Biomathematics
Dr. Ranjith Padinhateeri
Department of Biotechnology
Indian Institute of Technology, Bombay

Lecture No. # 02
Graphs and Functions

Welcome to the second lecture of biomathematics. In this lecture, we will be studying about graphs and functions. So, this is the second lecture, where we will discuss more about graphs and more about functions. As you might remember, in the first lecture, we mentioned that we actually, basically, described the idea of functions, and we said that functions and graphs are related, and we also showed that many of the biological systems– many of the… we also discussed many graphs that you would get in a typical biology experiment. So, there are growth curves; there are… there are molecular motors walking along microtubules, and many other cases.

So, in this lecture, we will go and try and understand some of those graphs. So, the mathematical idea behind those graphs is– the equations behind those graphs– is what we will try and understand in this lecture.

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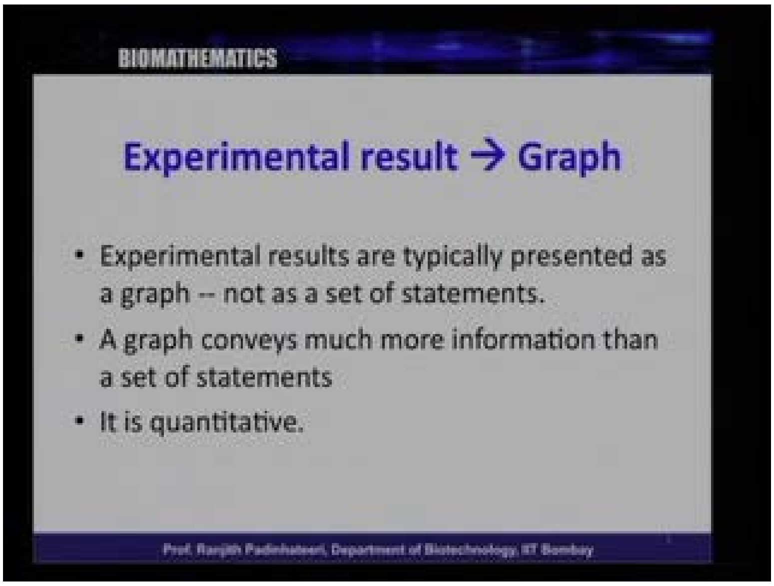

So, essentially, in this, what we discussed last time is as seen in the slide; we said last time that experimental results– the relation between experimental results and graphs. So, when you do an experiment, what you do is basically, you get a set of data and you plot this data as a graph. So, you get some, for example, like let us say you are measuring the length of actin as a function of free monomer concentration. Then the more the concentration, the more the growth speed is– actin will grow faster.

So, you can, for example, plot the growth velocity versus the concentration. So, you will have a… you do the experiment and you vary the concentration and you measure the growth velocity, or you could vary the concentration and measure the average length of the filament. So, depending on which experiment you are doing, you will be getting a table where concentration is in one axis and length or something else in the other axis.

So, basically, you will get two tables with two columns and you plot this– these data points that you get– into a graph. The reason you are plotting into a graph is, as we said last time, is that a graph can convey much more information than a set of tables or a bunch of statements can convey, and it is very… it is quantitative; it can precisely say a few things about the system that we are studying.

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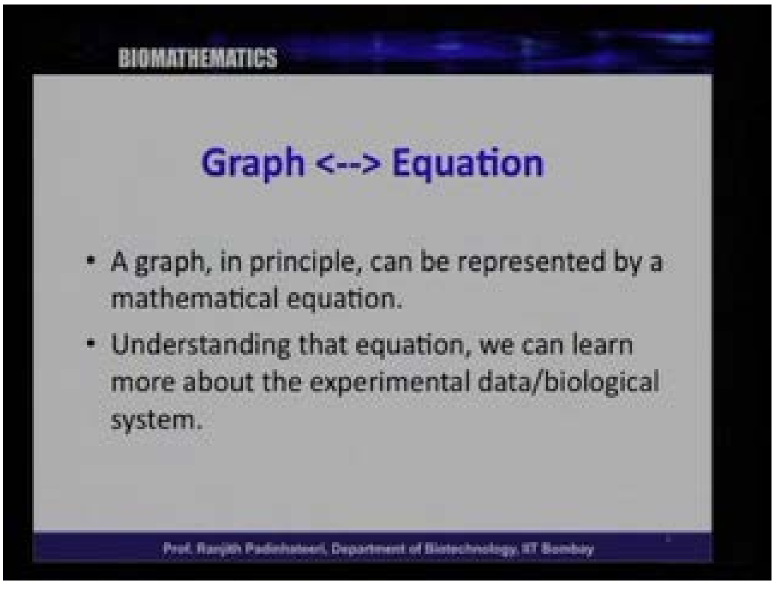

So, in this lecture, so, as we said, there is some relation between graph and equation. So, every graph that you plot can be represented by an equation. So, this equation could be very complicated, but the in principle, every graph that you plot can be fitted or represented by an equation, and understanding that equation, basically, will help us understand the biological system in more detail.

So, that is our aim; that is why we understand the equations. We want to study the equations, and therefore, understand the system biological system of our interest in a better way. So, as we say here, in this lecture, we will learn how simple equations can be plotted as graphs. So, we will take… we need simple equations and see how we can plot them, and how can we connect this to a graph.

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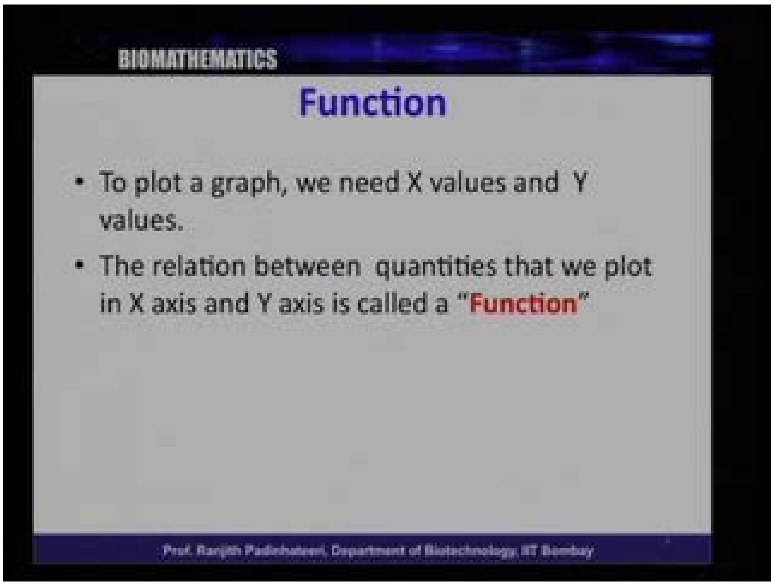

So, graph and equations– that is what we are going to study in this… in this lecture. So, as we said before, to plot a graph you need an X-axis and a Y-axis. So, some values– some column, which is X-axis– which we will plot in the X-axis– and some other column, which we will plot in the Y-axis. So, a relation– there is a relation between these quantities in the X-axis, I mean the quadratic in the Y axis. For example, there is a relation between concentration and the length of actin filament. So, or in other words, there is the position of something is related to time.

So, there is a relation between what we plot, typically, in a graph in the X-axis and what we plot in the Y-axis. So, this relation is mathematically called a function. So, I… we said this last time that function is essentially, is basically the relation between what we plot in the X-axis and what we plot in the Y-axis.

