Biomathematics

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Lecture No. # 13

Differential equation part 2

Hello. Welcome to today's Biomathematics lecture. We have been discussing differential equations. So, in the previous lecture, we discussed the first part of differential equations; very simple, ordinary differential equations. Today, we will have the second part on differential equations. So, the title of this lecture is Differential equations part 2. We will go ahead from where we stopped last time, in the last lecture…So, where we discussed simple differential equations of the form…

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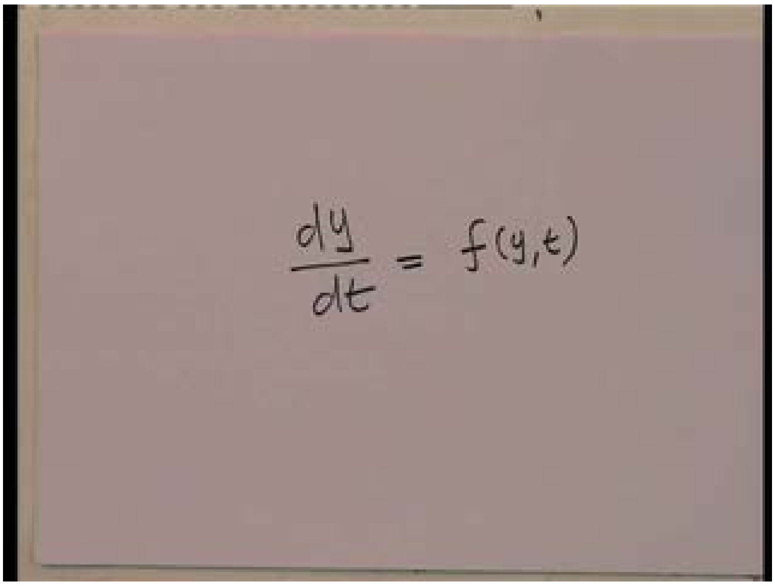

So, let me write here. The simple differential equations we wrote, like, something like, d y by d t is some function f, either of y or a t; something of this form. So, such differential equations, as we said, is called ordinary differential equations and today, we will go and see...So, we already discussed a few examples, few different cases, which is including bacterial growth. And today, we will go and see, which are the, which are the contexts in

Biology, where in Biology, you would need to think of differential equations. As you all know, one of the important things in Biology, as well as in Physics, in general, or in general in nature, is a relation between energy and force. So, you might have heard of potential energy, like electrostatic potential energy, when, where charges attract with each other; so, the force with which they attract and the energy, they are related to each other. So, they are related through a very simple differential equation in some sense. So, today, we will discuss, first, we will discuss the relation between the force and energy.

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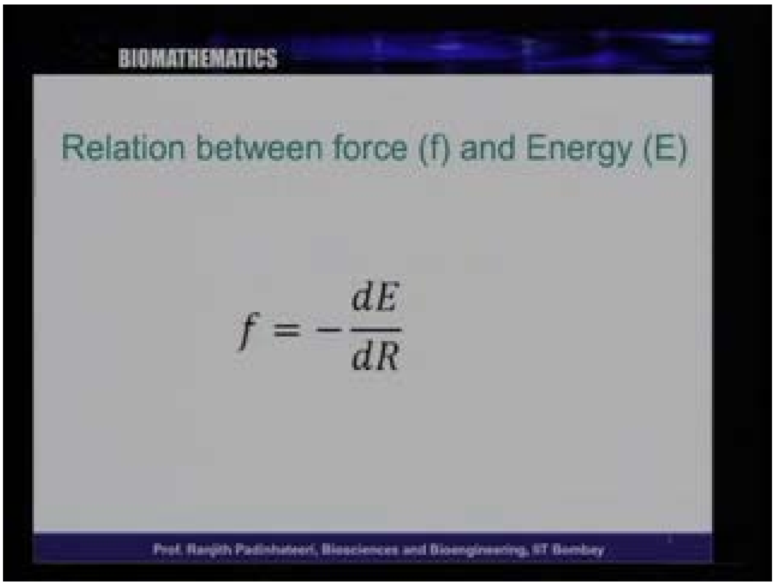

So, let us look at the relation, the relation between force f and the energy E. So, this energy here, could be potential energy. So, some places you might see this v. So, here, I am using the notation E for the energy; typically, you might see at some or some other, in some books, you might see, it is v or many other u; in some other places, you might see it as u; but essentially, what we are representing is the energy, the interaction energy. We will, as we come to different examples, we will see what E is. So, the difference, the relation between f and E, that is, if you know the E, the way to calculate f is to calculate minus d by, d E by d R is f; that means, the first derivative of energy d E by d R with a negative sign will give you the force. So, if, this is like a differential equation. This is like a differential equation, d E by d R is equal to minus f. So, one can integrate this equation and get energy, because if you know the force, you can calculate the energy by this, by solving the simple, the ordinary differential equation.

So, let us think of some simple energies. So, in Biology, like, there is all energy associated with proteins, their configurations, so on and so forth. But here, we will take a very simple case. So, it turns out that, one simple way of thinking about proteins is think about spring. So, spring is something like, an extend and compress and all that; proteins can also be pulled out. So, it can be extended in some sense; they get folded. So, they can come to a comeback state, from where you can pull, pull out the proteins. So, essentially, the simplest way of thinking about protein is to think about springs and charges; those are the two simple interactions, that one can think of easily and they correspond in some sense with proteins. So, we will, in this course, to understand the concept, we will think about the simplest example, that is a spring.

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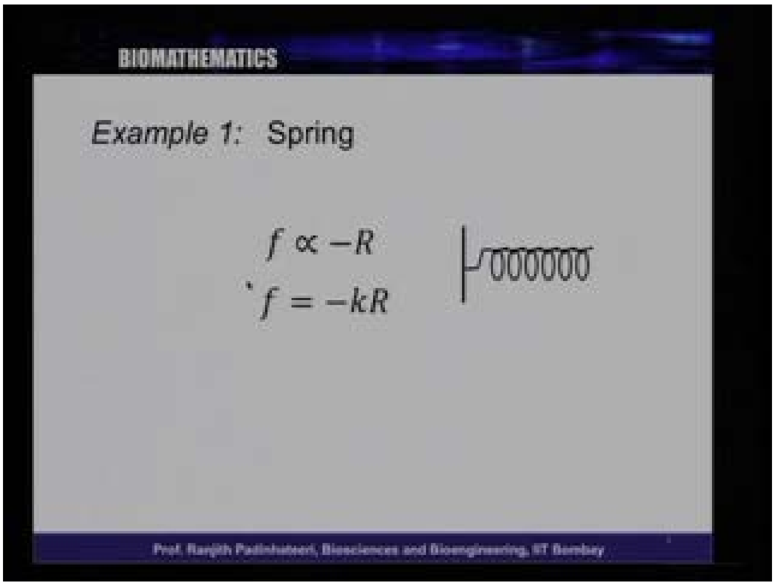

So, let us take the case of a spring and have a look at this slide. The spring has force that is proportional to the distance. Now, what is force and what is distance? So, what is the distance here? So, let us, let us try and understand a bit. So, have a look at this drawing here. So, you have a drawing here. Though, this distance…So, now, you, you have a spring, which you have some certain distance here, which we call R. So, it turns out that, the more the distance, the more the force is. So, you can always, all of you have taken a spring or go and take a spring and try pulling it; try applying a force. The more you pull it, the more force you have to apply; to hold it in a far distance, to hold it at a distance R, you have to apply less force; to hold this two ends of this springs' farther, you have to apply a larger force. So, it turns out that, the force and the distance are proportional to each other.

So, that is why, the force is proportional to R, the distance, but it turns out that, when you hold it, your experience, experiencing a force in the opposite direction. So, to show this direction, we have minus sign; the spring force is proportional to minus R. So, this is what, this is the basis of what is written here, in this slide, f is directly proportional to minus R. So, the more the force, the more the distance, this between two ends of this spring, if it is R, the more the distance, the more force you have to apply, but the direction is in the opposite. So, there is a minus sign. So, we know that, f is, you can remove this proportionality by, you can equate this with a constant k, where k is, you can think of k as a spring constant, which depends on the property of this spring. If the spring is made of some particular material, you will have particular k; if the spring is made of some other material, you will have some other k. So, k essentially, depends on the material property of the spring. So, we know that f is minus k R.

