Combustion in Air Breathing Aero Engines
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Lecture – 51
Aero Gas Turbine Combustors IV

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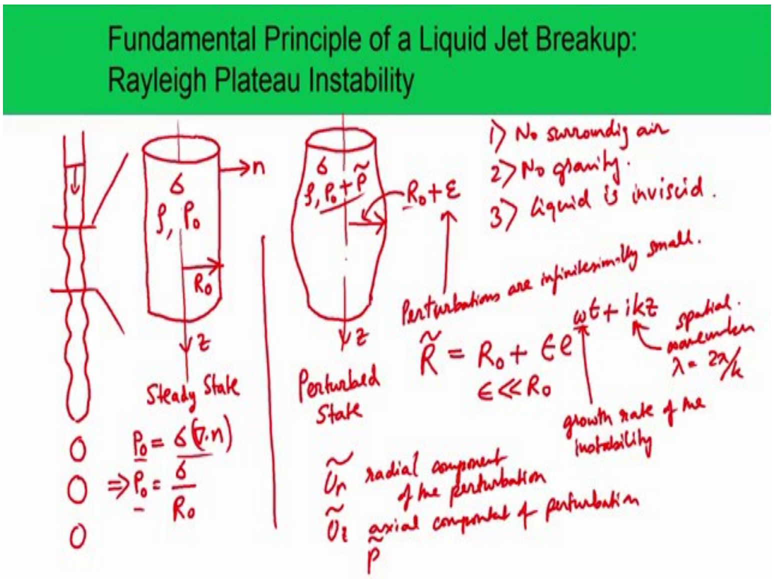

So, the fundamental principle of a liquid jet breakup is given by essentially what as you have seen that Plateau initially discovered this and Rayleigh analyze it using the hydrodynamic stability theory. So, we will take that up here and do the analysis. So, it is also the mechanism by which as I said that it is a very fundamental mechanism and you can watch it in your kitchen sink that if you open the tap like this and then you will see a liquid jet coming up. And then it will develop some perturbations like this, and then these perturbations will go. And then and at some point of time, it will essentially pinch off and create a droplets like this.

So, this is what you can essentially see in your own kitchen sink and, but why that happens that will be analyzed through this thing. Once again as you see that as you have as I have impressed upon you that the actually the atomization of this actual liquid jet process inside a combustor is quite involved. It is involved different kinds of shapes and sizes, but once again the whole as you have seen the whole philosophy of this course is that we deal with a very complex phenomena, but then we try to look into what is the basic fundamental process inside. Because without that you cannot be event you have absolutely no clue how to or you have absolutely no way by which you can understand the more complex phenomena in a more complicated environment. So, this is the thing we will approach we are going to take.

So, as you see there are this is how the droplet break except, but the equilibrium state if there was no instability, the equilibrium state of a segment of this if I just take up would have been and I assume it here would have been something like this. It is a cylinder right I mean cylinder. And I define the surface tension of this liquid cylinder by sigma density by rho and the intense pressure inside, which is uniform throughout my P 0 and this is the surface normal, which is pointing outside. And it has a radius which is given by R 0. This is the equilibrium state or the steady state.

But then as you have seen here that this does not remain steady like. This is a base state it actually in reality what happens is that when this liquid jet comes out. They are infinitesimally small disturbance everywhere present from numerous sources noises from numerous sources that act on this jet. And this noise essentially gets amplified in certain wavelengths certain particular wavelengths and those creates a disturbance those gets amplified to for create certain perturbations on this nice cylindrical state.

So, then we will consider perturbed state of this jet or a smaller perturbation problem. And this is of course, this is the cylinder and this perturbed state is R 0 plus epsilon, this is R 0 the best state in this perturbed state radius at this sinusoidal oscillations the maximum perturbed not the power maximum, but the at a certain point of time this perturbation is given by R 0 plus epsilon. Once again the properties remains same sigma, rho, p 0 plus p tilde. Sigma and rho remains same, but now because this has developed some perturbations and these perturbations has led to essentially change in the curvature. So, as a result of that the pressure inside changes and this perturbed pressure is given by P 0 plus P tilde.

