Combustion in Air Breathing Aero Engines

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Lecture - 36
Reacting turbulent flows V

Hello, welcome back. Moment: why moment because you see the first moment is essentially the mean the second moment is the variance. So, essential essentially the goal of this work is to have very good as possible or as realistic as possible governing equations for the moment of the different flow and the combustion properties of the different flow properties. So, we want to have governing equations for the first moment of velocity, which are essentially the rans equations in favre average form. We have a now we have want to have a moment of mean equation, or governing equation for mean species mass fraction or we have want to have a governing equation for mean temperature.

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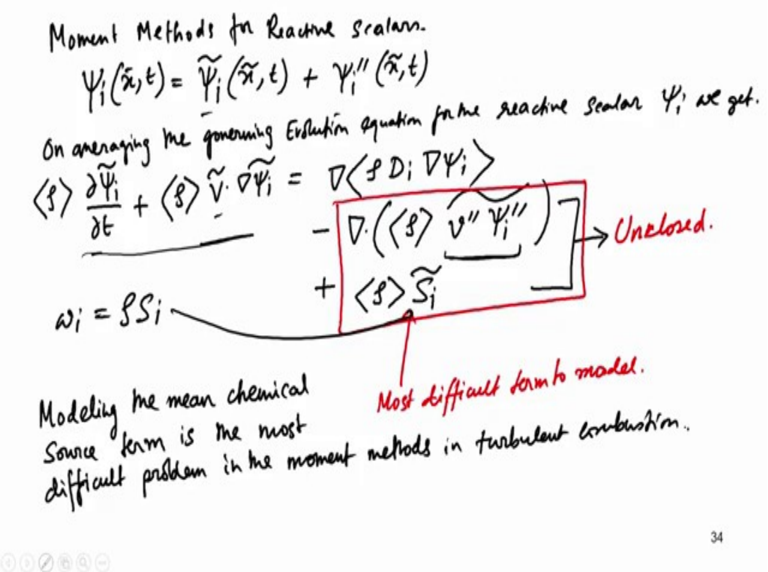

Moment Methods for Reactive scalars.

$$\psi_i(\bar{x},t) = \tilde{\psi}_i(\bar{x},t) + \psi_i''(\bar{x},t)$$

On averaging the governing Evolution equation for the reactive Scalar ψ_i we get.

$$\langle\rho\rangle \frac{\partial \tilde{\psi}_i}{\partial t} + \langle\rho\rangle \tilde{\underline{v}} \cdot \nabla\tilde{\psi}_i = \nabla\langle\rho D_i \nabla\psi_i\rangle \boxed{- \nabla\cdot\left(\langle\rho\rangle \widetilde{\underline{v''\psi_i''}}\right) + \langle\rho\rangle \tilde{S}_i} \rightarrow \text{Unclosed.}$$

$$\omega_i = \rho S_i$$

Most difficult term to model.

Modeling the mean chemical Source term is the most difficult problem in the moment methods in turbulent combustion.

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So, that is why this is called the moment, moment methods for reactive scalars. So, once again what we can do is that to moment for moment methods what we can do is that we can just decompose this reactive scalar variable into the favre averaged variable and the fluctuation term. This is the favre average term this is the fluctuation term. Now if you average the previous equation in this manner. And we introduce this favre variables what we will get is the following on averaging the we get mean density temporal term convective term.

So, here this mean the favre averaged reactive scalar variable is really transported convicted by the mean velocity. And we have the mean velocity mean density sitting in front and we have this thing. This is the species diffusion term or the reactive scalar diffusion term which contains which is diffusion as well as thermal diffusion minus. So, this term may or may not important and this term does not really haves a much of a problem as such, and now the fun starts what is the problem of averaging the reactive scalar will start here.

So, this is one term. You see similar to Reynolds systems it is a covariance of the velocity fluctuation and the reactive scalar fluctuation this is not closed you have a governing equation or now you can even assume that we have a governing equations for v tilde because that comes from the momentum equation, but you do not have a governing equation for this. As we average once again as we average psi tilde you a land up with this equation for the covariance of v prime and psi prime, but this is not all.

The second problem is this one. Averaging the source term that is, it can be either the average of the heat release rate term or the average of the species production and consumption rate. Both of these 2 terms are unclosed and require closure and especially in context of combustion, this is the most difficult term to model. This is the most difficult term to model.

So, we can write down. So, what is rho Si essential s is nothing, but the density times a source term. So, that is what this Si is so averaging of that how do you do. An averaging of that and why it is problematic because we will see soon that it involves lot it involves the it is also a non-linear term it involves the and it involves like coupling between species and temperature through this through this Arrhenius rate.

So, how do average that that is actually the biggest problem. So, write it down itself because it is so important that modeling the mean chemical source term is the most difficult problem in the moment methods in turbulent combustion. Where is the difficulty? The difficulty will see soon. So, if these terms were closed now you have a governing equation which describes the mean velocity and we have a governing equations which describes the mean scalar.

