**Combustion in Air Breathing Aero Engines**
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**Lecture - 59**
**Combustion in Scramjets-II**

Welcome back. So, here we will do a very basic steady 1D analysis of gas flow without friction, but with heat addition essentially really flow. So now, before we go into this why do we want to do this of course, the process that happens in a scram jet is very complex.

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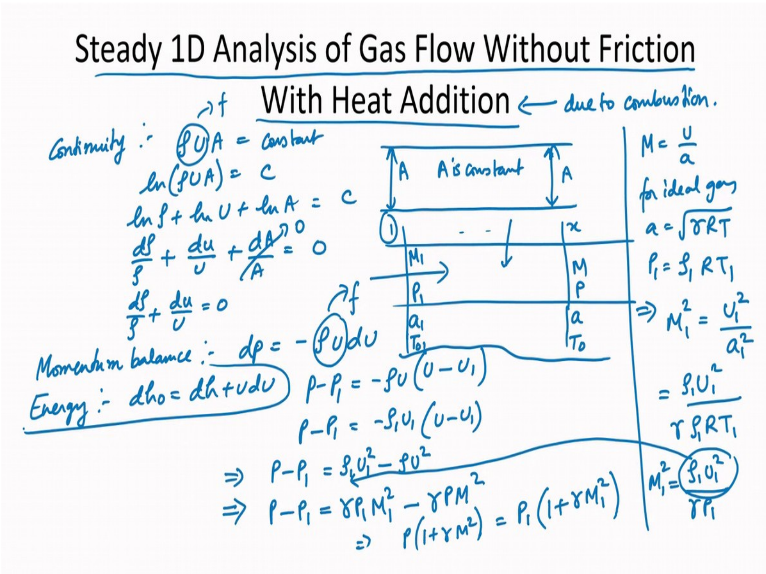


But this will tell us some very important design considerations in a scramjet engine. And phenomenologically what happens when you add heat into a supersonic flow. So, by doing a very simple analysis will get some insights. So, that is why this analysis is important. So, this is what we want to know that we will do a 1D analysis of course, you can question that that the actual flow inside a scramjet it is not 1D, but still it is the gradients or the flow is predominantly in the in the axial direction. And we can incorporate this heat addition even friction effects in some cases by in a simplified 1D flow.

So, this will give us very important design insights with a with a with an generate by doing a 1D analysis of the gas flow without friction, but with theta addition. So, it is a basic very basic condition that one can think of in a scramjet combustor. So, of course the heat addition now this can happen due to due to combustion, and as it happens in a scramjet engine ok.

So, let us do this analysis. So of course, we need the governing equations. So, the more first governing equation that you need is continuity, which is the conservation of mass. And in this 1D flow you can write rho times U times a is equal to constant. Rho is the density U is the velocity a is the cross section radium. And we can write ln of rho U a is equal to another constant. And we can write ln rho plus ln U plus ln A is equal to c and then we can differentiate we can write d rho by rho plus d U by U plus dA by A is equal to 0.

Now, here our the flow through which the tube through is the flow is happening is constant. So A, this is also A, this is also A. So, the area is not changing. So, if area is not changing then this term goes to 0. And we can write d rho by rho plus d U by U is equal to 0. Now let us consider the momentum equation. Here of course, if we write in the form f is equal to ma then f is my pressure gradient we can write dp dx or if I just write dp, is equal to minus rho U times d U mass per unit volume times the acceleration. And I can write the energy in just the form of dh 0 is equal to dh plus U d u, well as h 0 is the technician enthalpy not total enthalpy.

Now, if I go to this if I keep this apart for now if I (Refer Time: 04:30) go this go into this and if I say that this is my constant area duct, and this is my station 1. And I am interested in whatever is happening at any point. So, this is this is my station x essentially. So, then the flow is happening from this from the right to the left. So, one is my inlet station. So, if I integrate between one and any point. So, what I get is that P minus P 1 is equal to minus of rho U times U minus U 1. Why I can write like this? Because this quantity rho U is fixed is equal to f. From here itself you see that rho U is equal to fixed that is a mass flux, because the area is constant the mass flux is essentially constant ok.

So, I can also write P minus P 1 is equal to minus rho 1 U 1 times U minus U 1. Then this is just an artifact of continuity as you know. Or I can write P minus P 1 is equal to rho 1 U 1 squared minus rho U squared that is right.