Now here at the initial state. So, talking about curvature, let us find out how we can relate the pressure to the curvature and that can be related by young Laplace's equation which is given by P 0. In the steady state, P 0 is equal to sigma times divergence of normal which is nothing but the curvature. And this implies P 0 is equal to sigma by R 0 ok. So, here the radial in this cylinder in this steady state this radius is R 0 and the pressure is p zero. So, p zero and we can neglect essential outside pressure you can even consider the vacuum by the way the assumptions here in this it is at this point it is time to state the assumptions before we go into this then there is number one no surrounding air, number two - no gravity. So, this number three is that liquid is in viscid.

So, there is no surrounding air, then there is no gravity and then the liquid is in viscid. And in that set a state essentially you can we can write that because of pressure P 0 pressure is P outside is 0, we can write that P 0 is equal to sigma times n is equal to sigma times n and as a result the P 0 is equal to sigma by R 0. This is the young Laplace equation where do we essentially relate the pressure inside the liquid cylinder to its surface tension force. Essentially to balance up just like a pressure vessel to balance the pressure you must generate like a hoop stress and the surface tension essentially acts like that.

So, whereas, in this perturbed state, now we can if you consider the small perturbation, so as you see that we have written it as a R 0 plus epsilon where epsilon is a very small quantity. So, which means that in our perturbations are basically infinitesimal that is a infinitesimally small, perturbations are infinitesimally small. And these perturbations essentially happen in the z-direction whereas, this is R, and this is z, and this is z. So, we can say that now the now our perturbations are essentially R 0 plus epsilon. And we can essentially write it like that that R tilde, we will we will consider the evolution of infinitesimal perturbations on the interface. And the reason is that this will because if the perturbations to be small, we will be able to linearize our equations if the perturbations become large and the equations will be non-linear and we cannot do the analysis.

So, to make the system amenable for analysis mathematical analysis will keep the infinitesimals, we will keep the perturbation to be small and with that we will be able to do the linearized hydrodynamic instability analysis. So, we write that R tilde that is this now the perturbed column the surface takes the form R tilde is equal to R 0 plus epsilon times e to the power of omega t plus i k z. So, of course, we see that we have assumed perturbed state. So, this is essentially sorry this we will write this is a different notation actually, this is this to a different notation.

So, this whole thing is essentially this e and this is our epsilon. So, this whole thing can be consider be like this. So, then we consider that in the z directions the perturbations has a sinusoidal oscillations and the wavelength of those or the wave number of those oscillations is given by K. And in time domain we do not know whether it will be sinusoidal or not, it can be exponentially growing if omega is positive, it can be a damping if omega is negative. And if omega is complex it will lead to sinusoidal oscillations in timeouts, so which we do not know which we should come out of the analysis.

And of course, here the constraint is that for the perturbation to be small your epsilon should be much, much smaller than R 0. And as such essentially the omega is this is a growth rate of the instability we can write this. And k is the wave number a special wave number. And the wavelength for this is essentially lambda should be is equal to two pi by k. Now, we can denote once we have these perturbations on R on the radius of course, the velocity as you have seen that the pressure has been where the pressure there is a pressure perturbations.

So, u r which is the velocity which is the radial component of velocity that will also have a perturbation say of u r tilde, which is the radial component of the perturbation, and u z is tilde is the axial component of the perturbation. And similarly, P tilde is the perturbed pressure as you have seen here. So, u r is now essentially the radial velocity is given by u r plus the or u r 0 plus u r tilde; u z now is given by u z 0 plus uz tilde; P is given by P 0 plus p tilde that is these are perturbations on the base state, so that is how it is. And then we expect then if we put this things in the Navier-Stokes equation of course, you see that there is no in this momentum we can assume the density to be constant. And if we do that and if we neglect because this terms are essentially small. So, we can essentially neglect the non-linear contributions from the perturbations.

