Biomathematics

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Lecture No. # 24
Fitting a Function to Experimental Data

Hello Welcome to this lecture on Biomathematics. We have been discussing ideas from statistics that are useful to understand various biological phenomena or the various things in Biology. In this lecture, we will learn something that is pretty much used every day in laboratory. That is, if something that you all do in laboratory is or we all do in laboratory is basically, take some experimental data, you get some data points and you fit some curve. either fit a straight line or some curve. So, these days you use computers. You just give the data to a computer using an excel sheet or various other softwares. You just give a data, click somewhere, fit a line, it will just give you a nice straight line that fits to the data.

But, what is going on behind the computer? What is the principle behind this fitting? How do they fit it? So, that is the question. So, essentially we will be finding out how do we fit a function to an experimental data. So, under the statistics, our title for today is fitting a function to experimental data. So, that is essentially, what we are going to discuss.

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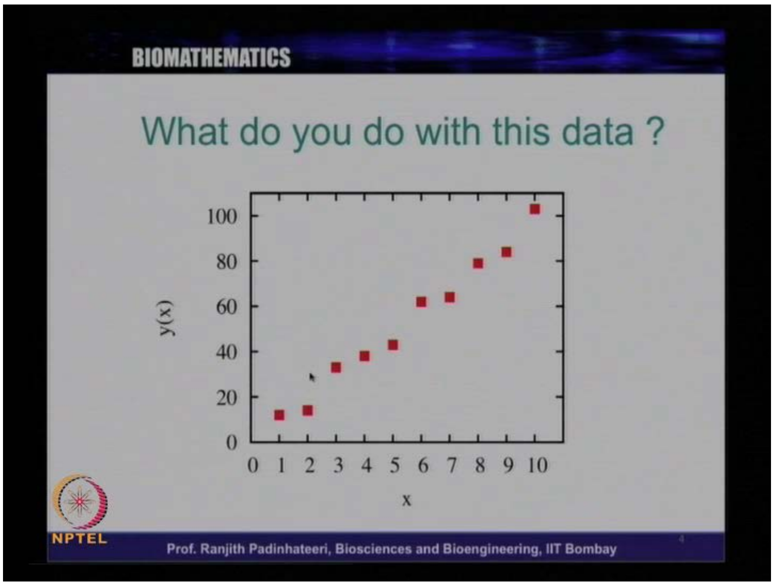

So, now let us go back and let us think about a typical situation in a lab. What happens ? You do an experiment, you get some data. So, this is something that typically you get. You get some points. So, by looking at this point, before concluding anything, like before knowing anything about this, some tendency is basically to join the data points.

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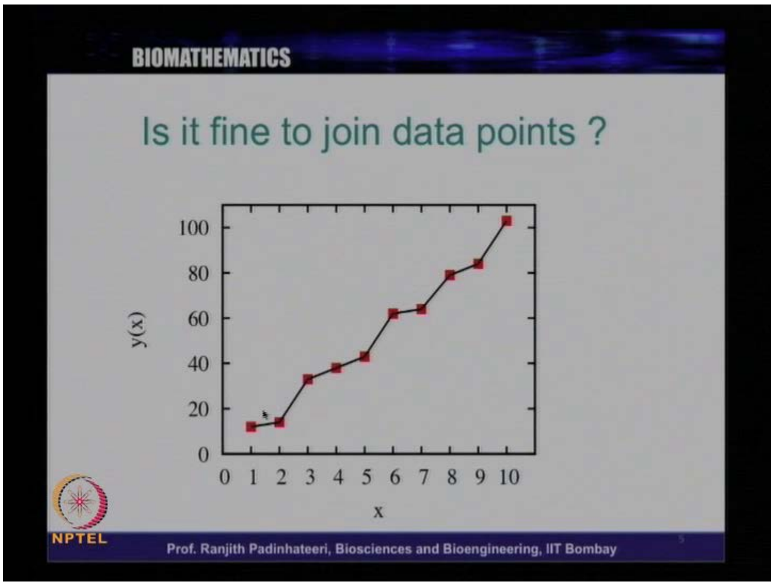

So, Is it fine to join points like this? The answer is no, It is not correct. It is incorrect to actually join the data points just with some curve like this. It does not mean anything. So, one has to know some idea about what these data points must be, to fit really. So, first of all we have to know, whether this will be a straight line or what is the function that you need to fit this particular data point.

So, for that, you have to know something about the phenomenon going on behind this experiment. What is actually going on? So, basically, when we get some data points like this, first we have to know what is going on behind this experiment and what is actually the phenomenon is. Depending on that, we can make some guesses or we can know sometime that, what is the function that this must be following? And, if you know the function, we can fit that function to this data.

So, let us take the example of polymerization of actin, which we have discussed in the cases or polymerization of any polymer for that matter, like actin, microtubule, and etcetera. There are many polymers in Biology. And, this is some simple example to imagine. This is nothing complex, either. So, I just wanted to point out that, I want to take this example because it is very easy to imagine.

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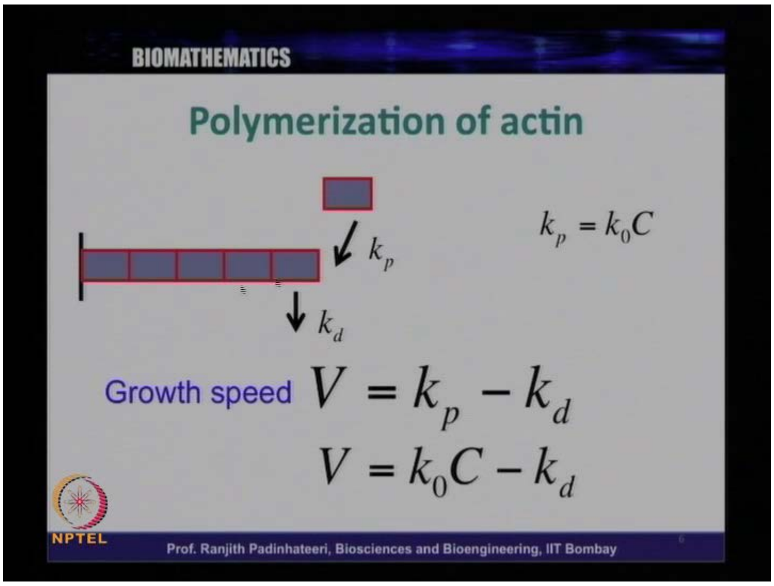

So, as we discussed some time ago, like in some lectures in Calculus, we have been discussing some simple things about polymerization of actin. So, what happens is, you have a filament and either, you can polymerize or depolymerize. So, you can polymerize with some rate K p; we can depolymerize with run rate K d. So, we have only two events polymerization and depolymerization. Now, the question is what is the speed or the velocity with which this filament will grow?