Now, let me define a mach number here. So, mach number is equal to U by a U is the flow velocity a is the sound speed. And if ideal gas for an ideal gas a is equal to root over gamma RT. R is the gas constant. And of course, you know P 1 is equal to rho 1 or T 1. So, then it implies, I can write M 1 square if I multiply I can write M 1 squared is equal to U 1 squared divided by A squared, sound speed is essentially I can write as essentially A 1 squared. So, this is essentially U 1 squared divided by gamma R T 1 T 1. If I multiply both sides by rho 1 and rho 1, then I get essentially rho 1 square by U 1 squared is equal to gamma P 1 ok.

So, I can have M 1 squared is equal to this. Rho 1 e 1 square divided by gamma P 1. This implies P minus P 1 is equal 2 gamma P 1 M 1 squared. Because what I am doing is that I am substituting this thing into here. And similarly this is essentially gamma P M square. So, this implies P times 1 plus gamma M squared is equal to P 1 times 1 plus gamma M 1 squared. M is the mach number at this point M this is the inlet mach number. P is the pressure here M P 1 is the pressure here. Similarly A 1 is the sound speed here a is the sound speed here.

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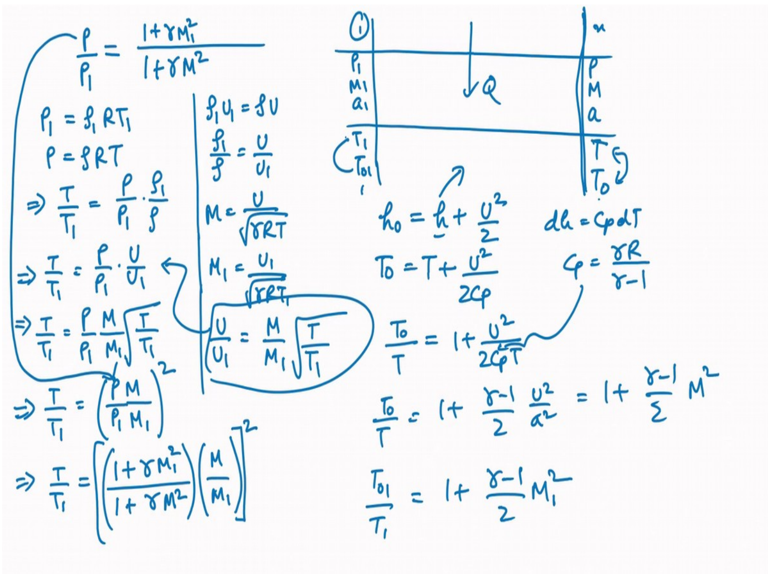

Now, as we will see that we will essentially consider that there is some heat addition here. So, this is this will be the T 0 stagnation temperature here and this will be the stagnation temperature here. So now, we can use this to write P by P 1 is equal to 1 plus gamma divided by 1 plus gamma M square. Once again remember that this is my cross constant cross section duct this is my inlet point which is one this is P 1 M 1 A 1 T 1 T 0 1. And this is P M A T T 0. And this by any point x and of course, there is some heat addition there can be some heat addition which can which essentially increases this temperature t from T 0 1 to T 0 which will come.

Now, let us come to temperature. So, to develop temperature equation we can once again go back to the ideal gas law. P 1 is equal to rho 1 R T 1 and P is equal to rho RT. So, therefore, T by T 1 is essentially P by P 1 times rho 1 by rho. Now if you remember rho 1 U 1 is equal to rho U by continuity. So, this implies rho 1 by rho is equal to U by U 1. So, this implies T by T 1 is equal to P by P 1 times U by U 1. Also now you can replace this U with mach number, but when you bring in mach number there will be additional temperature that comes. So, as you know M is equal to U divided by root over gamma R T. And M 1 is equal to U 1 divided by root over gamma R T 1. This implies U by U 1 is equal to M by M 1 mach number at M mach number M at x divided by mach number M, at M 1 at point station 1 ok.

So, U by U 1 is equal to M by M 1 square root of T by T 1. Now we can substitute this full thing to here this gives T by T 1 is equal to P by P 1 times M by M 1 times T by T 1, and this implies T by T 1 is equal to P by P 1 M by M 1 whole square. So now, I can use this relationship and put it here, this will imply T by T 1 is equal to 1 plus gamma M 1 squared divided by 1 plus gamma M squared ok.