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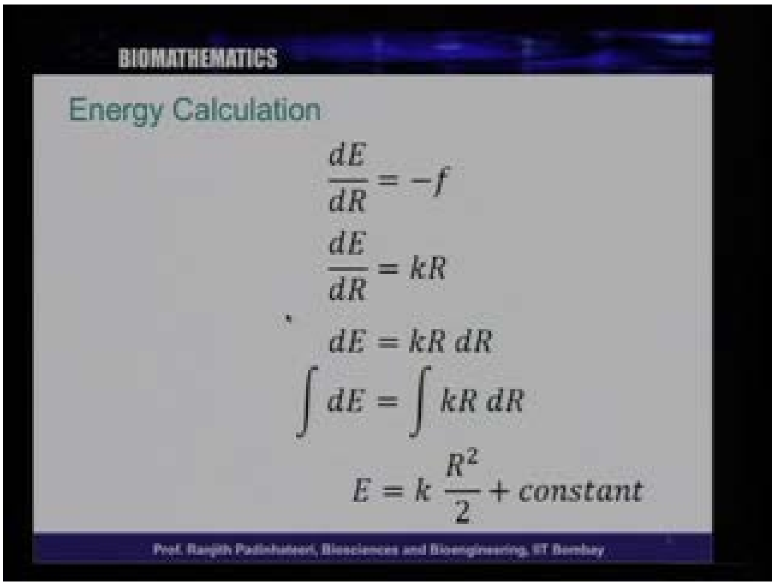

Now, let us go back to the old relation, which we, which we saw previously, and think about this. So, we saw this relation that, d E by d R is minus f, that is what we said. This is the relation between energy and force. Now, if we substitute f is equal to minus k R here, what you would get is, d E by d R is equal to k R or d E is equal to k R d R; that is, you can take this d R on this side; I can multiply both sides with d R in this part of equation and I get d E is equal to k R d R. Now, we integrate both sides, integral d E is integral k R d R. So, what you get is, E is equal to k R square by 2 plus constant.

So, this is what the integral of, integral of d E is E; integral of k R is, k is k and R is a constant, integral of R is R square by 2. So, basically, what you would get is, half k R square plus constant. So, half k R square plus constant. So, what you get is, energy is half k R square plus a constant. We always saw that, this constant is a part of the constant of integration. Whenever you do indefinite integral, you have to have a constant, which now depends, which can be fixed by boundary conditions, if you wish.

If you know the boundary conditions, the energy, the energy needed to pull from here to here, if know the, if you say that, then, you can fix this constant. But otherwise, a constant, you can always add a constant to an energy. It turns out that, what matters is the derivative of energy, because in experiments, what you can measure is always force; it is difficult to measure energy. So, whatever be this constant, whatever be the value of this constant that we are dealing with, the derivative of this d E, when you calculate the d E by d R, the constant will go to 0. So, you can put any constant you want in principle and still, you will get the same force. So, now, let us quickly plot this and see.

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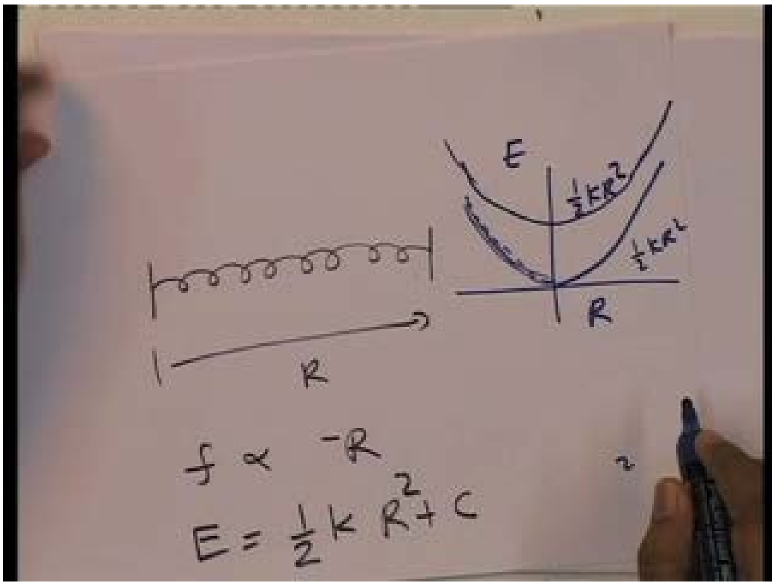

Let us get a feeling for this plot. So, let us have a plot here. Let us plot energy versus R. So, you will get a parabola k R square. I am not plotting this left hand side, because in this case, R is only positive for a spring. So, this is, this is what half k R square. Now, even if you add a constant to this, the shape of this curve will not change. So, let us have a look at this plot once more; even if you add a constant to this, the curve will become something like this. So, essentially, it will get shifted, but the shape, the shape of the curve will not change. So, this is, this is half k R square and this is the half k R square plus a constant. So, let, let me plot this little more carefully here. So, let me plot this little more carefully in the next one.

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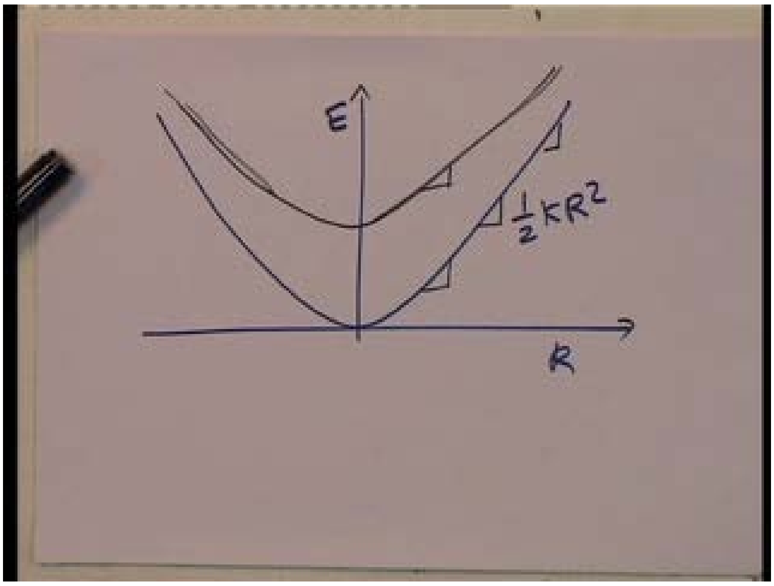

So, let us plot here. So, this is half k R square, which is energy versus R. Now, we can add a constant to this and if you plot that, you will get something like this. This is exactly the similar curve, parallel to this. I have not drawn it very properly, but it is just like adding the constant everywhere; the shape of the curve will not change. So, essentially, what you want to learn is that, this is the force, which is the derivative of this curve at every point, if you calculate the derivative at any point, if you calculate the derivative here, you should get the same derivative; because, you are essentially adding a constant. So, basically, that, the force will not change. Now, you can also do this by saying that, you can pull the spring from this place to this place and you can ask the question, how much is the energy caused to pull this spring from here to here. So, let us do that, let us do that, here. So, have look at this slide here. So, if you look at the slide, what you have this equation is integral d E is integral k R d R. So, now, let us write that here.

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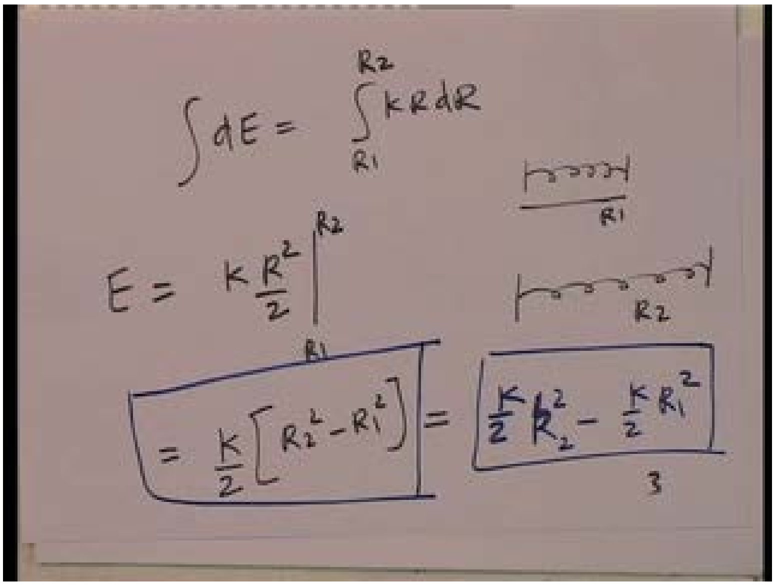

So, let us write integral d E is integral k R d R. Now, first you had this particular R, which is, let me call this R 1. Now, I am pulling this spring to a greater distance, which is R 2. So, I am pulling from R 1 to R 2. So, I have this spring, which is R 1. Then, I am pulling to a distance, length R 2. You can ask the question, how much energy it ((cost)) to pull from R 1 to R 2. So, then, this integral goes from R 1, R 2. So, this is, energy is, this is energy, and here R 1 plus R 2. So, k R square by 2, now, in the limits R 1, R 2. So, if you have, this is the integral, integration with the limits R 1 and R 2, so, this is done by, you can write integral of k R d R is k R square by 2 and you have to apply this limits. So, the way of applying limits, as you learnt is that, k into…So, this 2 is also constant. So, I can take this 2 out.

So, basically, R 2 square minus R 1 square. So, this is the energy. So, this is the energy needed to pull a spring from R 1 to R 2. If this is R 1, this R 2, the energy needed to pull the spring from R 1 to R 2 is k by 2 into R 2 square minus R 1 square. So, this is the answer. So, this can be written as k by 2 into, where k by 2, you can say, k by 2 R 2 square minus k by 2 R 1 square. So, you can also write like this, k by 2 R 2 square minus k by 2 R 1 square. So, this is exactly same as that. So, this is basically, the energy needed to pull the spring from R 1 to R 2. So, this is essentially, the energy difference. If you look at this equation once more, little more carefully…So, let us write it here, separately.

