Combustion in Air Breathing Aero Engines
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Lecture - 44
Turbulent Premixed Flames III

Welcome back. So, so far we have seen that basically the mechanics by which turbulence can essentially enter inside the flame structure and distort it. So, we have seen that that in different regimes that is especially demarcated by karlovitz number. So, when the karlovitz number is small. So, the essentially the laminar structure of the flame is retained of course, it can be stretched that is you see and we have also discussed this concept of stretch in the sense that, when you have this tangential stretch when you have essentially flow non uniformities along the tangential plane of the of a flame surface or when you have a flame that is curved And which is moving with the flame speed.

So, in that case of course, you can have flame stretch. And in those regions basically when the karlovitz number is less than 1 we can essentially consider this flames the local flamelets the local flame structure to be essentially that of a stretched laminar flame. Because in the karlovitz number is less than 1, your flame is bent your flame may not be straight or planner laminar as such it can be bent, but and there can be some flow non uniformities also. So, the flame can be stretched which will essentially lead to increasing the flame surface area.

But that does not mean that the turbulence has essentially created some structural change inside the flame. Now of course, that can happen we have seen from the regime diagrams that this just this considering or abstracting this a turbulent flame as an ensemble of stretched laminar flamelet as you as I said that this works in a regime where the karlovitz number is actually less than 1 or damkohler number greater than 1, but in a regime where the karlovitz number is very large this really does not work well, because it is not only the flames are stretched.

Of course they are stretched that is true, but in more than there is there are the fact that they are stretched there is some inside disturbance of created by turbulent that is turbulent flame turbulent eddies or small scales turbulence in the sense that kolmogorov sized eddies can enter inside the flame structure and create disruption, but you have to understand that the flame is just not a benign object it is not a passive structure it is got strong heat release. So, when there is strong heat release there is strong gas acceleration also. So, it is possible that the turbulence can also be destroyed by the flame. So, it is a 2 way coupling process ok.

So, on one hand you can have turbulence impinging on a flame and stretching wrinkling folding it and even changing it is structure at multitude of length and time scales. On of the other hand you have basically can have a regimes or you can have basically have situations where the turbulence can be essentially also destroyed or can be changed by the by the heat release rate and subsequent gas expansion by the flame. So, these 2 are basically competing effects and these 2 are have been a kind of in a simplified manner vary much simplified manner. We have essentially simplified them and these complaining effects in this regime diagrams ok.

Now, then of course, we discuss the concept of stretch and next we go into this flamelet nor models for premixed combustion in turbulence. So now, the basic assumption of the flamelet models is that that basically these are typically applicable for low karlovitz number large damkohler number flames, that is basically these are applicable for situations where we do not consider any disturbance or distortion of the inner flame structure or the or the preheat zone structure of the reactions zone structure of the flame by the impinging turbulence. So, we just consider that the flame is essentially wrinkled and it is wrinkled at a multitude of length and time scales.

And we and that that of course, has still it changes the global properties of the flame, because in once it is the flame is wrinkled it is surface area is much larger and then on the surface area is larger than it can consume more fuel air mixture burning a time and it can on statistically it can propagate at a much faster rate. So, still this that description is not trivial it has to be one has to consider that if that things, but here we will consider essentially this flamelet models, because those are much simpler and explaining going into this regimes where the turbulence distorts the flame structure that is that request much more complicated our complex analysis of turbulence came flame interaction.

So, here we will restrict our self with flamelet models, but there are basically 2 types of flamelets models that we will discuss.

