Combustion in Air Breathing Aero Engines
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Lecture - 35
Reacting turbulent flows IV

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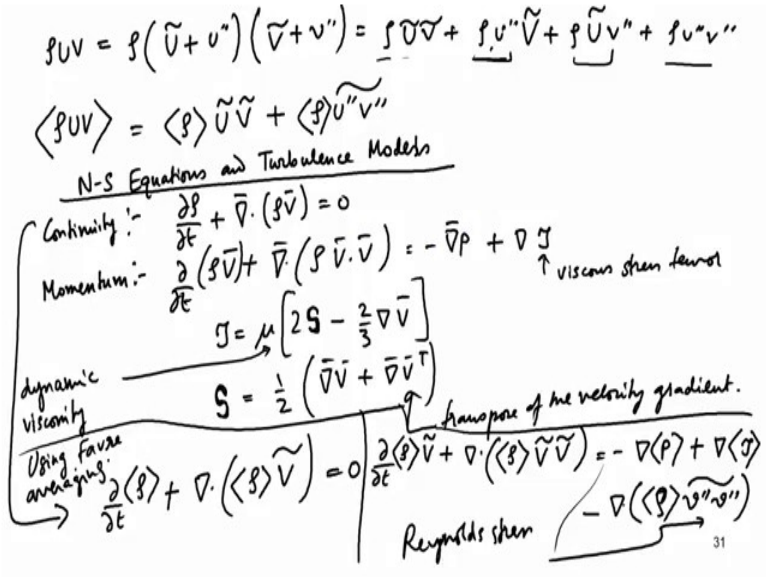

Now, one of the approaches that the closures are introduced in this is by the introduction of the concept of eddy viscosity. Now I am using I am doing all these things because of course, if you want to do turbulent combustion suppose you want to solve a turbulent combustion inside an engine. You have to solve the flow also unlike in the in the laminar flames, where we were discussing like we were approximating the enter flow by that pressure gradient is small and the by the contrary the rho u is equal to rho u. So, rho 1 e 1 is equal to rho 2 u 2.

And here of course, you have to consider the detail flow and to model the detail flow this Reynolds average Navier-Stokes equation on the favre average Navier-Stokes equation is one of the one of the possibilities. And of course, we will introduce the combustion models, but at least to for the first thing (Refer Time: 00:57) that you need to solve the average continuity equation and the averaged momentum equation. So, that is why we are doing all this things.

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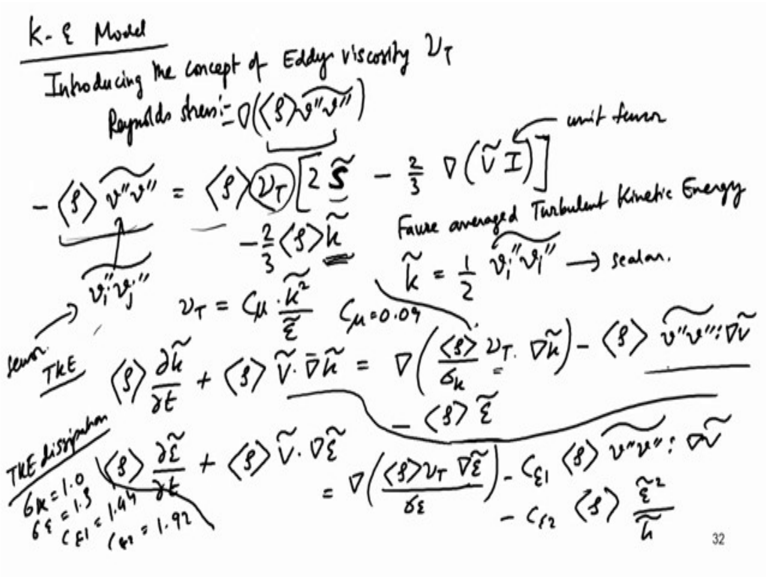

So, this is here essentially solution of the momentum equation at the average momentum equation, you cannot simplify it and you have to solve it along with the your combustion equations. So, that is one important thing in turbulence that you should keep in mind.

