

Combustion in Air Breathing Aero Engines
Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture - 47
Turbulent Premixed Flames VI

So, welcome back. So, we have discussed extensively about turbulent flame speed and about different experimental setups to measure those essentially. So, now we will go back to essentially the turbulent flame modeling and one of the most apart the G equation model one of the most prominent model has been the BML model which is a Bray-Moss-Libby model proposed by Bray-Moss-Libby. And yes essentially a classic model in turbulent combustion and it starts with this thing, it starts with the definition of a progress variable, which is essentially non-dimensionalized reactive scalar variable.

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Bray-Moss-Libby (BML) model

BML model is a **classical model** in premixed turbulent combustion $c = (T - T_u)/(T_b - T_u)$ or $c = Y_p/Y_{p,b}$

PDF of c is assumed as $\frac{P(c, x, t)}{c^{c \rightarrow 0}} = \alpha(x, t) \delta(c) + \beta(x, t) \delta(1 - c)$
 with $\alpha(x, t) + \beta(x, t) = 1$
 $\tilde{u}(x, t) = (1 - \tilde{c})\tilde{u}_u(x, t) + \tilde{c}\tilde{u}_b(x, t)$

α and β are probabilities of finding unburnt and burnt mixture, respectively

BML model helps in describing the unclosed terms of Favre mean progress variable \tilde{c} transport equation

$$\langle \rho \rangle \frac{\partial \tilde{c}}{\partial t} + \langle \rho \rangle u \cdot \nabla \tilde{c} = -\nabla \cdot (\langle \rho \rangle \overline{u''c''}) + \tilde{\omega}_c$$

So, the advantage is that if you define c that is the progress variable in this manner that is c is equal to T minus T_u T_u is the unburnable temperature and T_b minus T_u whereas, T_u T_b is the burned gas temperature over the turbulent flame. And typically if it is a if there are not much heat loss and this T_b will correspond to that adiabatic flame temperature and T_u unburned is the unburned gas temperature these are fixed values these it is not fluctuate. So, you will see that we can define c is equal to T minus T_u and T_b minus T_u in this manner and then the advantage is that c always varies between 0 and 1.

So, we can also define c in terms of a provoke in terms of a in terms of a mass fraction of a product whereas, Y_p is divide by Y_p , b . So, the PDF of we can assume the PDF of c as in this manner that is we can say that the PDF of c is essentially given by this is a PDF of c at any point x and t , x is the position vector of the point is given by α at the point x , t which is a parameter. And this is a delta function which delta function is when c is equal to 0, it is equal to 1; and when c is equal to any other value, it is essentially equal to 0. So, you can define this. So, d of x is equal to 1 and is equal to 0; when x equal to 0 and for x not equal to 0. So, this is our delta function is defined.

So, this essentially this delta function assumes the value one when c equal to 0 and plus this is given by β times x , t times delta function of 1 minus c . So, when you have c equal to 1, then this due delta function assumes a value 1 and otherwise is equal to 0. So, this is equal to 1, when c equal to 0 and this is equal to 1 and c equal to 1, and this is equal to 0 in all other cases that is the point. And an α plus β is equal to 1 that is how the probability distribution will works, and this is essentially it is a PDF which is given something like this. So, this is c this is c equal to 0, this is c equal to 1, this is α , and this is equal to β . So, this is how that the PDF of essentially c looks like.

And this is assuming of flame this is essentially this type of model is only valid for a for in the corrugated flamelets regime where you say only c only varies between 0 and 1 and any other values is not possible. So, of course, c is equal to 0 corresponds to the reactance and c equals to 1 corresponds to the products and this way we can basically figure out where the location of the flame is. So, and we can using this definitions you can also define a Favre averaged velocity at any point and that is given by 1 minus c tilde c here is the Favre average times u tilde plus c tilde times u tilde.