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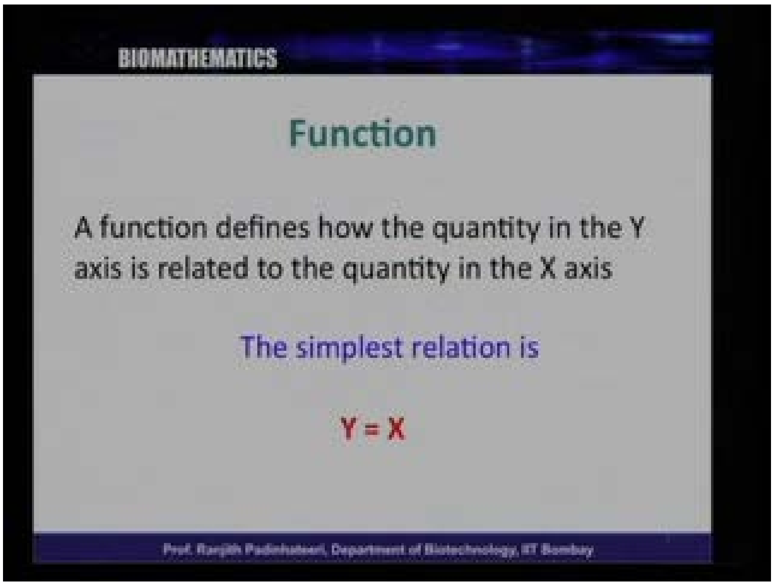

So, what is the simplest relation that you can think of? The simplest relation that you can think of is nothing but Y is equal to X. So, if we define a function as the quantity is… is how a quantity in the X, Y-axis is related to the quantity in the X-axis. The simplest relation is, basically, Y equal to X.

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X	Y
0	0
1	1
2	2
3	3
4	4
5	5
6	6

Now, how do we plot this Y equal to X as a graph? So, we will go and learn this today. So, let us say that you have a column with X values and Y values. So, what is the simplest values? X equal to 0; Y equal to 0.

So, like when X is 0, Y is 0; as you can see in the slide, when X is 0, Y is 0; when X is 1, Y is 1; when X is 2, Y is 2; when X is 3, Y is 3; 4,4; 5,5, and 6,6. So, this is the easy here points, and like a see… see to understand because for every X value the Y value is the same.

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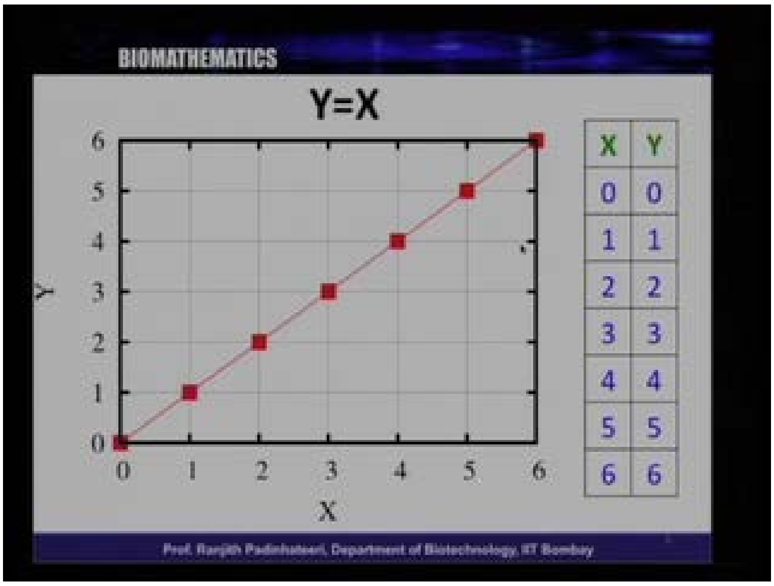

X	Y
0	0
1	1
2	2
3	3
4	4
5	5
6	6

So, this is the simplest function you can think of, and let us see how this is plotted. So, see the graph what we have plotted here. So, in the X-axis, we have X, which is 0, 1, 2, 3, 4, 5, and 6; in the Y-axis again, we have Y values– 0, 1, 2, 3, 4, 5, and 6.

Here, in the first point, the red square– fill square– red filled square is the points corresponding to these values. So, the first value X equal to 0; Y equal to 0. So, the (0,0) is represented by a point here, and the next one– the (1,1) in this table. So, what does it mean is that for X value 1, correspondingly, the Y value is 1.

So, if this is what it means for X value 1, the corresponding Y value is 1; similarly, for X value 2, the corresponding Y value has to be 2. So, if you start from X equal to 2, which is this, corresponding Y value is 2. So, you can see, X– where X value is 2, Y value is 2; similarly, when X value is 3, Y value is 3– the corresponding Y value is 3. The simple, as you all know, the way to plot a graph is keep putting dots for each pair of values, and you can connect them.

So, there are some details over how to connect these values, which we will discuss completely in a different set of lectures, but for simplicity, today, let us connect these values with a line. So, basically, what you will get is essentially a line. So, this set of… set of dots connects like a line. So, Y equal to X is a straight line. So, this is the simplest straight line you can think of: Y equal to X.

So, what does it mean, then? So, as we can see, Y is equal to X is a line which goes from through (0, 0). So, we learn the simplest graph in simplest function. So, what does it… what do we say here? Y is basically a function of X.

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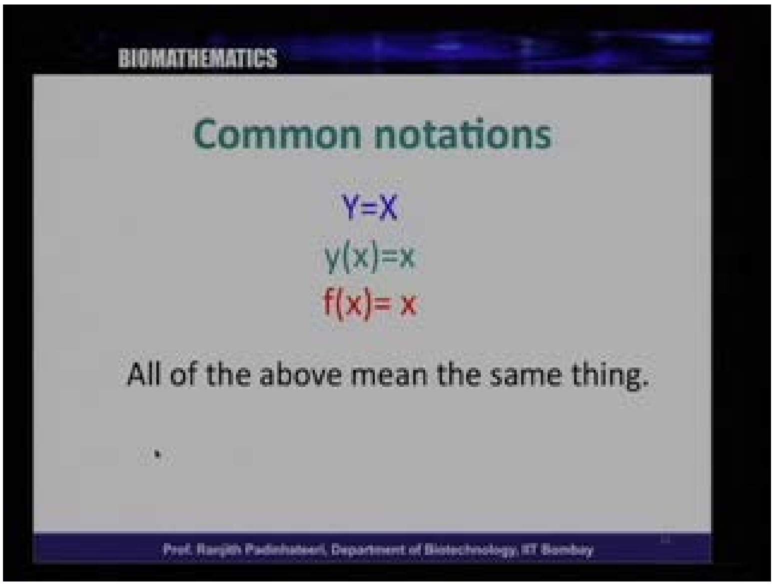

So, typically, this is shown in different ways in different books. Some books say Y equal to X; some other lecturer would say y of x equal to x; some other place you would see f of x equal to x; as you can see here, all this means the same thing. So, when a y bracket x– what does is it means is that essentially, is that y is the function of y depends on x. So, the y value varies depending on how x value varies.

Similarly, when is a f bracket x; that means, the f– the function– f stands for function and the function depends on the X value, and how it does depend? It depends like x. So, this the simplest way of saying as. So, this is one way of saying Y equal to X, or there are three different ways where… which we can see in different places, as you, for example, if you deal– see a… read a book, so the book would have returned y of x equal to x, or some books would read– write f of x equal to x.

So, all of this is essentially what we just saw– this curve– all of this means this curve; that means, for a given value of X, there is a Y value which is same. So, now, let us look at the next function. So, the next function is Y equal to 2X. So, when you say Y equal to 2X, what does it mean? The Y value is twice that of the X value.

So, let us look at this table which is here: for X value of 0, we have Y value which is 0; when the X value is 1, Y value is 2, which is double. So, that… that is what this 2X is means– twice that of the X value. When X value is 2, the Y value is 4, which is 2 times 2 is 4; similarly, 3, 6, 4, 8, 5, 10. So, for every X value, the Y value is 2 times that of the X value– this is what Y equal to 2X means, and let us sees how it is plotted.

So, in the plot here, you have X-axis and Y-axis. So, for every number in the X-axis, there is a corresponding Y number. So, there are these vertical and horizontal lines. So, they basically, as you know, you might… you know from your schools that how to plot a graph, and for every X value there is a corresponding Y value.

So, let us have a look on the… if you… if you look at this, you will see that when X equal to 0, Y is 0; for X equal to 1, Y is 2. So, let us look at this graph: when X value is 1, if you go from starting from 1, if you go up, you will reach the point which is corresponding to 2. Similarly, for X equal to 2, the corresponding Y value is 4.

For X equal to 3, the corresponding Y value is 6; when X equal to 5, the corresponding Y value is 10. Similarly, 6 has corresponding value 12; 7 has 14. So, basically, we have put a dot in for each pair of these values and connected them.

