

Combustion in Air Breathing Aero Engines
Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture – 34
Reacting turbulent flows III

So hello welcome back. So, in the last class we have discussed about the mechanics of how basically large scale eddies or the kinetic energy of the large scale is transported across the different scales and finally, it is dissipated into the smaller scales. But so far we have restricted our discussion to non reacting turbulent flows, and we have discussed a few of the things, but now it is time to move on to the basic topic of interest that is turbulent combustion.

(Refer Slide Time: 00:49)

General Structure of Reacting Turbulent Flows

- Effect of turbulence on combustion: Increases burning intensity through enhanced mixing and increased flame surface area
- Excessive turbulence intensity levels can cause local extinction
- Effects of chemical reactions
 - Heat release
 - Laminarizing: $Re \sim \rho/\mu \sim T^{-(1+\alpha)}$ ($\alpha > 0$)
 - Supplies energy to sustain turbulence
 - Baroclinic torque: Turbulence generation $\rightarrow \nabla(1/\rho) \times \nabla(p)$
 - Flame front instabilities: Turbulence generation

29

So, the general structure of turbulent flows you see when you have turbulence in combustion it has basically two way effect, there is like interaction happens in (Refer Time: 01:01) scales. So, different kinds of things the convection to transport turbulence enhances diffusion and also it affects the reaction the mean reaction rate as we will see later, but to the first order what it does is that it increases the burning intensity to enhance mixing and increased flame surface area. Because when you have flame in turbulence the turbulence essentially stretches and folds the flame at a multitude of length and time scales and the (Refer Time: 01:27) increased and when the (Refer Time: 01:28) is

increased the mass flux and if the mass flux per unit area remains nearly constant, then the overall burning flux essentially increases and as a result of that the turbulent flame speed becomes greater much greater than the laminar flames, but that is for premixed combustion. If on nonpremix combustion of course, also turbulence helps in combustion because it is essentially helps in mixing.

So, turbulence has a positive effect in combustion, but at the same time it can also lead to local extinctions, if the if the stretching due to turbulence is too hard it is to. So, yes as I said that excessive turbulent intensity turbulence intensity that is e' or u_{rms} or $URMS$ can cause small $URMS$ can cause local extinction also.

Now, effects of chemical reaction when you have heat release it has a it kind of have a re laminarizing effect because the Reynolds number is essentially $\rho v d$ by μ , and to the first (Refer Time: 02:23) ρ by μ it has a T to the power of minus 1 plus α dependence whereas, if we say that ρ depends as 1 by T and μ depends as to 2 minus α where α is greater than 0 .

So, with the Reynolds number essentially decreases when you have combustion and, but at the same time it can because of the flow acceleration and the large density gradients it creates it can essentially produce also supply energy under certain circumstances. So, sustain the turbulence for the towards one of the mechanisms can be like the baroclinic torque that which is essentially the cross product of the one by density times the pressure gradient gradient of one by ρ there is 1 by density times the gradient of pressure. So, when this density gradient when density and pressure the density gradient and pressure gradient are misaligned, then you have a this baroclinic torque which essentially produces vorticity and also there are other things like a flame front instabilities which can also help in turbulence generation.

(Refer Slide Time: 03:28)

Favre Averaging

Reynolds decomposition (for incompressible flows)

$$U(x,t) = \underbrace{\langle U(x,t) \rangle}_{\langle u \rangle \equiv 0} + \underbrace{u(x,t)}_{\langle u \rangle}$$

$$\langle (u')^2 \rangle = \langle (U - \langle U \rangle)^2 \rangle = \langle u'^2 \rangle - \langle u \rangle^2 \neq 0$$

Favre averaging (for variable density flows)

$$U(x,t) = \tilde{U}(x,t) + u''(x,t)$$

where, $\tilde{U} = \langle \rho U \rangle / \langle \rho \rangle$ and $\langle \rho u'' \rangle = 0$

$\langle u \rangle \neq \tilde{U}$
 $\tilde{U} = \langle \rho U \rangle$

Favre vs. Reynolds averaging

Reynolds:

$$\langle \rho UV \rangle = \langle \rho \rangle \langle U \rangle \langle V \rangle + \langle \rho \rangle \langle u'v' \rangle + \langle U \rangle \langle \rho'v' \rangle + \langle V \rangle \langle \rho'u' \rangle + \langle \rho'u'v' \rangle$$

Favre:

$$\langle \rho UV \rangle = \langle \rho \rangle \tilde{U} \tilde{V} + \langle \rho u''v'' \rangle$$

