Biomathematics

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Lecture No. # 28
Fourier Series

Hello! Welcome to this lecture on Biomathematics. In the last lecture, we discussed about Fourier series. So, in this lecture we will continue discussing about Fourier series. So, lecture is on Fourier series.

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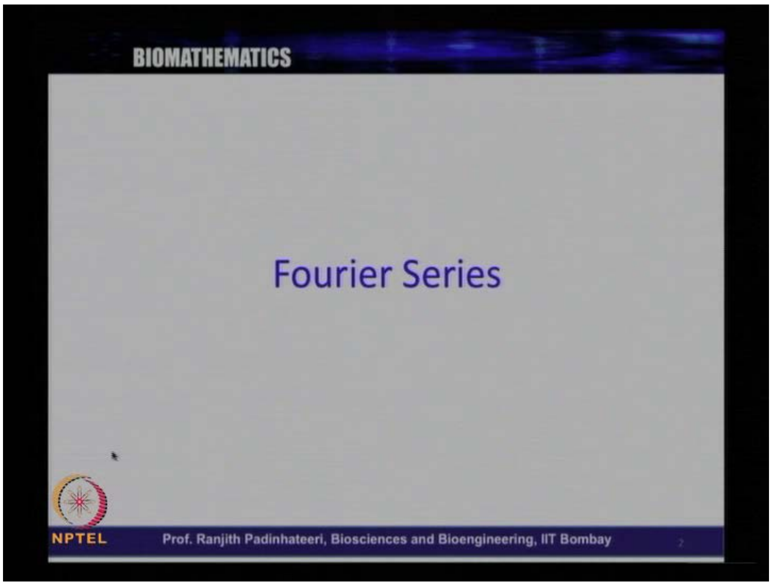

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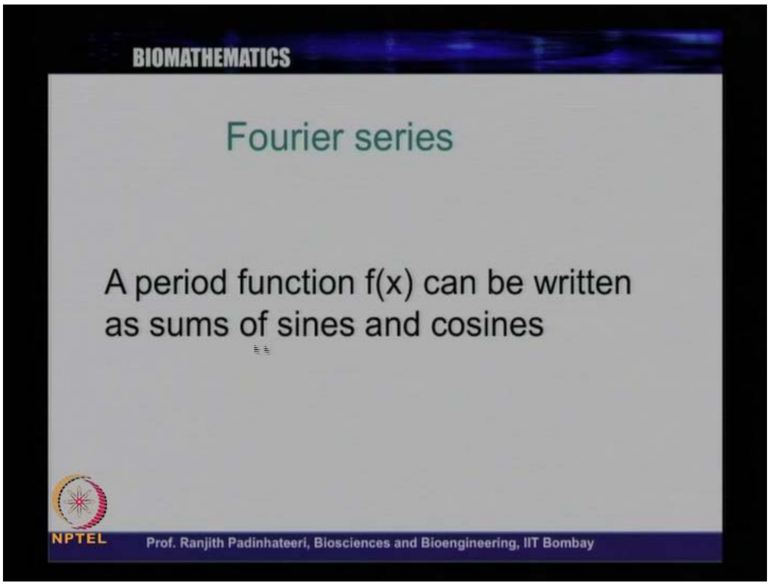

We mentioned that as a periodic function. Any periodic function can be written as sums of sines and cosines. So, this is an essential idea of Fourier series, which we discussed last time.

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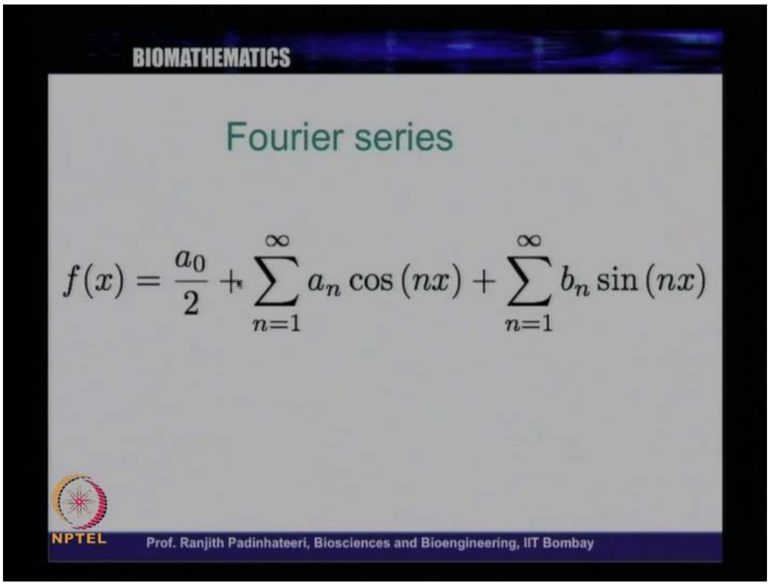

And, we also said that, this means that any function f of x can be written as sum of cosines, sum of sines and combination of some coefficients a 1, a 2, a 3, a 4; similarly, b 1, b 2, b 3, b 4 and a 0.

So, if we have a set of coefficients, f of x can be expanded in terms of some number times, cos, something else multiplied with sine. So, you can expand any function in any periodic function in this particular manner. So, what does this mean? How is it, why it is important to Biology? If we think about it a bit, think of various things in Biology. You can see that there are many things that repeat like the life itself, like day-to-day activities, kind of repeat. So, as we were discussing in the beginning that there are many things in nature and particularly in an alive living being that is, periodic. So, if you want to represent many of them for example, concentration of many proteins depending on the time like various cyclic, it just goes in a cyclic manner. So, they just increase decrease, increase decrease.

So, they kind of go in a periodic fashion. In cell cycle in some sense, the same things repeats like the cells divide, grow, divide grow. So, essentially, there is also some kind of a periodicity. It is a periodic event again, with some particular times periodic. It is kind of, cells keep dividing depending on the conditions and so on. So, again there are different phases and at some particular phase, cells divide and then again grows, divides.

So, it is also can be thought of as a periodic. In mathematical language, in principle, it can be described as some kind of periodic function; some particular events in a cell cycle. For example, some concentration of a particular protein as a function of time might be increasing and decreasing in a kind of a periodic manner.

So, this may not be just simple sine and cosine. So, it might be a complicated periodic function. But, whatever be that, however complicated that be, it can be written as a sum of sine and cosine. That is an essential message. We also know like for example, like in the last lecture, we described a function which is like this.

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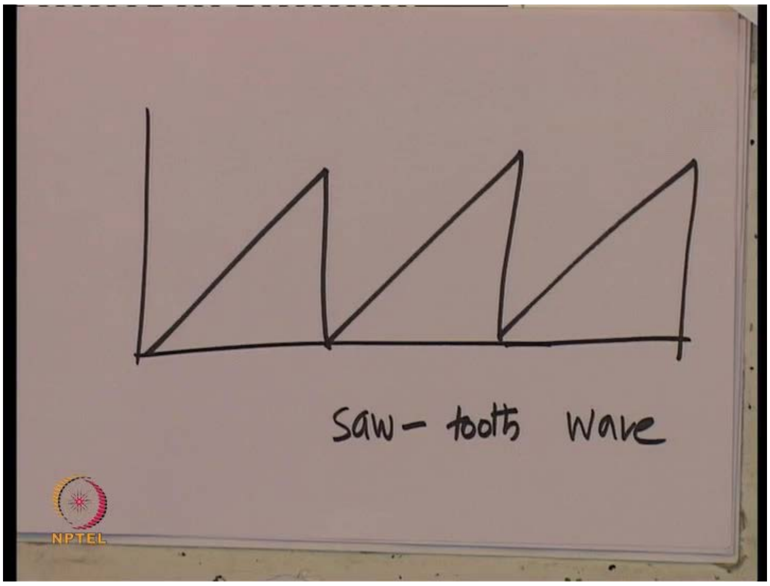

We call it as saw-tooth wave. So, we described such a function. We called it saw-tooth wave. When you look at for example, E C G or many bios, many of the machines that work in our body like… And, if you look at the signals from them, sometime various fourier techniques have been used to analyze the signals, which we will come to, we will discuss in a bit more detailed in the coming lectures where you learn little more than fourier series, you will learn something called fourier transformation. At that point, it will be more interesting discussion, but however, some kind of periodicity is again seen. And, some of the signals from heart or various machines that work in our body, sometimes you might be able to find some kind of periodicity.

Even, if you look at the DNA sequence like the repeat of a g or a t or g c, they might also have some kind of periodicity. So, there are people have been studying, whether they repeat in any kind of regular fashion depending on the gene region or in gene intergenic regions. So, we do not know, in some particular cases they do repeat in a particular manner.

So, even they can show some kind of a periodicity. So, wherever you see some kind of a periodicity that information can be translated in to a mathematical language. And, that these techniques that we used now are called Fourier series and Fourier transformation, can be used to study those things. So, that is the bigger picture of why we are learning Fourier series and Fourier transformation.

So, last time we learnt saw-tooth wave. Today, we will learn little another kind of a wave. That is the aim of today's lecture. But, let us quickly discuss, remind ourselves as what we discussed last time.

