Biomathematics
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Lecture No. # 29
Fourier transform

Hello, welcome to this lecture on biomathematics. In the last lecture, we have been discussing Fourier series and how to find Fourier series for a some particular functions where we said that any periodic function can be represented as some of sine's and cosines and today, we are going to discuss something related to that but, little more generalization of this where our today's topic is called Fourier transforms.

So the topic is Fourier transform; and this is Fourier transform is essentially, it is related to Fourier series but, it is the much generalized version of this little more it is more general and it is not for periodic function also its more general. And, this has much wider application in some sense and this can be also done for continuous cases where in the previous cases we use to write sumes and here only we can write only most integrals and we can also write sumes here too but, today we will discuss some integrals that way continue with the wave. We have written we will come and see and we will see this in a minute but, the topic of today is Fourier transform. Now what, why Fourier transform? What is Fourier transform? Etcetera, we will come and we will see this.

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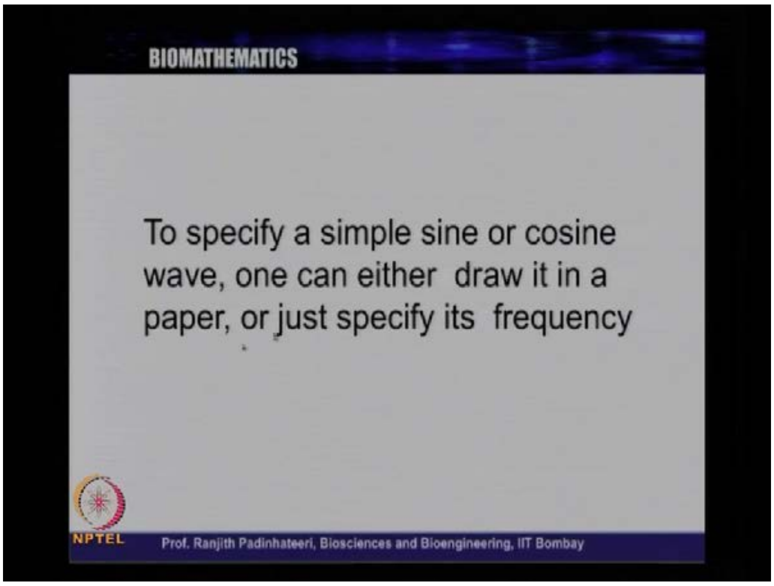

But, to say let us take this particular example of a sine wave or a cosine wave. So to specify a simple sine or cosine wave one has to draw either it in paper that is draw it in space or just specify its frequency; both of it I want to tell you that like sine x or sine 2 x sine 3 x, what do I do?

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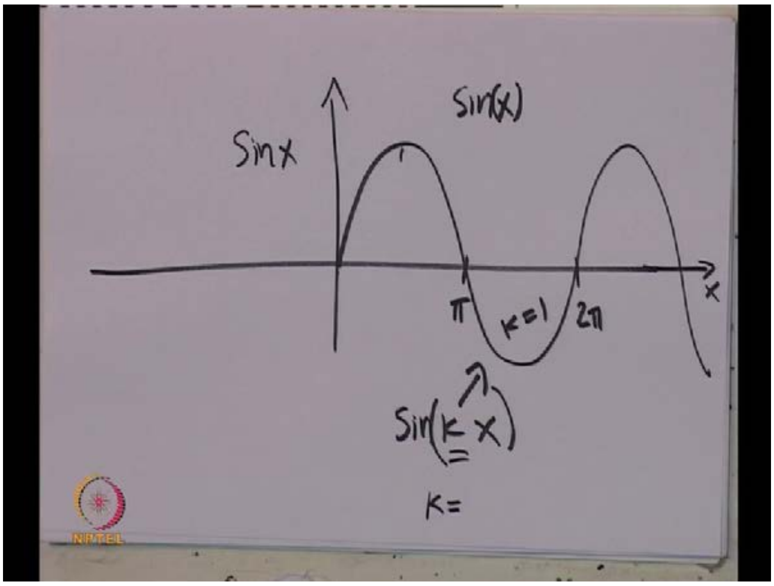

I just draw this so sine 0 is 0; so I draw like this so this is pi by 2 this is pi and this is 2 yeah this is 2 pi.

Similarly, so this is basically sine x now because its reach from 0 to it completes the cycle in 2 pi when x is equal to pi so this is x and this is sine x when x is equal to 2 pi it comes from the complete cycle.

This is essentially sine a x now either you can draw this or you just say some body that is a sine wave with so frequency one so what is frequency? We can define so you can define in general sine k x so here k is 1 so here k is one because this is just sine x; so you can just specify k because if you specify k you can have a different if k is 1 you have sine x if k is 2 something else k is 3.5 some other kind of a wave.

So just by specifying either I can say it is a sine wave and with k 1 or 2 everybody will understand immediately the shape of it or you can just show this shape and then show this show either you can show this shape. Then people will understand this is sine x or you can say that sine wave with it is a wave with k is a sine wave with k equal to 1 then also people will understand this. So there are two ways of telling people either you can just show the function by plotting it or you can just say that is sine wave with k equal to sine k x with k equal to 1 then sine we this.

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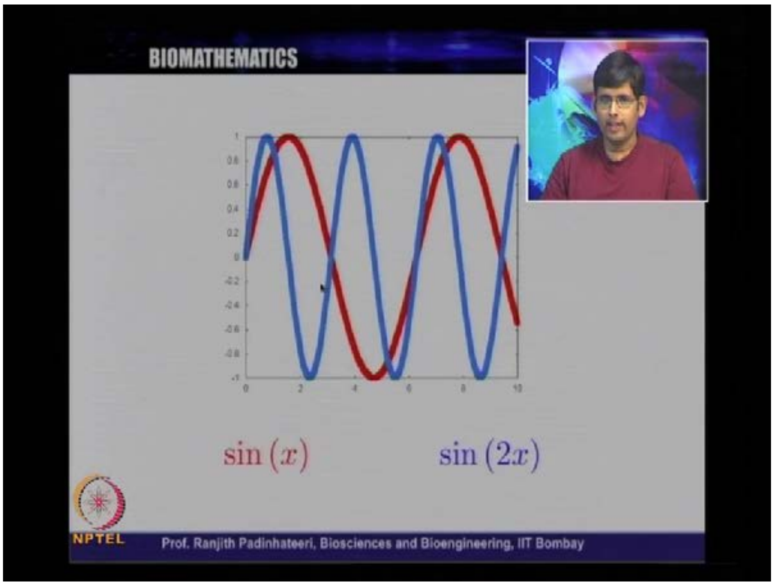

So look at here what we are plotted is the red color is sine x so in the blue color it is sine 2 x so here k is 1 here k is 2 if here talking about sine wave and we say that sine wave with k equal to 1 people will immediately understand this radical or we can say sine wave with equal to 2 then we will understand that it should look like this blue or you can plot this and see then also people will understand this is actually sine x and sine 2 x.

So there are two ways of representing it one you can represented as you can specify the function as a function of x or you can specify something which is function of frequency which is k so what is k k is essentially this frequency here so we define as k as spatial frequency.

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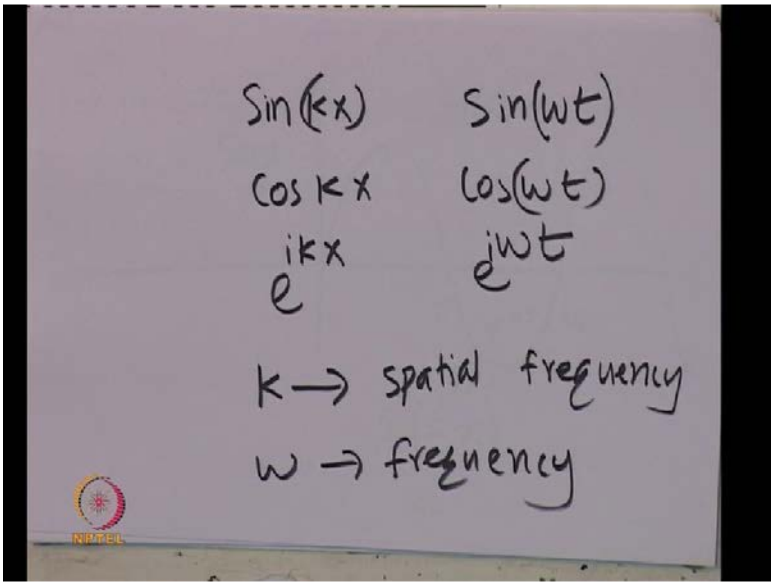

If we have sine k x or cos k x or even e power I k x which is the combination of sine k x and cos k x you can call x as spatial frequency we can do exactly same thing with time you can either say sine omega t cos omega t e power I omega t omega is also frequency is temporal frequency, we can say both, whatever we said. So, for either apply for k or omega so there is either space or time and there is corresponding frequencies. So, now what is the deal if we have this? how is it related to Fourier transform?

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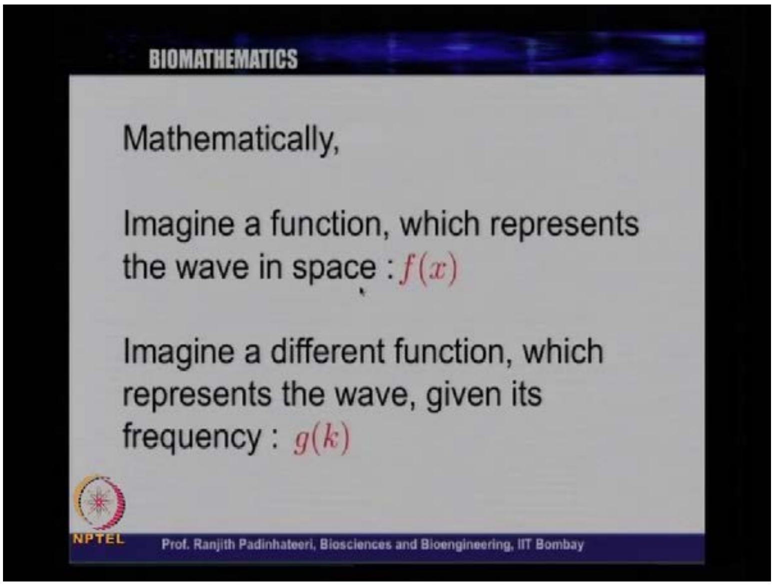

Mathematically if you imagine a function and this represents the wave in space so you can have sine x cos x so f of x is a function which represents the wave in space is in paper is 2 d space.

