Combustion in Air Breathing Aero Engines
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Lecture - 45
Turbulent Premixed Flames IV

So, hello friends, welcome back we were talking about turbulent premixed flames and we were discussing various aspects of its various physical aspects of its we discuss that regime diagrams, and how by using simple scaling relationships we can identify different regimes of turbulent flames in particular the most important are the corrugated flame length and the thin reaction zone of the thin reaction zone regimes or the reaction sheet regime as such, and most of the it was specific function happens in between these two regimes. If there is some can happen in the distributed reacted distributed reaction zone regime also.

Anyways now, we were proceeding to develop basically the models that can describe the propagation of the turbulent flame and as you see that in the; from the beginning video that we saw if you just go back this video that we saw that.

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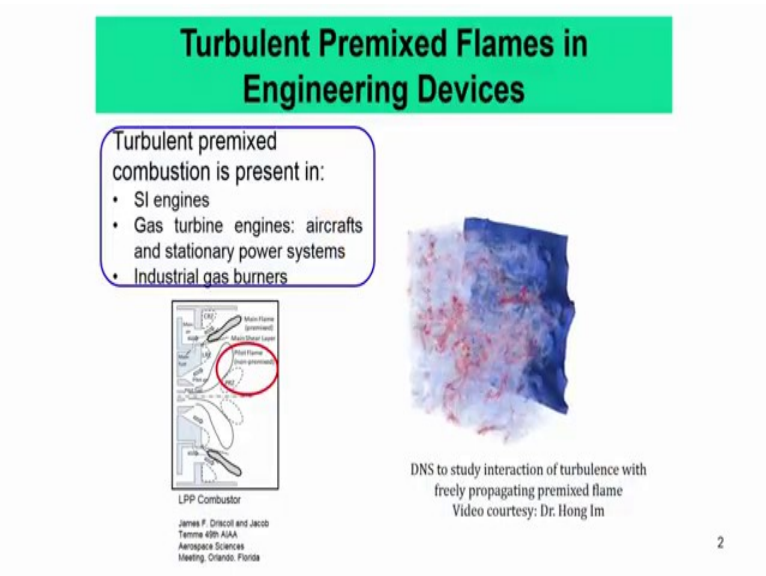

We can understand it appears we can understand turbulence flame interaction by basically considering what happens on different iso surfaces inside the flame, iso surface of temperature inside the flame. Then the task could be to essentially describe the evaluation of this iso surfaces under the impact of turbulence, large scale turbulence to small scale turbulence.

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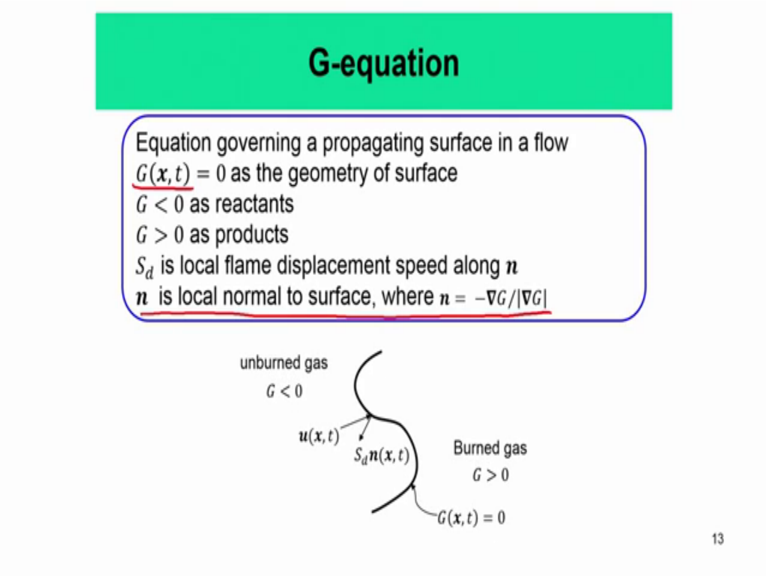

And for that we essentially went to describe you the G equation I will just take a revisit at it.

And basically this is an field equation G equation is a field equation, where you can describe a G field everywhere inside the inside your domain. And then you basically select one of those iso surfaces of that G field, say that iso surfaces given by G x t is equal to 0 as you see here, and that essentially these propagation and movement and stretching and folding of these iso surface mimix, essentially the dynamics of a turbulent flame surface. And we say that if G is less that equal to 0 as a reactant G is greater than 0 is a product then the local flame the local surface G must propagate with the local displacement flame speed where as displacement flame speed is of course, rate of the propagation, the propagation rate of the surface with respect to the local fluid velocity.

And this propagation happens along the local surface normal which is normal to the surface and then whereas, this n normal is given by the is given by this expression can be found out from the G field itself, which is given by minus of gradient of G divide by mode of grad G and then this is the representation the cartoon representing the G field.

