**Combustion in Air Breathing Aero Engines**
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**Lecture - 33**
**Non-reacting turbulent flows II**

Welcome back. So, as you have seen very exciting things happening in turbulence that is turbulence the whole mechanics works like this. That the mean or the or the energy of the mean flow the kinetic energy of the mean flow is taken away and that is used for creating turbulent kinetic energy and that clearly shows up in this equation.

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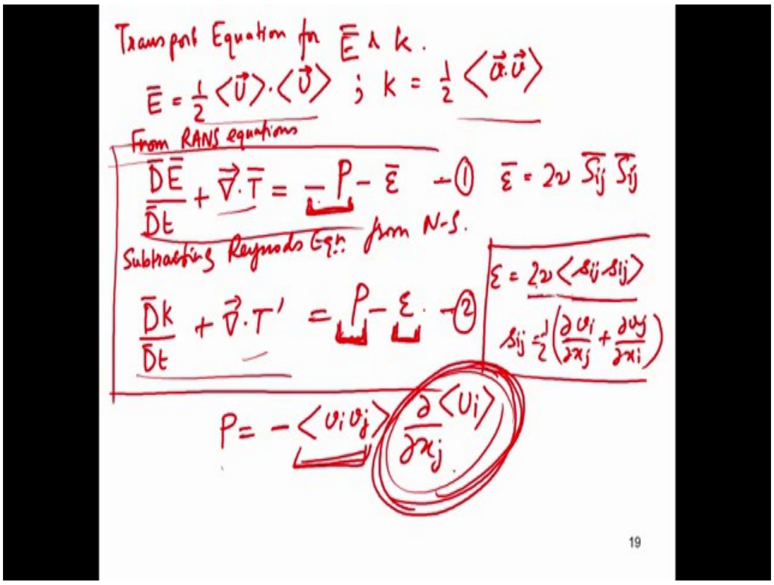


That you have seen that the transport equation for the transport equation for the for the kinetic energy mean flow, and the transport equation of the turbulent kinetic energy where p was acting a appearing as a as a sink term for the form on and the source term for the latter. And of course, this is happening through this through this production which contains the Reynolds stress times the mean velocity gradients and the then it is happening then it is dissipated at the small scales by this thing which is the dissipation rate turbulent kinetic energy dissipation rate, but the how does this happen.

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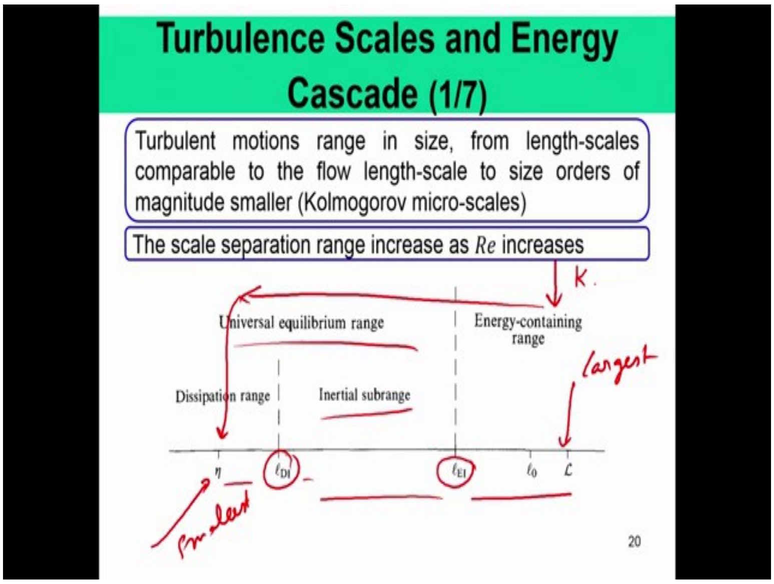


For that we need to go into the turbulent scales and energy cascade. So, as we have seen that from the jet image itself from the turbulent jet is that the turbulent motions range in size from length scales comparable to the comparable to the to the flow length scales that is a flow length scales can be for a jet can be the width of the jet to the size orders of magnitude smaller with to the size which is order of magnitude smaller. So, the smallest scales if the if you have a jet like this, the largest eddies you will see is basically the essentially of the order of the size of the jet, but the smallest eddies this then becomes actually cascades into small and smaller eddies and these smaller eddies are actually much smaller.

So, if this is of the order of say 10 centimeters. This larger steady for a jet then this can be of the order of few microns given that Reynolds number of course. So, that is how this thing happened how does the turbulence convert this energy that is coming into the large scales and how is that going into the small scale. So, that is the whole concept of this cascade and the small scale smallest scales at which this the smallest scales of turbulence and we call the kolmogorov of micro scales which will come later.