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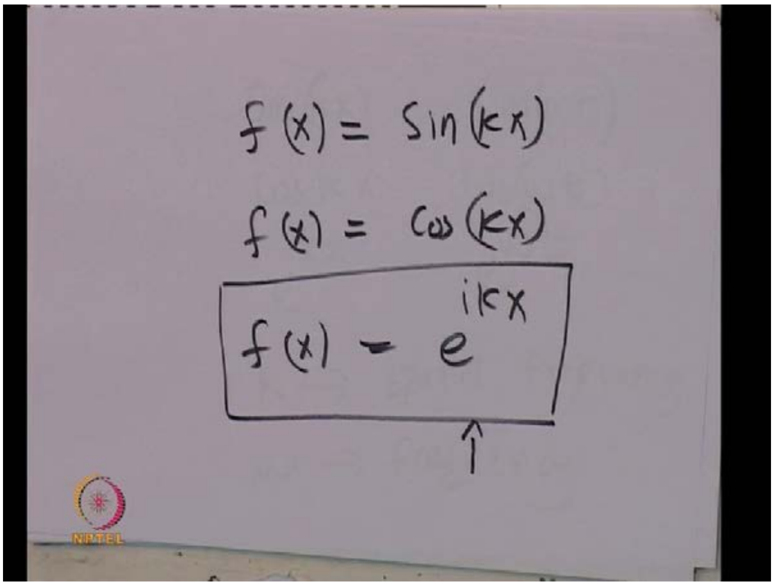

So any f some f of x you can imagine an f of x which is repressing a wave so f of x could be sine k x f of x could be cos k x f of x could be e power I k x. So this is the most general f of x which is representing a wave because this is the combination of sine and cos so given that you have a combination of sin and cos which is e power I k x then this represents a wave so that is the function which represents the wave in space so if you plotted you will get it a wave.

Now, you can also imagine a different function which resembling that wave given is frequency so given k you can represent a function and say you can say that this will be a wave with this particular frequency.

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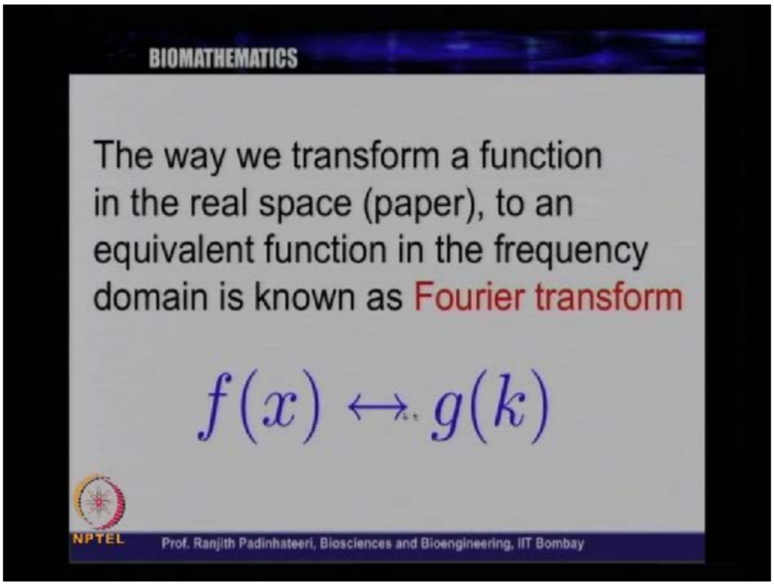

Now the relation between this f and g the way we transform a function in real space or in paper to an equivalent function frequency domain is known as Fourier transform. So, the relation between f of x and g of k is typically called a Fourier transform. We will see what exactly this relation is but, one can imagine a relation between f of x and g of k and this relation can be is called Fourier transform. Now, what is the relation between f of x and g of k? The relation between f of x g of k is the following.

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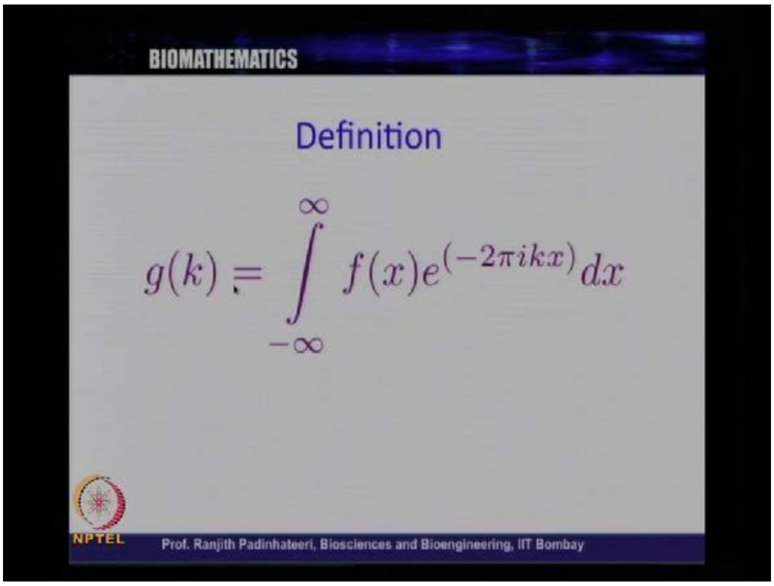

If you know f of x you can calculate g of k in the following manner you multiply with e power minus 2 pi I k x and integrate with x integrate over x. So this is the definition of g of k so you integrate over x in the answer will be a function of k only so for a given value of k there is a g of k a for a given value of k you can find out many values of for different values of x here e power I 2 pi k x and then you multiply with the f of x and integrate it you get g of k note that this is from minus infinity to infinity. Yesterday when we are discussing we define something called c n.

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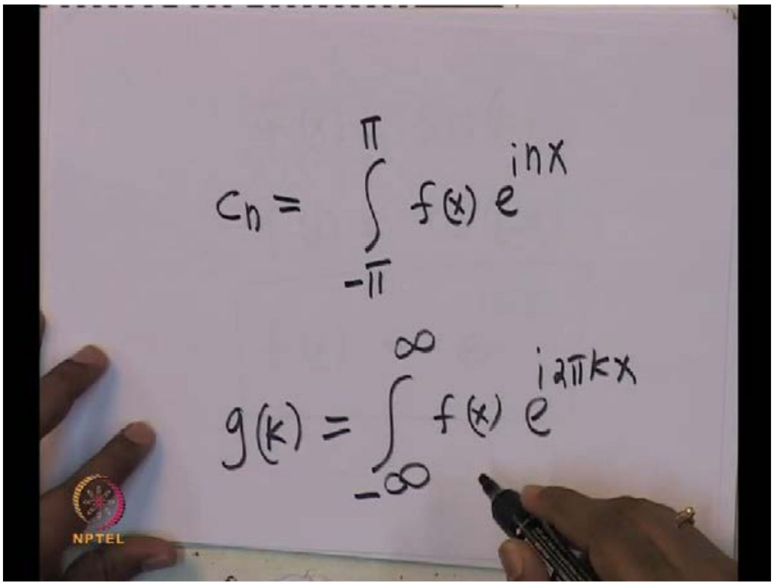

We defined something called C n which was minus pi to plus pi f of x e power I n x here today g of k which it is minus infinity to infinity f of x e power I 2 pi k x.

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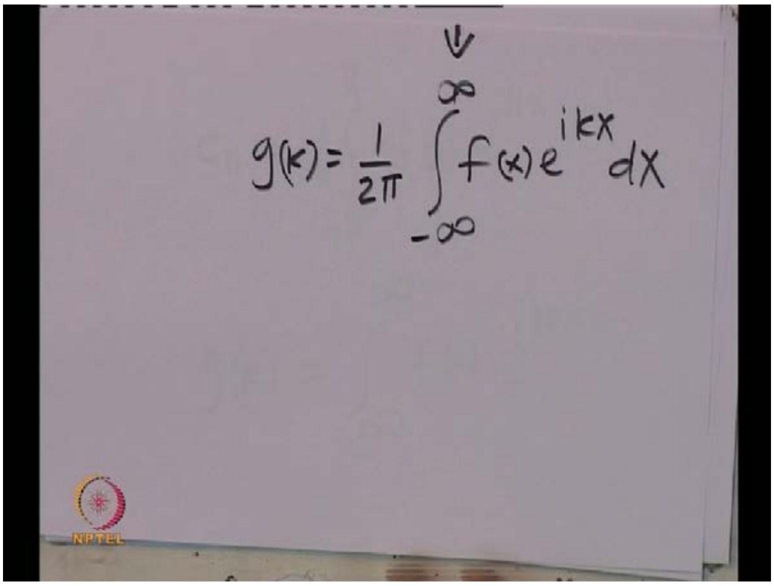

Small difference like this essentially you can write this; you can also write if you want g of k as 1 over 2 pi in to minus infinity to infinity f of x e power I k x d x this is also a definition so one can define c of n this particular way there is a 1 over pi here we study also.

So there is a relation we can see that there is a generalization of this there is an f of x but, the limit as gone from minus pi to plus pi to it is minus pi to plus pi yesterday here it is minus infinity to infinity in the Fourier transformation.

You can understand this and this is the definition of g of k now we have g of k so what what is this all mean we can define some g of k what is that all mean now let say that I want to say that there is a wave sine wave or cosine wave with frequency two.

