Biomathematics
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Lecture No. # 23
Understanding Normal Distribution

Hello, welcome to this lecture on biomathematics. Currently, in the section we are discussing statistics relevant to biology. And, we discussed about averages, standard deviation and some kind of distributions. So, we said that data, the simplest thing that we can learn from data is average. And then standard deviation and, in fact, to present the data instead of presenting a set of numbers we can present it in a distribution, which will have more meaning. Which will convey much more than a set of numbers would convey? So now, we discuss various distributions, we discussed distribution and said that typically many things in nature fall into something called “Normal distribution”.

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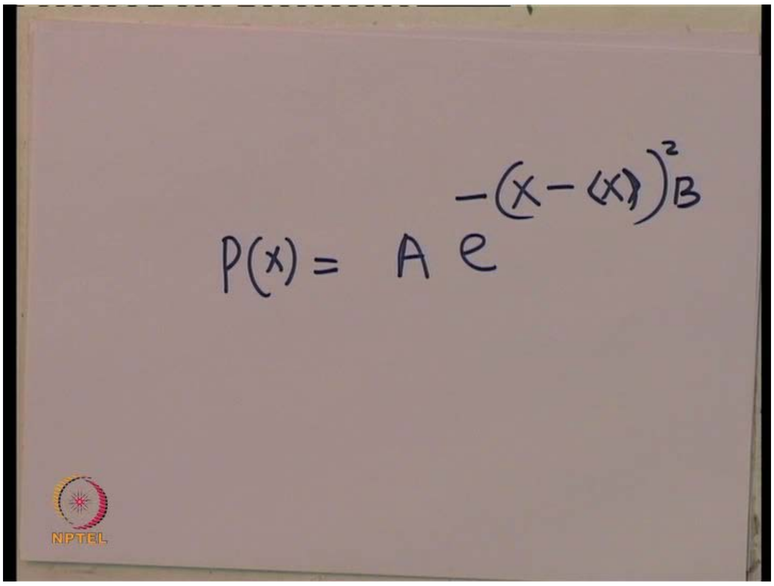

So, now, today we will understand what normal distribution is. So, we just said in the last lecture that normal distribution has a form, P of X is equal to A e power minus X minus X average whole square with some constant B.

So, this is this particular form normal distribution has this particular form. We will try and understand more about this in this lecture. We will pretty much understand more let more things about this. So, title of the lecture is. So, this is statistics itself. And the specific title of today's lecture is understanding normal distribution. In this lecture we will try and understand pretty much everything about normal distribution. Like, whatever you need to know, have a know about normal distribution will try and understand this lecture. At the end of this lecture I hope you will know pretty much you will be very confident to deal normal distribution, and pretty much learn everything about normal distribution that point should known. So now, what what do we what what is the simplest that we should learn about normal distribution.

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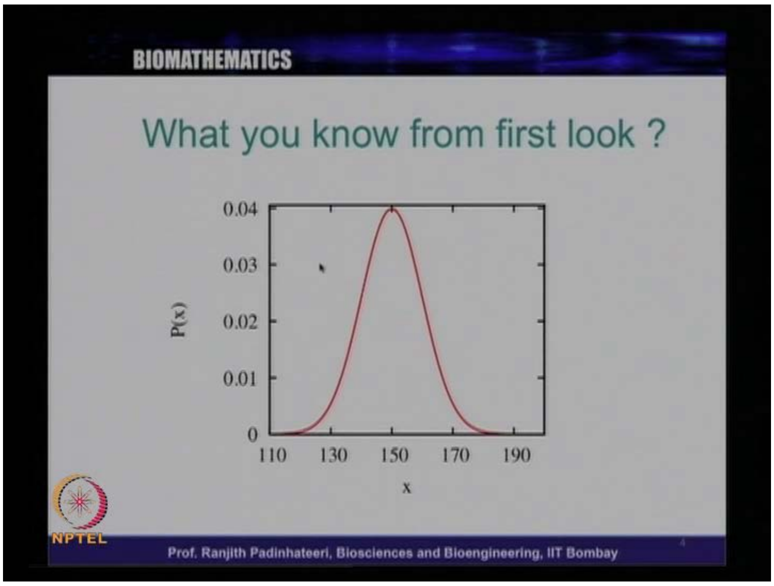

So, let say you have given a normal distribution, which is like a normal distribution we said this bell like shape .What the what can we know from the from a first look. What do you know from a first look, what all things you can learn from this? You know that the probability is high, around 150. So, we had yesterday discussed let say this is about the height of a is the probability distribution of the height of students in a particular class. So, X is 150 let say this is 150 centimeters.

So, this then you know that the probability has a peak the maximum number of students will have around height around 150, and as we go to either side the probability of finding students with lesser than 150 and more than 150 kind of decreases. So, this is something which we can know from the first look as we discussed last time. But, what more can we know from just looking at it?. Can you tell look at here can you tell what is an average?

By looking at the curve can you tell what the average is or what is the standard deviation? Well, you should be able to do this or you should be able to do pretty? So, you can at least roughly say by just looking without doing any measurements. Just by looking at the axis of the graph the range of the graph you can tell and just looking at the graph itself you can tell roughly what is the standard deviation is and pretty much accurately what is a average?

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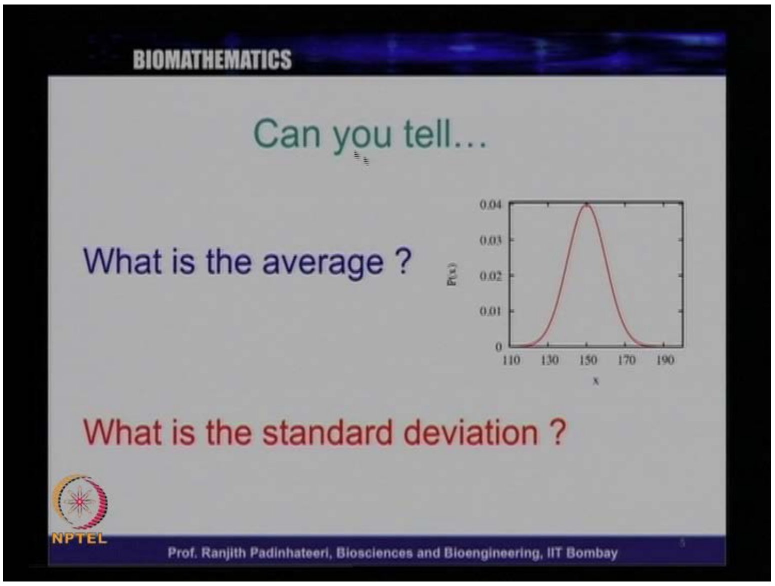

So, that is the question here can you tell what is the average and what is the standard deviation. At the end of this lecture you should be able to do this.

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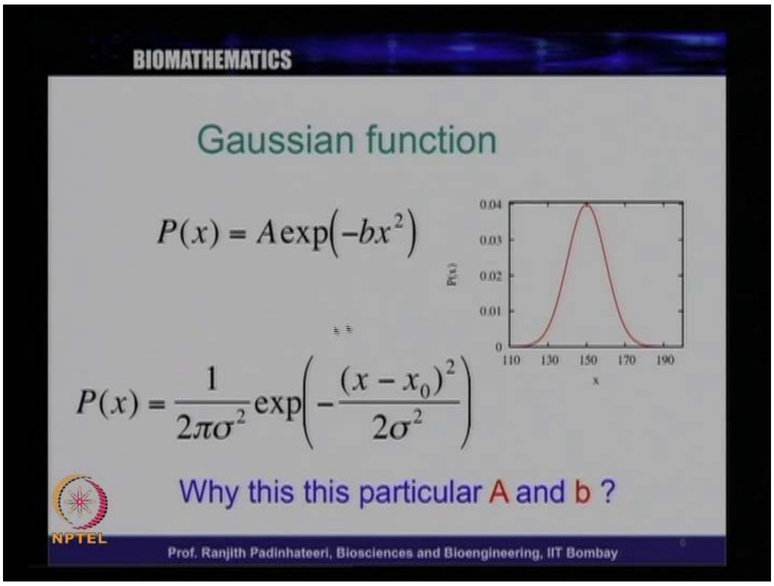

Now, we know that, this normal distribution is a Gaussian function is a mathematical function is called the Gaussian. It has this particular form A e power minus b x square. So this is this distribution this if you plot this A e power minus b x square for a some given value of A and given value of b you will get this curve. So, there are just two parameters that describe this normal distribution. One is So, we will discuss about that, but just by plotting this function you will get this Gaussian this particular bell shape.

By plotting this using a either you can take numbers take a value for A and b and for different values of x you can plot it and you will get this, but, in many cases you must have seen that normal distribution is presented in this particular way have a look at here. It might be you might have seen that is presented in a way that P of x is one by two 1 by 2 pi sigma square exponential x minus x0 whole square by 2 sigma square. You must have seen this. So, what is this relation between this and this, it is clear that A is 1 by 2 pi sigma square and b is 1 over 2 sigma square and x 0 is 0.

So, there is what the relation between the by comparing this and this, we can know that one by two pi sigma square is written as A, 1 over 2 sigma square is written as b and X0 is 0. So, why this particular form of A and b why is A is 1 by 2 A is, A can be anything, but why is it related to the sigma here the sigma is standard deviation. So, for normal distribution sigma is standard deviation. So, why is A and b related to standard deviation. So, that is why this particular form why is A is 1 by 2 pi sigma square and b is 1 over 2 sigma square. So, right you can know this just by comparing. So, let us let me write it here.