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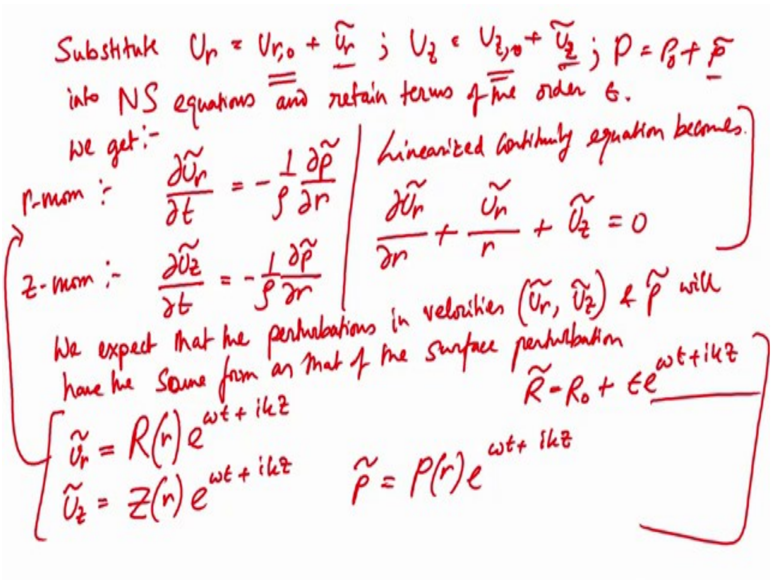

And what we do is that if we substitute u r tilde, u z tilde, p tilde or substitute the current let us write it clearly. These are the steady state quantities and these are the perturbations and P into Navier-Stokes equations and retain terms of the order epsilon that is this small terms. Then we get the r-momentum equation becomes, and the z momentum equation becomes. This is the consequence we can only write this because we have assumed the perturbations to be small and that is why these non-linear terms like rans terms do not come here that you must pay attention too. And the linearized continuity equation becomes this one.

Now, we have already assumed the form of the perturbation for the radius. Now, we can expect you can expect that the disturbances of the perturbations in the velocity and pressure also will have the same form of the surface disturbance. We expect that the perturbations in velocities and pressure will have the same form as that of the surface perturbation which was. So, then in that case our u r tilde and u z tilde and P tilde, this part of quantities are given by this small, this is the amplitude which is a function of R omega t plus i k z. Similar, this form the amplitude is only a function of r and the rest is a function of z and t. Now, if we substitute this, if we substitute these equations into that is if we substitute this into the r-momentum equation, z-momentum equation and the linearized continuity equation, what we will get is the following.

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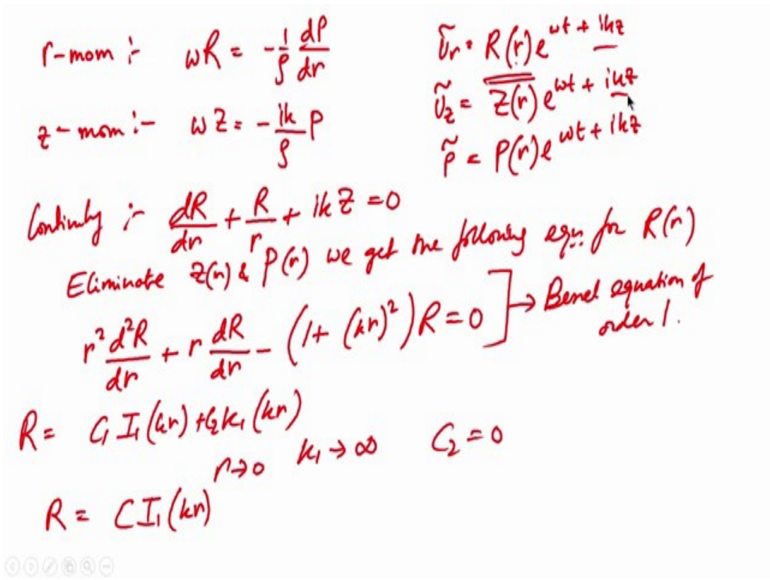