So, and as you have seen that just from the image from that the jet image we have seen that the scale separation what do you mean by scale separation by scale separation we mean that the we can say that the ratio of the largest scales of turbulent motion to the smallest scales of turbulent motion and that increases as Reynolds number increases. So, what happens is that. So, at the largest scales we have this, if this l is the largest scale and say this is the smallest scale.

So, at the larger scales we have the energy containing range. So, we where the energy is essentially injected where the turbulent connect is injected by the production mechanism and then it basically travels through a whole range of scales through this range of scale call the inertial subrange, which is essentially we will show that is universal in nature and until and unless it is dissipated into this dissipation range to the smallest scale will be the eta.

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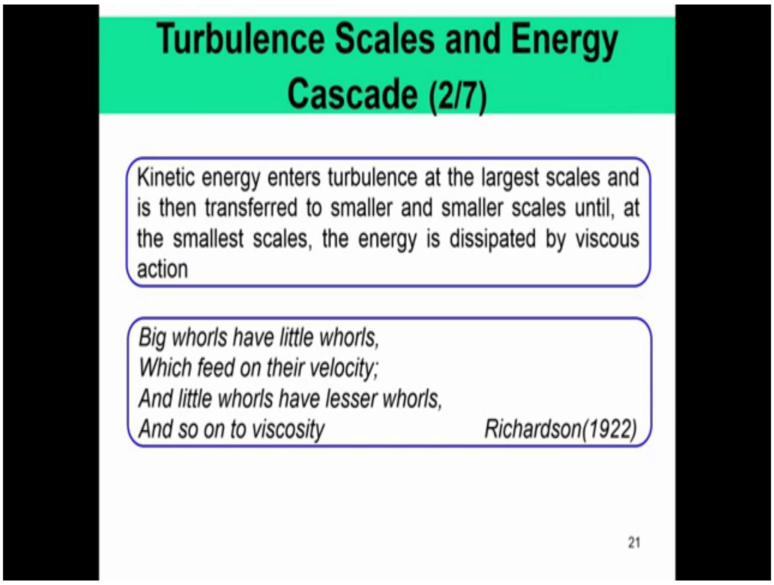


So, this demarcation between the energy containing range and the inertial subrange we will call this as lei and the demarcation between the dissipation range and the inertial range will because ldi. So, that is how the definition scope now kinetic energy as we have seen that the kinetic energy enters turbulence at the largest scales; and is then transferred to smaller and smaller scales until at the smallest scales the energy is dissipated by viscous action. So, that is the whole point. So, that a kinetic energy enters turbulence of the larger scales and is then transferred to smaller and smallest scale something at the smallest scales the energy is dissipated by this viscous action

So, this concept was realized by Richardson, actually by much earlier than Kolmogorov and he were up and down this point that big holes have little holes which we on their velocity and little holes of less a holes and so on their viscosity. So, it is a nice point by which this whole picture is essentially base basically put in which convey to the general audience.

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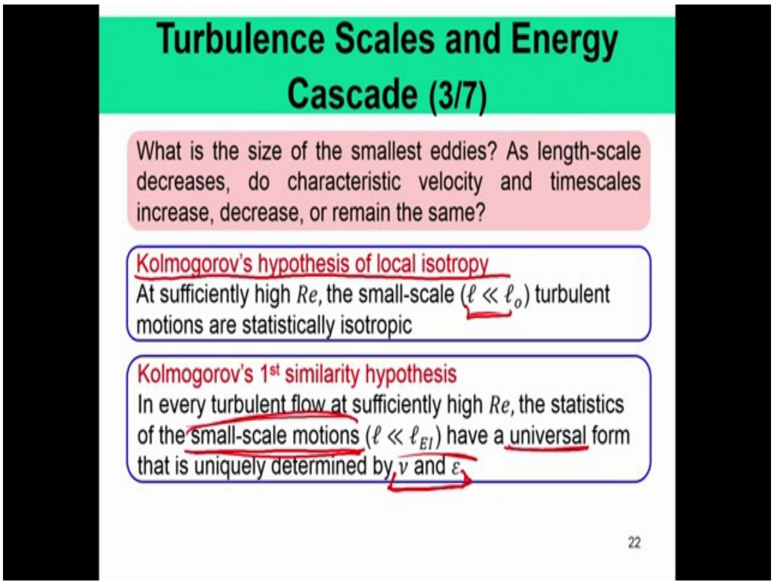


Now, then the question is that then what is the size of the smallest eddies are the universal in nature. How can you relate the size of the smallest eddies to those of the larger eddies? As length scale increases do the characteristic velocity and time scale increase decrease or remain same, these are important questions to answer.