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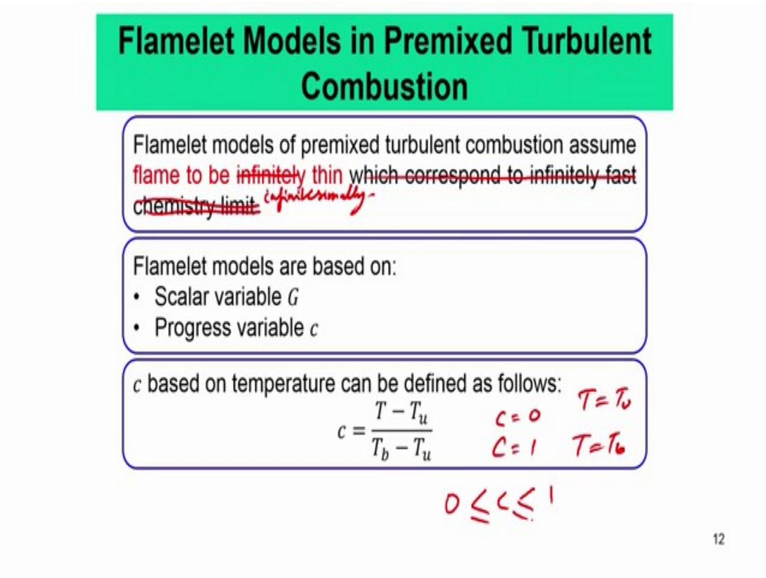

And the inherent assumption is that that the flame has infinitesimally thin infinitely infinitesimally thin which correspond to infinitely fast infinitely fast chemistry limit. So, it is a basically infinitesimally thin rather than infinite infinitesimally thin. And which correspond to the infinitely fast chemistry limit. So, this is the assumption that the flame is very thin compared to any structure of any turbulent flame structure. So, it is the characteristic length scale of the flame is smaller than any turbulent structure.

And it corresponds to the infinitely fast chemistry and. So, the flamelet models are based on the 2 things that is the progress that is a scalar variable G and the progress variable c. And basically the (Refer Time: 05:37) assumption is that we do not even need this thing that is it is need not be infinitesimally correspond to fast chemistry. We just need to consider this flames that are infinitesimally thin compare to the any structure and turbulence. So, and this we will consider for basically 2 approaches one is for that we will use this G equation model and the progress variable model ok.

So, we will we will come into this what the G equation are and these are the 2 basically models that will consider the scalar variable G and the progress variable c. Now we will come to this, but are the c is essentially a non dimensional temperature difference. And this is T minus T u divided by T b minus T u. So, as you can see that the c can only attain a this T since this T can attain a minimum value of T u. So, that the time c is equal to 0 when T is equal to T u and it can attain a maximum value of 1 that is that will happen when T is equal to T b. So, the between c basically varies between 0 and 1 and this is what we will consider.

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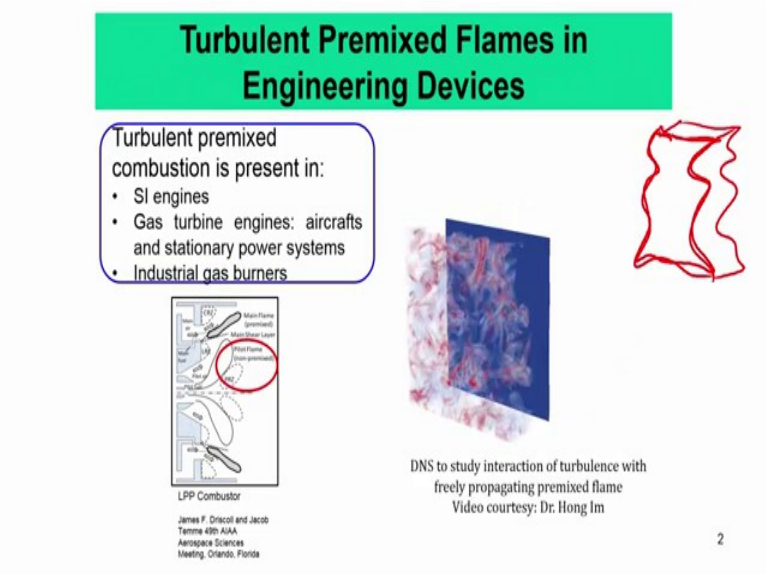

But before that we go into this G equation ok.

Now, this is a little conceptually very interesting thing. So, the things that you have seen once again we go back keep going back to this thing, you see here. This is what we are showing here is essentially a surface that we have taken out of the flame. So, the actual flame if we is essentially thick. So, if you just. So, so the actual flame is thicker. So, it can be like if we take the structure it will be something like this, there will be like a it will be a thicker object, but what we have taken is that we have just taken one surface out of it. And one surface out of it I am just showing that how the turbulence interacts with the flame.