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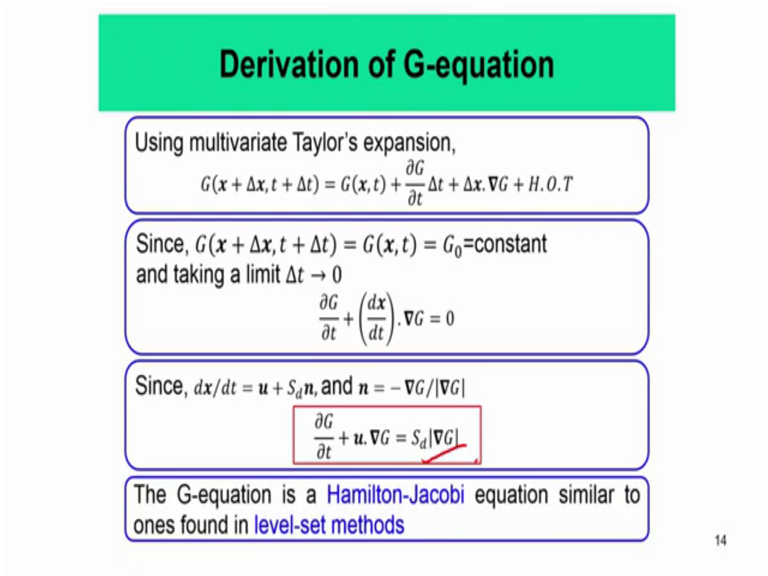

So, then we could derive this and the final G equation was given like this d G d t partial d G partial d t plus u dot gradient u dot u vector dot divergence over the acting on the G field, and that is equal to S d times mode of grad G of course, this is a non-linear term.

Because its carries can be essentially written as square root of grad G dot grad G that is what the non-linearity comes from of this equation, then there are specific methods its cannot be directly solved by normal central differencing of normal finite differencing method, and that there are special methods available for doing this thing of course, solving the G equation that you that is not we will not discuss here.

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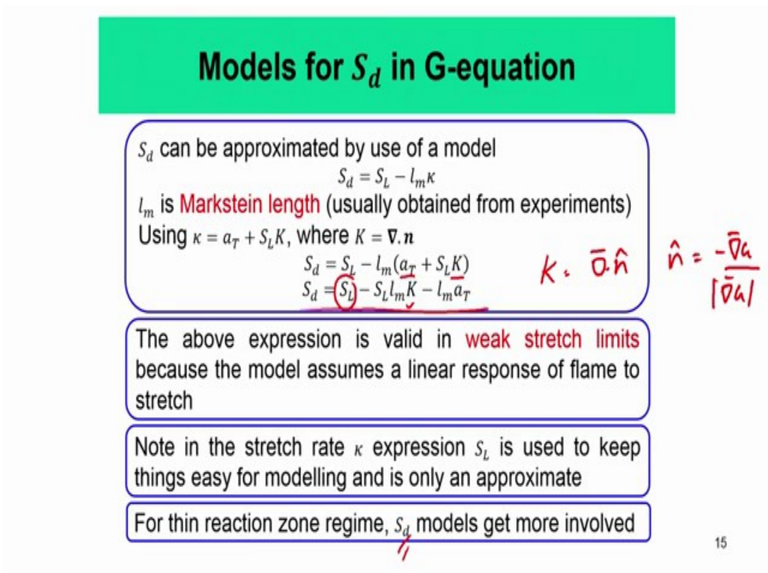

But then one needs to find a appropriate model for S d also there is a local displacement flames, how will you find that because that is a that goes into the input that is an input into the G equation model right into the G equation. So, that can be given as a function of the local stretch rate the local tangential strain rate which is A T and local curvature.

And this is an expression that we can use whereas, a curvature can this this curvature can be obtained from the geometry of the surface once again, this this curvature is essentially is the divergence of the normal vector. So, once again if you know the normal vector which is which we have given as minus of grad G divided by mod of grad G. So, from that you can find this out and A T comes from the by projecting the local strain rate into the tangential direction, which also you can find out from the tangential direction you can find out from the local regimental flames surface, which is given by the field G. So, this is how the local property local geometry of the surface.

And local flow properties are essentially captured into the propagations speed itself also. So, whereas, of course, the to the first out of the propagation speed depends on the planner laminar flame speed, and then what we see here is that the local curvature and the local tangential strain rates can cause deviation from the planner laminar flame speed, but that needs to be accounted for in the model for the local propagation speed that goes into the G equation. Now of course, another things is that this is valid in the weak stretch limits, but yeah we can use that with certain constraint, as long as a stretching rate is not too high. So, this works well the in the corrugated flame let regime where the turbulence is not too strong.

But people have done some are extra collations into the thin reaction zone regime also. So, of course, as you see that the S d models get involves in the thin reaction zone regime or the reaction sheet regime.