And of the kolmogorov hypothesis of local isotropy is these are now what we will do is that. So, to understand these things we will introduce Kolmogorov's different hypothesis, basically we will introduce 3 hypotheses and the first hypothesis is this Kolmogorov's hypothesis of local isotropy. And this t is that are sufficiently high Reynolds number the small scale of turbulent motion are statistically isotropic and it can be revised to the first state that at sufficiently high Reynolds number the small scale turbulent motions, which are away from the walls are statistical isotropic means that you define a statistical quantity which is invariant upon translation and rotation of your coordinate system.

So, that is what is isotropic. So, it does not have any direction since. So, it is whichever direction rotated state is the statistical the statistics says stay say that is very important it is not the instantaneous field we are talking about is the statistical field we are talking about so, but this scale has to be is much smaller than the energy containing scales. So, that is important. So, you see that is what is very important over here. So, this using this we can go to the second Kolmogorov's of the of the first similarity hypothesis. And he says that in every turbulent flow at sufficiently high Reynolds number the statistics of the small scale motion. This is what is very important? The statistics of the small scale motion have universal form and that is uniquely determined by the kinematic viscosity and the turbulent kinetic energy this is dissipation rate.

So, this is very more important that this is a small scale motions are independent of anything are universal are. So, this 3 is towards important small scale motion are universal and determine by kinetic energy are by the kinetic energy dissipation rate and kinematic viscosity, but it is only the small scale motion or the large scale motions.

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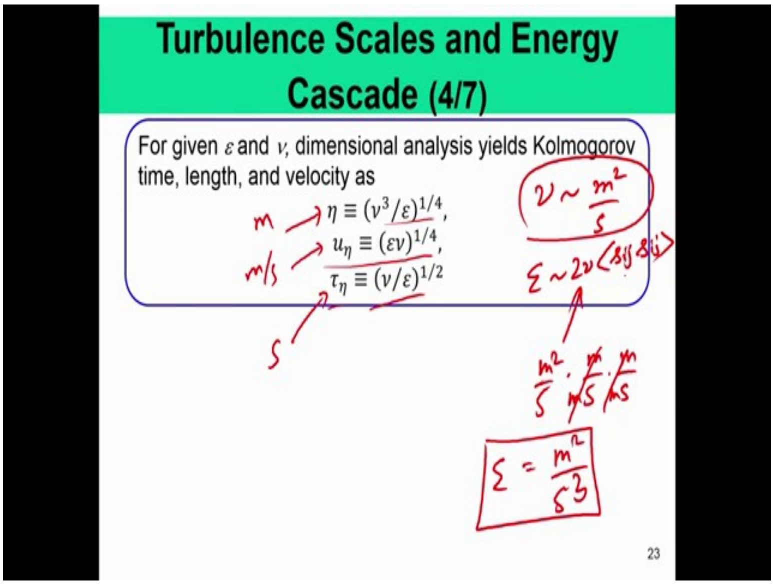


So, what does that mean; that means, that that to be small scale motion if you consider the smallest scales of turbulence to be the Kolmogorov length scales and the smallest scales of velocity or the velocity the Kolmogorov of velocity scales and the Kolmogorov of times scale. So, we can only determine this from this quantity using kinematic viscosity and dissipation rate.

So, by dimensional analysis kinematic viscosity is as a dimension of meter square per second it is essentially the same dimension of diffusivity. Whereas, epsilon is essentially has a defer as a unit of you see it is about what is what it is essentially, twice nu times sij times sij right. So, this has unit is of meter square per second. So, this has unit is of meter per second times meter per second sorry this is not meters per second. So, this is meters this is meter per meter second and meters per meter seconds of this cancels. So, this is essentially has a unit of meter square per second cube.

So, dissipation rate has an unit of meter square per second cube and kinematic viscosity has a unit of meter square per second. So, then and whereas, this Kino this will have a dimension of meters this will have a dimension of meter per second and this will have a dimension of one per second have a have a dimension of second. So, if. So, then we using this quantities how can we form this scales. So, this we EK this can be found only if it is nu cube by epsilon to the power of one-forth it is Kolmogorov of velocity scale can only be formed if it is epsilon times nu to the power of one-fourth and tau it can be only form this epsilon by dissipation to the power of one-half right.