What is this growth of a speed or what is the speed with which this filament will grow or shrink? So, the growth velocity or speed can be written as, K p minus K d; what is this? This is the polymerization rate minus the depolymerization rate. Now, what is polymerization rate? Polymerization rate is actually proportional to the free monomer concentration.

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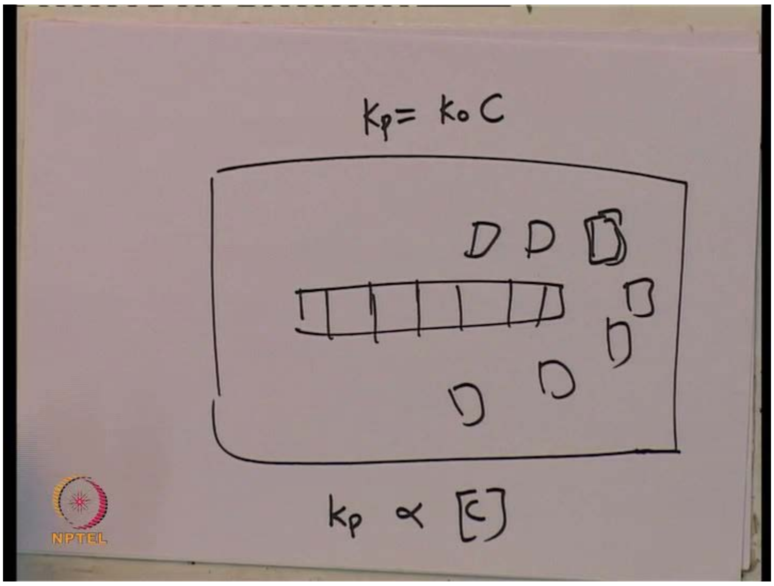

So, as we said some time ago, if you look at here, like you have a container and you have a filament and you have a lot of free monomers. If there are no monomers, this will not polymerize; if there are lots of free monomers, it will polymerize faster. So, the polymerization rate is proportional to the concentration of free monomers. So, I represent concentration as C; where C is the concentration of the free monomers. So, I can write K p the polymerization. It is some constant; K 0 times the concentration of free monomers.

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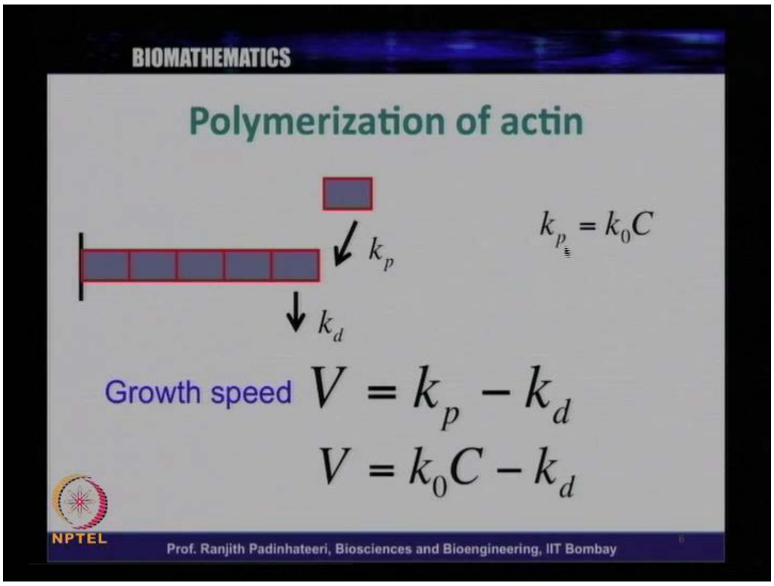

So, that is what is written here. K p is K 0 times C. So, you can search to that here. So, the velocity of the growth speed can be written as, K 0 C minus K d; where K d is the depolymerization rate. So, this is the simple formula that you can get for polymerization speed. How the polymerization speed is related to the concentration of free monomers in the solution? The more free monomers; more polymerization will happen. Now, you can plot a graph between V and C.

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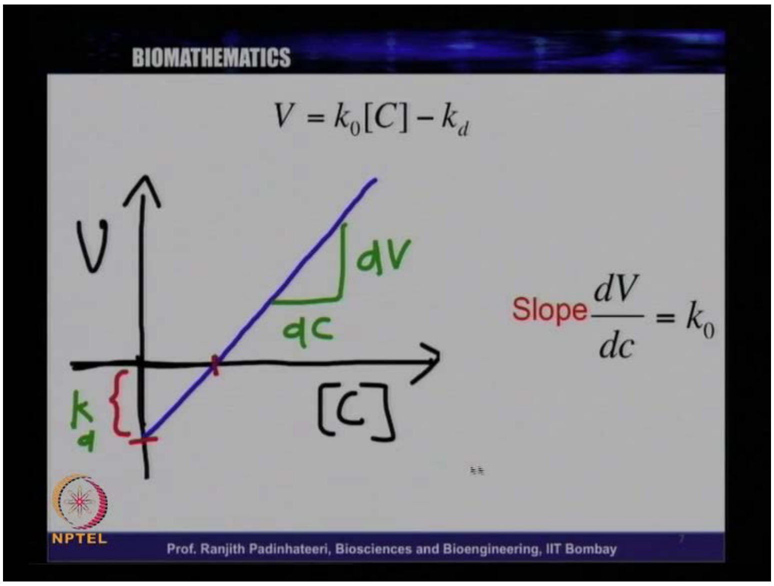

What do you expect? So, this is like y is equal to m x plus c. If you can plot V along y-axis and C along the x-axis, what you get is a line like this. So, this is, V is equal to K 0 C minus K d. So, where C is the concentration of free monomers, V is the speed you essentially expect the straight line, the slope of the straight line is the K 0, which is the intrinsic rate of polymerization and the Y-intercept will be like K d. This is something that we have already mentioned.

So, let us imagine that you are doing an experiment of polymerization of a filament, and what you are measuring is this speed of polymerization as a function of concentration. So, this is what you are measuring in the lab. Let us, imagine that you are measuring the speed of polymerization of filament as a function of free monomer concentration. And, you know that, so essentially from the Physics of header, from the Science; if you look at the Science, Kinetics is behind it. We can know that essentially the polymerization kinetics will have a straight line; if you plot V versus C, it is a straight line. So, for every value of C, the V if you plot it has to be straight because this is the formula. So, we expect this to be a straight line theoretically.