This is this follows where as we with the actual definition of α and β essentially the probabilities of α and β essentially the probability of finding the unburnt and the burnt mixture respectively. So, this is what α and β are? So now the one can derive an equation of c just like what you can do is that you can since you have a temperature equation you can put in the simplifications that is your different terms the radiation term is non is equal to 0 and you invoke the distinct same diffusivity. And the Lewis number equal to one assumptions and you can from the temperature equation you can just immediately derive this c equation, because c is algebraic function is just a some constant times temperature right some constant times divided this is c is only just a

algebraic function of a linear function of temperature. So, immediately you can from the temperature equation you can equation you can derive an equation of c .

So, in this manner, you can derive the equation of c and then just like an equation of or the reactive scalar, one can immediately derive a transport equation of c which is c average c , this is a Favre average c . And one can define derive an transport equation of c tilde which is given by this is the average density times $d c$ tilde $d t$ plus ρ times u this will be tilde actually, this is u tilde times gradient of c minus this. Once again this scalar transport terms comes this u prime u c prime and then once again these are the fluctuating velocities and then this is the divergence of this term and you have the then you get this ωc tilde which is the μ reaction rate this is the Favre averaged mean reaction rate for this variable c . Of course, this is an equation for this derive this averaged equation for c from that of a reactive scalar. So, of course, you will get a reaction rate term. So, this is the distinct difference between the mixture fraction model in the mixture fraction model. Even if you derive define a variable which goes between z and one that was essentially a passive scalar variable; whereas, this is a reactive scalar variable.

In reactive scalar variable of equation has other problems. So, the first problem is essentially that you have to have this closure of this different terms and the you have to have the closure of the mean reaction rate and then you have to have the closure of this scalar transport term, so that is the problem. So, I will show you the problem here. So, these are the two unclosed terms that we need to take care.

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Closure problem for $\widetilde{u''c''}$

$$\widetilde{u''c''} = \frac{\langle \rho(u-\bar{u})(c-\bar{c}) \rangle}{\langle \rho \rangle}$$

$$\Rightarrow \widetilde{u''c''} \sim \underbrace{\bar{c}}_{>0} (1-\bar{c}) \underbrace{(\langle u_b \rangle - \langle u_u \rangle)}_{>0}$$

$\widetilde{u''c''} > 0$
 If we invoke the gradient transport assumption

$$\underbrace{-\widetilde{u''c''}}_{<0} = D_t \frac{d\bar{c}}{dx} \quad \text{Countergradient diffusion.}$$

Countergradient diffusion \rightarrow Turbulent mixing + gas expansion.

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So, let us say we look into this term we look into this term and these two terms separately. So, if we have these two terms have a model good model for this two terms then the whole problem of this transport equation of the mean c is solved. So, then the problem is that that if you define this is your term to be closed U prime C prime and that is equal to average of ρ times you can write it like this and c minus c tilde. This of course means this right. This is an exactly it means this just we have in invoked the definition of U prime and C prime. Let us (Refer Time: 08:17) let us write it down here.

So, this is the closure problem for this quantity. So, this can be exactly written as in terms of the normal ensemble average it can be written as this is just the definition of the average right. And then we can model this as we can write this implies as equal to this can be scaled as like this from here. So, this part and then this part comes here and this part can be written like this by scaling arguments on the overall. So, of course, this quantity is this is positive, this is c , c is always greater than 0 and c tilde is always greater than 0 and c tilde is always less than 1. So, this quantity is greater than 0. Whereas, U_b average is of course, greater than U_u average because of gas expansion the burnt gas velocity is must be greater than the unburnt gas velocity this is of course, for a one-dimensional statistical planar flame that we have to talking about here. And then this implies that this is the positive quantity and this is a positive quantity. So, then it means U prime C prime this is a positive quantity.