So, here we introduced this k epsilon model, where we introduced the concept of introducing the concept of eddy viscosity. This Reynolds stress terms which was the inside of the words this so this Reynolds stress terms to have this, and of course, we had a gradient in front like that now if we call this as Reynolds stress then we model that by the following. Nu t this is just a model. Under the so here the strain tensor is essentially favre averaged, this I is an unit tensor. So, this is how you introduce this thing minus there is a 2 third whereas, k is the turbulent kinetic energy favre averaged.

So, this term is essentially equal to you can write it as v i v j double prime double prime this thing. So, here it is written in this simplified form, but are actually it includes both the cross terms also. So, essentially this is the tensor this left hand side whereas, k is actually a scalar because of this thing because k is defined as that is the favre average. Turbulent kinetic energy is defined as this way. Favre average turbulent kinetic energy is defined as k tilde is equal to half VI double prime VI double to tilde.

So, this is a scalar defined that a point, whereas, this is a tensor of course, scalar is also 0 without a tensor. So, anyways, but is this is a full it is a second order tensor with the 9 elements inside it this Reynolds stress. Now this we can define this nu t which is essentially a kinematic eddy viscosity. So, you see what we have done here we have taken this Reynolds stress term and say that it is essentially directly proportional to the favre average strain rate tensor and this proportionality constant is essentially this thing call the when nu t is essentially which is the eddy viscosity.

So, this is the important concept of eddy viscosity and that is needed because use of found that when we do the when we introduce the favre average in the Navier-Stokes equation. We are basically left with this term which does not have any closure. So, of course, this appears elsewhere show you and in the when the in this thing itself also. So, as you see here. So, essentially you will have strain rate inside this so the and if this term is essentially proportional to the this what is inside is essential motion to the nu t times the standard tensor, then this can be coupled with this term which contains the normal viscosity. And then we can add the effective viscosity we can add the normal kinematic viscosity and this eddy viscosity together to basically form a effective viscosity, but typically in turbulent flows I mean the effect of viscosity in normal viscosity in this 2 will be much smaller than the turbulent viscosity turbulent eddy viscosity.

So, this is the how the how the modeling is introduced, this is the first modeling that we introduce here that is in the k epsilon model and we have not gone into the k epsilon model. Yet we will come to that because this is still will see how to basically do this because you see that we have still this term that is a k what equation, which we need to basically have a model have an equation for.

So, to close this is so this is the bottom line is that this is still not close because we have got k. So, we need an equation for the governing equation for k tilde that is the favre averaged turbulent kinetic energy, and this will be given by the following now before that we can write down nu t as essentially c mu times k tilde square, by mu epsilon whereas, c mu the value is taken typically to be naught 0.09.

So, that is the thing now. So, of course, you see that we have got then 2 terms in nu t itself you put here and this goes into this whole thing goes into the this average Navier-Stokes equation. So, essentially you get a get 2 more terms that is k and epsilon. So, we need 2 and closure for those and those closures are essentially provided by the k epsilon model because you have k and epsilon here. So, that we have a governing equation of turbulent kinetic energy which can be exactly derived just like the way we derived it in the previous class where for of course, for that was for constant density flows.

So, this is little more complicated, but the basic essence is similar then the left hand side you have the transport of k, this is equal to mean density there also range sigma k, is essentially a model constant times nu t grad k, tilde minus v prime v prime double dashed where all systems contracted with divergence gradient of v tilde minus mean of density times dissipation.

So, essentially this is which was we have is essentially the production term and this is the dissipation term in the turbulent kinetic energy. So, and of course, is you remember in the production term you had this was preceded with the with that Reynolds stress terms and then you have the velocity gradient we have the velocity gradients. So, you get similar things here all right here also.

So, that is the point that is this, this equation that is the kinetic energy equation the kinetic energy transport equation which you essentially need to have a closure for the previous Reynolds stress terms of the eddy viscosity closure; so which you need for this itself. So, for that is it can be essentially an exact equation with some model constants because of the introduction of these things, but this the next equation is that is the that is this epsilon equation of our turbulent or tk dissipation turbulent mean turbulent kinetic energy dissipation rate. This equation this is an essential and ad hoc equation and there is no direct reason direct ab initio you cannot we cannot derive this ab initio and the other this comes from the fact that this dissipation should also look similarly similar to the previous equation.