So, this is basically another straight line. So, if Y… X equal to Y is equal to X is the straight line, Y is it will 2X is also a straight line, as you can see here, and in… in… in the coming classes, we will try and understand what exactly is the difference between Y equal to X and Y is equal to 2X because this is important, and so, but as… as of now let us compare these Y values– Y is these two curves.

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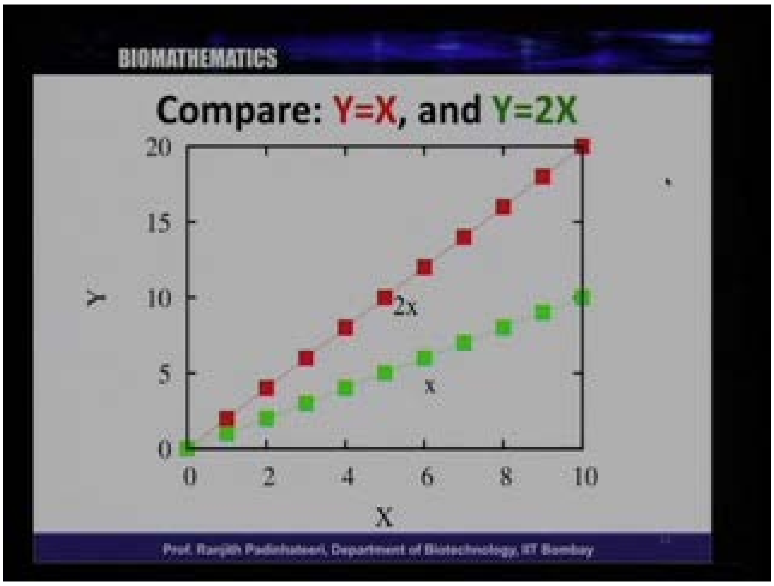

So, let us look at the next slide where we have compared Y is equal to X and Y is equal to 2X. So, if the red curve here, as you see here, red is Y is equal to as you what is written on the top is wrong… what actually is that the green curve is Y equal to X-axis marked here, and the red curve is Y is equal to 2X, which is… which is showed in the red.

So, what is showed in the top is change the color the color code is not correct. So, Y equal to X is shown in green and Y equal to 2X is shown in red. So, as you can see, there is difference when Y is X is 10, the Y value is 10; for the Y equal to X curve and when X is equal to 10 the Y value is 20 for the red curve which is Y is equal to 2X.

So, as you see in the graph, the graph is marked Y equal to X and 2X. So, you can see that the corresponding values for each of these X values, you can see corresponding X value and 2X value, and you can see that the second the red curve values are twice that of the green values. So, this is the basic difference between X. So, always, 2X is larger than X, which we know, which is evident from the graph except for X equal to 0 values.

So, we will come back to these two curves and comparison more in the coming lectures, but so, right now, we understood that Y equal to X is the straight line; Y equal to 2X is also a straight line. So, when…. So, in general, Y equal to 3 X, if you plot, it could be another straight line; Y equal to 4 X is another straight line. So, Y is equal to any constant times X is all a straight line and all of them will pass through 0.

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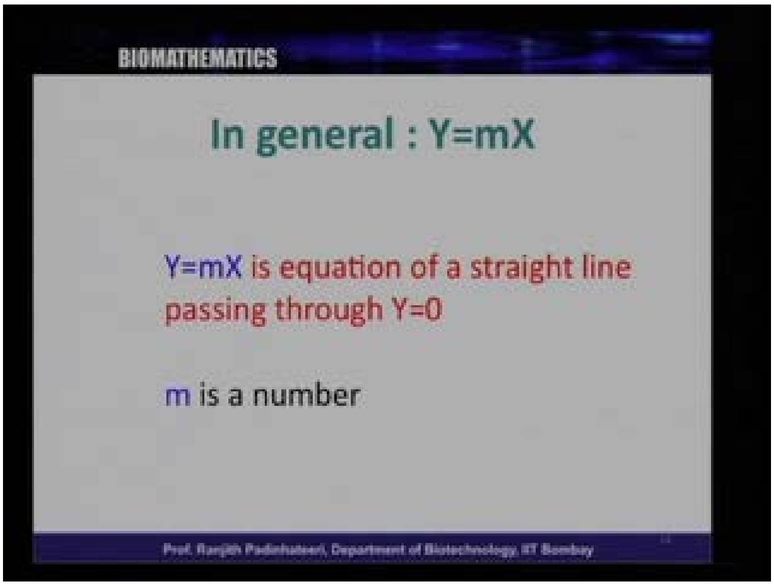

So, in general, we could say that Y equal to mX is basically equation of a straight line that passing through Y equal to 0, and when you say Y equal to mX, where m is any number, so 1X, 2X, 3X, 4X, it would even -5X; m could be negative number, and we will see later how this… when m is a negative number, how does this graph changes.

But for today, let us stick for m equal to… yes today, we will stick for m values positive; we will only take m is equal to 1, 2, or positive values, and we… we can try taking negative values and try plotting it and see how it is different from the values we saw. So, so far, we saw that Y equal to a constant times X is a straight line that is passing through Y equal to 0.

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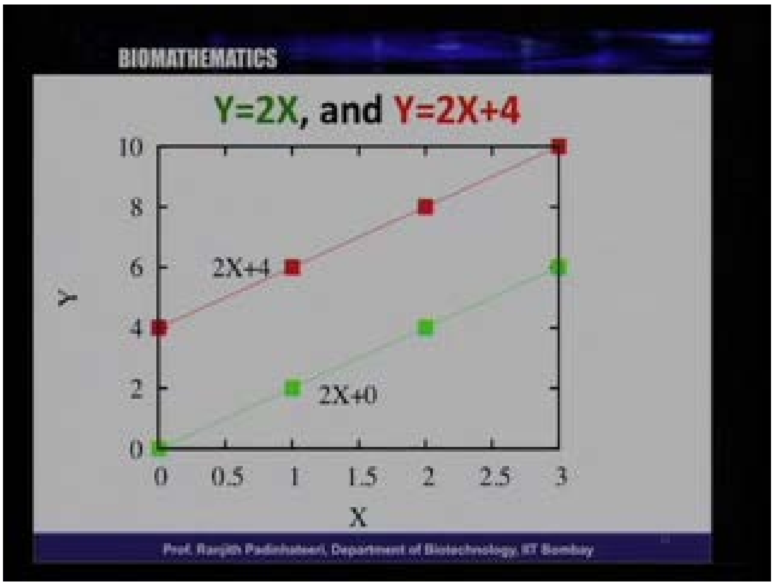

Now, let us see, if you add a constant to this, let us say Y is equal to 2X plus some constant, how does it change? In the next graph, you will see this relates to the next graph. So, next graph, what we have plotted is Y is equal to 2X plus 4 is in the red; the red dots represent a curve, which is Y equal to 2X plus 4 and the green dot is the curve, which we already saw, which is Y equal to 2X.

So, this is… basically, this graph is basically a comparison between Y is equal to 2X, which can be written as 2X plus 0, and Y equal to 2X plus 4, which is the red– the red dots– and the line– line points– represents 2X plus 4.

So, as you can see, at X equal to 0, the green has value 0; on the other hand, the red has value 4. So, what does it mean is that the curve Y is equal to 2X and Y equal to 2X plus 4 are shifted; they are shifted by certain amount– the value 4 here. So, the difference between Y equal to some constant times X, and constant times X plus some other constant, is that they are shifted by some values.

So, it is only in a difference is only a shift, as you can see in the slide, as you saw in the slide. So, in general, any curve which is Y is equal to mX plus c, where m and c are some constants, will remain a straight line. It is just that it we you can it can shifted from, so that could pass through different values in the Y-axis.

But it does not matter– whatever be the value, as we take it will be a straight line. So, that is… that comes… so, the… since us now we understood functions like Y equal to X, 2X, 2X plus 4. So, let us generalize this Y is equal to mX plus c.