Now, if we invoke the gradient transport assumption, we should have written minus $U' C'$ averaged is equal to a turbulent diffusivity times $d c / dx$. Now, then what happens is that because this quantity is greater than 0, this one quantity is less than 0 this is greater than 0 and of course, this is also greater than 0 because it goes from 0 to 1 from one (Refer Time: 11:40) greater than one. So, essentially we have a conflict. On the right hand side, you have your entire term is essentially greater than 0; and the left hand side you have a term which is less than 0. So, then it means then this is not there is no gradient assumption, but there is counter gradient, there is not this assumption of this, this gradient transport is invalid for this type of turbulent flames in terms of the progress variable when you write in terms of the progress variable. So, progress variable does not allow you to invoke the gradient transport assumption, because there is counter gradient diffusion that is taking place instead of the gradient diffusion. So, this phenomena is called counter gradient diffusion. And of course, this becomes less important when your $U' u'$ is less than $S L$.

When your $U' u'$ is much greater than $S L$ that is when turbulent is very strong. And then of course, when the reason is that that this counter gradient diffusion is essentially considers both things, it considers turbulent mixing as well as gas expansion. So, the actually when you apply this gradient transport assumption, you consider only turbulent mixing. You need a gradient transport assumption to only model turbulent mixing, but here gas expansion because of this $U' b$ is greater than $U' u'$ this is complicated the picture, so that is what has made things more complicated.

So, this is essentially combination of this counter gradient diffusion contains both turbulent mixing plus gas expansion. So, this has to be kept in mind if you used model with this thing as if you want to model this progress variable average progress variable transport assumption and want to close this unclosed terms. So, then you have to be mindful of the fact that the in this c domain, you have got strong counter gradient diffusion and that needs to be accounted. So, you have to find out situations when you have to use counter gradient diffusion and we have to use gradient diffusion.

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Bray-Moss-Libby (BML) model

A simple model for $\overline{u'c'}$ was derived by Veynante et. al. (1997)

$$\overline{u'c'} = \tilde{c}(1 - \tilde{c}) \left(\frac{\gamma}{1 - \gamma} S_L - 2\alpha u'_0 \right)$$

A model for the mean chemical reaction rate is:

$$\langle \omega_c \rangle = \frac{\rho_u S_L^0 l_0 g(c)(1 - \langle c \rangle)}{\tilde{L}_y}$$

where $\tilde{c} = \int_0^1 c P(c; \mathbf{x}, t) dc = \beta(\mathbf{x}, t)$; l_0 is stretch factor; g is a coefficient and \tilde{L}_y is crossing length scale which needs to be modelled

Alternatively $\langle \omega_c \rangle = \rho_u S_L^0 l_0 \Sigma$
 where Σ is flame surface density: flame surface per unit volume

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So, another problem of course, is the closure of this the same problem is of course, the closure of this u' c' and it was closed on this manner. So, this one \tilde{c} times one minus \tilde{c} times \tilde{L}_y and this is essentially contains the gas expansions as well as the turbulent mixing in this form. So, this was close by Veynante and this was form that was taken. So, this was the form proposed by Veynante which was used to close, this unclosed term; and this appear to be a good model for this for over to tackle this counter gradient diffusion problem.

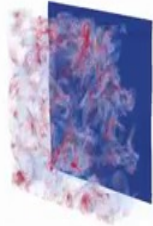
Now, of course, you still need to close for this term that is the mean reaction rate and that can be closed by this way. Whereas this model was proposed to that this ω_c ensemble average is equal to unburnt gas density time planner laminar flame speed times are stretched factor times some this constant g and coefficient and times c times $1 - c$ divided by L . So, there are a several parameter that you see has coming. So, whereas a size 0 is a stretch factor g is a coefficient L y is a crossing length scale which needs to be modeled. Or alternatively, I mean this thing actually feeds into this thing you can model this whole thing as this entire thing something called flame surface density which is the essentially the flame surface per unit volume. So, this much for the turbulent flame, so what we have done I mean if you just go through a recap of this whole turbulent flame thing, the most important thing is of course, as you see here once again.

