Biomathematics

Dr.Ranjith Padinhateeri

Department of Biotechnology

Indian Institute of Technology, Bombay

Lecture No. #10

Integration

Hello. Welcome to this lecture of Biomathematics. So far, we have been discussing differentiation and its application to various Biological system. In this lecture, we will go to a new topic in Calculus called Integration. So, today, we will have the first part of the integration. So, the topic is integration. Today, we will have the part 1, the first part of this integration and it will be, just like we had a set of lectures on differentiation, we will have a set of lectures on integration and its applications in Biology. So, before starting the first lecture on integration, let us take an example and see, why do we need this technical, why do we need to learn this integration; and then, we will learn what is integration, how to do it. So, first, an example.

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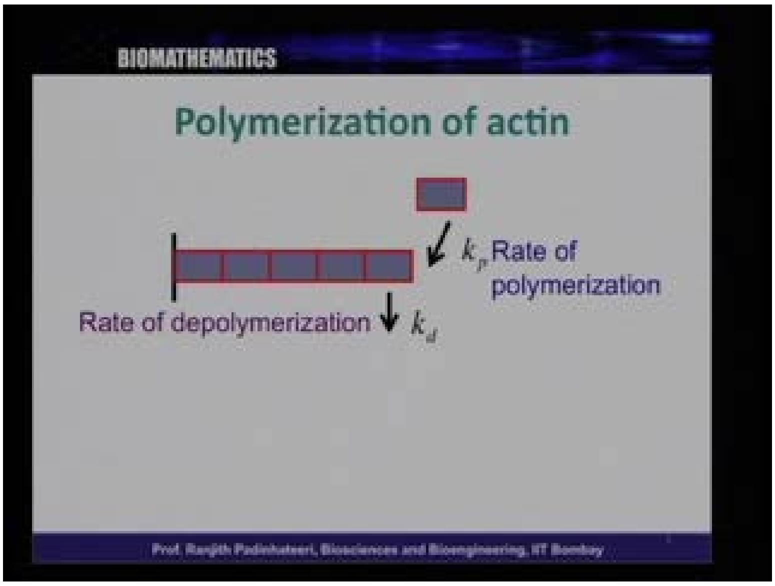

So, consider the case of polymerization of actin filament. This is something which we have discussed before. Basically, what happens is that, you have a filament, as seen here and one end of the filament is blocked and the other end is polymerizing with some rate k p, which is the polymerization rate, rate of polymerization and it can de=polymerize, the monomers at this end, at the end can de-polymerize with some rate k d. So, k p and k d are polymerization rate and de-polymerization rate of a filament. So, what does it mean? Rate means event per second. So, rate is something 1 over time; it is like, per second.

So, k p is, it is like, k p is 10 means, 10 monomers, 10 units will be added per second. So, 10 per second is k p; k p is equal to 10 per second means, 10 monomers will be added per second. And, k d is like, let us say, 5 per second; that means, that 5 monomers will be on an average, removed in every second.

So, once you know the polymerization rate and de-polymerization rate, you can figure out, how fast this polymer will grow or shrink. If the polymerization rate is much larger than the de-polymerization rate, it will, polymer will grow; if the de-polymerization rate is larger, you can imagine that, the polymer will shrink; the actin will shrink.

So, this much we know. So, known the k p and k d, we can know the change in length.

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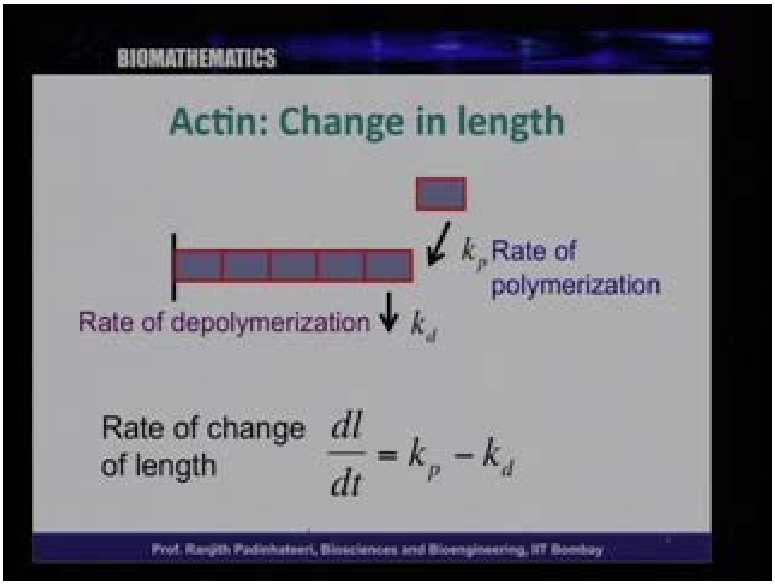

So, the how does the length changes with time, is related to k p and k d and we know that, it is related in the following way. Have a look at here, rate of change of length the d l by d t; that means, d l is change in length, d t is change in time.

When the time changes by a small fraction d t, that length will change by a fraction d l and this d l by d t, the change in length, or the rate of change of length is nothing, but k p by k d, k p minus k d, multiplied by, of course, the length of the monomer, (( )).

So, basically, here, what, the l represents the number of monomers. So, the number of, l is the number of monomers; in this case, 1, 2, 3, 4, 5, here. So, the, how does this number l, which is the rate of, this number depends, the length depends on this number. So, d l by d t, how does this length, which is expressed in the number of monomers…

So, the length of this now, is 5 monomers. So, how does this change with polymerization and de-polymerization, and that is depicted by this equation, which is d l by d t is k p minus k d.

Now, here is why we need integration. If we know this much, if we know that, the rate of change of length, if the length varies with time in this particular, if we know the d l by d t, how do we get l.

That is the question and the, to get l from d l by d t, we need to do a, we need to do a set of, first, we need to do some calculation and that calculation is called integration. So, that is why, we need to learn the integration. Any such equation, wherever you have a differential…So, such equations are called differential equations. So, d l by d t, whenever there is a derivative at one side…So, any, an equation, which involves a derivative is called differential equation.

We will come and learn about differential equations later, but, just keep in mind that, when you have derivatives involved in an equation, you would typically call it differential equation. So, when you have such an equation and you want to calculate the l from this equation, you have to do this, I mean, we have to do this calculation called integration. So, we would call it, we would call it integration. So, once we integrate, we will get l.

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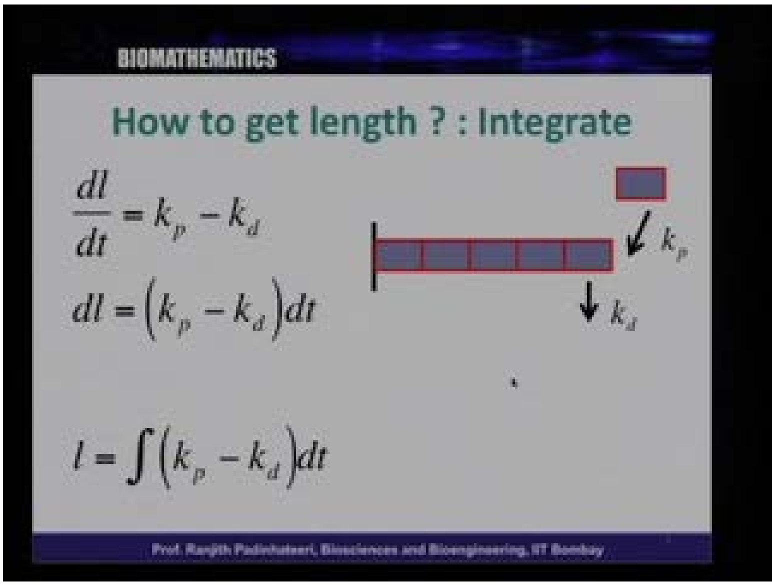

So, this is what is depicted in next slide. So, how to get the length, the answer is integrate. Now, what do you mean by integrate? So, have a look at here, we know that d l by d t is k p minus k d. So, we can immediately write, d l is equal to k p minus k d into d t. Now, if you want to get l,so, the change in length, if you want to l, you do this, in the, you do integration. And, the symbol for integration, so, this (( )), this calculation of integration is represented by the, this symbol, this particular symbol here. So, this is, this means integration. So, if you integrate this k p minus k d d t, integral of k p minus k d d t, will give you the length. So, this symbol is called integral. So, this is the basic idea. How do we do this, we will come and discuss this. So, the basic idea is that, if we know derivative of something,

we have to do this integration, to calculate the function. So, this is the basic idea. So, then, that, immediately leads to a question. In fact, it can be written as two different questions, but essentially...

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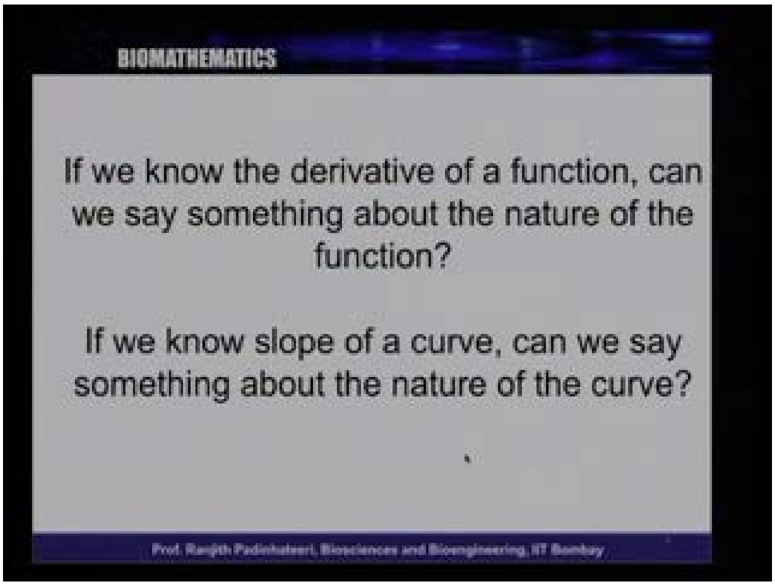

So, let us…This immediately leads to the following question. So, the question is this. If we know the derivative of a function, can we say something about the nature of the function? So, in this, in the case, we know the d l by d t, the derivative of length with time, d l by d t we knew; if we know the derivative, can we say something about l itself.

