Combustion in Air Breathing Aero Engines
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Lecture - 32
Non–reacting turbulent flows I

So yes, we were talking about the probabilistic description of turbulent flows and we have just shown this we have just shown this momentum equation.

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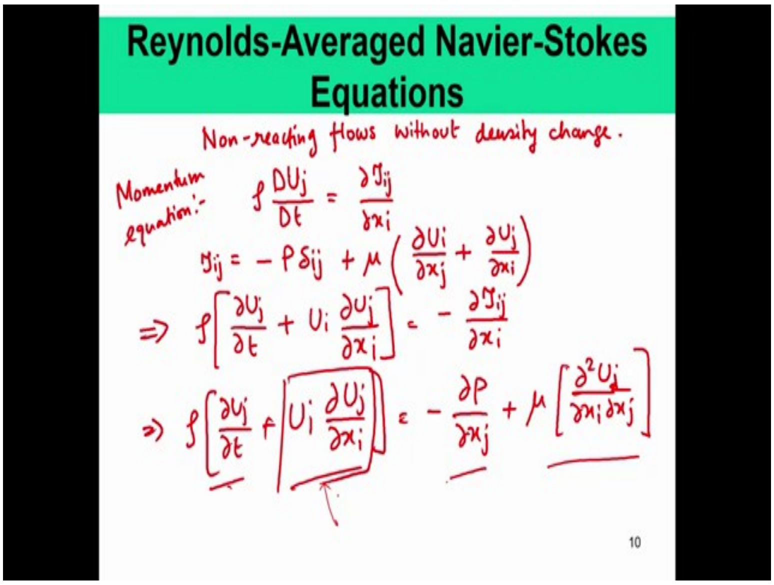

And of course, this can be simplified to or expanded to the dou U j dou t plus U i dou U j dou x i is equal to minus this will be dou P dou xj plus mu times this part that is. So, this is the thing which you have expand of course, so, this is the temporal acceleration there is a convective acceleration there is a pressure gradient and this is the viscous terms. So, this is the whole term which contains turbulence which creates turbulence this non-linear term.

But of course, then after that it gets coupled with all of these terms and as we see later that this term is also the most difficult problem that it poses in modeling turbulent flows.

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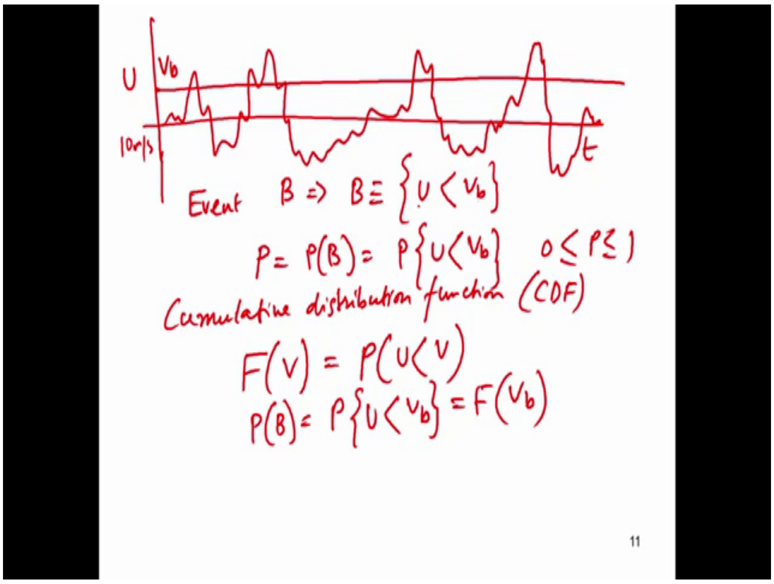

So, now to have a probabilistic description of that we will let us consider we have a; with the anemometer we are measuring this velocity field at a particular velocity at a particular point in time using this hot wire anemometer and we are measuring the velocity U actually. So, this is say the U mean or something this is say value of whatever it can be like this mean value is about 10 meters per second and then the velocity is fluctuating like this.

That is typical in a you know turbulent flow it has different scales, but at the center its random now we can define a new variable which is called the sample space variable which say that we define this value of this variable as equal to V b and then we can say that the we consider an event B we consider an event B which corresponds to b is equivalent to U less than V b.

