**Gasdynamics: Fundamentals and Applications**

**Prof. Srisha Rao M V**
**Aerospace Engineering**
**Indian Institute of Science – Bangalore**

**Lecture 34**
**Converging and Diverging Nozzle Operation**

We are looking at varying area ducts and in the last class a very important concept of choking in varying area ducts was introduced in the context of converging nozzles. And there we had discussed that it is not sufficient to just look at a variation of area variation of velocity you are also have to look at pressure ratios that have to be provided across the nozzle. You have to provide correct pressure ratios across the nozzle. So, that nozzle operates in a certain desired way.

If pressure ratios are changed then mass flow rates can change but mass flow rate will not change infinitely there is a limit and that limit is when at the exit of the convergent duct the Mach number becomes equal to 1 when that happens there is an information cut off because no pressure information from downstream can propagate upstream. The upstream portions of the nozzle will not know that there has not been a change in downstream pressures.

And as a consequence everything else becomes constant like mass flow rate becomes constant for given stagnation, pressure, temperature conditions. But if you change stagnation conditions automatically mass flow rate and pressure at the exit will change but given that the nozzle is always operating in choke condition. Then increase in stagnation pressure will lead to increase in mass flow rate increase in pressure.

And that pressure may not correspond to the ambient pressure or back pressure. So these are some of the highlights of what we discussed the last class. And that sort of discussion should be carried over to our current discussion on convergent and divergent nozzle operation. So C - D nozzles is what as termed and we looked at varying area ducts.

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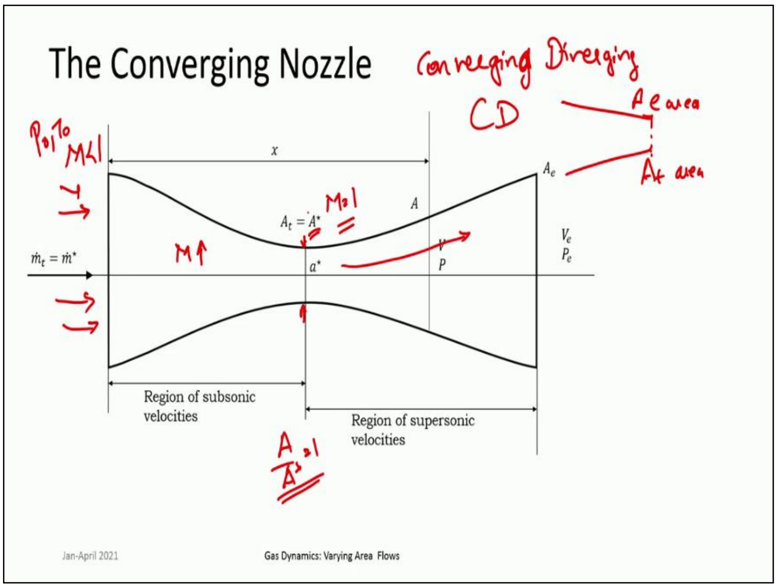


And we saw that if we have to convert a subsonic flow to supersonic flow starting from subsonic flow you have to go to supersonic flow then the only way you can achieve that is through a C - D nozzle that is this is converging diverging or C - D nozzle. So you can see the schematic of a C - D nozzle given over here. Here if the intake at the inlet you have Mach numbers less than 1 then the conversion portion accelerates Mach number. So, Mach number increases.

And it can continue to increase until the minimum area where it should achieve Mach number equal to 1. This is something we saw at the convergent nozzle that the maximum Mach number you can achieve in a convergent section is Mach number equal to 1. Further if you change anything it will not change the Mach number at the convergent section. So, if you need to further increase Mach number beyond Mach number equal to 1 then you have to attach a divergent duct.

Because a divergent duct is the one which increases Mach number in supersonic conditions. So if you attach divergent duct then the flow will further accelerate to supersonic condition. So to get flows from low velocities to all the way to high velocities you need a C - D ducts convergent divergent nozzles. The behaviour of convergent divergent nozzles is some ways similar but many ways different from our discussions on converging nozzles.