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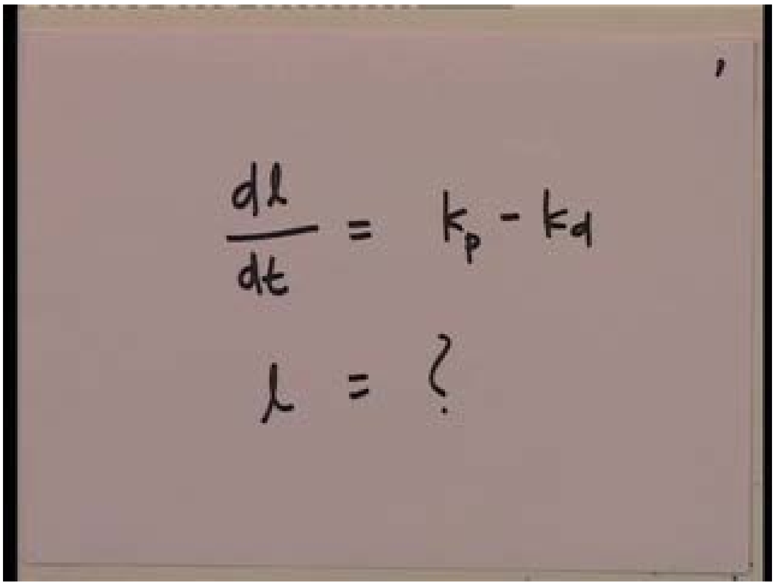

So, just to get you this idea, we had here, d l by d t. We had k p minus k d. What is l? So, this is what, this is asking. If we know derivative of a function, can we say something, about the nature of the function. Or, the.

same question, you can ask in a different way. d As we know, derivative is nothing, but slope of a curve. So, if we know the slope of a curve… So, any function can be represented by a curve and if we know the slope of a curve, which is like, slope and derivative are same.

So, if we know the slope of a curve, can we say something about the nature of the curve? This are the same, these are same questions. And, this two questions are essentially the same, written in different ways. So, the answer is yes, we can know and we have to do this calculation called integration and if we do this integration, we will get this answer. We will get answer to this question.

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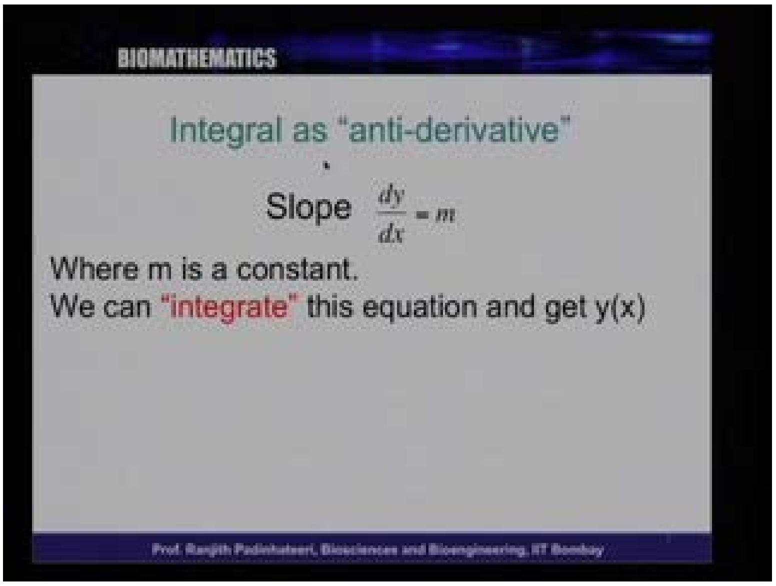

So, let us look at the next slide. So, integral or integral is…So, if we want to do the integral, integral is nothing, but the anti-derivative; that is, the opposite of derivative;

just opposite of a…Derivative was the act of finding slope. Previously, we said, if we know the curve, we can find the slope and that technique, the way of finding the slope was called derivative.

And here, if we know the slope, we have to calculate the nature of the curve itself, the function itself. So, that is the opposite action, opposite, opposite process of derivative. So, one could call integral as a anti-derivative.

So, let us say, we know the slope d y by d x is m. So, m is, where m is a constant. So, if we know that we have a function, whose derivative is just a constant, a number, what, we can integrate this equation and get y. So, we know d y by d x and we can ntegrate this equation and get y, and let us see, how do we do this.

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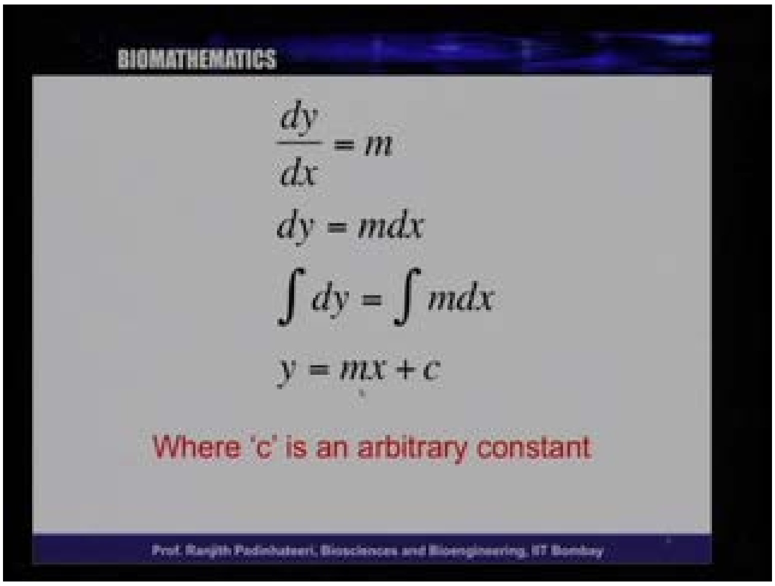

So, this slide shows how do we do this. So, we have d y by d x equal to m. So, let me just take you through, what do you mean by d y by d x equal to m. So, what do you mean by d y by d x equal to…Let us have a look at here.

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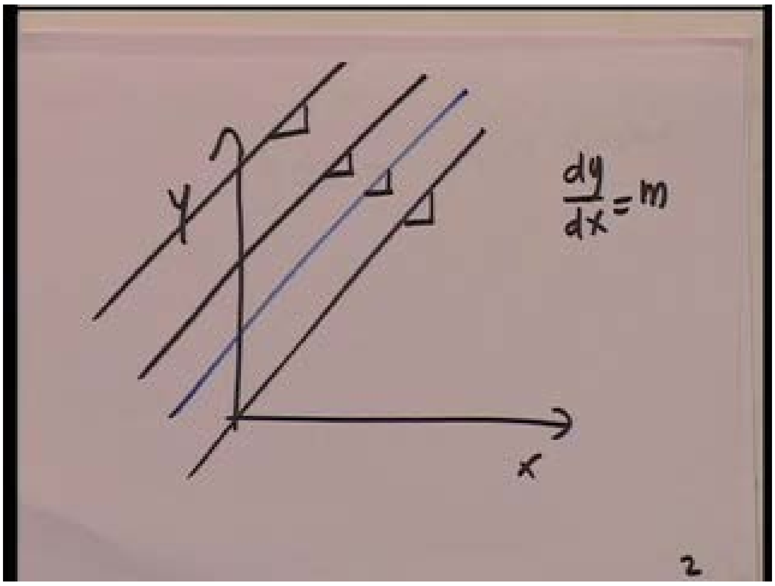

So, you have X and Y. So, when you say d y by d x equal to m, we have a curve whose slope is a constant always. So, you have such a curve and this is the slope here. At any point, if you find the slope, it is a same Slope. So, this has a constant slope.

So, such a curve has slope as a constant. So, d y by d x is m; it is a constant. From knowing this, one can write, d y is equal to m d x. We can take d x this side and write, d y equal to m d x and you integrate on both sides, integral d y is equal to integral m d x.

So, basically, the answer of this m d x is, m is a constant, integral of d x is x itself plus an arbitrary constant. So, any time you do an integral like this, if you have, such integral is called indefinite integrals, where, we will, we will distinguish between indefinite and definite integral soon, but if you do an integration… So, 1, the important thing to do, first thing to remember is, the first thing to remember in integral is that, integral of d x is x plus a constant. So, that is the integral of d x. So, if you, from this rule, by understanding this, we can understand that, integral m d x is m times x plus a constant.

So, what we have is that...So, what we have is that, y is equal to, integral of d y is y, is m x plus c. So, this is the straight line. So, y, you know that, y equal to m x plus c is straight line and the, the, the 1 we drew here, the y equal to m x plus c line, this is a straight line and this is basically, the answer. But, as I…In the slide here, we say that, c is an arbitrary constant.

So, c could be anything; c could be 0, 1, 2, 3, minus 1, minus 2, minus 3, anything.

So, what does this mean? This means that, if you integrate this d y by d x equal to m, you can get a curve like this, or a curve like this, or a curve like this, or a curve like this. Because, we do not know, what c is; we only know about the slope. The slope of this and slope of this, slope of this, slope of this, all of these are same slope.