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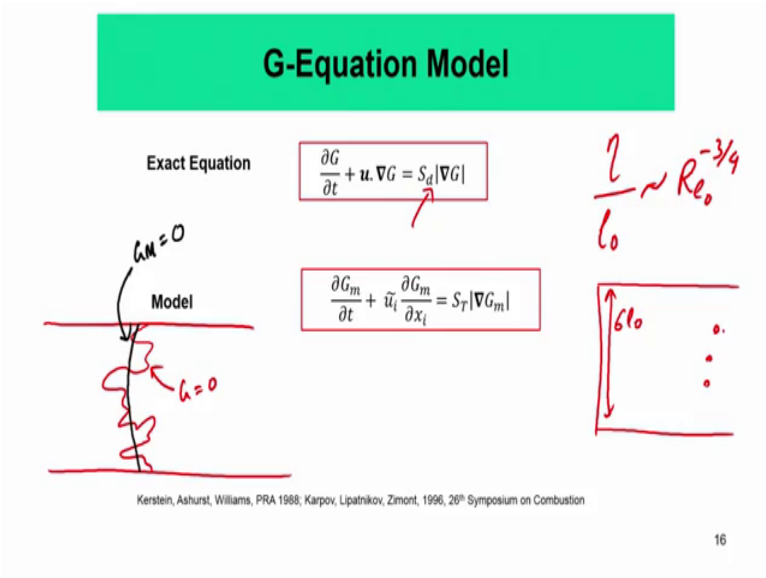

Now how do you use this for turbulence? Now as you know the biggest problem of turbulent combustion or turbulent or modeling turbulent flows is that your eta, that is kolmogorov length scale by the integral length scale scales as Reynolds number to the power of minus 3 by 4 right. So, as the Reynolds number goes big the eta l 0 ratio becomes smaller and smaller right whereas, the thing is that suppose you are solving an engine which is a domain, which is the largest length scale is say 6 times l 0 that is the integral length scale.

So, but if the Reynolds number is very large say if the order of 50,000 your eta will be extremely small very very small of the order of few microns. So, in a problem is that then you need to grade basically from all the way from this 6 time of l 0 which can be of the order of say 100 of centimeters to this few to this length scale of few microns. So, your largest domain size has to be of the order of few say 100 centimeter and yours smallest length scale in your mesh if you want do by CFD you has to the grad size as of (Refer Time: 06:53) has to be of the order of few microns right. So, that is the problem I mean that is why we cannot solve the what happens inside the engine using just by solving all the continuity.

And momentum temperature species equations as well as the constitutive relations as is, and that is why we need to seek models which describe their mean propagation mean properties or their filter properties is as it is done in LES larger dissimulation. So, that is that is the thing. So, similarly that is why we went to do the rans thing rans model where you do not need. So, fine grades because you are only interested in the evolution of the average quantities. So, whereas, will the as I said in the alias we are only interest in the evolution of the filtered quantities, now those are not the instantaneous quantities. So, you lose some in knowledge of thing what happens instantaneously, but then we are more interested in the average statistical behavior of an a engine rather than what exactly happens in rate for instants.

So, if, but the problem as you have seen that that closure model, closure problem whereas, this there are closure problems at different stages for example, we have seen that there is closure problem in the Reynolds system there is closure problem in the turbulence scalar flux terms, there is closure problem in the mean reaction terms. So, there are closure problem at different stages. So, same thing happens in a turbulent premixed combustion also. So, if you have this even of you have this exact equation for the propagation rate of the flame surface, which is governed by say G even if you assume that that this G equation can describe the propagation of the flame surface exactly, but then that is if we have a very good model for u and you can accounting for density variation somehow in the flow field, but given that your G field if you have a if you have turbulent flame this turbulence flame will be wrinkled at multitude of length scales right.

So, you basically you need to even if you solve this G equation, you need to solve for this largest length scale which is again of the order of 6 times l 0, approximately right because your integral length scale is one sixth of the domain size, and your smallest scale would be on. So, fluctuation will once again of the order of eta or the flame thickness whichever is smaller something like that. So, your computation recommends still is very huge right. So, what people try to do is that people try to solve for an averaged G field, that is you do not average exactly over the G field that is not really done, but you just try to describe an equation. So, I will let me just draw this once again.

So, the idea is that suppose this is your domain and this is your statistically one dimensional turbulent flame speed, which is wrinkled at multitude of length scales and you do not have the resource to solve a such a huge mesh of an another thing of course, is that even though this scale separation is eta by l 0 grows to the power of Reynolds (Refer Time: 09:59) to the power of three by 4. When it comes to mesh it becomes it is always three d machine turbulence, because turbulence is essentially three d. So, this scale separation essentially gets not it grows to the power it grows in a cubic manner because your smallest scales has to be of the order of delta. So, whereas, your largest scales is order of your smallest scale is of the order of eta and your largest scale is of the order of 6 times l 0.

So, if you have to reduce your scale your smallest scale by10 times. So, your eta now is essentially your delta is essentially one tenth of the previous delta is. So, what happens is that, your actual grid size or the number of grid points increases by 10 to the power three not just by 10, because it is a if you assume it is a cubic domain of the order of that depending on the domain size domain shape of course, but it grows in a cubic manner. So, that produces immense computational load which is really not possible even. So, the current gas turbine combustor if you want to solve it by direct numerical simulation, where you will simulate all the way from the large scale to the small scale, that is simply not possible.