So, this is the ratio of temperature at this point T divided by tone. And of course, there is some heat being added Q, which essentially has increased this T by from T 1 to T, but will not explicitly consider this t or over this q will just consider in terms of the temperature raise and how does that relate to mach number. So, that that given the mach number is M here and the given the mach number is M 1 here how is T 0 and T 0 1 are related, that is what we would like to find. Adjust as you find that how is t and T 1 are related that if the mach number is M here and the mach number is M 1 here. So, then the t and T 1 must be related by this manner. Of course, the 2 T 1 the T 1 2 t change has been inflicted by some external agent which is can be heat addition or rejection, yes which can wear as the case may be ok.

Now to develop the relationship for stagnation temperatures of course, stagnation temperature is essentially a measure of the total energy of the flow both the static temperature plus the kinetic energy. So, which is better quantified by the stagnation enthalpy h 0 is equal to h plus U square by 2. So, this is the stagnation enthalpy or total enthalpy you should not confuse these total enthalpy with the total enthalpy when we defined as like the standard state enthalpy plus enthalpy of formation, or the sensible enthalpy plus the enthalpy of formation. This is only this enthalpy is essentially contains can contain both the sensible enthalpy and enthalpy of formation ok.

So, this is the stagnation enthalpy which is the essentially this total enthalpy this enthalpy plus the kinetic energy. So, this enthalpy can contain the sensible enthalpy plus the enthalpy of formation. So, that you have to remember. So now, we can of course, write dA dT is equal dh is equal to Cp dT. And we can write T 0 is equal to t plus U square by 2 cp. So, then this T 0 by T is equal to 1 plus U square by 2 cpT. You can write this. And of course, if you remember Cp is equal to gamma r by gamma minus 1. So, we can put this Cp value here and we get T 0 by T is equal to 1 plus gamma minus 1 by 2, U square by s square is equal to 1 plus gamma minus 1 by 2 M square. So, this is a very standard relationship which you know from gas dynamics. So, T 0 by T is equal to 1 plus gamma minus 2 by M square gamma, gamma minus 1 by 2 times M square.

Now, similarly you can also write. So, this is at this point T 0 by T the ratio between these 2 things, we can write T 0 1 by T 1 that is a ratio between these 2 things as similarly 1 plus gamma minus 1 by 2 M 1 squared. So, we can write we can we can essentially write that T 0 1 will come to this plot later do not worry about it. So, will write that T 0 1 divided by 1 plus gamma minus 1 by 2 M 1 squared is equal to T 1 ok.

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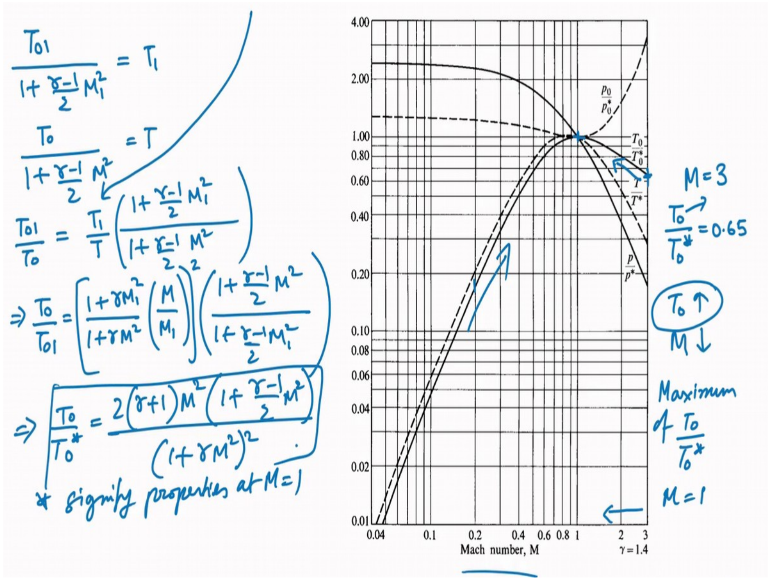


And T 0 divided by 1 plus gamma minus 1 by 2 M squared is equal to T. So, we can write the ratio of T 0 by T 1 T 0 1 by on T 0 1 by T 0, as essentially T 1 by T times 1 plus gamma minus 1 by 2 M 1 squared 1 plus gamma minus 1 by 2 M squared. And then if you invoke this relationship into this thing T 1 by T what you get is essentially I will write down the final expression T 0 by T 0 1 is equal to 1 plus gamma M 1 squared by 1 plus gamma M squared times M by M 1 whole squared times 1 plus gamma minus 1 by 2 M squared divided by 1 plus gamma minus 1 by 2 M 1 squared.