So, all the possibilities of this U is being less than V b is captured in the event B. So, then we can calculate the probability of the event B and that is given by P is equal to P B is equal to probability of essentially U being less than V b. So, this U this is also corresponds to U this is also V; V here it is not a different velocity it also corresponds to U it is just call this V is will be essentially generalized with the sample space variable of U.

So, U V happens in the probability space where does U happens in the actual physical space in time and of course, here the probability will be t less than equal to one now we can define a quantity called cumulative distribution function or which in short the acronym of which is CDF and we can define it by a capital fv is essentially probability of U less than V.

So, this is the probability and essentially then which we see that the probability of the event B is essentially probability of U being less than V b is essentially f the CDF of f is equal to taking the value vb. So, this is the thing then. So, this is how we define the cumulative distribution function and then we can go into essentially the probability density function which we call which we define as which we define as fv of the sample space variable is equal to dfv by dv.

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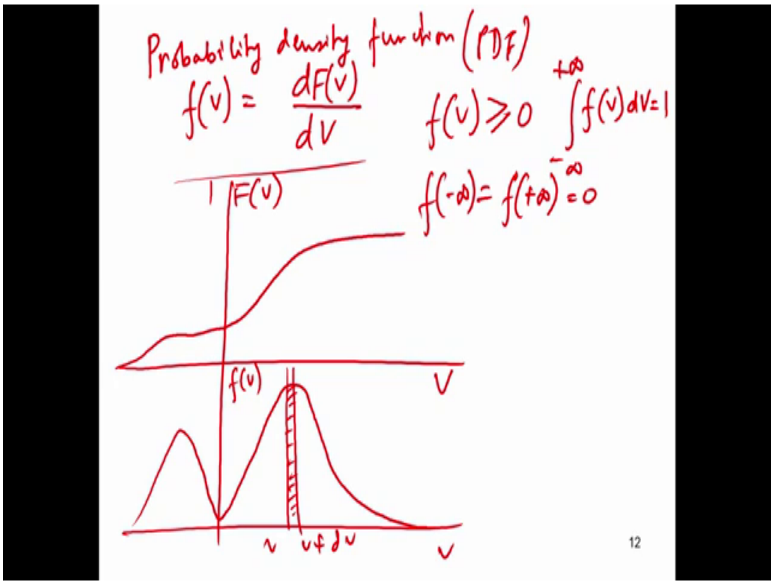

So, here f is the cumulative distribution function and the probability distribution function is essentially defined as the derivative of this cumulative distribution function in the sample space variable space. So, this is the definition of the PDF this is what is called the PDF. So, this is the best definition of the PDF that you can have and it essentially replaces a probability density of this sample space variable essentially lying between V and V plus dv.

So, of course, there are some criteria that is this fv is always greater than equal to 0 because they are giving no probability negative probability density and of course, the integral of minus to plus minus minus infinity to plus infinity fv dv is equal to one and of course, also at f at minus infinity is equal to f at plus infinity and that is is equal to 0 right.

So, now if you have a cumulative distribution function like this. So, say this is this is fv cumulative distribution function and this is the probability density function you will have this is the derivative of this it will look like this and then this is the probability of and this is the sample space variable V the probability of V happening between V and V plus dv is essentially the probability density fv.

So, this is the thing then now using these things we can define different things like we can essentially define different statistical quantities which are essentially the mean and the different kinds of the mean and the moments different moments of the of the probability distribution function and the first most important thing is the essentiality defining the mean will come to this things later.

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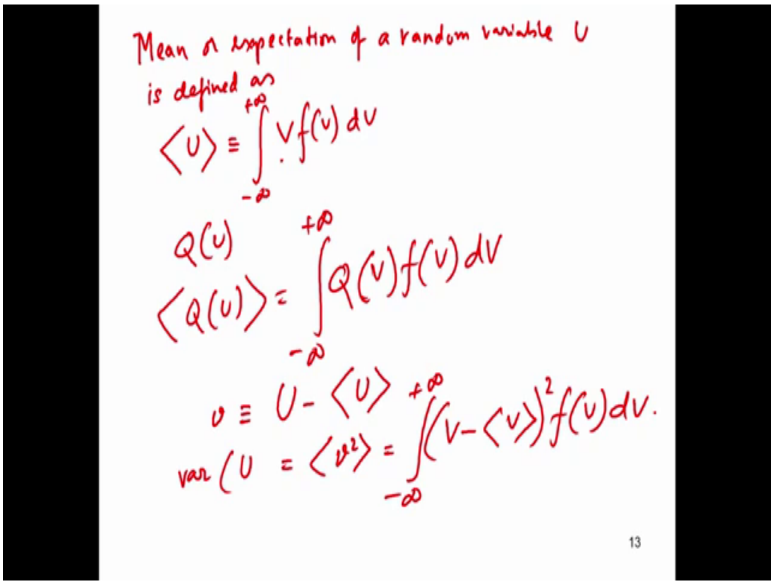