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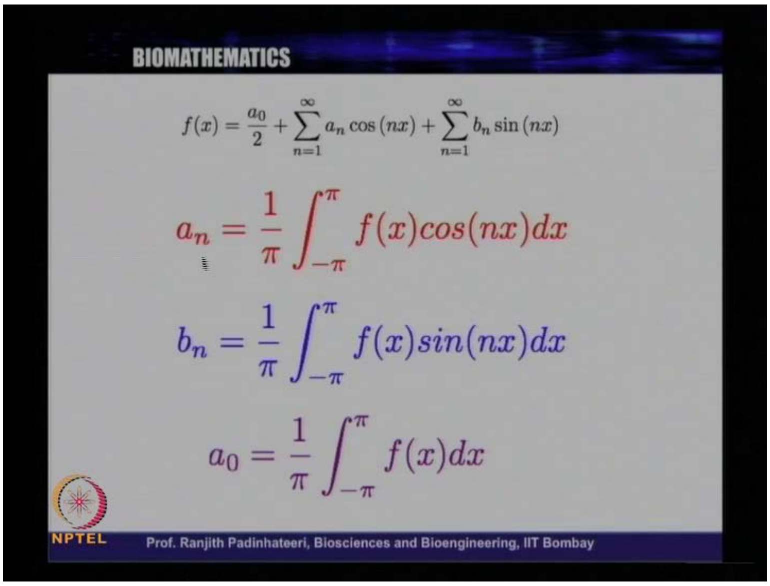

We said that if you have a function, which is the combination of sine and cos; the coefficients a n and b n can be found out by integrating that function, multiply with cosine f of x cos n x integral minus pi to plus pi will be a n. And, f of x sine n x minus pi to plus pi will be b n; for all values other than... So, this n starts from 1 to infinity. And, for a 0 this is just f of x d x. So, this is the way we can get a n, b n and a 0. And, if we know a n, b n and a 0, function can be expanded in this particular way.

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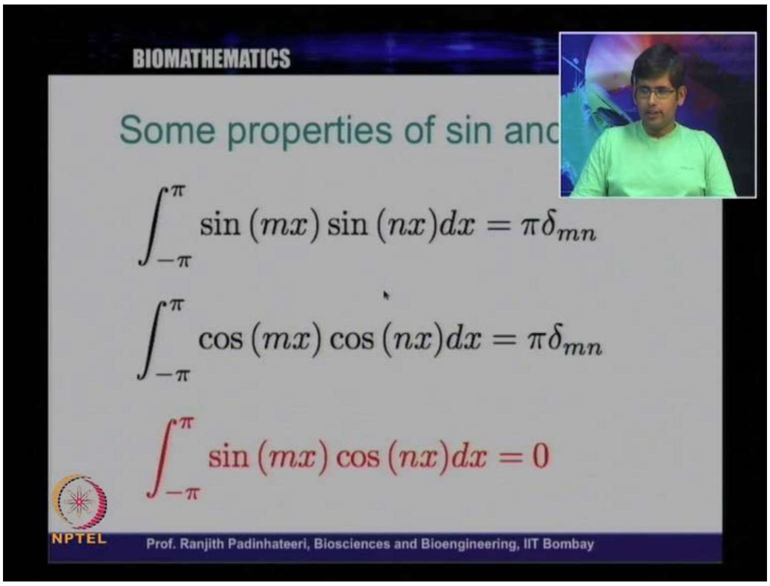

So, we also saw that this was the reason, why we could do this kind of a special property that the product of sine and sine m x and sine n x is delta m n. What does it meant? It is that, only when m equal to n you have this 1. And, in all other cases, this integral was 0. And, if m not equals to m this integral is 0; similarly, here integral cos m cos n x d x is 0 when m is not equal to n; and it is equal to pi when m equal to n; similarly, here and here, this is always 0; so, this integrals sine m x cos n x is always 0.

So, this property has being used to do this. And, this kind of properties can be called a kind of orthogonal function. So, sine and sine m, this is we can call it orthogonal property, orthogonal functions. So, we will come back to this a bit more today, later. But, before that, let us discuss. So, last time we discussed the saw-tooth wave. Today, let us discuss a different kind of a wave.

So, let us take another complex, another little more extreme case. Let us discuss just to convince you that, anything can be written as a combination of sine and cos.

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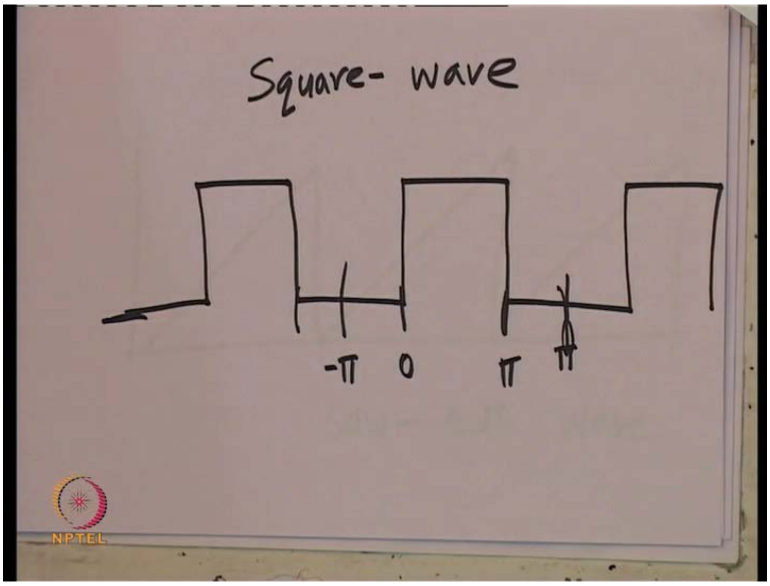

So, let us think of some function which is like this, which is repeating. So, if you do this, we can call this as square wave. So, we can call it as square wave. So, this looks like squares. So, what is it happening? So, let us call this as minus pi and call this as plus pi. Sorry, let me call this minus pi; let me call this plus pi. Let us call this 0. Sorry. Maybe I should write it little more in a different way.

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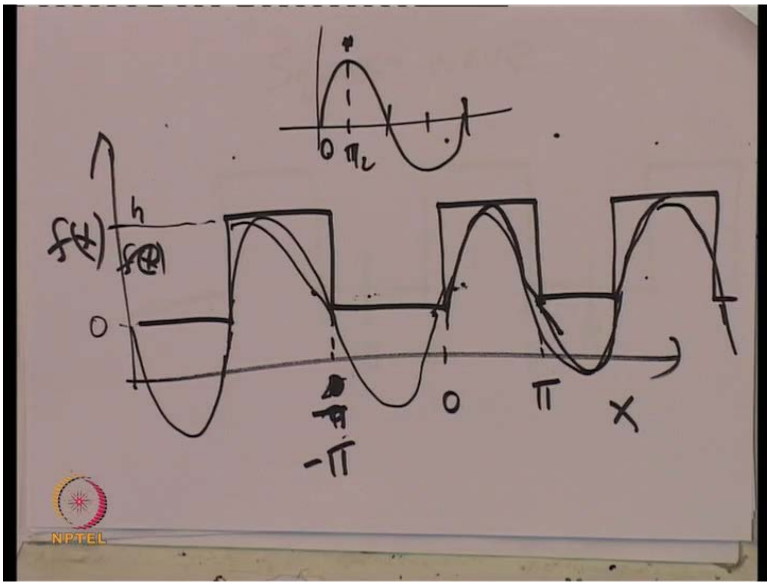

So, let me draw this in a different way. So, when we draw this, so we can take any where 0 and pi. But, for simplicity, let me take this as this particular point as 0, let me take this particular point as minus pi, let this particular point as 0 and this particular point as plus pi.

So, this is f of x and this is x. So, this is f of x and this is your x; when x equal to minus pi to 0, this is a particular value. And, suddenly from 0 to pi, it has a different value. So, let me call this as 0; this base line. So, from minus pi to 0 it has a value 0. And, let me call this particular x value as h. h is this particular value. So, for minus pi to 0, the value of function is 0. And, suddenly from 0 to pi, the value is h. So, this is, if I plot this separately here, it is little more carefully here it is like this.

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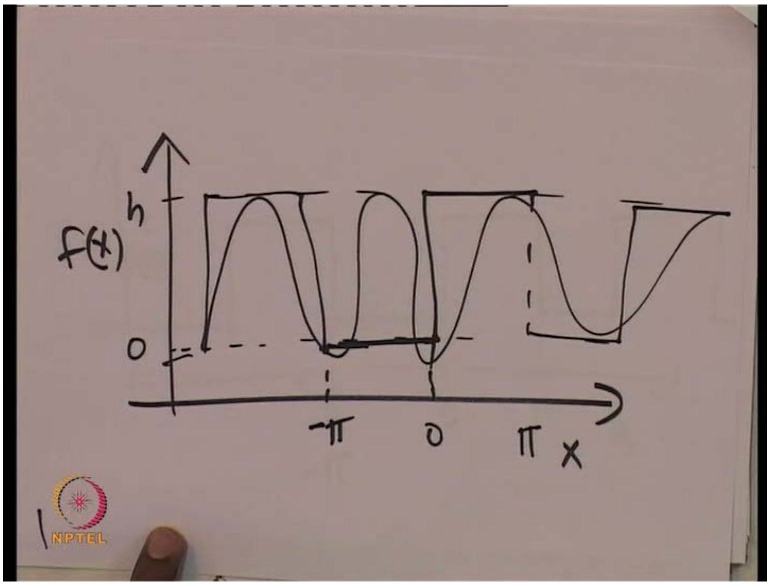

So, you have here this is x, this is f of x. And, if you mark here, so let me mark this as 0 and this as h. So, from 0 till some particular point, suddenly it becomes h. So, somehow minus pi to 0, it is 0. And, 0 to this particular point is pi, it is some value h. then, again it repeats. It repeats at here also. So, in this kind of a square wave, how do we write this particular wave in terms of sine and cosine? Sine, we know typically looks like this, something like…So, if I look carefully, I plotted at 0; sine is 0 and at pi by 2 is 1 and at pi it is… So, if I just look at sine of x, it looks something like this. So, where this is 0, this is pi by 2, this is pi, this is 3 pi by 2, 2 pi.