So, slope is equal to m. So, any, each of these curve will have a derivative, d y by d x equal to m. Each of these curves has a derivative m; that means, the integral of this, integral of m, will give you any of this curve. So, we do not know wwhich, all these curves are answer to this or when you integrate this m, you can get any of this curve. So, we will try and understand what is this, any of this curve, what does this imply, what is the implication of this. But, for the moment, you understand that, if you know the…If somebody tells you that, draw a curve with slope m, what would you do? You would draw any of these curve. So, if you tell you, draw a curve with slope m, if you draw any of this, you get the correct answer; you are doing the right thing. So, essentially, that is what is happening here; when somebody tells you to integrate d y by d x equals to m, essentially, that person is asking you to draw a curve, with derivative or a slope equal to m.

And, we do it; any of this is answer. So, in other words, the integral of this is y equal to m x plus c, where c could be anything, any constant.

So, there is some freedom here to choose c and we will come back and understand about this later. But, for the moment, just realize, understand that, this is the way one will, one do the simplest integral and you also remember that, integral d x is x plus a constant. So, now, let us go to this, a more general rule. So, when we learnt about derivatives, we learnt that, derivative of x, we learnt is a constant; then, we learnt derivative of x power n as a formula and then, we could use that, different places and then go ahead and do the derivative of many, many functions.

So, we will use a similar strategy here. So, we will try and learn, what is the integral of x power n. Since integral is the anti-derivative, the opposite of derivative, if somebody is asking, what is the integral of x power n,so, we can ask you the opposite question, if you find the derivative of something, will you get x power n. If yes, that is the integral . So, let us have a look at the next slide.

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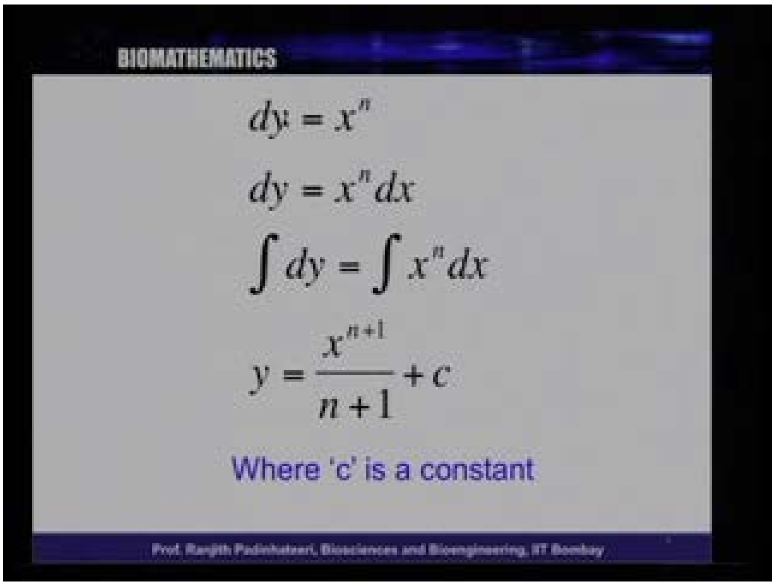

So, we have d y is, d y by d x is x power n. Sorry, this is the, what I wanted to write here is that, the d y by d x is x power, sorry, d y by d x is x power n; that is what I wanted to write there, basically, what, let us go back and try and do this.

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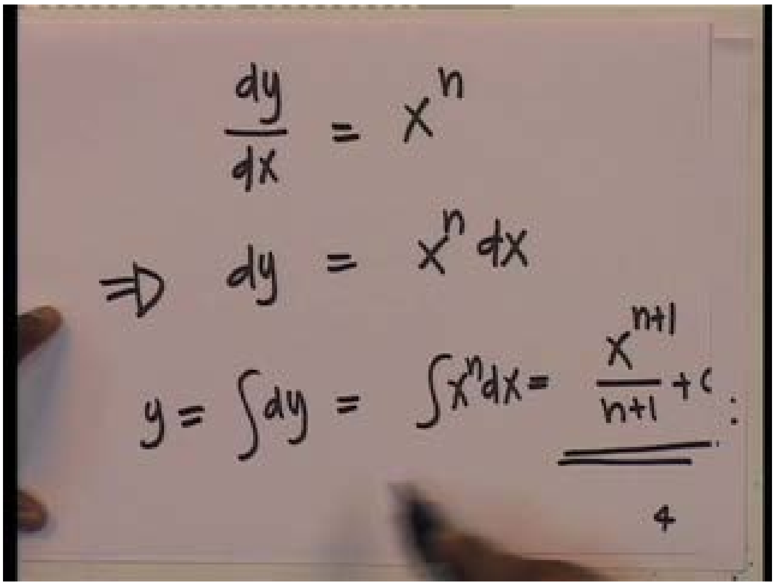

d y by d x equal to x power n. This would imply that, d y is equal to x power n d x. So, this is what? So, you have a slope of a curve d y by d x is x power n, where n could be any number, 1, 2, 3, 4, 5, 6, any number; even minus 1, minus 2. So, if you have such a thing, then, integral of, you can integrate both sides.

So, you use the symbol, integral of d y is same as integral x power n d x. And, the rule is that, x power n has an integral, which is x power n plus 1 by n plus 1 plus a constant. So, this is the integral of x power n,

which is x power n plus 1 by n plus 1. So, where c is an arbitrary constant, like we did last time. Any time we do an integral, it such an integral, you have to add a constant; because you are only said about the slope; you are, you have no idea about the y intercept or anything. So, you have only idea about slopes. So, just slope alone will not give you a exact position of a curve.

We need one more variable and that variable is c. So, what you should understand from this particular slide is that, integral of x power n, the rule is that, integral of x power n is x power n plus 1 by n plus 1.

So, now, since, since integration is the anti operation of differentiation, let us try and check, whether the derivative of this, whatever answer we got, will we get, if we find the derivative of this answer, will we get back the answer x power n. So, let us, let us try and understand what am I trying to say.

So, what we are trying to say is that, d y by d x is…Let us have a look at here. d y by d x is x power n; d y is x power n d x. So, integral d y is integral x power n d x and the answer we wrote that, x power n plus 1 by n plus 1 plus a constant c.

So, this is the answer. So, now, let us find the derivative of this X power n by n plus 1. So, will we get this back to… So, if we find the derivative… So, basically, to get the y, we have to integrate d y. What you will get is y.

And, which is nothing, but this. So, we got y.

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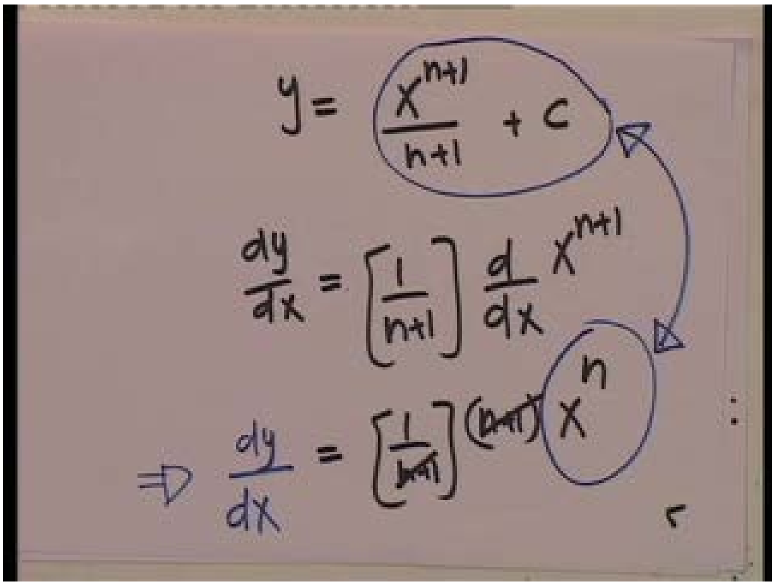

So, we got, we found that, y is equal to x power n plus 1 by n plus 1 plus a constant c. What do we get if we find d y by d x? d y by d x is a constant is 0; so, this is irrelevant;,

because, derivative of a constant is 0. What is derivative of 1 over n plus 1 is a constant. So, let us take out this, 1 over n plus 1 out, because that is a constant. Now, what you have is derivative of x power n plus 1.

So, if you go back and look at the rules of the differentiation, the rule is that, if you have n plus 1, that comes down. So, 1 by n plus 1 here, and the derivative of this is, n plus 1 times x power n plus 1 minus 1; n plus 1 minus 1 is n. So, this and this cancels. So, the answer you finally get is, x power n. So, this is the answer.

So, the d y by d x, finally, you get is x power n. If you have y is equal to x power n plus 1 by n plus 1 plus a constant, if you find the derivative of this, you will get x power n. In other words, if you have x power n as the derivative, the integral of that, is this. So, this and this are connected; they are complementary; they are…So, derivate, integral of this, is this and derivative of this, is this.

So, derivative of x power n plus 1 by n plus 1 plus a constant is x power n and integral of x power n is x power n plus 1 plus, by n plus 1 plus a constant.

So, this complementary operation, this is called integral or integration. Just by knowing this formula for x power n plus 1, one can learn many def, integral of many functions.

Just like we made use of the formula for x power n, derivative of x power n to learn the derivative of e power x, sin x, Cos x and so on, we can do similar things here.