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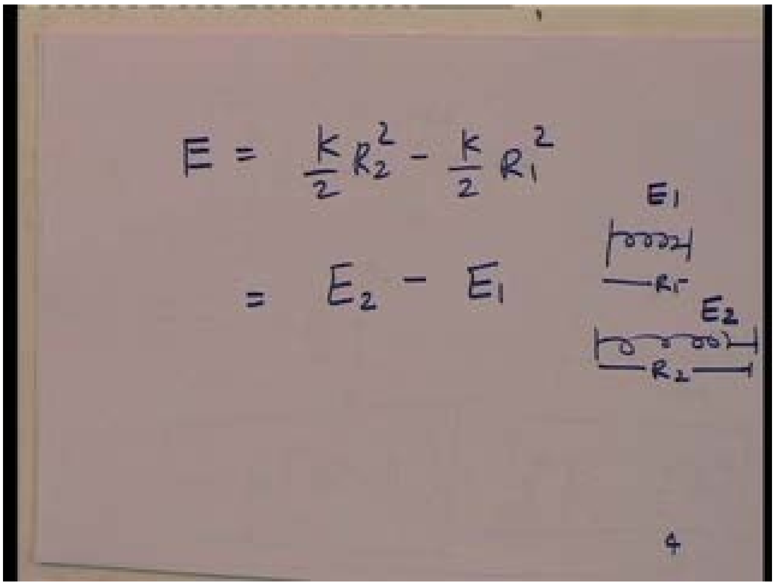

So, let us write, let us write that, what we got is energy is k by 2 R 2 square minus k by 2 R 1 square. So, this is nothing, but energy to keep this...So, this is an one energy, which is E; let me call this E 2 and this, may call this is E 1, where E 2 and E 1…So, this is R 1 and this is R 2. So, this is, the energy of this spring is E 1 and this, energy of this spring is E 2. So, then, that, the energy difference is basically, E 2 minus E 1. So, basically, what you find is that, the change in energy, the energy needed to pull from here to here, is the energy difference between this two and this thing, you can get by integrating this particular equation, this particular differential equation. So, this is the simplest case that we learnt. Now, let us go to a next case, for example. So, let us, which is basically, charges.

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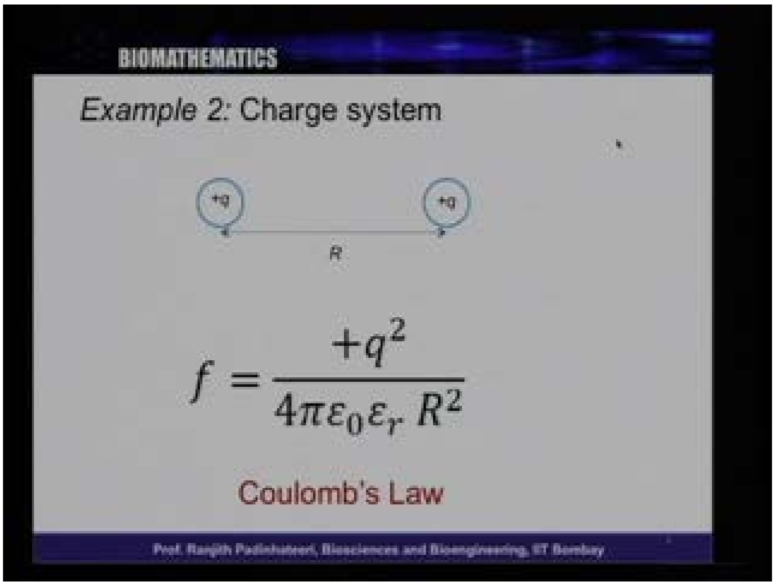

So, as we said, we, if we learn about springs and charges, we learn a lot about Biological system. So, let us take the simplest case of a charge system, which is two positive charges at the distance R apart. So, this are two charges plus q and a minus q and the distance between them is R. And we know that, in such a case, what is force. The force is given by Coulomb's Law and it is, f is q square divided by 4 pi epsilon 0 epsilon r R square, where epsilon 0 and epsilon r, you know as the permittivity of, at vacuum and relative permittivity. So, you know, you know this constants from your school classes. So, where capital R here is the distance between this two charges and this is the force and this is the famous Coulomb's law. Now, known this, how do we calculate energy of the system, energy this charged systems. So, the energy of this charge system can be calculated the same way, by integrating the equation as you see here.

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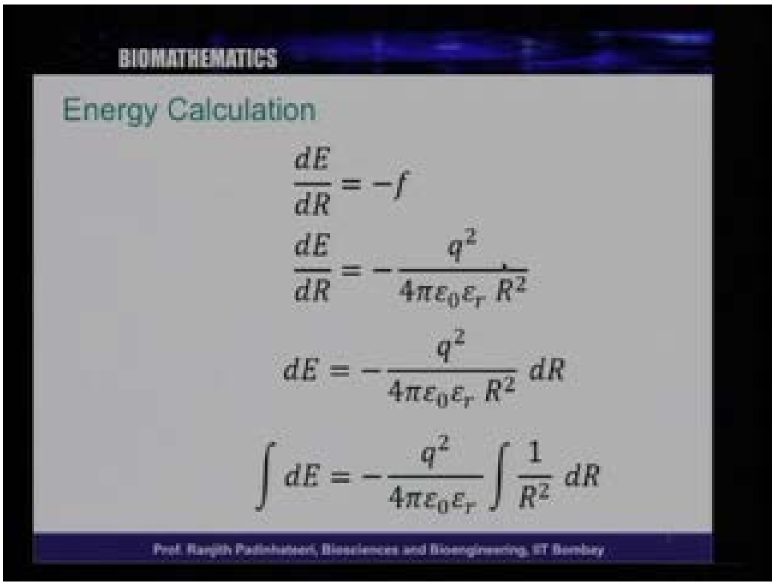

So, the equation here is, d E by d R is minus f. So, d E by d R is minus f and the f we know. So, let us find out what happens. If you substitute this f here, you get q square by 4 pi epsilon 0 epsilon r R square, that is what you substitute for f; this is what you get. So, essentially, you can take this d R this way, this side, or multiply both sides by d R and you get d E is q square by 4 pi epsilon 0 epsilon r R square d R. So, this is basically, rewriting this equation and now, we can integrate both sides. So, q is a constant; 4 is a constant; pi, epsilon 0, epsilon r, all of them are constants. So, you can take out all of this and you can have just integral of 1 over R square d R. So, what is integral of 1 over R square d R? We know that, integral of R power, x power minus n or 1 over x power n is, if you have integral of x power minus n, it is x power minus n plus 1 divided by minus n plus 1. We know this.

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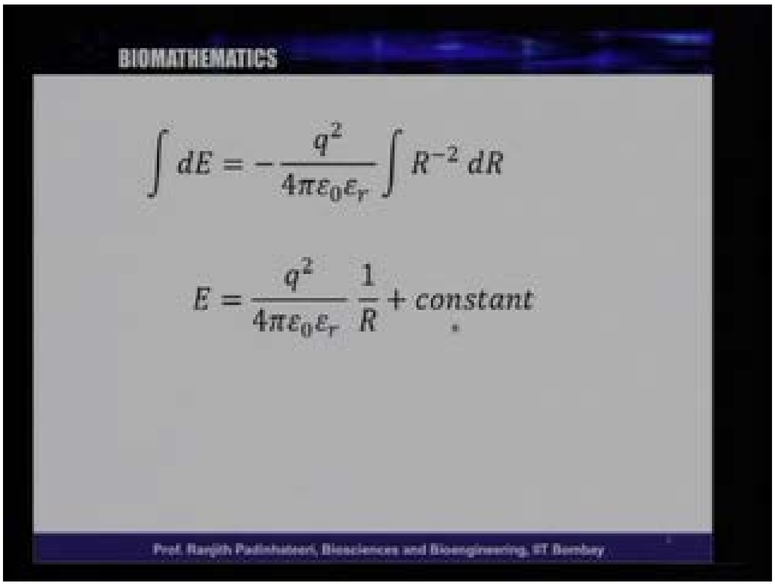

So, let us have a look at it. So, if you do this integral of R power minus 2 d R, that is what you have to calculate.