So again the terminology should be borne in mind. Now here in convergent nozzles the exit area and the throat area this is $A_t$ exit area and throat area they are the same. So the minimum area is the same but in a convergent divergent nozzle the exit area is greater than the throat area. So when we talk about this C - D nozzles and we talk about throat area it refers to the minimum area and if the C - D nozzle is producing supersonic Mach numbers then necessarily the Mach number at the throat has to be equal to 1.

So if Mach number at throat is equal to 1, $A_t$ a throat is equal to $A^*$ that is the star area. So $\frac{A_t}{A^*} = 1$, this is very important. And the concept of mass flow rate choking is relevant here also because you have a minimum area here. And you are giving a certain $P_0$ *and* $T_0$ to this particular nozzle. So that it operates and if you have given enough $P_0$ *and* $T_0$ such that $\frac{P_t}{P_0} = \frac{P^*}{P_0}$, then Mach number at throat becomes equal to 1, $M_t = 1$.

And mass flow rate becomes is choked. So, m dot is choked. So you should understand the choking of mass flow rate should be understood in proper context if your $P_0$ *and* $T_0$ is fixed and mass flow rate chokes then whatever may be the downstream conditions it cannot change mass flow rate. But if you vary $P_0$ then your mass flow rate will increase linearly.

So, this has to be understood very clearly. So, but now there is a distinction between exit velocities, exit Mach numbers and exit pressures and throat velocities, throat Mach numbers and throat pressures. So, that brings in some additional discussions in convergent diversion nozzle. So now similar description as we started off you are looking at not only the area ratio you are looking at pressure ratio that is happening within these nozzles.

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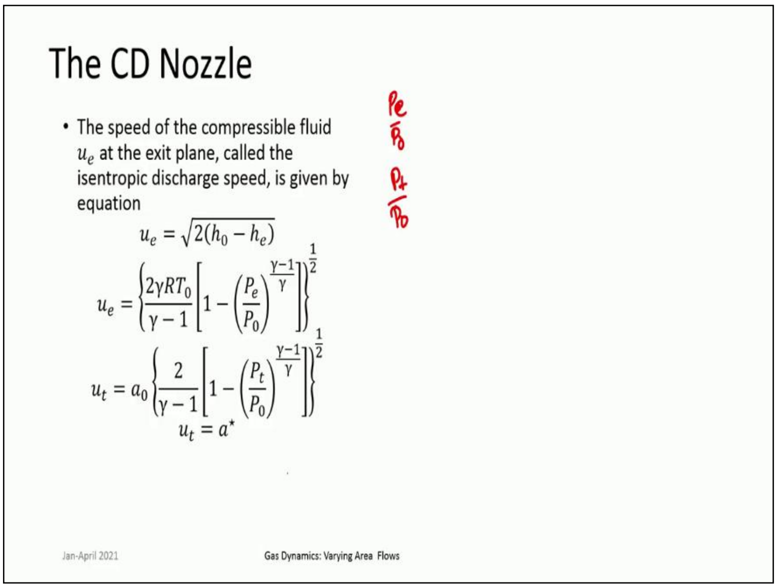

And you express velocities in terms of pressure ratios this is what you had done earlier also in the previous class. So now this is $u_e = a_0\left(\frac{2}{\gamma-1}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right)^{\frac{1}{2}}$ that is exit pressure $\frac{P_e}{P_0}$ and at throat the pressure is $\frac{P_t}{P_0}$. And the pressure varies continuously along the nozzle it is an isentropic flow you can use isentropic relations to know what is how the pressure temperature density they change through the nozzle.