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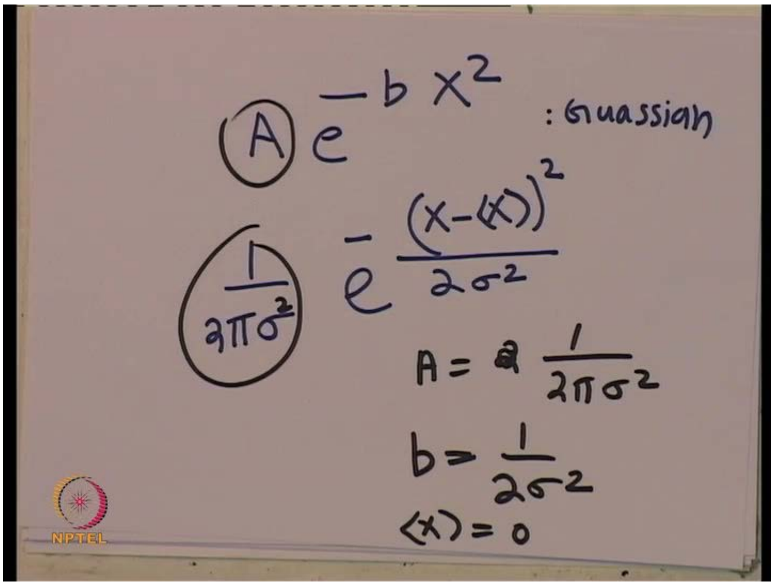

So, what here one function here is an A e power minus b X square. And this also one by. So, this is the Gaussian function. when normal distribution is written it is often written that if you look at any book normal distribution is written such a way that, X minus some time it is even written as X average Whole Square by two sigma square. So, if you compare this and this it is clear that this is A. So, A is A is one over sorry this is sigma square here 1 over 2 pi sigma square and b this is 1 by 2 sigma square and X average in this case is 0. So, this is the if you compare this and this just you get this particular thing.

Now, X average can be anything, but A is always 1 by 2 pi sigma through 1 over 2 b is 1 over 2 sigma square. So, why this particular, how sigma is coming here? Why is it suddenly sigma coming here? Why this particular form for A and a and b we will understand in this lecture. So, let us start with the simplest case.

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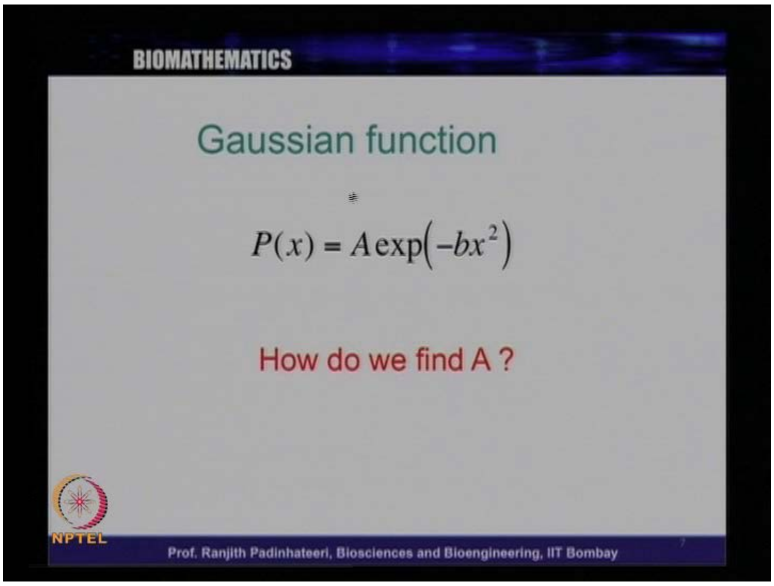

So, the simplest question is so, if you have this Gaussian function. So, if you have a Gaussian function P of X is A e power minus b X square how do we find A. So, this is the function of normal distribution. If you plot this function you will get the normal distribution a bell shape curve. Now from this, how do we find A. So, that is the question. So, let us think of it.

So, this is probability distribution P of X what is this mean? This means that the probability of finding having a value of X is this. So, probability is this. So, what is the total probability, total probability is always one. Why? Because, like if you have a you know that in the case of head and tail of half is the probability. So, half plus half probability of finding head is half, probability of finding tail is half. Half plus half is one.

Probability this is something that you know total probability as to be one. In the case of marks that we discussed in the last lecture , if you have a probability of finding one mark, probability of finding two marks, probability of finding three marks, probability of finding four marks, up to probability of finding 100 marks, the total probability has to be one. Because you will surely find a student with probability, you will find even if all students got 100 marks probability of finding hundred is the sum will be one let us say all students get 53 marks, all students get 60 marks still the sum total probability of finding all the marks is one. So, the total probability has to be always one that is the definition of probability.

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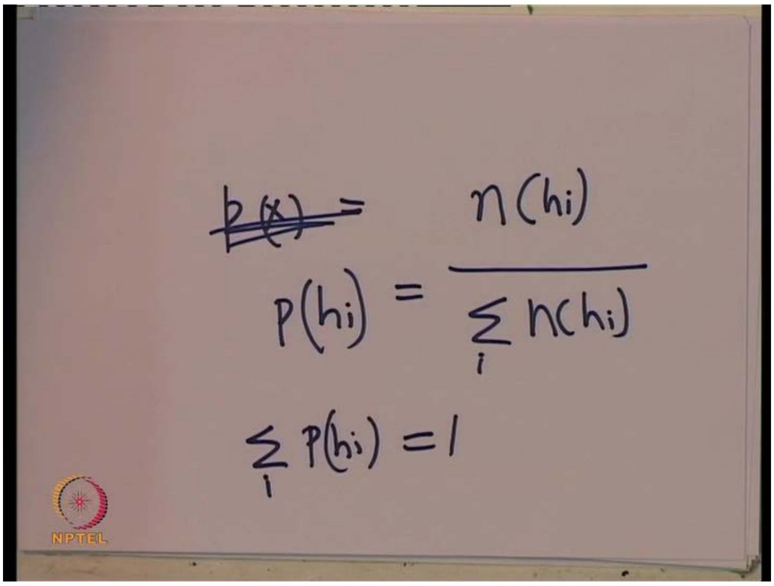

So, when we define probability the other day yesterday, in the previous lecture we said that P of X when we wrote. Where we had written that we even in the case of h we had like in the case of P of h i. We had written, that this is can be written as the peak is the function. So, you had you had n of hi into sum over i sum over i n of h i. So, sum over i if you divide this. So, now, if you do now sum over of P of h i this has to be 1. Total probability an always has to be one. So, knowing this, knowing the total probability as always to be 1.

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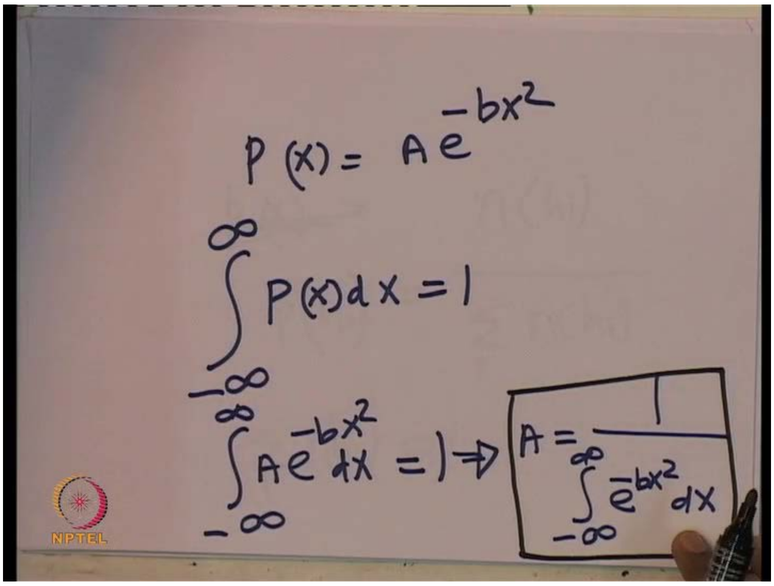

We have this function P of X is A e power minus b X square integral minus infinity to infinity P of X d x has to be one. Minus infinity to infinity P of X dx has to be one. What is this mean? This means that, minus infinity to infinity P of X is this. So, A e power minus b X square dx has to be one. A e power minus. So, this is if this is the case A is you can take an outside, if A is outside this would imply that, A is equal to one over minus infinity to infinity e power minus b X square dx. So, just by knowing the integral of this function, we get A is one over e power minus infinity to infinity e power minus b X square dx. This is A. So, we get a now, how do we how do we calculate this integral? what is the value of this integral? So, let us quickly find out the value of this integral.