So, that is this term is equal to divergence of mean density nu t. So, this is once again this is an ad hoc equation which is required for closure and, but this works well in certain flows like turbulent shear flows event density wording to some extent of course, as you see that because the modeling thing has a lot of ad hoc constants, and here you have sigma k is equal to 1.0, these are the different model constants that are typically used in commercial softwares.

So, to summarize as you see that when you will have basically here do a trying to deal with the flow. So, to deal with the flow you need to at least you need to solve 2 equations that is units of the continuity equation and you need to solve the you need to solve the momentum equation. So, in non reacting flows of course, in constant (Refer Time: 11:40) flows in CFD, that that directly do not solve the continuity equation it is comes from the pressure partial equation from the momentum equation itself. And then you can then implement the divergence of v equal to 0 criteria, but in density varying flows typically you solve the continuity as well as the momentum equations; and then in an average form.

nd of course, So, in an average form continuity equation does not have any problem as you saw that is it can be this is the form of the average continued equation with favre variables, favre average variables and then when you introduce the then when you do the same thing for the momentum equation you see that, of course, these Reynolds stress systems emerging from the emerging from this the non-linear or the convective acceleration term. And then you have these emergences of this Reynolds stress terms which are not closed. So, to close them we introduce this model, which is to which is essentially says that this Reynolds stress terms is essentially proportional to the strain rate favre average the favre average Reynolds stress terms is as usually proportional to the strain rate tensor. And that is the favre average Reynolds stress term it is essentially proportional to the to the favre average Reynolds stress favre average strain rate tensor and then that is that proportionality constant is given by something called the eddy viscosity.

now eddy viscosity introduced has of course, this k squared tilde divided by epsilon I which can be defined in this form and then of course, you need to solve for you this k and epsilon are enclosed, and that leads to the introduction of this of this turbulent kinetic energy of this k epsilon model where we have 2 equations for the for the turbulent kinetic energy and where we have one equation for the evolution of turbulent kinetic energy and one equation for the evolution of the turbulent kinetic energy dissipation rate. Of course, if the for the turbulent kinetic energy evolution equation that can be derived in a very systematic manner as we have did previously, which leads to the which shows us the production and the dissipation terms, but then there is no such governing equations for turbulent kinetic energy dissipation rate at least in the in the scope of that the k epsilon model, and then that leads to the introduction of this epsilon model in ad hoc manner and which leads to the these several model constants.

But at least with this frame work I mean it works well for essentially for like a turbulent jet or other shear flows it does not work too well for density wearing a turbulent jets, but especially there are other problems with non reacting flows, but at least this gives as an idea about how we can approach how can I approach a turbulent combustion modeling at least the flow parties can be settler to some extent with this with this rams models, of course, there can be more elaborate models like large eddy simulation etcetera, but that we not to go right now.

So, up to this point we have had very solid you know we have shown you that to model the flow at least you have the continuity, and the and the average momentum equation enough for the to close the average momentum equation you need the where this eddy viscosity closer and for the eddy viscosity closer you need the favre average turbulent kinetic energy equation and the dissipation equation, but at this with this the flow can be flow can be described, of course, you cannot describe the flow totally now still because you do not have know the density and densities. Of course, depending on temperature and that is how essentially the combustion couples with turbulence through essentially this density variation at one level, that it is combustion creates heat release heat relates create increases some temperature reduces density and density goes into your continuity and your momentum equation.

So, this is one level of coupling of course, you will see that this leads to other level of coupling also because it because combustion is essentially leads to consumption of the species and that has other sorts of couplings and to how it couples a temperature. So, that will come in this form, but now at least we have the flow sorted out. So, now, we will go into because this is a turbulent combustion course we will go into essentially will go into looking for we go back to the species and temperature equations.

and we will see how we can tackle those and if those can be tackled in these manners in manners which is similar to the 2 the eddy viscosity type of closures, we will see that these are more problematic it this eddy there are more complexities that arise when you try to apply this kind of closures for the temperature and the species equation, but we will have to deal with that.