So, that is the thing. So, this is how the different scales of turbulence are formed by eta u eta and tau eta. So, these are very important things and these are essentially universal in nature. So, that is what Kolmogorov's claim is and it is actually it turns out to be true using this we can have a very well good definition of a essentially what are the smallest scales of motion are and they, but this has to be stead this as statistical in nature which should not be confused with instantaneous observations.

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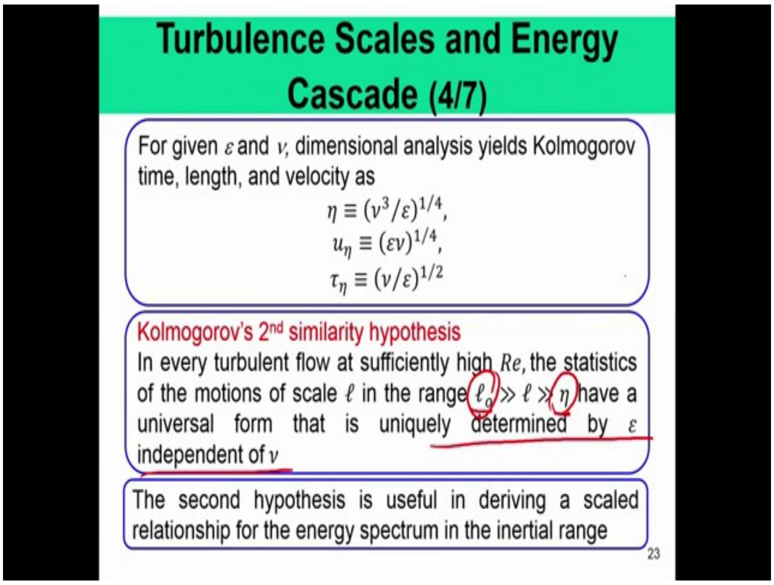


So, far we have talked about that far from the walls small scalar isotropic, yes and Secondly, he introduces the first similarity hypothesis we said that yes when the flow is at large Reynolds number. So, these are very important all the these hypothesis holds that very large Reynolds number. So, at a large Reynolds number the statistics of small scale motion have a universal form and they are uniquely determined by kinematic viscosity and kinetic energy dissipation rate. So, he only talks about small scales what happens at the intermediate scales.

So, the intermediate scale he says that in every turbulent flow at sufficiently high Reynolds number the statistics of motion of scale l, which is an intermediate scale that is which large between your largest energy containing scale l 0 and the dissipation scale which is Kolmogorov of length scale which is far away from both. So, this intermediate scale l which is or this intermediate range of intermediate scales l, which is far off from which is far smaller than the energy containing scale which is of the size of the largest energy continues scales is the order of the say the jet width, and at the same time which is which is also much larger than the kolmogorov of length scale they have a universal form and that is determined by turbulent kinetic energy dissipation rate only and it is independent of kinematic viscosity.

So, this is very important. So, the smallest scales are determined by kinematic viscosity and turbulent kinetic energy dissipation rate intermediate scales which are smaller than the largest length scales or the hydrodynamic scales. So, let us call this hydrodynamic scale l 0, which is smaller than hydrodynamic scales yet much larger than the Kolmogorov's length scale those are determined by the turbulent kinetic energy dissipation rate only why is that. So, why did kolmogorov say this things what does it actually mean; and as you will see that the second hypothesis useful in deriving a scaled relationship for the energy spectrum at the inertial range.

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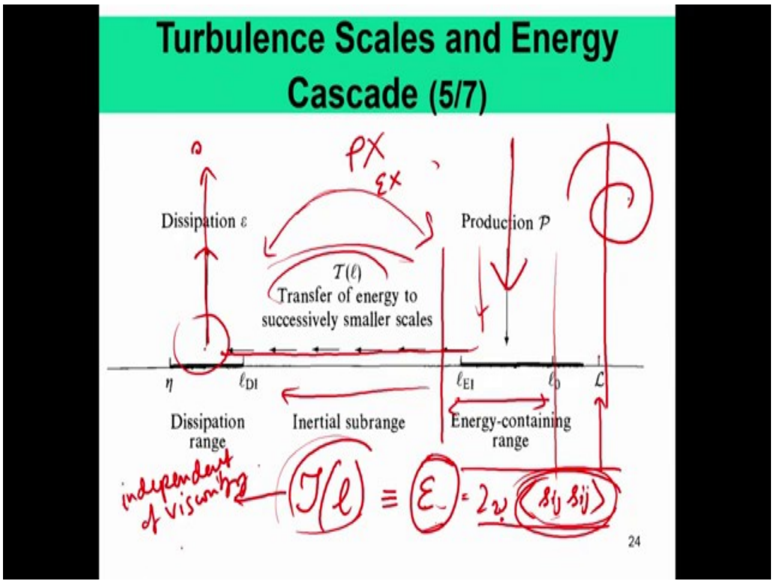


So, this is what he is meaning these are the largest scales like this. So, this is largest and this is smallest. In between you have this intermediate scales. So, what he have what is happening is that and this is the largest length scale which is like the size of the combustor or the size of your pipe or something like that, l which is the largest flow length scale this is the large ld length scale l 0. So, production through production turbulent kinetic energy enters into this flow at this energy containing scales through this production mechanism you have already seen that.