So, what if we consider that the flame is infinitesimally thin compared to turbulence and that the entire reaction is happening inside this infinitesimally thin object.

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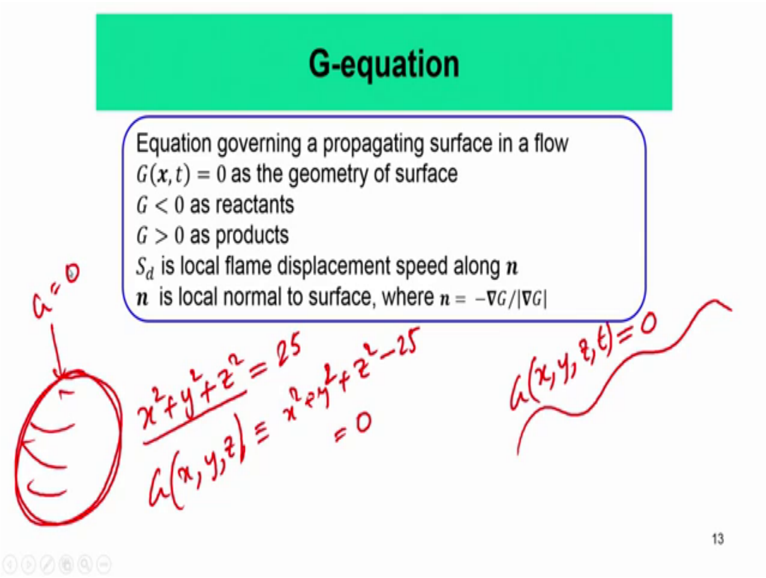

Of course, it is it is a 2 dimensional it is a complex surface. So, where the thickness of the surface is infinitesimally thin and the reaction is concentrated in that infinitesimally thin surface. And this surface how do determine describe a surface as you know a surface can be determined by a level set fellow. So, what; that means, is that that if you have a surface like this I say that that the this function G x y z t this is determined by the surface that G x y z t is equal to 0 or is equal to is equal to a any constant G 0 as such.

What will let us say this We determined by the surface a G x y z t is equal to 0. As a very good example is that that if you want to determine if you have a sphere in space how do you how do you describe it? We describe it by this thing that you say that is x square plus y square plus z square is equal to some say if the radius of the circle is 5 unit is square is equal to 5 square. So, this is my say this is equal to is equal to 25. So, this then this whole thing is my G. So, then I say describe the surface of the sphere as G x y z at because if it is not moving as essentially x square plus y square plus x square. And I can put 25 also on this side and this is equal to 0 ok.

So, this is how then the then the whole the surface of this surface of the sphere is essentially determined by this thing G is equal to 0. So, this is how we essentially come to this G equation description. This is essentially a level set description. So, you consider different. So, you consider a function which is distributed in space and by selecting this ISO values of these functions that is in this case the ISO value of the function is 0, by selecting proper ISO values of this function you determine a you can you can describe a complex surface you can describe a convoluted shape of the surface.

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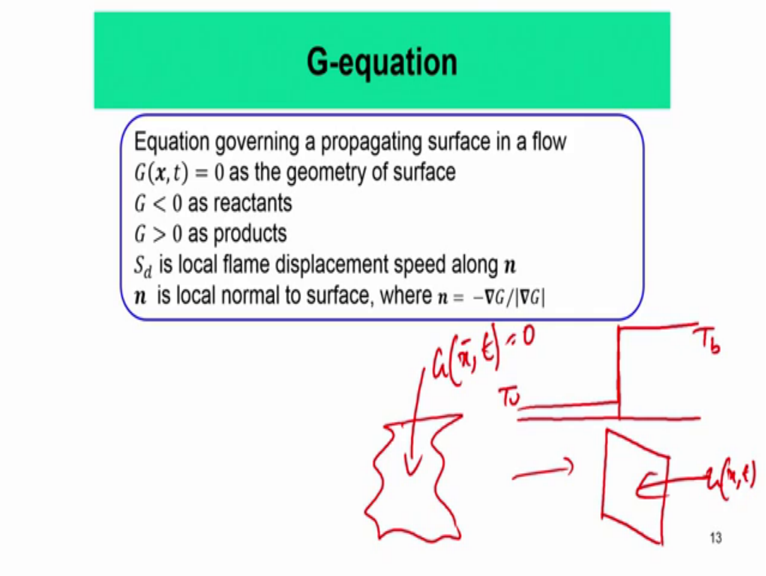