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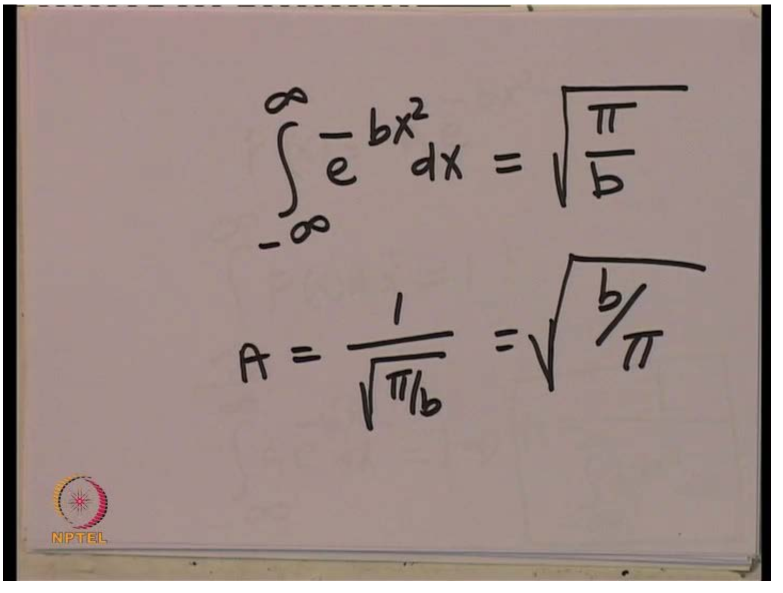

So, at this A to begin with I am just tell you the answer, that, minus infinity to infinity e power minus b X square dx is , root of pi by b. How do we get it? Later we will discuss how do we get it? How do we do this integral, but for the moment just understand that the answer of this integral is root of pi by b. If you have a function e power minus b X square and if you integrate minus infinity to infinity the answer is, e power minus b X square is a root of pi by b. We will show this is the answer in it, but further moment just take my word this is the answer if this is the answer A is one over root of pi by b. It is root of b by pi this is A.

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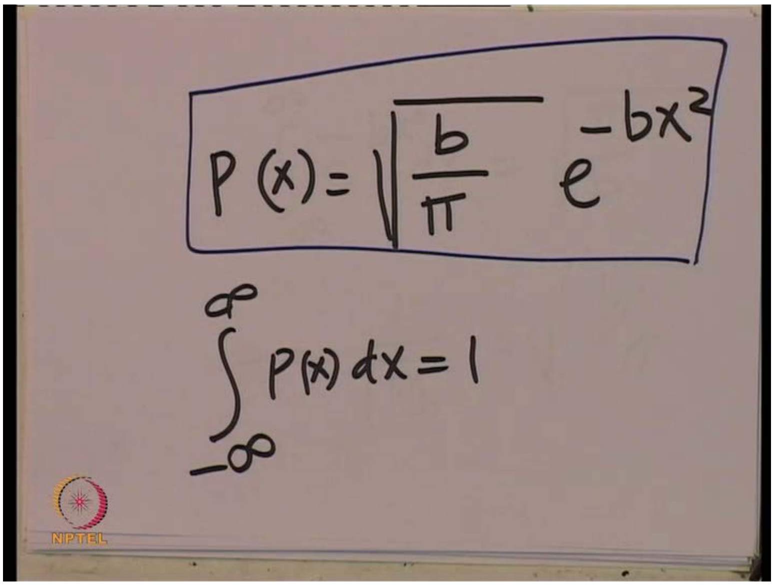

So, our probability distribution, if this A the probability distribution P of X basically is can be can be written as, P of X is equal to root of b by pi into e power minus b X square. So, this is our P X and surely we can see that minus infinity to infinity P of X dx is one. So, this is the Gaussian function and A we found already A. So, this is the full form of the Gaussian.

Probability distribution normal distribution as this particular form well. So, now, this b by root P by pi is written such a way that, minus the total integral is one. Well now what do we want to find out? So, we want to we had we had this first question, what is standard deviation?

So, for the moment we will consider to begin with, we will consider this particular form for the probability distribution P of X e power minus b X b by root pi e power minus b X square. Now then we will find out for this particular case, what is the standard average as standard deviation? So, we know the definition of average and standard deviation what is the definition of average and standard deviation?

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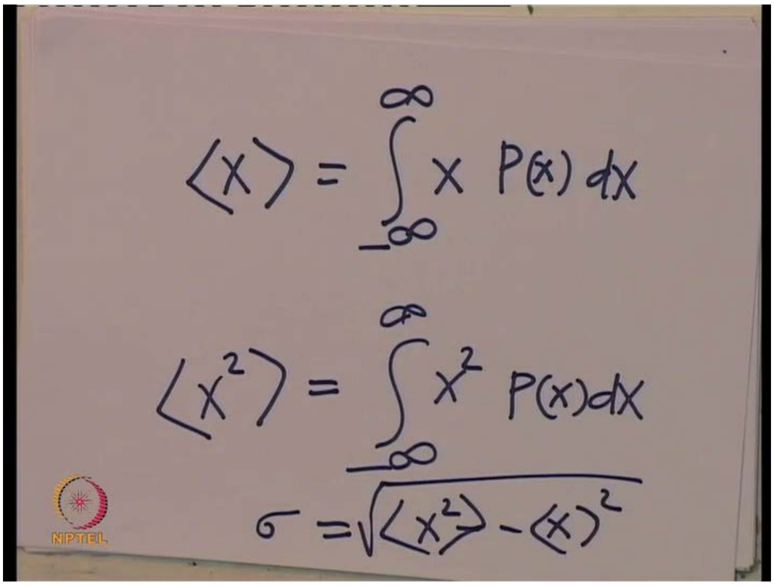

So, the average as always. So, in the in the previous lectures we had defined the average X average is defined as, minus infinity to infinity X P of X d x. So, just remember this is the always the definition of X average. If you're given a P of X the probability distribution the way to find X average is this. You multiply with X and integrate. So, just remember this is the is the definition of X average .If P of X P of X has to be the integral of P of X is one. So, the process we just did a few minutes ago to calculate integral of this find out A is called normalization. So, this process we had here finding of A. This process is called normalization. So, this is the P of X is the normalized function, if P of X is normalized function, this is the definition of X average. So, we have X average and definition of X square average is minus infinity to infinity X square P of X dx. This is the definition of X and X square average.

And we know the standard deviations sigma is nothing,, but X square average minus X average square square root. So, this is the standard deviation. So, we have to find out X average we have to find out X square average and we then we can find out standard deviation

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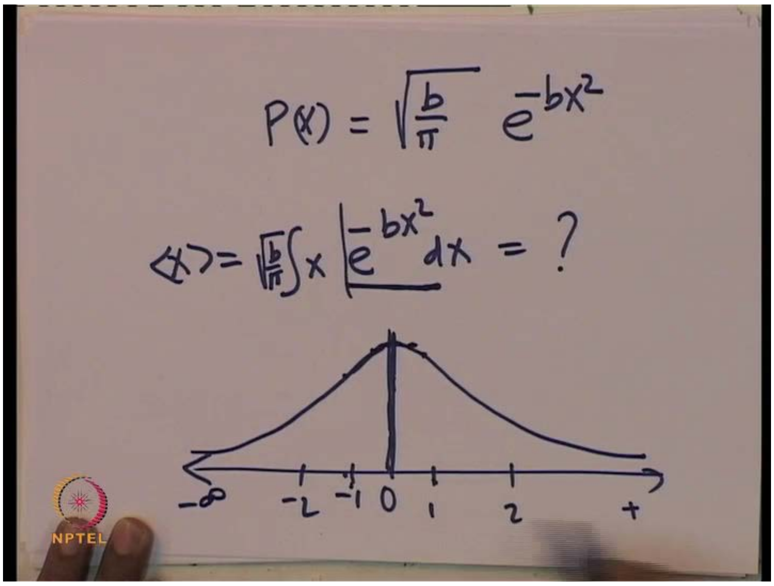

So, we have a P of X here, and our P of X is root of b by pi e power minus b X square. Now we know, what is X average. X e power minus b X square and root of b by dx. Root of b by pi is the constant so, I can write write it outside. Now it turns out that, what is the answer of this? Well without doing this real calculation just by looking the function plotted you might be able to know the answer. So, let us just see how the function is. So, this is 0 this is plus X and this is minus x. So, this is minus one, minus two. This is 1, this is 2 and this is some function, which is symmetric on both sides. It is this way is the same this way is the same.

So, you have a function. This is exactly symmetric on both sides, exactly same. Symmetric on both sides, that means, whatever the value of the plus one is the same value at minus one. So, the e power minus b X square if you find out, at plus one is same as the value at minus one.

So, the only thing that differs is x. So, when you do this integral for every you multiply with one first let us says X will have a some value positive value 0.1 and integral is nothing, but a sum. So, you multiply with point one and multiply with minus minus 0.1 and sum of them same values you multiply with 0.1 and minus point one and you sum you sum your boundary get 0. So, since it is symmetric and there is some minus infinity to plus infinity, the answer of this is 0. Because for every positive value there will be a negative value which is canceling. So, if you take one term and this if you into expand this, if you integrate this integrally if you write as a sum. So, let us write this quickly.

