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Reacting turbulent flows VII

So then, we have to see that whether this gradient transport assumption still holds or not. And for that we will introduce a very simplified reaction scheme where we will not consider the all this linear this exponential form etcetera, will just and we will leave in neglect essentially.

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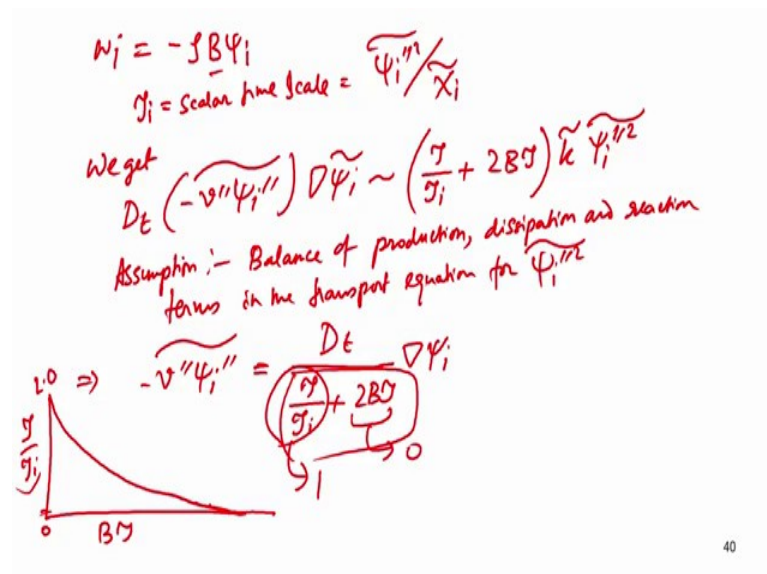
The exponential form and just say that for dependence on temperature we just consider a very simplified reaction, where this thing is shown by is given by react to the production and consumption (Refer Time: 00:42) species is given by minus rho times b times psi I.

So, this linear reaction scheme where we have just totally ignored the exponential part will see that, what how we can show that this is done does not hold. And then of course, if you go back to that the ti that is the scalar time scale, which is psi I double prime square average by the scalar dissipation rate. And if we know set this production equal to dissipation equal to this covariance of velocity fluctuation and scalar fluctuations we get. Of course, we get that 1 plus some additional contributions due to the due to the be since your chemical times scale now becomes important.

So, here we have assume basically the assumption is that balance of production dissipation and reaction terms in the transport evolution equation of so then this gives you get the dt, but you get additional contributions from this. So, and actually in this case you will see that your T by tau, i with b tilde varies like this. So, b is essentially comes from this your reaction term this is 2.0.

So, this is b tilde b tilde equal to 0. So, I will see that that when does the gradient transport assumption hold, if the gradient transfer assumption will hold, when this is equal to 0. And this is equal to one or rather this is equal to is equal 2 this thing. So, so this will hold only then, but you see that the problem is that when b tilde becomes large this thing becomes very, very small.

So, then when these 2 are small that is when these 2 are very small, then this equation when this when this is small then this becomes big and then this becomes small then this becomes big. So, these 2 becoming simultaneously becoming equal to one this quizzed does not really happen. So, as a result of this the gradient transfer assumption fails in most cases when you try to apply this for the reactive scalar.

So, basically and you see that as what happens is that as b tilde increases this T by tau I becomes goes to goes to 0. So, as this goes to 0 of course, but then this then this goes to become small and this where the smaller the quantity the far and more far and far is being deviated this assumption is will gets deviated from the basic gradient transfer assumption. And physically this results from the change of the scalar fluctuations that is brought about by the chemical reactions

So, chemical reactions actually what it does is the chemical reactions consume the scalar. So, when there are consume the scale the local scale of fluctuation reduces because the scalar can go to the value can go to 0 or it can reduce. It can be reduced as well, but it can go to it can it can change from the actual scalar value that was in place. So, that chemical reaction introduces scalar fluctuations and as a result of that this gradient transporter, this there is a result of the scale of fluctuations are not independent of the reactions the reactions contribute to the scale of fluctuation they can reduce it and as a result of that the gradient transport assumption for non reactive scalars may not hold for the reactive scalars.

So this was the, here of course, this will be this will be favre average. So, this was the thing. So, gradient transport assumption is a problematic thing for a for a for combustion and you will see that sometimes is in combustion we assume that gradient transport or counter gradient transport as a situation may be.

So, next what will do is that next we will go into some practical modeling approaches that is used for turbulent combustion, and that one uses to solve large situations as it using this. So, these approaches are typically implemented in commercial codes like fluent etcetera and it is fluent. So, we will just go were this to see that how by introducing simple assumptions, we can have a reasonablic rather crude closure for the for the source for the mean source terms.

So, that will proceed in the in the following. So, for that now; so do that closer of course, you will go into better closures will go into more fine models like the transported pdf model. And the and the conditional moment closure models the g equation model and the flamelet models in the later non premix flamelet in premix flamelet process so, but to give you a just a flavor of how the commercial codes work and how this modeling for the main source term is done.