So, this is how you can go with this G equation description. It is essentially the equation governing a proper is essentially a equation for governing a propagating surface in a flow. So, this is evolves from this level of description by osher and sethian in the late 90s and early 2000s and then this combustion community adopted this. So, we said that that suppose you have a suppose you if you go back to the previous description of the laminar flame structure. So, in the infinite infinitesimally thin reaction limit it is this in this is a structure. So, this is T u this is T b if we go to 2 dimension it will be a surface like this ok.

And now if you have turbulence this surface can be like this. That that is what we just saw right this one can be a complicated surface and we determine this surface as G x this is the position vector x t is equal to 0. So, this is the description of course, if this is can also be describe as G x t equal to 0. So, initially my surfaces whatever that is that this is this surface whether it is planner or convoluted this is described by my function which is G x t is equal to is equal to G x t is equal to 0.

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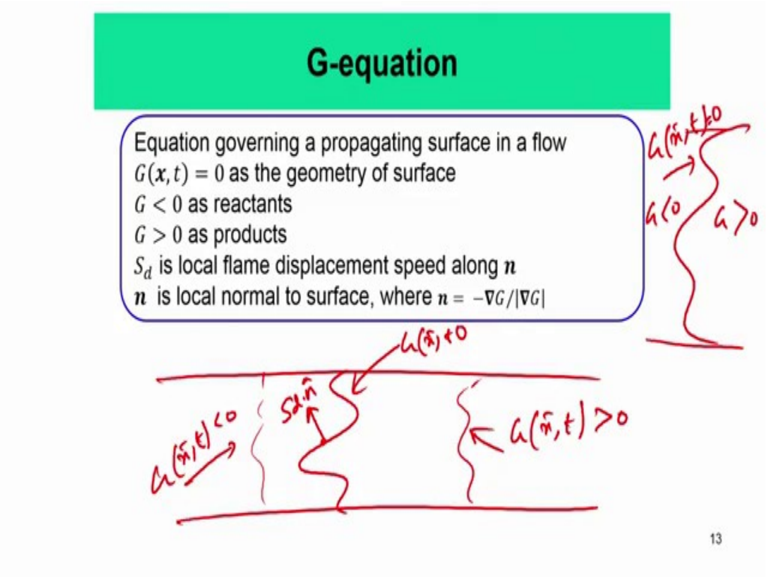

So, now we can we say that this when these G is this the flame surface the flame surface which is essentially reacting propagating surface.

Of course in G equation description we do not consider any reaction as such, but it is a propagating surface. We say that this and the surface is described by this level set by value of 0. So, this way you say that this surface is essentially is given by G equal to 0 G x t equal to 0 and this here my G value is less than 0. And in the product side which is downstream of the surface my G value is greater than 0.

So, this is this is the description the so if this is my flow inside a channel and this is my flame which is given by G x t is equal to 0. So, they are also can be G values here there also can be level set values here. So, these are given by all G x t less than 0. And these are given by G x t greater than 0. So, this is the G g x t G equation description. And of course, at this point the surface moves and the surface moves of course, with a local flame speed which is equal to the velocity of the surface relative to the local fluid velocity, ok.

So, this is the flame speed with it is move and what is the in the normal direction of course. So, this surface is propagating with the local flame speed in the direction normal to itself. So, this is the thing.