So, you have a governing equation which describes essentially mean species mass fractions and as well as the mean temperature. So, those mean values you can at a point we can consider those to be available. So, mean species mass fraction is available mean temperature is available will that be enough for us to obtain the mean of this of this of this chemical source term the answer is no. Why it is no? We will see.

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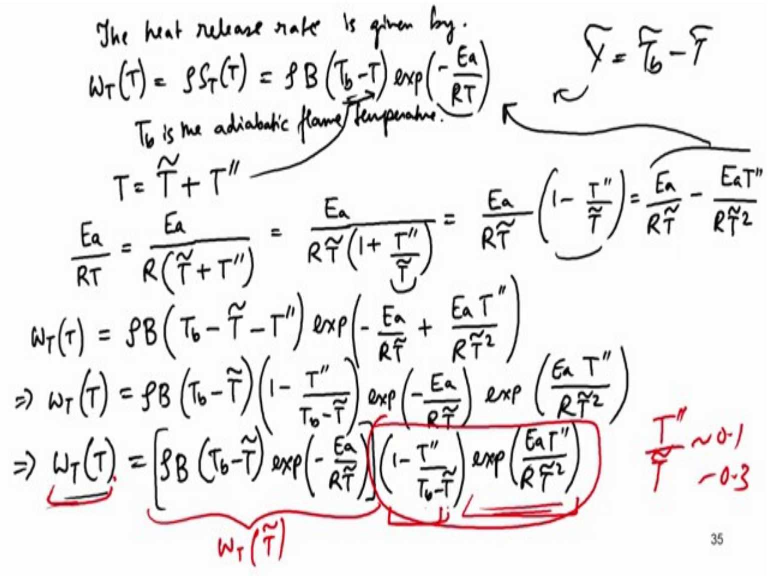

The heat release rate is given by.

$$\omega_T(T) = \rho S_T(T) = \rho B (T_b - T) \exp\left(-\frac{E_a}{RT}\right)$$

T_b is the adiabatic flame temperature.

$$\tilde{Y} = \tilde{T}_b - \tilde{T}$$

$$T = \tilde{T} + T''$$

$$\frac{E_a}{RT} = \frac{E_a}{R(\tilde{T} + T'')} = \frac{E_a}{R\tilde{T}\left(1 + \frac{T''}{\tilde{T}}\right)} = \frac{E_a}{R\tilde{T}}\left(1 - \frac{T''}{\tilde{T}}\right) = \frac{E_a}{R\tilde{T}} - \frac{E_a T''}{R\tilde{T}^2}$$

$$\omega_T(T) = \rho B\left(T_b - \tilde{T} - T''\right) \exp\left(-\frac{E_a}{R\tilde{T}} + \frac{E_a T''}{R\tilde{T}^2}\right)$$

$$\Rightarrow \omega_T(T) = \rho B\left(T_b - \tilde{T}\right)\left(1 - \frac{T''}{T_b - \tilde{T}}\right) \exp\left(-\frac{E_a}{R\tilde{T}}\right) \exp\left(\frac{E_a T''}{R\tilde{T}^2}\right)$$

$$\Rightarrow \underline{\omega_T(T)} = \underbrace{\left[\rho B\left(T_b - \tilde{T}\right) \exp\left(-\frac{E_a}{R\tilde{T}}\right)\right]}_{\omega_T(\tilde{T})} \boxed{\left(1 - \frac{T''}{T_b - \tilde{T}}\right) \exp\left(\frac{E_a T''}{R\tilde{T}^2}\right)}$$

$$\frac{T''}{\tilde{T}} \sim 0.1 - 0.3$$

Now, so, considered for a very lean mixture; consider the heat release rate is given by source term is equal to rho times B times y, but if you remember for premix combustion we had something like y tilde is equal to Tb tilde minus T tilde. So, now, to just to remove the complexity when you introduce y tilde it essentially adds to the complexity because now you have to deal with 2 different variables; so y as well as t. So, let us assume that our heat released rate term is now given by introduction of this, it is now given exclusively in terms of temperature and it is this form, whereas Tb is the adiabatic flame temperature.

Now this is very important this analysis very important. Because this really tells you that why is turbulent combustion difficult, why is closing this chemical source term or the obtaining the first moment of this chemical source term is so difficult this is the this, analysis clearly show will show you that. So, what the how do you show that? So, now, we can write T is essentially T double T tilde plus T double prime.

So, this is the once again T with the compose T into a favre averaged variable T tilde plus the fluctuating variable T prime double prime. So, now, we can consider this part itself and these part is given by Ea by RT. So, we can write Ea by RT as Ea divided by r times T tilde plus T double tilde. And this can be written as Ea by RT tilde 1 plus, T double tilde y T tilde this you can write and then using binomial expansion an approximation for that we can write this as when this quantity is smaller than 1. Of course, T double prime tilde in turbulent flows it is normal to expect the air temperature would can be very high 500, 600 up to thousand Kelvin 1000, 2000 Kelvin.

And the temperature fluctuations are definitely smaller than the mean temperature, we can consider that to be true and if not so this is needs even further complexity, but at least let us considers that. So, we can and these term this I can be given by Ea by RT tilde minus Ea T double prime by RT tilde square. So, now, if you put this into this equation, and write this also put your this thing in this is also we get this is just algebrium.