So, similarly, if you just go from here, sine would look something like this. And then, it comes to minus 1, and comes back here, and then goes like this. So, sine with typical sine curve will look like this. So, this will be very different from… So, sine wave will look like this and square wave is very different. So, how do we make this sine to a square or a cosine to a square or a combination of sine and cosine to a square? That is the question. So, we will use the same idea. So, we use this particular idea here that it can. This, if you write in this particular way a n, b n and a 0 can be found out by this formulae. So, let us define what f of x in our case is.

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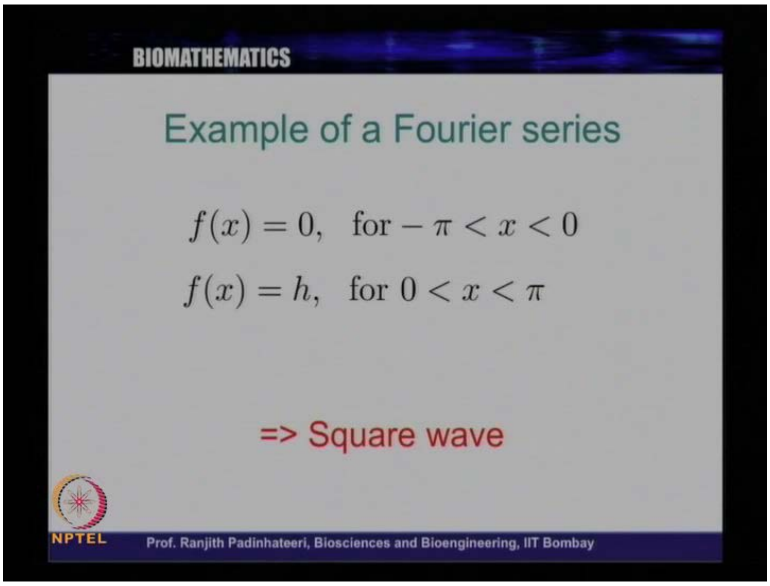

So, for a square wave, first it is between minus pi and 0. When x is between minus pi and 0, f of x is 0. So, that is what it is here. That is what it is going to be here. Between, minus pi and 0, f of x is 0. And, it is h between 0 and pi. It is h, this particular value between 0 and pi. So, f of x is equal to h between 0 and pi. So, between minus pi and 0, it is 0; between 0 and pi it is h. h can be any value. It could be 1, 2, 3 and 4. But, for simplicity, I will take 1 here. But, you could take any particular value. So, f of x has some particular value 1 or 2 or 3, which is h for some interval, half of the interval and the other half it is 0. So, this can be called as square wave. So, please plot this yourself and see, as I realize this is needed square wave.

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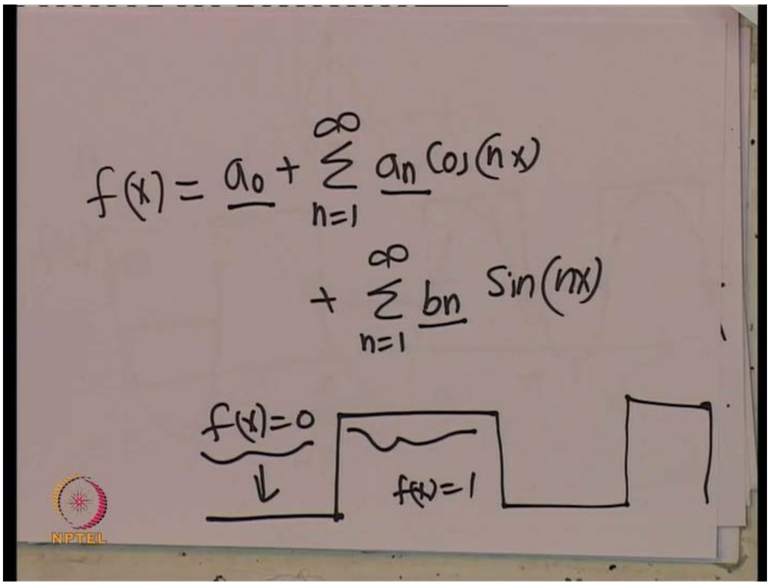

Now, if you know this f of x, as we defined previously we can find out a n and b n. And, if you know a n and b n we can write f of x as a 0 plus sum over n is equal to 1 to infinity a n cos n x plus, sum over n is equal to 1 to infinity b n sine n x.

So, if you know a n and if you know b n and if you know a 0, we can write the f of x. So, we know f of x now. Because we just said that, f of x is 0 in this regime. And then, it becomes 1. So, in this regime f of x is 0 and in this regime f of x is 1 or h. Since it is again 0, then becomes 1, so this is your square wave. So, square wave will have this particular definition for f of x. So, if you know f of x, now what we want to do? We want to calculate the coefficients. So, we want to calculate the coefficients. So, the way to calculate the coefficient is... We, let us, look at this definition once more here. So, there according to this definition here, a n is given by 1 by pi minus pi to pi f of x cos n x d x.

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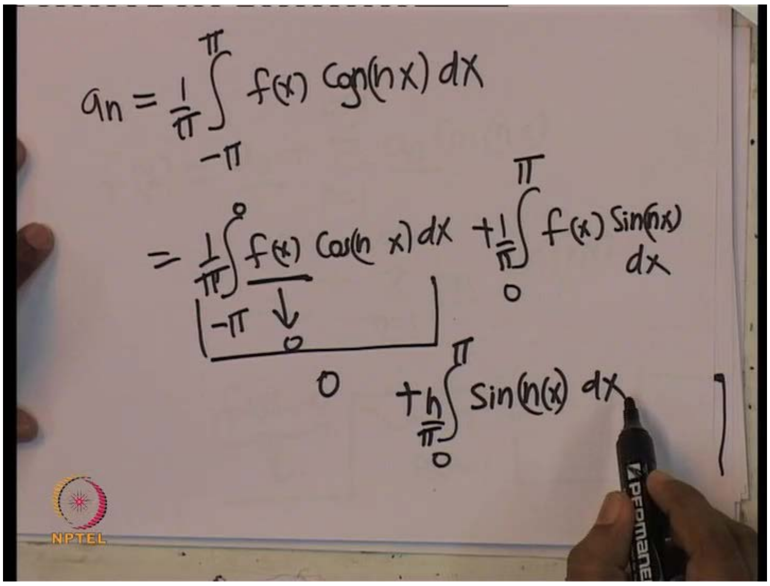

So, we have a n is equal to minus pi to pi f of x cos n x dx. But, f of x is 0 between… So, this I can write in two parts that is, first minus pi to 0 f of x cos n x d x. This is the first part. Plus 0 to pi f of x sine n x d x, this is the second part. And, between minus pi and 0 we know that f of x is 0. So, this whole term becomes 0, plus between 0 and pi we know that f of x is a constant, let us call it h, which again take outside; so, h into sine n x d x. So, what is 0 to pi sine n x d x? So, there is overall the 1 over pi here. So, there is an overall 1 over pi everywhere. So there is a 1 over pi here. So, there is a h over pi in to 0 to pi sin n x d x. So, what is 0 to pi? So, what we want to find out?

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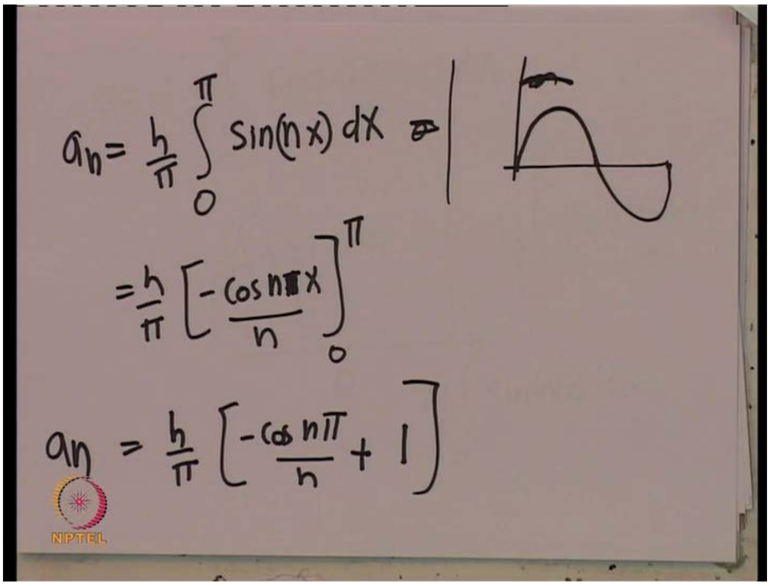

What we want to find out is integral h by pi into 0 to pi sine n x d x. This is here a n. Now, what is this? Sine n x, if you plot sine 0 is 0. So, between 0 and pi, it looks like this. And, minus, so, between 0 to pi, it has this particular value. So, this is h by pi into, integral of sine is minus cos, so minus cos n pi by n x by n in the limit 0 to pi. So, this is h by pi into minus cos n pi by n minus cos 0 is 1. And, there is a minus plus and it is 1. So, this is the answer you will get for a n.