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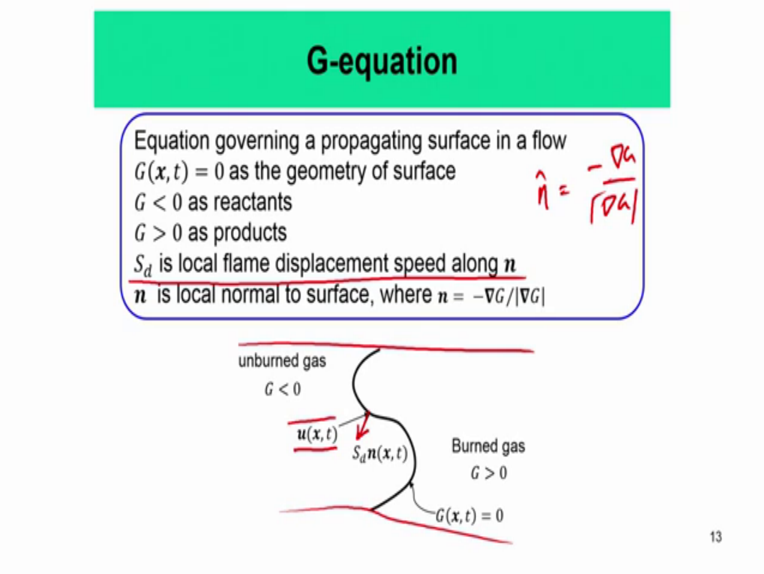

And then we can of course This thing that we see that it propagates a local displacement flames with a normal vector. And how can you find the normal vector? The normal vector is find out like minus G divided by mod of grad G. It can be true is it is true for any scalar function of for which you are seeking the normal it is the definition. So, this is the description that we have this is the flames if this is my chamber this is my configuration. You have this is my G x t is equal to 0 ok.

This is my flame and this is my here my G x t is equal to 0, and on the left hand side we have unburnt gas. You have G is equal to negative that is the level set value of G is equal to negative in this one and on the right hand side we have burnt gas where the G is essentially positive. And here the it is basically propagating due to 2 effects, the surface is moving and convoluting or stretching whatever you call whatever it is happening is due to 2 effects number one there can be it is stretching or wrinkling due to the flow non uniformities which is created by if there is non uniform flow upstream that is if there is a turbulent flow upstream which is non uniform, instantaneously and in point wise so this u x t.

So, it is moving due to 2 effects number one in u x t and which is impulse by turbulence and it is moving due to it is own propagate propagation due to S d and x t. So, these 2 effects must be in built into the into the G equation model alright. So, then we need to find out essentially what is the G equation model.

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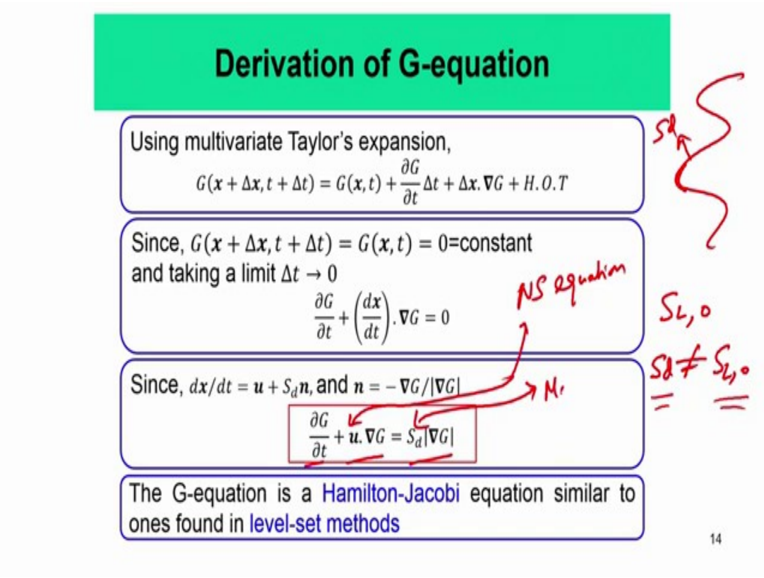

So, what we can do is that, just like we can do the taylors series expansion. So, we can write this thing that is G at x plus delta x and the T plus time delta T. We expand around the G value at x and T and that is given by since G is a function of both x and T is given by partial d partial d G partial d T or dou z dou T times d T plus delta x times gradient of G plus higher order terms which we neglect this is the first order taylors series expansion.