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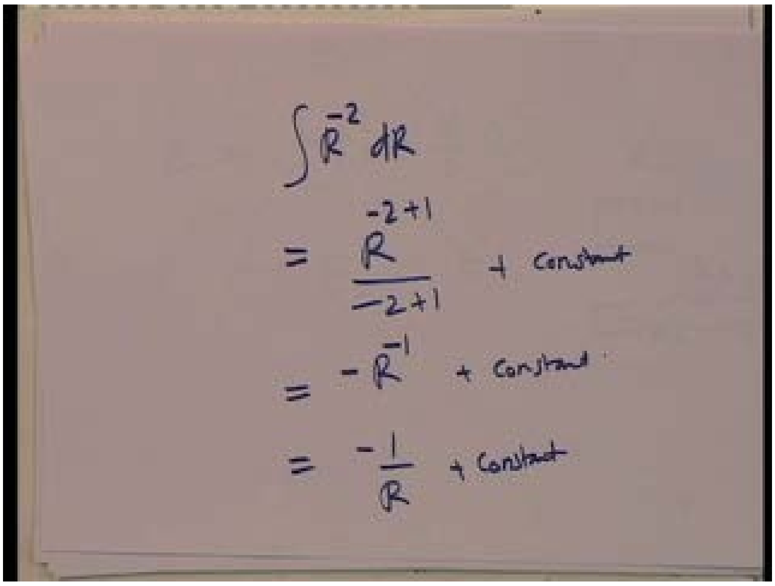

And, we know that, integral R power minus 2 d R is R power minus 2 plus 1 divided by minus 2 plus 1 plus a constant. And, this is R power minus 1, with the minus sign. This is what integral R power minus 2 d R. So, essentially, you will get R power minus 1 is 1 over R with a minus sign plus a constant. So, have a look at this, here in the slide. You get a minus sign extra. This minus sign goes away and q square 4 pi epsilon 0 epsilon r remains the same and R power minus 2 becomes 1 over R plus a constant. So, this is basically, you know that, this is nothing, but the potential energy of two charges. So, the potential energy, when you have two charges, the potential energy of this system was q square by 4 pi epsilon 0 epsilon r by R. So, this the potential energy. Then, do not confuse this, this E is here, E is not electric field; this is potential energy. I just use a symbol E for energy. So, some, some places u; in the text book, you might see this as v. Typically, this is what is used, v. So, you will see that, this is, this is typically written as v. So, some places, you will see that potential energy…

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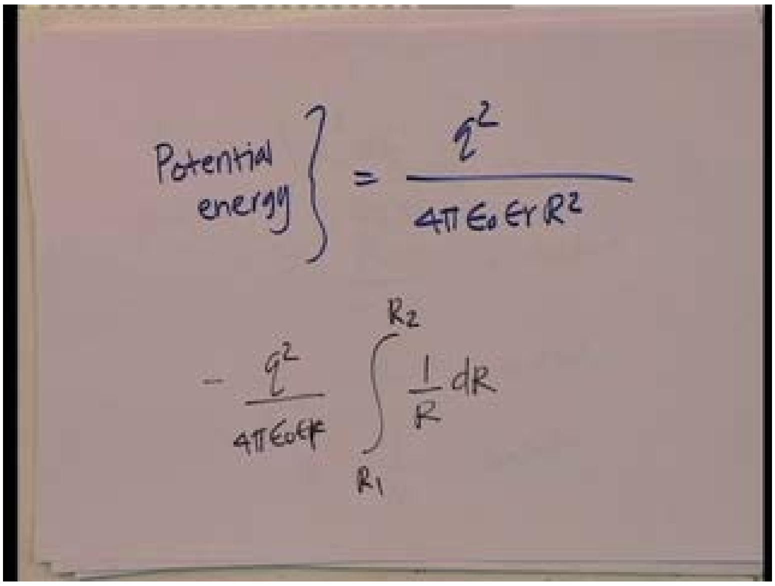

So, let me write, potential energy of two charge systems is q square by 4 pi epsilon 0 epsilon r R square plus a constant and that constant, can be anything; that does not change the force. Whatever be the constant, again, the force remains the same, because, the derivative of energy is force and derivative of any constant is 0. So, again, you have the force. Again, here also, you can do this integral in the limit. You can do, what we had here is, if you look at this slide, what you have here is, integral minus q square by 4 pi epsilon 0 epsilon r, integral 1 over R d R. So, you can do this from R 1 to R 2 again; this is the same (( )). What are we asking here? We are asking, if, we have charges this distance and if it move, if you move this charge from this distance to this distance, how much energy it cost, to move this charge. So, again, you can do this. So, if you do this integral…

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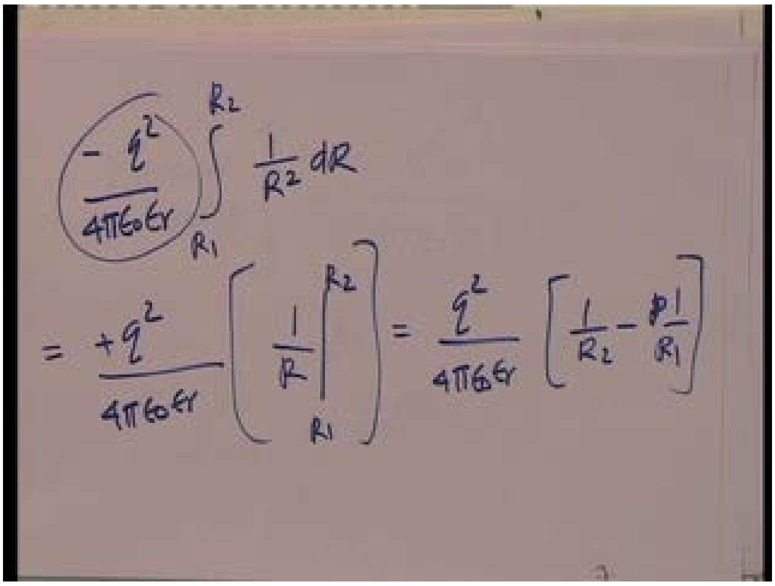

So, let us have a look at what you end up here. Basically, what you have to do is, q square by 4 pi epsilon 0 epsilon r is a constant. R 1 to R 2. 1 over R square d R, this is what we have to do. So, in the previous one, we had integral 1 over R square. So, this is, this is the energy we have. So, integral of this, in the limits R 1 to R 2 is nothing, but…So, this is equal to, if you do this, this constants, this constant will exactly remain the same, q square by 4 pi epsilon 0 epsilon r and integral 1 over R square, as we saw, is 1 over R with a minus sign. So, we had a minus sign here.

So, that minus sign goes with the plus, it becomes plus here, in the limits R 1, R 2. So, you have to apply a limit to this 1 over R. So, essentially, what you would get is, q square by 4 pi epsilon 0 epsilon r into 1 over R 2, 1 over R 2 minus 1 over R 1. This is what essentially, the potential energy difference, the energy difference between two points; or, in other words, the energy needed to bring this charge from R 1 to R 2. If you want to move this charge, let us say, this proteins keep moving; when they change, when they their configuration, the charges get, has to move from one place to another place. So, that much energy has to be put in to move one charge from one place R 1 to another place R 2. So, this is basically, what we have seen here is, the energy needed for a charge to move from R 1 to R 2. Again, we got this by solving this simple differential equation.

So, this is all a part of calculus, where you can apply all this ideas, simple ideas of integration or differentiation, to understand about forces, bacterial growth, energies and motion. So, when we talk about motion, the next question is, how does this differential equation governs motion. So, that is what, something we ill, we will, we will come and see later in this talk.

But there is another simple differential equation, which we have to understand. So, one another thing that is often comes in Biology, is concentration. So, concentration varies with, within the cell or concentration can vary within the, within a particular regime, region. So, when this concentration varies, a diffusion can happen; that you know. So, our aim is to try and understand this diffusion, from this perspective of differential equations. So, there is something called diffusion equation; but at this moment, immediately, we will not go directly to the diffusion equation, which is a bit complicated for beginners. So, first, we will try and understand, how the concentration gradient or the difference in concentration, how do we represent that, through a differential equation.

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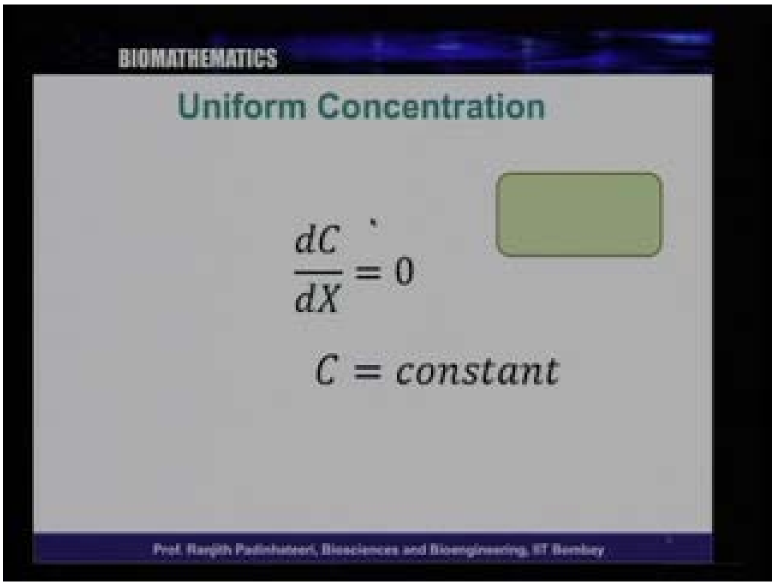

So, let us have a look at this particular, this slide. So, let us imagine such a cell having uniform concentration of something. So, this is the green thing here, which is, let us say, some protein; it is almost uniform all over. So, what does, if we want to make this statement mathematically, that this is uniform all over, the way to make the statement is exact following; that is what written here. We can say that, d C by d X is a, is 0; that is, the c does not change which x. So, here the x is this distance.