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Turbulent Premixed Flames in Engineering Devices

Turbulent premixed combustion is present in:

- SI engines
- Gas turbine engines: aircrafts and stationary power systems
- Industrial gas burners



DNS to study interaction of turbulence with freely propagating premixed flame
Video courtesy: Dr. Hong Im



LPP Combustor

James F. Cincolli and Jacob Tereno 49th AIAA Aerospace Sciences Meeting, Orlando, Florida

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I will show you this video that you see this turbulence interacts with this flame at a multitude of length scales and its stretches and folds this thing. So, it is important to understand that and the most important property is that this turbulent flame propagates much faster than the laminar flame. But before going in to that what we need to address is that the different regimes in which the flames can behave in a very different manner. There are regimes in which the flame properties can dominated over the turbulent flow properties, but there are regimes in which the flame properties, the turbulence properties can dominate over the flame properties and that can change the flame structure.

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Regime Diagrams

Turbulent $Ka = \tau_L / \tau_\eta$
 Using $Re_\eta = 1$ and $\nu = D$, we get $Ka = (\ell_L / \eta)^2$
 Using $\eta / \ell_o = Re_o^{-3/4}$ and $Re_o = \frac{u_o' \ell_o}{\ell_L S_L}$, we get

$$Ka^2 = \left(\frac{\ell_o}{\ell_L}\right)^{-1} \left(\frac{u_o'}{S_L}\right)^3 \quad (2)$$

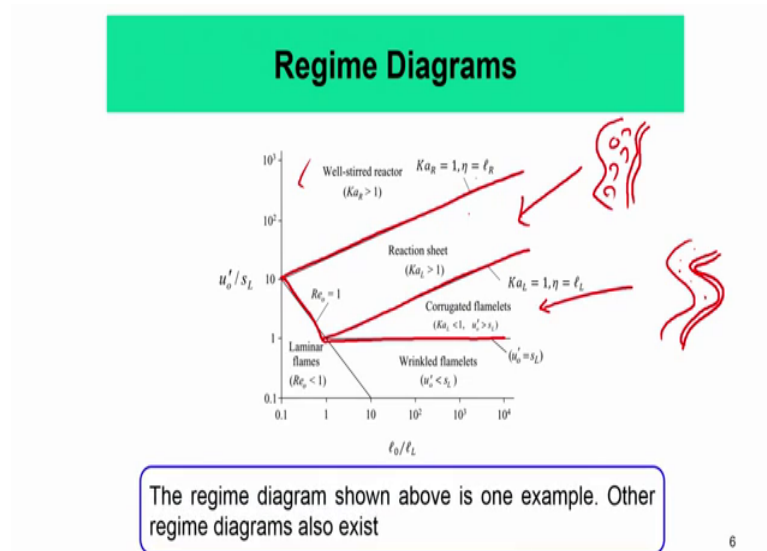
Turbulent $Da = \tau_o / \tau_L$
 Using $\tau_o = \ell_o / u_o'$ and $\tau_L = \ell_L / S_L$, we get

$$Da = \left(\frac{\ell_o}{\ell_L}\right) \left(\frac{u_o'}{S_L}\right)^{-1} \quad (3)$$

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So, we went onto this thing called the regime diagrams where we obtain different scaling laws, where we obtain different non-dimensional numbers not exactly scaling laws non-dimensional numbers in terms of this non-turbulence and flame parameters.

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And we obtain this regime diagram and which was plotted in u' by S_L and flame integral length scale by flame thickness. So, with this these two are the most important regimes we identified for turbulent flame most practical regimes. So, in this corrugated flame sheet regime in this regime what was the thing was that the flame was essentially wrinkled by turbulence. And but the both the preheat zone structure as well as the reaction zone structure was essentially similar to that of the laminar flame except the fact that it was bent except the fact that it was stretched. But all this properties this in internal properties could be consider as same as the laminar flame. And this was the condition when your Karlovitz number was essentially less than 1; Karlovitz number less than one meant your essentially your flame thickness was smaller than the Kolmogorov of length sale which is a smallest scale of turbulence.