Now, this is we can write of course, for any function and G is of course, a function. Now since what you have to remember is that, that the value of G at x plus delta x and T plus delta T if it is of the same surface, then the value of G does not change; so to give you an example. This is my chamber. So, this is my say G x t and this is equal to 0. Now when this surface will move or I will use the same color actually. We say after sometime or in a very short after some small delta T or some big delta it moves G x plus delta x times T plus delta T ok.

It has moved this much. The value it will take because it is the level set function is still equal to 0. Another good example is to describe you is this thing that suppose we have this sphere, it is a spherical flame which is given by G as you remember x t is equal to 0. And this G is essentially here we know the function in case in this case we may not know the exact functional form because it is not a regular surface. This is equal to 0. Now after some time say this surface this flame expanded and it become like this. So, this is G x t x and this value is still equal to 0, because it is the same level set surface, ok.

So, the value of the value that is taken by G, the value that is taken by G this function that does not change, which is equal to 0 or which equal to G 0 in this particular case it is equal to 0 which is essentially a constant. If you are denoted by G 0 it is we can it is an arbitrary value or if you denote by it 0 it is 0. And if you take the limit say delta T tends to 0 let us put it itself instead of G 0 that is a creating a confusion, ok.

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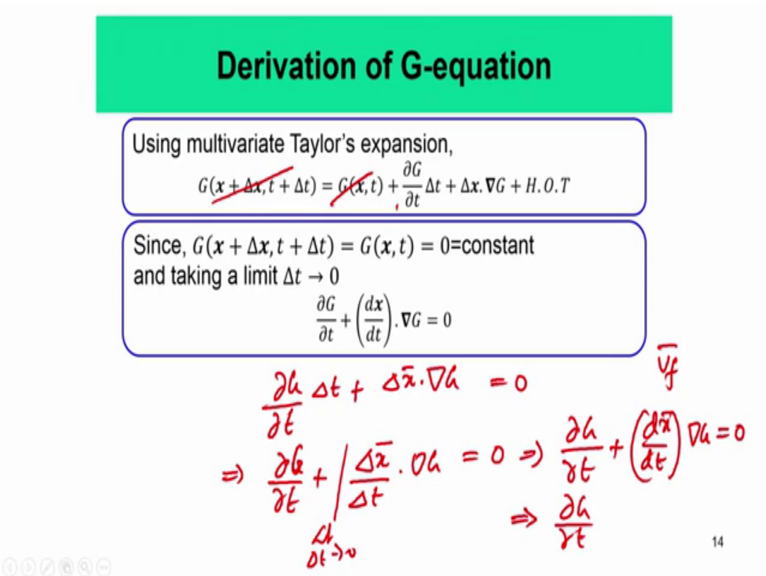

So, if we now take a limit in this thing. So, these 2 terms basically cancel out because these are both equal to 0. So, we are left with essentially dou G.

So, in this thing we are left with essentially dou G dou T times delta T plus delta x times grad G is equal to 0. This is if we take delta T downstairs what is this delta x delta T? This is nothing but the rate of propagation of the surface itself. And now if we take limit delta T tends to 0 which is equal to v f that is the velocity of the flame. This is not the flame speed this is the velocity of the flame surface or the velocity of this surface G and now if we decompose this.

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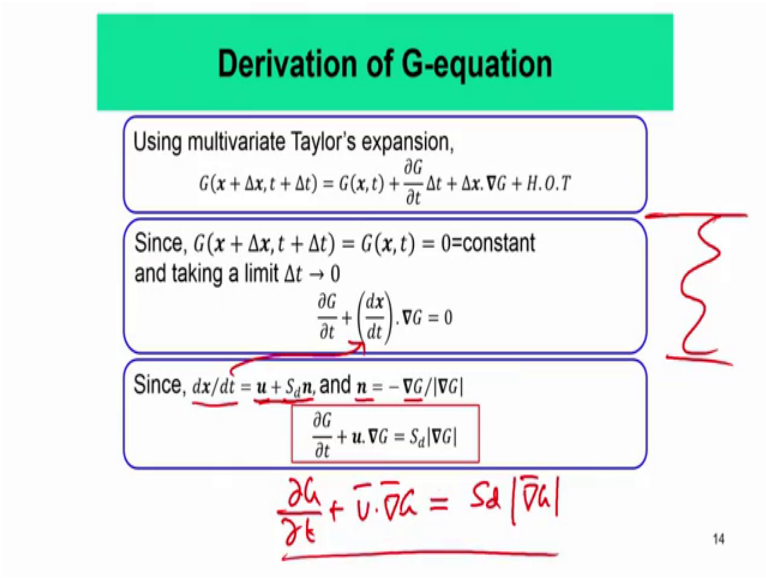

Now, if we decompose this d x d T is nothing but it contains contribution from 2 things. Why does the flame move? The flame moves because if there is a fluid velocity.

