coli. So, to begin with, let us say, you have 1 e coli and in about 20 minutes, the e coli will divide into 2. So, you will have 2 e coli. So, this is the first generation, the 1 generation. The next, so, 1, 20 minutes (( )); so, 2 power 1; the number of bacteria is 2 power 1, which is 2. Now, in the second generation, which is this, another 20 minutes if you wait, roughly, you will get 4. This will divide into 2, this will divide into 2. So, you have this 4. So, which is 2 power 2, the second generation. So, everything is divided into 2; that is, this 2 and this power is for the generation; this is 2 power 2, you have 4.

In the third generation, you have 2 power 3 number of bacteria; that is, this will divide into 2; this will divide into 2; this will divide into 2; this will divide into 2. So, we have total 8 bacteria, which is 2 power eight. So, 8 e coli which is 2 power 3, equal to which is 2 power 3 equal to 8. So, this is the, this is the way bacteria grow, 2 power something. So, because everything is dividing into 2, so, this is 2 power some factor. So, this, we will call it exponential growth, because it is 2 power, 2 in the exponent. So, the number, the bacterial number increases as 2 power some factor. So, this is exponent, you, you exponentiate like, this is the power. So, this is called, anything in the power is called exponent.

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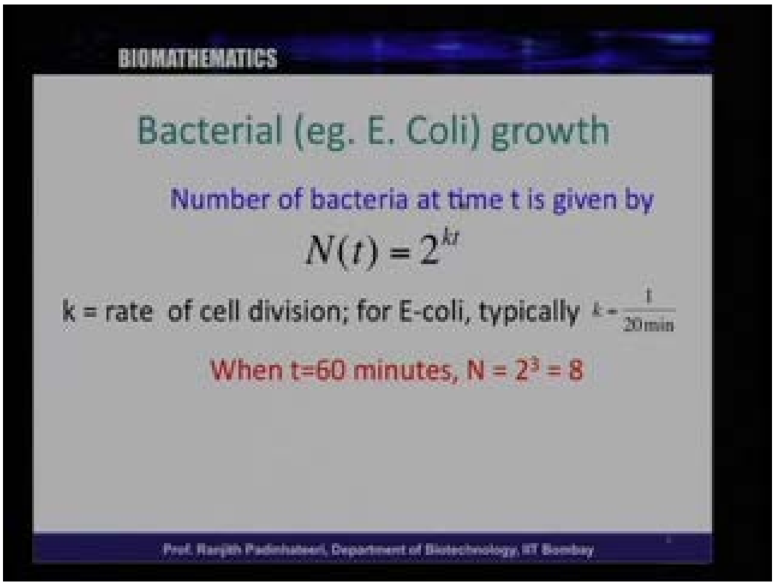

So, some exponent of 2 is the number. So, now, as we had, we had said this last time, in the, in the slide, the number of bacteria at any time t is given by 2 power k t. So, here we have k t in the exponent, where k is the rate of cell division. For bacteria, it is typically, 1 in 20 minutes. So, k is 1 over 20, t is the time. So, if you take t in time in minutes, k is 1 over 20 minutes, this is N of t is e power k t, which is t by 20. So, here you have t divided by 20. So, let me write it again more clearly. So, then, at any time t is 2 power t divided by 20, where t is time in minutes. So, this is the kind of formula, which, this is the kind of function which we discussed last time, the number function for the bacterial growth. So, known the t, you can calculate this function. So, for example, let us take, t is equal to 200 minutes; that is, like, after 200, how many bacteria would be there. So, let us go back here. So, N at t equal to 200 minutes is equal to 2 power, t is 200 minutes; 200 divided by 20 is 10; so, it is 2 power 10, so, that will be 2 power 10 bacterias, but…