What does that mean? Let us simplify; for simplicity do this thing called e before just going to an example just we what we saw here is that if you know f of x you can calculate g of k through this particular relation and.

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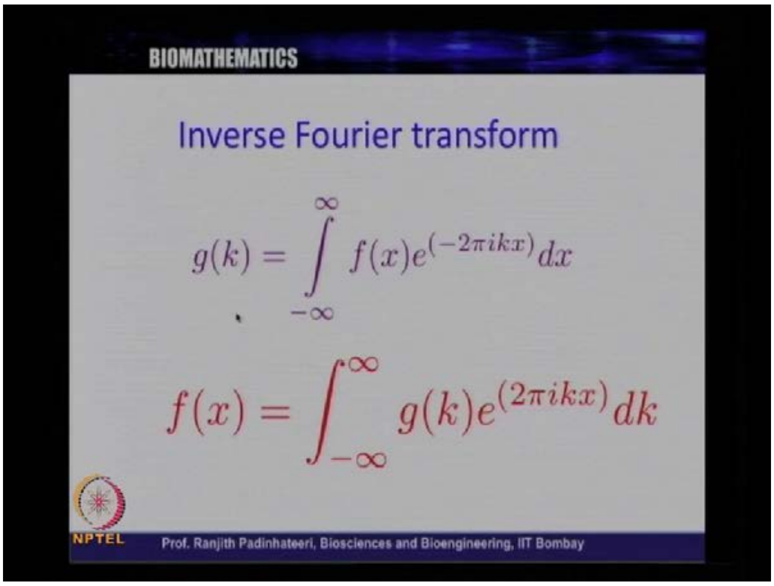

If you know g of k you can calculate a f of x by inverting this and this is called inverse Fourier transform that is how to calculate f of x given g of k so it is exactly same thing but, this minus becomes plus here it was f of x e power minus 2 pi I k x here it is f of x.

Sorry here it is g power g of a g of k e power 2 pi I k d k so this is the plus sign here so there is a difference here and integral over k will give you f of x so this is the way of knowing f of x this is the way of finding g of k and if you know g of k you can also find f of x in this particular manner

So this is the complete Fourier transformation and inverse Fourier transformation Fourier transform and inverse Fourier transform so you know Fourier transform and inverse Fourier transform and let us do an example let us now think of a little more carefully about this now we can take.

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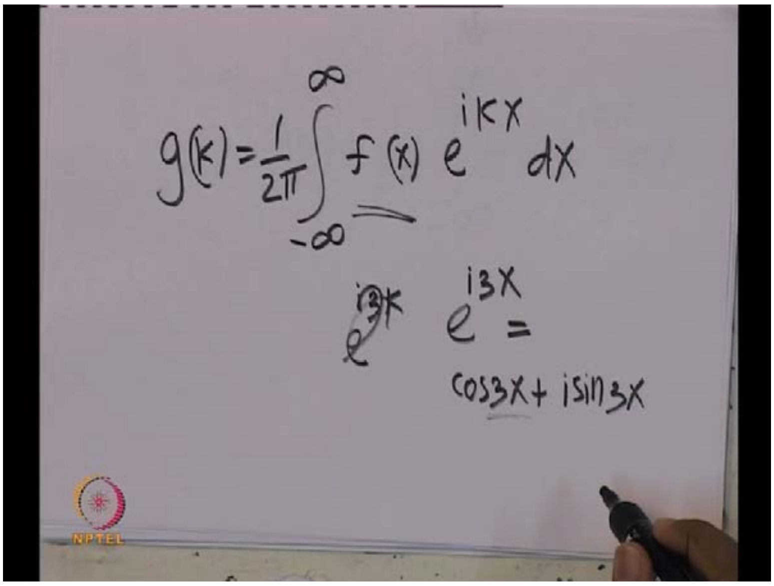

Let say we have f of x we see what will we see we just saw that integral minus infinity to infinity f of x e power I k x d x is what we call g of k with the 1 over 2 pi I can define also this way or I can define 2 pi right 2 pi here also there are two ways of defining as I said.

So I will some time use this definition now let us say that there is a f of x that we know so we want to consider a wave f of x is a wave so let say you want to represent like waves sine three x or cos 3 so the general form of sine 3 x is cos 3 x is like e power I three sorry e power I 3 x which is essentially cos 3 x plus I sine 3 x. So this is the most general form of wave with frequency 3.

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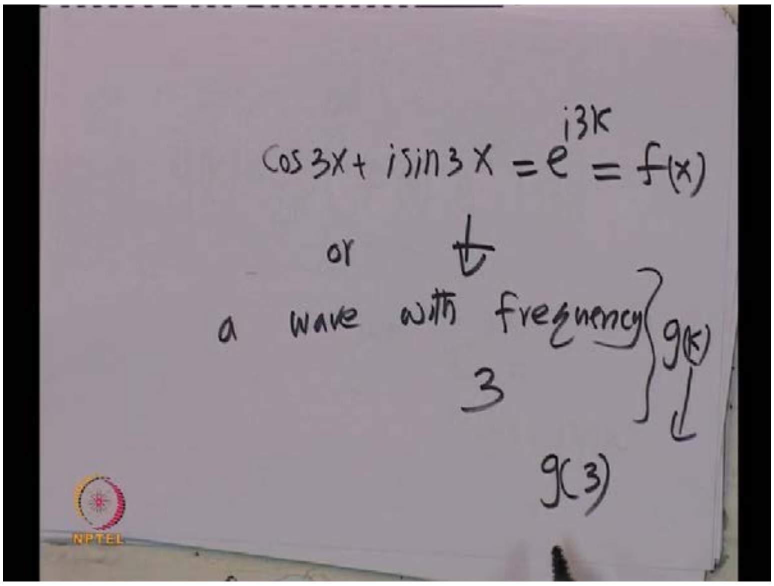

We can say that either cos 3 x plus I sine 3 x or you can say that a wave with frequency three so we have those two ways of saying either I can say cos 3 x plus I sine 3 x or I can say a wave with frequency three so this is e power I 3 x and this I will call f of x.

Now I want to go and find out this function which is my g of k which is here k is 3 so I will get essentially g of 3 so I want to find out how do we say this mathematically so to find out what is the way of saying this mathematically that is it is a wave with frequency three the way to do is Fourier transform of this.

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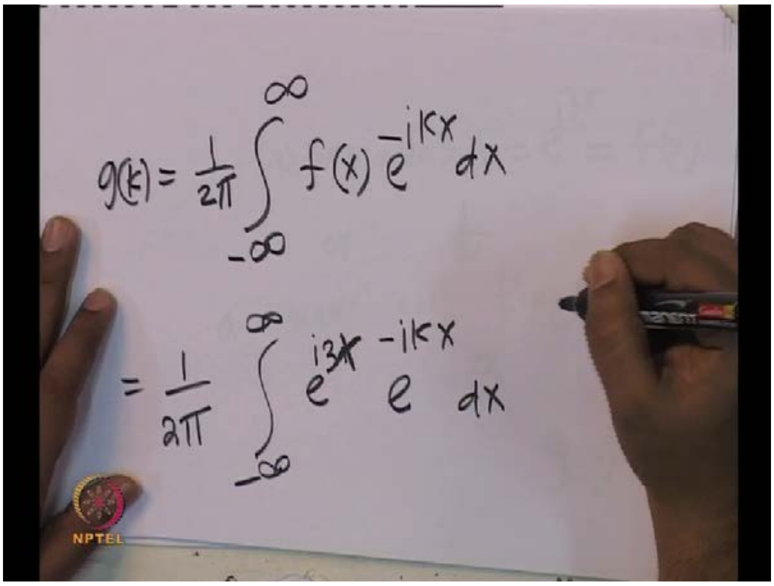

So let us do this Fourier transform of integral minus infinity to infinity f of x e power I k x d x with 1 over 2 pi this is equal to 1 over 2 pi minus infinity to infinity f of x can be written as e power I three x e power I k x d x.

Now, e power three x in to e power I k x can be written as sorry there is a minus sign this is I am just going to use definition with the minus sign because that is what we discussed so this is my g of k. So this is my g of k; now what to, how do I get g of k so how do we can do this integral what is this integral?

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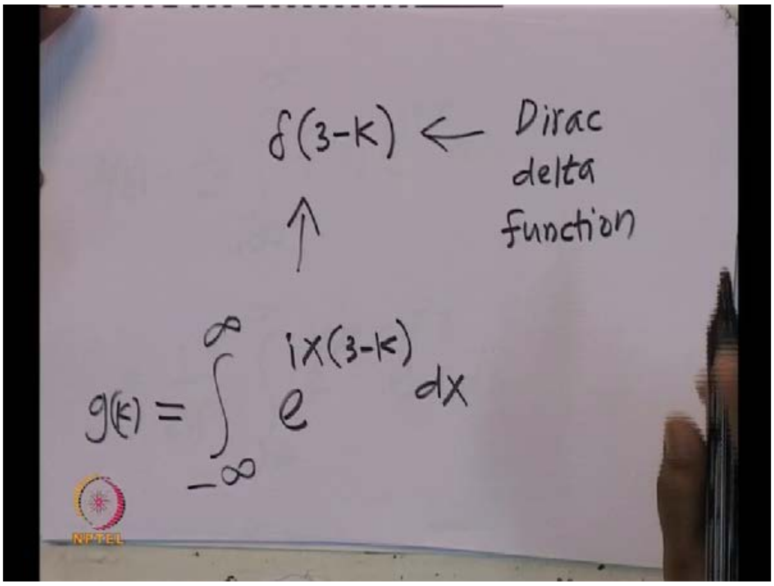

So I can write this as integral minus infinity to infinity there is a g of k is equal to e power I 3 x minus e power I k x so e power e minus into e power minus b is e power a minus b so this can be written as e power I e power I x in to three minus k into d x. So 3 minus k can be this can be written this particular way. It turns out that yesterday we were saying so the answer to this something called dirac delta function that we discussed yesterday briefly I will tell you little more about what is the dirac delta function the answer of this is delta of three minus k what is that mean.