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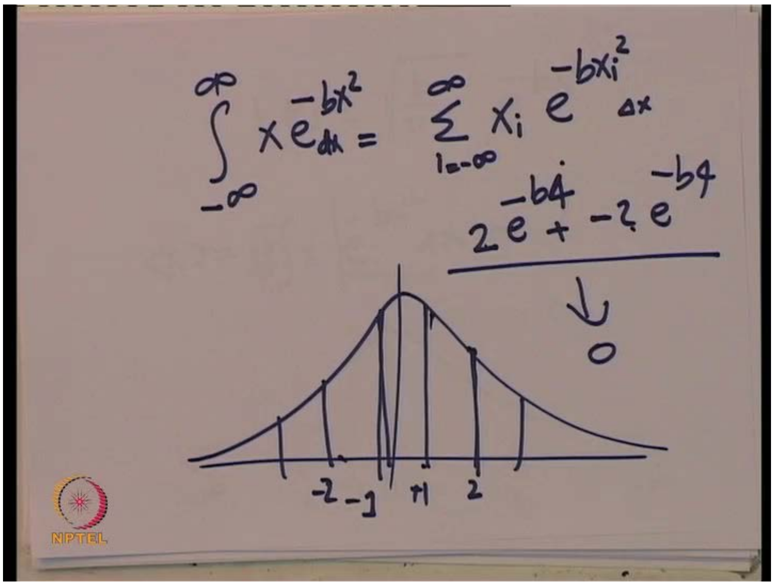

So, integral minus infinity to infinity X e power minus b X square can be written as sum over i X i. So, let us call i is equal to minus infinity to infinity X i e power minus b X i square delta x. So, this is dx if you wish. So, this can be written. So, now, for let us now let us have a look at this function. When X i is so, the many many values. So, let us call this minus 1 minus 2. So, if X when X i is x. So, let us look at this value 2 and minus 2. So, at this particular 2... So, what do you get two into e power minus b X square X 2 square this is 4, plus minus 2 e power minus b 4. So, this is zero. So, just by summing both sides, you are multiplying with plus 2 multiplying minus 2. So, for every plus value here, there is a minus value multiplying on the other side. So, total of this integral is 0. So, this is one way of saying it. So, it turns out that the X average is 0.

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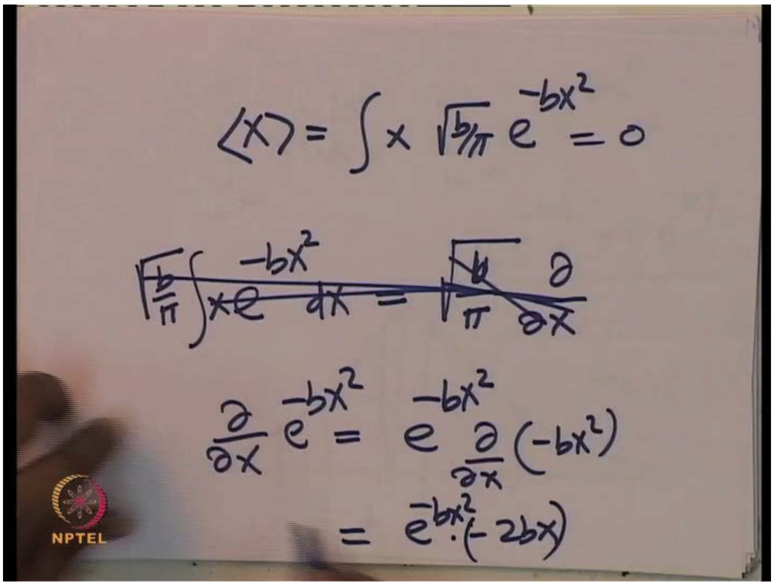

So, X average which is integral X root of b by pi e power minus b X square is 0. Now, there is a rigorously showing this, which we will, the is actually not way very difficult at all see. So, let us let if you want let us do that rigorous way. So, what we want, we want root of b by pi e power minus b X square X d x. I can write this as root of b by pi del by del x of. So, let us look at let us look at this. So, what is del by del x of e power minus b X square. This is e power minus b X square into derivative of minus b X square. So, this is e power minus b X square minus 2 b X. So, by just by knowing this, just by knowing that, that e power minus b X square into minus 2 b X this is into times this is the derivative we immediately.

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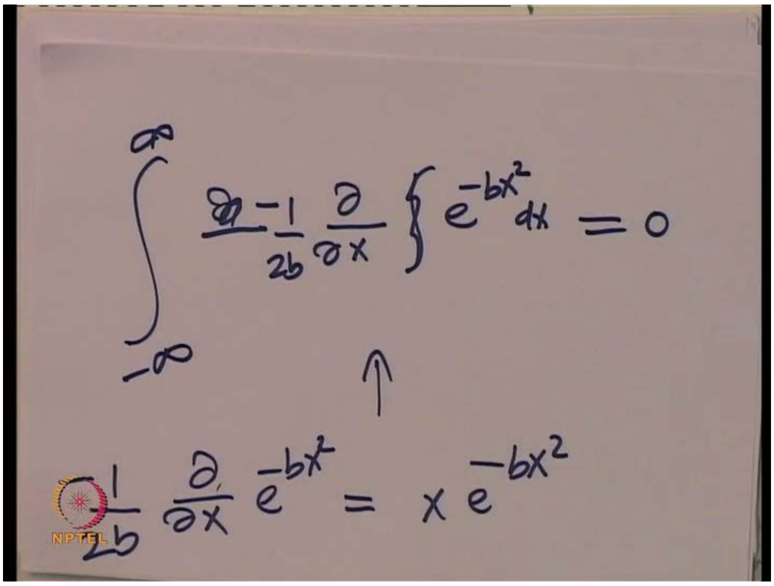

See we can immediately see that just from this, I can say that del by just by this you can see that del by del X of e power minus b X square divided by 1 over 2 b. So, I take one over b the other side is equal to X e power minus b X square. So, X e power minus b X can be written as, minus 1 over 2 b del by del X of e power minus b X square. So, you substitute this there and, do the integral yourself you will find the answer there it is in deeds 0. So, I will not do this clearly you can do this yourself just by substituting the fact that, X e power minus b X square is, del by del X of e power minus b X square minus 1 over 2 b because this you know. If you do this you will get this.

So, let such that let us do that. So, let us substitute this here. So, what we want del by del X 1 over 2 b with a minus sign integral e power minus b X square d x. So, this integral actually written outside. So, this is 0. You can convince yourself that, minus infinity to infinity, this is 0. Do it yourself and convince yourself that this is lead 0. Ok to next question is, how do we calculate X square average?

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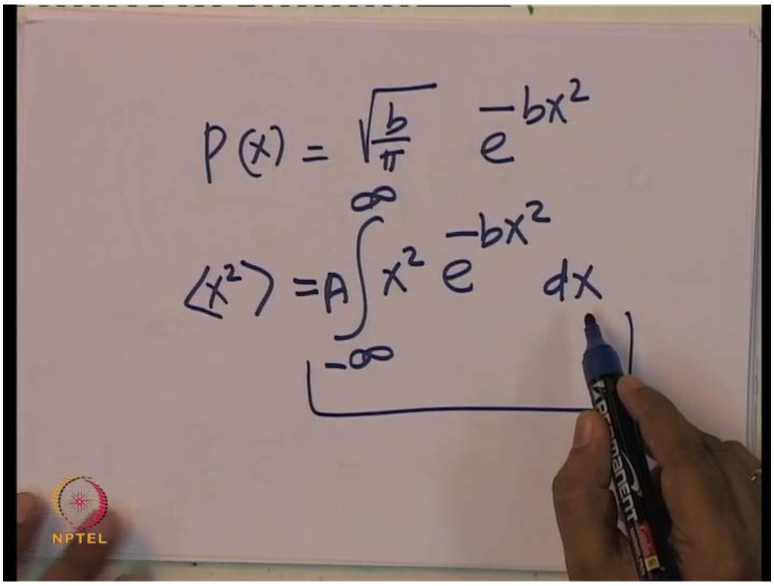

So, we have P of X is root of b by pi e power minus b X square. So, for most of the time I will write a sometime I write root of b by pi as A for convenience. Ok. So, what you want to find out is that X square average, which is nothing, but X square e power minus b X square minus infinity to infinity d x. So, we want to find out X square average now. So, we know P of X is root of b by pi e power minus b X square and X square average is A into A is root of b by pi which is an I write is as A sometime minus infinity to infinity X square e power minus d x. So, this is the definition of X square average. Now, we want to find out why it is X Square. So, first let us do this particular integral. We know that a is root of b by pi. So, let us calculate what is integral X square e power minus b X square d x?

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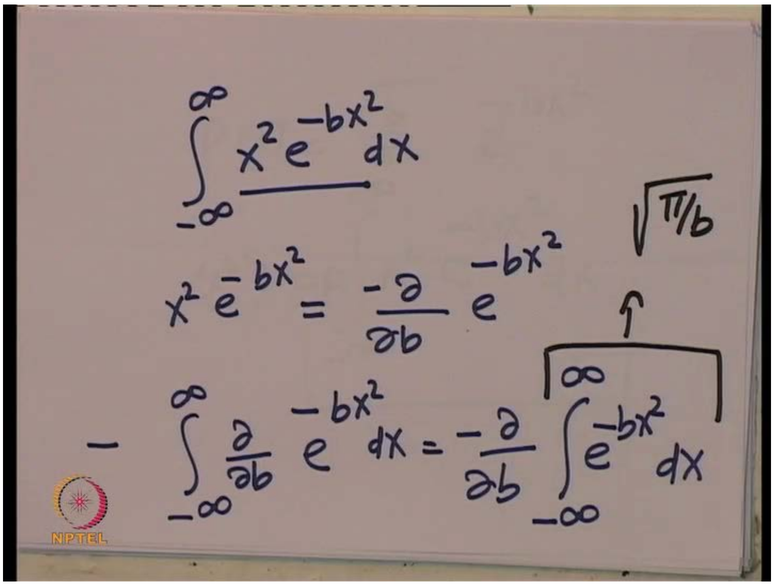

So, let us let us do this calculation. So, what do you want to calculate? We want to calculate integral minus infinity to infinity X square e power minus b X square dx. To do this we do a standard trick which is, we can write this particular function X square e power minus b X square as minus del by del b of e power minus b X square. So, if you derive if you if you find the derivative with respective to b and multiply with the minus sign, you need get this. If you can if you know del by del b of e power minus b X square is e power minus b X square times the derivative of this which is X square and with a minus sign.