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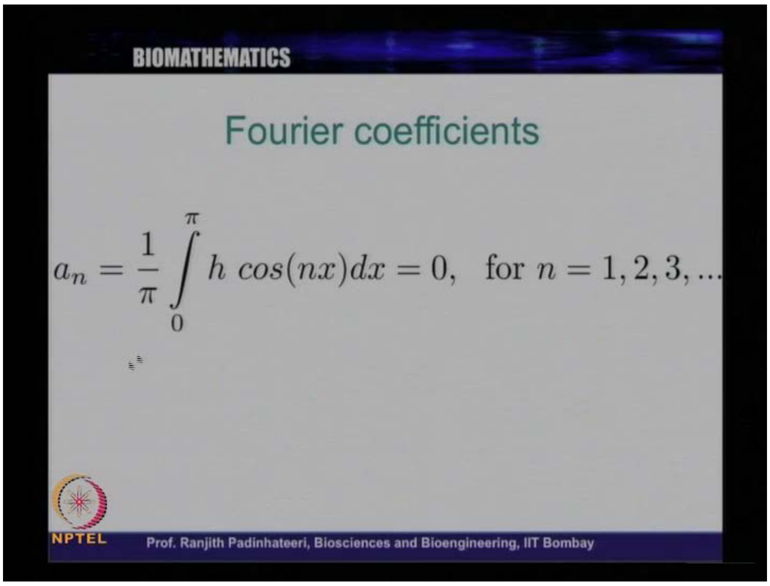

So, a n is h by pi 1 minus cos n pi by n. So, let us look what is the answer, which we get here. So, a n is essentially what I wrote here; 1 over pi 0 to pi h cos n x d x. sorry. This is b n. sorry. What I got here is b n because the sine definition is actually for b n. So, what we, this is actually b n. So, there is a minor error which came here. So, let us redo this calculation. Let us redo this quickly. So, according to our definition, let us do this. So, let us go back and check once more because when we made a minor mistake and we rewrote it.

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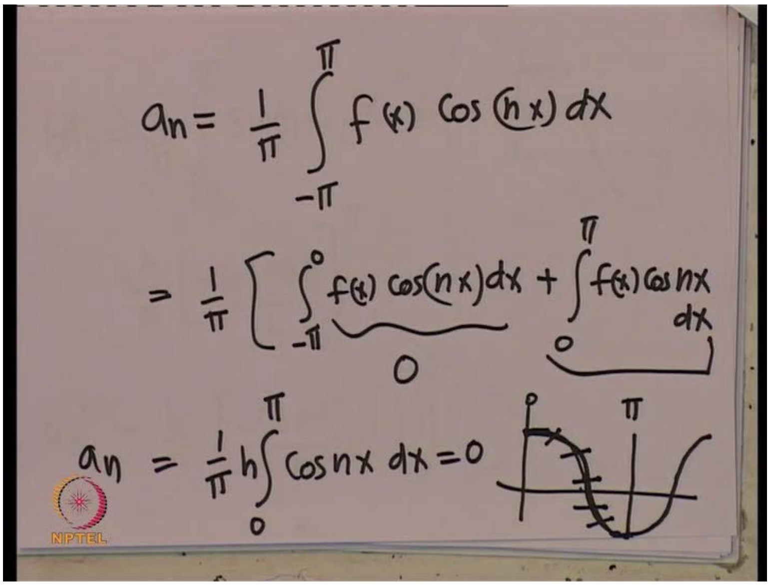

So, the correct answer is, a n is 1 over pi integral minus pi to plus pi f of x cos nx dx. Now, we know that f of x is 0 between minus pi and 0. So, this whole thing can be written as 1 over pi into, as we said minus pi to 0 f of x cos nx dx plus, 0 to pi f of x cos n x d x.

So, this part f of x between minus pi and 0, this f of x is 0. So, the whole term is 0. And, the only term which is this, which is 1 over pi into integral 0 to pi f of x. f of x in the 0 to pi is a constant. So, let me call that h, so that h can be taken out. So, this is h into cos n x d x.

Now, what is cos n x for any value of n? It will be between 0 and pi. So, let us plot this for n equal to 1. So, cos 0 is 1, so it will just, cos typically goes like this. So, this is pi. So, between 0 and there is a positive part and there is an equal negative part. So, positive part and negative part will totally give you a 0. So, the sum between 0 and pi, this is pi, will be sum of positive and negative same numbers. So, there is a, for any negative value there is a positive value also, for each value. So, for there is a plus 1 there is a minus 1, there is for each positive value, there is a negative value. So, 0 to pi cos n x d x is essentially, 0; so, a n is essentially 0, a n is 0. So, that is what we find. So, this is what written here. So, a n terms turn out to be equal to 0 for n is 1, 2, 3 and all that.

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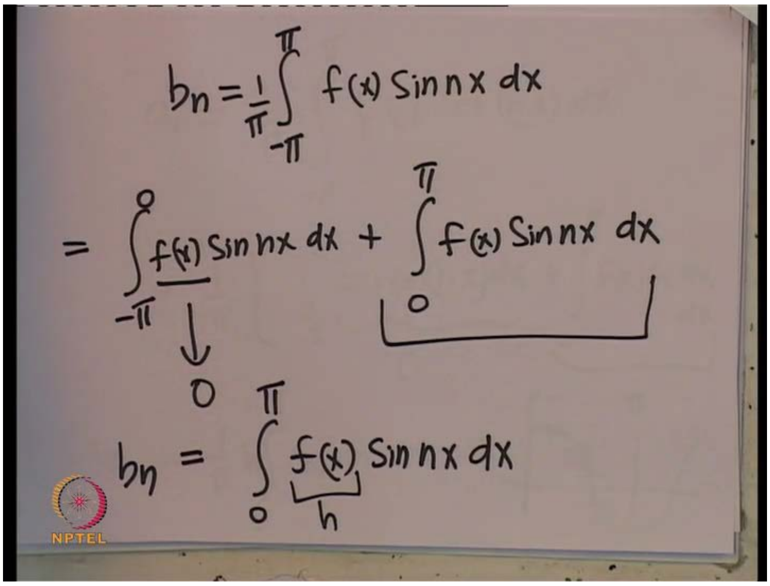

What is now b n? So, the definition of the b n is again, b n is equal to integral 1 over pi minus pi to plus pi f of x sine n x d x. As we said, saw before, this can be written as minus pi to 0 f of x sine n x d x plus, 0 to pi f of x sin n x d x. So, f of x between minus pi and plus pi, this f of x is 0. So, this whole thing is 0. So, then what you have is this. So, this is, b n is essentially is equal to 0 to pi f of x sine n x d x. So, now what is 0 to pi f of x sine n x? So, f of x is essentially a constant. So, let me call this is just as a constant h. So, I can take this h outside.

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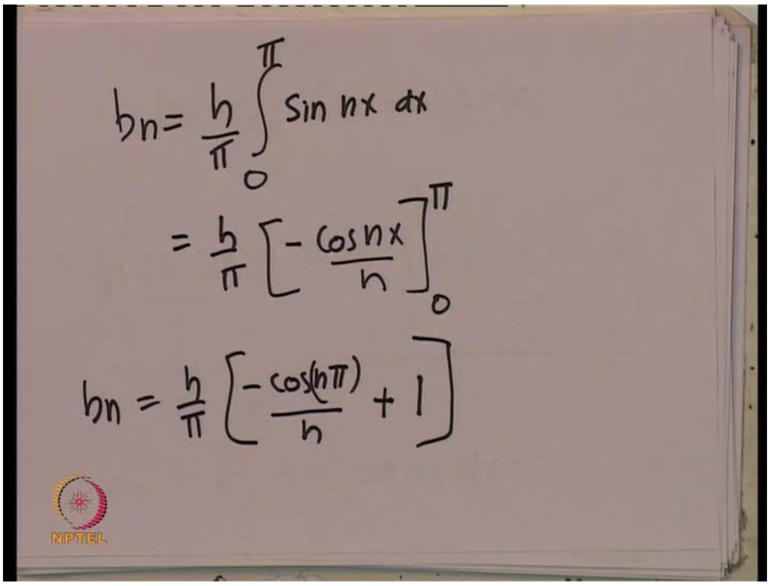

So, what you have is essentially is b n. This is what we said before is. It is h 0 to pi. There is a h by pi here; there is a 1 over pi here throughout. So, there is a 1 over pi. You carefully do this. There is 1 over pi, which comes here and becomes h over pi, and then there is 0 to pi sine n x d x. So, what does integral of sine n x d x, which is minus cos n x by n and this limit, 0 to pi.

So, at pi, now what is this? This is h by pi into minus cos n pi by n minus. And, there is a minus sine again and this is plus and 0 cos n 0 is just 1. So, this basically b n tells you about 1 minus h by pi into 1 minus cos n pi by n.

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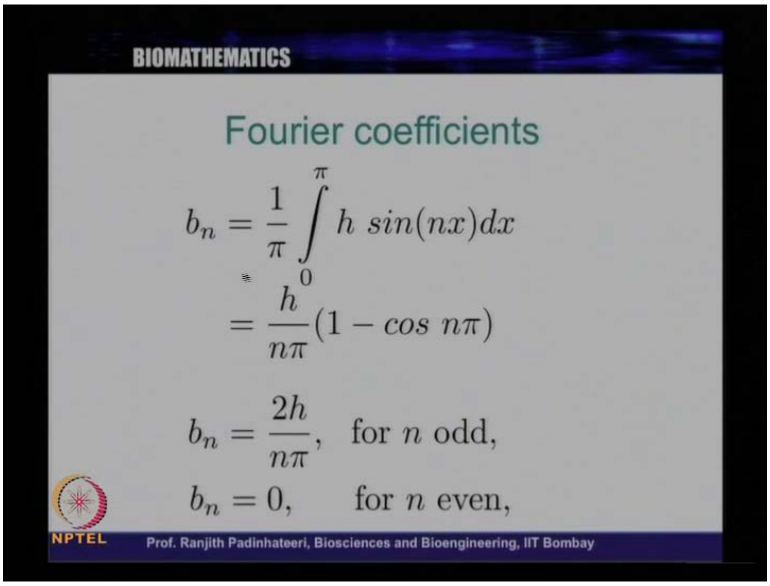

So, that is what is written here. b n, if you do this carefully as we just did it. The definition is 1 over pi f of x sine n x d x. And, f of x is just a constant h. You will get h by n pi into 1 minus cos n pi. So, that is what you will get.