See what you are trying. What you are trying to do here, you are trying to write a the heat list in terms of average temperature, because out of favre average temperature is a function of favre average temperature. Because once we have that your favre average temperature can be there is the governing equation for favre average temperature through that equation for the n plus 1 at equation for the average reactive scalar equation. So, if we can write it in terms of that then this equation will be closed. If you can write it now say we exclusive in terms of that, but can we do that.

So, then we get rho b you can write this Tb minus T tilde times exponential of minus Ea by RT tilde. We can separate this out times 1 minus T double tilde times Tb minus T tilde times into the power of Ea by T double tilde times RT tilde square. So, this whole part you can write this essentially as has T tilde as the heat release rate at the average temperature this whole thing. So, this is the heat release rate term at a particular temperature and that is equal to the heat release rate term at the average temperature, but that is not enough you have got more terms which are these.

So, these term itself might be this term, itself might be the heat release rate or average temperature, but this is as what terms in this can be positive or negative, but this is definitely positive right. And this is actually also positive because 1 minus T double prime divided by this is definitely smaller than Tb the adiabatic flame temperature and this is much smaller. So, this is always less than one. So, this is this is positive of course, this is positive. So, the heat release rate is always positive, this term is always positive I am sorry. So, this term is always positive this term is always positive and, but this term can be greater than or less than one depending on the situation.

But this term is strongly it can contribute heavily to this, this to this actual heat release rate what the heat release rate in terms of the mean temperature. And this term is nontrivial because you see it has got an exponential on that. So, it is an exponential of T double prime divided by T prime T tilde whereas, T double prime by T tilde can be of the order of 0.1 to 0.3. So, these creates a major addition to the to these things.

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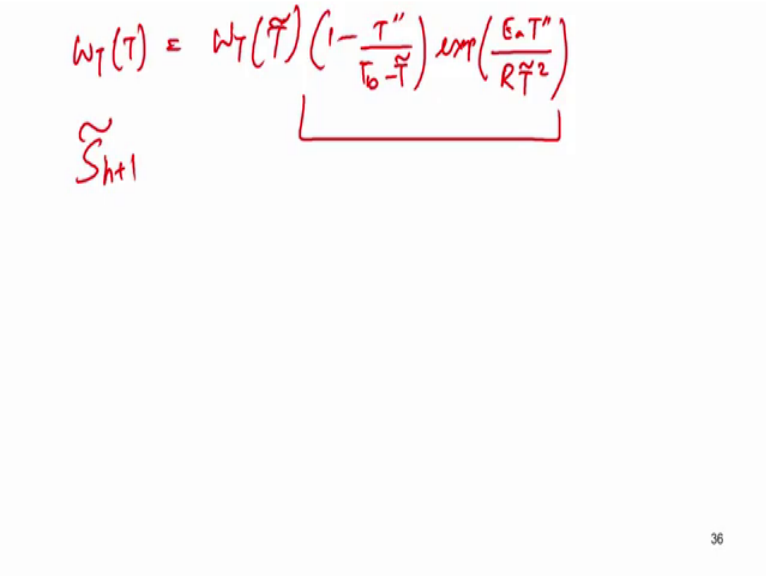

$$\omega_T(T) = \omega_T(\tilde{T})\underbrace{\left(1-\frac{T''}{T_b-\tilde{T}}\right)\exp\left(\frac{E_a T''}{R\tilde{T}^2}\right)}_{}$$

$$\tilde{S}_{n+1}$$

So, to again right that we can write omega T, at T is not equal to omega T at T tilde that is even if you T know T tilde the and the most important thing is that even if you know T tilde you do not know the mean heat release rate you do not know the heat release rate at that at a at a particular point, even if you know T tilde because there are fluctuations also contribute to the heat release rate. So, this is essentially equal to 1 minus T double prime divided by Tb minus T tilde times exponential of Ea by T double prime RT tilde square.

So, this additional contribution makes this source term or this mean Si term that arrived. This is for the n plus 1 a term unclosed. So and there is no direct model for this so how do you incorporate this in your analysis? So even if you know mean T mean temperature at a particular point that does not give you the mean heat release rate and as a result just doing the moment of methods will not give you the mean temperature itself because the mean temperature is once again coupled to the mean heat release rate. So, this is the problem of turbulent combustion that the mean that the source term because of it is exponential behavior is unclosed and this temperature fluctuations causes a non negligible and non negligible contribution on the heat release rate term when it is written in terms of the mean temperature.

So, this is a major problem. And if you go back to this so this was this is the problem we know understand that if we consider this averaging this average transport equation for the for the for the reactive scalar this is one point where the problem comes, but we will now next take up that, why this is also problematic and we will see what can be done to alleviate these problems and then we will take up simplified models which can be used to basically model the source terms and model these terms. So, that is will be taken up in the in the next class. So, this is the problem for the closing the heat release rate term in terms of mean temperature.

So, next will take up the problem of basically the turbulent transport the will take up the dissipation and scalar transport of the non-reacting and linearly reacting species, so till then goodbye.