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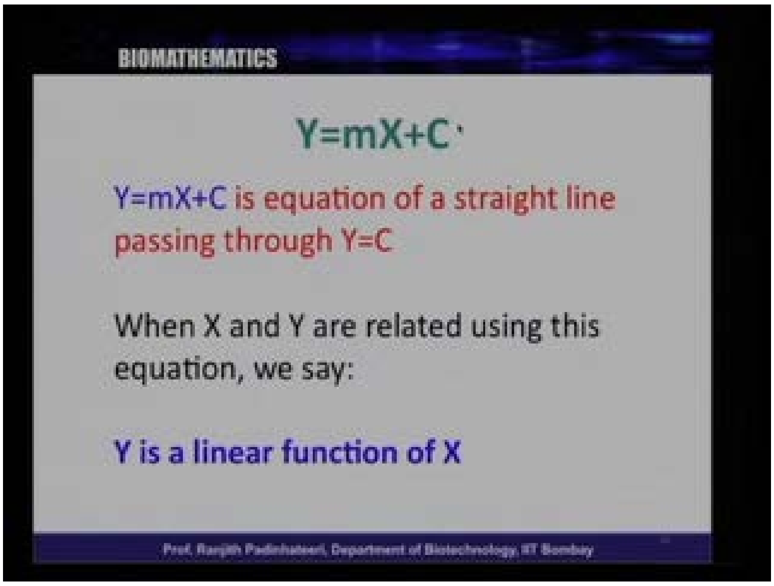

So, let us see the… the slide here, which says Y is equal to mX plus C is equation of a straight line passing through Y equal to C. You saw that m value has 1 and 2 and C value has 0 and 4, the previous slide where we saw and that this… this is the general equation of a straight line. When X and Y are related using this equation, the equation of the kind Y is equal to mX plus C, we will say that Y is a linear function of X.

So, when the… the term linear itself means it is like going along a line. So, when somebody says Y is a linear function of X, what they mean is that the relation between Y and X, if you plot a graph, it will look like a straight line. And if somebody else says a linear function, it only means that the… the relation is a straight line; if you plot a graph you will get a straight line. When two quantities are linearly related, when you plot a graph, you will get a straight line.

So, you could think of many examples where things are straight lines. For example, you… for example, if you have a circle, the circumference of a circle is 2 pi R. So, the radius and the circumference are related linearly.

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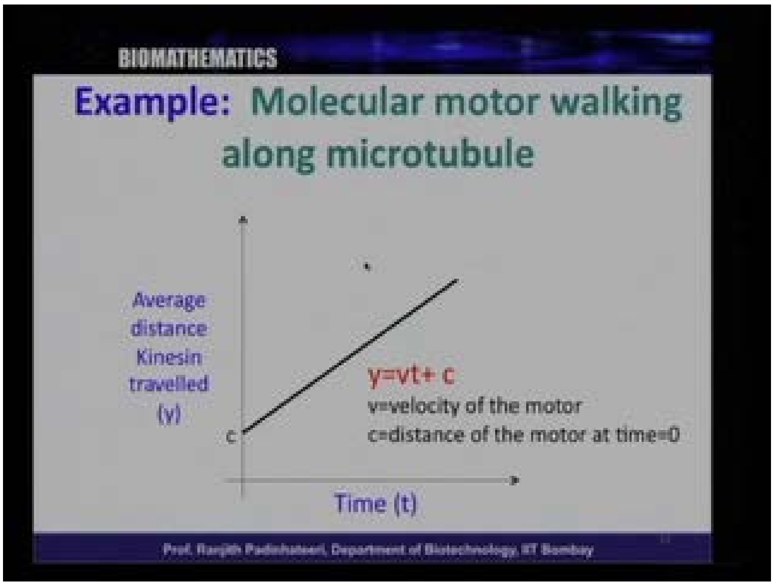

So, so, you could take another example from biology. So, let us take an example from biology where a molecular motor is walking along a microtubule. So, in the cells, microtubules are typically starting from near centrosome and going to the periphery–periphery of the cell, so, from the center to the periphery. So, microtubule will be walking from one end to the other end of the microtubule.

So, if we plot the distance from the center towards the microtubules travelling where the… sorry where the motor is travelling, you will in some… in some cases, you will get a straight line, and average… the average distance motor travelled from the center is essential… and if you plot the time, you will get like you will get essentially a straight line. So, let us look at the graph. So, what is plotted in the Y-axis in the graph is basically the average distance the kinesin travelled from the center versus time and you get a straight line.

And this equation is y is equal to v t plus some constant, where v is the velocity of the motor and c is the distance of the motor. So, this you would have to keep in mind that this is not the instantaneous distance, but this is the average distance. What does it mean is that you do many experiments and look at the average value, you will get the straight line.

So, this is the one example from biology were you have a linear relation between the position and time connected through velocity. We will come back to this example and understand this in detail in another context in another lecture, but in this lecture, it is… is enough to say that there are many functions that are Y and X; two quantities in biology are related linearly, and when they plot, you will get a straight line.

So, you could think of many, many experiments you did, where you got a straight line. In each of these cases, the relation between the quantities in the X-axis and the Y-axis are linear. So, now that we understand the linear function, let us go and try and understand the next case which is a quadratic function.

So, what is an… is the next case. So, we… as we understand Y equal to X, the next simplest thing is Y is equal to X square. So, let us go to the case Y equal to X square; we will have a look at the table. So, when you say Y equal to X square, what does it mean? For every X value, there is a corresponding Y value which is square of that.

So, the square of 0 is 0 itself; the square of 1 is 1 itself; the square of 2 is 4; the square of 3 is 9; the square of 4 is 16; square of 5 is 25. So, you have five values, and five X values and five Y values. The Y values are basically the square of X values. So, now, let us see how do we… if we plot this, how it would look. So, before seeing the plot, think about it. So, you had Y equal to X is a straight line; now, how would it the Y equal to X square would look like?

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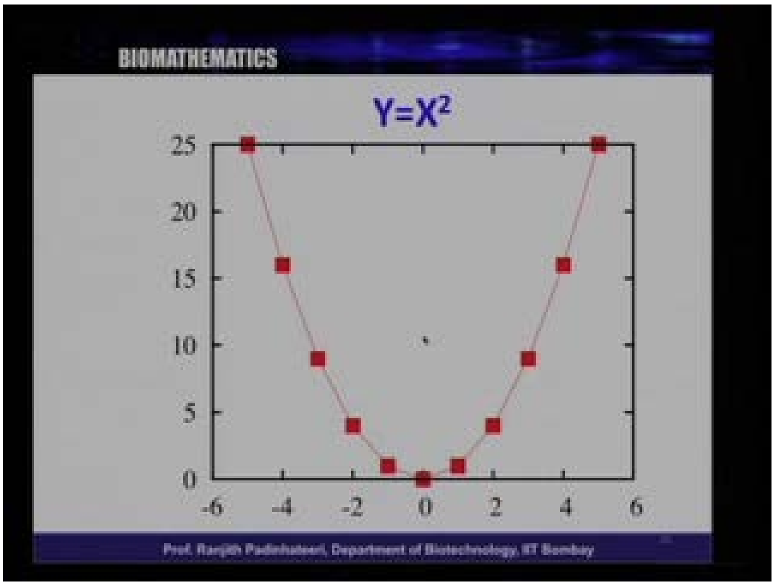

So, think for a minute and then have a look at this slide here. So, what you have here is basically for every X value, you have plotted the Y value as we tabulated here, and what you would get is not a straight line, but a curved line. So, there is a curve; there is a curvature. So, the Y equal to X square is slightly curved. We will compare soon the Y equal to X and Y equal to X square, but you can clearly see this curvature, and for every value of X, you have a Y value. So, Y equal to 1 has 1, 2 has 4, and 3 has 9, and so on and so forth.

So, you can see from the graph that Y equal to X square is a… is a slightly curved, is not a straight line. So, as you see here, this is how we plot Y equal to X square, and if we plot the negative plot, so, we had only taken positive values for X.

But if we take negative values for X, you would also get similar. So, you will have, for example, let us have a look at this graph Y equal to minus 2. You have 4– minus 2 square is 4. Similarly, plus 2 square is also 4. So, this curve is essentially symmetric to about 0, if you go to the right of the 0 or left of the 0, there is positive values or negative values. Y values are the same; when Y is 4, there are two points; when Y is 9, there are… nine is just below 10, 9, there are two values, because minus 3 square is also 9; plus 3 square is also 9.