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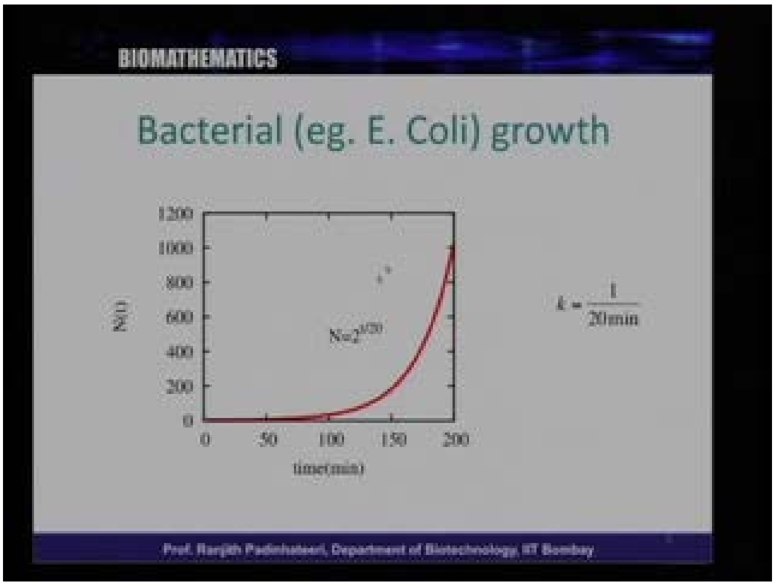

So, let us see, how we can plot, when we plot this, this function, N is equal to t into 2 power, 2 power t by 20, what do we get? So, have a look at here. What we get is, this kind of a function. So, this is what, N is equal to 2, N in the y axis and time in the x axis. So, after 200 minutes, you have about, more than 1000 bacteria. So, you can just calculate yourself and see, what is 2 power 20. It has to be some number above 1000, where 20, 1 over 20 is the rate of growth; 1 over 20 per minute, that is the rate of growth. So, now, what we want to understand here? We want to understand something about the derivatives; something about the slope of this, this curve. So, if you have a curve, which, as you see here, which is exponential. So, let us, let us just plot this curve here. So, we had time and number, and if we plot it, it is like, exponentially growing curve. And, if we calculate slope at any, every point, the slope at each point is different; the slope here is different; slope here, slope here; if you calculate the slope at each point, you can try calculating this slope; you will find that, this slope itself is varying. So, if we want to get some idea about this slope, it is not very easy from this curve, because, it is always difficult to calculate the slope of such a curve; like, the slope at each place keeps varying. So, slope of this curve, calculating is slightly difficult.

So, we can, people do a trick. Now, the trick is the following. So, we can use this mathematical identity. So, what we know here is, we know here, we learnt that, the function we have was N is 2 power t by 20. Now, if you find log of this, if you find the logarithm, log of N. So, log N is log of 2 power t by 20. Now, it turns out that, log of, log of a power x is nothing, but x log a. So, this is the, this is the rule of log, log of a power x is x log a. So, now, using this rule here, log of t power 20. So, this is 20. So, let us say, log of t power 20 is…So, a here is 2; x is t power 20; this is our x. So, log of N is x log a; x is t by 20 into log 2. log 2 is a number; 20 is a number; t is the variable in the x axis. So, you can write it (( )) t times log 2 divided by 20. So, basically, log N is t times log 2 by 20. So, this is log N is t times some constant; log 2 is a number, 0.31, around 0.3 is log 2. log…

If you look, you can look at log 2 in a calculator. I think this is 0.310 or some 3 0 1 0 or something like that. So, it is about 0.3. So, log 2 is a number; 20 is the number. So, this is, whole thing is a number. So, log N is some constant number times t. So, now, let us try plotting. Instead of plotting N and t, let us plot log N and t. So, previously, we had plotted N in the x axis and, sorry, N in the y axis and t in the x axis and we got a curve which is N and t; a curve is look like this; and, we had difficulty in calculating the slope; it is not very easy. One has to calculate slope at different points and it is not, it is not easiest way, easiest, easiest function to calculate the slope. Given the experimental data, calculating slope was little difficult. On the other hand, let us look here; let us plot log N versus t. So, let us plot these two things in the x, this in the x axis and this in the y axis. So, the expression we have is, the expression we have is basically, the expression we have is, log N is t times log 2 divided by 20. This is the expression we finally got and if we plot this as a y, this as x times some constant m; this is some constant.