So, then what happens is that after this l EI when it enters into the inertial subrange it is basically behaves like an inertial system, where it is not acted upon by any external force. So, in these inertial subrange you neither have the effect of viscosity not you will have any kind of a other force. So, this behaves in a kind of an inertial manner. So, essentially this kinetic energy that comes in enter here. These travel through all these length scale until it is being dissipated here. And in this range there is no production in these intermediate range there is no production. So, production is equal to 0, there is no dissipation in these intermediate ranges. Production happens at the largest length scale dissipation happens at the smallest length scale. So, these full this full range is essentially we have like a concern in a in a conservative manner. And actually it can be shown that the angular momentum is essentially constant and there is no external torque of these eddies that can be shown, but we will not go into that.

So, if there is no source of sink in this in this intermediate scales in terms of in terms of the of the energy flux. So, it means that across the scales the energy turbulent kinetic energy flux at each given length scale remains constant. Because this is the only point where it is actually coming in and this is the only point where it is being dissipated. So, this whole breakdown procedure is essentially inertial where this eddies break down and there is no source or sink in this. So, there is no production no dissipation.

So, that the turbulent kinetic energy flux crossing the length scales which remains constant. Now then if you have a dissipation process at the end, so then it means that the rate whereas, this dissipation happens at the smallest length scales, then it means that the rate of the turbulent kinetic energy flux which crosses this different length scales must be equivalent to the dissipation rate so that a steady flow of turbulent kinetic energy across scales can be maintained.

So, then it means this tau well which is the transfer of kinetic any of energy to successively smallest scale that can be equated to the dissipation rate. So, that is why the dissipation rate is independent of the kinematic viscosity. So, that is why even though the dissipation itself contains dissipation itself is given by twice nu sij times sij, but the fact is that because dissipated dissipation is almost an identity that dissipation can be equated to this transfer of energy across successively smallest scales, which is the which is the transfer of turbulent kinetic energy across this small scales.

So, these thing is essentially independent is essentially independent of viscosity. And because dissipation rate has to be equated to the turbulent through this transfer of turbulent kinetic energy, this transfer rate this dissipation rate it becomes essentially independent of viscosity though it is you see is explicitly dependent on viscosity to get through these things. So, it means that. So, the only way that can happen is that that if you change the viscosity, this quantity also changes and just in such a manner that this whole quantity becomes independent of viscosity. So, that is the whole beauty of turbulence that is all beauty of high Reynolds number fully developed turbulence the production happens at lengths at production is of turbulent kinetic energy happens at large lengths scales then this whole turbulent kinetic energy travels through these different length scales. And when it travels through those different length scales there is no source or sink, there is no dissipation mechanism there is no production mechanism at this intermediate length scales which is the inertial subrange.

So, it is untouched this flux is untouched flux remains constant across different length scales and then when it goes into the smallest length scales it is essentially dissipated into the in to thermal energy and the rate at which this dissipation happens must be equated to the turbulent kinetic energy rate. So, consider like a pipe if you the amount by continuity equation the amount of if there is no mass generation or loss through the pipe. So, the amount of mass we put in at one end must be the amount of mass flow rate at the other end.

So, it is similarly this continue to maintain the continuity of turbulent kinetic energy this like which is happening through this pipe is across different length scales. So, this in a turbulent kinetic energy is passing through this different length scales and is dissipated at the end and this dissipation at the end therefore, can be equated to the transfer of this rate at which this energy is successively formed into small and small scales. And that is why this transfer rate becomes essentially independent of viscosity. Because there is no viscous action in this one and because the epsilon can be equated to the transfer rate epsilon essentially becomes independent of viscosity though it is very paradoxical in the sense that it is essentially dependent on viscosity, but the fact is that this strangers adjusts in such a manner. And that is it depends on because independent of this viscosity actually it can come out here also that why this small scales are formed you see that in the large Reynolds number turbulent. Suppose we consider make a flow more and more turbulent by reducing the viscosity.