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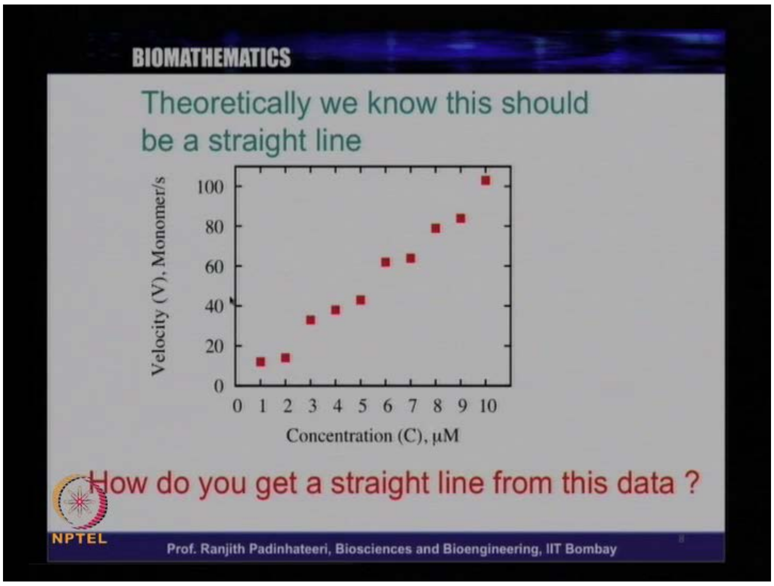

So, theoretically we know that, this should be a straight line. So, we have this particular data point and we know that, this should be a straight line. But, how you get a straight line from this data? So, what we have now is that the just data points. If we have such data points, how do we get a straight line from this?

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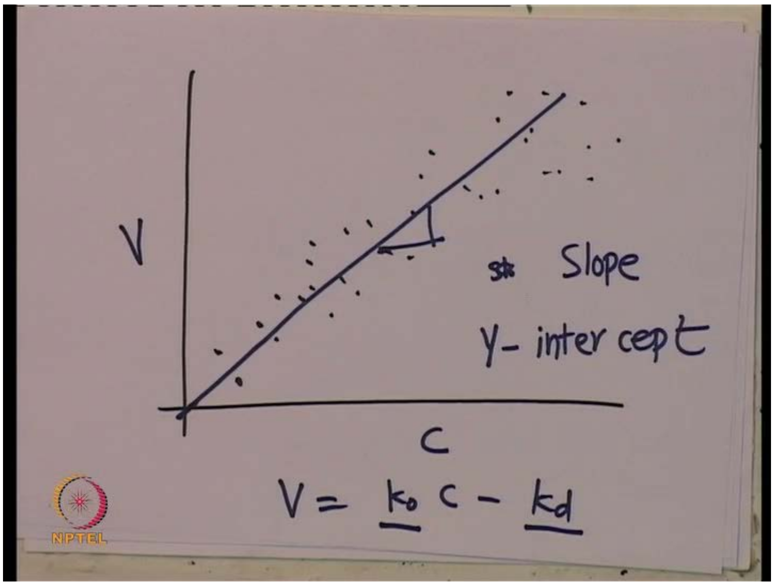

So, what we have is essentially some points, experimental data points here and there. Now from this, we have to get a nice straight line; that means you have to know something about the slope and something about the Y-intercept. So, you have to know two parameters: the slope and Y-intercept. So, we have to know two parameters to get this straight line. We have to know the slope; we have to know Y-intercept. If you know these two parameters, you can get a straight line. And, for some value of slope and some value of the Y- intercept, this will fit precisely to this experiment. It will fit very nicely to the experimental data.

So, the aim is what is software? The softwares that you use typically what they are doing? They are doing essentially, finding a straight line, finding slope and Y-intercepts such that, it nicely fits.

So, when I say nicely fits, what is that mean? So, there is something, there is a specific meaning for when I say it fits very nicely, something is fitting very nicely to the data. There is a specific meaning; there is a precise meaning mathematically. So, we will see today, what exactly we mean when we say that something is fitting precisely or something is fitting nicely or when we say best fit. So, the best fit means there are some mathematical meaning for this. So, we will come to that in a few slides.

But, at this point, I want all of you to understand that if you have a set of data and if you know what is going on behind these experiments, experiment, that is, if you know this function, whether if the data is expected to be, theoretically, if the data is expected to be a straight line or if it is a parabola; if it is an exponential curve. If you know this particular function and if you have a set of data, we can typically fit this data to this function. And, how exactly we do that? We will discuss in a minute.

But, at this moment we have this data. And, here we are taking example of an actin polymerization. So, from this example, we expect that if you plot V versus C, we expect that V is equal to K 0 C minus K d. So, we need to know two parameters K 0 and K d. If we know these two parameters, I can plot a straight line.

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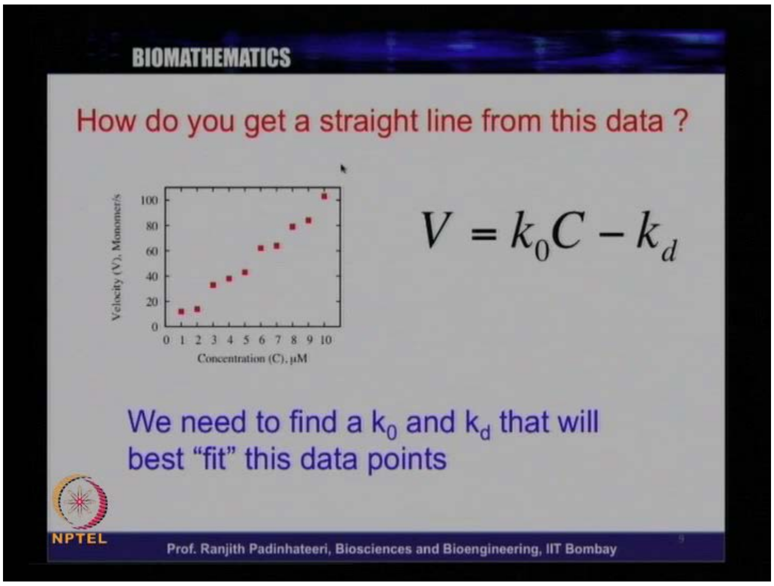

So, that is what essentially, we are saying here. To get a straight line from this data, we need to find K 0 and K d that will fit in the best way, neither in nicely fit to this data points. In an ideal case, we will expect all of them very nicely to go on top of each other in a straight line, but this is not the reality. So, there is, we have to say something was best fit. How do we define best fit? How do we define nicely fit? So, what do we expect from the common sense.