The first we need to understand what is defined by the mean is essentially the mean or expectation of a random variable U is defined as mean of U essentially minus infinity to plus infinity V times fv dv of course, integral minus infinity to plus infinity fv dv is equal to one. So, V times fv dv. So, when we are taking the moment the first moment with V thus defined as the mean.

So, the mean of a sample another variable Q which is the function of U is can be given by mean of Q U is nothing, but minus infinity to plus infinity Q V times fv dv and the fluctuation of Q is defined as small U which we represent by this U with a bend in the top is equal to U minus mean U this angular brackets mean ensemble averaging which we have defined before and the variance of U. So, this is the thing alright.

Now, using these concepts we are suited to go into what is Reynolds decomposition. So, by the Reynolds decomposition we essentially decompose velocity or any other variable as such by with in this manner that is we define this fluctuating velocity U vector U fluctuating vector at x at the position vector x vector at time t is equal to capital U vector ups sorry is defined as capital U vector at x vector time t minus U vector mean of that at time t.

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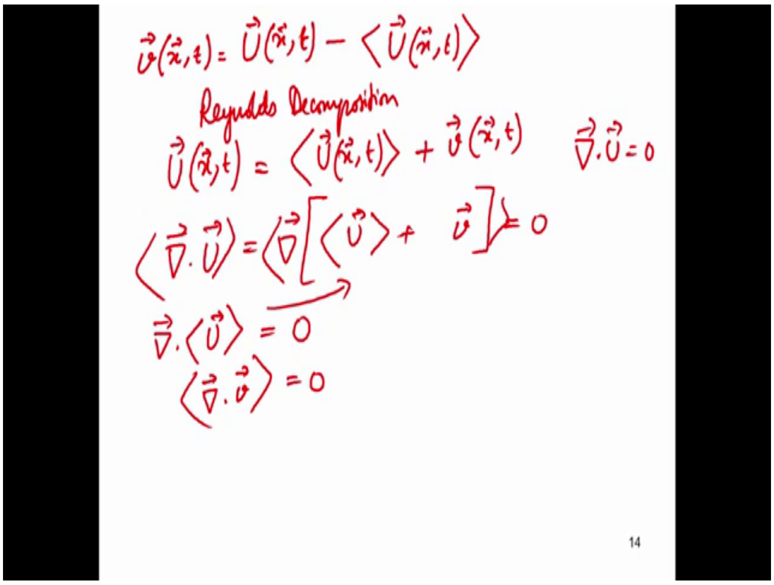

And then this is the Reynolds decomposition is this thing that is this is essentially decomposing the U vector into this manner. So, now, then of course, the continuity equation when then is no density variation becomes divergence of U vector equal to 0. So, then we can write the same thing that is divergence of U vector this is nothing, but divergence of this is small U and that must be equal to 0.

So, then it means if we take averaging on both sides of course, averaging commutes with the derivatives. So, divergence of mean U is still remain 0. So, then this implies then this implies that divergence of mean of the divergence of the small U vector is also equal to

0. So, this is a very important thing that is the divergence of the that is the divergence of the the mean vector is equal to 0 and of course, the mean of the divergence of the fluctuating velocity is also equal to 0.

So, now we can apply these things into the into the momentum equation at this, let us consider the left hand side and also then the right hand side.