So, you need to use a model that is why we have industry essentially uses rans models and some there are also aliases also merging as one of the possible models to basically simulate turbulent combustion in practical engines. So, one of this model that was proposed by Karpov lLpatrikov Zimont about 20 years ago was this say the mean flame surface propagation. So, this is not average, but what it shows tries to says that is a I will not my I do not have the solution to grid resolution tool basically simulate all these small scales. So, what I will do is that I will have a mean of flame surface, which can be considered to be essentially the average location of all the is instantaneous flame surfaces. So, one average of course, you see that this flame surface will fluctuate around this this mean.

So, this mean is essentially given by G m is equal to say; this is another of course, a different function which is a given by G 0 whereas, this thing this is equal to instantaneous which is given by G which is given by 0. So, or this can be also given by G 0 also given by 0 actually. So, this is the propagation of this G m which is the; which is suppose will be the flame the surface which is averaged over all possible locations of the instantaneous flame surface. So, this is the essentially the mean flame surface model and of course, now you have a (Refer Time: 13:13) is essentially propagating as a result of two things, it is now propagating due to the mean flow ok.

Which is U i average for the average and then this itself propagates with the instead of the instantaneous and local displacement flame speed, it for this thing propagates with the, this thing propagates to the with the mean propagation velocity which will emerge to essentially turbulent flames speed in the as you will see later. So, this is the this was my what my domain was this is the instantaneous flame surface and this is the mean flame surface straight because this is statistical planner and. So, it now behaves almost like a one statistically planner laminar planner one d flame, but of course, its structure is very much different from the one d flame.

Because it is essentially average for example, how I it should be different is that it you will see that there are two major points of difference, a number one this thickness of the flame will be much thicke, thickness of this average flame will be much thicker than the instantaneous flame and two it will propagate much faster because as instantaneously as its surface area is much bigger than the planner laminar surface area, it will essentially consume a lot more of the fuel layer mixture than a planer laminar flame can consume and as a result the result in velocity of the result and speed with which it will move is given by this turbulence flame speed and improve much much greater than the planner laminar flame speed. So, this is the thing.

So, now we need to understand basically this our concept called ST now if in a planner laminar flame or in laminar flames the most important property is laminar flame speed for a turbulent flame or a turbulent premixed flame, the most important property is the turbulent flame speed. So, what is it and how does it to behave. So, this is a very important topic of which that has been for the last several decades, and it is still kind of an unclosed problem, but we need to understand this because turbulent flame speed is for what governs the propagation stabilization or any such dynamic behavior of a turbulent premixed flame.

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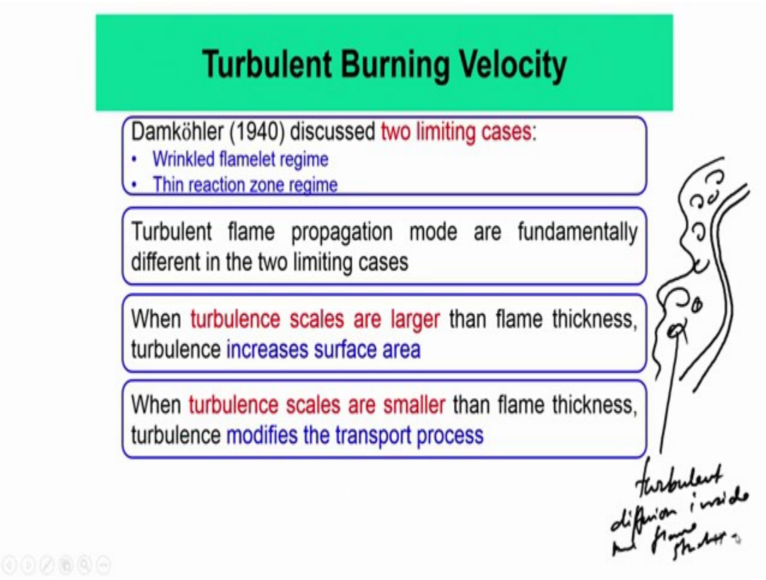

So, turbulent burning velocity, it has the research of turbulent burning velocity is essentially originates from the works of Damkohler.

Who suggested two limiting cases and one is the wrinkled flamelet regime and another is the another works in the thin reaction zone regime, which are the two regimes that we have seen. So, that the turbulent flame essentially the turbulent propagation modes are fundamental different in the two limiting cases the when the turbulent scales are larger than the flame thickness, that is in the corrugated flame regime or the wrinkled flamelets regime where there is no disturbance from the preheat zone structure by the turbulent eddies, then just turbulence increases the flame surface area by due to tangential straining, without essentially distorting or disturbing the inner flame structure. But when turbulence scales are smaller than the flame thickness then turbulent can modify the transport processes also.