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We will go look into this class of like the eddy break up and eddy dissipation models.

Now, these are from spelding: spalding introduced the eddy break up model in 1971. And this is a very solid I mean unlike the eddy dissipation models, this is a very solid physical reasoning. So, he said that that the turbulent mixing can be viewed as a cascade process and this mixing process is essentially is basically the most important factor that that drives that drives the reactions we can assume that the reactions are so fast that the whole process of turbulent combustion to be essentially mixing controlled.

So, essentially the reaction rate in turbulent combustion then we do not have to consider the individual reaction rates, whether the in turbulent combustion the average reaction rate would be have in essentially governed by the average mixing rate. So, that is the idea. So, he says that the turbulent that is the eddy break up model says that the turbulent mean reaction rate is governed by the mixing, as mixing is the rate determining process that is the mixing time scales are much larger than the reaction time scales which can be considered as a resulting instantaneously or rather infinitely fast reactions and essentially the whole process is governed by turbulent mixing. So, the average reaction rate is essentially the mixing rate.

So, the turbulent mean reaction rate of products is given by is equal to rho times a C EBU which is a constant times epsilon by K times Yp to the power of half. You see this thing rather if you if you go back what was the definition of over scalar dissipation rate what was the modeling for our scalar dissipation rate.

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So, this was the modeling for over scalar dissipation rate right, as you see here this thing is looks very close to that.

So, essentially the scalar dissipation rate has a has a unit of 1 per second, and we here this is also when you this does not have an unit. So, essentially when it is epsilon by K this essentially gives you time scale because you see epsilon is a as a time you need of meter square per power second cube, and K has a unit of meter square per second, so epsilon by K itself as a unit of 1 per second. So, this gives you a time scale itself and say they say is saying that this epsilon by K times the Yp fluctuation that is the variance of y product fluctuation variance of the Yp prime is essentially this essentially gives you a mixing time times this dissipation rate by turbulent kinetic energy by turbulent kinetic energy, is essentially gives you a mixing a time scale and inverse of a mixing time scale and that is essentially proportional to the mean formation of the mean is equal to the mean reaction rate of the mean consumption or mean production rate of the products.

So, that is what he is saying and of course, Yp double prime. This one is the variance of Yp double prime of essentially a variance of Yp, whereas, Yp is given by mean Yp plus Yp double prime. So, this is not favre average. So, we see the what I saying is that this epsilon by K times Yp double prime square. This whole thing is essentially kind of though this thing has a scalar dissipation it as is essentially the scalar dissipation out of the products and that is essentially proportional to one by time.

This is slightly different also because scalar dissipation it has a squared here which is not here. So, this is what the eddy break up model is it says that the essentially this the mean reaction or the mean production rate is essentially equal to the mean mixing of the mean mixing rate and that is essentially inverse of the mean mixing time scale.

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$$\underline{EDM}$$
$$\langle Y_p''^2 \rangle^{\frac{1}{2}} \sim \langle Y_p \rangle$$
$$\nu_F' F + \nu_o' O \rightarrow \nu_p'' P$$
$$\langle w_F \rangle = \langle \rho \rangle A \langle Y_F \rangle \frac{\varepsilon}{k}$$
$$\langle w_{O_2} \rangle = \langle \rho \rangle A \frac{\langle Y_{O_2} \rangle}{\nu} \frac{\varepsilon}{k}$$
$$\langle w_P \rangle = \langle \rho \rangle \frac{A \cdot B}{1+\nu} \cdot \langle Y_P \rangle \frac{\varepsilon}{k}$$

So, that it is now there is another model of the eddy dissipation model, and this was born by Magnusson and he replaced this Yp double prime they replace this Yp double prime whole square half with the mean mass fraction.

that is in the mean Yp and they define this things if the if they consider. So, that if they consider reaction like a single step reaction plus Yo going to Yp double prime p. So, then this mean of the consumption rate of the fuel is essentially equal to mean rho times a is a modeling constant times yf times epsilon by K and this mean consumption rate of o 2 is essentially equal to mean the density times a times mean Yo 2 by nu times epsilon by k.

And mean wp is essentially equal to mean density times a times b by 1 plus nu another modeling constant and Yp times epsilon by k. So, this is the eddy dissipation model, which actually now though it is does not have much physical meaning it actually includes this reactions through this through this one step reaction mechanism. So, this is the eddy break up and the eddy dissipation models, and then there is one model which is the eddy dissipation concept.

Now, before that we can also use the detail finite rate chemistry, and write that is of this these reactions as a function of temperature mean temperature. So, then of course, it does not introduce any models, but then the problem is that then you where then you basically miss out the contribution that can come from the fluctuations of temperature, so then that and then as well as the scalar fluctuation. So, that is the downside.

So, the eddy dissipation concept is essentially the eddy dissipation concept is essentially this extension of the edn, and is contains that the detailed reaction it can account for the detailed reaction kinetics though this is also good and does not always have very solid physical arguments.