We will get the momentum equations will be in this form that is after we substitute this equations, we will get the momentum equations to be this form. You should work it out and I just showing you the final result. This is the amplitude p r because this p is only a function of r. Because if you remember that U r is equal to R r e to the power of omega t. If this is so then the z momentum equation becomes we get complex notations because we have in z you have got ikz which you do not have in r. And then the continuity equation becomes dR dr, this R is the amplitude, where a small r is the coordinate radius is a radial coordinate. So, if we eliminate z r and p r we get the following equation for R r and that is given by ok.

Now, this one is your governing equation. So, basically this equation contains comes from the continuity equation as well as which has been where the different terms of pressure and the z the pressure amplitude and the z and the amplitude of u z has been replaced by the momentum equation. So, this equation becomes essentially the Bessel equation of order 1. And it may be written in terms of modified Bessel functions of the first and second kind that is it can be written like as a sum of I 1 k r plus k 1 k r. But as you know that as r tends to 0, where of course, it is the equation is to be defined your k 1 tends to infinity, so then this means C 2 is equal to 0. As a result of that r becomes is equal to C I 1 kr once again k is the your wave number which comes in the z direction all right.

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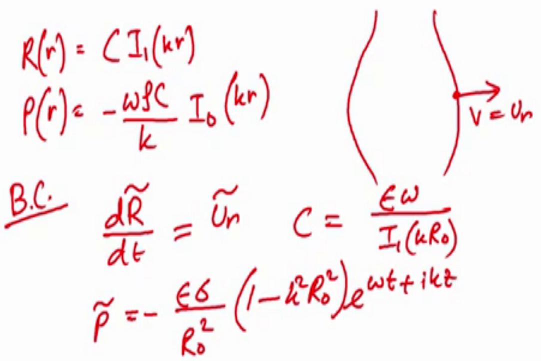

So, now that we know that your R r is given by the Bessel functions of first kind. We can obtain a pressure also like this. So, we have got the with this R r the amplitude of the surface fluctuations and the amplitude of the pressure fluctuations in the radial directions as the Bessel functions of first kind. And now what we need to know is that to but we do not we have basically in this things we have basically constants are there. So, to eliminate the constants, we need the appropriate boundary conditions.

So, what can be a boundary condition the boundary conditions can be that if you this was the perturbed state the velocity of this surface is essentially equal to the u r velocity. So, once again this velocity here v is equal to u r and that v is nothing but your dr r tilde dt. So, the boundary condition is this that your dR tilde dt is equal to your ur tilde. We approximate it like this and then with this we can find out the constant c to be essentially is equal to E omega times I 1 k R 0. And then if you do this the pressure balance, I will not go into that and then we can obtain this the pressure perturbation solution explicitly which is given by this thing epsilon sigma by R 0 square and 1 minus k square R 0 square times e to the power of omega t plus ikz. This we can do that.