Because we see essential turbulence has a very strong diffusive character whereas,, but it is says that that diffusion arises essentially due to the by conviction and the essentially due to the convective terms and due to the Reynolds rate terms, which are essentially very much responsible for the diffusive nature of turbulence because of the turbulence fluctuation. But you see that in a laminar flame essentially the both the diffusive molecular diffusion processes as well as reaction are very very important. If you remember your flame speed was essentially the geometry mean of the reaction rate and the diffusivity thermal diffusivity right. So, now, if you have very strong turbulent diffusivity, which essentially overrides your thermal diffusivity or molecular diffusivity then this turbulent flames.

This turbulent diffusivity can become an equal important or even more important parameter than the turbulent diffusivity than the molecular diffusivity in this propagation rate or this global propagation behavior of the turbulent flames. So, in the preheat zone structure, if this is the now the distorted of preheat zone distorted flame shape whereas, in the thin reaction zone regime like this let me just draw it. So, this is the preheat zone structure. So, where now we have eddies of different sizes distorting it, then these eddies will essentially pre act can create turbulent diffusion inside the flame structure. So, that is one possibility.

Of course when you look into this in a statistical way that is if you now think that your flame is as a has a your you can essentially covert this entire different flame structures into essentially a statistical one d structure, then also you will see that this turbulent diffusion will play a major role in essentially controlling the diffusive processes in this inside this preheat zone of the turbulence premixed flame. So, when what we want to say is that that when there corrugated flamelets regime, that the turbulence scales are much larger than the flame thickness then turbulence only increases the flame surface area.

Whereas if the turbulence scales are smaller than the flame thickness that is in the thin reaction zone regime, the reaction to regime will can turbulence can modify the turbulent process also. So, in this one we just only need to calculate the increased flame surface area whereas, in this one we might need to even calculate the enhance diffusion processes that happens inside that turbulent flame structure.

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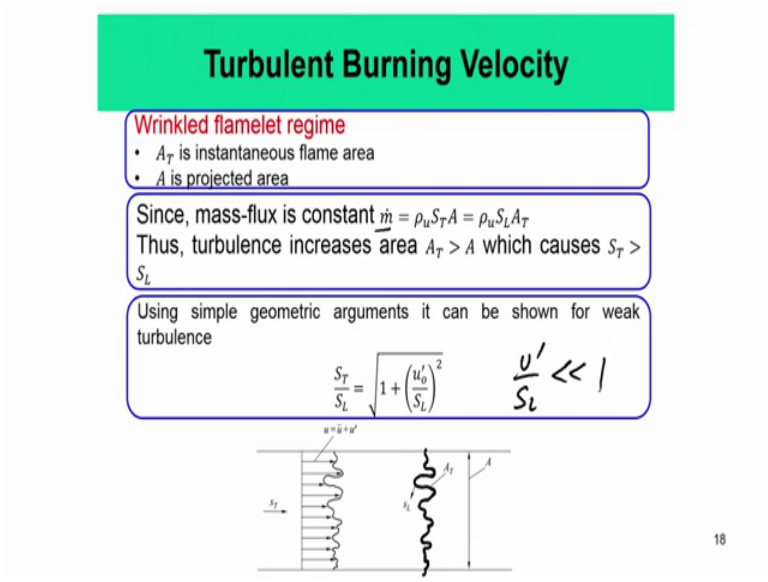

So, then the wrinkled flamelets regime that A T if we consider that the A T is the instantaneous flame area whereas, the a is the projected area that is what I mean is that the if have a flame like this ok.

So, this is in three dimensional of course, the flame will be will be three ds and then this increased area. So, now, you just have an increased curve length over the planner laminar flame, which was a planner. So, this is the laminar flame which is say which had a area of say A L say it is a its a its a cubic cuboid channel, in which is the flame is propagating. So, then this wrinkled area is has the surface area A T of course, are wrinkles surface area and a given in the same cross section will have much more area than the planner area. So, this wrinkled area has an (Refer Time: 20:28) has a surface area of A T whereas, the planner laminar area A L or the project area has an area a or has an area a only.

So, then we can what we can do is that we can find out the relationship between a ST and a SL and as a function of A T and A. So, what we can says that lets consider this is to be a cuboid in which this turbulent flame as you see here is stabilized its statistically stationery at this location; that means, it is essentially fluctuating about this constant about this line a line or area a. So, for this as you if you remember that for a laminar flame to be stabilized or stationary at particular location, it must the reactance must enter into it at a velocity which is given by the planner laminar flames speed SL.

So, by the same argument to keep this flame this turbulent premixed flames stationery at this particular point, the average velocity with which the reactance must enter into the flame is ST. There is in the other way that if you allow this to be free if you allow this to be a freely propagating flame, on average this flame will move to the reactants will the average propagation rate of this flame into the reactance will be equal to ST. So, ST is essentially kind of a you will see that it is a average propagation rate of the turbulent flames of the turbulent flame ok.

So, now to compute that in terms of the area what it will be is can be found out from here that is if I say that lets compare the mass flux is. So, the mass flux then if this flame is stationery then the rate at which the mass flow rate in to the flame surface is given by rho u times local SL I consider the local flame speed to be essentially constant times A T whereas, this is the flame surface area. Now this is the mass flow rate into the flame surface and now if of course, this is the mass flow rate that is going into the flame surface and of course, this is coming from the upstream mass flow rate, and if that has to be then the mass flow rate is being fed at a velocity at a speed which is equal to the turbulent flames speed, and this because the flame is essentially statistically stationery.