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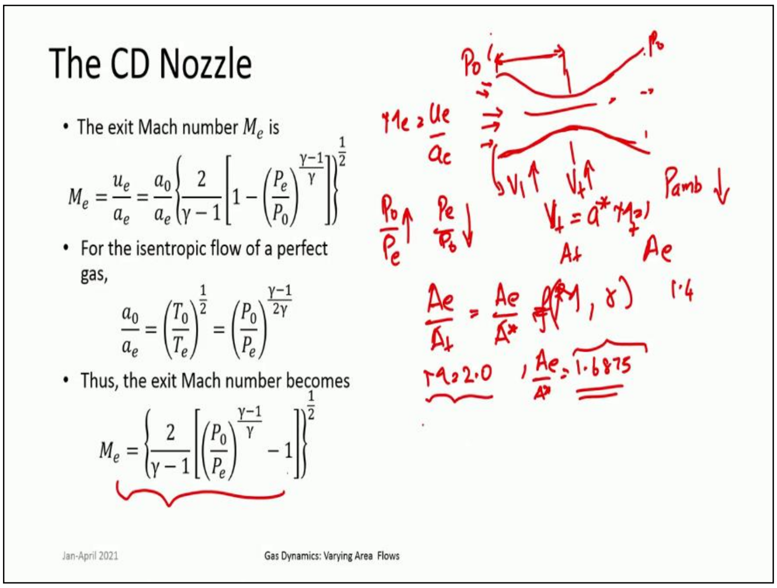


Now. So, what is the exit Mach number? Exit Mach number is $\frac{u_e}{a_e}$. Now this is a completely varying flow. So you have to calculate these quantities separately at the exit a is not constant from the inlet to exit. So that has to be really understood properly that exit and the exit flow continuously undergoes changes. So Mach number at the exit can be represented $M_e$ by this particular equation $M_e = \left(\frac{2}{\gamma-1}\left[\left(\frac{P_0}{P_e}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right)^{\frac{1}{2}}$.

So now you can understand from here that if I have a given $P_0$ and I start with a variable convergent divergent duct $P_0$ is given. Initially at the exit also it is $P_0$. So $\frac{P_0}{P_e}$ is one that means Mach number is zero there is no flow and now I start reducing the back pressure or ambient pressure that the nozzle is exhausting to. So, it is exhausting to an ambient here so, $P_{ambient}$.

So $P_{ambient}$ started to reduce. So, basically $\frac{P_0}{P_e}$ will increase or $\frac{P_e}{P_0}$ will decrease. So it can be either way always we look in terms of pressure ratios they are important then we look at how the C - D nozzle behaves? So similar to previous discussions once you start decreasing this back pressure or ambient pressure flow starts happening and as pressure ratio is increased more and more flow can happen. So, mass flow rates and velocities increase through the duct even at the exit they will increase.

So mass flow rate continues to increase. In all these sections the flow at this condition is subsonic but there is a limit. So, this cannot happen all the while. Because you started with a subsonic flow and there is a convergent duct. So as you keep on adding more and more flow velocity at the $V_1$ will start increasing that means $V_t$ will start increasing and there will come a particular point when $V_t$ will become equal to $a^*$.

That is Mach number at $V_t$ will be equal to $a^*$ or Mach number will be equal to 1, $M_t = 1$. So when that happens then this nozzle becomes choked. So further on there can be no changes in this section of the nozzle. So, there can be changes downstream. So, we will start discussing that with a diagram.