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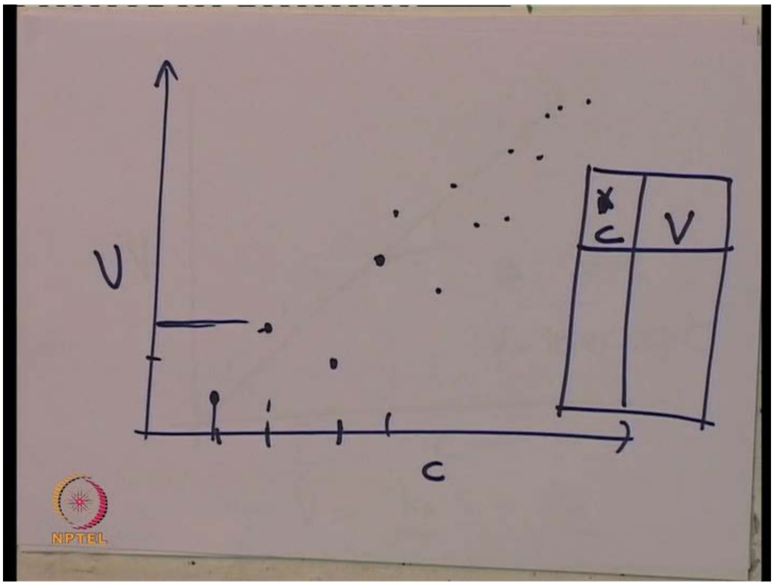

So, what we have here is that, what we have here? We have for every x value; there is a y value. So, there is a data point here for another x value, there is another data point here, so this is the x value; there is a y value. For every x value, so there is a next x value, there is another data point here; for a given x value there is a y value, there is another x value there is s data point here. So, for each x value, there is a y value. So, essentially what you have is a table x, which is the C the concentration here. And, velocity for every concentration there is a velocity value, which you get from experiments.

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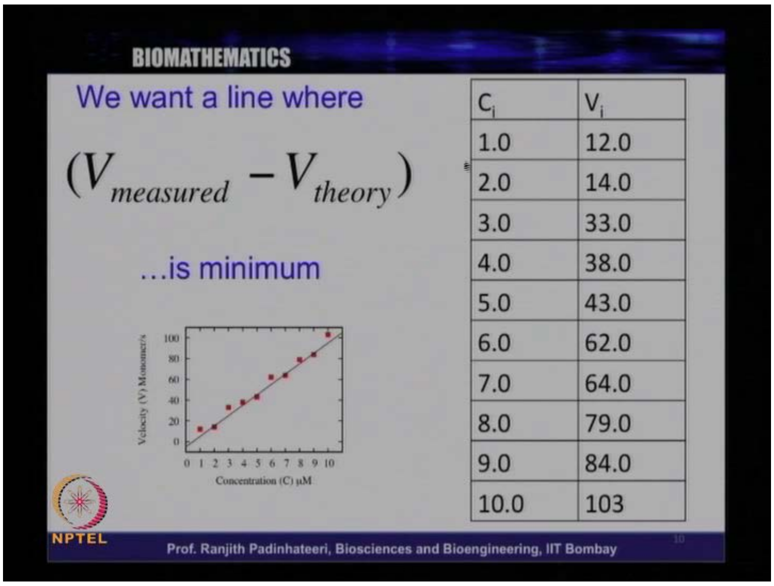

C_i	V_i
1.0	12.0
2.0	14.0
3.0	33.0
4.0	38.0
5.0	43.0
6.0	62.0
7.0	64.0
8.0	79.0
9.0	84.0
10.0	103

So, essentially what we have is a table here. The table looks like this. Look at here. So, for every experiment, this table you get from experiments. For there is a C i and a V I; there are like i is data, there are 1,2,3,4 there are 10 data points. So, i represent first data points. i is equal to three; means third data points. So, that means this particular point. So, this is first point, second point, and third point. Third data point is this. So, for every value of i, there is a V i.

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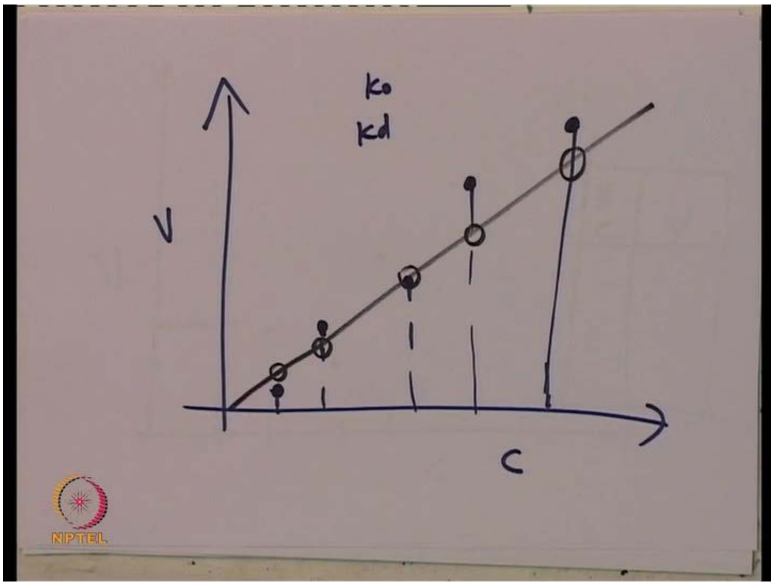

Now if you know this K d and K c, so what we have here is, for every concentration experimentally you have a velocity. So, this is one concentration value, another concentration. So, let us take a few points for each of this concentration. There is a particular y value that you get from the experimental data. Now, for theoretical curve, let say the theoretical curve goes something like this. Let say we get a best fit, which is something like this let us imagine. Here also for every C value there is a y value, which is this is theoretical. Whatever, I show in circle that is the theoretical value; whatever I show as dot, the field circle. This is the experimental value.

So, these are experimental values. This experimentally you got. And, this must be the theoretical value because when we finally get a straight line, let say for actin, if you know K 0 and K d, you can get this straight line. So, you get this theoretical value. So, we have a theoretical value and an experimental value.

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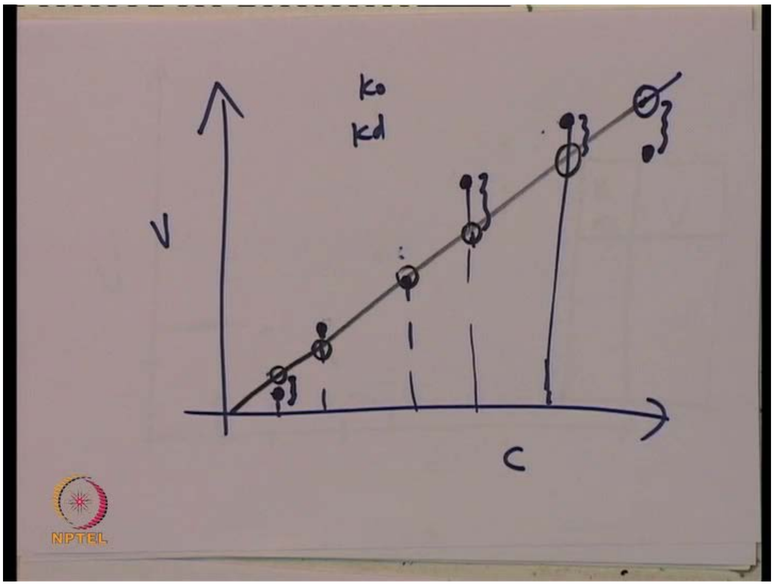

And, in the ideal case in an ideal scenario, what do you expect? What do you except is that in an ideal case. What do you expect? You expect that theoretical value and experimental value will be exactly on this case and know each other. That means they will do exactly fall on the straight line. But, this is an ideal case.