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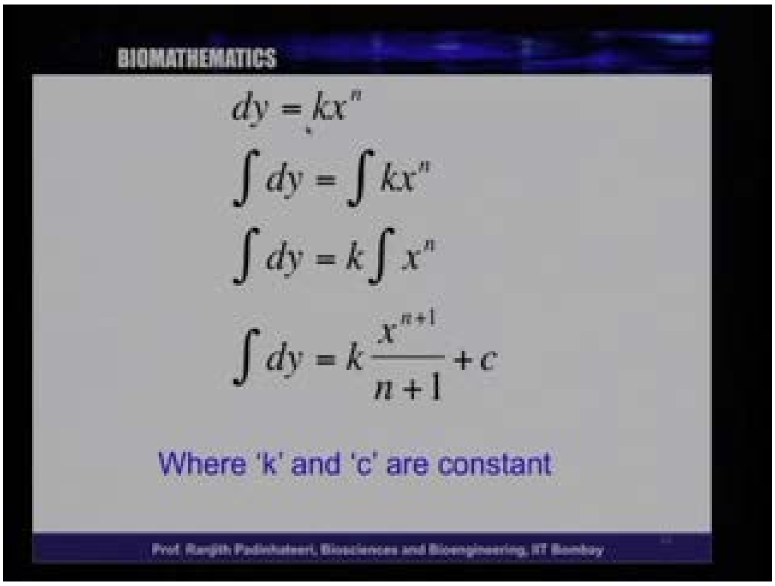

But, just before that, we have to just know one more thing, which is, if we have d y by d x is k x power n… So, here also, I wanted to write, d y by d x is k x power n, sorry.

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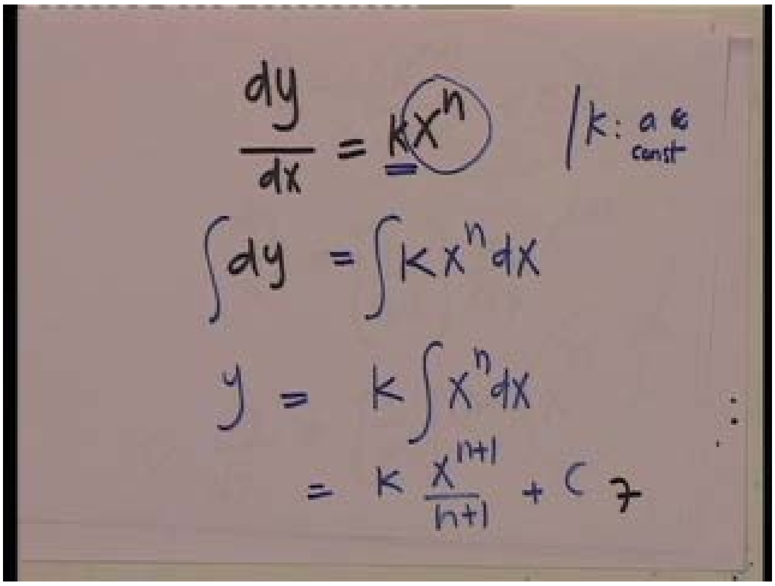

So, what I wanted to write there is d y by d x is k x power n. Then, what is y, that is the question. So, to get y, you have to integrate; d y is k x power n.

So, since k is a constant, which is independent of x, you can take this k out and write k integral x power n and you have to have a d x here. So, let, let, let me write it here, clearly. So, what is, some of this things, here are, there are some type, tight, typographical errors in this slide.

So, let me write this carefully here. So, the correct thing is, what I am writing here. d y by d x is x power n. So, d y, sorry, k x power n, where k is a constant. So, k is a constant, it is a number. So, d y is integral k x power n d x, sorry. The, there is no integration; you can integrate both sides. So, I can take this x, d x this way. So, d y k x power n d x and I can integrate on both sides. So, this will give you y.

So, y, integral d y is y. If k is a constant, so, you can take it out;x power n d x. And, we know the answer of x power n d x, which is x power n plus 1 by n plus 1 plus, I have a constant c. So, basically, what does this say? This says that, if you have a constant k here, you can take this out of this integration process,and then, do the integral of the rest of this x power n. You can do the integral of x power n alone; and the answer you get, multiply with k, is the answer for this. So, this is same thing as in differentiation. If you have a k times some function of x, you can take k as a constant and find the derivative of the function x alone.

So, this rule is applicable for differentiation and integration.

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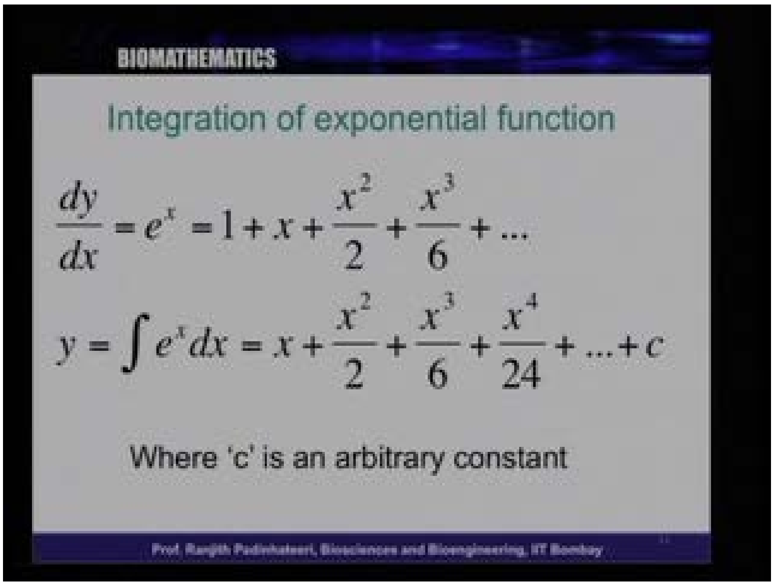

So, once you know this, you can go to this next one, which is integral of, integration of exponential function. So, how do we find the integration of an exponential function?

So, basically, you have d y by d x is e power x. Now, what is e power x? As we know, e power x can be, x, written as 1 plus x plus x square by 2 plus x cube by 6 plus so on and so forth. So, e power x can be written as an infinite series. As we learnt in earlier lectures, e power x can be written as an infinite series' sum.

And, which is, the first few terms of this infinite series is 1 plus x plus x square by 2 plus x cube by 6 plus so on and so forth. From this, if you find the integral of this, integral of this e power x, we have to integrate. So, y is basically, e power x d x integral. So, as we know, just have a look at here.

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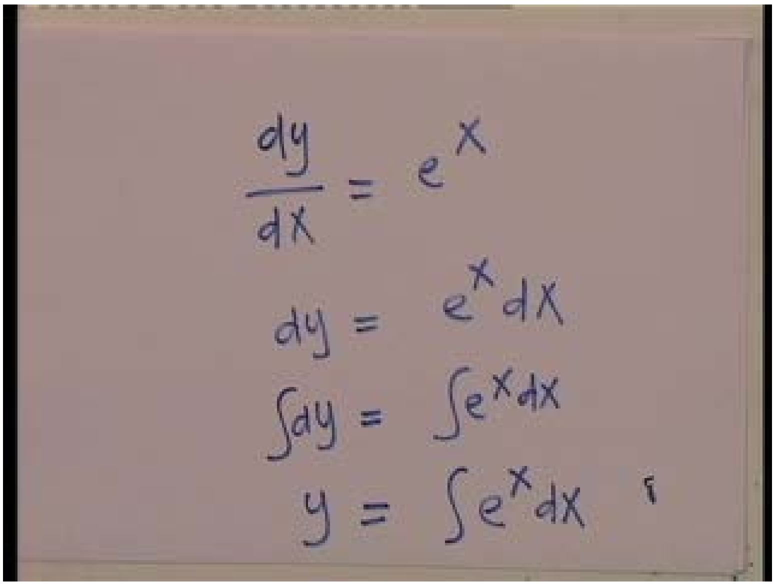

So, d y by d x is e power x. d y is e power x d x, you can take this, this side. So, I can integrate on both sides; that is, integral d y is integral e power x d x. Integral get d y will give you y. So, y is equal to integral e power x d x. Now, what is integral e power x d x? So, let us have a look at this. What is integral e power x d x?

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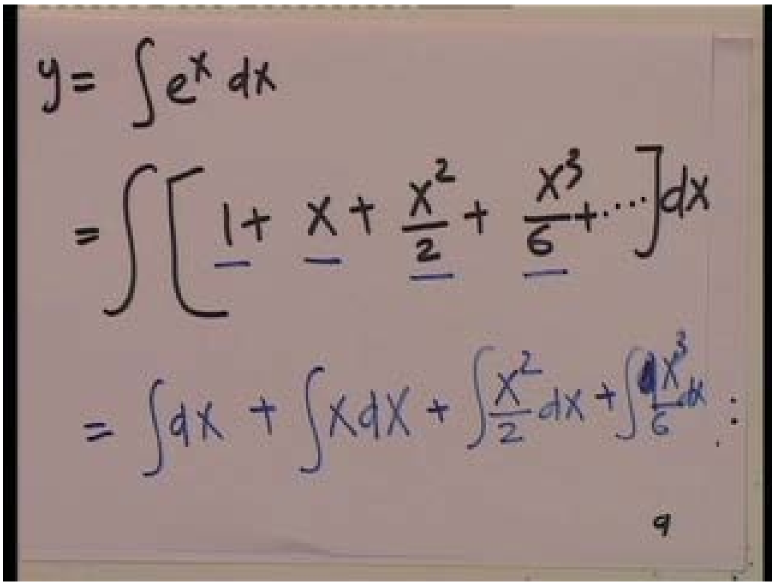

So, integral, the y is integral e power x d x. So, this is, integral e power x is 1 plus x plus x square by 2 plus x cube by 6 plus dot dot dot d x. So, this is your y. Now, as we know, we did for differentiation, each of these term you can separately integrate. So, as we know, 1 into d, the first term is… So, each, you can find the integral of each term and sum of them. So, the integral of the first term. Let us have a look at the first term. 1 into d x, that is the first term. So, the first is, integral of 1 into d x, which is d x, plus the second term is x into d x.