Whereas, in the reaction sheet regime, you had basically structure where your essentially your preheat zone structure to be disturbed by the smallest turbulence eddies this was because your Karlovitz number was greater than 1. Karlovitz number greater than one meant your flame thickness was essentially greater than the Kolmogorov of length scale which is a smallest scale of turbulence and as a result of that your smallest scales of

turbulence could penetrate into the preheat zone and distort the structure there. So, in this regime it is made to understood that along with molecular diffusivity your turbulent diffusivity might be important.

Also there is also another regime where well stirred reactor regime where your Karlovitz number your reaction Karlovitz number was essentially greater than 1. So, it meant that your now your Kolmogorov of length scale is essentially smaller than the even the reaction zone thickness of course, you remember the reaction zone thickness divided by the preheat zone thickness was equal to the 1 by Zeldovich number which is very small number as such, but still if you Kolmogorov length scale. And the preheat zone on if Kolmogorov of length scale is smaller than the preheat zone thick on and the reaction zone thickness then it is expected then the Kolmogorov radius can even distort the reaction zone and completely disturb the distort the flame structure.

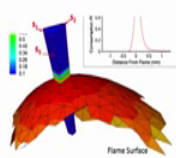
And the whole flame is essentially is like a well stirred reactor, but still there is there is no experimental verification of this regime though there are experiment of verification of this two regimes, but these average regime boundaries should not be taken with too much. This is not (Refer Time: 19:31) as such, and this boundaries are kind of smudged and but this gives a idea about what kind of structure is to expect. So, these are the different things that we had went through.

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Other Definitions of Flame Speeds

Flame Consumption Speed:

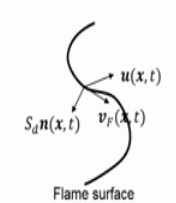
$$S_c = \frac{\int_{\Omega} \dot{\omega}_F d\Omega}{(\rho Y_F)_{react} A_{ref}}$$



Flame Surface

Flame Displacement Speed:

$$S_d = (v_F - u) \cdot n$$



Flame surface

Surface propagation

fluid motion

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And then we are define that flame speed you see that this way by this by this process we essentially moved from our understanding of the laminar flame planner laminar flame that we developed in the previous classes and which allows us to directly smoothly graduate into turbulent flame. So, this is the turbulent consumption speed which that is that is flame displacement speed which is essentially the rate of propagation of the surface this is the rate of propagation of the surface surface propagation with respect to the local fluid velocity. So, S_d is essentially relative velocity of the surface with respect to the local fluid velocity measured in the direction normal to it.

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Flame Stretch

Premixed flames can be sensitive to non-uniformities in the flow, flame curvature, and flame unsteadiness

The effect of above non-idealities over flame is quantified through the stretch rate of flame

Flame stretch is defined as, $\kappa = \frac{1}{A} \frac{dA}{dt}$, where A is the area of an infinitesimal element on the surface

$$\kappa = a_T + S_d \nabla \cdot \mathbf{n}$$

where, $a_T = \nabla \cdot \mathbf{u} - \mathbf{nn} : \nabla \mathbf{u}$ is the tangential strain rate and $\nabla \cdot \mathbf{n}$ is the mean curvature

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So, this is the fluid velocity and then we conserve into the concepts of flame stretch and which is essentially the rate of change of surface area per unit original surface area and we saw that there are rate of flame stretch has two parts a tangential stretch rate and the S_d times the curvature.

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Flamelet Models in Premixed Turbulent Combustion

Flamelet models of premixed turbulent combustion assume **flame to be infinitely thin** which correspond to infinitely fast chemistry limit

Flamelet models are based on:

- Scalar variable G
- Progress variable c

c based on temperature can be defined as follows:

$$c = \frac{T - T_u}{T_b - T_u}$$

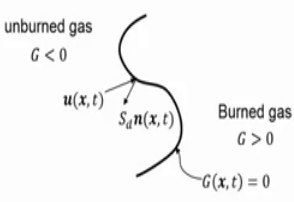
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So, essentially then we went into the different flamelet models and which was like scalar variable g and progress variable c .