But, in reality it will not exactly fall on this straight line. They will be straightly deviated from this straight line. So, for each point this will be deviated from this straight line, here this will be deviated, some other point here. So, the theoretical value is this and the experimental value is this. So, this deviation will be this. For some other case, here also the deviation is this sometime. So, for each experimental value, we will have some deviation from the theoretically theoretical value that you will get from the best fit.

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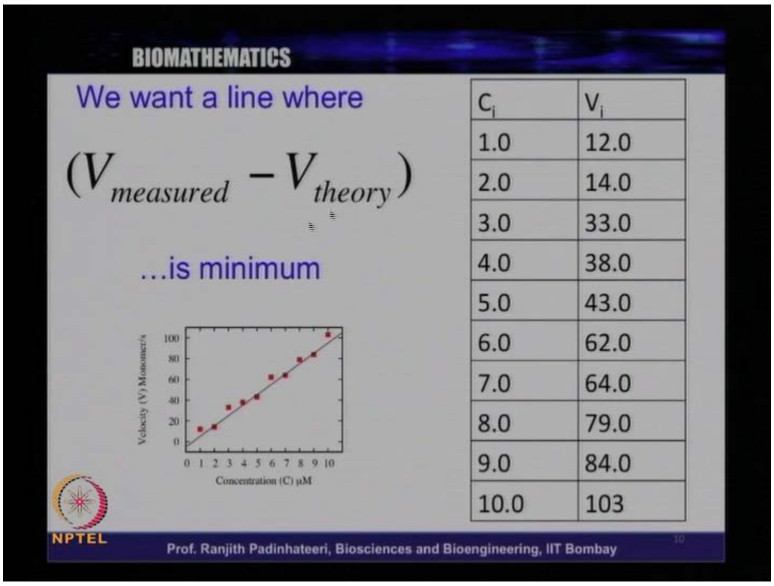

So, what we want is that, we want the measured velocity and the theoretical velocity. The difference between these two should be minimal. So, you should minimize this. That is what, commonsense we have told you. When something is a fit, when something is a nicely fitting, then there will be little difference. The difference between the measured value and experimental theoretical value should be the fit value or the fitting value. So, when I say V theory, this is the value where the velocity which you get from the fitting; that means, this line you has. So, for every C, there is a value. The line represents the value. So, this fitting value is we get from fitting. And, the measured value should be there. There will be some difference where this reference should be minimal. So, what we expect?

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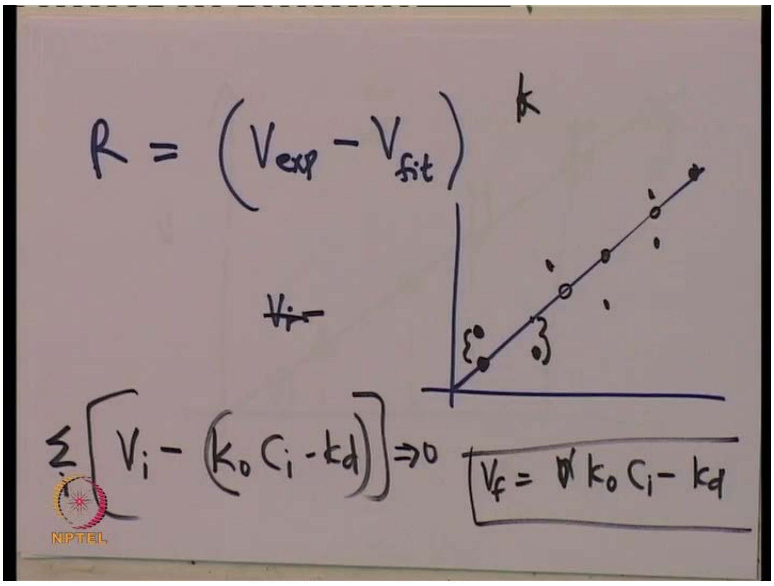

We expect that V experimental minus V fitting; this has to be minimum. So, let if we define this as some function R, this function R has to be minimum. So, you know what experimental value, that is, the points are. So, V experimental is V i. We have this V i, we have here in this table. Look at this table, which is our experimental value. 12, 14, 13, 33, these are the experimental values. There are ten experimental values.

So, if you look at this graph here, so the experimental value we represent by some…So, now, this is the theoretical value corresponding to the fitting value. Fitting value is what you get. This is the fit value from the fitting. So, for each point, there will be some value from the fitting. So, this difference, here this is this difference. This difference should be minimal. So, V i minus, what is the theoretical value? So, V i is the experimental value, the theoretical value will be K 0 C i minus K d. This is the value we get from fitting.

From fitting we get that, V fitting is K 0 C i minus K d. If you know K 0 and K d, we can get this V f. So, V i minus V f should be minimal; that means, V i minus this, has to be minimal. But, for every value, there is a difference. And, this all the total difference should be minimal. But, if you look here, some of these are positive. This, if you subtract this from experimental value, then you will get a positive number here. Here, you will get a negative number because this is more and this is less. So, the sum of this, typically sum of positive and negative, you expect these two go to 0. So, if we just minimize this, sum of this you get 0.

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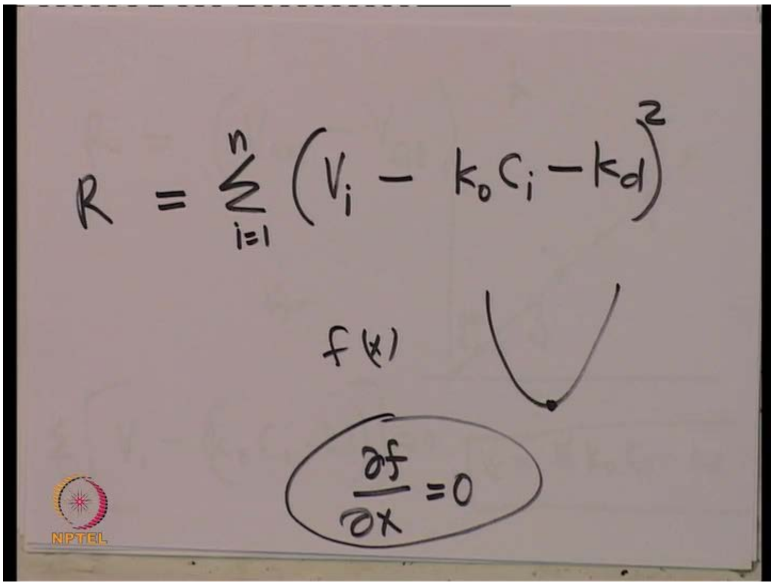

So, what we want is that the better way to minimize the square of this; that means, if we minimize sum over I, experimentally measured V i minus theoretically or by the V i we get from the best fit. The difference square i is equal to 1 to n; where n is the number of data points. So, if we minimize this particular function, let me call this function as R. So, this function R if we minimize, we get this value for K 0 and K d.