So, this one goes becomes smaller, but the thing is that I mean if the dissipation rate has to be in essentially kept constant. So, what will happen is that then this dissipation rate the dissipation rate has to be held constant. So, then this sij must increase. So, what is sij sij is nothing, but the gradient of the derivatives of the fluctuating velocities are strain rates. So; that means, the strain rates must increase the same strain rates can increase only by reducing your length scale essentially and that is why that becomes that the dissipation rate essentially remains the same and whereas, your size of the scales becomes smaller and smaller.

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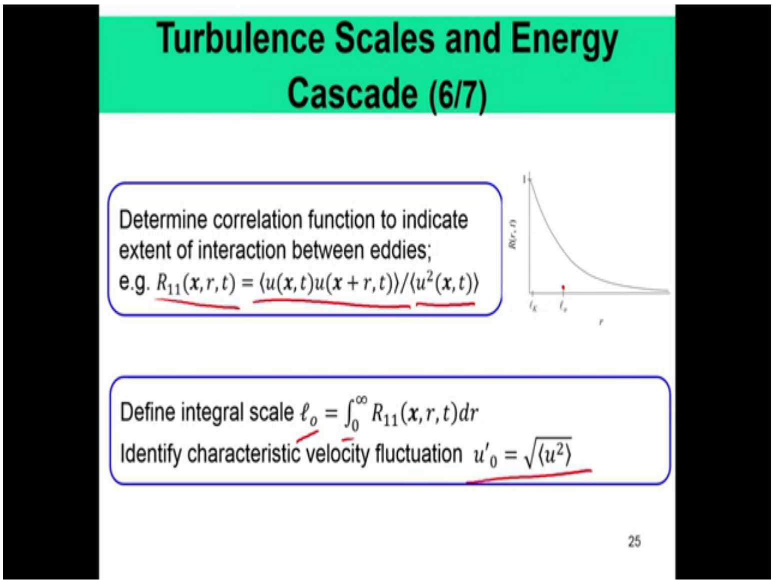


So, that is the basically the basic mechanism of turbulence. Now here we can estimate different length scales how do we estimate the large this l length scale essentially or the characteristic length scale in the inertial range. So, we can define a correlation function to do indicate the extents extent of interaction between eddies, and that is defined by this r one xr quantity which is this autocorrelation function, average of u x t times ux plus r which is shift at a same time divided by the variance of u and this is the autocorrelation function behaves like this. Where your that is it is correlated at itself, but this correlation reduces. So, that is the important point that if it was turbulence was noise then the autocorrelation function would have been a peak like this would have been a delta function which is not.

So, this is then the this becomes the length scale here lk and then this becomes the integral length scale and you can define integral length scale l 0 essentially as 0 to infinity R 1 1 x r dr and we can also identify a characteristic velocity fluctuation by essentially the rms of velocity.

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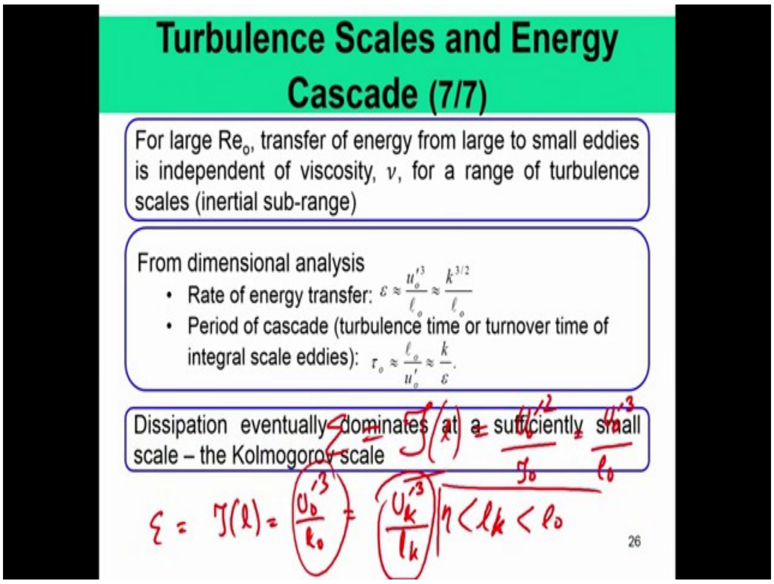


So, these are the definitions, we will use in the latter. So, essentially your for large Reynolds number transfer of energy from large to small it is independent of viscosity nu for the range of turbulence and that is the inertial subrange. And that is what we have learnt and because the rate of energy transfer you see that we have equated epsilon is equal to tau and this tau at any length scale