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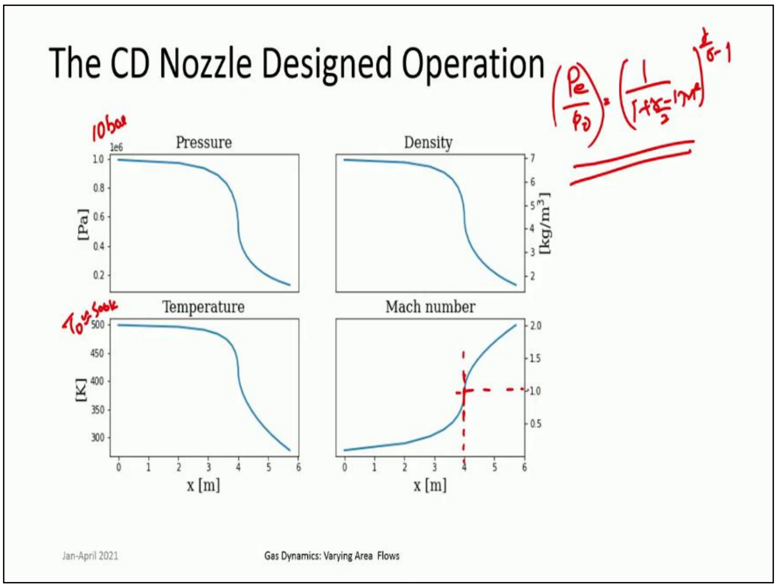


So, what is known as a perfect operation of the nozzle is when you keep the correct pressure ratio such that. So, this nozzle has a given area ratio. So this nozzle has a correct relationship between $\frac{A_e}{A_t}$. So $\frac{A_e}{A_t}$ when it is properly choked it will become $\frac{A_e}{A^*}$ this corresponds to some Mach number. So this is a function of some Mach number Mach number and $\gamma$. So $\gamma$ if you take 1.4

this is corresponding to a particular Mach number. So if for say for example Mach 2 nozzle $M = 2.0$ then $\frac{A_e}{A^*}$ is 1.6875 this is known.

So if you have a nozzle with a given area ratio it should produce a Mach number of 2.0 but when will it produce Mach number of 2.0? It will produce a Mach number of 2.0 when you give the appropriate pressure ratio across it. And that pressure ratio is you can calculate $\frac{P_e}{P_0}$ that will be $\frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$ this is approximately in the range of $\frac{1}{8}$ $\frac{P_e}{P_0}$.

So or $\frac{1}{8}$ ; or $\frac{P_0}{P_e}$ is in the range of 8. So if you give the correct pressure ratio for that particular Mach number and for the given area ratio then the nozzle operates perfectly. There will be no problems and there will be an isentropic flow pressure will continuously decrease continuously across the nozzle starting. So if here it is taken as the starting pressure or $P_0$ is taken as 10 bar.

So $1 * 10^6$ and it decreases reduces to the exit pressure. Similarly density and temperature starting $T_0$ is close to 500 K. And if that correct pressure ratio is provided for Mach 2 flow then it provides the exact value that is Mach 2 at the exit. And this is the location where there is the throat which is at $x = 4$ . So, where Mach number becomes equal to 1 so that particular operating condition is known as the designed operating condition for the C - D nozzle.

For a given area ratio of the exit area ratio of the C - D nozzle to achieve a certain supersonic Mach number there is a certain pressure ratio that must be provided that pressure ratio is $\frac{P}{P_0} = \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$ this is from isentropic flows. If you give exactly this particular pressure ratio you will achieve the correct Mach number at the exit that is designed operation.

But there are many ranges of operations between all these designed operations sometimes pressures can increase also.

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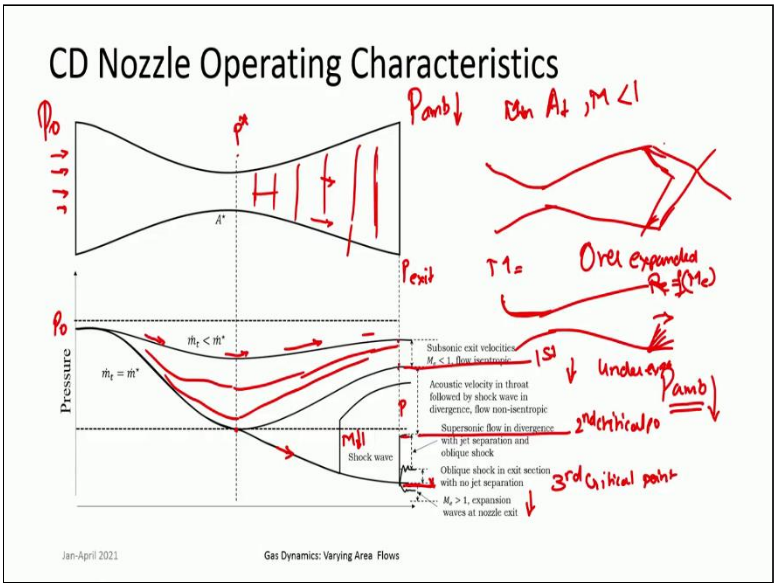