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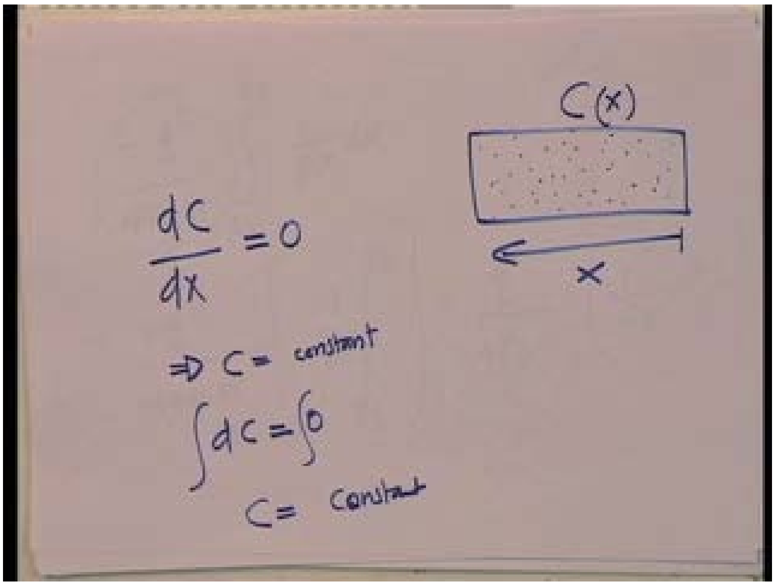

So, so, let me, let me draw here. So, we had…Have a look at this drawing here. So, we had, let me call, this x as this distance, for convenience, here. So, this, starting from here, the x goes in this particular way. So, all over here, the concentration is pretty much the same; let us say, uniform concentration everywhere. In that case, what you want to say is that, the concentration does not change with x. So, if you say C as a function of x does not change, that means, d C by d x, the change in C, when you change x is 0 ; that means, the concentration does not change. This only means that, C is a constant. How do you know that, because you can integrate this equation; I can multiply both sides with d x. So, you will get d C equal to 0, because, if you multiply 0 with anything is 0.

Now, you can integrate this and you can integrate this. So, 0 is anyway a constant, as there is the constant of integration. So, you will get C is equal to a constant. So, that is how you integrate by this equation. So, essentially, what we did here is that, we conveyed a message that, the concentration is uniform across this cell or this particular region that we saw and this is mathematically explained, this is mathematically expressed this way that, d C by d x is 0 . Now, let us, let us look the next case, where it is not uniform, but it is slowly varying. How do we express that?

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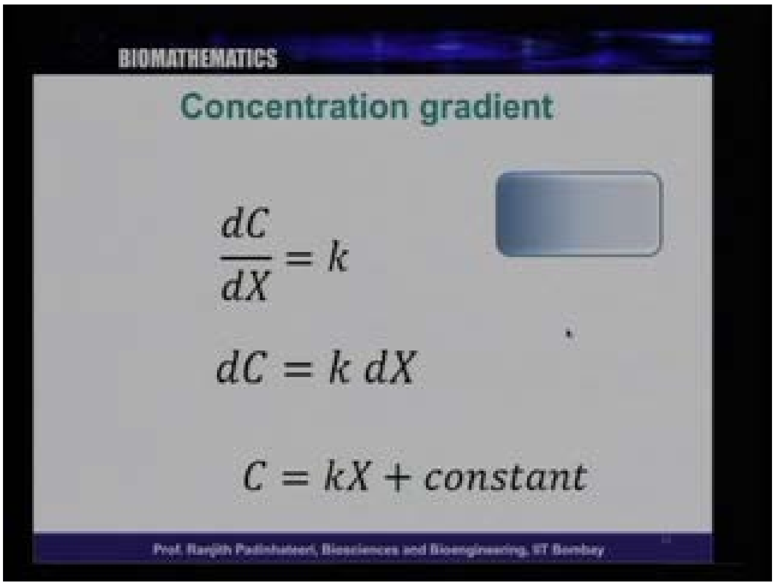

So, have a look at the next one. So, from here to here, the concentration is slowly increasing. Here, the concentration is less; here, the concentration is more. So, the concentration is slowly increasing. So, how do we say that, the concentration is slowly increasing? So, the way to say this is that, d C by d X is a constant; so, that means, the change in the concentration, when you change x is a constant. This is a simplest thing, constant change in concentration. And, you can integrate this equation, as we always do; multiply both sides with d X. So, d C can be written as k d X and you can integrate both sides, like we do. So, C is a constant; integral of d C is C; k is a constant; integral of d X is just X. So, k X plus a constant. So, that means…So, uniform, it is a concentration gradient. So, there is a concentration gradient.

So, in this point…So, this thing is called a concentration gradient. The d C by d X, the derivative of concentration. But, when you talk about the real gradients, there is something more we have to understand; we have to incorporate the idea of the vector, the direction. So, whenever we talk about the force, and whenever we talk about gradients, we have to include the direction, somehow, in this, they all, there is a gradient in a particular direction. As we go from here to here, there is a gradient. We have to bring this idea of gradient here. In, we will discuss that, how do we bring about this idea of direction here. We have, we have to incorporate some ideas from vectors. So, the vectors that we discussed, we will, we will see how do we incorporate the idea from vectors, to port, to stay, to say here, to incorporate the direction in this gradient. The, essentially, we want to say that, the concentration is increasing in this particular direction; in which direction is concentration changing.

So, we have to incorporate this idea of a vectors here, to exactly, mathematically convey this idea. So, we will discuss, how do we do that, in another lecture; but at this moment, we basically, just want to understand this differential equation d C by d X is a constant, and the integral of that is C is equal to k X plus constant and that is what the integral is. So, we know this much. Now, we will go to a different things. So, we said, concentration is one of the important things in Biology. Another thing is, the very important thing is the force, but there is force associated with motion, motion. So, we saw force associated, force and energy. Now, we, what we will see is that, force and motion; because things move around; molecules move around; and, when molecules move around, in Biology, you have to add some force.

And, how does this force and motion are related. Because, basically, for example, you know that, protein, which is completely straight, they fold up into a folded configuration. So, the protein which is unfold, which is not in this fold, which is in some random configuration, they all come together and fold. So, there is some force, leading force, making this protein fold. In other words, the protein molecules are moving in a particular direction, so that, they end up in a folded configuration. So, how does this force and motion, how are they related? And, they are related through a differential equation and this is what we will see today. So, let us have a look at this. So, before discussing this, we should understand one thing about differential equations.

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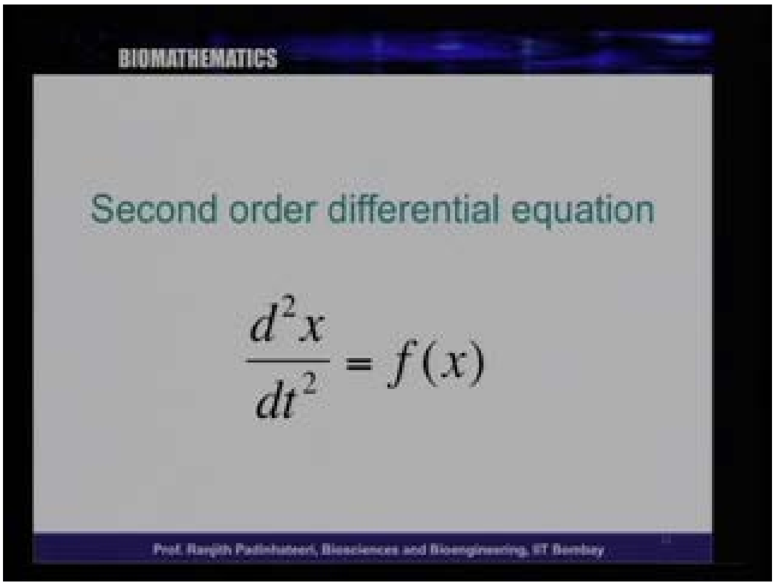

So, there is something called second order differential equation. So, what we have discussed so far is called the first order differential equation. So, let me just say, what I mean by first order differential equation once more.