For 16, minus 4 square is 16; plus 4 square is 16. For 25, there are two values because minus 5 square is 25; plus 5 square is 25. So, essentially, what does it say is that Y equal to X square is a symmetric function; does not matter X is negative or positive if you have same values for Y. So, it looks symmetric around X equal to 0 if you take X equal to 0 as a vertical line, and if you look both sides, it looks similar; it is symmetric.

So, this is basically simple as quadratic function as Y equal to X square. Now, let us compare. So, we… we saw Y equal to X and Y equal to 2X, and we saw that 2X is goes above Y equal to X in the graph.

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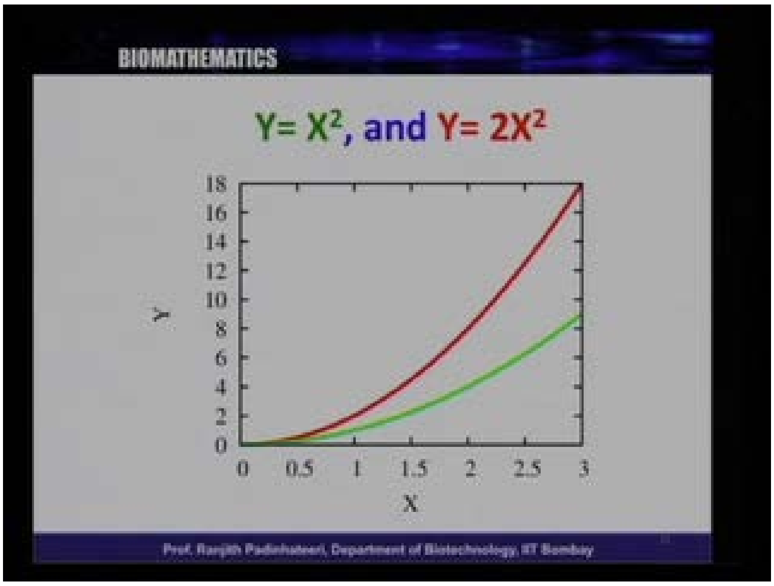

Now, let us compare Y equal to X and Y sorry Y equal to X square and Y equal to 2 X square. So, let us have a look at this graph here. So, when Y is equal to X square, you have this green line here, which go… this is Y equal to X square, and the red line is Y equal to 2 X square. As you expect, Y equal to 2 X square goes above Y equal to X square because these red values are twice as that of the green values. For example, let us have a look at 3. When it is 3 X equal to 3, here, X square is 9; where Y equal to X square– the green curve– this is at 9, which is just above 8; you can see here just below 10. Then, 2 X square is at 18, because 2 times 9 is 18.

So, this is… this is at 18 and…, but at 0, both X square and 2 X square zero values. So, starting from second point 0, they divert– they go to different values as we go along the X-axis. So, this is the comparison of Y equal to X square and Y equal to 2 X square.

So, now we learned two functions: a linear function Y equal to X and a quadratic function Y equal to X square. Now, let us compare. Let us plot these values in the same graph; let us see what do we get.

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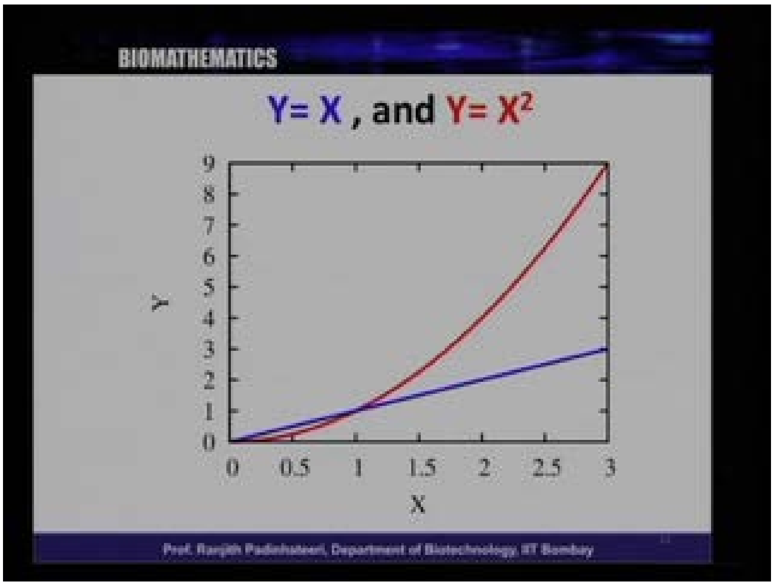

So, let us compare Y equal to X and Y equal to X square, and see the graph here in the slide. As you can see, the blue curve is Y equal to X is a straight line; the red is Y equal to X square. So, the red is curved and the blue is straight, and the interesting thing to note here is that below 1, so, this is the value at one at 1, as you can see, X and X square have same value at 1 square is 1 itself. So, this the curve; these curves meet at 1. They also have same values at 0, because X– 0 and 0 square is same– 0.

But between 0 and 1, the linear function, which is Y equal to X, is larger than X square. Above 1, Y equal to X square is larger than Y equal to X. So, X square is larger than X when X is larger than 1, and X is larger than X square when X– the X– is smaller than 1. So, at X equal to 1, they cross each other. So, this is an interesting point that you… you should keep in mind that when you compare X and X square, function having higher power will have a lower value below 1.

We will see this when we compare X and X square and X cube later, but always keep in mind when X is less than 1, X square is smaller than X. If this is… as you can… you can see, .5 has a square .25; .5 square is .25, and we know that .25 is smaller than .5. So, when we… when we plot– when we take any value of X below 1– and plot the X square will be always below the X, smaller than X.

So, this is one interesting thing and important thing to keep in mind. This will be very useful at some stages later and… but above 1, always, X square will… X square is larger. So, this is some interesting nice things about functions that one should keep in mind and this will be very useful as we go along.

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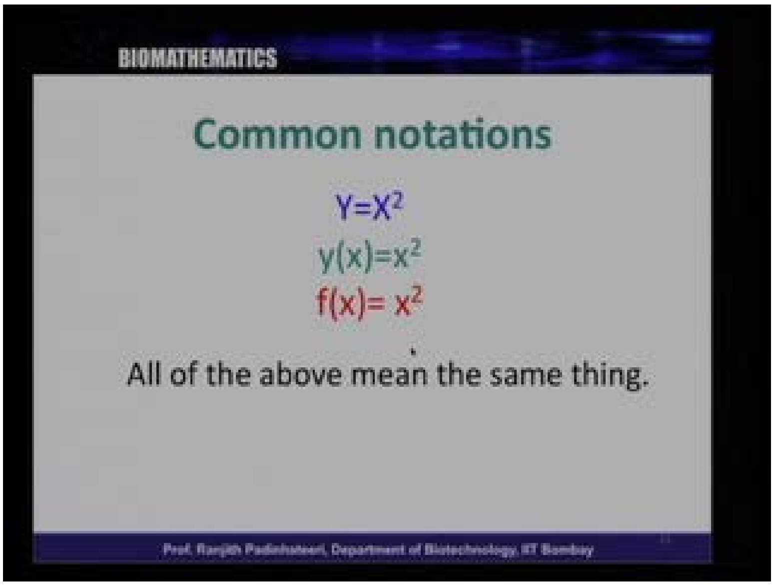

So, once we understand, as we saw last, previously, we had seen that we… we represent Y equal to X, Y equal to X square. We had two functions now, and as we saw, previously, that we represented this in different ways; we represented it has Y equal to X, y of x equal to x, and f of x equal to x. Similarly, you can represent in a different way here– three different ways you can represent, like in some books, you would see Y equal to X, y of x equal to x, f of x equal to x.

Similarly, in the X square case also, you could see as you see in the slide here, some people would represent y of x equal to x square, f of x equal to x square. As I said before, all of these mean the same thing; this all means that the y varies like square of x. So, the y, or in the function f, varies like the square of x. So, this is some, as you see, look different looks you would see different things, but always please understand that they all mean the same thing.

So, now that we understand a quadratic function, think where all have you seen this square– X square– behavior. The quadratic… the quadratic function– have you seen it somewhere? I am sure you have, and the simplest thing is area– area is square of radius– 4 pi R square. It is a formula which you remember from your school days.