All because of the and or because of the local flame speed. So, u plus S d and n is equal to minus n is equal to n vector is equal to minus grad G divided by mod of grad G. So, then if we just plug this thing here what we get is this thing the G equation dou G dou T plus nu vector grad G is equal to S d times mod of grad G the mod of grad G comes from this normal actually. So, this is the G equation alright. Now using this what is the advantage? Using this you can describe basically this is essentially a field equation using this you can describe the motion of a complicated flame surface. Or the evolution of a propagating surface under the influence of a non uniform flow fit, that is the thing. You can describe any complicated surface with this.

So, this G equation is essentially a Hamilton Jacobi equation similar to the and is essentially one outcome of the level set methods. So, this is the G equation modeling that we have taken up. So now, this thing we can you can we can use essentially to describe the motion of convoluted surface and this is this is typically once again if you go back here that this sort of motion, which you see that essentially once again I keep coming like this because the beautiful video it captures essentially the how turbulence interacts with a flame. So, you can capture this kind of a surface formation and deformation that happens due to this due to presence of turbulence and using the G equation.

Though this is not a G equation such this video is obtained this video is obtained by this video is obtained by doing dynamical simulation with all detail chemists and everything but you can capture this sort of motion of a flame using the G equation. So, that is the very powerful nature of G equation. And next we will take up the progress value variable modeling. So, you see we have just obtain this is G equation and one most important term in this G equation is the right hand side which is actually a non-linear term that is S d times mod of grad g, but here you need basically you see this yes we have this presence of this local displacement flame speed it is the local speed with which the flame propagates with respect to the local flow velocity in the direction of the local surface normal.

But now the thing is that what is the value of this S d? What is the flame displacement speed. Now if it was a planner laminar premixed flame then this S d would have been the exact value of the planner laminar flame speed that we had obtained. So, if we remember that we obtain this value f 0 square is equal to lambda by c p times p c I mean I am not go into that it was a f 0 square is equal to lambda by c p times b c times e to the power of minus of arrhenius number divided by lewis number time times lewis number divided by zeldovich number squared divided by zeldovich number. So, this was this we that was a expression for the planner laminar flame speed.

And we have obtained expressions for we have obtained experimentally or using computations we can obtain. This values for different for more complex situations where we consider detailed chemistry. So, that is this is this one can obtain a value of S l zero, but this S d this displacement flame speed of a convoluted flame structure like this in a turbulent flow field which is of interest is not equal to S l 0. Because of course, you can see that the flame is not planner and be the fact is that this there is there can be a curvature for example, you see here there is a curvature and of course, because you have there can be flow on non uniformities also.

So, when you have these 2 things then what you have seen already that there is a flame stretch. And what this flame stretch does? What this flame stretch does especially when the lewis number is non unity is that it causes a different it causes the differential diffusion of thermal flux of heat flux and the scalar diffusion. And So, that the scalar flux the species flux which carries essentially the enthalpy of formation as well as the thermal flux these are not matched. As a result of that the local temperature changes and when the local temperature changes the local flame speed also changes.

So, as a result of this when a when a flame is stretched and the lewis number is not equal to one then depending on the sign of stretch whether it is positive or negative. And depending on whether the lewis number is greater or greater than 1 or less than 1 the local flame speeds can change. So, this is a very important thing. So, when the local flame speed change of course, you see that your displacement flame speed is not equal to the planner laminar flame speed. And you need to basically have models or basically have to have some database by which you can have this local displacement flame speed.