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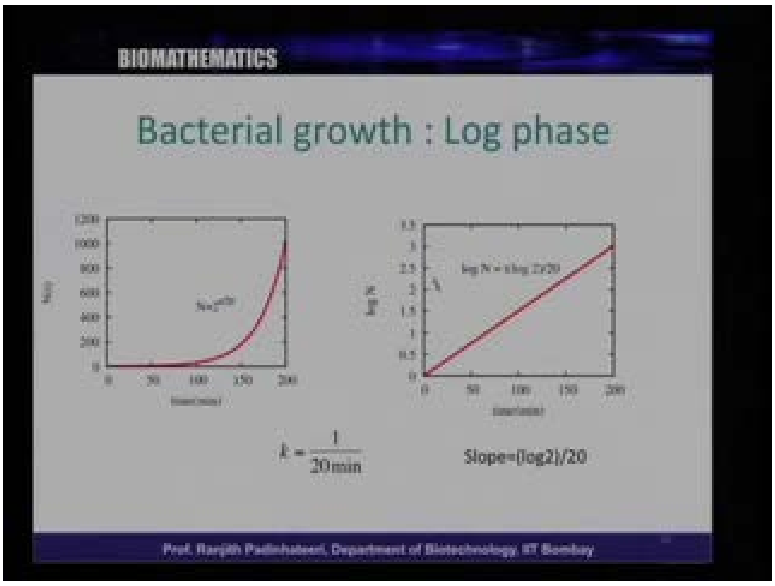

So, you know this, what is this. This is the straight line, what you would get is a straight line. So, let us see this plot. So, this is what, look at the plot in the left hand side, which is N is equal to 2 power t by 20, and the y axis, in the right hand side, you have log N versus time and you get a straight line. And, the slope of this straight line, this straight line is passing through 0, because there is no y intercept; y is equal to m x is the line, straight line passing through 0 and the slope of this or the derivative of this is very easy to calculate. And you will see that, the slope is log 2 by 20. So, if you know this log 2 by 20, immediately, you can say that, 20 is basically, that is the rate, the rate of growth. So, from this, measuring this, you can immediately calculate the rate of growth, which is 1 over 20. So, let us redo this, a little more in general. So, in general, the expression for… So, now, for equally, it is 20 per minute; 1 per, sorry, 1 cell division in 20 minutes.

So, it could be like, for some other, yeast it could be something else; for some other organism, it could be something, some other number. So, the rate of growth, in general, could be any number K. So, it could be 1 per minute; it could be 2 per minute; it could be 10 per minute. So, let us call K as the rate of growth. So, which is something per second or something per minute and we will have N is 2 power K t. So, this is the formula we had; k is 1 over 20. So, we had t by 20 before, for e coli. Now, if we take log on both sides, we have log N is equal to log of 2 power K t. So, log of a power x is x log a. So, this is, this gives that, log of N is nothing, but K t times log 2. In other words, t times K log 2. So, the slope…So, if you plot log N in the y axis and t in the x axis, the slope you will get is K log 2. So, slope you will get is K log 2. So, if you plot…So, let us consider this. So, you have this equation, which is y is equal to…So, the equation we have is log N is equal to t times K log 2.

So, this is y is equal to x times m, where m is this quantity, which is K log 2. So, as you can see, as we saw, if you plot this y and x, y equal to m x is a straight line and this slope which is m, which is K log 2. So, now, if you look at this here, if you look at this slide, what we had got is, the slope as log 2 by 20. So, the slope we had got from this is log 2 by 20. So, instead of K log 2 here, is the slope we got log 2 by 20; that means, K is 1 over 20; this means K is 1 over 20. So, just by calculating the slope, so, by plotting log N, so, y is basically log N; so, y is equal to, y is nothing, but log N; so, log N versus time, x is time. So, basically, we plotted log N versus time. So, by plotting log N versus time, the log of the quantity, we got the slope by just calculating the, this slope of, derivative of a straight line; derivative of the straight line is the easiest thing to calculate. So, here again, we were using the idea of…We wanted to calculate the slope. We want to calculate the derivative, to get basically, the rate of growth and by doing this, what we did was, for doing, to do this in the easiest way, we plot the log. So, this also tells you, how to calculate the derivative of 2 power K t. So, now, let us quickly, again think about it. So, what we did is essentially, calculating the derivative of 2 power K t and by calculating the derivative, we got the slope which is K log 2.