So this delta is called dirac delta function dirac delta function this was named after famous scientist called paul dirac now what is this mean what is this delta of three minus k what is that mean it means that this function as a value only when k equal to 3 whatever inside this bracket as we equal total of this will be 0 so when k equal to 3 this will become whatever inside this bracket will become 0. So when k equal to 3 this is the value everywhere else this as no value.

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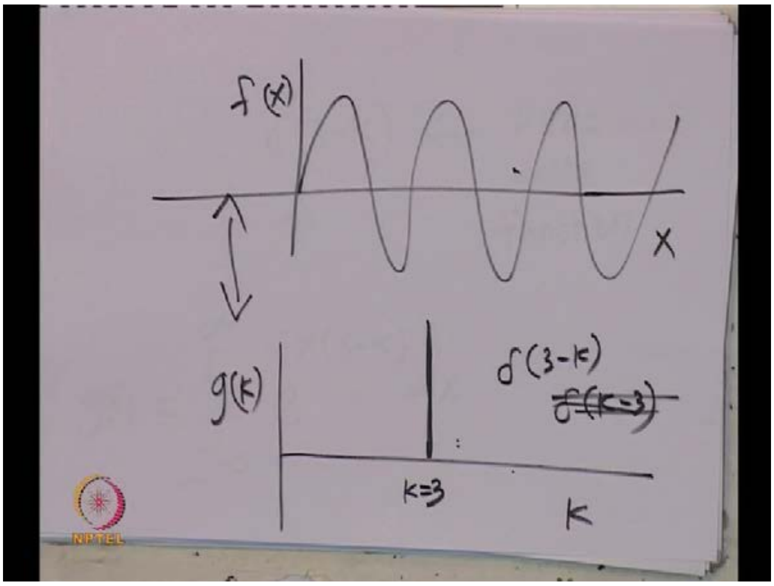

If I plot e power I 3 x cos three x sine k x real so like you might get some function like this now this is my f of x versus x now if I plot my g of k versus k so this is my f of k which is cos k x plus I sine k x.

So I can plot actually real part and imaginary part separately but, essentially it look like a wave and the Fourier transform of that is just one line this point is k equal to 3 so only at k equal to 3 so this is f of x and this is g of k which is delta of k minus 3 or 3 minus k they are equal.

So delta of k minus 3 that is only when k equal to 3 you have this value everywhere else so it was written delta of 3 minus k so let me write the same thing delta of three minus k so only when k equal to 3 you have a value.

Nowhere else you have this value but, everywhere else the function is 0 so this is representing the function in real space this is representing the function in the Fourier space so mathematically if you know about Fourier transforms one can either draw this function completely like this or you can just draw a line like this both represents the same thing same information.

So this is giving you an information delta of 3 minus k is of information is saying that it is a wave with only frequency value 3 only frequency value 3.

So that is only here so again represent either like this or either like this so this is the relation between this 2 g of k and f of x the way of finding this knowing this you can find this and knowing this you can find this is called Fourier transform. So, this is called the Fourier transform.

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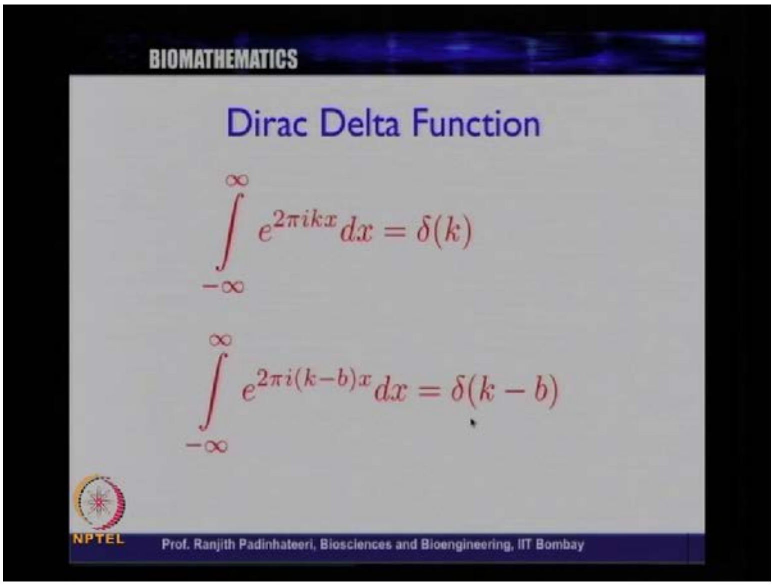

So what we just discussed is something called dirac delta function so you can also define it as e power 2 pi I k x d x so we can define it as delta of k and if 2 pi I k minus b in to x d x is called delta of k minus b. So, you we had b equal to 3 that is because e power I k minus 3 b 3 x so essentially k minus 3 x so essentially we had delta of k minus three in this so this is if you plot delta of k minus b.

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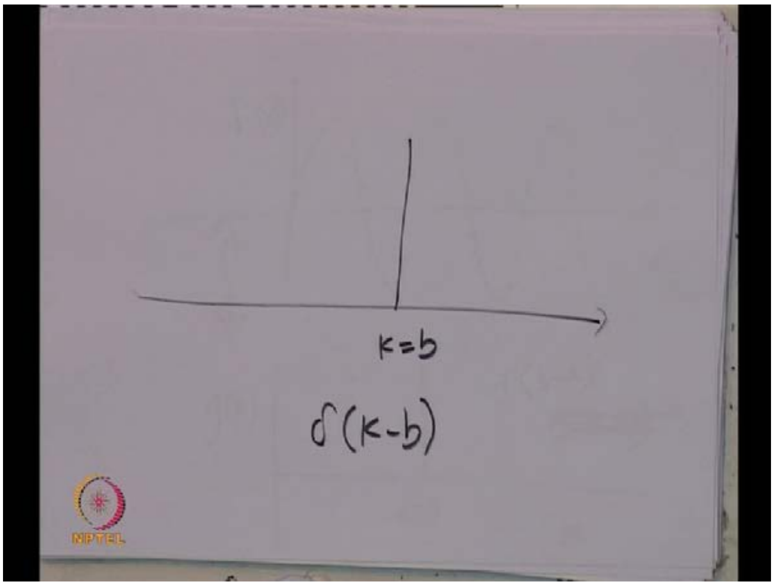

So this is k equal to b and this function is delta of k minus b. Only value the function as only value at k equal to b everywhere else the function is 0 such function is called dirac delta function just value only at one point now what are we why should we learn Fourier transform at all why Fourier transform.

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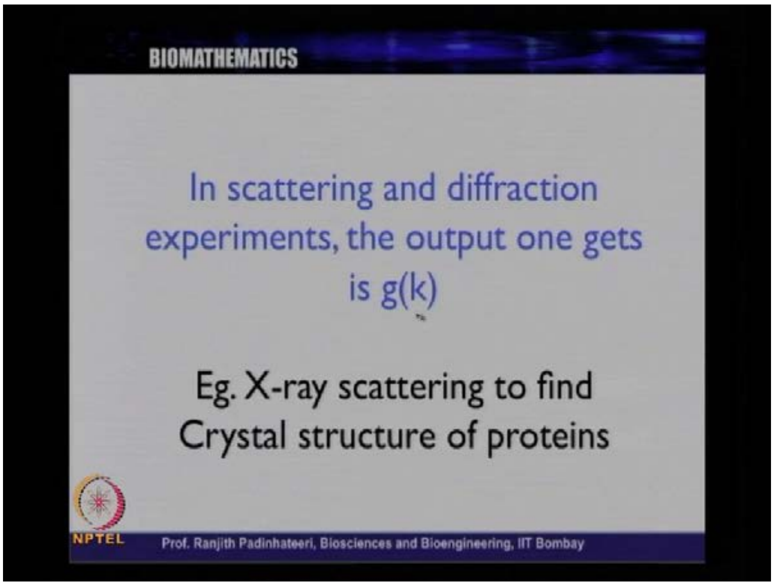

It tells out that why we should learn for at a one is this one reason is this so it tells out that in many scattering in diffraction experiments the output one gets is g of k so we just discuss g of k which is a function in the k domain in the frequency domain.

For example, if you do x-ray scattering experiments to find out the crystal structure of a protein typically what you would get is some answer in the k domain so if you can ask for both who do x-ray scattering or familiar with x-ray crystallography you will see that you will they will tell you that what they will get essentially is something in k space or g of k some value some function in the k space.

So because so this is also true for other experiments is scattering experiments you do let say you want to find out the protein then people find something called as structure factor and various other things in when you do scattering experiments to find polymer information about this structure of polymers or any structure for that matter when people do scattering experiments what they find what they get essentially as an output is something called structure factor which is essentially g of k.

The function in the Fourier space so if you know g of k you have to really find out if you know g of k if you really find out the structure of the protein which is f of x you have to do this inverse Fourier transform.

So you have to know this Fourier transforms essentially to get the real curve in 3 d space if you know some cryptic information like information in the Fourier spaces like you now some you know some cryptic information and then you can convert it in to real phase information like if somebody tells you. If you do not know Fourier series and if somebody tells you.