But here the main focus of our discussion on turbulent flows will be essentially towards the modelling of the turbulent flows, and for non-premixed and premixed combustion as when we go later will go into different kinds of the mechanics of the turbulent flame interaction also. But one of the biggest challenges in turbulence is or in turbulent combustion is that how can we use reduced order models to basically describe the turbulent combustion. Of course, if you do not want to use a model one can use something like direct numerical simulation. I will go to this one can use direct numerical simulations where essentially you resolve your simulation from the larger scales to the smallest scales of course, the scale separation as you know l by η is essentially proportional to the Reynolds number to the power of $3/4$, and hence as the Reynolds number grows which is very high in practical engines your scale separation is very large; that means, you have to have a very large number of grid points as well as a very large number of time steps to basically resolve all your spatial length scales as well as the time scales of the problem, and that creates prohibitively expensive demand for prohibitively expensive computational power.

So, in practical engines you cannot do DNS on them one can do. So, the basic approach would be to basically do the RANS simulations that is you are basically use the Reynolds average Navier-Stokes equation or Favre average Navier-Stokes equation you will see what; that means, forward is been by Favre averaging you will leave some sort of average Navier-Stokes equation and try to describe the average flow field instead of the

instantaneous flow field. And but as you will see that this also had some problems, but still in the industry this Reynolds average Navier Stokes equation or averaging averaged average resolving the average flow field. So, CFD has been of great interest and that is widely used in the industry.

So, in this class itself you will go into the different how this is done how to use this kind of like average Navier Stokes equation, an average species equation, and average temperature equation how that can be used for describing the a turbulent combustion phenomenon as such and. So, in this class then this next two hours class will essentially focus on this and next two hours model will essentially focus on this RANS how we can use RANS or this average Navier Stokes and average species equation, governing equation is average temperature equations towards basically solution of the flow fields, and then we will move on to the different and then we will move on to the other basically more sufficient modeling approaches.

So, first we need to introduce a different kind of averaging called Favre averaging because just if you average the flow field like we did for previously, that is you just take the arithmetic mean of the flow field or what is ensemble averaging, then we land up in problem in when density varying in flows which involves density variation.

So, for flows which involves density variation, we will introduce a different kind of averaging. But before we go into the different kind of averaging you just recap what we meant by what was the conventional averaging. So, the conventional averaging was done by done towards Reynolds the composition, where we took this flow field instantaneous flow field $U \times t$ and then we averaged it then we averaged it in this way and then this is the mean component and this is a fluctuating component of course, the mean of the fluctuating component was 0 and we defined the variance in this manner.

Now, we introduced a different averaging called Favre averaging. So, what is Favre averaging? Favre averaging means that here we decompose a flow field of a variable density flow that density is not essentially constant you can vary with space and time. So, for those kinds of flows we consider again this velocity at a particular position given by the position vector x vector and time t , and that we decompose as this \tilde{u} plus u'' .

So, how where here you tilde is not just. So, \bar{u} here u mean was just averaging over you mean, but here \bar{U} which is not equal to what you mean is not equal to \bar{U} whereas, \bar{U} is defined in this manner, that is its essentially density waited averaging divided by mean of density. So, this is this thing and when you do that this is the definition of the fluctuation that is mean of the density times you prime is average to when you average that, by the convictional averaging then there average is equal to 0.

So, why do we do this? The reason is that the reason is the following that if you in the momentum equation. So, in the as you know the turbulence the heart of turbulence comes from the non-linear term of the momentum equation, that is the second term on the left hand side right.

So, if we do our Favre average if you do a conventional Reynolds averaging on this non-linear term of the momentum equation which is essentially this of the rather it is a gradient of this, but when we do the averaging because gradients and averaging computes. So, when we do that we will decompose it into this manner, will get this thing when we do a \bar{u} when we do a normal ensemble averaging on the on this term that is ρ times U times V and we average that what we get is that average ρ times average U time is average V and then of course, this terms all these terms we get we essentially get 1, 2, 3, 4, 5 terms and now this goes to 0.

Whereas if you do if you essentially do a Favre averaging what you will get is this thing that is if you do a Favre averaging if we introduce Favre variables, and then you do the average of this same term what you get is mean ρ times U' V' plus ρ' U prime double prime plus V' prime.

It can be easily shown in this manner that is in this manner itself.

So, we just introduce some white pages will need lot of those in this class because I will do this mainly this derivation semantically.