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G-equation

Equation governing a propagating surface in a flow
 $G(x, t) = 0$ as the geometry of surface
 $G < 0$ as reactants
 $G > 0$ as products
 S_d is local flame displacement speed along \mathbf{n}
 \mathbf{n} is local normal to surface, where $\mathbf{n} = -\nabla G / |\nabla G|$



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And then we went in to G equation where we show that flame propagation can be described by the motion of an iso surface of that variable. And then this iso surface locally propagate with this flame displacement speed and then we can derive

the g equation and once again we need to put in some inputs into the G equation and which came from the our previous models.

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Models for S_d in G-equation

S_d can be approximated by use of a model

$$S_d = S_L - l_m \kappa$$

l_m is **Markstein length** (usually obtained from experiments)

Using $\kappa = a_T + S_L K$, where $K = \nabla \cdot \mathbf{n}$

$$S_d = S_L - l_m (a_T + S_L K)$$

$$S_d = \cancel{S_L} - S_L l_m K - l_m a_T$$

$\kappa = \bar{\sigma} \hat{n}$ $\hat{n} = \frac{-\bar{\sigma} a}{|\bar{\sigma} a|}$

The above expression is valid in **weak stretch limits** because the model assumes a linear response of flame to stretch

Note in the stretch rate κ expression S_L is used to keep things easy for modelling and is only an approximate

For thin reaction zone regime, S_d models get more involved

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Where your S_d can be the local input into the G equation that is this needs to be input that as a model and that can be input by the which is which we show that this depends to the leading order of on the flame speed planar laminar flame speed. And then of course, on the curvature and the tangential strained rate. Of course, this was in the weak stretch limit and then we went into model for the turbulent flame.

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Derivation of G-equation

Using multivariate Taylor's expansion,

$$G(x + \Delta x, t + \Delta t) = G(x, t) + \frac{\partial G}{\partial t} \Delta t + \Delta x \cdot \nabla G + H.O.T$$

Since, $G(x + \Delta x, t + \Delta t) = G(x, t) = G_0 = \text{constant}$ and taking a limit $\Delta t \rightarrow 0$

$$\frac{\partial G}{\partial t} + \left(\frac{dx}{dt} \right) \cdot \nabla G = 0$$

Since, $dx/dt = u + S_d n$, and $n = -\nabla G / |\nabla G|$

$$\frac{\partial G}{\partial t} + u \cdot \nabla G = S_d |\nabla G|$$

The G-equation is a **Hamilton-Jacobi** equation similar to ones found in **level-set methods**

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And then you showed that why the turbulent burning velocity is important we showed that the turbulent by considering.

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G-Equation Model

Exact Equation

$$\frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = S_d |\nabla G|$$

Model

$$\frac{\partial G_m}{\partial t} + \tilde{u}_i \frac{\partial G_m}{\partial x_i} = S_T |\nabla G_m|$$

Handwritten notes:

$$\frac{l}{l_0} \sim Re_0^{-3/4}$$

$6l_0$
 \vdots

Kerstein, Ashurst, Williams, PRA 1988; Karpov, Lipatnikov, Zimont, 1996, 26th Symposium on Combustion

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The mass flow rate into the wrinkle flame surface.

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Turbulent Burning Velocity

Wrinkled flamelet regime

- A_T is instantaneous flame area
- A is projected area

Since, mass-flux is constant $\dot{m} = \rho_u S_T A = \rho_u S_L A_T$
 Thus, turbulence increases area $A_T > A$ which causes $S_T > S_L$

Using simple geometric arguments it can be shown for weak turbulence

$$\frac{S_T}{S_L} = \sqrt{1 + \left(\frac{u'}{S_L}\right)^2} \quad \frac{u'}{S_L} \ll 1$$

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We can define and averaged turbulent flame speed so by this way.