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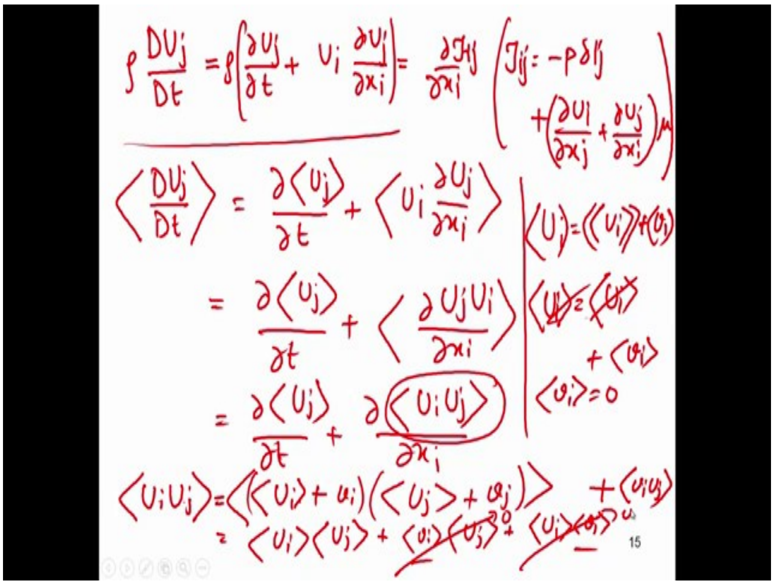

So, the left hand side; so, this is the full momentum equation that is the left hand side of the momentum equation which is equal to del del x i tau ij and where as tau ij defined as minus P del ij plus del U i del xj plus del U j del x i oh by the way I mean here we had a the density is there. So, here also you should have the density. So, this is the dynamic viscosity.

So, just considering the left hand side if you mean a if you consider the mean of this thing that is nothing, but right. So, of course, if you take this inside you can write this as and this becomes this one now then the question is that how to decompose this and you can see that then this thing becomes we just consider this part only.

and this is nothing, but. So, these are the mean of the fluctuating components and of course, the mean of the fluctuating component is equal to 0 which can be shown by the fact that if you say U i is equal to mean of U i plus fluctuating U i if you take the mean for all these things mean of the mean stays the mean that is U i is equal to mean of U i plus mean of U i prime. So, this cancels and this means that mean of U i is equal to 0.

So, by that same token this also becomes 0 and this becomes 0. So, what you are left with is essentially what you are left with is this thing that is mean of capital U i capital U j is essentially the mean of U i times mean of U j capital capital plus the mean of U i times U j that is the covariance of the fluctuating components of the velocity. So, this is what makes things complex.

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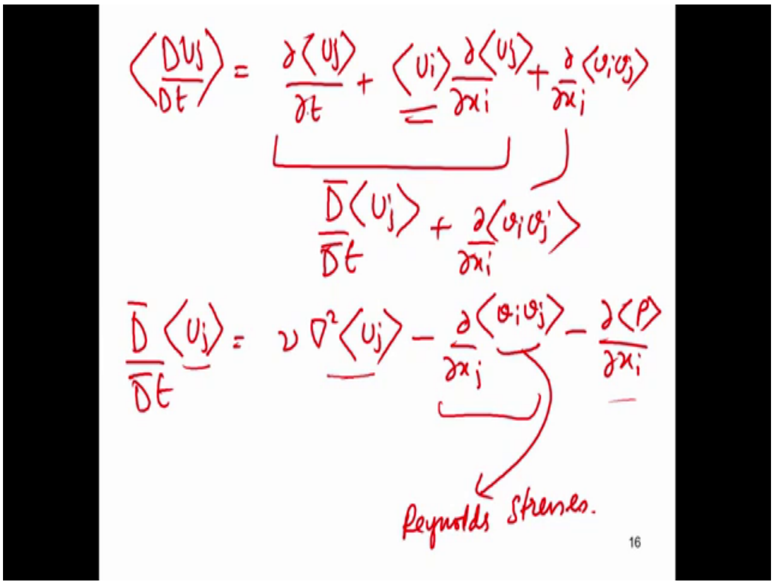

So, now; so, then this equation becomes like this we have to introduce some more slides. So, then this equation becomes of course, remember mean averaging commutes with derivatives. So, this quantity is now convicted with a mean velocity plus this gradient of the covariance. So, we can represent this full term as d bar d bar d bar full material derivative, but with bars, but is this is now convective with the mean velocity of U j plus this one that is.