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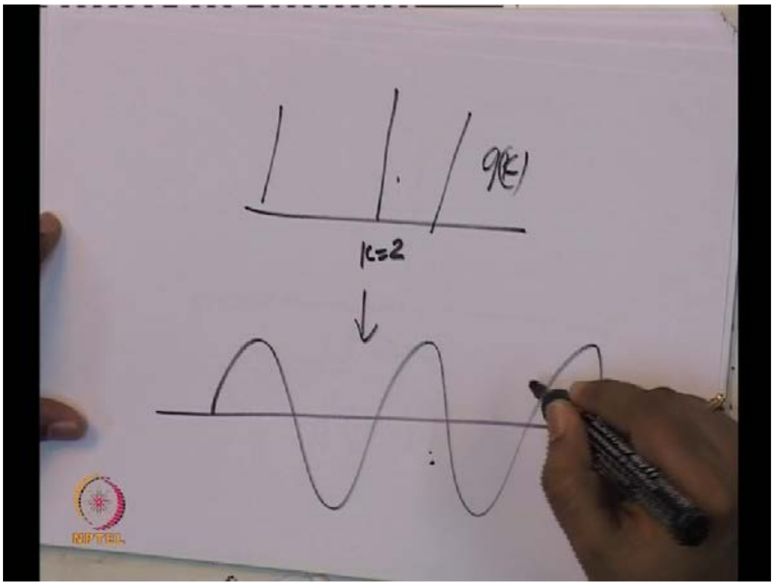

What you get is this with k equal to 2 you need some expertise to convert actually this means it is the wave like this you need some expertise to know that this is essentially what you if you get the output g of k as this the wave as to be like this. You can sometime can g of k as like this then the wave as to have some particular form so all this information how if you know g of k in a particular manner.

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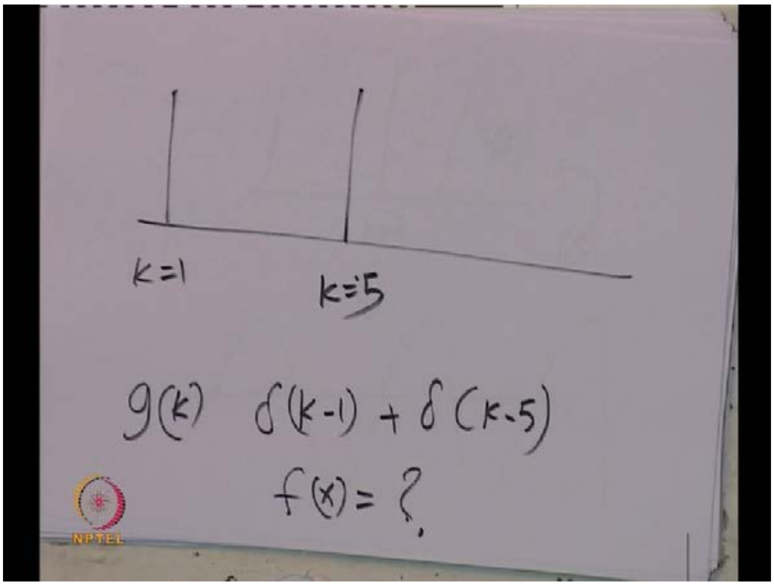

If you get two peaks let say you get like one here one here. So let say this is k equal to 1 and this k equal to 5 you get two peaks what is that mean in the wave space so what are we getting here we are essentially getting delta of k minus one and delta of k minus 5 then k equal to 5 above value and k equal to 1 above value.

So this is the two functions as this is your g of k and then you have to ask what does how does the actually the wave look like so to find such answers to find to answer this 1 has to know how do we do the Fourier transform of this to get this. So Fourier transform of delta of k minus 1 if you do.

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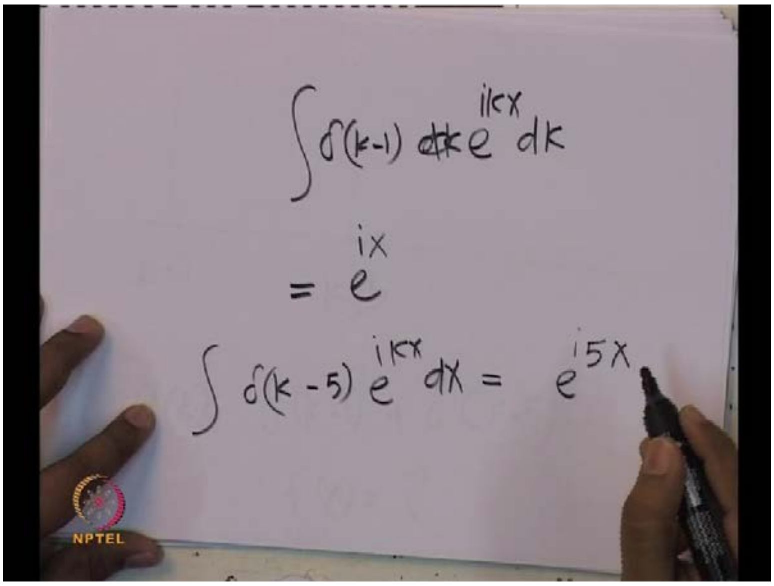

So you can do delta of k minus 1 in to integral d k e power I k x d k will give you essentially e power I only when k equal to 1 you have value so you get e power I x. Similarly, only when k equal to 1 you have values so you get e power I x similarly, when it is delta of k minus 5 e power I k x d x what does mean this means that only when k equal to 5 you have this function so this answer of this is e power I 5 x. So, if you have such two peaks the answer f of x is essentially e power I x plus e power I five x this is essentially cos x plus cos 5 x plus I sin x plus I sine f x so this is essentially this is the real function and this is the function in the Fourier space.

So knowing this how can we convert to this that is what essentially Fourier transform tells you given some peaks from that you have to convert to some real functions which as some meaning which we can think of now how do we know this delta of k minus 5 e power I k x is e power I 5 k 5 x. So the definition of dirac delta function.

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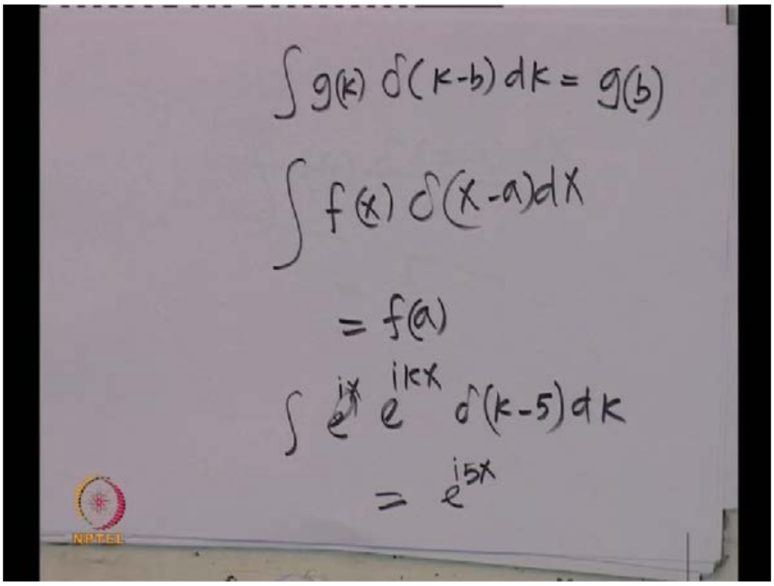

When you define there is one definition is f of x delta of k minus a d k is essentially f of sorry wrongly wrote this. So, integral f of x delta of x minus a d x is essentially f of a or also you can write integral f of x plus e power I x sorry f of x was so if we have this f of x was e power I k x and there is delta of k minus 5 and you have d k then this is essentially e power I 5 x instead of k you should substitute 5 and this is the answer. This is essentially the definition or if you want write g of k delta of k minus b d k is essentially g of p. So this is the definition of the delta function dirac delta function which means that this is 0 everywhere on the except at k equal to b everywhere else this is 0.

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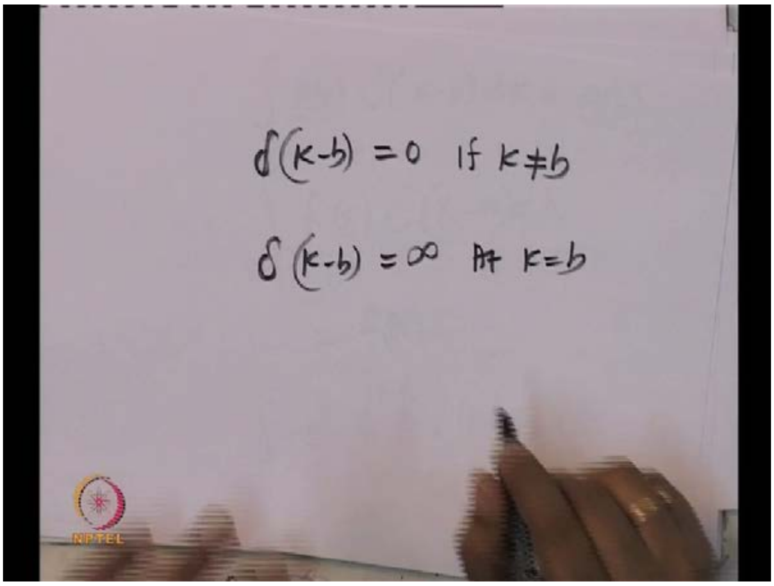

So delta of k minus b equal to 0 if k not equal to b delta of k minus b is actually infinity at k equal to b it has a value at k equal to b so this is the dirac delta function that we have. Now, there is so one use of this is that so what when you do x-ray crystallography or search scattering experiments what you will essentially get is g of k and then you have to get some idea about sine x how the curve look like ok.

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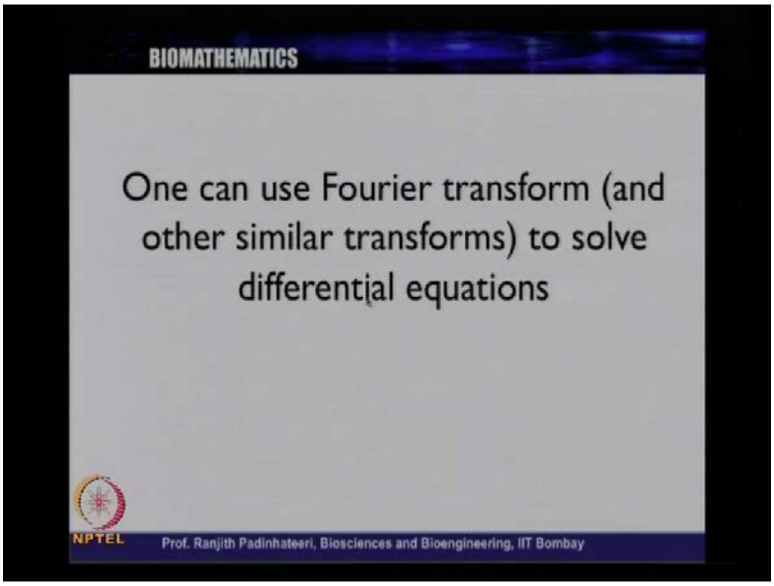

Now, one can use also Fourier transform you can also us Fourier transform and other similar transform to solve differential equations so if we have a differential equation and if you most of the time if you do Fourier transform it becomes easier to handle this Fourier transform this differential equation and solve it.