So of course, you see that in the G equation that there are 2 things of course, that you have the there are 2 things actually you have the transient term you have the convection term and you have this propagation term. So, this is the transient term this is a convection term and this is the propagation term. So, there are 2 unit is which are required. So, one unit is of course, one input 2 inputs that are required. So, one input is of course, you have to specify the local fluid velocity. So, this can be obtained by solve solving either the continuity on the momentum equation simultaneously or if you have a solved u field you can put that, but of course, then you cannot have thing that when the where you should your u should change.

So, it is better to basically solve this u and this coupled with the momentum equation. And then you can essentially then you can have an equation then you need an input for the displacement flame speed also. So, these 2 inputs are required. So, this can be obtained from the navier stokes equation and this can be modeled. So, this S d can be essentially modeled. So, how do you model it?

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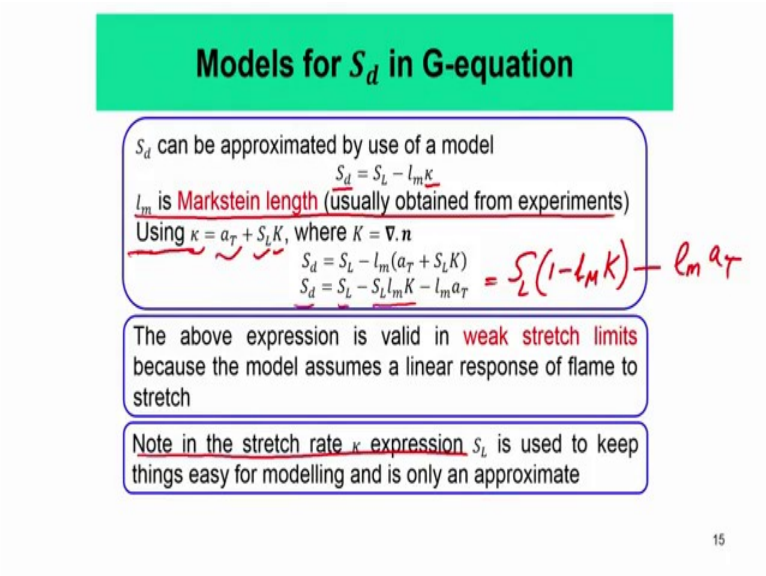

So, you can basically model it by using this linear stretch model. It there can be numerous complicated models also, but basically it assumes that the local flame structure is that are same as that of a laminar flame.

So, the you can say that this local flame speed S d is essential is equal to S l, which is the planner laminar flame speed minuses constant called the proportionality constant call the markstein length times curvature time stretch rate sorry. So, this is stretch rate and essentially this is the markstein length usually which is obtained from experiments or simulations. And this curvature this stretch rate sorry this kappa the stretch rate is essentially contains this tangential strain rate which arises from the non uniformity of the flow velocities along the tangential plane of the flame surface, and times the planar lamina flame speed S l, here the there is this is a essentially a model and times the curvature. This is the k is the k is essentially the curvature ok.

So, this is a model one model that you can use. So, once you plug this in what you get is essentially that that a S l minus this thing one you once you plug this in you essentially, get S d is equal to S l minus S l well minus l m k minus l m T which can be essentially, we can write it as S l times one minus l m kappa by the curvature minus l m times a T there are different forms available actually this is little more complex it is actually one can find is f square log f square is equal to minus 2 sigma where that is the stretch. But let us not go into that in this in this simplified model. So, the above expression is actually valid for weak stretch limit is because the model assumes a linear response of limit to stretch.

And the we have to note that the stretch rate expression we note that the stretch rate expression that is this kappa in S l is used to keep things easy for modeling and it is the only an approximation. So, this is the thing. And So, you see that we have obtained the G equation and which needs basically 2 things one is the flow velocity and one is the local flame speed. Local displacement flame speed the flow velocity can be obtained from the navier stokes equation and whereas, this S d equation is can be modeled. And these there are different techniques to solve the G equation, but there are actually very specialized techniques to solve the G equation.

But the one has to be very careful with the solution there are many intricate things and we can refer any levels at book if you want to solve this, but this gives you a very nice idea of how to basically have an idea of the overall flame structure overall complex flame structure in turbulent flows. And the next step we will take up this progress variable approach where we will look into the Bray-Moss-Libby model.

So, till then thank you.