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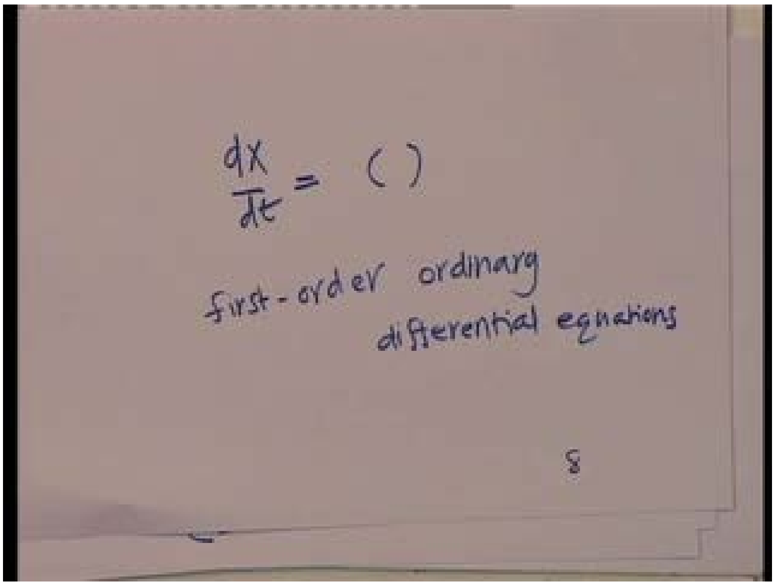

So, we said things of the form d X by d t is something. So, here, there is, we only use the first derivative. So, such things are called first order, ordinary differential equations. We will see what are, we will see some other kind of differential equations. So, but, whatever we saw so far are first order, ordinary differential equations. Now, we will go and look at something called second order differential equations. So, have a look at this slide. If we have d square X by d t square is something…So, there is a d, this is second derivative. So, such things are called ordinary differential equations. So, now, we will immediately come and see what are this examples, what is the example of this, because, when we say about force and motion, we have to deal with second order differential equation, which is one level up from what we discussed so far. So, now, how do we learn about force and motion? So, we all know that, something that connects the force and motion is this famous equation called Newton's equations, where Isaac Newton proposed this equations in, way back in 60, late 60s, like, in 17th century and early 18^{th} century. So, in late 1600s and 1700s, Newton discussed about this equations. So, the equation that connects the force and motion are called Newton's equations.

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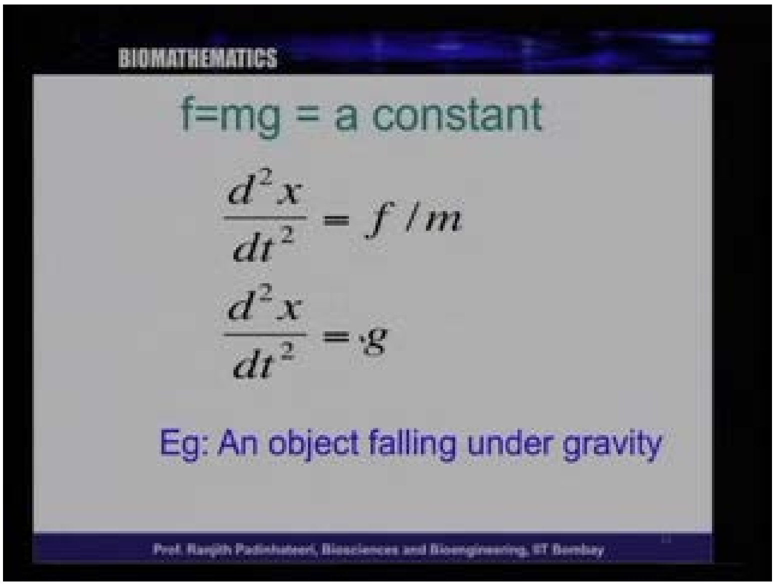

So, let us see an example. So, we can think of an example, which is d square x by d t square is some, some constant f by m. So, this f is a force.

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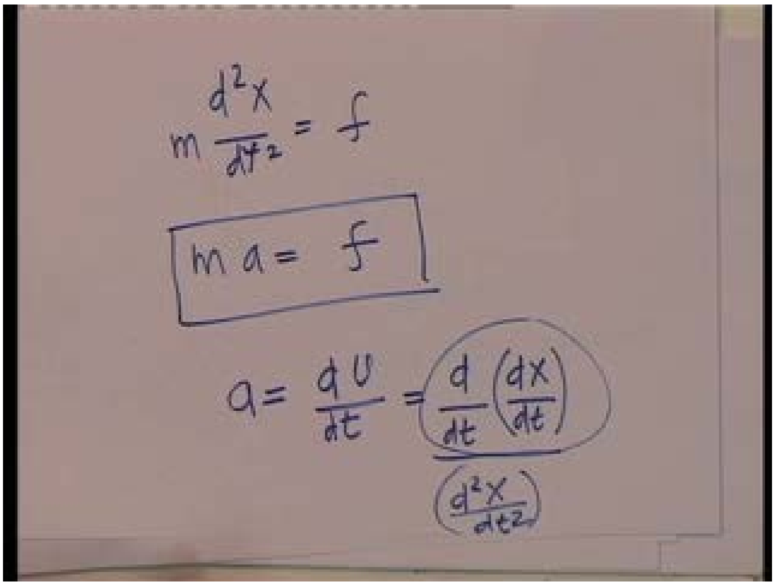

So, Newton’s equation is essentially, as you all know, is that, d square X by d t square is m. So, this is called Newton’s equation. So, you all know that, f is equal to m a. This is the Newton’s equation that, as all of you know. Now, a, acceleration can be written as d v by d t; that is, a is acceleration and acceleration is change in velocity by change in time, d v by d t; and, v can be written as, v can be written as d X by d t, that is change in position divided by change in time. So, if we, d X by d t is v and d by d t of this is acceleration...So, this whole thing can be written as d square x by d t square. So, this is essentially, d square X by d t square, this whole thing. This whole thing is written as d square X by d t square. So, essentially, acceleration can be written as d square X by d t square and acceleration is force by mass. So, this is basically, Newton’s equation. And, the simplest Newton’s equation you can think of is something falling under gravity. So, let us say, this pen is, if I leave the pen, it will fall under gravity.

So, the falling of this pen or any object that is falling under gravity, can be described by this equation. If you say f is equal to m g, if I say f is equal to m g, I get this equation, which is d square X by d t square equal to g, which is a simple equation, which is basically, equation of, Newton’s equation for an object falling under gravity, where g is acceleration due to gravity, which is the constant, which is 9.8 meters square per second. So, g is a universal constant for, in Earth, which is 9.8 meter per second square. And, once you know this g, we can solve this differential equation d square X by d t square, the second order differential equation and get a solution for X, where X is the position of the object falling. So, let us try and understand little more, what is this equation actually mean. So, let us try and understand little more.

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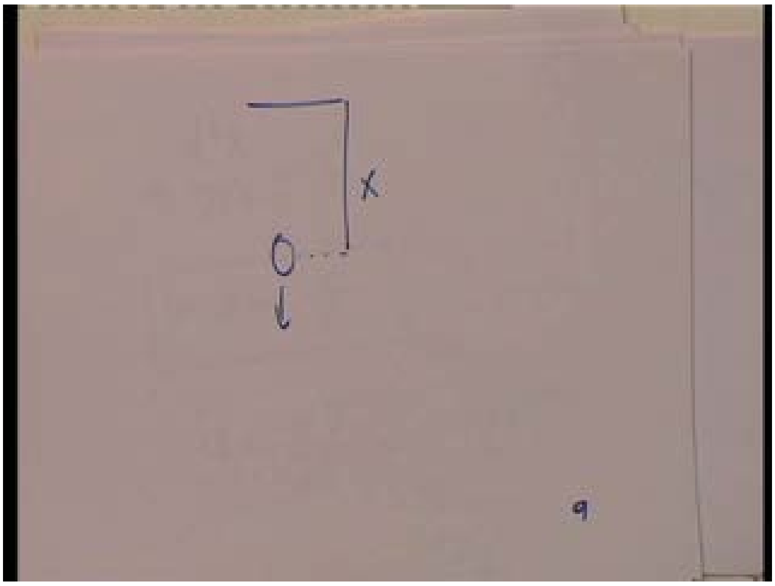

Let us say an object…Let us have a look at here. Let us say, an object is falling from here. And so, this is the object, which is falling. It started falling from this particular position. So, let X is the distance from the, from the initial position, at any point. So, sorry, I, it, consider X as the distance from…So, X can be thought of as a distance from any point. We will come and discuss, what is X, basically. X is basically, the distance of this object falling. You can measure either the distance from the ground up, or from a, some fixed point. So, we can, we can decide, where you want to take this distance.

But, X is essentially, this distance and how does this X, this distance vary as the function of time. So, that is what, this equation is described. How does this distance, for example, the distance from the ground, how does it vary as the function of time; or, distance from a fixed point, how does this distance vary as the function of time, when this object is falling. So, that is, this equation describes. So, again, let us have a look at this equation, d square X by d t square. How does this X, which is the distance from the Earth, for this particular object that is falling, is changing with respect to time, that is what the question we are asking. So, it turns out that, this can be integrated the way we did for all other cases.