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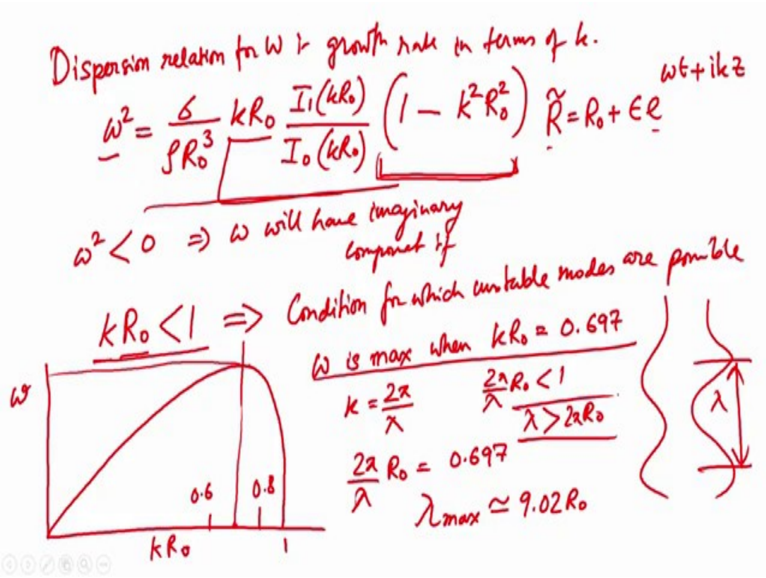

Next is the very important thing that is our entire goal of this analysis is to obtain the dispersion relation. What is the dispersion relation? Dispersion relation is the expression for your omega that is the growth rate. So, the growth rate is a relationship of the growth rate omega in terms of the wave number k. So, we get the dispersion relation for omega or growth rate in terms of k and that is given by you can solve it yourself, it is very simple. Just algebra even though this equation might look intimidating this is the most important term. The whole equation is important in terms of quantitativeness, but qualitatively, this is the most important terms. Why, because you see our R - the perturbation R this was given like this epsilon e to the power of omega t plus ikz.

So, if omega is positive which means that the growth rate, if omega is positive then this R will have an exponential behavior? So, it will continue to increase. Whereas, if this omega is negative it will have a damp behavior; whereas, it will be unstable if omega is complex. So, now you have a dispersion relation omega squared is equal to given by this thing. And all these things are positive. The only thing that can be negative is this one that is omega square will be less than 0, this means omega will have imaginary component if k R 0 is less than 1. This means, this is the condition for which unstable modes are possible; of course, we know from experience that this liquid jet can never just diverge into can this perturbations can never grow infinitely large, so that is a invalid condition it can dam down or it can happen, it can be unstable.

So, under what conditions it can be unstable. It can be unstable if the disturbances if the wave number of the disturbances times, the radius the steady state of this unperturbed state radius is less than 1. So, of course, we can then plot this thing. And if you plot this dispersion relation as k R 0, we will find that its only less than one conditions are important. So, it has a plot like this, this is one and it maximizes that a value of this is 0.6, this is 0.8. So, this is a maximum value which it attains and this value which gives and this is my omega actually. So, this is omega by k R 0. So, omega is max when k R 0 is equal to 0.693, 697, so that means, when k R 0 is equal to 0.697. So, for all k R 0, k times R 0 less than 1 omega is unstable.

And of course, if you just the plot; no that this part is also actually important. So, if you just plot the real part of it, you see that this is max when k R 0 is equal to 0.697. And this means that the growth rate will be maximum when k R 0 is equal to 0.697. Now, what does that mean k is of course, your 2 pi by lambda which is lambda is better because a lambda is wavelength. So, essentially this perturbations when you have then this is the wavelength lambda. So, this means that when 2 pi by lambda times R 0 is equal to 0.697 then the omega is maximum at that lambda. So, then that lambda is given by or if we call the lambda max that is given by 9.02 R 0.

So, now, you see where the 2 pi comes from this. So, if you just put this curve at the 2 pi essentially comes from this criteria that if we just take any lambda and we write that 2 pi by lambda. So, the criteria for this jet to be unstable essentially is 2 pi by lambda times R 0 should be less than 1, this means that your lambda should be greater than 2 pi by R 0. So, for all two lambda greater than 2 pi by R 0 as long as the lambda the perturbation wavelength is greater than the circumference of the jet, the jet becomes unstable. So, of course, that is the that is the thing. And of course, it maximizes when the lambda is equal to 9.02 times R 0. So, using this we can even have a breakup, we can have a breakup time for the jet.