So, this just by using the fact that X square e power minus b X square is minus del by del b a derivative of this function e power minus b X square, we can instead of X square e power minus b X square we can substitute this back and write it as minus infinity to infinity with a minus sign here, and del by del b of e power minus b X square d x. So, del by del b is independent of X because b and X are completely different variables. So, I can take this outside. So, this is minus del by del b of minus infinity to infinity e power minus b X square d x. So, what did we do ,we wrote integral X square e power minus b X square d X as minus del b of del by del b of e power minus b X square dx minus infinity to plus infinity we know the answer to this. We just told some time ago, that answer to this particular integral minus infinity to infinity e power minus b X square dx is root of pi by b. So, this is root of pi by b.

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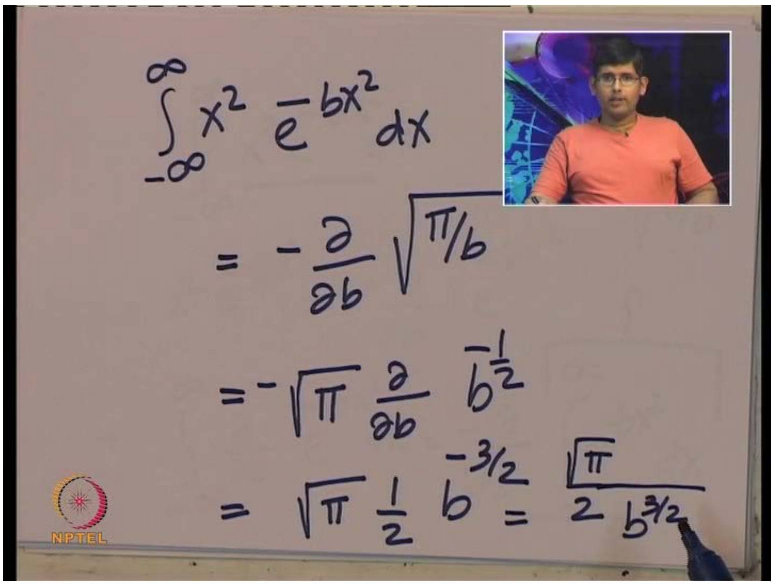

So, the answer, the full answer that is if that is root of pi by b integral minus infinity to infinity X square e power minus b X square dx is nothing but, minus del by del b of root of pi by b. what is this? Is the root pi by there then del by del b of root one over root b. One over root b can be written as b power minus half. So, this is nothing but, root pi into derivative of b power minus half is minus half is the is the minus sign is a minus half. So, this a half and minus minus cancels. So, plus half b power minus 3 by 2.

So, this is the answer of this. So, this can be if you wish can be written as, root pi by 2 into b power 3 by b power minus 3 by 2 can be written as one over b power 3 by 2 and there is a half and there is a pi. So, integral X square e power minus b X square dx is nothing, but root of pi by 2 b power 3 by 2.

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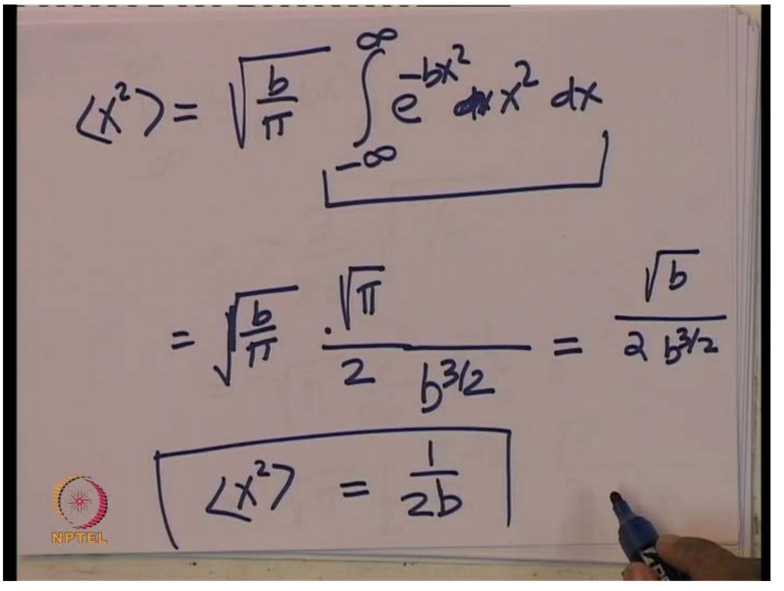

Now what do we want, we want X square average. X square average is defined as, as we said, you have to multiply this with root of b by pi which is, our integral minus infinity to infinity e power minus b X square d x X square d x. So, this part we just found this as. So, this is equal to root of b by pi root of b by pi into this just we found this has root pi by two b power 3 by 2 by two b power 3 by 2. So, b power 3 by 2 is what? So, b into b powers 3 by 2. So, let us write b power 3 by 2 here b power. So, let us write b power 3 by 2 here b power 3 by 2 it can be written as b into root b if you wish.

So, this can be written as b into b. So, root pi root pi cancels. b root b and b power 3 by 2 they together produce. So, if you wish root pi and root pi cancels. this can be written as root b by two b power 3 by 2. Root b is b power half, so, if you write root b as b power half this is 1 over 2 b. So, X square average is over two b. So, what did we find? We found two things . We found that X average of a Gaussian function of this particular problem? Normal distribution if you look at here if you look at this slide, you will see that X square. So, once we find that X square average is one over 2 b and we have found that X average is 0.

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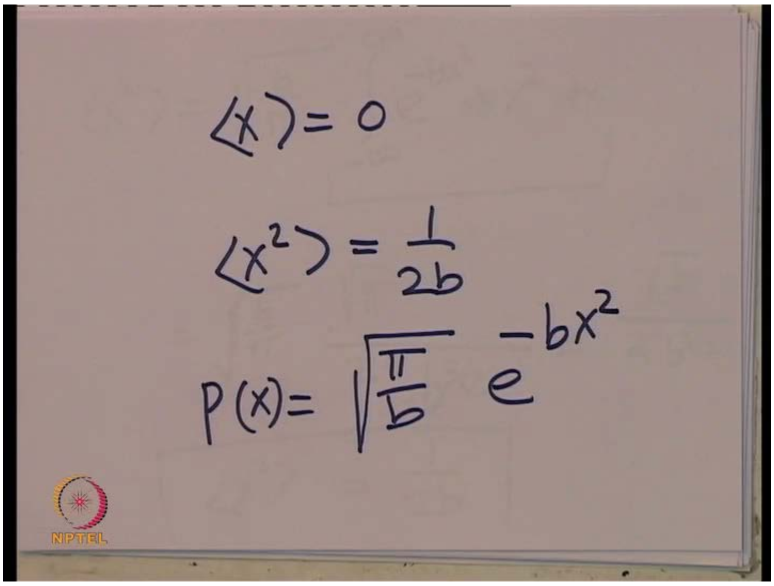

So, we found two things we had found X average is zero, X square average is one over two b, and what do we have, we have P of X is root of pi by b e power minus b X square.

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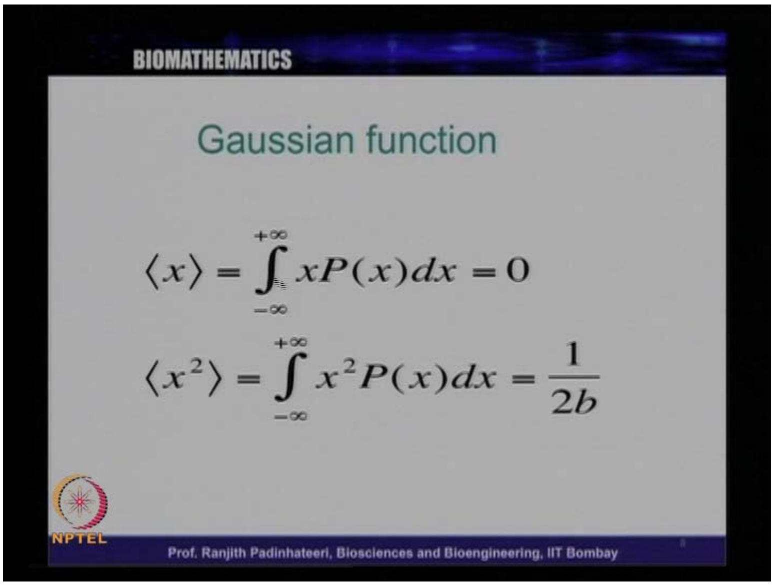

So, here this is what written in slide roughly here, that, X average is X P power P X dx which is zero and square average is X square P of X dx is 1 over 2 b.