So what happens then? So, this is the operating characteristics of a C - D nozzle. So this is a character this is schematic representations of how the pressure varies as a consequence mass flow rate as a consequence Mach number how do they vary? In this case we are considering that we are having a fixed $P_0$, $P_0$ is fixed and $P_0$ is fixed at a certain value and $P_{amb}\ or\ P_{back}$ pressure is reduced.

So when we start the way we start this is that you start that with the same pressure as P 0 then there is no flow across the nozzle and then $P_{amb}$ is reduced. When it is reduced the mass flow rate starts increasing velocities increase they can continue to increase and pressure decreases all through the nozzle. So very smoothly decreases an isentropic flow it smoothly decreases all through the nozzle.

So you consider one specific case. So, the inflow is now subsonic. So it is a convergent area the velocity increases. But the pressure ratio that is has been provided is not sufficient to choke the nozzle at minimum area. That means at minimum area first condition is minimum area at A route throat area Mach number is still less than 1 that means mass flow rate is not choked. So it is still subsonic and then it faces a diverging area that implies that now the pressure will start increasing in the diverging area Mach number decreases.

So that is the plot that is given over here max the mass flow rate is less than choked mass flow rate for that particular nozzle. Now as you continue to decrease $P_{amb}$ you will get various plots here but they will all be not choked. So you can see that mass flow rate will continue to increase. But once it achieves that particular $P^*$ value at the throat. So, the throat it has achieved $P^*$.

That means mass flow rate becomes choked mass flow rate becomes a constant and there can be no change to the upstream conditions. So, all the cases between these 2 points so, these 2 points you can continuously have a change in pressures in the upstream as well as downstream but the moment the Mach number one is achieved at the throat upstream does not change upstream of from the throat will not change.

So this particular point when the first choke condition is achieved at the throat is known as the first critical point of C - D nozzle operation. But does that mean that you will get supersonic flow immediately? No because you do not. So, for getting complete full supersonic flow your ambient pressure must be much lower corresponding to that particular Mach number. So, if that is not satisfied then the flow will continue to remain subsonic in this region.

So, that when first critical point is achieved flow increases from subsonic speeds becomes Mach 1 at the throat and then it becomes subsonic at the exit in an isentropic fashion. So, that is first critical condition. You now the further pressure is increased actually pressure is decreased ambient pressure is decreased as ambient pressure continues to decrease we first talk about the next critical point that is you decrease ambient pressure to such an extent that the flow now accelerates in the in the divergent section it becomes supersonic.

So, the flow accelerates and it can continue to accelerate until the exit because now see once the flow becomes supersonic the downstream conditions do not affect the upstream so, easily. So, it can go all the way supersonic till the exit of nozzle till the exit. So, that is Mach number equal to it will just be achieving Mach 2 at the exit. But if the pressure condition do not satisfy the required pressure condition for fully optimally expanded flow which we have discussed just the previous case then what happens is you get a shock at that point.

We consider the maximum limit is that you consider a normal shock occurs over there. So, immediately the flow becomes subsonic after the normal shock. So, that particular point where you have full expansion in the divergent portion to the exit Mach number according to designed area ratio but a normal shock stands at the exit converting it into a subsonic flow that a condition is known as the second critical point.

And third critical point is when there is no shock it is a completely shock free flow you have provided the correct ambient pressure for a designed operation. So, that is the third critical point that third critical point is over here this is third critical point. So, first critical point is here first critical point and second critical point is here critical point where you get the full supersonic flow in the divergent portion.