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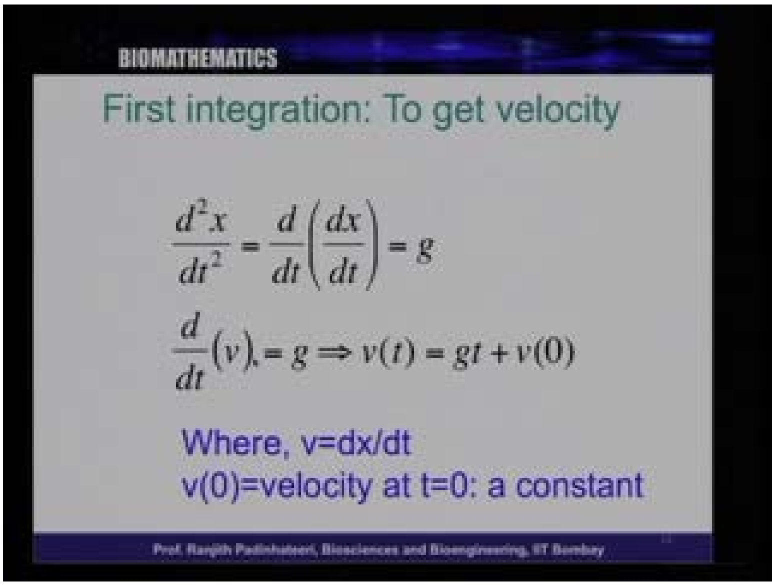

So, let us have a look at this next slide. So, this slide says, d square X by d t square can be written as d by d t of d X by d t, as we saw. Once we write this in a particular way, this is basically, equal to g. This is our equation. Now, d x by d t, as we said, is change in position by change in time. It is nothing, but velocity v. So, v is d x by d t. And, this is, d v by d t is equal to g. So, this equation becomes d v by d t equal to c, g and as we…This is a simple, ordinary differential equation that we have been seeing all in this class, in the previous class. And, such an equation has an, immediately, we can see that, such an equation has a solution, which is v is equal to g t plus constant. And, the constant, as we said previously, is basically, the velocity at time is equal to 0. So, essentially, you can write, v of t is equal to g t plus v of 0. If you have confusion, let me do this little more carefully. Let me do this little more carefully. How did we do this?

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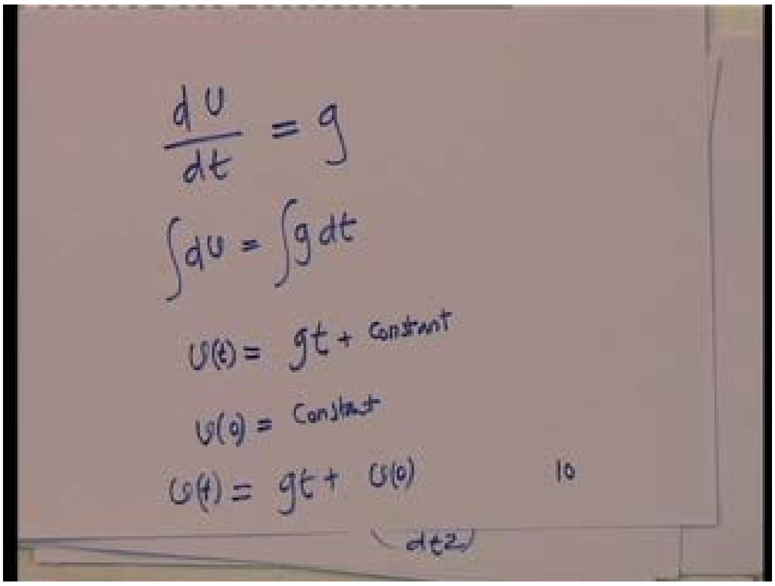

So, d v by d t was g; this was our equation. So, now, you can integrate this by d v is g d t and integrate both sides. So, basically, v is equal to, v as a function of time is equal to g t plus a constant. Now, how do we calculate this constant? Put time equal to 0. At time is equal to 0, what you have, v of 0 is equal to v of, v at time equal to 0, is this constant. So, essentially, what we have, you substitute this thing back in the above equation. Then, you get v of t is equal to g t plus v of 0. So, this is essentially, what I have written here. This is exactly what I have written here, v of t is g t plus v of 0. So, you know, as this is falling, the velocity seems to be increasing. So, if you plot this g as a constant, it is like, y is equal to m x plus c. This is a straight line. So, that means, as an object is falling, its speed is increasing; its velocity is increasing. So, that is what it, that is what it says; that is what the first part of the solution tells you that, as an object is falling under gravity, its speed is increasing; that is what, the first part of this equation says, v t is equal to g t plus v 0. Now, this is only the fist integration we did and we got just the velocity and to do the one integration, we need to understand one constant of integration, v of 0.

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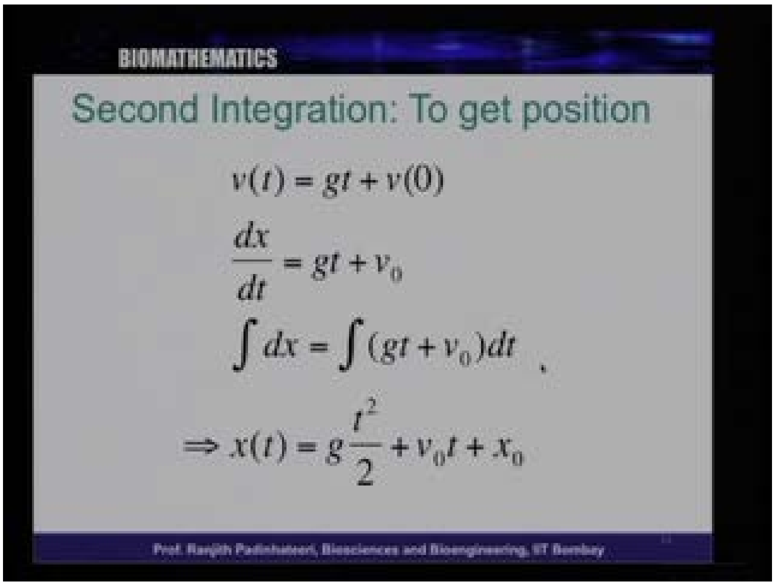

Now, let us do the second integration. The second integration is…So, we have, as of now, we have v of t is g t plus v 0. Now, we know that, v is nothing, but d x by d t. If we substitute v of t is d x by d t here, we have another differential equation now. Now, we can integrate this differential equation by, as we…This is again, is the first order differential equation. You can multiply both sides with d t. So, what you will end up is, integral d x is integral g t plus v 0 d t; that is what you get. So, you know that, integral g t is g t square by 2 and integral v 0, which is a constant, is v t, v 0 t. You have and you have another constant, which is, we call x 0. Essentially, it turns out that, x 0 is the position at time equal to 0. This is something which you can simply see, just as we typically do. So, let us have a look at here, how did we do that.

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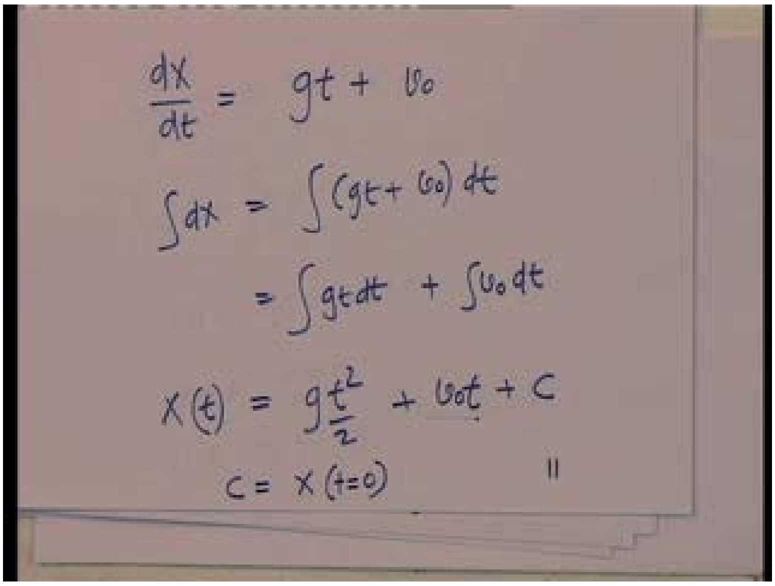

So, we had this equation, which is d X by d t is equal to g t plus v 0. And, we multiply both sides by d t and integrate it. So, integral g t plus v 0 d t and the first part of this is integral g of t d t plus integral v 0 d t. This is equal to, integral g t d t is g t square by 2 plus a constant and v 0 is integral v 0 t plus some other constant. So, this two constants together, let me call this constant C, as a constant C. Now, this C…So, basically, what you get is x of t. Now, when you put t equal to 0, this term goes to 0; this term also goes to 0. So, C is nothing, but x equal to 0. So, C is nothing, but x at t equal to 0. So, you substitute this back here; so, you get this particular thing, which is x of t is g t square plus v 0 t plus x 0.