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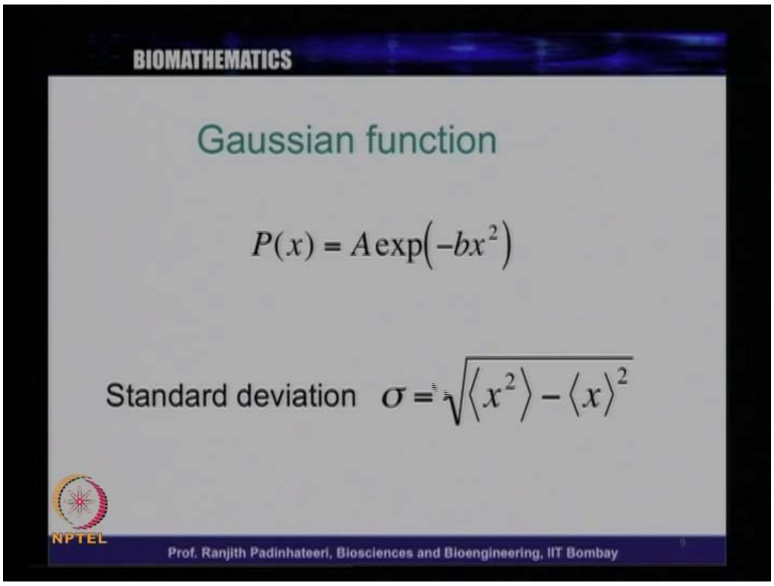

So, and what is the standard deviation if you know this two? If you know this the standard deviation is, X square average minus X Average Square. So, the standard deviation here X square average is this, X average is zero. The standard deviation is X square average root itself.

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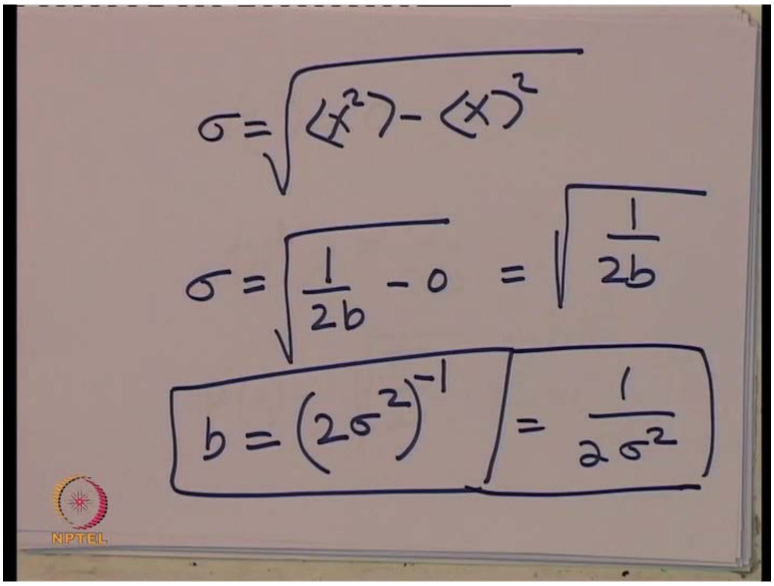

So, standard deviation is. So, the standard deviation is sigma, that is X square average minus X average square square root. So, this is 1 over 2 b minus 0. So, it's like 1 over 2 b what is this imply From this implies a b is equal to two sigma square sorry b is equal to

1 over 2 sigma square one b is equal to two sigma square inverse. So, that is the sigma square is equal to 1 over 2 b b is equal to 1 over 2 sigma square. So, we get we get a relation between b and sigma . This gives you a relation between b and sigma. So, b is 1 over 2 sigma square, this is b. b is 1 over 2 sigma square.

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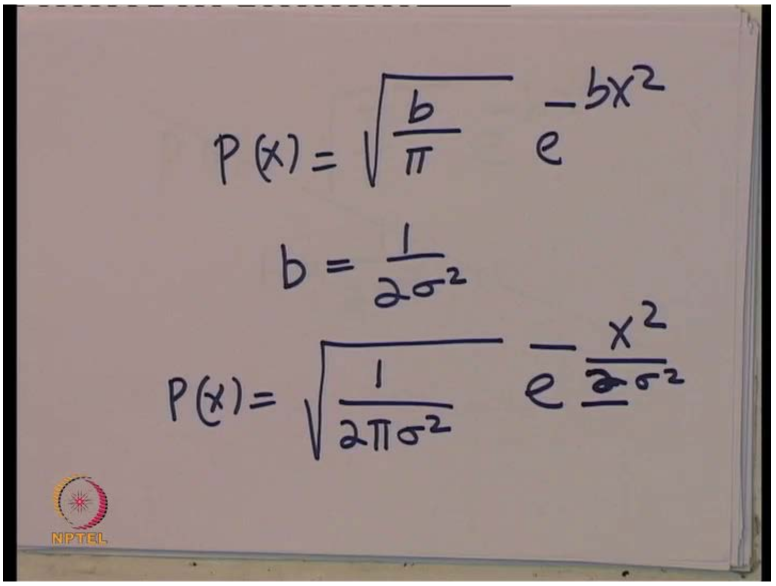

So, once you know the relation between b and sigma we can look back our function or function was, what is the P of X P of X was root of pi by b e power minus b X square and we just found that b is equal to 1 over 2 sigma square . By So, I substitute this back here, sorry this is a wrongly written. So, this is the distribution function with P of X is, root of b by pi e power minus b X square and we found that b is equal to 1 over 2 sigma square. If I substitute this power here, What you get? You get P of X is equal to root of b is 1 over 2 sigma square. So, you get 1 over 2 pi sigma square e power minus X square by 2 sigma square. This is 2 X square by 2 sigma square, this is 2, this is 2. So, P of X is equal to one over root of one over 1 over 2 pi sigma square e power minus X square by 2 sigma square.

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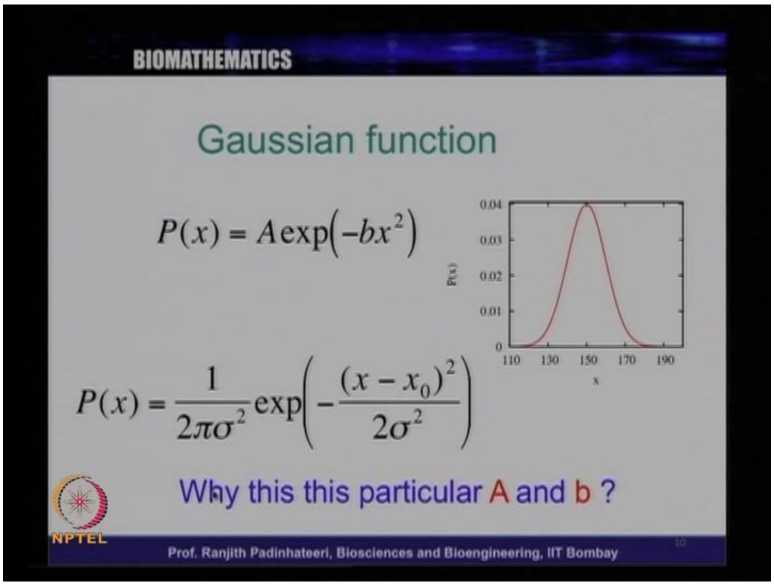

So, this is what precisely written here. So, then we have ask this question, why this particular a and b? We found that a is. So, there is there is a square root here sorry there is a there is a square root here which is forgotten here.

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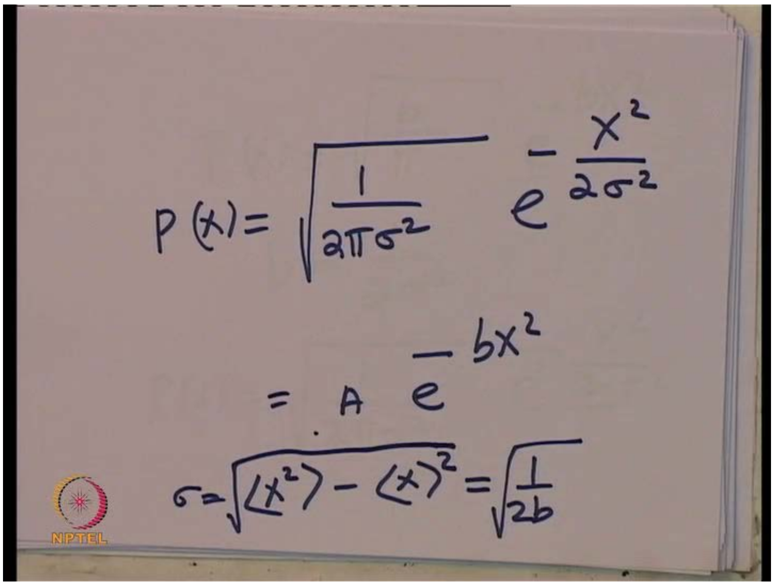

So, the correct form has it should be, it is P of X is root of 1 by 2 pi sigma square into e power minus X square by 2 sigma square. So, this should be the correct form. So, this is the precisely the reason, why sigma square is put here? Because if you just call it, A e power minus b X square the and we found that the if you calculate from the you take this function, and if you take this function and calculate the sigma that is X square average minus X average square. What we get is, 1 over 2 b ; that means, root of 1 over 2 b. So, what what we what a sorry. Sigma is root of X square average minus X average square and if you found this we found root of 1 over 2 b. So, that gives a relation between b and sigma. So, that gives P of X this particular form. So, this is the reason why P of X is particular form here sigma is coming here and here.