So, now you know that for cos n pi for even n; when n is even, that is cos 2 pi, cos 4 pi cos 0, cos 2 pi, 4 pi, 6 pi. They are all zeros. Sorry. They are 1, so cos 0 is 1, cos 2 pi is 1, cos 4 pi is 1. So, cos n pi, when n is even number 0 to 4, this is 1. So, this is 1 minus 1 is 0. So, b n is equal to 0 for n, when n is even. For n even, b n is 0; for n odd, b n is 2 h by n pi because when n is odd, this is essentially minus 1 cos n pi is minus 1 for n odd. So, 1 minus, minus 1 is 2. So, 1 minus 1 is 2, so this is 2 h by n pi. So, b n is 2 h by n pi, if n is odd and b n is 0, when n is even. So, that is the case here.

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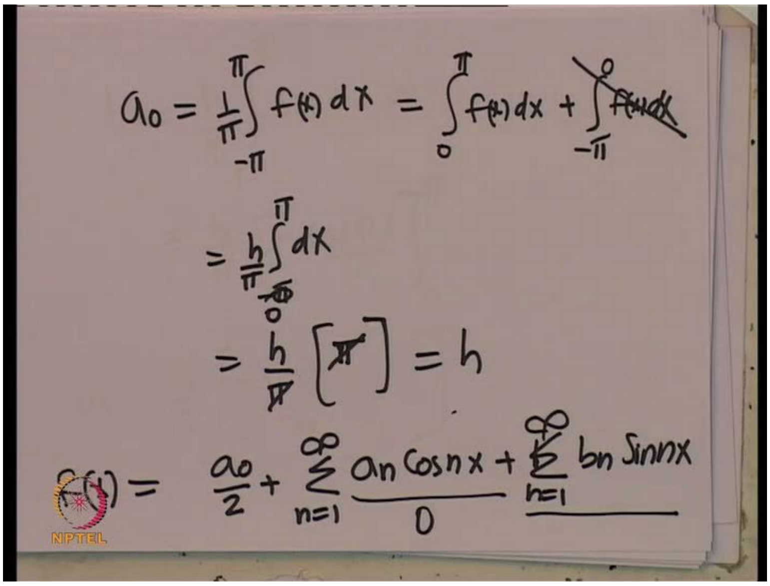

Now, what we want? We want a 0. What is a 0? So, a 0 is just minus pi to pi f of x d x. So, f of x is a constant. So, minus pi to pi in this case it is h. So, this is just simple. This so d x is x and this is h into pi minus, minus pi is like 2 pi. So, this is essentially, there is a 1 over pi here. So, there is a 1 over pi everywhere.

So, there is a 1 over pi here. So, there is a pi h by pi in to pi. So, essentially pi, so this is again only from 0 to pi. So, this is, this can be again written as 0 to pi f of x d x plus, minus pi to 0 f of x d. And, this is 0 because minus pi to pi f of x d x as we just saw is 0. So, this is only from 0 to pi. So, essentially this will come as pi. So, this and this pi cancels and answer is h. So, a 0 is h.

And, our series is f of x is a 0 by 2 plus, sum over n is equal to 1 to infinity a n cos n x plus, sum over n is equal to 1 to infinity b n sine n x. So, what did we find? We found that a 0 is just h. So, a 0 is first term is just h by 2. One term is h by 2. We found that, a n is 0, so the whole thing is 0. And, b n is, we just saw that, b n is 2 h by only for n equal to odd. It is non-zero value for all even value it just a zero. So, this is again the sum. This particular term b n is only non-zero for odd values of n. So, that means when n is equal to 1, 3, 5, and this is non-zero. Everywhere it is 0.

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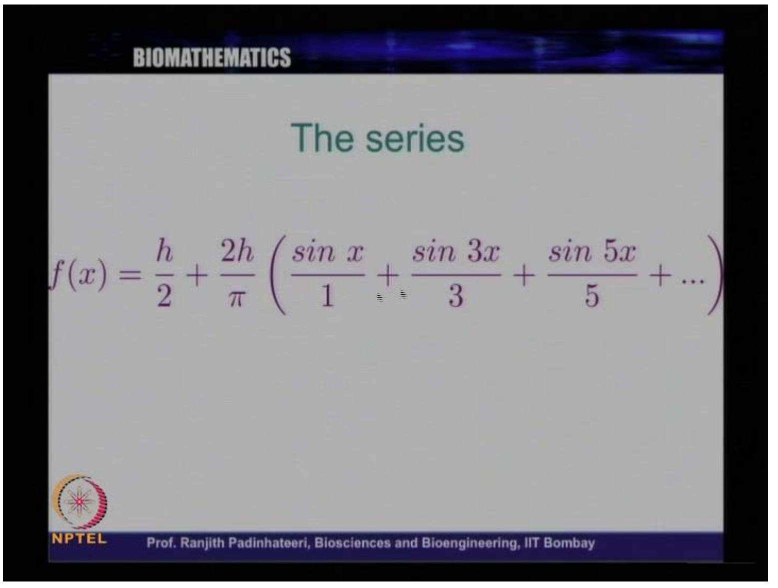

So, essentially what happens is that, you end up. If you do this carefully, you end up with this particular formula. The series for a square wave, sorry, is that f of x is equal to h by 2, which is a 0 by 2 times 2 h by pi into sine 1 x by 1 plus, sine 3 x by 3 plus, sine 5 x by 5 plus, dot, dot, dot, so on and so forth.

So, what does this? This is, a is 0. So, this is b. So, b is, b was basically, 2 h 2 by n pi. So, 2 by pi is taken out. And, 1 over n is here is this 1, 3, and 5. And, we found that this is only true for odd values. So, you only have odd values here.

So, if you apply all these together, you end up this series. So, this should represent a square wave; h by 2 plus 2 h by pi into sine x by 1, plus sine 3 x by 3, plus sine 5 x by 5, plus so on and so forth. It should give you like a square wave. Do you think it will give a square wave? Let us check. So, let us for simplicity, what is that f of x is just h by, the first time alone if you look at here, the first time alone is one term is just h by 2. So, h by 2 is anyway we know that, if we just plot h by 2 f of x, there is no surely a square wave. h by 2 is just a line.

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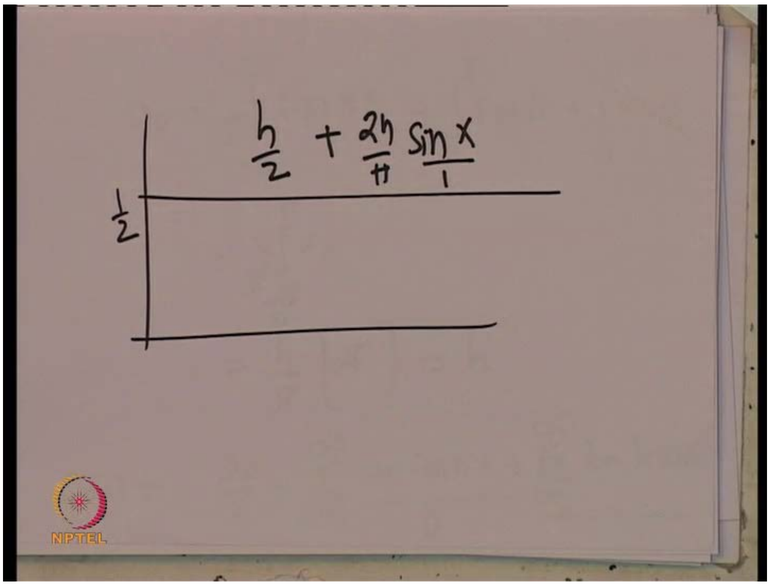

So, let us take h as 1. This is just line in passing through half. This is h by 2. So, first term alone will not give anything. So, this first term in the sine series, can it give you something? So, let us look at this first term that is, sine x by 1. Can it give anything? So, let us look at, let us, so, this is just h by 2. Now, add the first term which is 2 h by pi into sine x by 1, what is that look like? So, that is what is shown here.

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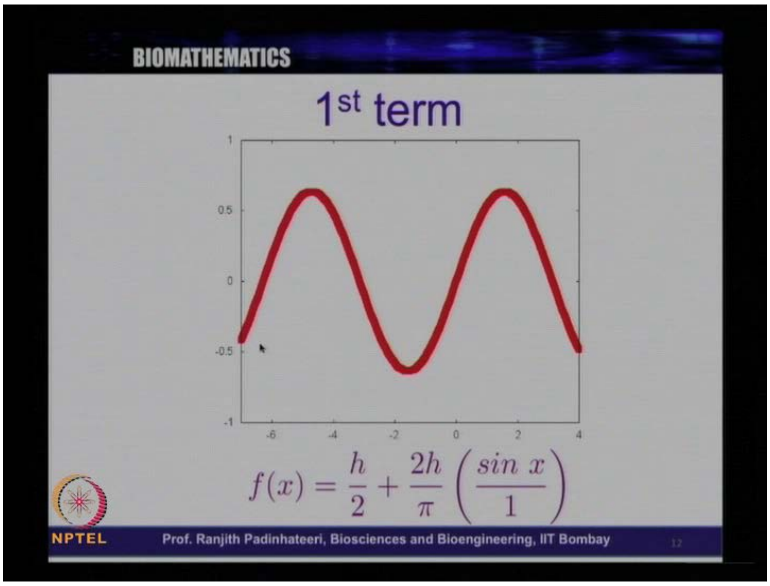

If you look at the first term alone, what you have is that, h by 2 plus 2 h by pi in to sine 1 x by 1. So, that is the first term. And, you can see that if you plot this, you pretty much get a sine wave because sine plus some constant. So, just a bit of shifted sine wave, shifted by a small value may be. So, this is just a shifted sine wave. As you can see, this is just a sine wave shifted from some particular value to some other value. So, you can see this here. So, first term alone, this is not surely like a square wave.