Now what is tau is essentially, we can equate it to like if it is entering into this eddy. So, we can equate it like turbulent this is energy transfer rate. So, it is essentially have must have unit is of energy kinetic energy per unit time. So, we can estimate this as u 0 divided by tau 0 and tau 0 we can write it as l 0 by u 0. So, we essentially can get u 0 prime cube divided by l 0. Whereas, this is the rms of velocity this is the integral length scale and this is essentially becomes this, but then we can actually write this as we can write epsilon is equal to tau l is equal to u 0 prime cube by l 0 l 0 is equal to u any l length scale as long as it in the inertial range divided by l or any length scale K l at any K we can we can write this for any essentially any intermediate length scale whereas, lk is basically lies between your l 0 and eta.

So, this is this how this equation I we can equate the dissipation rate to the turbulent kinetic energy transfer rate across different length scales. So, whatever this quantity even if you define this at the large length scale; this is also equal to the same quantity at the small length scales as long it is larger than the Kolmogorov of length and smaller than the your l 0 itself. So, that is the whole beauty of turbulence and then using this we can essentially show that of course, we have also shown that that the dissipation is essentially dominates at a small scale and the Kolmogorov of length scale.

now turbulent kinetic energy spectrum derivation now one very important quantity in turbulent turbulence is the kinetic turbulent kinetic energy spectrum. Now what is that. So, we can define now. Instead of instead of length scales we can define we can also define wave numbers whereas, wave numbers K is nothing, but 2 pi times the characteristic length scale.

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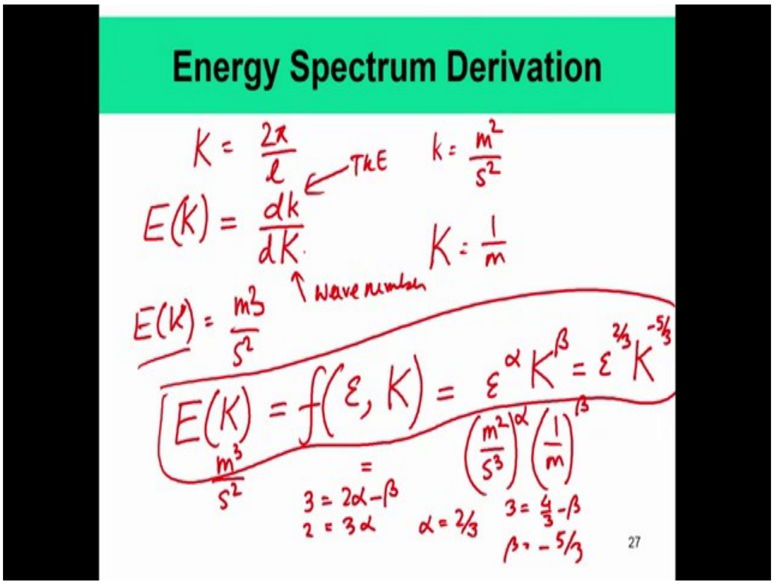


So, this is the wave number. So, we need to define this quantity we want to see how this quantity EK e of K behaves which is nothing, but the d of small K by d of capital K this is the turbulent kinetic energy and this is the wave number.

now for that we can just do the dimensional an analysis. Whereas, K has unit is of you see K has unit is of meter square per second square whereas, this capital K has unit is of one per meter. So, then this guy in K has unit of essentially meter square per second square divided by 1 per meter. So, it is essentially meter cube per second square. Now in the inertial range you have seen that the statistics will only depend on dissipation rate. And of course, this quantity can depend on K itself. So, EK . So, by Kolmogorov's second similarity hypothesis EK can only be a function of the turbulent kinetic energy dissipation rate and k. So, by dimensional analysis let us say this is epsilon to the power of alpha and K to the power of beta.

now this has unit is of meter cube second square and let us write down as this thing itself meters cube second square. And this is unit is of meter square per second cube to the power of alpha and this is unit is of meter to the power of beta. So, using this we can immediately derive 2 equations 2 algebraic equations that is for equating the dimension of meter, we get 3 is equal to 2 alpha minus beta and 2 is equal to equating the dimension of second we get 3 alpha.

So, alpha is equal to 2 by 3. So, epsilon the power of epsilon will be equal to 2 by 3. So, now, if you just plug in this we can estimate beta. So, 3 is equal to 4 by 3 minus beta. So, then it means beta is equal to minus 3, 3 here minus five. So, it becomes essentially could the minus 5 by 3. So, then it means K is you have nothing, but the dimension is e 2 epsilon to the power of 2 thirds times wave number to the power of minus 5 by 3.