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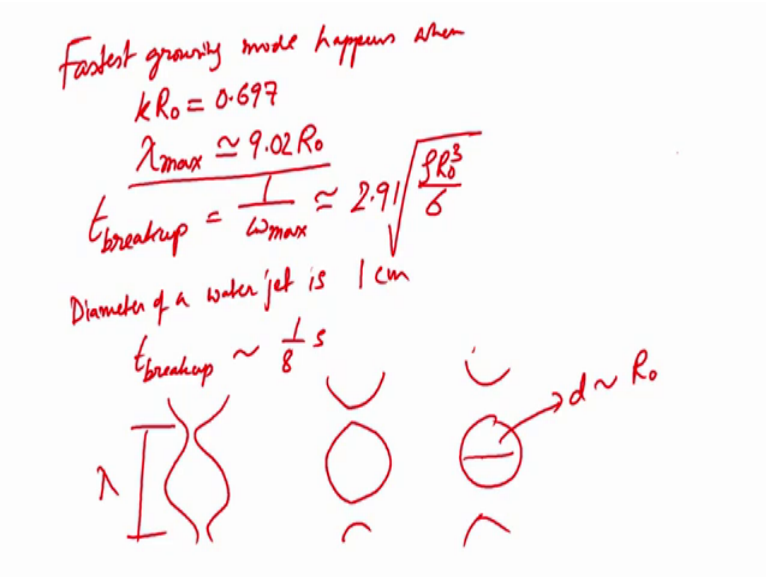

And do that we can obtain as if we try to introduce a time scale I am sorry. So, what we have obtained is that the fastest growing mode or the maximum the fastest growing mode happens when k R 0 is equal to 0.697 and that corresponds to a lambda max of 9.02 R 0. And if we correspond the time scale the t break up is equal to 1 by omega max then that becomes essentially 2.91 square root of rho R 0 cube by sigma. So, if you have take a water jet of 1 centimeter radius of diameter, so if a diameter of a water jet is 1 centimeter then this t breakup is about 1 8 seconds which is seems pretty consistent from our own experience.

So, this much is the Rayleigh plateau instability. And this is the fundamental mechanism by which a liquid jet breaks up even in absence of any cohesion, even in absence of gravity even when there is no viscosity in the liquid. So, that means that a liquid jet is inherently unstable because of the surface tension forces or that is present in the jet. And when that completes with this it is a competition between pressure surface tension forces and inertia forces that leads to the breakup of this jet at a characteristic line scale.

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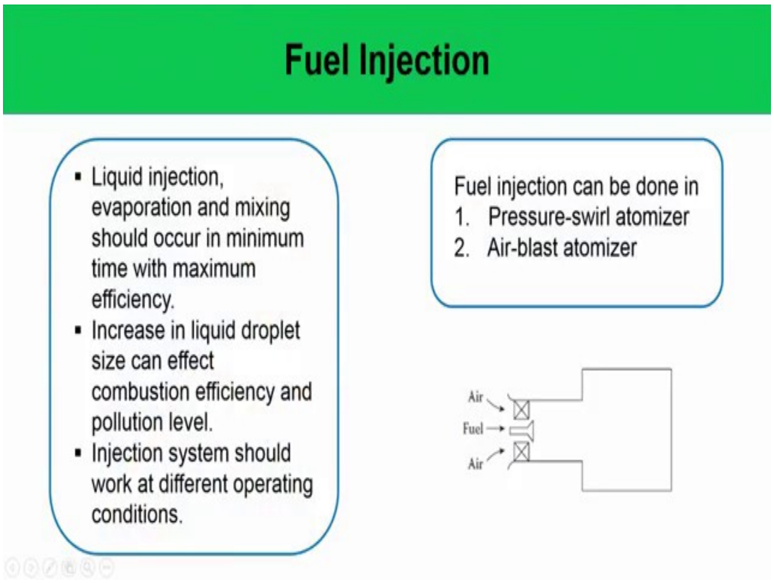

So, then now we will go into fuel injection. So, now coming back to the practical gas turbine engine where essentially you have to have something like this.