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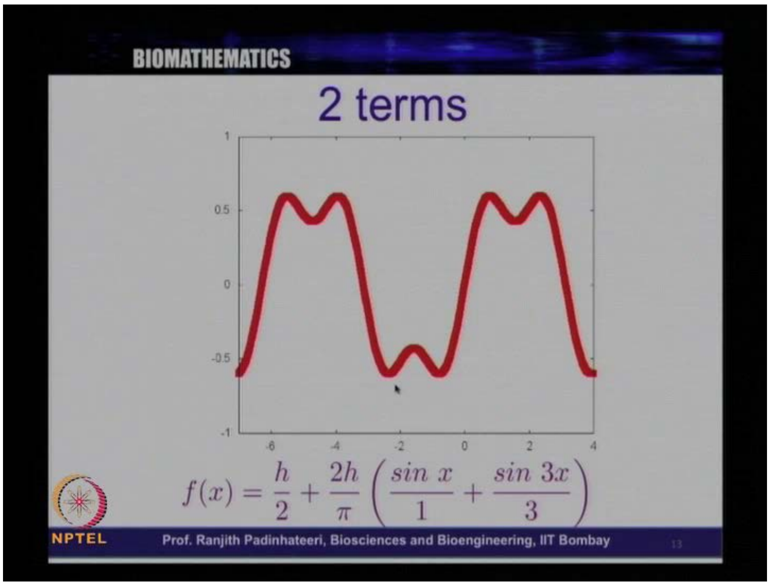

Now, let us look at another term, which is a second term. Add one more term to this. So, if you add one more term, what happens? h by 2 plus, 2 h b y pi plus, sine x by 1 plus, sine 3 x by 3, so, this is the new term that we added. It is straight quite a bit, but surely it is not a square wave. But, it is slowly is just some shift has happened. So, we added two terms. So, this is this term plus, this term plus of course, this term is a constant. We are not considering.

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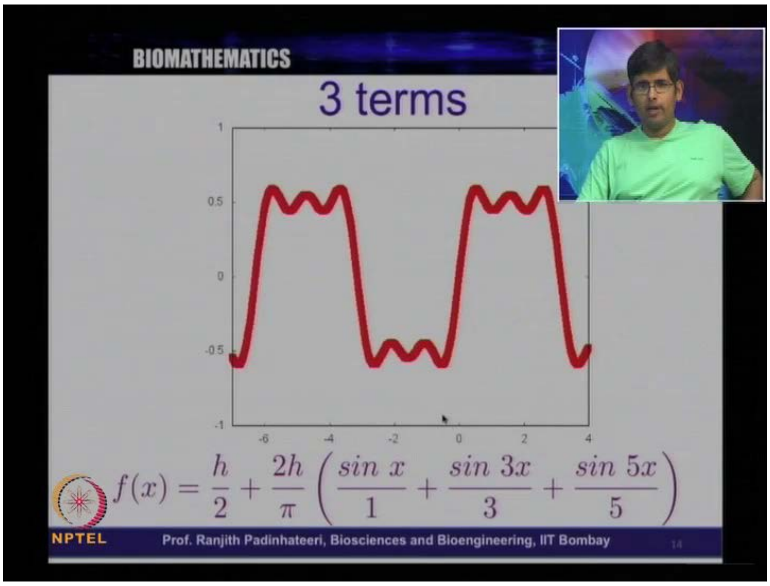

So, there are two sine terms essentially. Now, let us add one more term, three terms in the series. It is slightly better. So, you can see just coming. This shape of a square is about to come. It is just starting to come. Even after just three terms, you can see that the square is begun to emerge, even though it is far from a square. So, the f of x of this is essentially h by 2 plus 2 h by pi plus, sine 1 x by 1 plus, sine 3 x by 3 plus, sine 5 x by 5.

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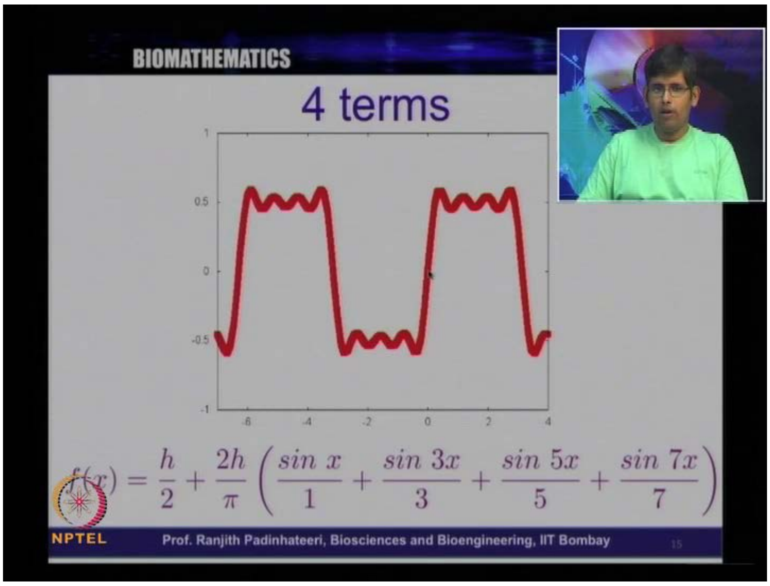

So, you can see that this interesting combination of sines, gives you very interesting like a function, which is not very easy to intuitively think that you will get this. Add one more. Sine 1 x by 1 plus is sine 3 x by 3 plus, sine 5 x by 5 plus, sine 7 x by 7; there are four terms, then you get a better, it is more approaching towards a square.

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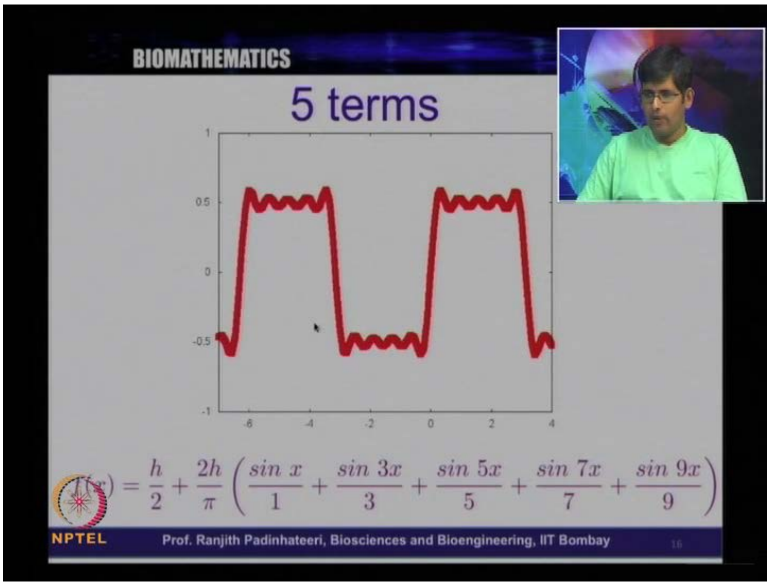

So, there are four terms and is somewhat is actually nearer to square. And then, you add one more term and you have something, which is little closer to a square. So, look at this sine 1 x by 1, there is sine 3 x by 3, there is sine 5 x by 5, sine 7 x by 7, sine 9 x by 9. So, essentially you have one, two, three, four, five terms inside this bracket; essentially, five plus one constant term. So, essentially five terms, which is x dependent terms and that gives somewhat closer to a square. But, actually it is far from a square. However this seems to look like, it is very far from sine, which is more like a square wave.

So, now if you add more and more terms, as this goes nine as you add eleven, thirteen, fifteen, seventeen and as we go to infinity, this will become a perfect square wave. So, that is the idea. But, it was intuitively, before we write down this principles, would not think that by just adding some set of sine, you will get a square.

This is very difficult to think of. And, given that sine and sine can be made to a square or a combination of sine and cos can be made square, rectangle, and saw-tooth, any shape can be constructed. Any periodic function can be constructed as a combination of sine and cosine.

Thus, so essentially that is the message this gives. We just saw that there are with five terms. You get pretty good decent square as you see here. So, if you have five terms, you get pretty decent square looking. This is somewhat looks like a square wave. So, we need at least five terms. And, if you add more and more terms, it becomes like a perfect square wave.

So, now let us see a movie of how more and more terms makes a better square. So, this is we are moving a one term, two terms, three terms, four terms, five terms. So, one term, two term, three terms, four terms, and five terms. So, one term, two terms, three terms, four terms, and five terms.

So, when you have five terms as you can see in this movie, just let us to see the movie here. So, as you can see in this movie the more and more terms, it becomes more and more squares. So, the movie is repeating. Let us watch it for a minute; so, one, two, three, four and five. So, the more and more terms, it becomes more and more square shape.