The second was, integral x d x. The third term is, integral of x square by 2 d x and the next term is, integral of, integral of x cube by 6 d x. So, these are the, these are the terms. So, if we find integral of each of these term, you will get the final answer.

So, let us have a look at, what is the integral of each of these term. So, let us have a look at here. So, let us look at this.

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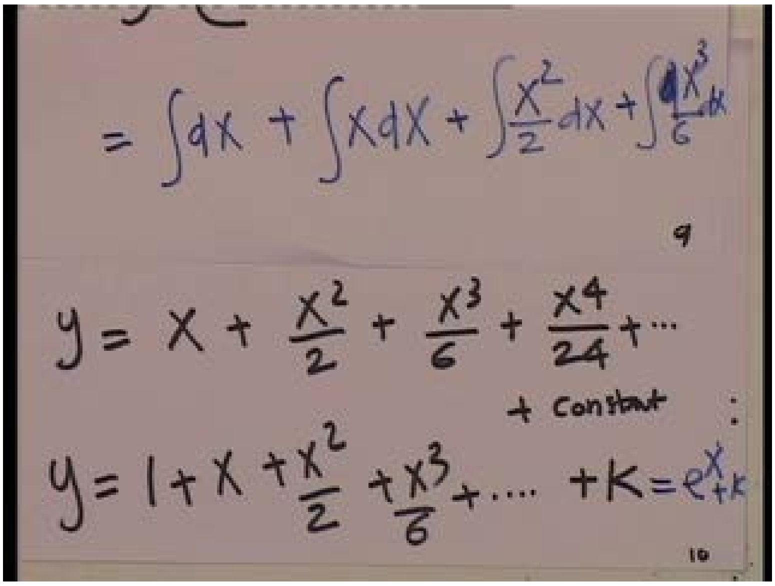

So, basically, you want the integral of the, basically, you want y is equal to integral of d x is basically x. Integral of x d x is basically, integral of x is, integral x d x is basically, x square by 2; integral of x square is x cube by 3; 3 into 2 will become 6. So, x cube by 6 plus; 3 will have a 4. So, x 4 by 4 into 6, 24, plus dot dot dot plus constant. So, this is the answer you will get.

So, you have a careful look at it. The answer you got is x plus x square by 2 plus x cube by 6 plus x power 4 by 24, plus, plus, plus a constant. Now, what is this constant? The constant could be any number.

So, any number k could be written as, 1 plus some other number k 1. So, let us write this constant as 1 plus k. It would be any constant. So, instead of writing 23, again, write it 1 plus 22. So, let me rewrite this constant as 1 plus a constant. So, let it rewrite this. y is equal to 1 plus constant k plus x, I rewrite, x square by 2 I rewrite, x cube by 6 I rewrite plus dot dot dot.

The many terms. So, 1 plus x plus x square by 2 plus x cube by 6 plus dot dot dot plus a constant k. This is the answer y and what is this? If you look at this, it will turn out that, this is nothing, but e power x plus constant k.

So, the answer is, integral of the, it turns out that, integral of e power x d x is e power x itself. Plus a constant k; some constant,it is like c; some other, some constant, some number; it could be any number; plus 123, minus 23, 100; any number, depending on k. So, it is, you can put any arbitrary number and you can know, the derivative of this is this.

So, derivative of e power x is e power x itself. Integral of e power x is e power x itself. So, once this is understood, the same way as you understand the derivative of e power x, take a paper and pen and go ahead and do, calculate the integral of e power x,

just by knowing the formula that, integral of x power n is x power n plus 1 divided by n plus 1. You can basically, go ahead and do this integral of e power x. Now, as we learnt it in differentiation, if we know this, we can, in fact, go ahead and do that of pretty much any trigonometric function, because, almost all trigonometric, all trigonometric functions, pretty much can be written as a power series of x power ns, x power something, right. So, I, let us have a look at cosine and sin, that is the next things, which we should look at. So, for a moment... So, next thing we will calculate is, we will calculate the integral of some trigonometric functions like sin and cos.

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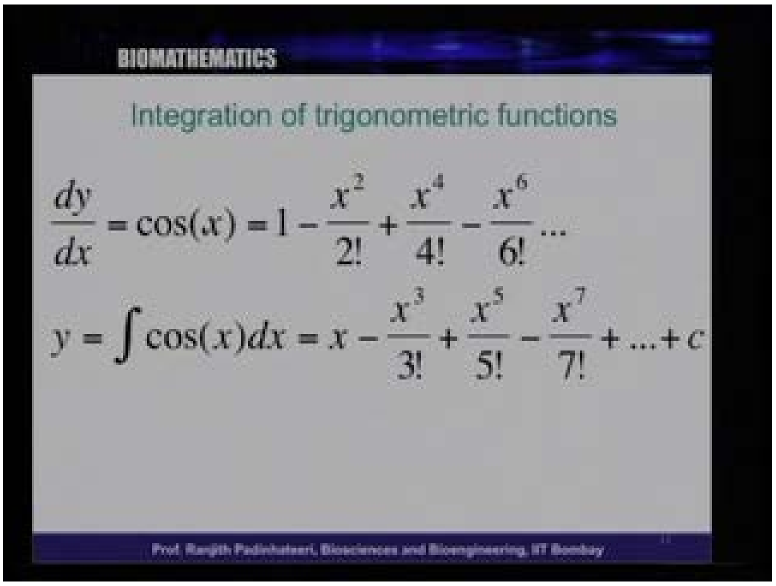

So, let us take this case here. d y by d x is cos x. So, cos is a trigonometric function, which we know, which is 1 minus x square by 2 factorial. So, as you know, this factorial means, this exclamation mark which will represent factorial, this means, like, n factorial means, 1 into 2 into 3 into upto n.

So, 2 factorial means, 1 into 2; 4 factorial means, 1 into 2 into 3 into 4; 6 factorial means, 1 into 2 into 3 into 4 into 5 into 6; 1 times 2 times 3 times 4 times 5 times 6. So, that is 6factorial. So, d y by d x is cos x , equal to 1 minus x square by 2 factorial plus x power 4 by 4 factorial minus x power 6 by 6 factorial and so on and so forth.

Now, the way to calculate y from this is basically, integrate this. So, integrate cos x d x. So, now, let us carefully do this once more, just like we did e power x. So, just like we did e power x, let us go ahead and do it.

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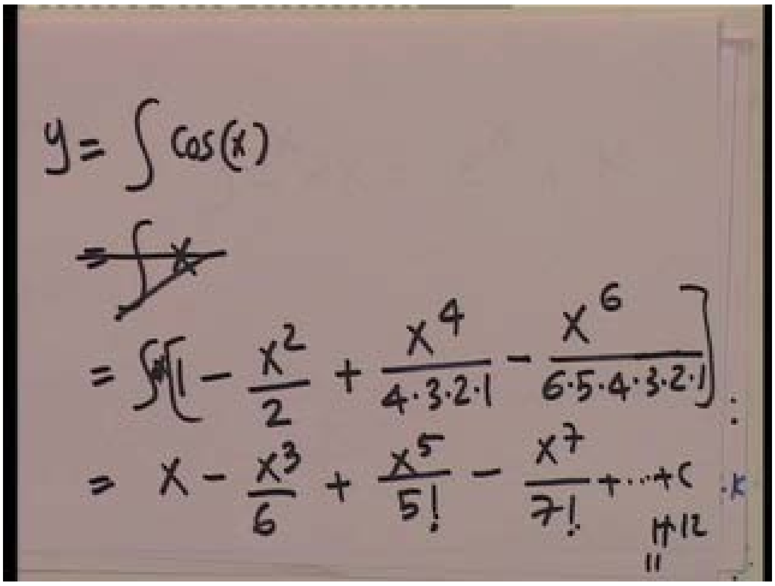

So, we know that, y is equal to integral cos x, which is equal to integral x minus, sorry. cos x is, can be written as 1 minus x square by 2 factorial; 2 factorial is 2 itself;

plus x power 4 by 4 factorial; 4 factorial is 4 into, 4 times 3 times 2 times 1. So, that is 1, that is 4 factorial; minus x power 6 by 6 factorial; 6 factorial is 6 times 5 times 4 times 3 times 2 times 1.

So, this is 6 factorial. So, this is the, and you have to do this integral of this whole thing. So, you find the integral d x. So, now, the integral of 1 is x; 1 d x is x. So, we can write, integral of minus x square by 2 will be minus x cube by 3. So, 3 into 2 is 6. So, x power 4 will become x power 5 by 5 factorial, minus x power 7 by 7 factorial and so on and so forth;

plus a constant c. So, if we do this, rewrite, do this, you will end up learning that, the integral of cos x is nothing, but sin x.