So we will see some examples of this in the coming lectures how can we solve Fourier transforms or similar way use Fourier transform or similar transforms to actually study and understand some differential equations so we will come to that now we will look carefully little more carefully in to the transformation we were discussing. So we were discussing essentially we were discussing that.

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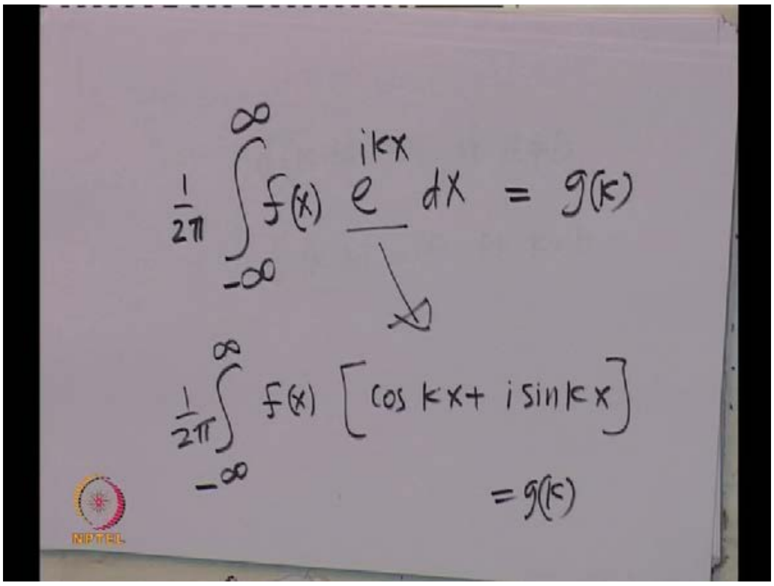

If you know f of x and if you have e power I k x and d x integral minus infinity to infinity 1 over 2 pi and we call this g of k. Now, one can write e power I k x as cos k x plus sine k x so this is minus infinity to infinity f of x in to cos k x plus I sine k x so this is 1 by 2 pi so this is also g of k because e power I x can be written as cos k x plus I sine k x; if you know cos k x plus I sine k x.

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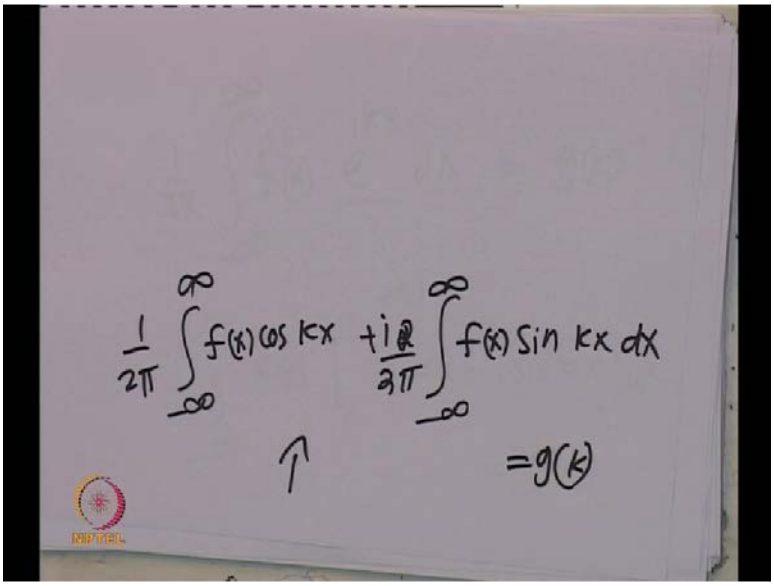

We can we can rewrite this essentially this cos k x plus I sine k x as one by two pi minus infinity to infinity f of x cos k x 1 integral plus I into 1 by 2 pi into minus infinity to infinity f of x sine k x d x. So essentially this can be written as a cos integral and sine integral so Fourier transform essentially like a cos integral and sine integral so this is essentially or g of k whatever this gives you g of k now so this kind of a form is called a trigonometric form if you like that is given this function f of x you can write this in to the form of sines and cosines. So this is called a trigonometric function.

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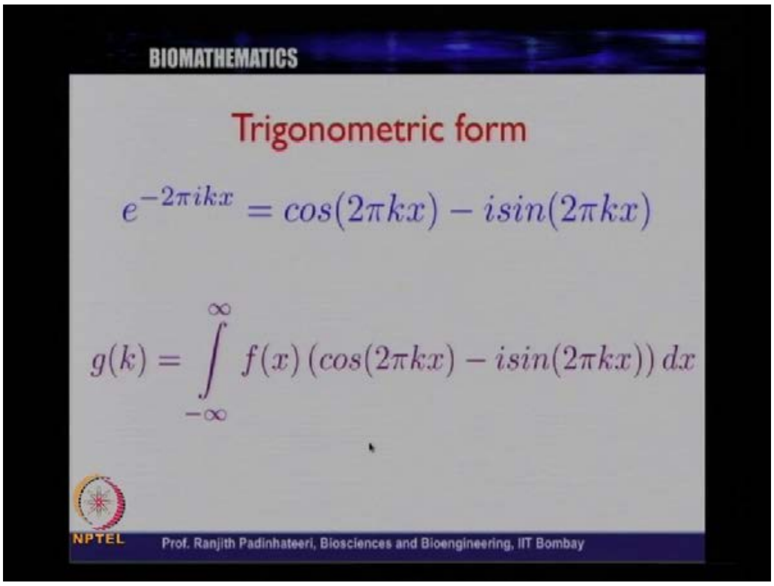

So this in the trigonometric form if you use the other if you use c power minus 2 pi I k x you can write it as 2 pi cos 2 pi k x plus I sine minus I sine 2 pi k x and g of k in that case if this is the definition you are following for Fourier transform like f of x cos 2 pi k x minus I sine 2 pi k x. So you can write f of x cos 2 pi k x d x minus I sine 2 pi k x d x so essentially you get this relation between f of x and g of k so what is that mean what is the advantage of having this.

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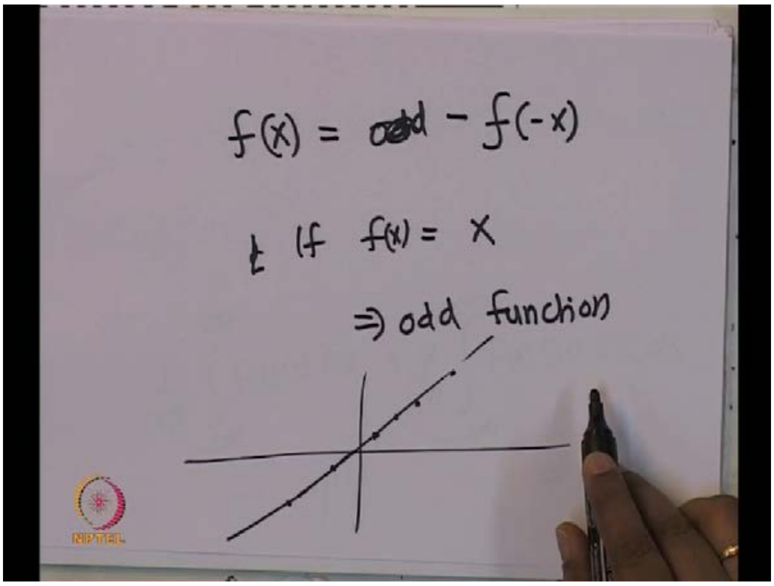

So let say that you have f of x which is let say an odd function so f what is that mean f of x is equal to minus f of minus x for example, if f of x is just x this is an odd function what is that mean when you put a minus sign is just minus of f of x so minus if you go to the negative part this is just a minus sign for x is negative the function is minus sign of the positive part. The negative of this is this negative of this is this; so, this is an odd function and in such cases when it is an odd function you have to just only the sine part remains the cos part goes to 0 so for an odd function the g of k it is can be written as 2 0 to infinity f of x sine 2 pi k x. This is simple property of sine function you can just check what is the property because what happens is that we just saw here that we can write this as f of x sine k x and f of x cos k x.

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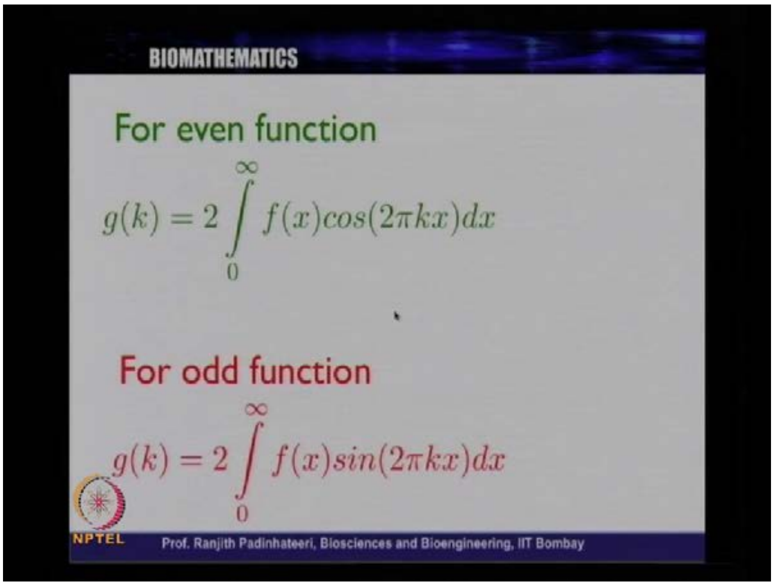

Now, you can use the property with an odd function and you will end up with that you can you can rewrite this particular way and if it is an even function you have to just do this cos integral. Why is this so? We will come and see but, this is you can take it as an exercise and just put f of x as an odd function you can use the original definition and take f of x is an odd function or an even function and then try using what you get you will find that one part goes to 0 and what essentially you have is this so for an odd function even function the definition becomes just this ok.