So, then we can equate this to rho u times ST times A and so, then my ST by SL essentially becomes A T by A. So, this is the reason why our ST must be greater than SL because your A T is much greater than A. So, once again to clarify that so, the rate this is essentially the rate at which this mass is being consumed this m dot is the rate, at which the mass is being consumed in this wrinkled over the entire wrinkled flame surface, and now of course, the appropriate velocity locally with which this mass flow rate is going in is essentially SL, which is the relative velocity between the flame local flame motion.

With respect to the local fluid velocity and that is why SL comes in. So, then we can equate it to an average speed with which the mass is must be fed, for the flame to be essentially stationery and that is given by essentially S t. So, we get that m dot is equal to rho u times SL times A T, and with which the mass is being fed into this thing, and then this we can equate that to essentially rho u times ST times A, and our then ST by we have got ST is the local is a global propagation rate of the flame surface or if this is statistically stationery it is the rate at which mass this fluid reactance is a speed at which this fluid reactance is approaching the flame surface the overall flame surface.

And then this is given by ST and then ST by S n is essentially equal to A T by A. So, then since turbulence increases the area A T is greater than A which crosses ST greater than

SL. Now in the weak turbulence limit weak can be shown that that for this ST by SL this essentially is square root about 1 plus u by u prime by SL square, but this is only true when u prime by SL is much much less than 1. So, this is true only in the very weak turbulence limit, and this is not very much practical in importance because we do not have situations in a practical engine which is weak turbulence weakly turbulence, may have situation in a certain engines which is strongly turbulent ok.

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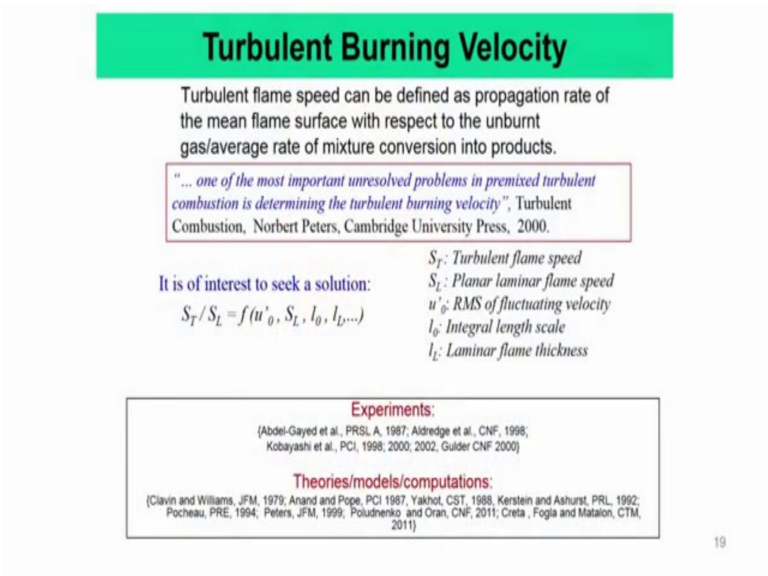

Now, we go into more.

Now, we go into more formal discussion and methods by which the turbulent burning velocity of the turbulent flames be rustic, and can be obtained because as you see that ST comes in to the model for the G equation to describe the average propagation speed, average propagation of the propagation rate of the average of the mean surface and it is used in different turbulent flame sheet models for validation also. So, just like laminar flame speed is an important parameter for laminar flames, turbulent flames is also very very important parameter for turbulent burning velocity, but there are problems which will see. The turbulence flame speed can be defined as a propagation rate of the mean flame surface, with respect to the unburned gas and average rate of mixture converted in to conversion into products. So, that was our ST ok.

This is what we described as ST and it is essentially it is still a kind of we still do not understand turbulent flame speed completely, and peter describes in his book as that this is one of the most important un result problems in previous turbulence combustion, is determining the turbulent burning velocity. And if ST is turbulent flame speed SL is laminar flame planner laminar flames (Refer Time: 27:28) u prime 0 this is r m s of fluctuating velocity integral length scale l l laminar flame thickness then we want an to see the solution of ST by SL as a function of u prime SL l 0 l, that is if you have a turbulent flow which is completely define by URMS and in to the length scale and of course, the other property is the fluid property is are given.

And you have a fuel mixture by which you where your flames speed and flame thickness are known for the laminar planner laminar flame, then can you with this parameters can you determine the turbulent flame speed as a function of the laminar flame speed. Now there was been of course, we have try to seek this and there is an numerous experiments and there is numerous model models and computations, but still this remains little bit unresolved and we need to do more research on this, and that is why I will give you some claims of what has been done ok.