(Refer Slide Time: 10:28)

$$\rho UV = \rho (\tilde{U} + u'') (\tilde{V} + v'') = \rho \tilde{U} \tilde{V} + \rho \tilde{U}'' \tilde{V} + \rho \tilde{U} v'' + \rho u'' v''$$

$$\langle \rho UV \rangle = \langle \rho \rangle \tilde{U} \tilde{V} + \langle \rho \rangle \tilde{U}'' \tilde{V}''$$

N-S Equations and Turbulence Models

Continuity: $\frac{\partial \rho}{\partial t} + \bar{\nabla} \cdot (\rho \bar{\mathbf{v}}) = 0$

Momentum: $\frac{\partial (\rho \bar{\mathbf{v}}) + \bar{\nabla} \cdot (\rho \bar{\mathbf{v}} \cdot \bar{\mathbf{v}})}{\partial t} = -\bar{\nabla} p + \nabla \cdot \mathcal{T}$ ↑ viscous stress tensor

$\mathcal{T} = \mu \left[2\mathbf{S} - \frac{2}{3} \nabla \bar{\mathbf{v}} \right]$

$\mathbf{S} = \frac{1}{2} (\bar{\nabla} \bar{\mathbf{v}} + \bar{\nabla} \bar{\mathbf{v}}^T)$ ↑ transpose of the velocity gradient.

dynamic viscosity

Using Favre averaging: $\frac{\partial \langle \rho \rangle}{\partial t} + \bar{\nabla} \cdot (\langle \rho \rangle \tilde{\mathbf{v}}) = 0$ ↑ Reynolds stress

$\frac{\partial \langle \rho \rangle \tilde{\mathbf{v}} + \bar{\nabla} \cdot (\langle \rho \rangle \tilde{\mathbf{v}} \tilde{\mathbf{v}})}{\partial t} = -\bar{\nabla} \langle p \rangle + \bar{\nabla} \cdot \langle \mathcal{T} \rangle$

$-\bar{\nabla} \cdot (\langle \rho \rangle \tilde{u}'' \tilde{v}'')$

31

So, now if we do this then consider again this variable that is the this rho times u prime v, that we can write as and that is equal to rho u tilde v tilde plus rho u double tilde V tilde plus rho U tilde V double tilde v double prime plus rho u double prime v double prime.

Now, if you take average of this what happens is that of course, you have averaging on this is need not be average because it is always averaged quantity. Now when you take average of this thing when you take average of rho times u prime u double prime then it goes over to 0. Similarly this average of this row and vwm that goes to 0 and this stays. So, the terms that stays is essentially out of the 4 terms only the 2 term stays. So, this is the advantage that is this is the thing that we get and this is of course, the similar to that Reynolds stress term, but it here it is a Favre average rather than normal Reynolds averaged and these are just all right.

So, now we can use this concept of Favre averaging. So, clearly the Favre averaging of course, is advantageous because it just reduced the number of terms from the from much more number of terms in the this non-linear term to this only two terms.

So, essentially the analysis becomes of this Reynolds or the or this Favre or this Reynolds average Navier stokes equation, but with Favre variables becomes much more simpler. So, we will use this Favre averaging for the rest of this class when we deal with

turbulent when the for the entire class of where we will deal with turbulent combustion. So, this is the averaging of choice that is used in turbulent combustion ok.

So, next we go back and apply these things in the Navier stokes equation and then we generate the turbulence models. Now the first of all of course, before we go to the Navier stokes equation we need to go into the continuity equations. So, here we go into the Navier stokes equations and so, first is of course, the continuity we cannot just consider divergence of v equal to 0 because now you have (Refer Time: 14:13) got density variation. So, the equation of that we need is this one this bar is not averaging this is vector then we have the momentum.

So, in the momentum equation you get the this left hand side acceleration term the temporal acceleration convective acceleration which is equal to the minus of the pressure gradient forces, plus divergence of the gradient of the strain rate tensor of this gradient of this viscous stress tensor essentially, we neglect gravity and this needs the closure equation which is given by this. This is the strain rate tensor or as the strain rate tensor is given by this is the transpose of the velocity gradient and ν is the dynamic viscosity here.

Now if you introduce Favre averaging here on this continuity and the momentum equations the equation we get are these. So, this becomes this is wood angular brackets its normal ensemble averaging, it is not Favre averaging class this term p vector. Favre average and later we will not actually use the V vector sign. So, without the over word this means that the Favre averaging on the V vector itself ok

So, this is the Favre averaged continuity equation and the Favre averaged momentum equation is the following. Normal averaging gradient of the mean shear stress tensor minus their we all stress terms, but here it takes a different form because of the Favre averaging. So, these are velocity fluctuations and this is the term.

So, this is the renown stress that appears in the Favre averaging momentum equation of course, here also the problem is similar that is we have an equation a governing equation of V tilde, but you while you introduced the averaging, you encounter this new unclosed terms which is V prime tilde V prime tilde and you need to have some kind of a modelling for that otherwise you cannot solve this equation. So, these are the concept of Reynolds stresses.