So, now, let us basically, this is another case, where something in mathematics, some idea in mathematics is used. Basically, the, to calculate the derivative, we had some algebra plus some derivative, finding slope is used, to calculate certain rates in Biology. So, here now…So, with this, we will stop the example of derivatives for the moment and we will go on to learn a few more other things today and go to the next topic, in the next case. So, let us now look at a few, another interesting cases of derivatives, positive derivative, negative derivative, zero derivative. So, let us, let us think about slopes, which are positive and negative all that. So, let us take a curve. So, we know that, straight lines have a constant slope. What about curves?

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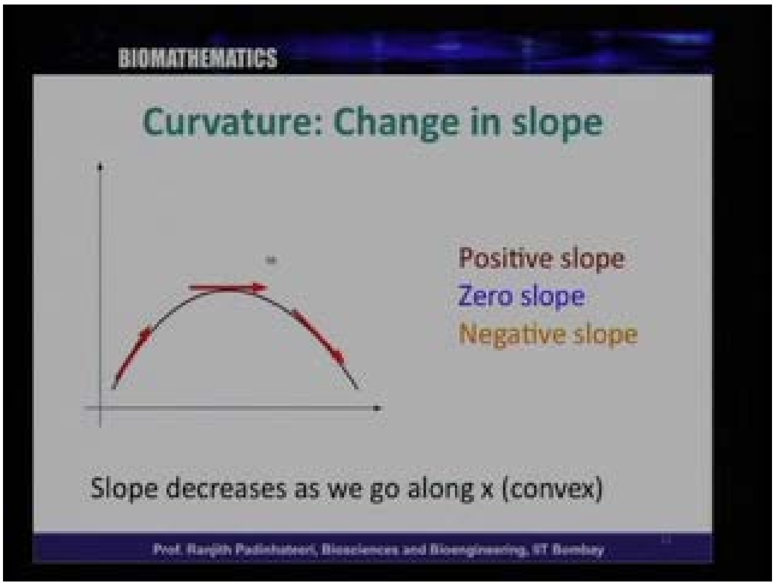

So, let us look at this following slide. So, basically, if you look at this curve and if you think about slopes at different places, what can we learn? So, let us, this, in this curve, if you look at the slope here, and if you look at the slope here, and if you look at the slope here, what do we learn? So, let us quickly look at the slopes. So…So, the curve we have, so, we have x axis and y axis. And, we have a curve, which look like this. And, let us calculate the slope at different places. So, let us first calculate the slope here. So, let us calculate the slope here. So, this is the point, which is that…So, the slope here is basically, change in x, which is delta x and change in y, which is delta y. So, basically, we want to calculate delta x and delta y and the slope is delta y by delta x. Without doing any calculation, we can say that, since delta x is positive and delta y is positive, the slope has to be positive.

So, this is some number, which is positive number and this is some number, which is positive number. So, delta, slope has to be positive. So, let us, in other words, let us do this. delta x, when I say, delta x is x 2 minus x 1. So, this is x 1; this x 2; and, this is y 1; this is y 2. So, the slope of the first case. So, in the first case, 1, slope is y 2 minus y 1 by x 2 minus x 1, which has to be greater than 0. If you do this, it is greater than 0, because, as x 2 is larger than x 1 and y 2 is larger than y 1. So, as you can see here, as you can see here, x 2 larger than x 1 and y 2 is larger than y 1. Now, let us look at the slope here. So, let us again redraw this curve for the convenience of calculating this, drawing this. So, let us plot x and y and let us draw this curve. And, if we calculate the slope here, at this particular point, where the, slope here. So, let us calculate the slope here. So, you have these two values; this is x 1 and this is x 2. But, for each of this x 1 and x 2, the y value is the same; the y value does not change. So, y 1 equal to y 2; for x 1 you get the same y; for x 2 you get the same y. So, what does this mean? This means that, y 1 and y 2 are same.