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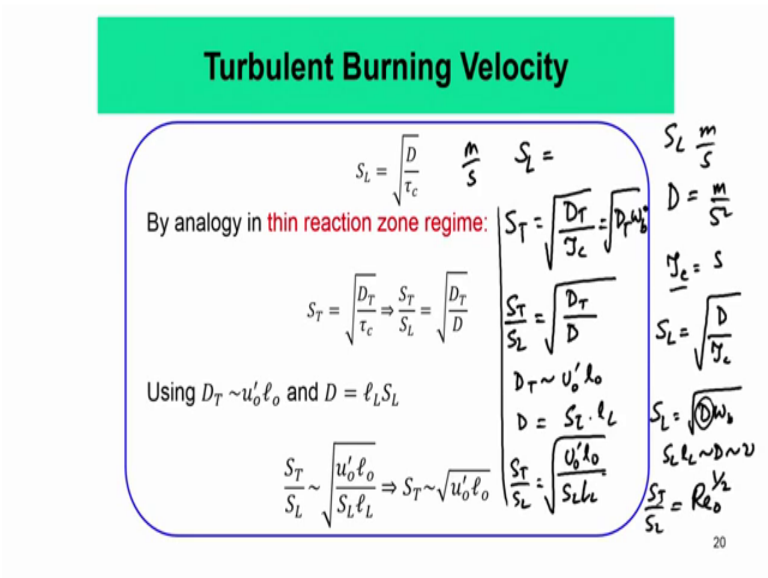

So, in the thin reaction zone regime, we have a very interesting idea from Damkohler he said that you know for a planner laminar flame the planner laminar flame can be written as it is a of course, you need meter per second.

And your of course, a planner laminar flame has been to say units meter per second and the diffusivity when Lewis number equal to one has an unit of meter per second square, and your time scale or the flame crossing time is essentially seconds. So, what we can do is the we can write this or this is a chemical time in terms of seconds, we can write this SL of course, we know that we have written SL as essentially D times omega whereas, D is the diffusivity and omega is a reaction rate, and now we just want to write this omega in terms of a time scale. So, in surface reaction rate we just write a chemical time scale which is a units of second.

So, then the by scaling argument we can write SL is essentially equal to D by tau c that is a thermal diffusivity by a chemical time; or we can write it as SL is equal to root of a d times omega b also. Now then what he says then by analogy in the turbulent for the turbulent flame speed in the thin reaction zone he is saying that Damkohler is saying that your now your diffusivity is not just thermal diffusivity or which is governed by molecular (Refer Time: 29:56) anymore. Now this did diffusivity that you have here must be replaced with turbulent diffusivity. So, what if we just replace turbulent diffusivity in the similar way and keeping the chemical time scale to be same or we can write this as essentially root of our d t times omega b 0 that is a local reaction rate mesh unchanged.

So, then we can write ST by SL is essentially equal to square root of d t which is a turbulent diffusivity by d. What is turbulent diffusivity? Turbulent diffusivity is if you remember that by the gradient transport assumption your u prime c prime average right this prime c prime this one is equal to minus D T times d c d x that was what we wrote. So, this is our c any species. So, this is the gradient transport assumption that we could write and hence by that is how the turbulent diffusivity essentially originates and hence if you now we have to find out what is a suitable expression for DT, now since DT is given by has an unit of once again of meter second square, we can write in terms of the large scale turbulent parameters as u prime time l 0.

So, this is meter per second this is meter. So, it is essentially meter per second essentially meter square per second and we write of course, D is equal to l L which is the which is SL times l L which is the local laminar flame speed (Refer Time: 31:29) flame thickness. So, then ST by SL you essentially get is u prime is 0 by l 0 divide by SL times l L and this is nothing and as you know that if we say Schmidt number equal to 1 this SL times l L becomes essentially u can be written as equal to d is equal to the kinematic viscosity. So, then this ST by SL becomes essentially. So, we can write SL times l L is equal to d is equal to nu. So, then ST by SL is becomes essentially equal to Reynolds essentially becomes to Reynolds number to the power of half ok.

So, ST by SL should be equal to then Reynolds number to the power of half. So, this way we can have a scaling relationship for turbulent burning velocity. So, this is a important thing. Now why is this so important for gas turbine combustors? Because as you have seen that from this definition itself the turbulent flames speed is can be defined as the propagation rate of the mean flame surface with respect to the unburned gas or average rate of mixture conversation into products this is one is the essentially a displacement turbulence burning velocity, another is a consumption turbulence burning velocity and. So, this turbulence burning velocity is a global in units of meter per second it is it gives an idea about what is the global consumption rate of the fresh meet mixtures ok.

Just like the planner laminar flame speed gives you an idea about, how the mixture is being consumed across the flame in units of meter per second. So, turbulent flame speed gives you two things it gives you an idea of the how the mixture is being consumed, how fast and be and consequently it also tells you how fast turbulent flame speed can propagate or reversely that to stabilize to statistically make a flame stationery, what is maximum velocity with which you can feed in fresh fuel air mixture. So, that is essentially governs your combustor entry.