So, the Reynolds formulation becomes the Reynolds equation becomes d bar j dt U j is equal to nu times del square U j which comes from the viscous stresses minus this term whereas, is the essentially the special gradient of the Reynolds stresses this is covariance of the velocity minus the mean pressure gradient.

So, you see that all terms the non-linear the linear terms that is this transient this transient term this viscous term when the special term these does not create any problems these are just averaged, but the problem becomes evolves from this non-linear covariance of the velocity fluctuating velocity components and then they have the special gradients. So, this is what the whole and the. So, the you see that the equation becomes essentially unclosed.

So, you have an equation of U u into have a new variable mean of U j which is everywhere here, but then you have introduced new variable cy uj. So, you need to have additional closures for this thing which cannot be solved on its own. So, this is called the Reynolds stress the Reynolds stresses.

Now, the thing is that the most once you have done this and then we have posed the problem of turbulence that why just its very difficult to represent turbulence in terms of its statistical moments that is if you try to represent this mean of velocity and try to derive an evolution equation of that we have faced with the problem that unclosed term emerges.

And. So, this is just for the velocity, but in turbulence it is essentially a matter of turbulent kinetic energy and so, we go on to derive define that.

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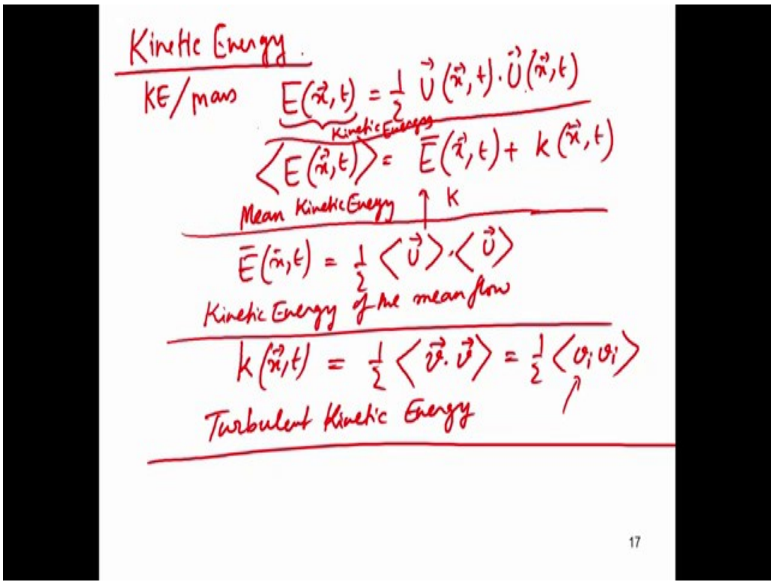

So, the kinetic energy now we go into kinetic energy these all these principles will be important to understand the basic features of turbulence and kinetic energy is particularly important.

So, the previous one shows with the basic problem of turbulence and you will use this kinetic energy concepts to describe the how the mechanics of turbulence works how the engine of turbulence works how it basically converts the large scales to essentially the small scales before the small scale turbulent kinetic energy will be essentially dissipated into thermal energy by the by the viscous action of the small scale stresses.

So, the kinetic energy of a fluid per unit mass is e is equal to half you can define kinetic energy in various ways. So, it is important to pay attention the mean of kinetic energy is essentially equal to once again we can just take the average of this and you will see again with there will be Reynolds decomposition.

So, essentially you can write it as the kinetic energy to the mean flow which is this c bar plus k that is the turbulent kinetic energy. So, what is this e bar is essentially nothing, but kinetic energy to the mean flow is nothing, but just like Reynolds decomposition we did that is the mean velocity times the mean velocity.

Whereas k. So, this one is will call actually kinetic and did not refer it is kinetic energy of per unit mass will refer will consider unit mass always and we will just refer it as kinetic energy. So, this quantity is kinetic energy and this is the mean kinetic energy this is this one bar.

This one is the kinetic energy of the mean flow and then we have the turbulent kinetic energy which is given by k small k which is k xt is equal to half of U fluctuation dot U fluctuation then averaged is equal to you can also represent by U i vector dot U i dot U i. So, this is the turbulent kinetic energy.