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Turbulent Burning Velocity

$S_L = \sqrt{\frac{D}{\tau_c}} \quad \frac{m}{s} \quad S_L =$

By analogy in **thin reaction zone regime**: $S_T = \sqrt{\frac{D_T}{\tau_c}} = \sqrt{D_T \omega_b}$

$S_T = \sqrt{\frac{D_T}{\tau_c}} \Rightarrow \frac{S_T}{S_L} = \sqrt{\frac{D_T}{D}}$

Using $D_T \sim u_o' \ell_o$ and $D = \ell_L S_L$

$\frac{S_T}{S_L} \sim \sqrt{\frac{u_o' \ell_o}{S_L \ell_L}} \Rightarrow S_T \sim \sqrt{u_o' \ell_o}$

$S_L = \frac{m}{s}$

$D = \frac{m^2}{s}$

$\tau_c = s$

$S_L = \sqrt{\frac{D}{\tau_c}}$

$S_L = \sqrt{D \omega_b}$

$S_L \ell_L \sim D \sim \nu$

$\frac{S_T}{S_L} = Re_o^{1/2}$


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And then we went onto define find out the turbulent flame speed for the thin reaction zone regime and we showed why turbulent flame speed is important.

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Experiments

$$\frac{S_T}{S_L} \sim \frac{A_T}{A}$$



$$\dot{m} = \int_{A_T} \rho_0 S_L \cdot dA = \rho_0 S_T A$$

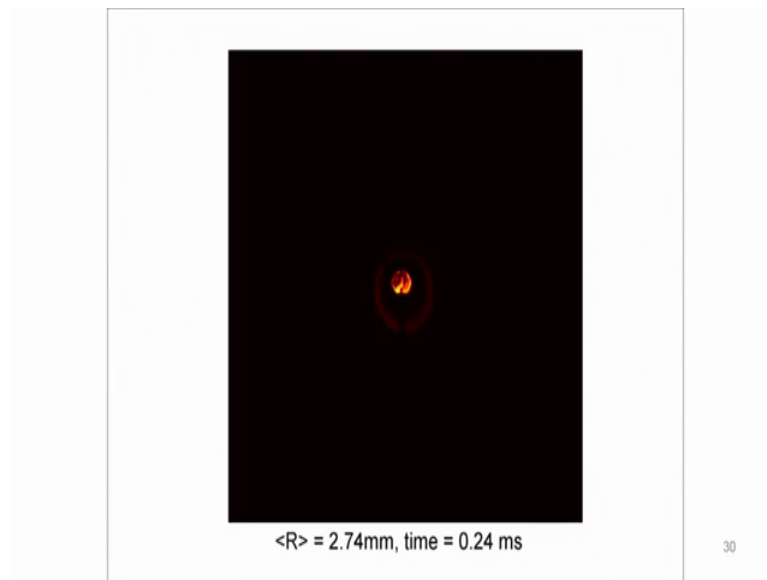
$$S_T A = S_L A_T$$

$$\frac{S_T}{S_L} = \frac{A_T}{A}$$

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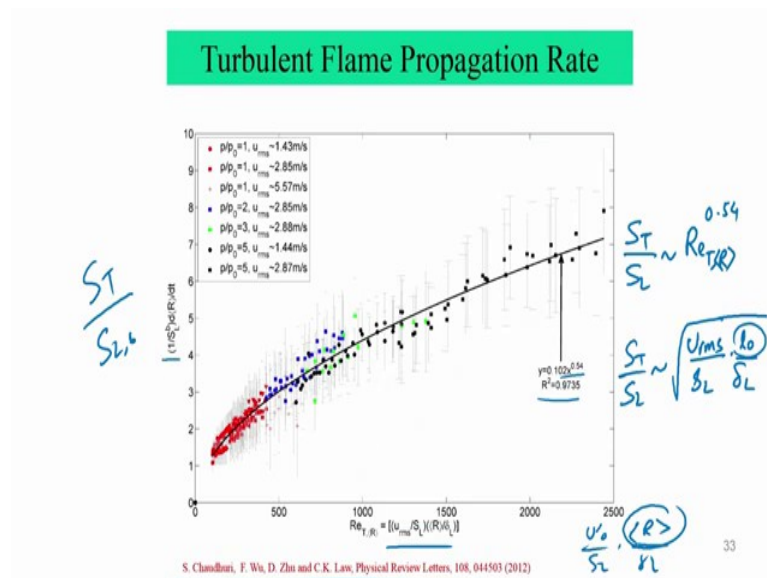
Then of course we showed you why we even recapped why turbulent flame speed should be proportional to the turbulent flame speed by local laminar flame speed should be proportional to the flame surface area by actual by projected area of the planner laminar flame surface area.

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Then we showed different turbulent flame speed experiments. The state of the art experiments and how the turbulent flame essentially propagates, and how the multi scale structure of the turbulent flame emerges under the effect of the turbulence.

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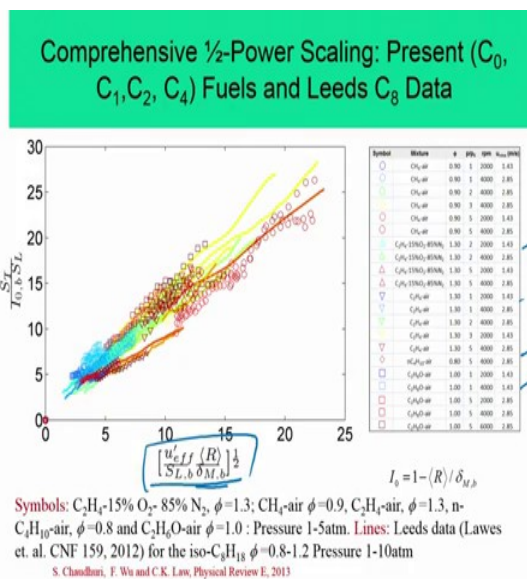


And then we showed you how why you for our expanding flame the characteristics length scale should not be the only the length scale of turbulence, but the characteristics length scale the flame should be the characteristic important. But the characteristic radius of the flame the average radius of the flame should be characteristic by length scale when