So, this is the essentially, how the Fourier series, the infinite series, the more and more terms you add, it becomes like a square wave. So, essentially you can write, make this square wave by summing sine 1 x by x plus, sine 2 x by 2 x plus, sine 3 x by 3 x plus, so on and so forth. Sorry. Sine x by x plus, sine 3 x by x 3 x plus, sine 5 x by 5 x 5 plus, essentially sorry for the…

So, this is essentially the series sine x by 1 plus, sine 3 x by 3 plus, sine 5 x by 5 plus, sine 7 x by 7 plus, sine 9 x by 9 plus, sine 11 x by 11 plus, sine 13 x by 13 and so on and so forth, will generate a better square wave, a good square wave.

Ok. So, we saw that saw-tooth wave and square wave, how we generate. Now, any function you can generate in this particular manner. And, one of the properties those we… So, as we said, we have this square wave as we generate from the series of sines. And, one of the properties that we used to write this Fourier transform, important property is this property is that we saw. That, if you multiply sine m x and sine n x and integrate, you get 0 or 1. Delta function something out chronicle delta, which we discussed. And, cos and cos you get again chronicle delta, and cos and sine you get this.

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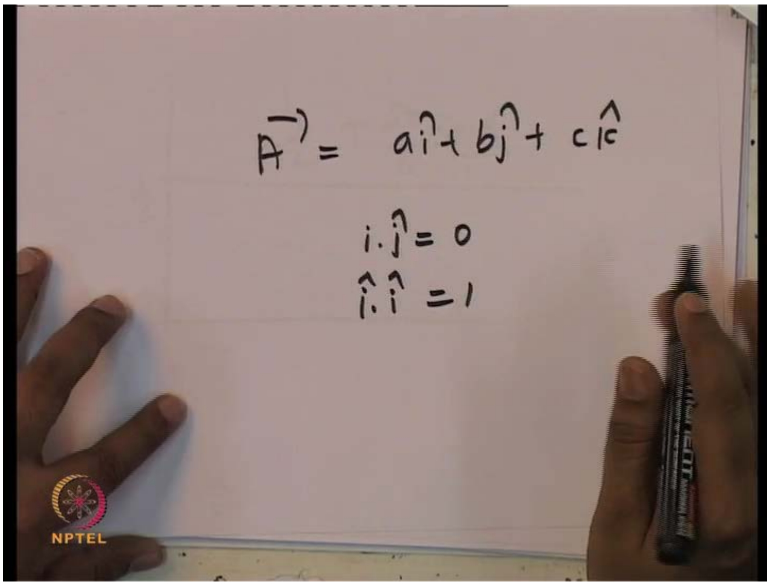

So, this kind of a property, wherever this we saw that, for when we also saw that, we also briefly said that whenever you have, whenever you write a vector a as a i plus b j plus c k. this i dot j is 0; i dot i is 1.

So, this kind of orthogonal relationship is needed to write this. Expand this in a series, so, but there is an interesting another set of functions like, which is related to cos and sines also can be used to expand the Fourier series. This is all this orthogonal properties.

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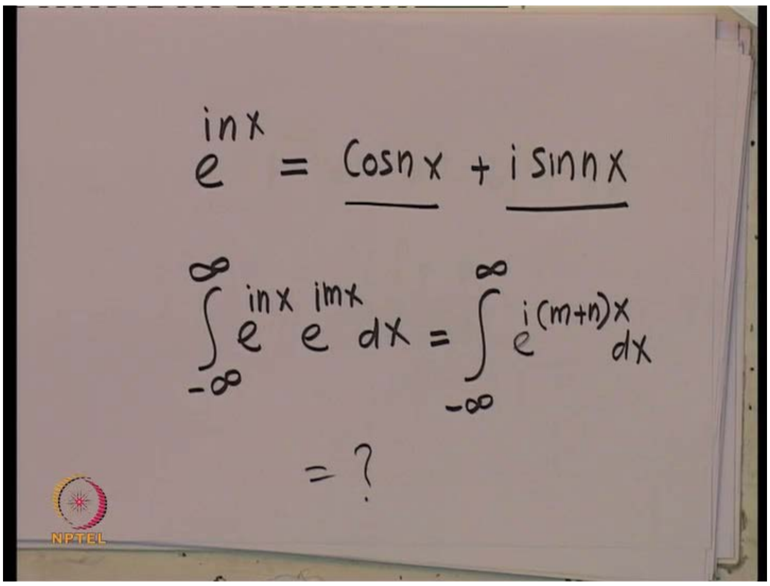

So, you know that, you can read e power i n x. You might know that this can be written as cos n x plus i sin n x. So, cos x plus i sin n x is e power i n x. And, integral e power i n x into e power i m x d x, minus infinity to infinity. This is basically, integral minus infinity to infinity n plus m. So, this is e power i into m plus n into x d x. So, what is this? What is the answer to this? What is an integral of integral e power i into some constant time x?

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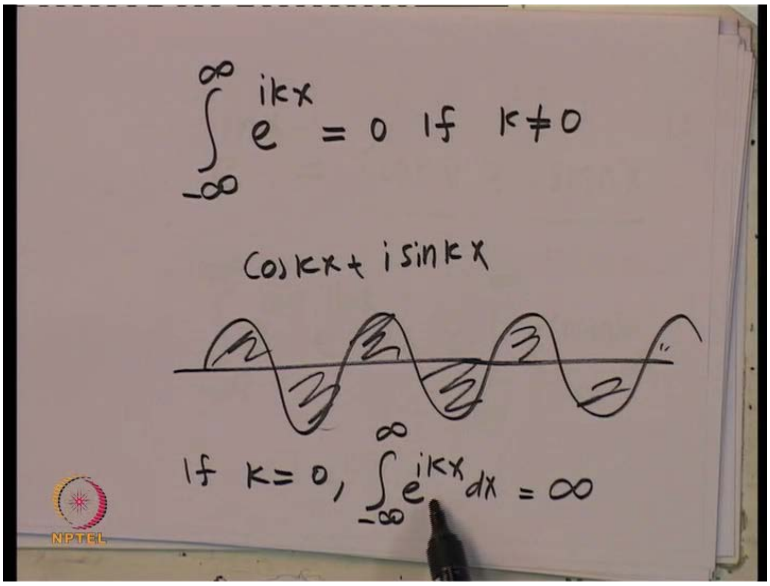

So, it turns out that integral minus infinity to infinity e power i k x; where k is some number, which is m plus n. It can be written as 0 if k is not 0. If k is not 0, e power i k x can be written as cos k x plus i sin k x and as you know, cos and sine will be the periodic functions like this. And, if you go from minus infinity to infinity, the sum area under this will be 0 because this a and this is positive part, there is a negative part, positive part, negative part. So, each, whereas for each positive part there will be a negative part. So, minus infinity to infinity, this cos and sine will be the positive part. And, negative part will cancel each other. So, this will give you 0.

So, whenever there is a k, non-zero k, it is like a wave. And, whenever is the wave minus in periodic wave, minus infinity to infinity the integral will be 0.So, this is 0 if k is 0 and this is not 0 if k is 0. If k is 0, integral e power i k x minus infinity to infinity. This basically, e power i and this is 1 and this is d x.

So, this is basically, essentially you will end up as infinity. So, this is 0 if k 0, infinity if sorry, 0 if k is not 0, e integral of e power i k x is 0, if k 0. Integral of e power i k x is infinity, if k equal to 0. If k is equal to 0, essentially integral d x. Integral d x is just x. And, x at infinity is infinity and infinity minus infinity, essentially two infinities, which is essentially infinity itself. So, this is infinity.

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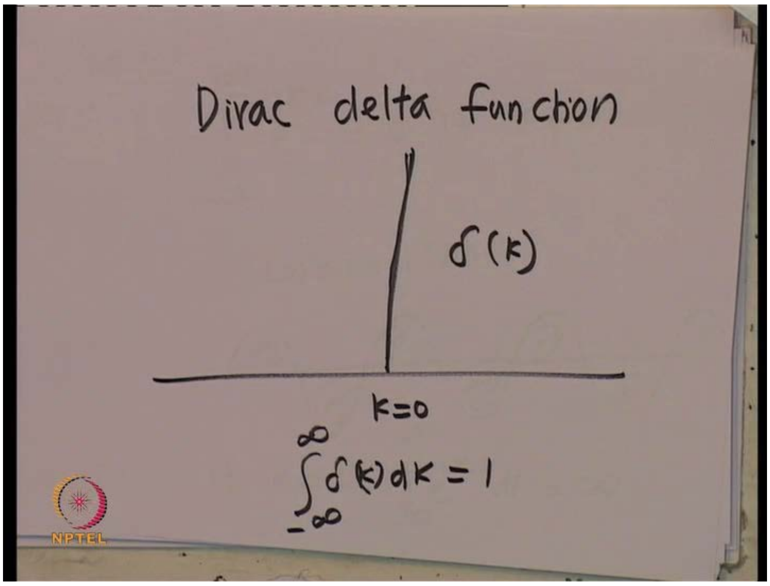

So, such property, such function can be written as something called the Dirac delta function. Dirac delta function is the function, which is infinity only at k equal to 0. And, for all values of k, so this is, I can write delta of k. It is written as delta of k. Delta of k has a value only at k equal to 0 is infinity and everywhere else it is 0.