But at the end a normal shock stands and third critical point is when there are no shocks and it is a correctly expanded flow now there are other conditions in between these critical points. So, if you consider. So, once the nozzle gets choked and you start decreasing pressure after the first critical point you continue to decrease pressure. Now what happens is there is a possibility that the flow can accelerate in the divergent portion.

So, it continues to accelerate here but the downstream pressure conditions do not support a fully expanded supersonic flow. So, in order to match the downstream pressure conditions in the downstream shock waves form the duct you can get a normal shock in duct and the normal shock can position itself along this duct and then after the normal shock Mach number becomes less it becomes less than 1.

And then you have an increase in pressure because it is a divergent duct it is a subsonic flow. So, all the way starting from first critical condition to the second critical condition you can have shock waves in the duct and they move from lower and lower strength. So, as they go higher and higher here outside the shock Mach number increases. So, the strength keeps increasing until the exit.

Once it reaches the exit you pass the second critical point then what happens is that you can have. So, now between suggest before it reaches the third critical point completely expanded flow correctly expanded flow if the pressure ratios are slightly lower that means you still have higher pressures then at the exit you get oblique shocks. So, you get oblique shocks the reason is that the flow has now expanded completely but the outside pressure conditions do not allow a completely expanded flow.

But the nozzle has no way of communicating because it is supersonic. So, as a consequence you get oblique shocks. When that kind of oblique shock is formed then that operation of nozzle is termed as over expanded operation over expanded operation. Now what happens if you can you can reduce continue to reduce the ambient pressure further down then what happens is that you get perfect exam expansion until the nozzle exit.

So, $P_e$ will exactly be equal to the corresponding pressure ratio for the nozzle Mach number that function of Mach number but the now the pressure here will be greater than the ambient pressure outside it will be greater than ambient pressure outside because you are further reducing ambient pressure. So, nozzle can continue to I mean the flow can continue to expand outside the nozzle.

So, it expands using by using by series of expansion waves and that operation is called under expanded operation. If these oblique shocks become very strong they can move within the nozzle and can cause the jet to be separated from the nozzle this is a interaction due to the shock and the boundary layer that is called severely over expanded nozzles severely over expanded operation.

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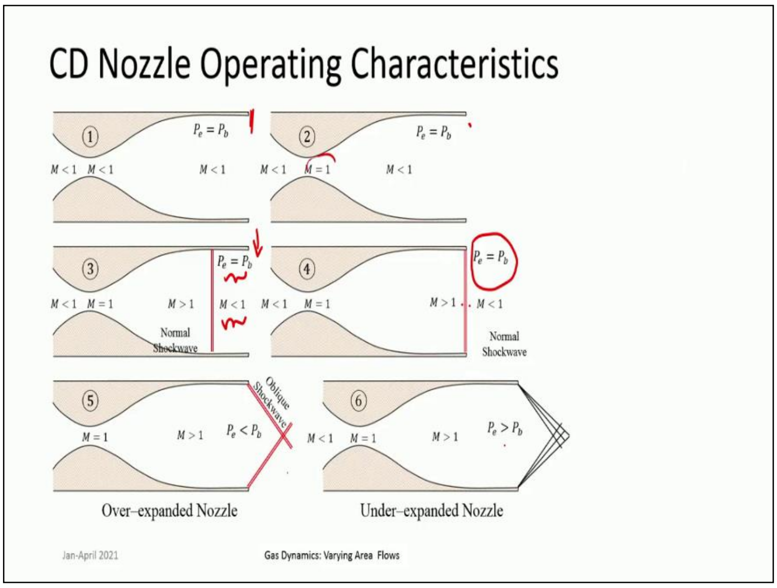


So, here is a schematic of all that we had discussed till. Now first condition here when flow is completely subsonic back pressure and exit pressure should be the same. Second condition is that the flow is sub flow accelerates to Mach number equal to 1 but is subsonic at the exit there is no case for a the back pressure does not support a fully supersonic flow. The third case is that you have continued to reduce the back pressure.