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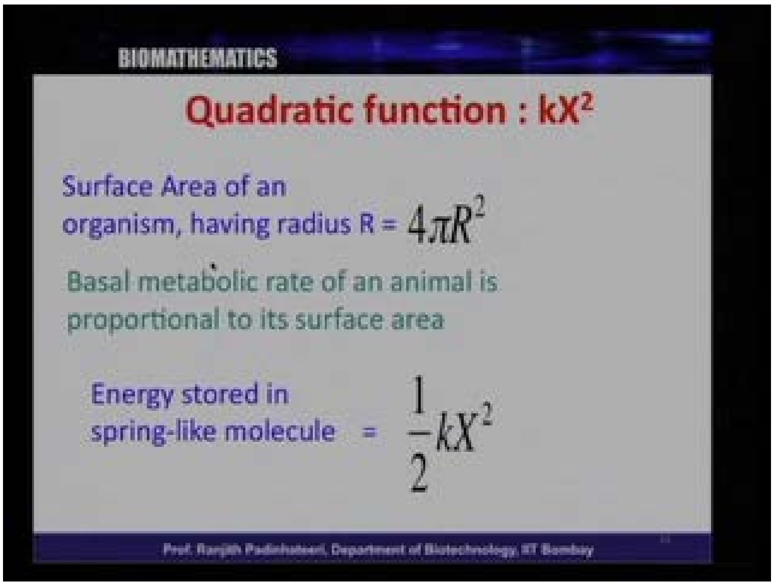

So, for example, the area– the surface area of an organism– their surface area means, if you have a spherical organism, the area– the surface area– is basically four pi R square again, and this is important, because as you see here, the basal metabolic rate, for example, of an animal is proportional to its surface area. So, basal metabolic rate will be a quadratic function of the radius– the more the radius, the metabolic rate could differ. So, it differs quadratically. Similarly, the energy stored in a spring-like molecule is half k X square, where the X is basically how much the spring is stressed. As you can see, the more as we saw here in the previously…, earlier, X square had a behavior, which is like increase like a curve.

So, as we… as I can draw here, we saw that when we plot X square, it was increasing. So, this is, let us say, half k X square. So, what does it mean? The more stretching– X is stretching– the more you stretch the spring, the more energy you need to give to stretch the spring. So, the energy of... So, this is true for any molecule. If you take any… any… any protein and stretch the protein, you have to spend more energy.

So, this is basically… the spring analogy is very useful for different biomolecules and the energy of stretching a biomolecule is quadratic function. So, the elastic stretching energy is a quadratic function. So, this is like metabolic– the basal metabolic rate– and stretching energy are two examples of a quadratic function. So, now we have seen this X, X square and the next simplest function is X cube, and let us try and see how if you plot X cube, how it would look like.

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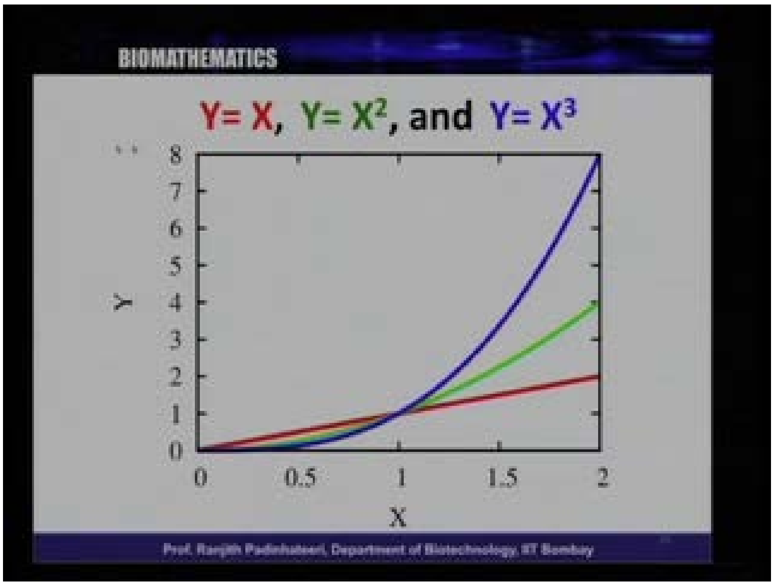

So, have a look at the slide. So, what you have plotted here is basically Y equal to X, Y equal to X square, and Y equal to X cube, and the red line where Y is equal to X, the green line– the green curve– is Y equal to X square, and the blue curve is Y equal to X cube, and as you can see here at 1, all of them meet; that means, the square at cube of 1 is 1 itself.

Below 1, highest, the X is larger than X square. The red is above green; X square is larger than X cube– that is a green is above blue. So, as I said previously, X square is larger than X cube when X is less than 1. But above 1, as you can see, the green gets larger as we go for larger and larger X. The green, sorry, the blue is growing much faster, which is X cube, and green actually grows faster than the red; this means the X square is growing faster than X.

So, for… for example, the… for the value 2 here, red has a value, when I say X equal to 2, Y is 2 for a Y equal to X; X square, it is 4; for X cube, it is 8, and the interesting thing you should note here is the way it grows– it grows much faster. So, when I say the blue is growing, the… when I say blue is growing faster, what does it mean? When you said what does it mean? To say grow is an interesting thing, and we will come and discuss in another lecture what is it means to say it grows faster. It is…. It is related to some interesting mathematical of mathematical property of a function, and we will discuss this in the coming lectures.

But just look at this graph and remember. Let us…. Let us have a look at this graph once more. So, just remember from this graph that X cube grows faster as we go along this; for larger and less X values, X cube goes faster. X square is not that fast, but faster than X, and X is the slowest. So, when I say faster– growing faster– there is some strict definition of this mathematical definition, which we will come and discuss another day. Now, we got introduced to this the cubic function. So, the cubic function is 4 pi. So, what is the simplest cubic function that you can think of– the volume.

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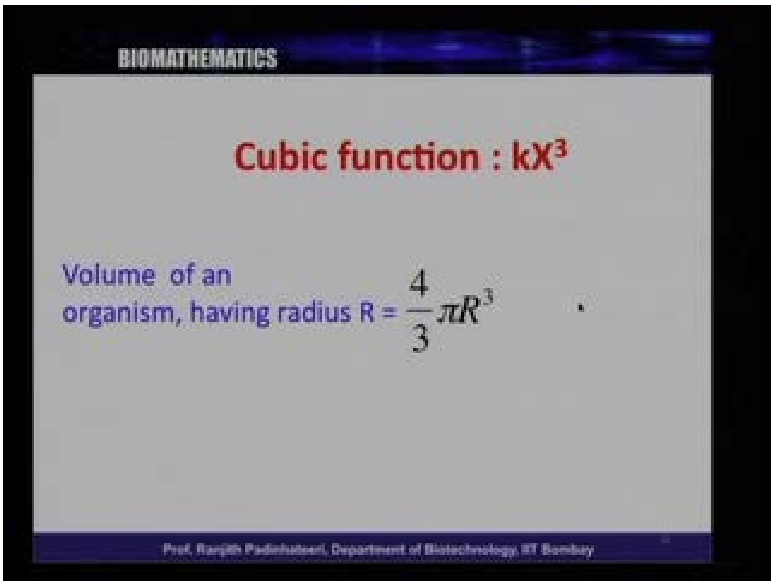

So, the volume of a sphere, for example, is 4 by 3 pi R cube– you all know. So, that is the simplest function– the cubic function is basically 4 by 3 pi R cube, which is volume of an organism, for example, having radius R is 4 by 3 pi R cube. So, you could think of other examples been the property– the biological property– goes like a cubic function. So, now, we understand X square and X cube– we could also imagine X power 4; X power 5; X power 6; X power 7. So, you could all… Now that we are familiar with this, you could plot all of them.

And one interest… you could also imagine various combinations of all these functions. So, you could define a new function, which is X plus X square. So, let us have a look at this– a new function, here. Let us Y is X is what we learnt; we also learnt Y is equal to X square. You could think of new function, which is X plus X square– this is the different function. You could think of another function, which is X minus X square. So, you can add and subtract these functions and you will get a new different function.

So, one can imagine may is a… Now, you could imagine the same thing with X cube. So, let us say you have Y is equal to X plus X square minus X cube, or you could say X minus X square plus X cube, or you could say Y is equal to X minus X square minus X cube plus X power 4.