So now we have this simplify this particular way so what did we essentially the message is that.

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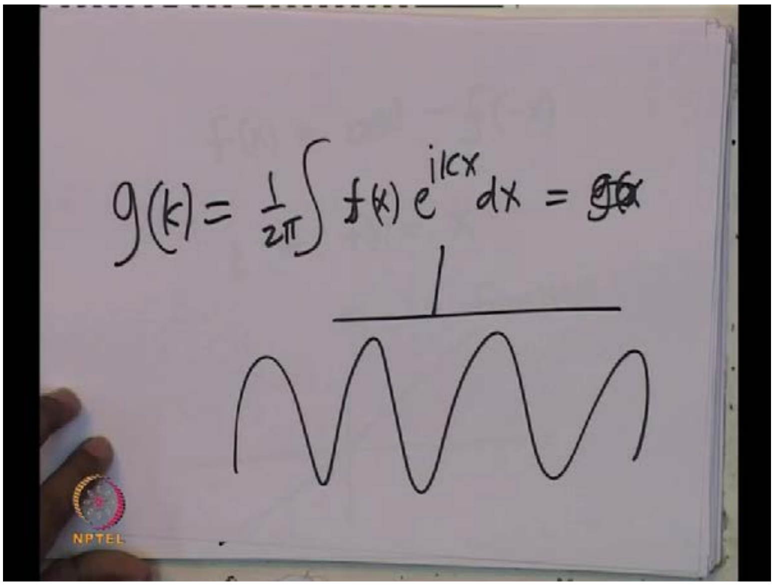

If you know a function f of x you can do Fourier transform and get a function which is function of k. And, we saw that if it is a wave then the Fourier transform of that is just a peak a just line like a delta function so every function as a its Fourier transform so it tells out the if the function is the Gaussian if the function is just the Gaussian function.

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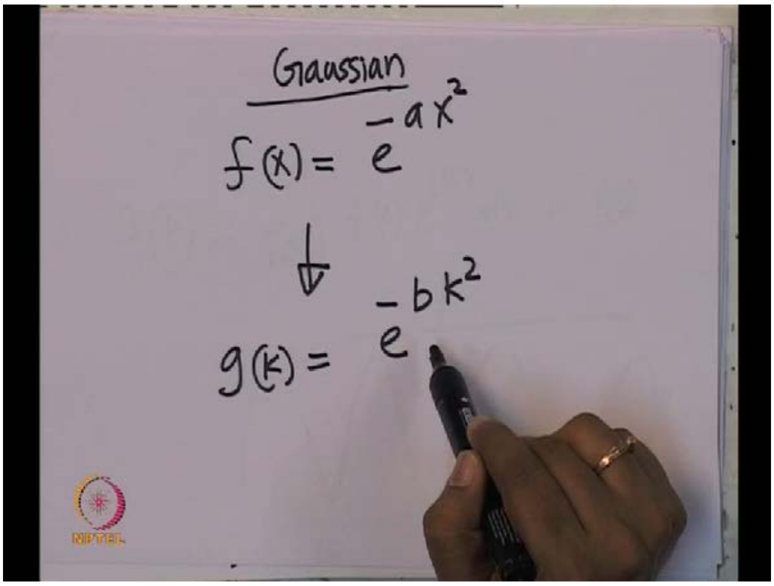

So let us say f of x is e power minus a x square its Fourier transform g of k tells out to be e power some other b k square it has a same form same functional form this is also a Gaussian.

So the Fourier transform for a Gaussian so this is it tells out that it is just this is self so how do we will try and do this later in another lecture we will try and prove that this is indeed this but, you can try doing this but, you can show that this is indeed the Fourier transform or if you know the f of x is e power minus a x square the g of k tells out to be this.

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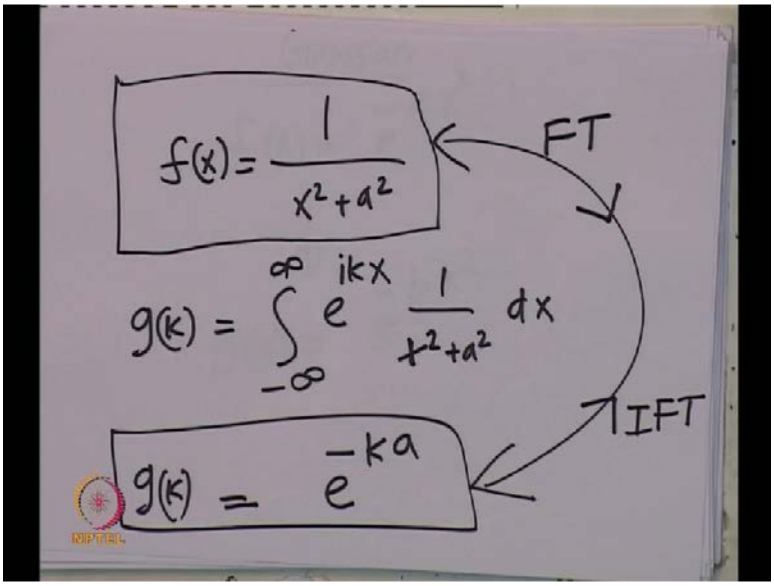

If you have something called Lorentz n which is 1 over x square plus a square let us say this is your f of x then what is g of k it is essentially minus infinity to infinity e power I k x in to 1 over x square plus a square d x and if you do this the answer you get turns out to be I thinks e power minus k into a.

You will if do this, you will get g of k which is essentially e power minus k; so this is your f of x. So, this is, we will see how 1s get all this f of x in g of k but, what I am trying to say here is that for every f of x there is a g of k 1 can calculate a g of k. This way of calculating this is Fourier transform. So, you do a Fourier transform to get this you get an inverse Fourier transform to go back. This has enormous use in different fields like especially like in bio medical engineering and many such fields like enormous usage where signals, many things there is a function of time many signals that is as a function of time which you get like some signals that you will be getting as a function of time; Which is some function of t will be can be converted to some function of the frequency omega which is defined as integral minus infinity to infinity f of t e power minus I omega t d t.

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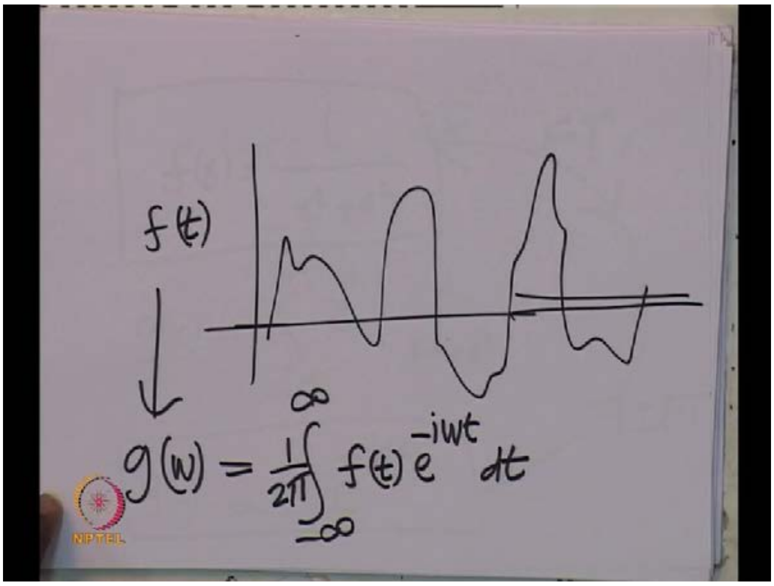

In this particular way, you can convert there is a 1 over 2 pi so this is some basic and convert the information in the t real space to a frequency space information so this is essentially converting information from one language to a different kind of a function. This is synonymous, we will discuss the use of this in the coming lectures but, at this moment it is sufficient to say that free atom transform is a very powerful technique this can be used in many different ways and it can be rewritten as a sine transform or a cosine transform. In some sense, if it is an odd function or even function depending on that so it has so on in crytallography scattering x-ray crystallography it is wide range of uses especially in all sorts of optics essentially all optical experiments in even including a even including like let say you know spectroscopy in spectroscopy Fourier transform is extensively used so to understand the principle behind all this one should clearly learn the basics how to do the Fourier transform.

So that is the important point which we should come, we should learn; so we will have some exercises related to this specified how to do we will also, in the coming lectures.

We will discuss how do we actually see; how do we actually do Fourier transform of various functions and we will also see how do we use this Fourier transform actually to solve differential equations. We will take some differential equations and solve them using Fourier transform or similar transformation. So I will briefly tell you one another transformation which is very close to Fourier transformation.