So, the assumption is that that reactions happen in fine scales which is true to some extent reactions happen on small scales. And the small scale is essentially this we represent we represent by star which is the fine scale. And the reaction volume the reaction length scale is given by zeta star this is the constant. It is given by this new kinematic viscosity times the favre average dissipation rate divided by the favre average turbulent kinetic energy squared to the power of one forth for dimensionality. And the this is the volume of the scales is given by of course, this cube.

And the reaction rates of course, are given by determined by the Arrhenius reactions. Arrhenius expressions and the time scales that is up to how much time the reaction will proceed is given by tau star is equal to c tau which is a model constant, nu by to the power of half. So, this is the time scale for the reaction. And of course, to find out the boundary condition or the initial conditions for the reactions on small scales the assumption is that pressure is equal to constant and T and Yi is of course, defined in a given cell in a computational cell. That is our initial T and Yi and the very reaction time that is the how much time you will integrate up to these reactions is given by tau star and

the integration is performed by some routines with I sat because this can lead to very stiff differential equations.

And the model for the source term, which is the most important thing that goes into the governing equations it was given by s Si tilde squared time scale minus times, Yi tilde minus y tilde. So, it is a mass fraction. So, this guy is essentially the mass fraction on small scales of species, i after reaction time tau i star. So, by this model we can essentially go into so we can put this inside our source term for this governing equation for the reactive scalars.

Whereas this thing can be always obtained by integration of the Arrhenius reaction rate Arrhenius type reaction rate constants etcetera, and those detailed things and we integrate essentially up to tau star as you said. So, by this we can include the detail chemistry into the into this source term in the in the models, but of course, the physical reasoning is not very robust and this is still is still accrued and the first order model. And later in later classes we will see that how we can revise those models and what different techniques can be used for modeling turbulent combustion.

So, in this class we have essentially have started looking into this, what we have done is we essentially looked into the density varying flows and introduced a new concept called favre averaging. And then with the favre averaging we derived the different averaged continuity equation derived the average momentum equation. We have showed what is the that we can now close those a momentum equation using those momentum equation when you do a when you introduce favre variables and you do an averaging you basically encounter unclose terms in the form of Reynolds stresses. We showed that how Reynolds stresses can be closed with the essentially with the help of eddy viscosities, and which introduces this favre average turbulent kinetic energy and favre averaged kinetic energy dissipation rate.

And for that we can have a governing equation of turbulent kinetic energy favre average turbulent kinetic energy and we can have a governing equation for turbulent dissipation rate, the turbulent kinetic energy governing equation can be obtained rigorously the dissipation rate equation is basically kind of ad hoc, but with that we can model simple force like jet flows etcetera. Next we considered because you are consider in turbulent combustion we consider basically this different kind of reactive scalars, where we put

that species and temperature into one vector equation, and we wrote one general equation and we saw that when we averaged that this where the closure problem arises.

So, the closure problems similar to the similar to the equation for the for the for momentum the closure problem arises in species in averaging the species equations also the biggest closure problem is of course, the modeling the source terms and here we have shown that the more source terms can be more releasing this eddy break up work the average source term can be. And of course, before that which showed that the average source term for the temperature equation which is the heat release rate equation cannot be written just in terms of average temperature because it also includes the contribution from temperature fluctuations.

So, that is the biggest problem of this kind of moment methods and then here we show that this weaker this, if we just forget about that we can use this eddy break up an eddy dissipation and eddy dissipation modeling an eddy dissipation concepts to have kind of like ad hoc closure for this source terms, of course, the eddy break up model is physically sound in the sense that it says that the reactions are too fast and as a result your average reaction rate in turbulence is essentially governed by the average mixing rate.

So, that is the that is the argument whereas, of course, one can use a finite rate chemistry also which is used in commercial softwares, but then there you do not introduce any turbulence chemistry interaction and they basically ignore the effect of this temperature fluctuations. So, that is one penalty one has to pay and then will looked into the there are other problems also where the turbulent transport term closure also has a problem. And we showed that how for nonreactive closure nonreactive species or nonreactive scalars we can use a gradient transport assumption for that that gradient transport assumption is justified, but even you for a simple reaction where the reaction rate depends linearly on the species without everything considering the exponential terms Arrhenius terms, we showed that that can lead to a lot of problems in closures and that does not hold under all circumstances.

So, here we have just started looking into turbulent combustion, we have looked into basically basic models and basic problems of turbulent combustion the foremost being the closure of the source terms the which arises due to the Arrhenius, once again due to

the Arrhenius dependence of the reaction rate the reaction rate is Arrheniusly dependant temperature that is the reaction rate is e to the power of minus ea by rt. So, that creates the problem and that is the biggest one of the biggest challenges in modeling turbulent combustion, but then modeling must be required because in if you have to model engines where turbulent combustion happens in the combustor, you cannot use direct numerical simulations where we essentially solve from the large scales to the small scales you have to solve for the averaged quantities. And when we see saw that when we try to define derive the governing equation for the averaged quantities we face different kinds of closure problems and which we try to circumvent in different forms.

So, next class we look into turbulent non premix flames and turbulent premix flames and the modeling approaches for that. So, till then goodbye.