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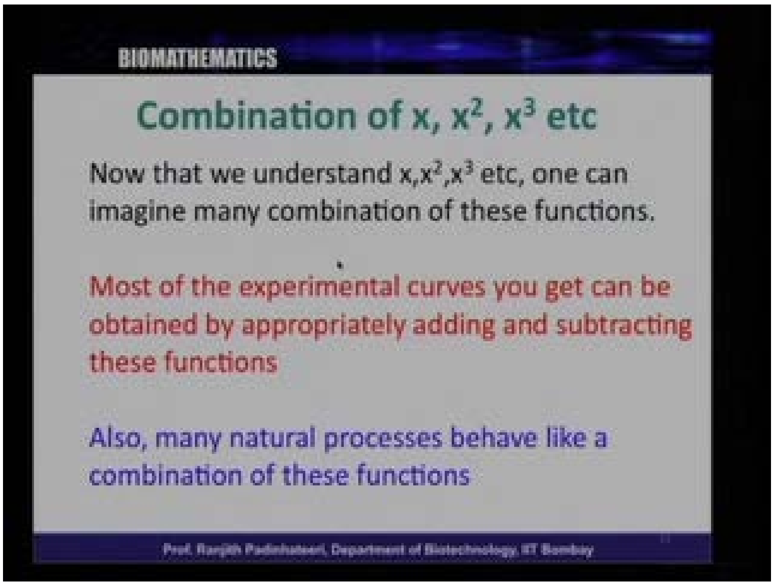

So, one could imagine various combinations of all these functions, and it turns out that as you can see here, most as you will see in this lecture, soon that many of the most of the experimental graphs you get when you do experiment, you get very complicated graphs and which is very different from what you saw now. X, X square are simple graphs, and… but you get a complicated graph– many complicated graphs– and it turns out that most of these graphs can be represented as a combination of this X square, X cube, X power 4, X power 5. Just like we saw, we can appropriately add and subtract these functions and you can get any other function. So, this is important this is very interesting feature

that….

So, this is very important to remember that any graph– almost any graph that you get– can be obtained by adding or subtracting this X square, X cube functions in different ways. So, what people call it linear combination? So, this is the combination of X square, X cube, etcetera, etcetera, will give you another function, and most of the function that you see in experiments can be represented as a combination. Also, many natural processes that you see can be modded– fitted– with this kind of combination of these functions.

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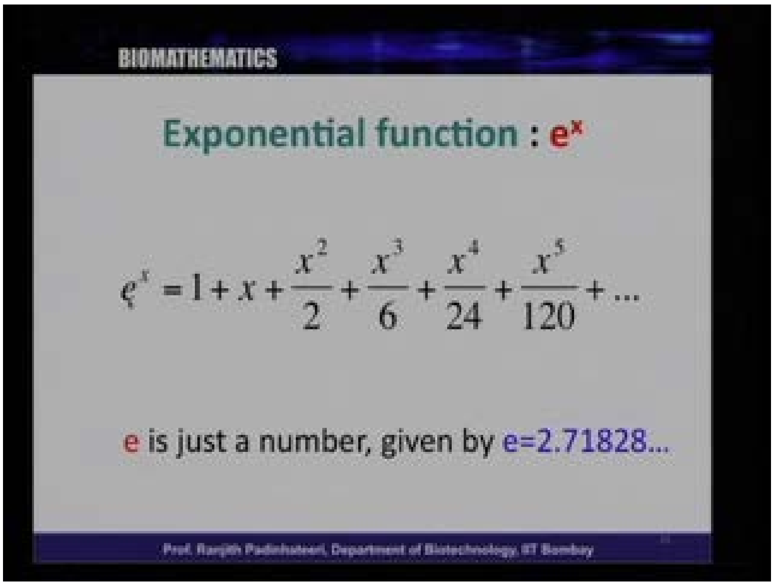

So, one very common, very simple, and very commonly seen function is called exponential function. So, you might have heard of… you might have seen it somewhere– e power x is called exponential function. So, this exponential function, essentially, can be written as some combination of series of this x, x square, x cube, x 4 and all that.

So, let us see how this exponential function is defined. So, this is, as you can see here, exponential function e power x is basically defined as 1 plus x plus x square by 2, plus x cube by 6, plus x power 4 plus 24 x power 4 divided by 24 plus x power 5 divided by 120 plus so on and so forth.

So, you can this is an infinite series. So, you have to some x power 4, 5, 6, 7, 8, 9, 10, and you have to… you have to keep summing. So, if you just keep summing, you will get this value of e power x.

So, where do we stop? So, in the coming slides, we will see how– where– to stop this. So, do we have to sum till infinity? Is it practical or can we stop at some point? We will discuss this in the coming slides, but as you know, e is just a number– it is a irrational number– which is 2.7, approximated 2.7– 2.71828 so on and so forth.

So, and e power x is basically, this is defined in this particular fashion as a combination of these functions– x square, x cube, x 4, etcetera,

and these values– coefficients– 1 over 2, 6, 24, etcetera, then just as for now, just remember that there is some combination– some particular combination of x square, x power 4, etcetera, and we will try and understand later that where this combination calls from and so on and so forth.

But for now, it is sufficient to understand that e power x is defined in this particular fashion; e power x is defined as 1 plus x, plus x square by 2, plus x cube by 6, plus x power 4 divided by 24, and so on and so forth.

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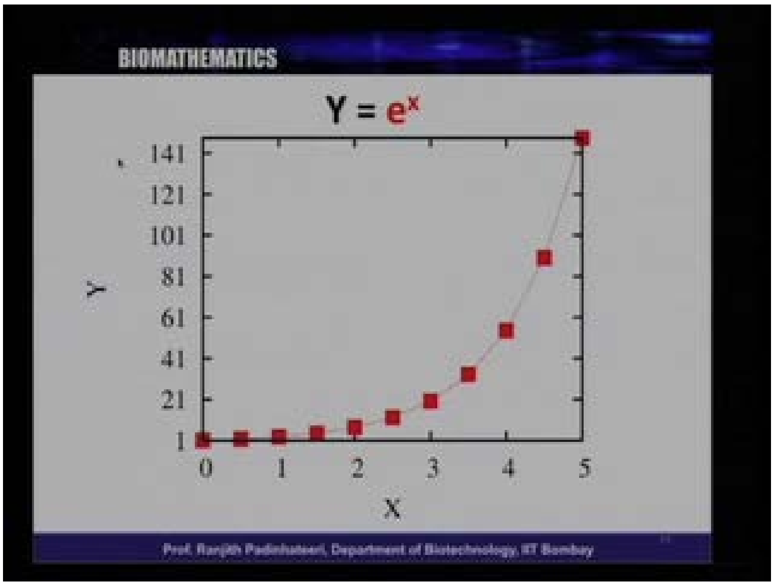

So, now, let us see how, if we plot this function x power… e power x, how does it look like? So, let us have a look. So, here is what we have plotted is e power x in the Y-axis and x in the X-axis. So, as we did previously, we can… we have… we can have table: x in one column and e power x in other column, and you will you can plot these points and connect them together; that is what we have done here.

So, each of these point represent some pair of values. For example, when x equals 0, e power x is 1, as it we can see, as we saw in the previous slide, if we put x equal to 0 everywhere, here all of these… this term was 0; this term was 0; this term was 0; this term was 0; this term was 0; here, just 1.

So, at x equals to 0, e power x is 1. So, that is why when x equals to 0, you have e power x is 1, and you should remember whenever you see a graph, you should remember two things: at the two ends, how– what– are the values.

So, x equals to 0, it is e power x is 1. As x equals to infinity or x grows, e power x grows like is a… is curved– it is like a curve; it is not a straight line, and you can see that it is increasing; it increasing rapidly. So, we will see later that is it larger than x or larger than x square and so on and so forth.

So…. So, as we saw, e power x is basically defined as an infinite series. Now, let us ask as question: where do we stop the series? Can we stop at x square, x cube? Is it correct to say e power x is 1 plus x, or is it correct to say that e power x is 1 plus x square, that is x square by 2? You start correct. Always.