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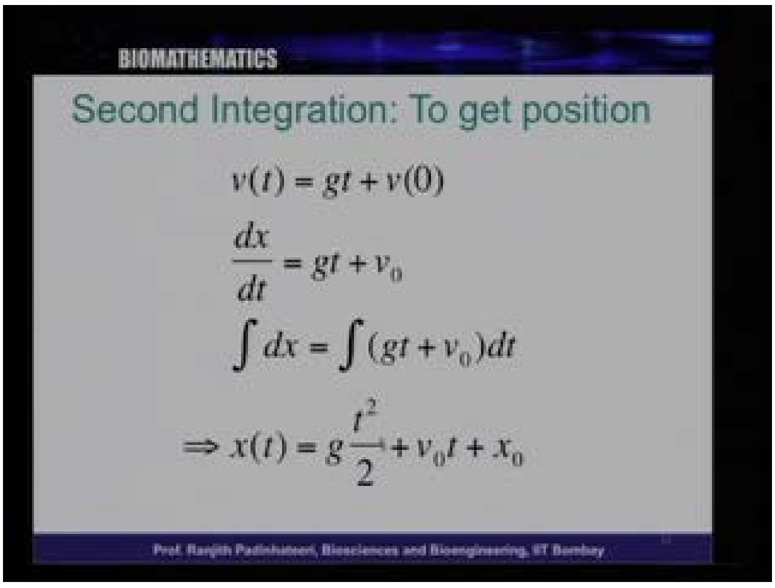

So, this is essentially, what does it say is that, if something is falling under gravity, how does the distance…So, distance is, let us say, you, I start, from this particular point, I drop this pen and the distance will increase. The distance from my hand to this pen will increase as the distance, as the time goes. So, then, what you get is essentially, x of t, which is, this equation is basically, gives you the position as the function of time. As the, after…So, you can ask this question by looking at this equation. So, let us have a look at this equation once more.

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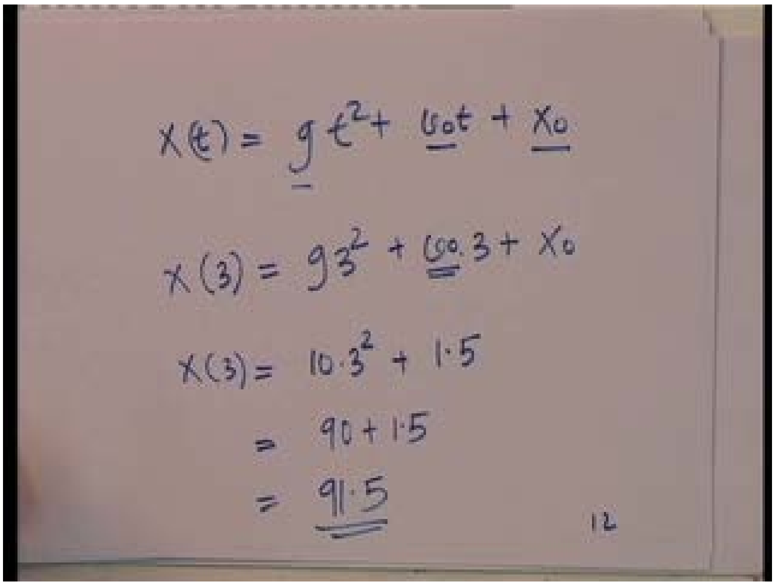

So, basically, x of t is g t square plus v 0 t plus x 0. If you know x 0, and if we know v 0, and we know g, which is the constant of, as gravity is a constant, acceleration due to the gravity is a constant. If we know this much, for any time t, we can substitute in this equation and get x of t. So, we can get x after 3 seconds, is equal to g into 3 square plus v 0 into 3 plus x 0. Now, if we know that, x 0 is 0, the, if we said that, x 0 is your height, let us say, you are putting from your height, which is, let us say, your height is like 1 and a half meters. So, let us say, you are dropping this pen from 1.5 meters, I can get x after 3 seconds is g, which we know is like, close to 10 meter square per second into 3 square plus v 0 and I am dropping it at 0 velocity; at the beginning, there is no velocity at all. So, v 0 is 0.

So, let us assume, in this equation v 0 is 0. So, this is 0 into 3 is 0, plus x 0, I, initial position is 1.5 meters. So, if you do this, what I get is, 10 into 9, 10 into 3 square is 9. So, it is 90 plus 1.5, 91.5. So, this is, there is a slight absurd thing here, which is…This is assuming that, the force is there forever, but we have, we have Earth after 1.5 meters. So, we have to take this numbers properly. So, same way that, you put from, like, from reasonably good height and within, before 3 second, you would have hit the ground in this particular case; because, the answer you get essentially, is 91.5 meters. This equation says that, if you have such a force, in 3 second, if there is nothing to stop, it would have gone 91.5 meters. It is a slightly absurd thing, because, you are only putting 1.5 meters height and then, it cannot go beyond that, because we have Earth. But if you take this from very, if you drop this pen from very, from a very height point, you can see that, it will fall down and this equation will be obeyed.

So, take or do this exercise by taking x 0 as, let us say, you are putting it from a top of a building and let us say, how far this would go, if you had taken all this constants properly. So, let, we will, we will do this exercise in the next class, but have a, go home and then, also think about what you would get by putting the right numbers in this equation. But as of now, you understood, how do we solve this differential equation.

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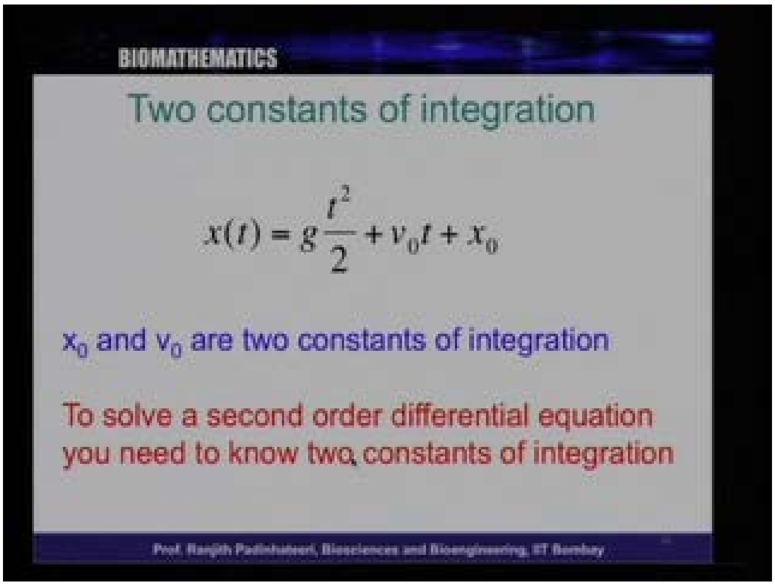

So, one point, important point to remember is, again, I am reiterating in the next slide, which is that, we have two constants of integration, v 0 and x 0. So, to solve any second order differential equation, any equation of the form d square x by d t square or d square y by d x square or d square something by d something square, any second order equation, if you want to solve it, you need to know two constants. To solve a first order equation, you need to know one constant. To solve a second order equation, you need to know two constants. Here, it turns out that, the two constants, in the case of equation of motion, Newton's equation, they are the initial position and the initial velocity. But in some other context, it will be something else. But you need to know two constants, basically, to understand, to solve a second order differential equation. So, basically, essentially, knowing this, let us summarize what we learnt today.

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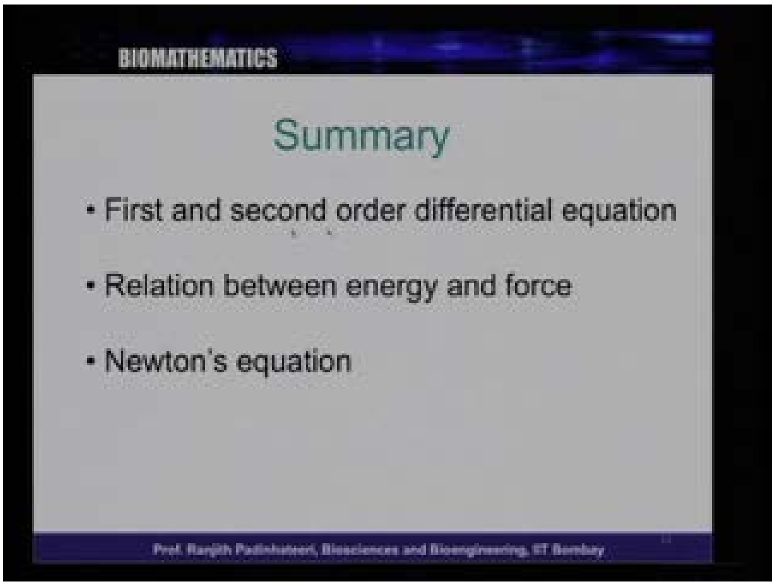

So, we learnt the first and second order differential equation. (( )) the second part of this differential equation is, what we discussed today. And, we said that, how the relation between energy and force can be expressed through a differential equation; and, how Newton's equation is essentially, a second order differential equation. So, we also learnt a bit about concentration, but in the, in the coming lectures, we will discuss more about concentration and understand.

We will go and understand diffusion; but to understand diffusion, we have to discuss, how does the concentration vary in a concentration gradient, for which we have to use ideas from vectors. So, we will discuss that, and discuss this in one of the future lectures. So, essentially, we will go slowly towards diffusion, because diffusion is an interesting phenomenon in Biology, which we want to understand using Mathematics. And for that, whatever tools we needed, we will develop as we go along and we will see those differential equations. In the coming classes, we will also solve many other types of differential equations and learn a few more different techniques to solve differential equations. So, with this, I will stop today's class. Thank you.