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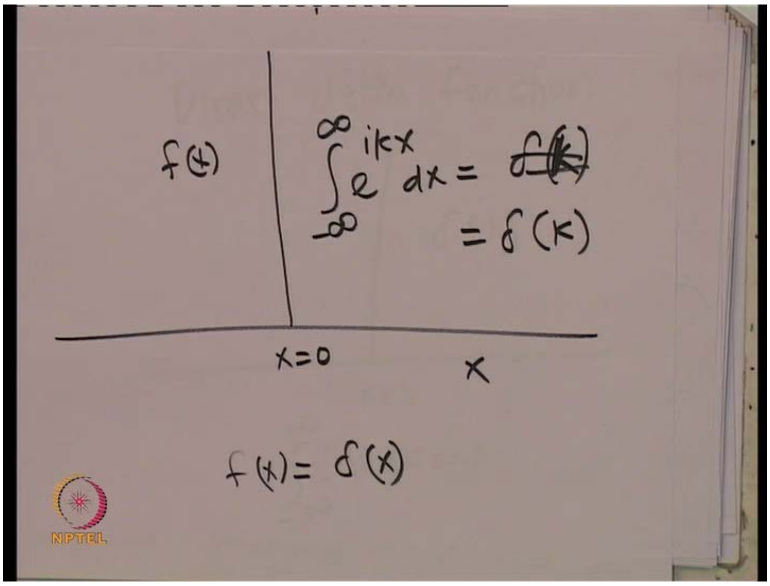

And, integral minus infinity to infinity, delta of k d k is 1. So, essentially any function, which has only value at a just one point and everywhere else, is 0. So, this is f of x and this is x. Only at x equal to 0 it is a value; everywhere else it is 0. Then, this function f of x can be called as a Dirac delta function. This is delta of x.

So, what we are saying is that integral minus infinity to infinity e power i k x d x is essentially at delta of x, sorry, delta of k; when k is, so this is delta of k. When k is 0, this is infinity; when delta is k is non-zero, it will have value, it will be 0. So, this is of, so, a similarly, so, this is somewhat similar to what case is here? Somewhat very similar to what we have here.

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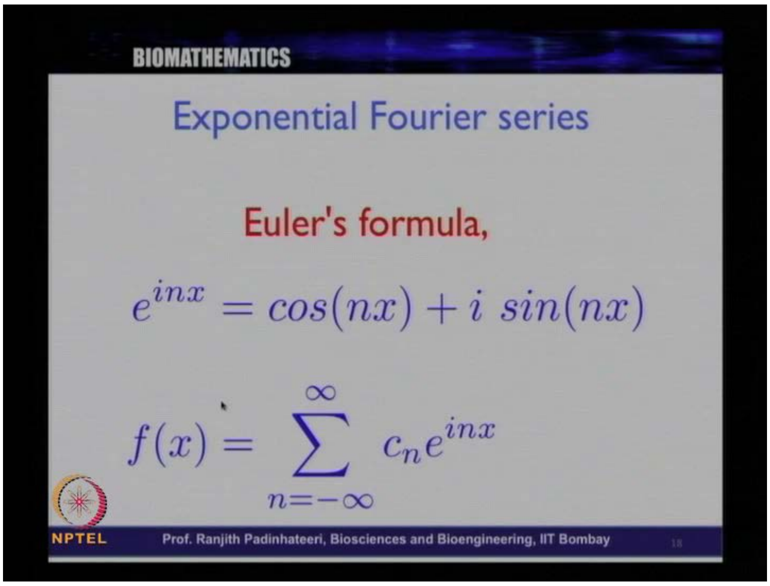

So, based on this property one can write f of x, as I can expand f of x in terms of e power i n x also. So, what we said previously is that, one can expand any periodic function in terms of sines and cosines. Here, we are saying little more. We are saying a periodic function f of x can be expanded in terms of e power i n x with some coefficients c n and where n goes from minus infinity to infinity.

So, the same thing we are saying in a different way. We did not have e power cos x and sin x. If you say e power in x is also can be used to expand this. So, any x function can be written as a sum of e power i n x. So, that is what essentially this is seeing. Any function, periodic function f of x can be written as n minus infinity to infinity, sum over n minus infinity c n e power i n x.

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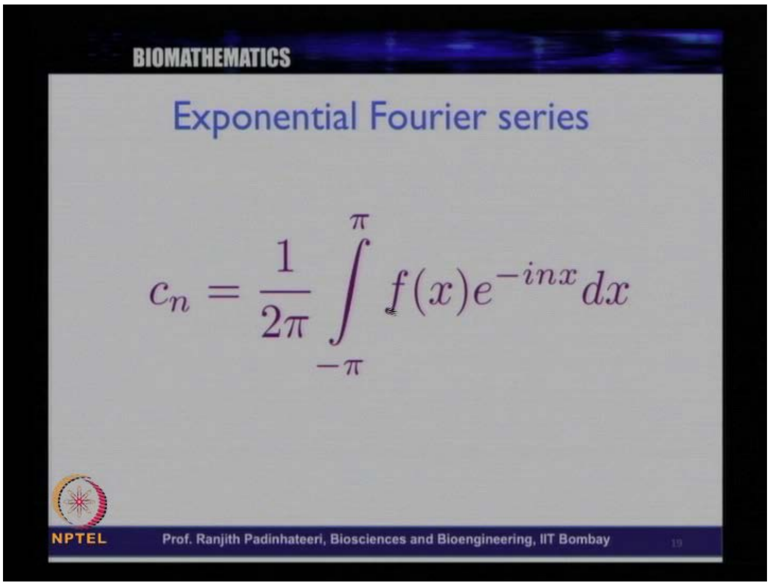

So, what, how c n to the… previously a n and b n, how are they related? They are related… c n is essentially can be obtained just like previously we used to do. f of x into e power minus i n x d x is c n. And, the c ns are related to a n and b n in this particular way.

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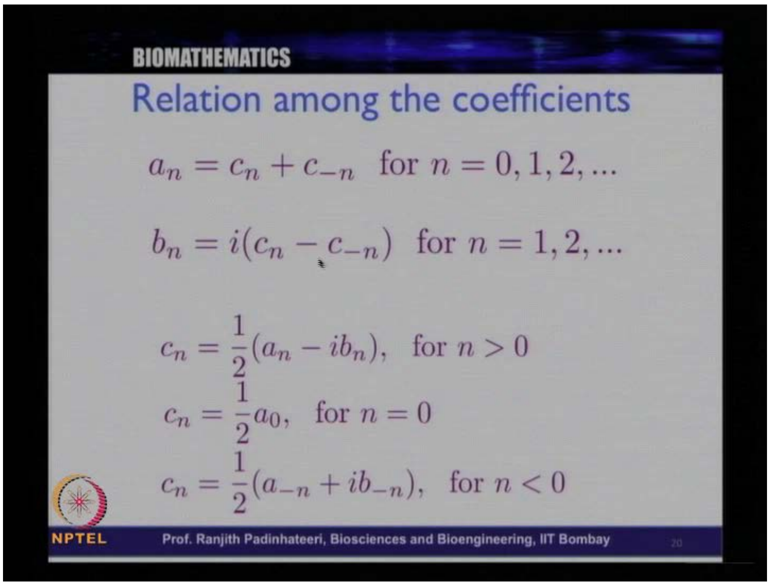

So, either you know, if you know c n again, calculate a n and b n. If you know c n, this is a, this is the relation for all a n and b n.

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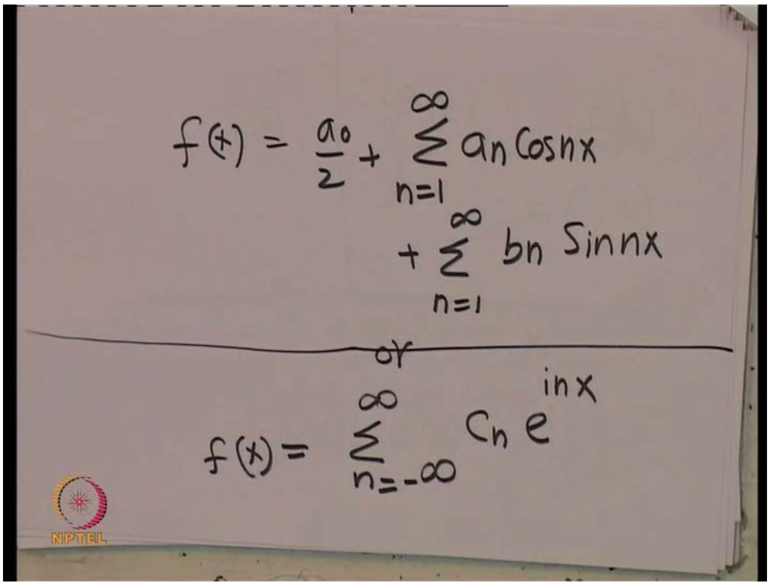

So, essentially, what we are saying here is that series, any function, any periodic function, either can be written as, f of x can be written as a 0 by 2 plus, sum over n is equal to 1 to infinity a n cos n x plus, sum over n is equal to 1 to infinity b n sin n x or f of x can be written as sum over n minus infinity to infinity c n e power i n x.

So, both are possible and you can calculate a n, b n. In that case, so this is same thing. Either of the top or bottom, you can use one of this ways. one way to write a periodic function is, expand the periodic function in terms of an infinite series and any function that we have seen in Biology pretty much, sorry, any, many function, many periodic functions that we have seen in Biology can be learnt and then understood and used using this particular series.

So, this is the techniques of Fourier series. Now, we will exchange this to something called Fourier transformation. And, this Fourier transformation is highly powerful both to understand many things and also to even to solve some differential equations. We saw that differential equation is very important in Biology.

So, this Fourier transformation is kind of an extension of this to a general case. And, somehow related to fourier series, this can be again used to understand many problems and Mathematics in general. And, essentially Mathematics related to Biology.

So, with this we will stop today's lecture. So, we discussed today another example of fourier series, square wave and the way of expanding fourier series using an exponential function e power i n x. So, with this, I will stop today's lecture. Bye.