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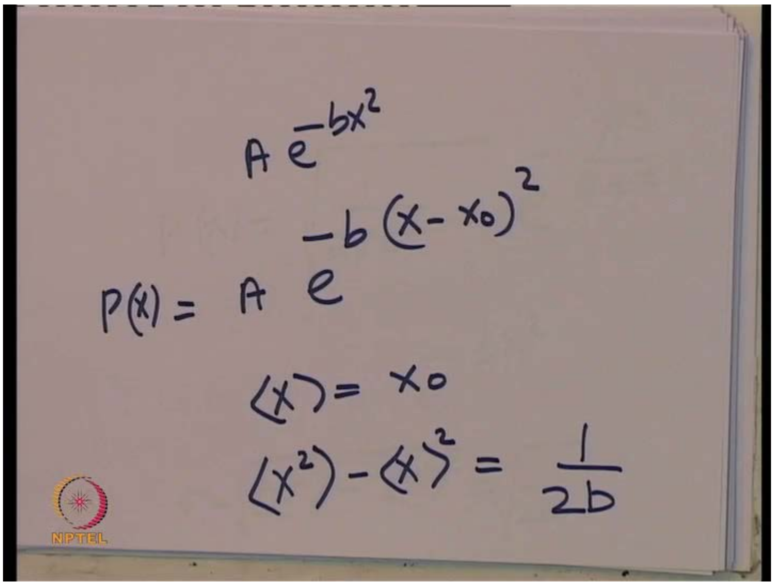

So, we found X average and X square average. Now, this is the function, we took was, if the function we took was A e power minus b X square if you have taken the function , P of X is equal to A e power minus b X minus X 0 whole square. Same answer you have got except X average you would have got X 0 and X square average minus X average square you would have got 1 over 2 b ok. So, this you can see yourself by just finding that X average in this case and the standard deviation is the variance X square average minus X average square is a variance.

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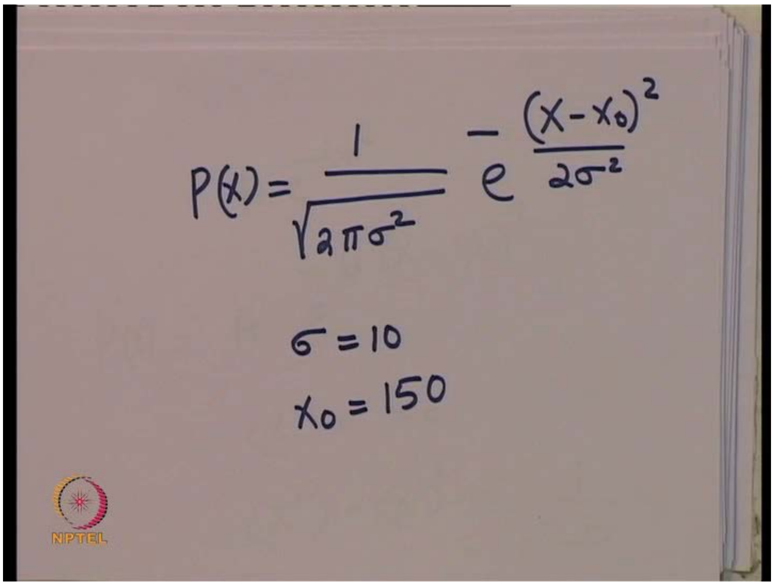

And variance is 1 over 2 b. So, the full function the full function the probability distribution function P of X, can be written as 1 by root of 2 pi sigma square e power minus X minus X 0 by 2 sigma square whole square. Now, given thus this has particular form, just by looking at this particular function, just by a graph of this function what all can you know? Just by looking at the graph of this function, what all can you know?

So, let me let me take some particular value, sigma is equal to ten. So, sigma square is hundred. And let me take X 0 equal to 150 just and so values and just just plotting this particular function by taking this two values you get this what we have proved earlier we get this. So, what is this? tell us, see have a look at this slide here. What does this tell us?

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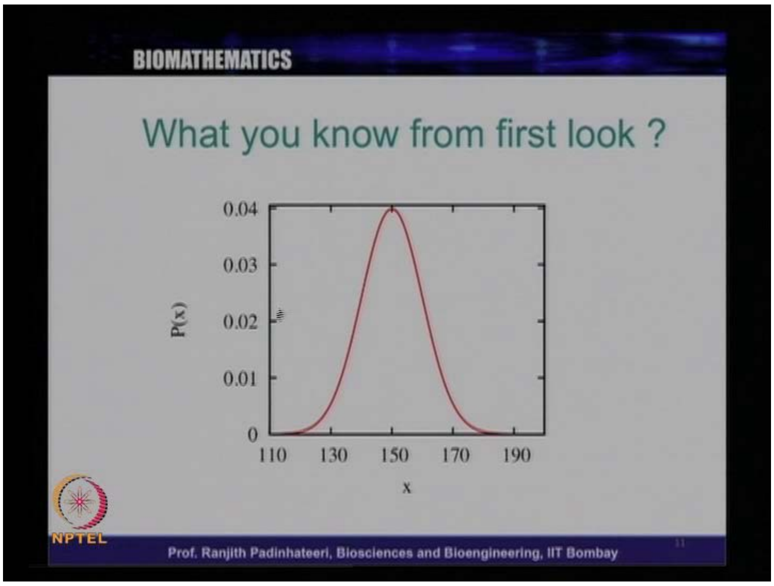

So, if you look at the slide here, this tells us that. So, this is exactly that is plotted here. So, what do you know from this look by just looking at this, the peak is at 150. So, the once in the peak is the point where is average value. So, X 0 is 1 by 150. So, X average is 150. So, X average is 150. So, what is this mean? X zero, the peak is at 150. So, just by looking at the peak, we can know what average value is. So, average values easy to find, just look at the peak, the position of the peak is 150 that will be the average of this distribution.

Now, how do we find the standard deviation? How do we find this sigma? So, just by look at this slide here. So, you look at this paper here. So, we know that P of X is one by P of X is one by two pi sigma square e power minus X minus X 0 whole square by 2 sigma square and X average is 150 which is X 0. So, just note couple of things, if you know X 0 and if you know the sigma, you know everything about P of x. So, you have to know only two things, it will be X 0 and you have to know sigma then you can know, you know, everything about P of x.

So, I told you if I find X by looking at the peak, we can find X 0 and how do we find sigma? We want to find the sigma. So, it turns out that sigma is related to the width of the distribution. So, if you if you look at here. If you look at the slide, the width this distance. So, this are related. So, how do how is this related the you can you can measure the width here, the width here the width here. We can measure the width of the distribution at various places. Which of this width will give you sigma, will this distance give you sigma or will this distance gives you sigma or will this distance give you sigma? So, this is the question which of this distance will give you sigma. So, let us look at this again the Gaussian function.

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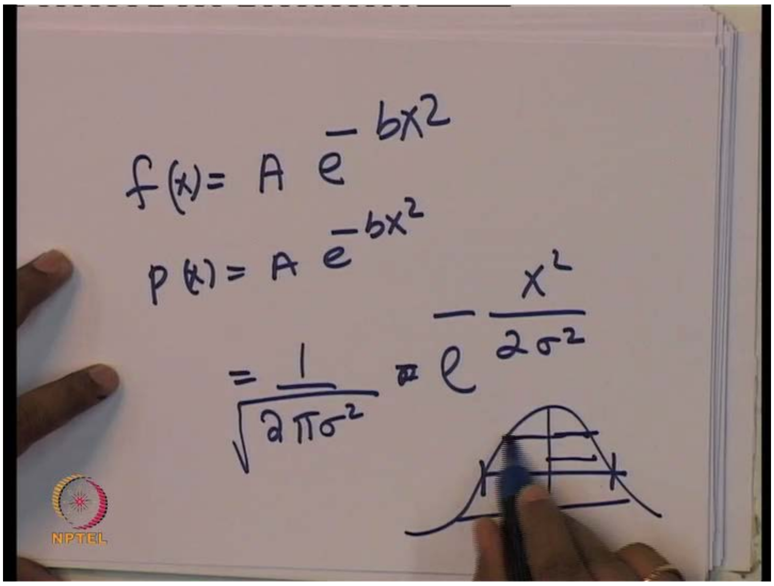

So,Gaussian function is A e power minus b X square. The Gaussian function is A e power minus b X square or P of X , if you wish. Now, this we know that, this is nothing but 1 by 2 pi sigma square square root into e power minus X square by 2 sigma square. So, for when will it let us say? So, when will it sigma will be equal to... So, what what we want to we want just by looking at the width by measuring this distance, we want to get the sigma, but where will you measure the distance that is our question we measure it here or will you measure it here. So, what we want? We want this distance. So, this is if this is X this distance is two X. So, we want two X to be sigma that is what we want, for a particular value of y.

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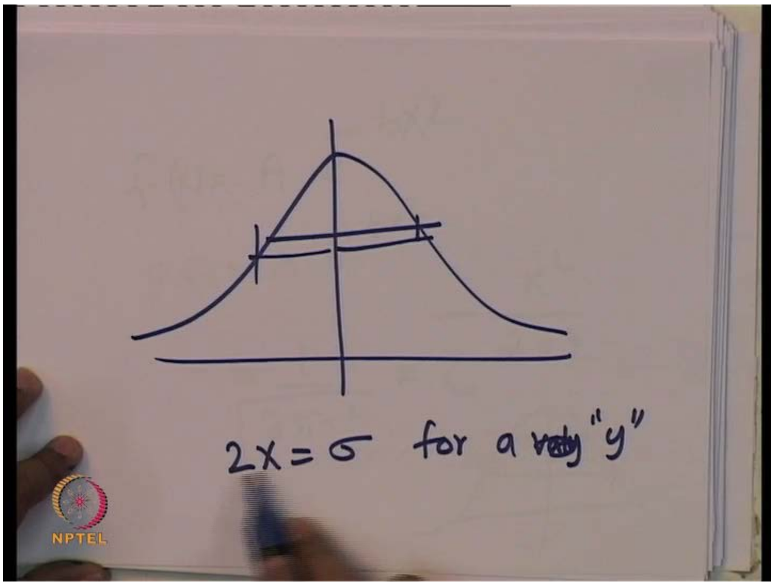

So, if you if you look at this. What we want? We want for a particular value of y for a for a given, we want this distance that is X and this is also x. So, the two X we want a sigma for a value of y. So, given a y, there is a place of there is a particular value of y, for which two X is sigma. What is the value of y, for which 2 X is sigma. So, let us first put 2 sigma is equal to 2 X and then calculate.