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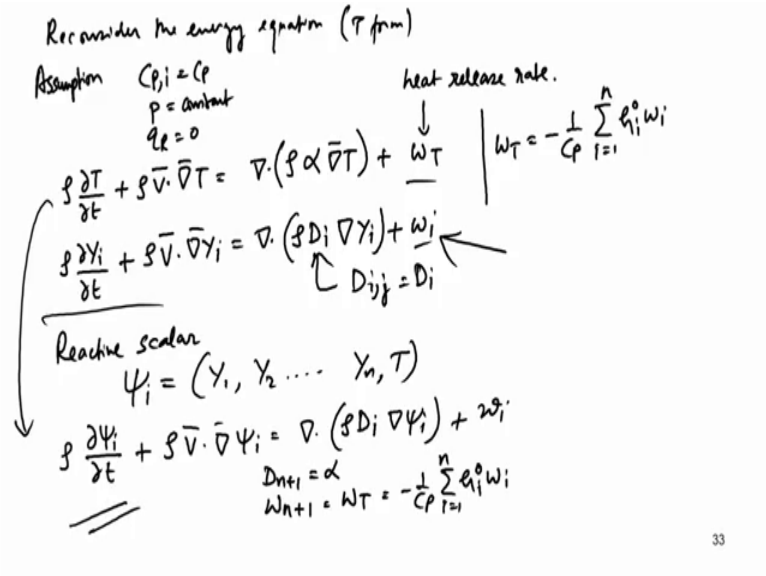

So, let us go into the temperature and the species equations. So, if we reconsider the energy equation in t form that is and with the assumption that that your CPi is equal to CP here p is equal to constant and your q radiation is equal to 0. So, we arrive at a simplified temperature equation which we devised which we obtained previously from the enthalpy equations essentially, left hand side temporal term convection term plus diffusion term thermal diffusivity cause heat released it.

And the heat related is of course, given by number of species enthalpy of formation reaction rate of the concern of the production and destruction of that species. And similarly the species equation is this one. These parts are mainly from we do not follow laws book anymore we are essentially going into peters say book and peters says notes.

So, you can you want to read more about this you can find this in peters is book turbulent combustion, where as we have assumed that d ij is equal to di. And this is of course, the corresponding species production and consumption or consumption rate. Now what we will do is that in the following we will use this term called reactive scalar. So, reactive scalar is nothing, but a vector which contains the mass fraction of all the chemical species and temperature. So, reactive scalar of course, as you see these are not passive scalars because you have got source terms here in our co source same terms in these things.

So, these are not passive scalars this is a reactive scalars and furthermore this also couples with the flow directly because of the density variation. So, this reactive scalar given by psi I equal to y 1 y 2 yn and t. So, there are n plus one reactive scalar. So, this is reactive scalar is essentially the vector. Now based on these equations we can write one governing equation for a reactive scalar which is given by this forum, which is essentially just simplifying these 2 equations in this form. Now for of course, I is equal to one to n Di is ti, but for n plus 1 Di is essentially alpha w n plus 1 or we write it like this . Dn plus 1 is equal to alpha w n plus 1 is equal to wt which is heat release rate minus 1 by c p times summation I is equal to one to n (Refer Time: 20:50) g i naught w I.

now. So, these are our governing equations for the reactive scalar for both the species as well as temperature. So, how can we apply can the question is can we apply this kind of averaging techniques, that we applied for the continuity as well as the momentum equations into these equations of the reactive scalar itself. That was very attractive then we can just only solve for the average flow and of course, if required for the variance flow where variance of these quantities also and then we have a very good understanding of entire things.

so, but it turns out things are not. So, simple as such even in the previous case when you introduce this concept of eddy viscosity that are some limitations itself that does not work too well because you are essentially mapping one tensor, which is this the Reynolds stress tensor to the to the to the strain rate tensor through one scalar which is this nu t. So, there is there are problems. So, that does not work too well, but you will see that, even if we can have a reasonably good description of the flow the problem here are even more fundamental when you try to do the averaging concepts for this, but then we still keep on improving the models and we will see how the models are done.

And we need to have a very good understanding of this, how this model works because this averaging of this averaged form of this continuity averaged from the momentum and average form are the of the of the of these equations of this reactive scalar equation and temperature equation are very much used in the industry, especially if you are a combustor designer or are an engine aero engine designer

So, we really need to do what are the things and what are the models, what are the problems with that models what are the problems with averaging. And how we can improve those kinds of things? So, that is the motivation for how this class. Now let us go into this averaging this is reactive scalars and in different forms. And basically this approach will be called the moment of moment methods favre average scalar.