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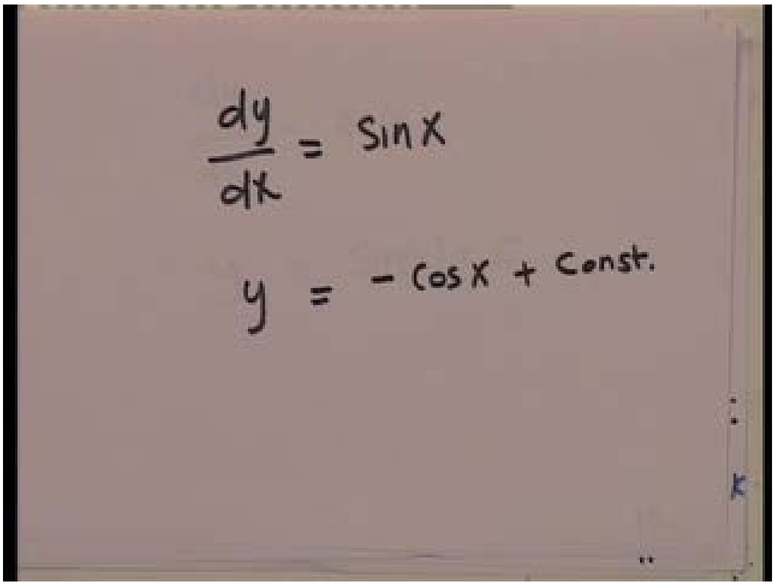

So, just like we know that, we had d y by d x is cos x, y is sin x plus constant. This much we know, because we can also guess this; because if the y is sin x, the derivative of y, d y by d x has to be cosx, because we know that, the derivative of sin x is cos x. So, similarly, if we say that, d y by d x is sin x, what will be y? So, the y will be minus cos x plus a constant c.

Plus a constant, because derivative of cos x is minus sin x, and with the minus sign, it will be plus sin x and the derivative of the constant is 0.

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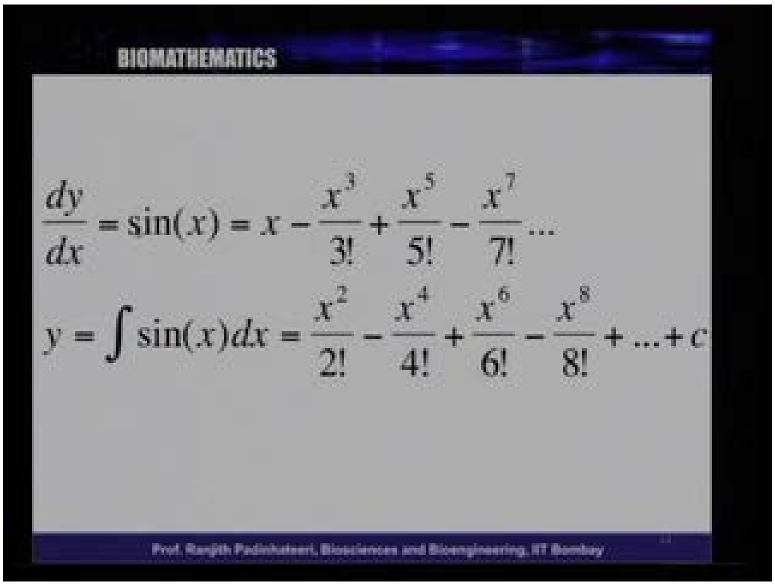

So, this, we can do like we did previously. Have a look at here. We can do derivative of sin x. So, sin x is x minus x cube by 3 factorial plus x power 5 by 5 factorial and so on and you can do the integral of this by knowing the formula we learned; and you will end up finding that, this derivative is nothing, but minus cos x plus a constant.

So, basically, what you learnt so far is that, we kind of understood, why do we need to do the integral, integration; because, if we know the derivative, or, if we know the slope of a function, or, the slopes at different points, we can draw the curve itself.

Now, instead of asking for the full curve, we can ask basically, let us say, we can ask the following question. Let us ask that, draw a curve which has slope m, a constant, between 0 and 1. Draw a curve of slope m between 0 and 1.

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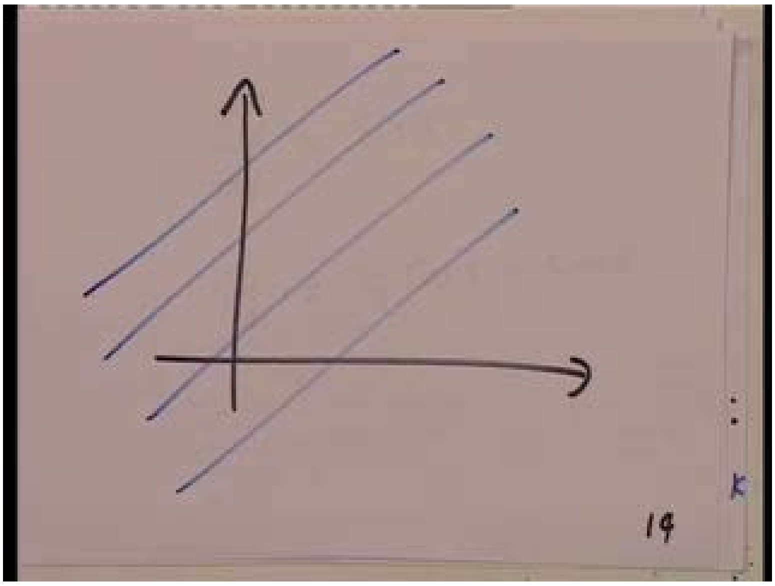

So, this is, we… Previously we asked this question that, draw…So, have a look at here. We asked this question, draw a curve of slope m and then, we drew many curves, because all of this curves will have slope m which is…So, we have a set of curves, which has slope m.

Now, we have to say, little more carefully. Draw a curve of slope m between 1 and 0, which has a particular value in 1 and some other value in, particular value in 0 and some other value in 1. You could ask such questions and one could put some limits of integration.

So, there are some integrals, which, you can integrate a function between two points. So, such integrals are called definite integrals and the way we would define, represent this, is like this.

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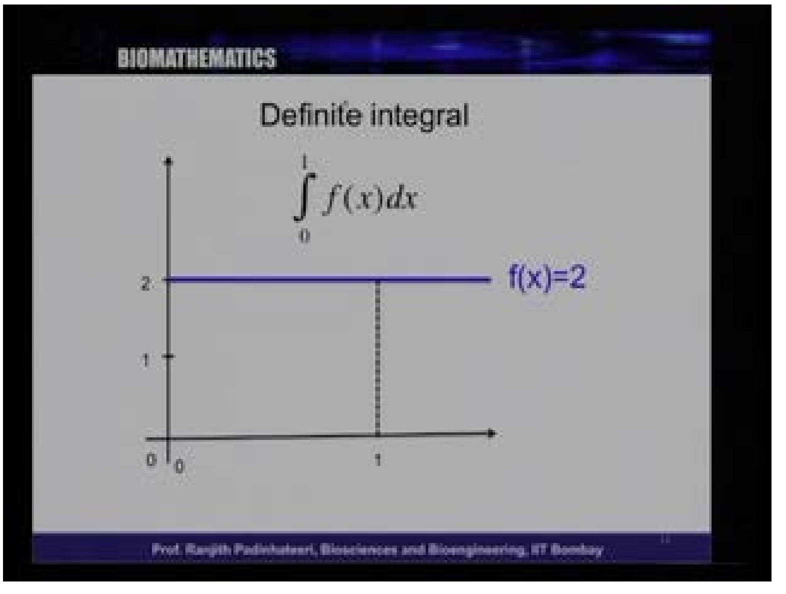

So, the definite integral is basically, you can integrate this function f of x from 0 to 1. So, now, here is an example of a definite integral. So, this is…Now, the function we have is f of x equal to 2 which is a line. So, this f of x is equal to a constant 2. Now, if we want to integrate this f of x, between, in the limit 0 to 1, that is, from here to here, we want the integral . So, if we have… So, what does this mean? When you, somebody writes integral 0 to 1 f of x d x, what does this mean?

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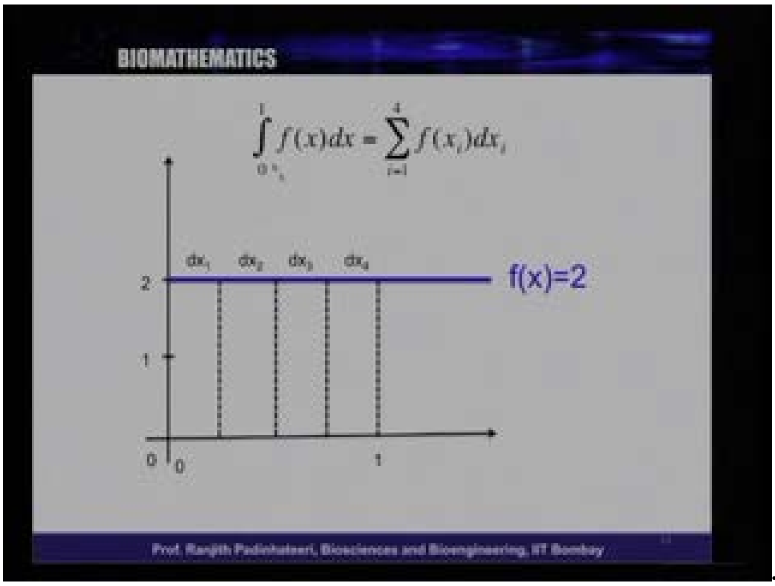

So, this means the following. So, have a look at this slide. So, integral f of x d x, you would see this at some point, some places; what does this mean is that, evaluate…This means that, this curve between 0 and 1 can be divided into…Let us divide, for convenience, this curves into many intervals, d x 1, d x 2, d x 3 and d x 4.

If we can divide like this, we can rewrite this integral as sums. So, integration is nothing, but summing; differentiation we had x plus d x minus x, so, where we have found the difference.

Here, we are summing it up. So, since the integration is the oppose, the co, anti-differentiation, we are summing up, here. So, we are summing up parts. So, f of x d x. So, if you expand this, what do we…So, what are we saying here? We are saying, integral f of x d x is nothing, but sum over I f of x I d x I, where d x I is intervals of d x 1, d x 2, d x 3, here.