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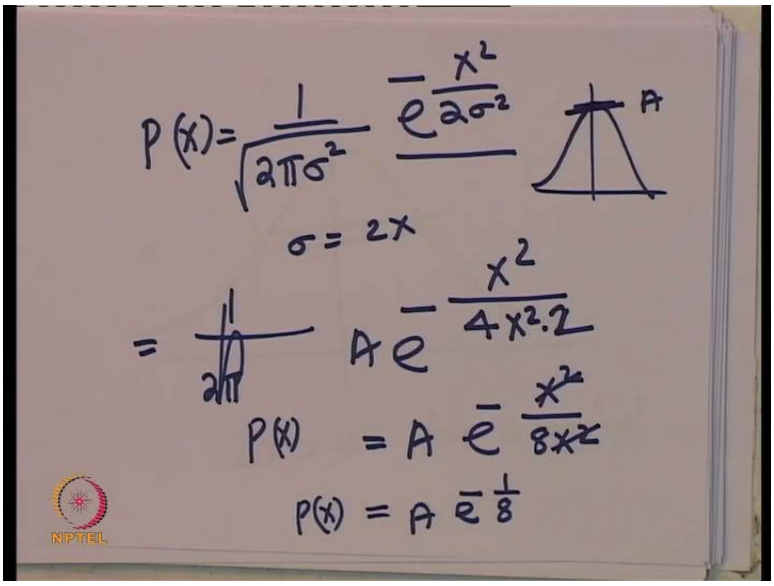

So, what we have, we have P of X is 1 over 2 pi sigma square square root e power minus X square by two sigma square and substitute sigma equal to two x. So, what do we get?

1 over a root of 2 pi. So, this we want is just, So, this particular part we have look at this particular part if you look at this what you get e power minus X square by 4 2 into 2 into 2 X square. So, 2 X square is 4 X square and again 2 into 2 into 2. So, what we get, this particular part. So, we get A let me call this an itself this part. A e power minus X square by 8 X square. So, we will get P of X is A e power minus a X square by 8 X square X square in to X square goes. So, what you get get A e power minus 1 by 8.

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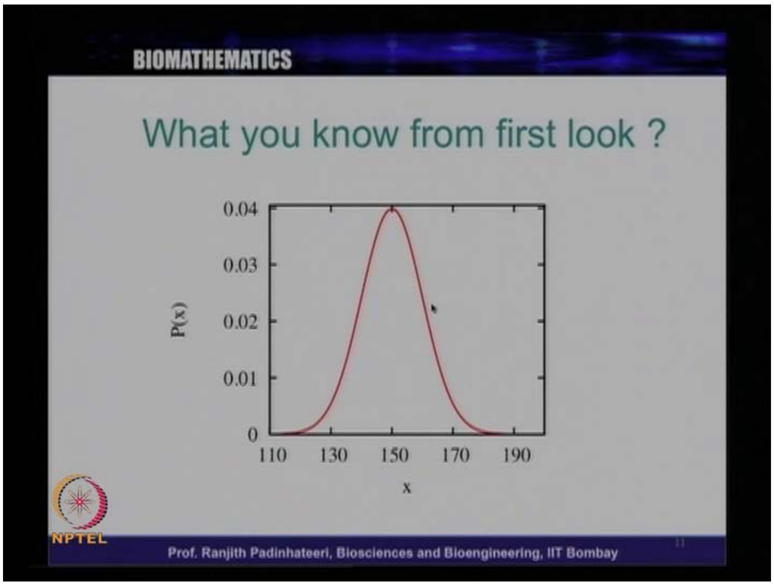

So, what is A? When X is equal to 0, if you put X is equal to 0 P of X is A. So, if you e power minus 1 by an if you multiply a with e power minus 1 by 8 at that particular value A is this as a constant is a maximum value, which we know A is the peak. So, A we know just by looking at it, because, if you have a function, this value is a this is the peak. When X is equal to 0, what is the value of y? That is A. So, if you multiply A with e power minus 1 by 8 that will be the y, where sigma is 2 X, if the width is equal to sigma. So, if you look at the distribution, if you look at this particular distribution and multiply this P with e power minus 1 by 8.

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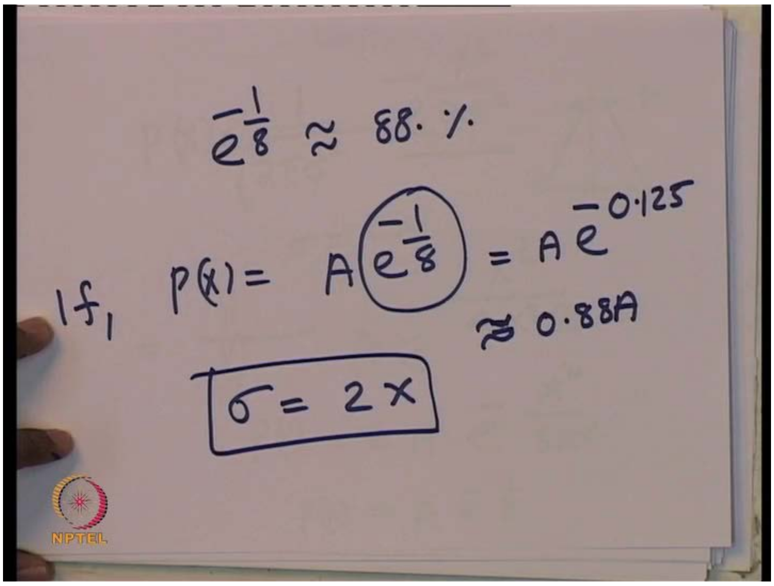

So, what is e power minus 1 by 8. So, it turns out that, e power minus 1 by 8 are approximately 88 dot some percentage. So, it approximately 88 percentage. So, if you multiply P one P of X is A e power minus 1 by 8, if what did we find? We found that, if P of X is a e power minus 1 by 8 sigma is equal to 2 x or in other words we found that, if sigma is equal to two X P of X is a power minus 1 by 8. So, if So, if you look at that eight if I look at 88 percentages from this peak. So, you multiply e power the peak by this particular number, which sends out to be 88t percent because, this is e 1 by 8 is around 0.125. So, point e power minus A e power minus 0.125. So, this is the coming 0.88.

So, this is something like 0.88 of A approximately. So, if you look at the 88 percent of this the peak, somewhere here. If you measure the width the 88 percent at this particular point, you will get the standard deviation. So, the width so, the peak at the peak the X value at the peak is the average and the two X the width, the width at e power minus 1 by 8 of the peak is the standard deviation. So, just by looking at it, we can know standard deviation as well as the average. So, to summarize what did we learn.

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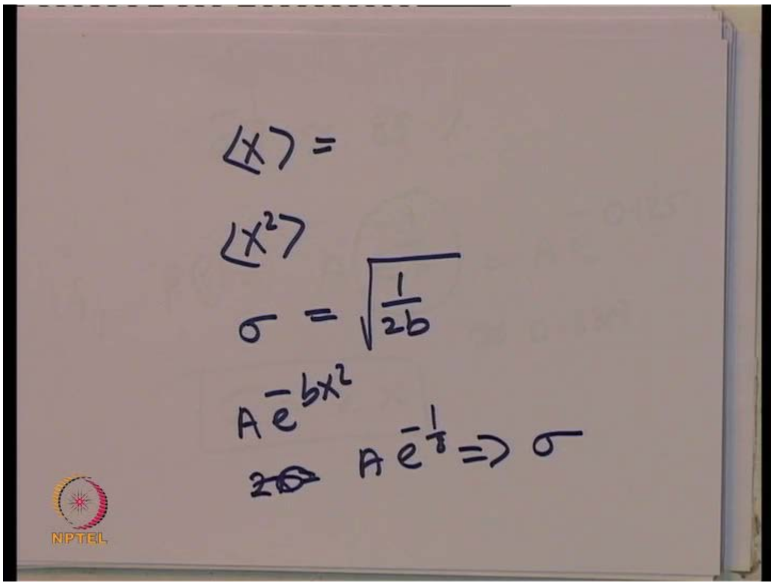

So, we learnt that we learnt that, X average if is we are what is we forward we learn how to calculate X average, we found out how to calculate X square average, we found out how to calculate sigma. So, if you have a particular function of the Gaussian function, we found that sigma is related to if you have a function of the form A e power minus b X square sigma is 1 over 2 b square root.

And we found that the peak is X average and the width at e power minus 1 by 8 times of the peak maximum value is 2 X, which is, at the at the 2 X a width at this particular point will give you the standard deviation. So, we learnt a new many things about normal distribution. So, use this information, use this ideas next time when you see a Gaussian distribution just look at where is the peak and the width at e 88 percent of the peak and also do yourself, what is the width, the full width at half of the maximum. We found that the width at 88 percent of the maximum is sigma.

What is the width at half of the maximum? You find out yourself? So, keeping keeping this things that will learned in mind that integral of e power minus d X square and average in X square average this will be very useful for various things in the future. So, discussing this, we will stop today’s lecture. Bye.