So, this is the origin of the famous minus 5 by 3 spectrums. So, we see we expect that in the inertial range the turbulent kinetic energy spectrum EK should have a dimension of epsilon to the power of 2 thirds K to the power minus 5 by 3. And does it really have it. So, actually it turns out beautifully to be. So, and this is one of the famous designs of turbulence that in fully develop turbulence for different configurations different thing channel jets etcetera is taken from popes book actually.

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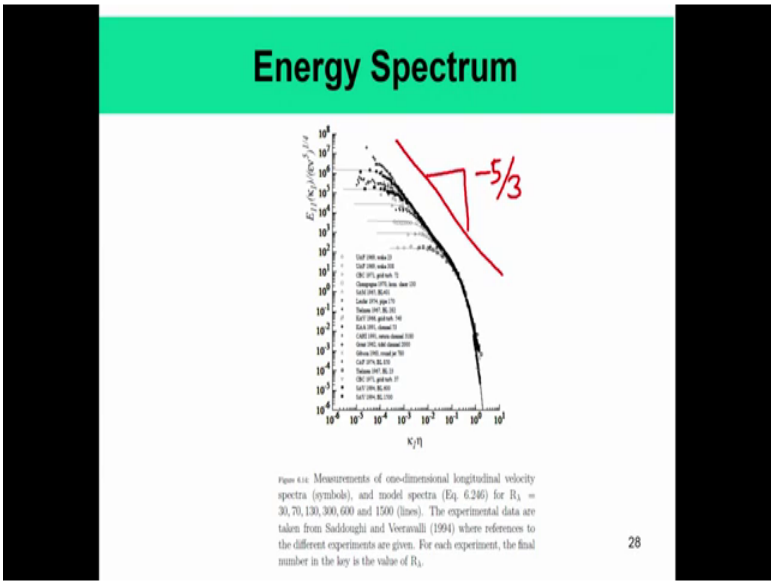

So, this has a slope of when you plot it in log scale this has a slope slop of minus 5 by 3. So, this is the (Refer Time: 24:35) famous minus 5 by 3 spectrum of turbulence.

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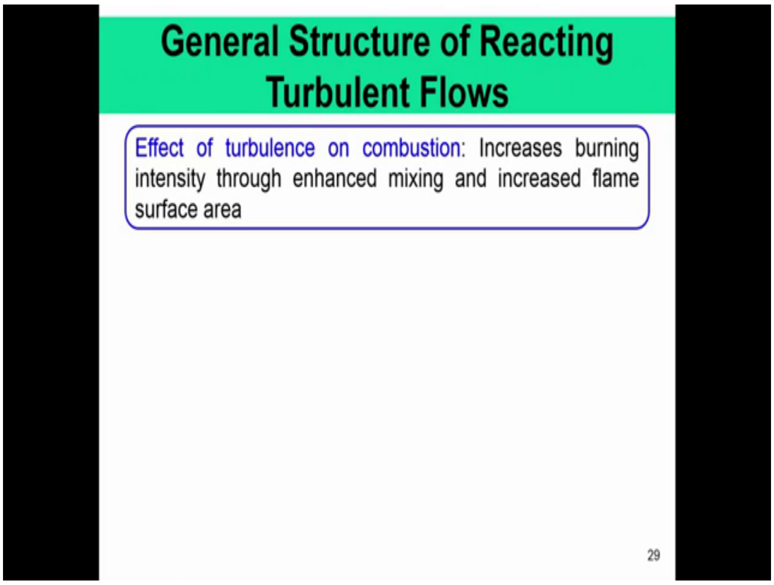


So, essentially we have looked into this about the whole spectrum and the whole dynamics of turbulence, how turbulence can basically sucks energy from the mean flow and then converts into turbulent kinetic energy and then how this turbulent kinetic energy which comes into turbulence at the large scales because of the mean velocity gradients. And internal stresses then the how that cascades into the smallest scale where it is being dissipated and the while it travels through these different scales we see that the flux of turbulent kinetic energy transfer rate remains constant. And because it remains constant it can be essentially equated to the turbulent kinetic energy dissipation rate and using Kolmogorov's first.

And second similarity hypothesis we can find out using Kolmogorov's first similarity hypothesis, we can find out the dimension or the equations for the parametric dependence of the Kolmogorov's length scale velocity scale and the time scale on epsilon. And dissipation rate and using second similarity hypothesis we can find out how this energy spectrum will look like.

So, that is all for fluid turbulence and using this concept, we will move on to turbulent combustion and that will take up in the next class.

So, until then thank you very much.