you want to define turbulent flame speed as a function of an Reynolds number. Because you have to remember that the turbulent flame speed is important turbulent flame speed is increased over laminar flame speed because you have the turbulent flame surface area is larger than a laminar flame surface area. Now the larger flame surface area can be caused by stretching and wrinkling by turbulence at different lengths and time scales.

So, only those scale of turbulence essentially which can disturb the flame structure disturb, create new area should be considered. And for a kernel for a spherical flame only those structures only those turbulence structures which are of the order of radius or smaller than the radius can affect the flame surface to create wrinkles. Whereas, structures turbulent structure which are much much larger than the radius does not serve any purpose in terms of generating new area, so that needs to be kept in mind.

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And that created this type of different scaling laws and then we obtained showed the new experiments in this turbulent flames. And then we showed bray Bray-Moss-Libby model and the closure problem of the Bray-Moss-Libby model where you cannot directly invoke the gradient transport assumption because c under goes counter gradient diffusion due to gas expansion and essentially this is an effect of both of turbulent mixing and gas expansion. So, whether it be counter gradient or gradient diffusion depends on which means turbulent mixing or gas expansion. And then we showed the models which can be

used for essentially defining the mean chemical reaction rate which is another the other closure problem here.

So, with this discussion of the turbulent flame speed, the discussion of the turbulent premix flame comes to an end. And we will essentially this part of this course that is where we discussed turbulent flames which essentially originated from discussion of turbulent flows to discussion of turbulent combustion then moving onto turbulent non-premixed flame turbulent premixed flames this part of the course comes to an end. Then we will move into the final part where we will discuss about practical combustors.

So, see you then and until then goodbye.