So, we know that mathematically if you have any function, the minimum of that function is can be calculated by, so, if you had a function f of x and if it had a minimum here, if you calculate del f by del x, here at this particular point del f by del x will be zero. So, if you have a function and if you want to calculate the minimum, what you do typically, is to find the derivative. So, if you find the derivative and equate that to zero, you will get that particular point where the function is minimum. So, we can do exactly the same thing here.

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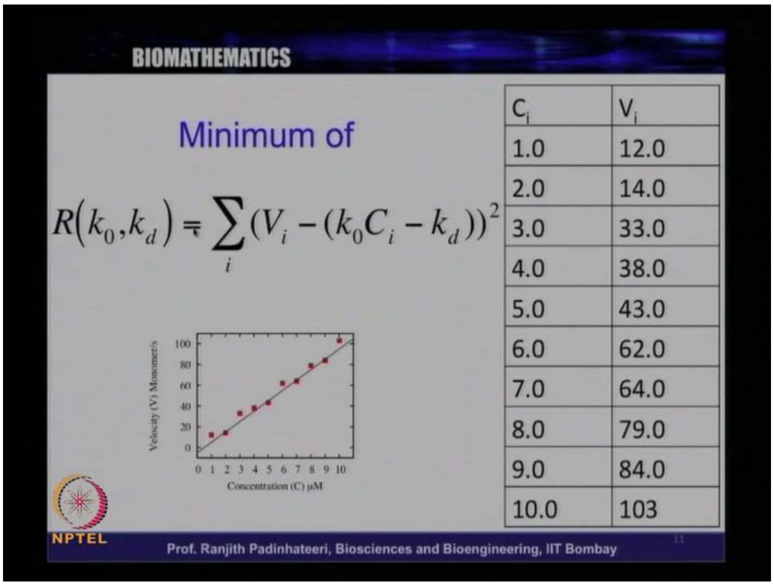

What can we do? So, essentially what we want? We want this R 0, which is a function of K 0 and the K d. This R 0 you want to be minimum, which is V i minus K 0 C i minus K d whole square. This is the difference between experimentally measured value and the best fit value. The velocity got from fitting and the velocity got from experiments, their differences should be minimal. So, that is, the best fit should be this fit, where this is minimal.

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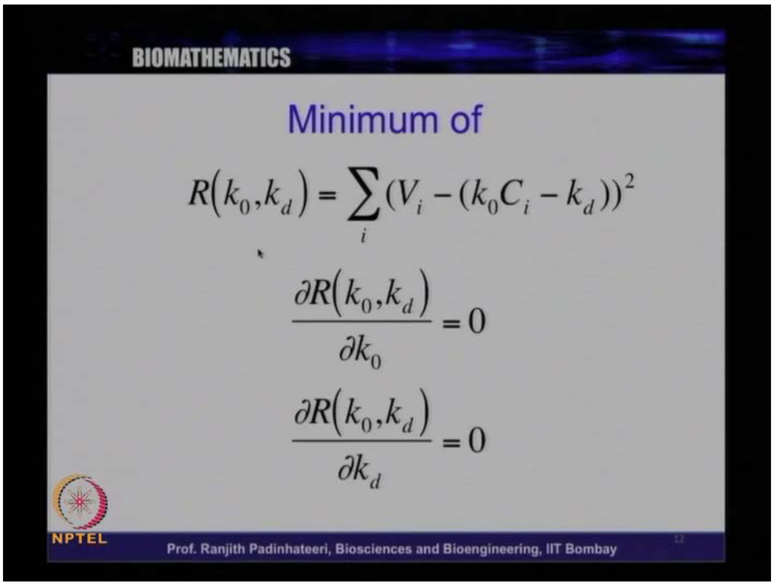

So, if we do that, that is, if you find the minimum of this, that is, we can find the derivative of this with reference to K 0 and K d. And, equate this to 0, then you will get the minimum of R. So, essentially what we want to do is that, we want to find certain value of K 0 and K d, such that del R by del K 0 is 0; and del R by del K d is 0. So, this is what, essentially we want. We want certain values of K 0 and K d, such that the derivative is minimum.

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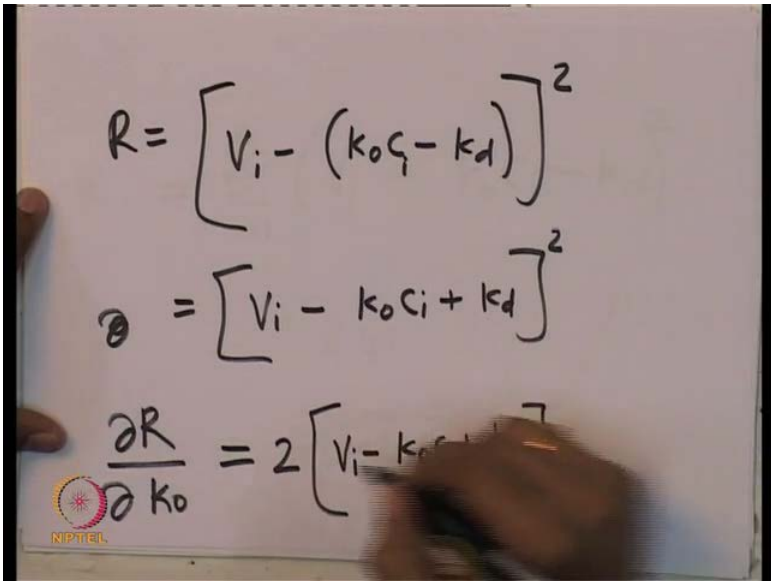

So, how do we do that? How do we calculate the derivative of this? So, let us let us look at this function we have here. So, what is the function we have? We have R is equal to V i minus K 0 C i minus K d whole square. Now, let us look what is, so this can be written as V i minus K 0 C i plus K d whole square. Now, we want… first, let us do del R by del K 0. Why it is del R by del K 0?

So, surely there will be a… The whole thing is the square. So, there will be two times this particular function; V i minus K 0 C i plus K d. So, do it yourself derivative carefully. So, this is the function, and find derivative of this. So, this square is this, then the derivative of this, so derivative of K 0 will be a minus C i. So, there will be a minus sign and there is a C i, so this is del R by del K 0. Now, what is del R by del K d? Why it is del R by del K d?