But it is not sufficient to produce full supersonic flow therefore you have shock waves in the duct even now the exit velocity is subsonic. So, it satisfies $P_e = P_b$ the final limit of this is that you get a normal shock at right at the exit. So, here completely it expands to supersonic flow but right at the exit it becomes subsonic even at this condition $P_e = P_b$. But beyond this condition if you decrease back pressure then $P_e \neq P_b$ ,need not be equal to $P_b$.

There is a perfectly expanded condition where $P_e = P_b$ that case when there are no shocks completely supersonic flow that is optimally expanded. But if there is it is not optimally expanded then you have 2 cases over expanded nozzle operation and under expanded nozzle operation. So, in our expanded nozzle operation the exit pressure is less than ambient pressure or back pressure.

So, in order to match pressure oblique shock waves form in under expanded nozzle operation the exit pressure is greater than the back pressure to match the pressure conditions expansion waves' form.

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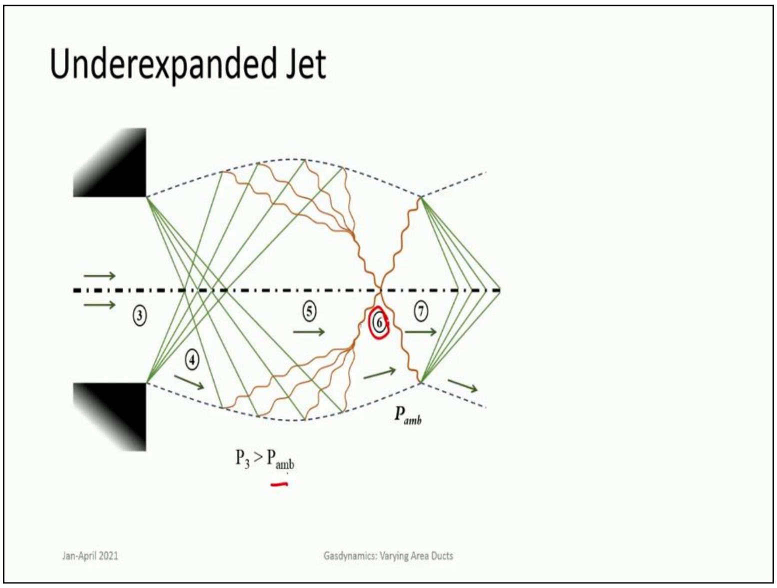


So, this is what we had discussed. So, the over expanded operation this picture should remind you of what we had discussed in the class on oblique shock wave reflections. So, this is exactly what happens to the jet after it leaves the nozzle even it is in over expanded condition the. So, in over expanded condition $P_1$ is less than the ambient pressure at this boundary of the jet and the ambient always pressure should be equal to $P_{amb}$. So, this is known as the free pressure boundary these oblique shock waves come in and this is a symmetry line.

So, here flow has been deflected towards. So, it has compressed. So, you see that the area of the flow has decreased but then it cannot go on doing this because it reaches the center line.

So, again further a reflected shock is formed. So, that it turns the flow parallel. So, when it when the flow passes through 2 such shocks you will find that $P_3 > P_{amb}$. So, as a consequence at this point of interaction of the shock with the free pressure boundary it has to reduce pressure.

So, that in $P_4$ it still remains $P_{amb}$ and as a consequence you get the expansion waves. Similarly in under expanded jets you can look at it in the sense of just the reverse $P_3 > P_{amb}$. So, to match pressures expansion wave form this is the expansion waves continues to expand the jet. So, in $P_5$ region your pressure will be much lower than ambient in order to match pressures in $P_6$ region these under these expansion waves reflect as compression waves which can form shock waves such that it still satisfies the boundary condition that it is $P_{amb}$ at the jet periphery.