Now this one is not a standard state can change. So, we can just write that, if we can replace this with it can be any state right. So, to maintain a standard reference state we can say that this M 1 is equal to 1 and this T 0 1 is equal to t star T 0 star. So, we can write that T 0 by T 0 star which refers to mark one condition is essentially if you put M M 1 equal to 1 what you get is essentially this following is 2 times gamma plus 1 M squared times 1 plus gamma minus 1 by 2 M squared divided by 1 plus gamma M squared whole squared ok.

So, the star signifying properties at M equal to 1. And then if you plot all these things we can also generate relationships for P 0 by P 0 star T by T star and T 0 by T 0 star, and if we plot all these things here as a function of this mach number in this log plot log lock naught mach number. And these are like different ratios you see, this tells you a very, very interesting thing. Let us consider this plot, this T 0 by T 0 star plot which is essentially a solution of this. This if you plot this you will get this. So, if you plot this , what you what do you get? So, suppose you are at a mach number of 3. Your T 0 your M is equal to 3. So, that will tell your T 0 by T 0 star is given by about what is this value this is about same 0.65 ok.

Now, due to heat addition your T 0 will increase. So, T 0 goes up. So, but all points to whatever T 0 must have fall on this line. So, if the T 0 goes up what happens? So, then if the T 0 goes up you have to basically you see are moving T 0 increases towards the left. So, you have to move in this direction. So, as you are moving in this direction what happens? You see your mach number is dropping. So, you see as T 0 goes up your M is reducing. And it can reduce you can show that for the T 0 2 2 continuously go up it can it can only go up to this point it cannot go up any further and this point which is the maximum of T 0 by T 0 star is given by M equal to 1 which is of course, obvious because star corresponds to the conditions where mach equal to 1.

So, you see that by heat addition the immediate consequence is that of heat addition is that your stagnation enthalpy goes up h 0. Which means your stagnation temperature goes up. So, if your stagnation temperatures at a mach 3 flow suppose this is your combustor entry condition at mach 3. So, if you are adding heat which raises T 0 it immediately progresses in this direction and you see it, can only you can add heat only up to this point where your mach number is equal to 1. Because you cannot add more heat because after that you see this line drops. So, you can only add heat up to the mach number 1, and that is the point beyond which you cannot add any more heat and that is where the flow is choked, and that is correspond to the sonic condition M equal to 1.

So, this is called thermal choking that is choking by heat addition. So, similar thing happens for here also that if you have as mach number of say point 3, and you are adding heat. So, you see you go towards the sonic conditions, but the thing is that here with the addition of a small amount of heat you can reduce your mach number drastically. So, that is the point because this slope is much smaller here you need to add a lot of heat to go to mach equal to 1, but in this side you have to have a little bit of it to even go to mach number 1. So, this is the consequence, why does this happen? This is the consequence of the fact that our (Refer Time: 24:00) is constant.

So, this constant area can only support as limited amount of heat addition or a limited amount of stagnation temperature rise. Beyond that the flow gets choked and this is thermally choked and you reach a mach number one which is of course, not wanted in a scramjet. So, what do you do? In a scramjet what you do is that, in the combustor you provide a little bit of diverging that is you increase the cross section area to prevent this thermal choking.

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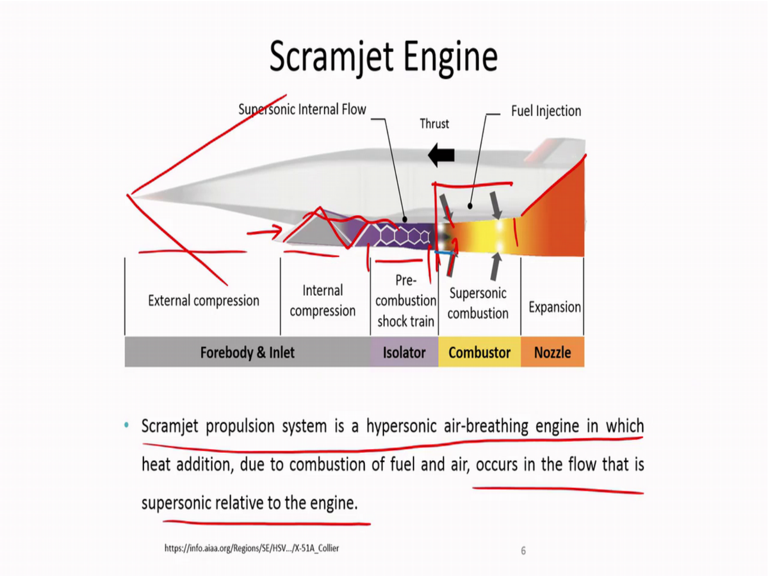


So now you see if you go back into these things into all these designs you see this is little bit diverging and comparison to this. This infinite view this schematic is diverging in comparison to this ok.