So, let us think about this, what does this mean. So, let us draw this curve of it. So, let us have a look at here.

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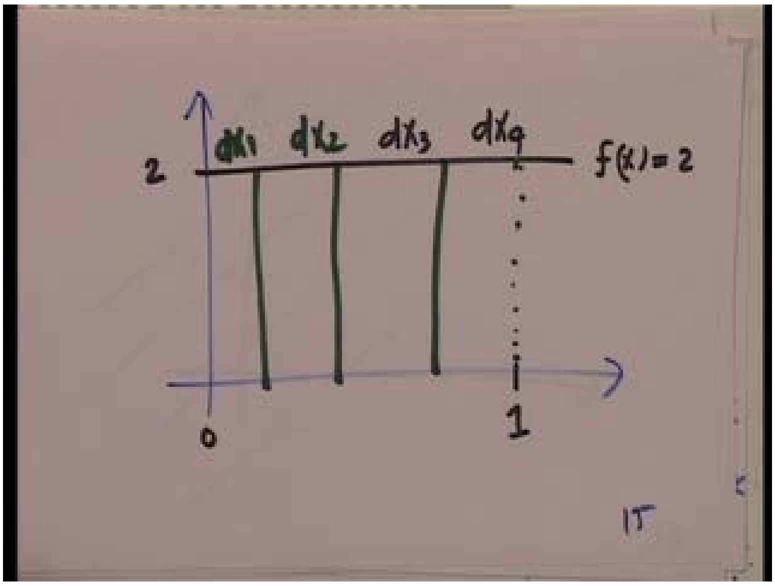

So, we have this curve, which is y equal to 2; function f of x equal to 2. So, this is 2. And, we want to find out the integral between 0 and 1. So, this is f of x equal to 2; this is this curve. Now, between 0 and 1, you can divide this curve into many small boxes. So, this is the point at 1.

So, you can divide this into many boxes. So, this is one box, one box. So, d x 1, d x 2, d x 3, d x 4. d x 1, d x 2, are intervals. Now, what the claim which we made is that, the claim we made here…

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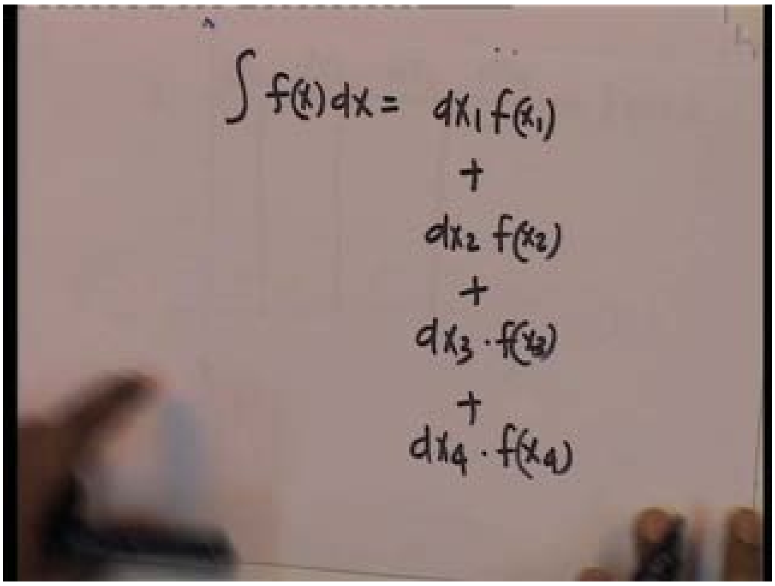

So, let us have a look at this. The claim we made is the following. We said that, integral f of x d x is nothing, but, take the first interval, d x 1 and multiply the function at this particular point x 1, somewhere in this interval. So, f of x 1, plus d x 2 into f of x 2, plus d x 3 into f of x 3, plus d x 4 into f of x 4.

So, this is integral f of x d x.

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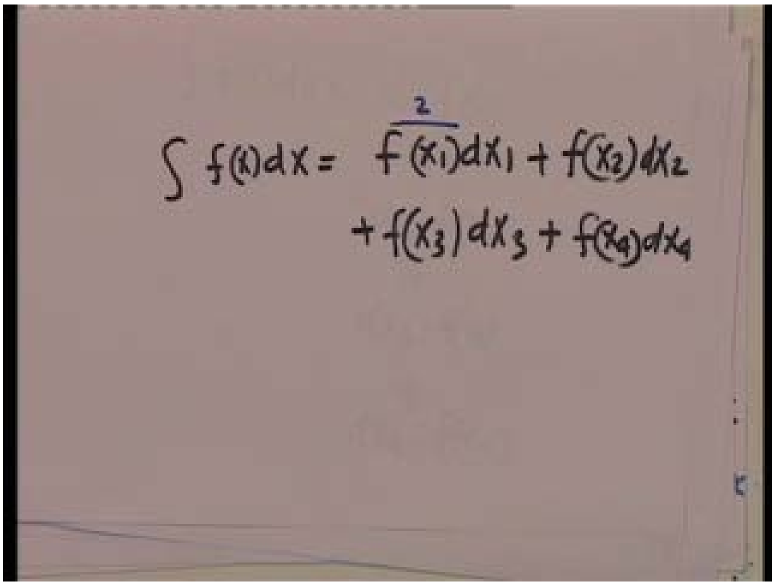

So, we said that, integral f of x d x is f at x 1 d x 1, plus f at x 2 d x 2, plus f at x 3 d x 3, plus f at x 4 d x 4. What is this f of x 1 into d x 1 means? So, f of x 1 is somewhere in the first interval. So, let us have a look at this. Somewhere in the first interval, we want to calculate the function value. f of x 1 is nothing, but 2. So, the function value f at any point x 1, anywhere here, is 2 itself. So, this is 2 into d x 1. So, basically, what we are calculating is, height into width. So, basically, what we are calculating is, the area of this box.

So, we calculate the area of this box, that is, f of x 1 into d x 1. f of x 2 into d x 2, d x 2 is this width and f of x 2 is this height, which is 2. So, now, we calculated, the second term is nothing, but the area of this box; third term is area of the third box and fourth term is area of this fourth box.

So, integral is nothing, but finding the area and finding the total area, under this particular curve. So, it turns out that, integral is nothing, but finding area under the curve.

So, have a look at here. Integration is nothing, but finding the area under this particular curve. So, we have this curve f of x equals to 2, and if we want to, if we can find out the area under this curve between two points, 0 and, we would say that, we found the integral of this, between these two points.

So, if we want to calculate the integral of a function between two points, if we can find the area between that, that enclosed, between the curve and this 0, the X axis, that is y equal to 0; the area between y equal to 0 and this curve, this particular area, will give you the integral.

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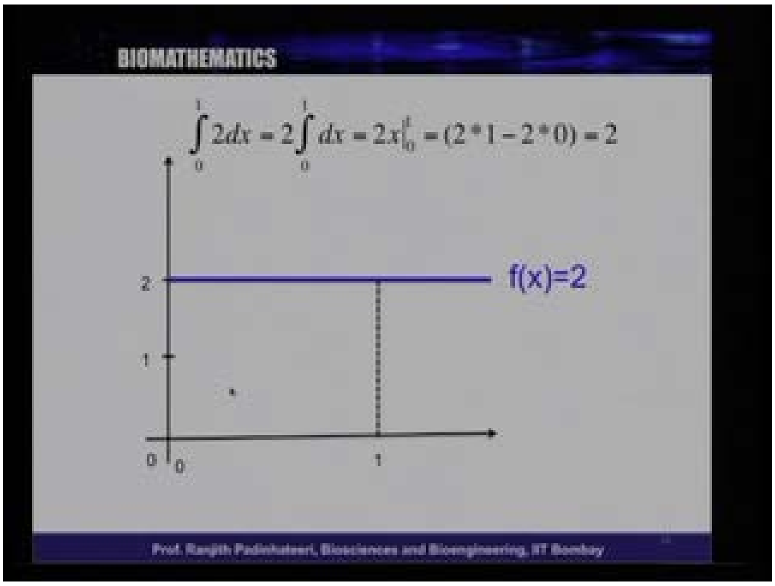

So, let us have a look at it. So, this is the, this area is what we have to calculate. So, what is the area? So, area is, this is 1 and this is 2. So, area has to be 1 into 2, which is 2. So, the answer has to be 2. Now, how do we find the answer? Integral 0 to 1 f of x is 2. So, 2 d x, that is what we have to find out.

So, 2 into 2 is a constant, integral d x. So, we know that, integral d x is x. Now, we want to evaluate this between this points 0 and 1. So, what does this, such a symbol means is that, this means that 2 into evaluate the x at 1, which is 1.

And, evaluate x at 0, which is 0. So, this is 2 into 1 minus 2 into 0, which is 2. So, this is what this means.

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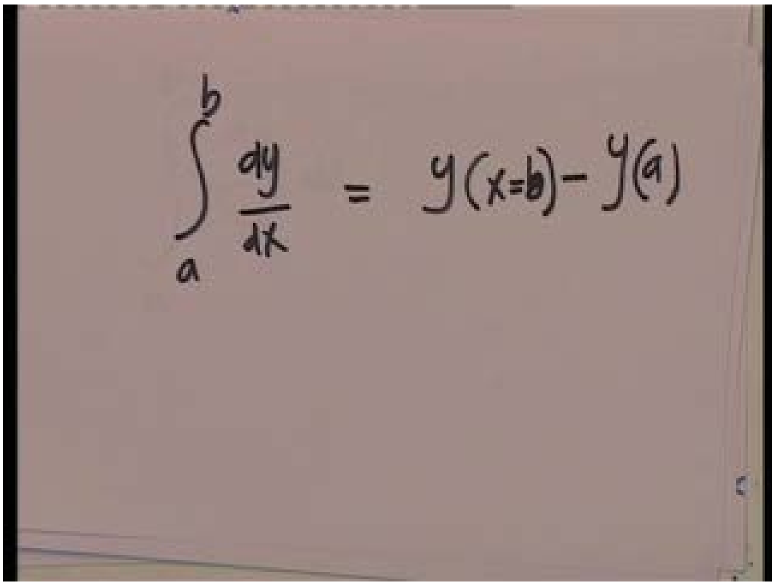

So, let us say, we want to find, we want d y by d x is our derivative, and we want the in, integral of this between a, b, sorry; we want to find out the integral of this. So, let us take a different example.