So, del R by del K d from whatever this is can be written as del R by del K d. So, if you know this particular V i, if you know this particular R, so this is our R; we can write del

R by del K d. So, there is same way. There is two times this function, which is V i minus K 0 C plus K d. And, that is it, K d as a constant. So, derivative of K d is 1. So, the only is a matter of one, so this is what it is. So, you can find that del R by del K 0 is minus two times C i times this. And, this is del R by del K d is two times V i minus K 0 plus K d. These both are equal to zero. So, this we want to find out the value of K 0 and K d, for which these two equations will be zero.

So, essentially this is written in this particular way. So, del R by del K 0 is minus two times V i minus K 0 C. If you take this bracket here minus outside, you can write in this particular fashion. So, do this yourself and see whether you are essentially getting this particular answer or not. Equal to 0, you get this. So, these are the two equations and two unknowns. So, let us rewrite this a little more carefully.

Let us rewrite this. Sorry, there is a sum over i here, which I have forgotten here. There is sum over i everywhere, there is a sum over i here. So, just like you can see a sum over i here.

Ok. So, what we get? What we get is two equations. So, let me write these two equations little more carefully. So, these two equations are minus 2 sum over i V i minus K 0 C i plus K d into C i equal to 0. Minus, sorry plus, there is a plus 2, if you wish plus 2 into sum over i V i minus K 0 C i plus K d equal to 0.

So, these are the two equations and you have two unknowns. Here, the two unknowns are K 0 and K d. So, you have two equations and two unknowns. This is one unknown, so two equations and two unknowns.

So, if you have two equations and two unknowns, you can solve this. So, you can take this inside. You can take this sum over sum inside, and rewrite this in a particular fashion. So, how do we rewrite it? So, if we rewrite this and rewrite the sum, what you essentially get is something like this. You can write this n K d plus K 0 into sum over C i plus equal to sum over i V i.

So, let us look at here is… So, the first term 2, I can divide both sides by 2. Something divided by 2 by 0 is 0 by 2 is 0, itself. So, this 2 goes away. Then, I can take this sum over i V i to the other side. So, what we have here is and is minus sign here. So, this is a minus V i. So, i can write this is sum over i V i; i goes from 1 to n.

And, there is K 0 C i in this, there is minus sign. So, there is K 0 into sum over i C i and K d into sum over i, this sum over n times. So, sum over i up to n is n. So, you have minus K d times n. So, you can write this... So, there is a C i here. So, here you have to take care of this C i also. So, there is when you multiply this C i with this K 0, we will get C i square.

And, when you multiply this K d, There you will get again a C i everywhere. So, essentially you get V i C i here and you get sum over i C i here. So, you get sum over the essentially what you get is? Essentially, you get is this particular equation here. What you get is K d into sum over i C i is K 0 C i square into C i V i and n K d plus K 0 sum over i C i sum over i V i. So, if you know this, if you know the sum over i C i from the experimental table, we had already this table with all the C i s and V i s. So, if you go back here, we had this particular table with all C i s and V i s. From this particular table, it is easy for us to calculate sum over i C i.

It is easy for us to calculate sum over i V i. It is easy for, again this is sum over i C i. It is, we can also calculate sum over i C i square. We can also calculate sum over i C i V i. Just sum over all the x-axis numbers in the first row in the first column, sum everything in the second column. So, these are all essentially some numbers. So, sum over i C i is a number because sum 1 plus, 2 plus, 3 plus, 4 plus, up to 10. This is summing over i C i.

Velocities are also numbers, so you can sum over everything. Velocity if the column the y the second column, we can sum from the experimental data. So, you can get this from the experimental data, you can get this from the experimental data, you can get this from this square and from experimental data, you can get this from experimental data.

n is also a number, how many times you did the experiments is ten times; so, these all some numbers. So, you essentially, what you have is that sum a 1 into K 0 plus a 2 into K d is equal to a 3; and b 1 into K 0 plus b 2 K d is from b 3. So, essentially two equations are two unknowns and a 1, a 2, a 3, b 1, b 2, b 3 are some numbers, which we can obtain from experimental data. So, we know a 1, a 2, and a 3 from experimental data; b 1, b 2, b 3 also from experimental data. Then, we can solve this equation, two equations are two unknowns and we can write K 0 and K d in terms of this a and b.

Sum something in terms of a and b. So, this is what essentially, they do in the computer. So, what do they will do? So, what do they get? They get K d is equal to sum over i V i into sum over i C i square minus sum, if you do the algebra properly you will find that K d is equal to sum over i V i sum over i C i square minus sum over i C i minus sum over i C i V i divided by n times sum over i C i square minus sum over i C i whole square, which we can rewrite in this particular fashion. You can just rewrite this. This is best if your wish because you can write this as V i average and so on and so forth.

This is a different notation. So, this is V average C. So, you can write it all this in a different notation of averages. And, we can rewrite this expression in a different way, if you wish. So, then you get this particular experimental value for K d, then similarly, we can get some value for K 0. We can say that K 0, we can find from the algebra, if you do the algebra properly. We can find that K 0 can be written as in terms of this experimental data, which is C i s and V i s. We know C i s and V i s. So, known some C i s and V i s we can get K 0 also.

So, essentially we can get K 0 and K. We can get, if you know the C i s and V i s. This is, essentially the experimental data. We can get K 0, which is sum of, summation of some combinations of C i s and V i s. And, K d is also some combination of C i s and V i s. and, if you know this experimental data, we can plot the curve that fits the best as, V fit is equal to K 0 C minus K d. So, V f is equal to K 0 C minus K d. We can find out a straight line which will fit the best way.

So, some particular value of K 0 and K d, you can find which will be the best fit because the deviation, from the total deviation from the experimental data and between the experimental data and theoretical, the fit data is minimum.

So, we minimize this deviation; so that, we get the best fit. So, knowing the constant, we can draw a line. So, we found, we briefly discussed, as we quickly discussed actually, known the experimental curve, how do we fit this data?

So, the same thing can be applied to any function. So, let say, you have some other function. So, this R, sometimes also called chi square. So, this is, sometime is called chi square fit. So, chi square is essentially, if you have an experimental data and x-axis and the y-axis, so for every, so, let say there is for every x i value, there is a y of x i. This is experimentally measured y of x i.

This can be some curve; if it can, we have some data like this also. So, it might be like fitting with some, need not to be a straight line. It might be fitting with the curve some function f of x, which could be let us say, a e power minus b x square. It could be even a gaussian or it could be any function. It could be a x square. So, here this looks like, more look like a x power n, some, in general some function, it could be any function. So, chi square is the experimentally measured by data minus this function, which is parameters a, b, c, etcetera. You can write these parameters.

So, this difference square, this is typically called as chi square. So, chi square is the experimentally measured function minus the function that will fit the data, the best way. So, this will be a, b. You can have any number of parameters. So, in our case we had K 0 and K d. We can have any set of parameters.