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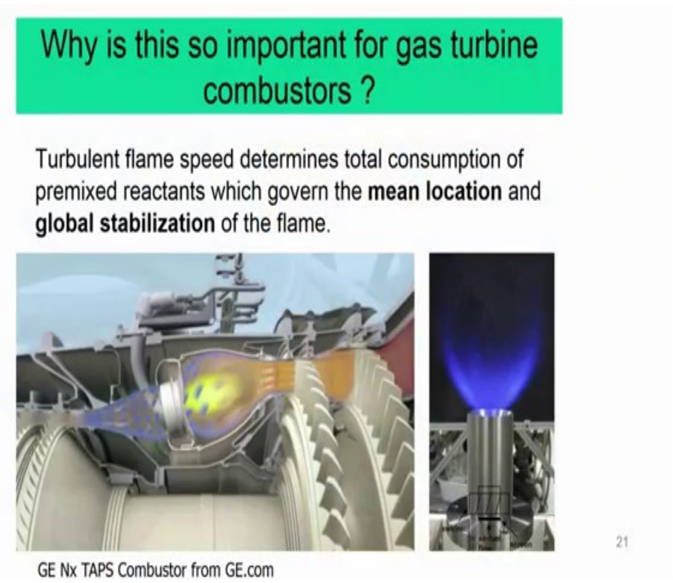

Speed in typical combustor your entry speed is a of the 40 meter per second of course, you have some aerodynamic means to stabilize, but this is far larger than 40 feet meters per second. Even more it can be 100 meters per second, but which is far higher than the laminar flame speed ok.

Which is of the order of say 30 centimeter per second, but as you are stabilizing a flame as 100 meters per second of course, you do aerodynamic means we create local low velocity regions etcetera, but still this flame should not have been stabilize if there are no turbulence. So, turbulence flame speed determines the total consumptions of premixed reactions and governs the mean location on global stabilization of the flame. So, in this engine even in a taps combustor which is a flame like this in a tabs combustor if you have two stabilizer flame you have to know what is a turbulence flame speed. So, I has to design this engine properly ok.

So, similarly if you have to stabilize this term lean premixed flames in this low (Refer Time: 34:38) burner, one is to know the turbulent flame speed because this turbulent this flame is being stabilized because the average velocity is equal to the turbulent flame speed of this flame. So, that is why it is important to know the turbulent flame speed and next will take up essentially the methods to analyze turbulent flame speed.

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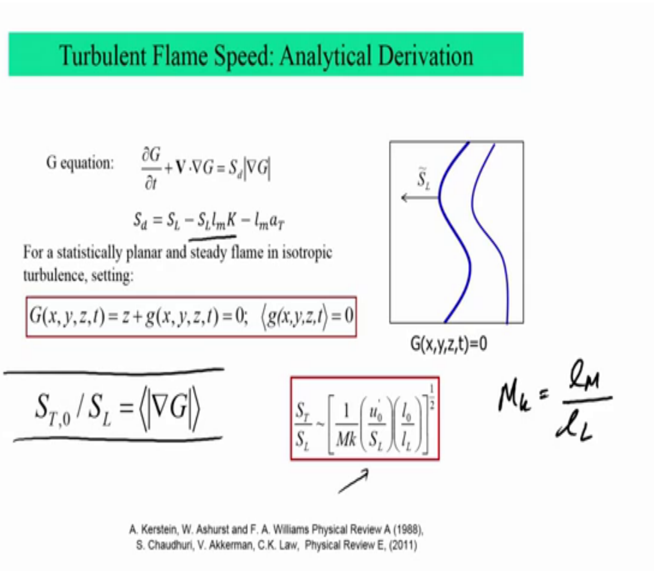

Now one can do some analytical derivation and do which we did and we took of the G equation and did to the spectral approach and which will not going to details, and this expression of turbulence flame speed in terms of the G field comes from the Kirstein Ashurst Williams where he takes a essentially an finite very long cuboid, and which is filled up with different of this surfaces.

So, he can shows that this ST 0 by SL which is the turbulent flame speed of a statistically planner flame which is stabilized which is stationery inside this statistically stationery inside this cuboid, and then he showed that then that is given by the though the volumetric average of this of the gradient of the mod of the gradient of the G field. So, and then using those one can arrive at a expression which is similar to the Damkohler, but not exactly same one can introduce this this stretch limits stretch effects, and then ST by SL is given by one by mast in number which is the ratio of your mast in length. If you remember that to the mast this is the mast in length which becomes. So, the mast in number here is essentially your ratio of mast in length to the flame thickness ok.

So, then this is given by for positive mast in number fuel air mixtures then the ST by SL is given by one by mast in number times c prime by URMS by planner laminar flame speed times integral length scale by the planner thickness planner flame thickness hold to the power of half. And then will show you some experiments in the next lecture to basically show that how one obtain its turbulent flame speeds using (Refer Time: 36:45) using laboratory experiments and how well does that effect into different kind of scaling logs, and how one can basically have good scaling logs for different kinds of a particular turbulent flame speed is in different configurations as well as for different fuel air mixtures, over different ranges of pressure and u r m s conditions so.

Thank you.