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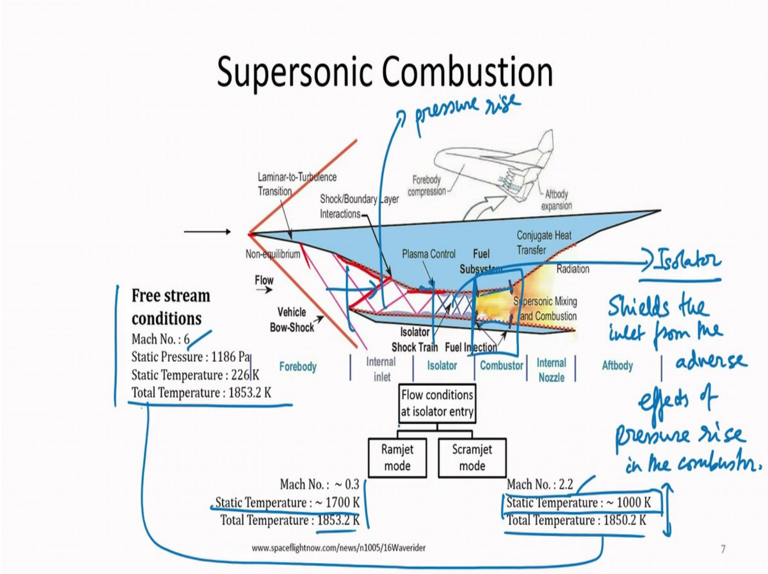


So, that is the reason to prevent thermal choking you prevent you provide the small amount of divergence into this scramjet combustor, and that emerges out the reason for this why should divergence should happen immediately comes out of from this Rayleigh flow analysis ok.

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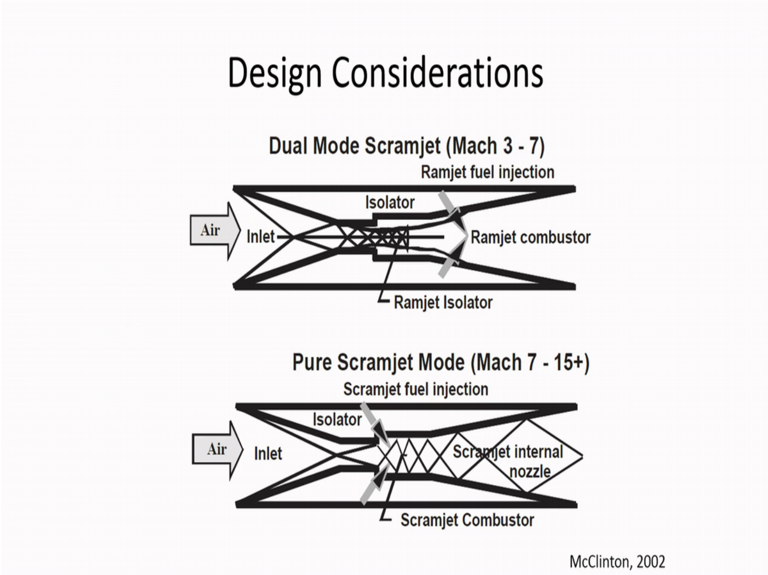


So, this is the whole idea that you should have in a scramjet you should have a you should provide a small amount of divergence. And then basically there is something called pressure area tailoring that one needs to incorporate that is how do you want the pressure to change in your combustor and that that is a little bit involved topic. And then will go into to take up to take those up well go into essentially the different aspects of some very basic aspects of a scramjet combustor design.

Basically the way how to do it, instead of actually providing the design methodology here because this course is about more fundamentals of the processes that happens in the scramjet engine, rather than the designing the scramjet engine itself. But I hope this Rayleigh flow analysis and that is this frictionless flow with heat addition, that is frictionless flow in which the stagnation (Refer Time: 26:14) energy stagnation temperature is increasing this kind of a flow this kind of an analysis tells you, that why you cannot you can only add increase the stagnation temperature up to a certain limit before thermal chocking happens. And to avoid thermal choking you need to provide some divergence into the scramjet combustor.

So, in the next class well go into the design considerations and basics and then will go into the different phenomena that happens in a scramjet combustor.

So, till then Thank you.