That means, y 2 minus y 1 is 0. So, the delta y is 0; that means, slope is 0. So, it tells out that, whenever we have a curve and at the maximum value of the curve, you have slope is 0. So, you had a positive, in this case, you had positive slope. So, you had slope, which is greater than 0; here you had, you have, slope which is equal to 0. Now, let us look at the next case, where we have delta x and delta y. So, let us again draw this. So, let us draw x and y and draw this curve and let us look at the slope here; the slope here. So, if you just mark x 1 and x 2...So, let us mark, this is x 1; the corresponding point is y 1. So, this is y 1. Now, let us mark x 2. So, this is the value x 2; the corresponding point here is y 2. So, according to what we learnt, the definition of derivative is d y by d x, which is, what, for whatever value of x 1, you have y 1. So, basically, slope is y 2 minus y 1 divided by x 2 minus x 1.

So, this is the derivative, this is the definition of slope. So, now, let, let us look at what is x 2 and what is y 2. So, as you can see here, x 2 is larger than x 1. So, x 2 minus x 1 has to be positive. So, this is positive, because x 2 is larger than x 1. On the other hand, y 2 is smaller than y 1. So, y 2 minus y 1 is negative, because y 2 is smaller than y 1. So, if you, this is a smaller number, compared to this. So, if you subtract y 2, y 1 from y 2, y 2 minus y 1, this will be a negative number. So, the slope here, which is the slope here, the slope here is negative, which is less than 0.

So, the slope is less than 0 here. So, we saw three cases. So, let us look here. So, the definition…So, this if you look at this thing here, here, you saw that slope is positive; here, at the maximum, the slope was 0; that means, even if you change x, the y was not really changing; here, the x was increasing, the y was also increasing; here, the x was increasing, but y was decreasing. So, the slope was negative. So, from a positive value, slope becomes 0 and then, becomes negative. So, the slope decreases, positive to 0 to negative, right. If you just plot the slope, if we just plot the slope and x, first, it was some positive value; then, it became 0; then, it became negative. So, the slope was somehow decreasing. So, this kind of curves, where the slope is decreasing, is called convex shapes. So, you can see convex shapes, where the slope or the derivative is decreasing, and when the slope itself is changing…So, as we all know, the slope is derivative. So, the derivative itself is keep changing. So, we can define something called curvature.

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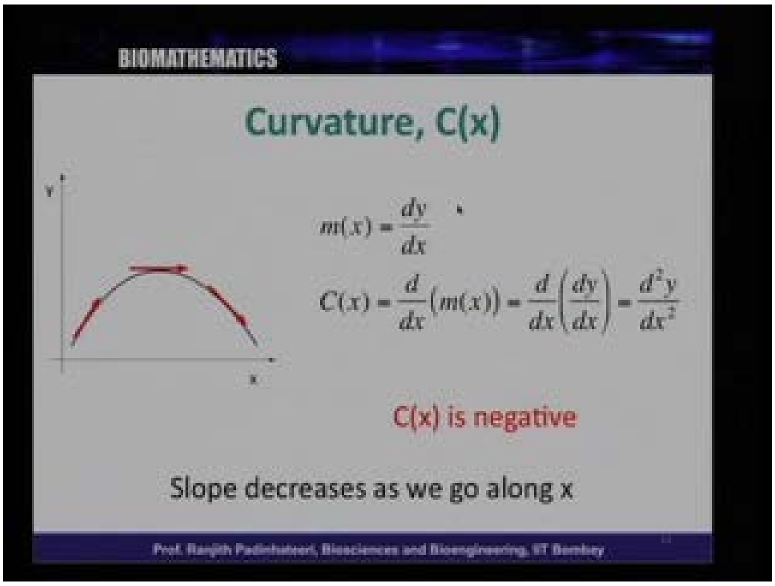

So, let us look at this next slide. So, we have d y by d x, which is the slope. So, the d y by d x is changing from positive value to a negative value, as we go along the x axis. So, as you can see that, the slope itself is changing. So, what does it mean? The m, the change in m, d m by d x, what is this d m by d x means, how does this slope changes. d y by d x is, how does the y change. How does the y change is d y by d x and d m by d x is, how does the slope change. So, the slope changes is, in mathematics, the slope change can be represented as d by d x of d y by d x, which is known as d square y by d x square; that is, the change in slope, which is derivative of a derivative. So, m of x itself, is a derivative. You again find the derivative of that, you will get d square y by d x square; that is called curvature. So, we will discuss about this later, but just keep in mind for the moment; curvature is nothing, but a second derivative, the derivative of a derivative; that is also has mathematical meaning. Mathematically, anything that is curved, has a derivative of a derivative. (( )) calculate the second derivative and you will get either positive or negative and what does it mean is that, it is curved.