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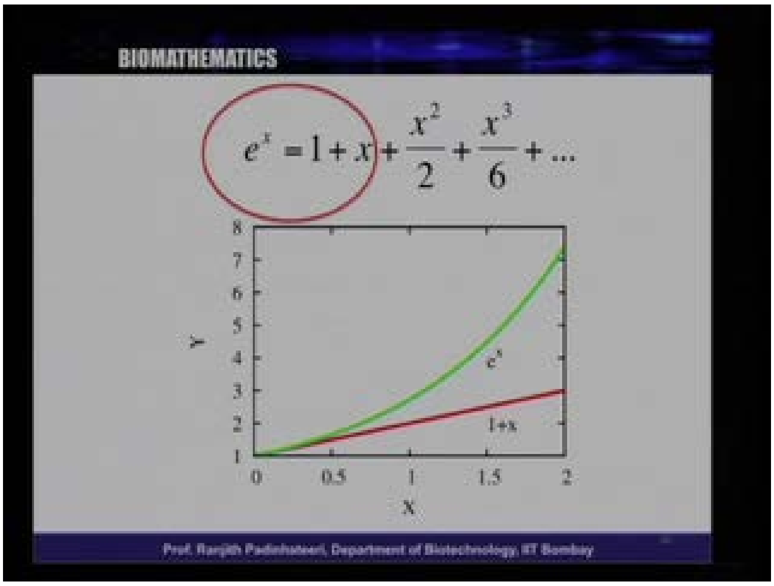

But at some cases, you can approximate e power x with one plus x and so on. So, let us look at this next graph here. So, what is plotted here is basically, in the Y-axis, you have two curves. So, green and red– the red is Y is equal to 1 plus x; the green is Y is equal to e power x.

So, when the 1 plus x is the straight line, it is a linear function; e power X is not a straight line. It is a curve, and so, as we can see, very close to 0, they have same values. So, this red and green meet close to 0, but as they go away from 0 they diverge. So, this goes like this and this x– 1 plus x goes… goes in this square. So, what does it mean? it means that close to 0, 1 plus x and e power x have same values.

So, it is to approximate e power x as 1 plus x when x is very close to 0. So, when x is very close to 0, one can say e power x is approximately equal to… that is what the sign means. The sign means approximately equal to; e power x is approximately equal to 1 plus x when x is very close to 0.

Now, if we take little more values like 1 plus x plus x square by 2 plus x cube by 6 and so on, what do we get? So, in this series, so we saw that e power x is a very long series– infinite series. If we take more and more terms, we took two terms: 1 and x 1 plus– two terms we took, and we saw that only when very close to 0, e power x, and this 1 plus x matches, and plus, x match when they are far away from 0, they diverge. So, now, let us take few more terms– 1 plus x plus x square by 2 plus x cube by 6 and so on and so forth, and see whether it matches when you go little more away from 0, whether the value– their values are same or not.

So, let us have a look at this graph. So, what is shown here is that the green is e power x; the red is here 1 plus X plus X square by 2 plus X cube by 6. So, this function– what is shown in this red ellipse– this is what is plotted here as a red line. 1 plus X plus X square by 2 plus X cube by 6 is plotted using a red line, and e power x is plotted using a green line, as we can see here, we know one pretty might. So, all… almost all the places, it they match. The function e power x, and this 1 plus x plus x square by 2 plus x cube by 6, they match with each other close to 0, and above .5 also they match.

It is a very close. Around 1, they start diverging and e power x goes faster than the sum of this quadratic, cubic, plus linear functions. So, two things we understand from here is that the more terms we take, the approximation becomes better.

So, one could write again that e power x is approximately equal to 1 plus x plus x square, plus x square by 2 plus x cube by 6, and the approximation is better here because we took more terms. Previously, we had only two terms, and here we have four terms.

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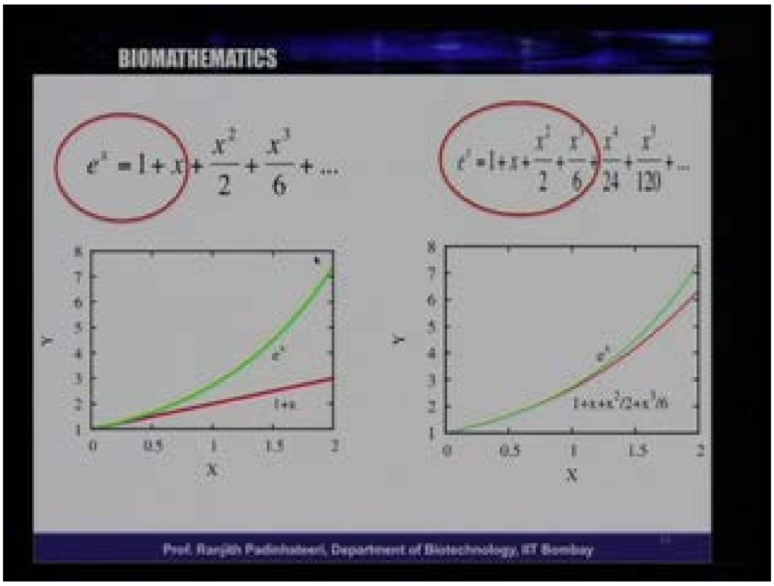

So, let us compare these two approximations. So, have a look at these graphs, and in the X-axis, in the left side, we have these two terms– 1 plus x. So, the approximation is with two terms, and in the right, we have approximation with four terms– 1 plus x plus x square plus x square by 2 plus x cube by 6, and you can see that on the right side is much better than what you see on the left side.

So, what you learn from is that if the more terms you take, the function becomes closer to the e power x. So, if you take all the terms, you get exactly e power x, and you also see that e power x is larger than the more the higher the x, e power x wins; e power x is faster than any of these combinations.

Like if you take the sum, e power x is faster– grows faster– is growing faster in a faster phase. So, we will come and see what it means to say that it is faster, slower, etcetera, but what we saw so far, today, is basically we will studied we… we saw some mathematical– simple mathematical functions– like Y equal to X, Y equal to X square, Y equal to X cube; we also saw that the combination of X square, X cube, X, etcetera, gives a sum of the functions, and some known– well known mathematical functions– like e power X can be represented as a combination of these functions.

And in the coming lectures, we will see how this combination gives rise to some very well known functions in mathematics. So, today we will stop at this point and in the coming class, we will take some biological examples, and we will see that how these examples– biological functions– that we plot as typically graphs in experimental data can be represented as a combination of these functions, and we will learn many more interesting functions, which is very important– relevant– in biology, and we will discuss their relevance.

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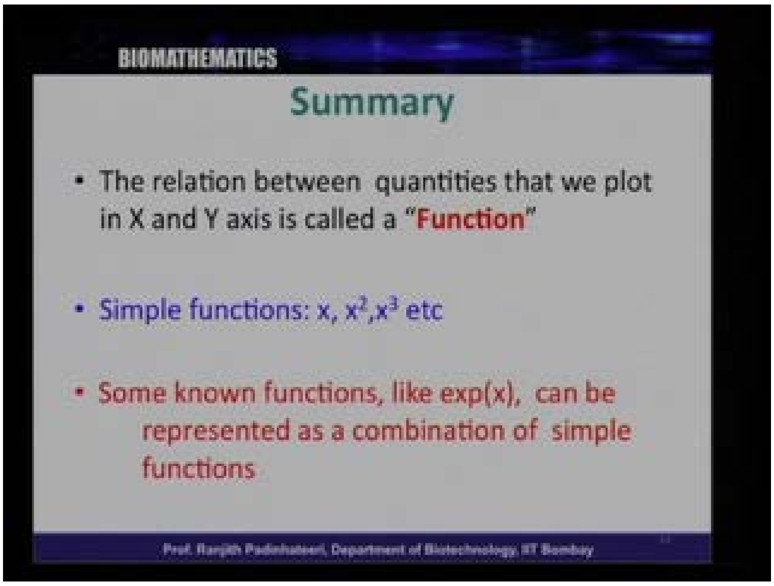

So, so… to summarize, the relation between quantities that we plot in the X and Y-axis is called a function, and we learnt many simple functions like x, x square, x cube, etcetera, and we also saw that some known functions like exponential of x– e power X– can be represented as a combination of simple functions like x square, x cube, etcetera, and in the coming class we will see many more interesting functions and see how they are related to some of the known biological experimental data. Some experimental data we get from biology experiments can be represented by as a combination of these simple functions, and we will see them in the next class. Thank you.