So, let us take, integral a to b, d y by d x and the answer of this is, y at x equal to a; you know that, integral d y by d x is y. So, you calculate the y at x equal to a and then, subtract from, sorry, y at x to b and subtract from y equal to x equal to a.

So, y at x equal to a. So, this will give you, this is the answer. So, this is the rule, which we want to learn now.

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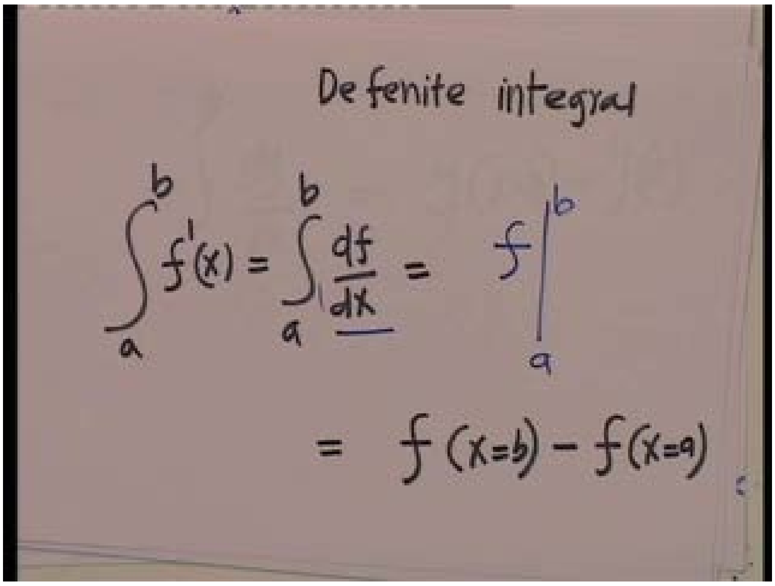

So, the next rule of… So…. So, rule of definite integration is that, the (( )) technique of calculating definite integral is the following. So, if you want to calculate the definite integral of some function f prime of x, which is d f by d x between two points a and b, first find the integral of this function d f by d x, like we did before.

So,integral of d f by d x is, integral of this d f by d x is f, we know. So, the integral of d f by d x is f; then, you can evaluate this f between these two points a and b. What does this mean is that, this means that, f at x equal to b minus f at x equal to a.

So, this is what I did. So, the answer of integral d f by d x between these two points a and b is, f at x equal to a minus f at, sorry, f at x equal to b minus f at x equal to a. So, this is the answer. So, now, let us do some examples for this. So, just like we did...

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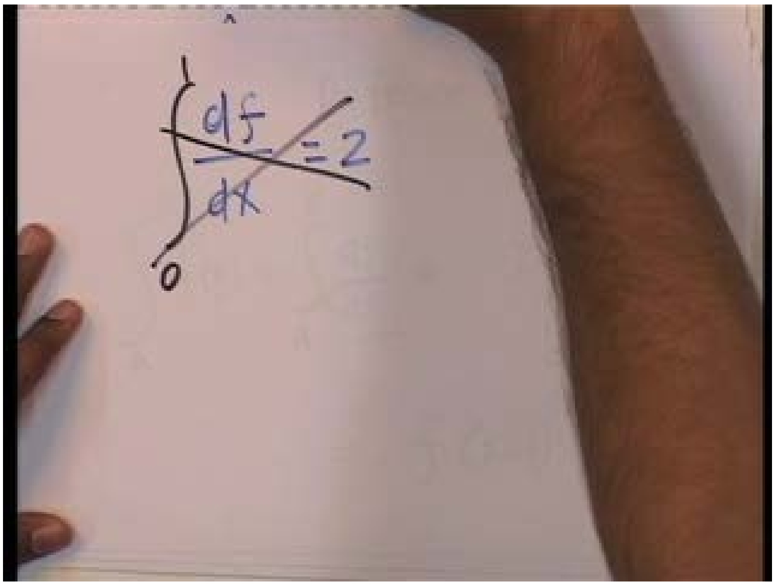

So, we (( )) get, we took the first example, which is d f by d x equal to 2. This is what, one example we took. If d f by d x is 2,

we want to calculate this between…So, integration of this between these two points 0 and 1. So, basically, let us go back to this example here.

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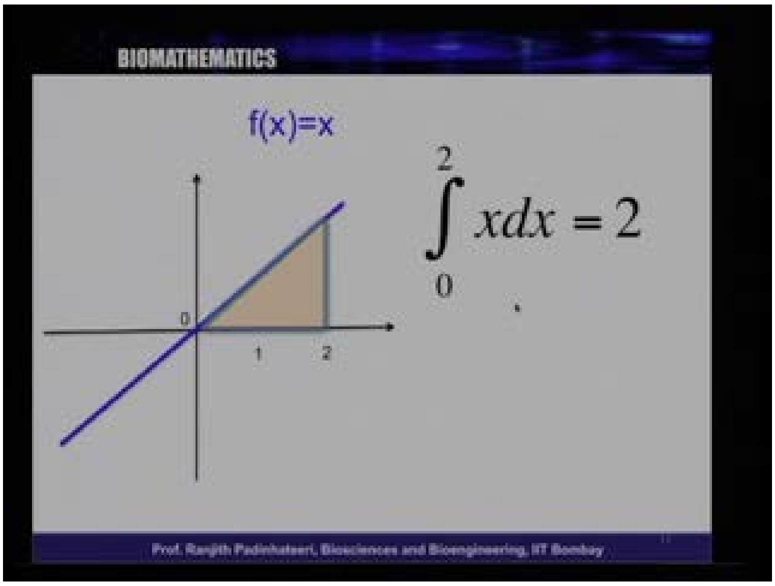

So, we had this function f of x. So, let us, let us go back to a new example, which is f of x is equal to a function x.

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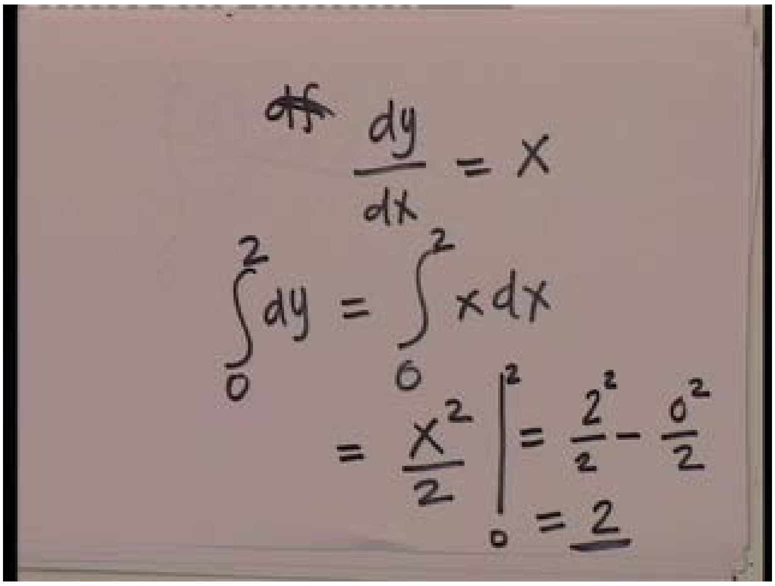

So, let us take this example, d y by d x equal to x. Now, you want to integrate this between, integral between two points 0 and 2; d y is equal to integral between 0 and 2 x d x. So, I can take this d x this way and x d x. Now, the integral of x, we know is x square by 2.

So, the way of integral of x d x is, x is x square by 2. So, x square by 2, at this two points 0 and 2. So, this represents, evaluate this x square by 2 at 0 and 2 and then, subtract; that means, this, what is, this is what it means.

This vertical line and 0 and 2, this symbol means that, evaluate x square by 2 at 2 first. So, if you evaluate x square by 2 at x equal to 2, 2 square, which is, which is 2 square by 2 minus 0 square by 2. So, this is basically, 2 square is 4, 4 by 2 is 2; the answer is 2.

So, have a look at here. Basically, what we want is, area under this particular, area of this shaded region. So, integral x d x between 0 and 2 means, you draw this function x and mark 0 and 2, and the area of this marked region will be the integral; and the way to calculate is, what we just described here; you calculate the integral and then, find the limits, and you will get the answer as 2.

So, this is the way of going, doing definite integrals; but this is just a peep, the look, took a look into the definite integrals, but we will come back and why do we need it and all that, we will discuss in the next class.

So, today, few things that should, should take back. One is, integration is anti derivative, which is the complementary thing of finding derivative. So, and 2, integral of x power n is x power n plus 1 divided by n plus 1. So, if we take these two things back, we will discuss the rest, the definite integral in detail in the next class; and we will, we will get in, we will, we will have a look at it more carefully and then, try and understand, where we can use this in Biology. So, today's lecture, we will stop here. In the next class, we will discuss more about Integrals. Thank you.