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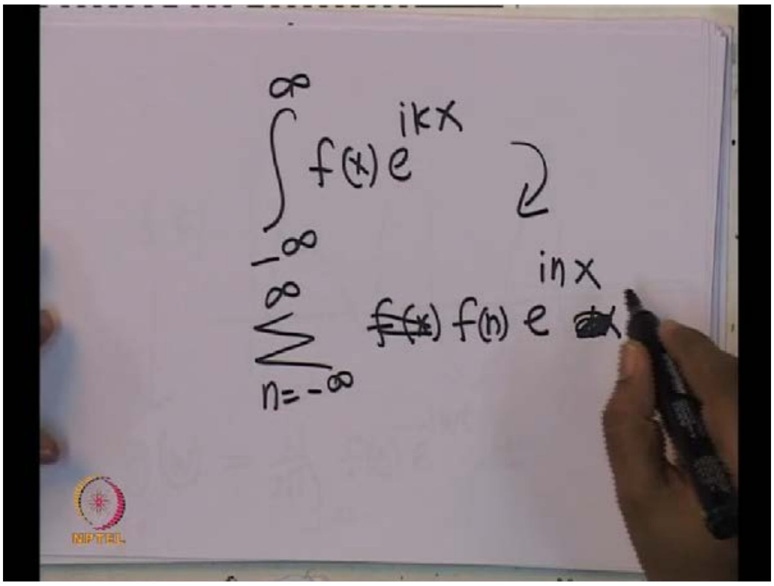

So, this integral minus infinity to infinity f of x e power I k x can be also written as some of n is equal to let say minus infinity to infinity some f of x sorry f of n e power I n x d x this is way also, sorry, no need of d x. We are converting this to a sum so f of n e power I n x; so, we can convert this in a way to a discrete form also. So there is an equivalent of trying equivalent transform; one can actually write in a discrete way.

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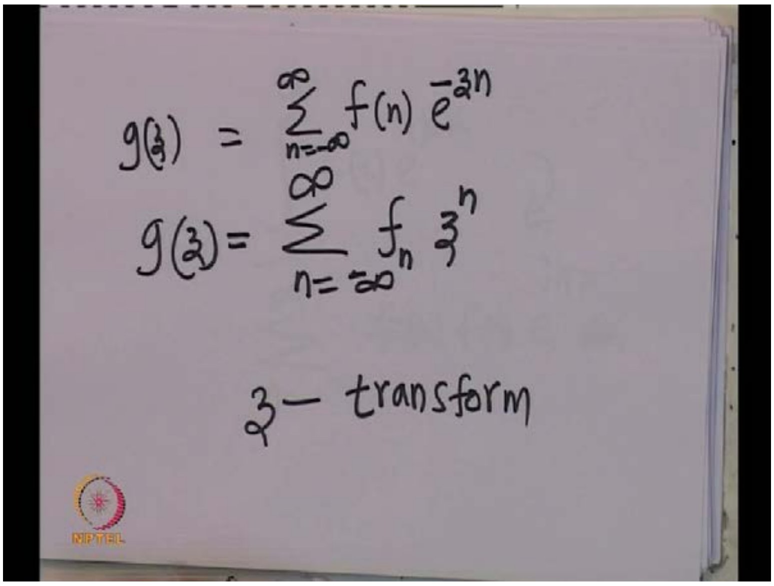

Some other transformation let us say, sum over n sum function f n z power n; so such thing can be written as some g of z. So, this kind of a transformation is also a generalized form of Fourier transforms. This is called g transform minus infinity to infinity. So you can also write this in a different way saying like, you can, if you can also write let say you can write this f of n e power minus z n sum over n is equal to minus infinity to infinity and you can write g of z.

So, this z you can also write e power minus z n; so there are different ways of writing this. Such transformation this sometime called z transformation but, there all essentially some kind of a generalized version of Fourier transforms. We will discuss all this when we need it; we might be able to use all this transformation transforms to do some simple studies on various differential equations. To summarize essentially, what we have to look is just this slide.

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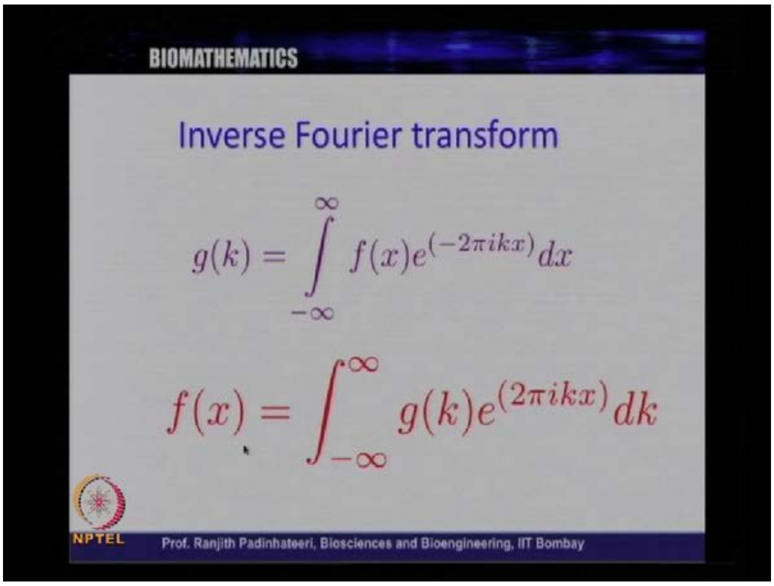

Here we have Fourier transform and inverse Fourier transform; you have a function f of x and that can be converted to a function of k g of k in this particular fashion and that can be inverted to get f of x back so if you know f of x you can het g of k if you know g of k you can get f of x in this particular fashion so this is the essential summary of Fourier transform and this is has lot of usage in different fields. So there are different way just one more thing I should say like some places there are different ways of writing this as we just saw.

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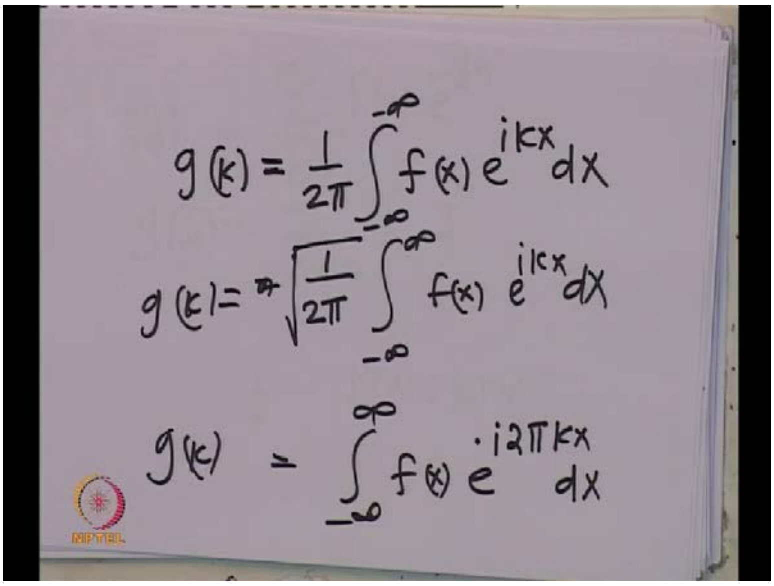

Some time you will write, you will see this written as g of k as 1 over 2 pi integral f of x e power I k x d x. Some of them it will be, some of the time it will be written 1 over 2 pi some of the time you will see this written minus infinity to infinity; all of this is minus infinity to infinity f of x e power I k x d x.

Some time you will see g of k as just minus infinity to infinity f of x e power I two pi k x d x so this some people write 1 over 2 pi some people 1 over root 2 pi so correspondingly the this all of this definitions are correct and they essentially the same but, the only thing is that for each of them the inverse transform is slightly different. So that we this as when you define inverse Fourier transform you do not have a 2 pi here both in transform and inverse Fourier transform you will have 1 over root 2 pi and here both inverse Fourier transform you will have this twp pi here.

Transform and inverse Fourier transform - so we can just stick to one of these definitions and follow this you can look at textbooks; you can see you may see any three of this; any one of these three. You might see sometimes this is sometime, this is sometime, this but, follow the same definition and its inverse definition.

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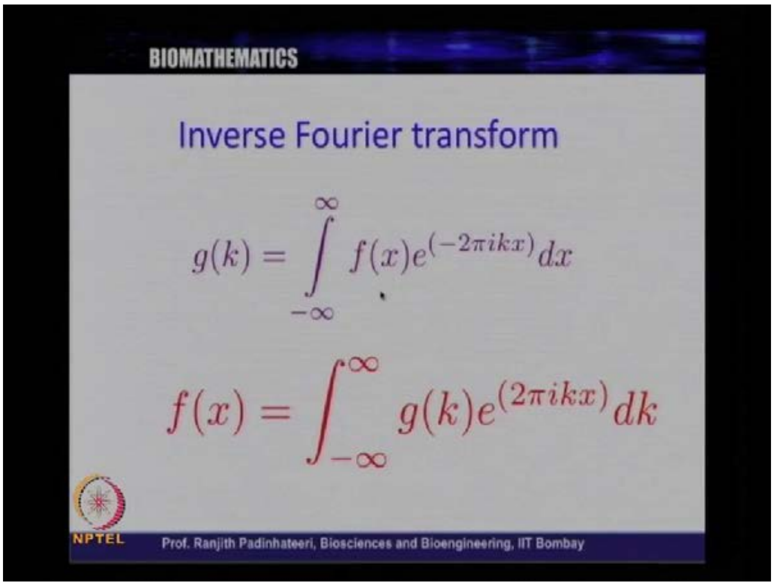

So here with definition which I use here in this slight is essentially f of x e power minus 2 pi I k x and is inverse is g of k e power 2 pi I k x so this is the pair and this pair and this complete. Similarly, if you write 1 over root 2 pi the inverse will have also 1 over root 2 pi here.

Depending on each of this, the inverse is slightly different look at the text book whichever the text book this is just a warning for you because when you look at the textbooks you might see any of this and you should not get confused. Just remember that any definition will have its inverse with correspondingly so just follow the same rule same definition for Fourier transform and its inverse definition from the same book be consistent with that that is all it is.

This is essentially the summary; to summarize you can define a Fourier transform and inverse Fourier transform and convert functions from real space to frequency space or but, sometime people call it k space; sometime people call it omega space frequency space, q space. So, you can convert a function from a real space to a frequency space and let us lot of use in different fields and we are just trying to understand the beginning of this. so, there it might be useful for biologists especially in the context of x-ray crystallography in the context of differential equations in the context of bio medical engineering and so on and so forth. So, with this we will stop today's lecture, bye.