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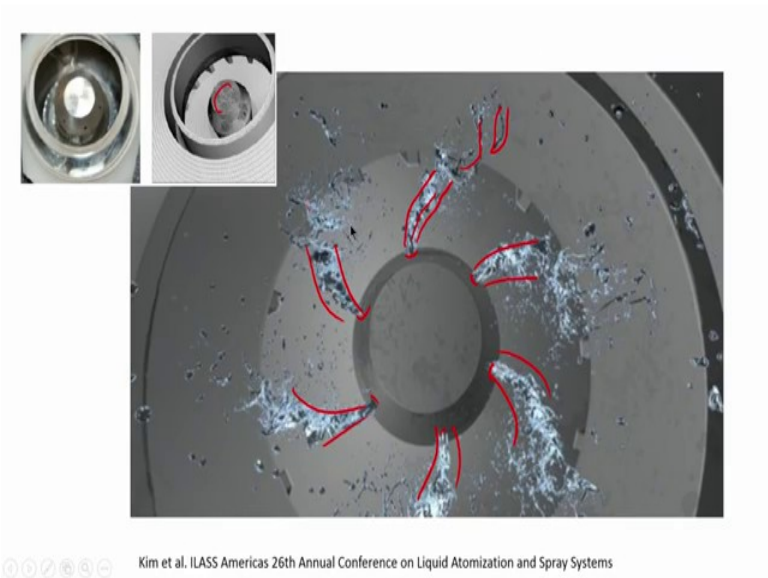

Where you send out the liquid jet and you expect that the liquid jet will break up because of this fundamental Rayleigh plateau mechanisms because of other Kelvin Helmholtz instability is Rayleigh Taylor instability is because of atomization you need to create fine droplets. So, the purpose as a engineer you have to then manufacture or you have to select atomizers or you have to select a proper fuel injection devices through which you can inject this these liquid fuels. And one more thing is that one more thing that you have to keep in mind is that that there can be different kind of injectors available, but these injectors, there are multiple like a conflicting requirements. Number one you need to have very small droplet sizes on one hand, but at the same time if you want to use two small sized orifices because of course, you see that the smaller the jet the smaller the droplets will be produced.

So, you from here itself you can understand that given this analysis that if you have if you have very small sized if we have a cylinder which is a very small size then the size of your then the break up length that this lambda will also be small. Because your lambda max is directly proportional to the R 0. So, what is happening is that you are creating this, I will say this is the largest perturbation that is happening on your liquid jet and this is the perturbations that is happening on the liquid jet. Now, where will it pinch off it will pinch off at this point. So, essentially this lambda this whatever volume of liquid is contained in this lambda will become a droplet.

So, after it breaks up in the next step can be like this. So, if I just show you like what is going to happen. So, this liquid jet is like this. In the next step, it can be formed this things and then it can form droplets like this in this in different times. So, this lambda the amount of liquid contained in this lambda will become a droplet. So, clearly if this lambda is small this, but it will you cannot have arbitrary small number because then it will not break up. So, this lambda max a typically it is a nine times or this nine times this radius of this liquid jet. So, this is a droplets that we will gain get eventually should be proportional this droplet size or this d should be proportional to your R 0 somehow.

So, what you get is essentially if your liquid jet is very thin if it is a small in size then you get a very small droplets right obvious, but. So, to make small liquid jets what do you have to do is that you have to use very small orifices, your orifice should be very very small how but of course, the physical you cannot make orifice very small suppose you want to make it 1 micron. But the problem is that some impurities will get unclog in this one micron orifice practically, so that is a problem practically. So, you can have a theoretical result, but to convert that into reality you need to put some more you need to do some more engineering, so that it can be practically implementable.