So, the curvature has second derivative. We will, we will come and if we have time, we will discuss, how to calculate second derivative, if by taking by some examples. But, for the moment, let us just understand that, there is something called second derivative, which is, how does this slope itself, change. So, the change in slope is d by d x of d y by d x, which is the second, d square y by d x square. So, now, you can imagine, just the opposite. So, look at the next slide. What happens here? If you look at, like, what I did, if you just do all this; just plot as a curve, calculate the slope here, calculate the slope here and calculate the slope here.

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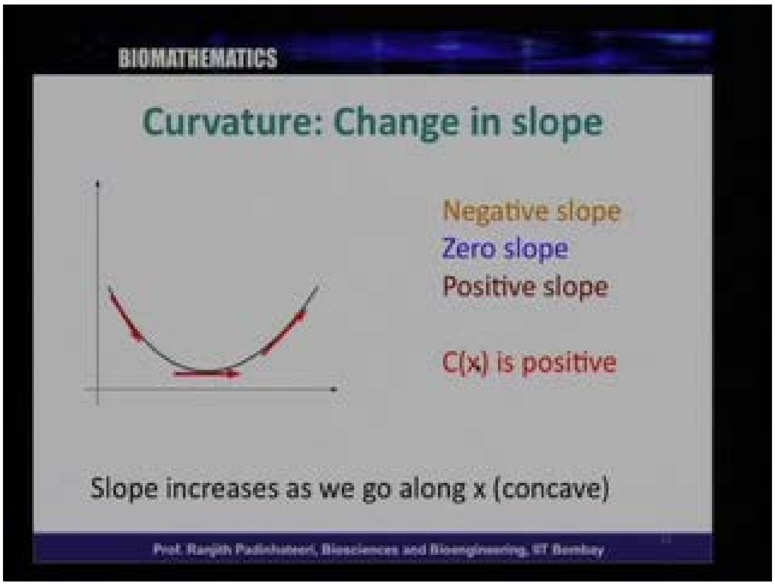

You will see that, the slope is negative here; the slope is 0 here; the slope is positive here. So, the slope increases as you go along. So, the C of x, which is the second derivative, is positive and slope increases as we go along x. So, such things are called concave shapes. If you have a curve in concave shape, we will have slope that is increasing, as we go along the x axis. So, just keep in…And another interesting thing, which we should keep in mind is, this derivative is 0 at the peak; wherever you have a maximum or minimum…See, here, you had a curve, which is a maximum here and the derivative here was 0; in the next one, you have a minimum here; the curve is changing from positive, negative slope to positive slope and it has to cross through 0 somewhere; and the point of 0 crossing, we will call point of inflexion, and the slope is 0. So, two things you should understand from this, the last few slides of curvature is that, the two take home messages, the two things you should change, you should understand is that, whenever you have a curved function, a function that is not a straight line, any function that is not a straight line, the slope itself varies and sometimes, the change in slope can be positive or negative.

So, that, you can calculate, whether the change is positive or negative, by calculating the second derivative. Second derivative, we will discuss, how to calculate this clearly, but just keep in mind that, we can calculate something called second derivative, which is d square y by d x square; that is, how does the first derivative, which is d y by d x changes; how does that first derivative change, that is the second derivative. And, the second derivative gives you the curvature and at the point of, whether it is maximum or minimum, whenever if a curve has maximum or minimum, at that particular point, the derivative is 0. So, this is the three things which you should mind, keep in mind; one is that, the slope changes as, for any curve, which is not a straight line, the slope will change and it can be positive changing slope or negative changing slope and wherever there is a maximum, the derivative will be 0. We will use this things in the next, coming lectures, in the coming lectures. So, for this moment, with this, we will stop with this lecture here and we will go and learn more things in the coming lectures. Thank you.