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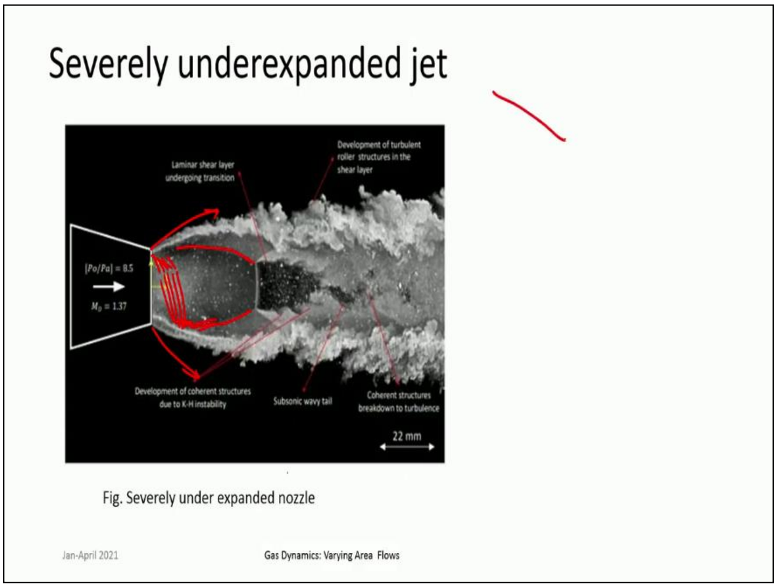


So, this is a typical severely under expanded jet produced in the laboratory. And here you can see the expansion waves the rapid expansion that happens. As a consequence there is a increase in jet area a lot of expansion waves are produced over here. These expansion waves go and meet the boundary of the jet over here and they form compression waves. And these compression waves actually coalesce together to form a shock here and that is the barrel shock here.

These shock waves are interacting in such a way that they cannot form a very regular kind of reflection that we just saw previously it forms a Mach reflection. So, this is known as max stem this is a case of a severely under expanded jet.

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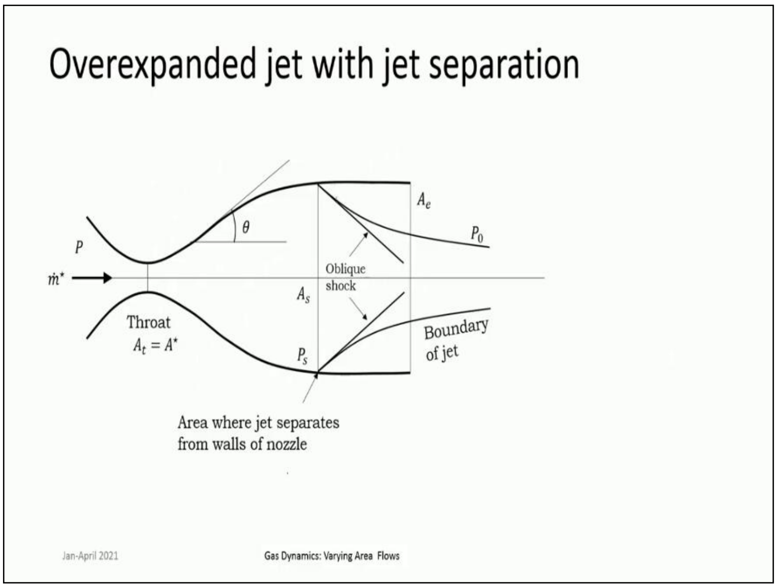


So, you can have cases where oblique shocks can move at corresponding pressure ratios into the nozzle and separate the jet out of the nozzle because of interaction of shock wave with the boundary layers along the jet. So, that these form the various different operating regimes for a convergent divergent nozzle and what we will do is understand them in more detail by doing a particular problem in the next class.

So, that you get clear about the relationship between area ratio pressure ratio and the way the nozzle operates. So, with this we close on how a C - D nozzle works various operating conditions and the relationship with pressure ratio and area ratio, thank you.