If you find this difference and square it, this function is called chi square. And, you minimize this chi square with respect to a, and equate to b equal to 0; you can minimize with respect to b equal to 0; if there is a C, you can minimize with respect to C and equal to 0. So, how many parameters you have? You have a, b and c. This is like K 0, K d and all that. If you do a differentiation with i equal to 0, you get that many equations. You can solve these equations. You can get the corresponding a and b that will be, for which this will be minimum. So, this is the idea. So, this is what is going on behind the experimental, behind the computer. This is what computer is essentially doing.

It is easy to do for a straight line; it is little more difficult if the f of x is the complicated function, but essentially, this is the idea. Now, what is this idea coming from? Why is this particular difference and square? Where is this coming from? Why this particular deviation being taken.

So, let us think about it a bit more deeply, why does this coming from? So, let us think about theoretical point verses experimental point, like what do we expect. So, let us see, we have this example velocity verses concentration. And, if you do for actin monomer, if you do for polymerization of actin, V is K 0 C minus K d. It is known that K 0 is about 11 micro molar inverse second inverse; that is, K 0. K d is depolymerized typically close to 2 per second. So, it is 2 per second.

So, theoretically what you get is, just a given these two numbers and draw a straight line. Now, if you do an experiment for this particular value of concentration, if I do an experiment I might get a point here; if you do an experiment you might get a point here; if somebody else does an experiment they will get a point here. So, if you, so, let us take the example, let us take even a simpler case. Let us take this particular value; C is equal to 1 micro molar. So, this is C equal to 1 micro molar.

What is K 0 C minus K d? It is K 0, so 11 into 1 is 11, 11 minus 2 is, so, this has to be 9. Theoretically, this value is 9. When C is one, theoretically the value is 9. But, some if you do an experiment, I might get the velocity 9.1 monomer per second; somebody else might get 8.9 monomer per second; somebody else if they do experiment they might get 8.8 monomer per second; somebody else might get 9.2 monomer per second. Now, you might get, if you do hundred experiments or a thousand experiments, you might get, if you fix the concentration as1 micro molar and look at what is the growth speed of actin at 1 micro molar. You might get different values if you do this experiment. If you repeat this experiment hundred times what are you doing? We are fixing C is equal to 1 micro molar and looking at what is the growth speed of actin. And, I do experiment hundred times or thousand times, each time I will get some value around 9; sometimes I get 9.1; sometimes I get 9.2; sometimes I get 8.8; sometimes I get 8.9.

Now, if I plot the value of V i got, so this what do I get? I might get some gaussian like distribution like this. This is one I meant to draw a gaussian like a normal distribution. So, this is 9. We expect to get 9 monomers per second. So, most of the time we will get 9; some of the time you will get 9.1; some of the time you will get 9.2. So, the probability of getting 20 or the probability of getting 0 is very small. So, we just say that is, if you can expect here, you can imagine that your deviations from 9 will have a normal distributions like this.

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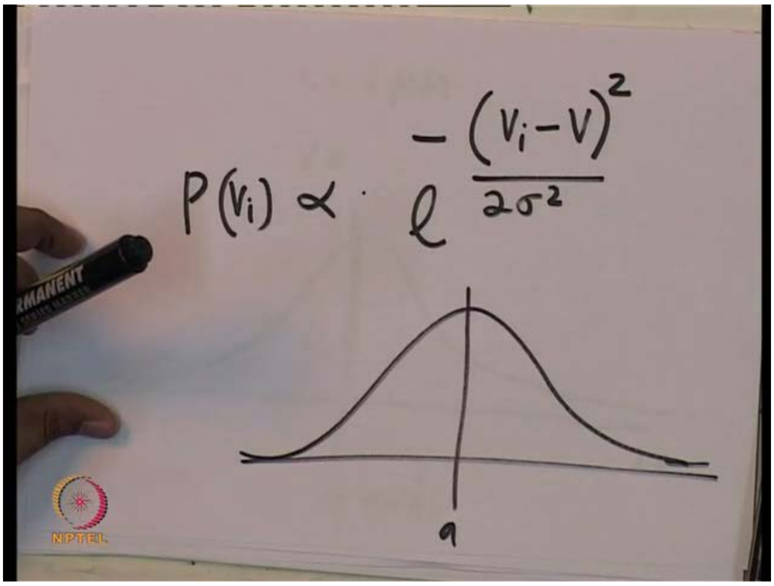

So, the experimentally obtained value and the theoretical value or the expected value or the best fit value, there the deviation of this, so, if you can find the deviation from this, but if you plot this, if you find the distribution of this measured value, you can expect to get a normal distribution or a gaussian distribution. So, what is the function? What is the formula for a gaussian distribution? We discussed that the formula for the gaussian distribution is the probability that you will find the velocity V i is proportional to e power minus V i minus some other V whole square divided by 2 sigma square. So, this is what you expect. So, where V is some kind of an average value; so, this was sometime you will get 8.9; some time you will get 9…So, this is, if you could draw a gaussian like this, this is 9; this is what, what you expect to measure the average value that you will get a distribution like this. So, this is the expectation. Now, if you want, so basically, what we want is to maximize this particular likelihood of getting this particular V i.

So, now this is for a particular measurement. So, you can say that for all measurements, for different values, you can expect this distribution to be a gaussian again or a normal distribution. Now, what is happening? You are maximizing the P. So, maximizing the P is like, it turns out that it is minimizing this difference. So, what we did is minimizing this difference, which is like maximizing this probability.

So, this is the basic understanding of this. But, I do not want to go into the details of this. I do not want to discuss this in detail. Why exactly this is coming? That it is, just to hint or direct you or just to point out that where it is all coming from. But, even from simple, some kind of a rational commonsense, it is clear that what we would need is the difference should be minimal.

Ok. So, what we essentially, so the deviation has to be minimum or actually equivalent of maximizing the probability. Even though I said about this, this particular case here, this is particular for a given experiment.

You might get a distribution of in this particular way. But, even you can extend this to a case of deviation and even the deviation will have as a gaussian distribution. And, you know and the probability you can find that essentially, what you end up is minimizing the deviation. It is like maximizing the likelihood, but if there is time, we will discuss this in the coming in another lecture. But, for the moment it is enough to realize that at the end of the ray, essentially what all those computers will be fit? What they are doing is, essentially they are minimizing this function called chi square, which is the measured value minus the fitting value, which is a function of the parameters square. And, this you can do mathematically by finding the derivative del chi square by del a, and del chi square by del b and equating them to zero.

So, this is basically the summary of this lecture. That, to fit a straight line or fit any experimentally measured value to a fitting function, this is experimentally measured value, this is the fitting function; you have to minimize this difference and this difference can be minimized in this particular way. So, there is sum over i. You have to minimize this particular function and this can be done in this way. And, that will give you a and b.

So, with this, we will stop today's lecture. So, this is essentially today's lecture was about fitting a line or a function to an experimental data. So, with this, we conclude today's lecture and we will discuss more about statistics in the coming lectures. Bye.