So, please pay attention to the different kinetic energy. So, this is the mean total kinetic energy that is the e xt is equal to half times U vector dot e vector mean kinetic energy is the is the when you average this mean. So, that mean kinetic energy has basically 2 components kinetic energy the mean flow and a turbulent kinetic energy. So, the kinetic energy the mean flow is given by half times U vector dot e vector half of U mean U vector dot mean of U vector.

And then you have the turbulent kinetic energy which is given by half of U vector U fluctuating vector dot U fluctuating vector which is half of U i wave, this is small; this is small caps. So, that is this is the small and this is a big. So, that is the point and then of course, from the Navier stokes equation one can derive an equation for one can derive an equation for the different for different kinds of kinetic energies.

And this is very important though i will not go into the derivation this understanding of this governing equation is very important because you will see that this provides you a very important understanding of the fact that; where does turbulence essentially originated from. So, where does turbulence these turbulence fluctuations come from.

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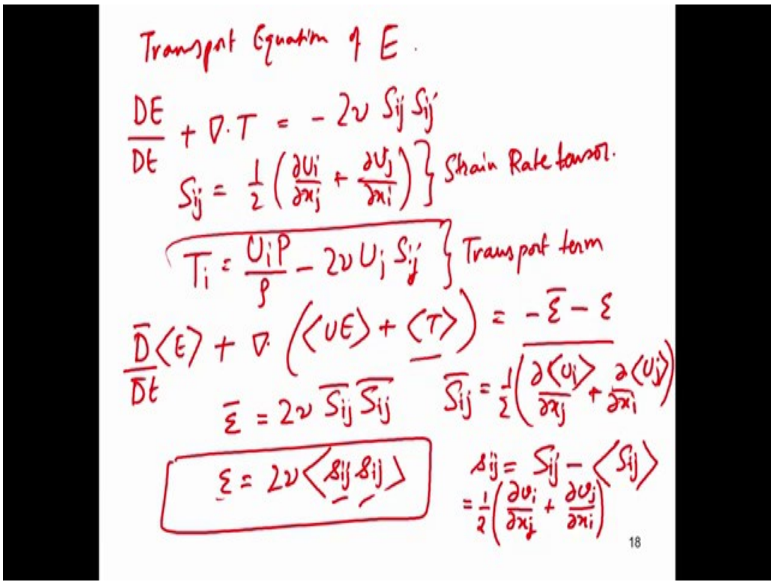

So, to understand that we will consider the transport equation of the first of e that is the kinetic energy and that can be derived as you should look into the derivation at forge book, but that can be derived at this de dt which is the full derivative plus divergence of transport term or a gradient of a transport term actually because this is a scalar equation minus twice nu S ij times S ij.

Now, S ij times S ij is nothing, but this S ij is nothing, but the stranger tensor which is given by half of duo U i duo xj plus duo U j duo x i whereas, ti is essentially U i times P by density minus twice nu U j times S ij this is a transport term which transfers the kinetic energy now if you average this one gets d bar d bar t dt is essentially i am not going into this, but I want to show you some final result or final form of this turbulent kinetic energy and the mean kinetic energy equation.

Which is very important times mean of ue plus average of this transport term is equal to minus of epsilon bar minus epsilon; so, this is once again the mean of the kinetic energy this is a different U times e and this is once again average of the transport term whereas, the transport term is given by this. So, this is essentially the transport and this is the strain rates or the strain rate tensor.

Now, this is very important because here these are essentially the dissipation. So, this is e bar is essentially the dissipation rate of the mean flow or the dissipation of the mean strains S ij times S ij bar whereas, S ij bar is nothing, but the average strain half of d U i d xj plus d d x i of d U j that is this is the average.

Whereas this is the mean dissipation rate of the or this is the dissipation rate of the mean energy essentially of the mean kinetic energy and this will be the mean dissipation rate of the turbulent kinetic energy which is given by twice nu average of S ij times S ij now please pay attention to particular to this term this is very important and will be useful throughout.

So, where ever we will talk about dissipation will rate will essentially mean this dissipation rate that is the dissipation rate of the turbulent kinetic energy now how this becomes the dissipation rate of the turbulent kinetic energy will come in the next slide whereas, this S ij fluctuating S ijs, this is the fluctuating strain rate tensor is essentially S ij minus mean of is equal to half or let us write this down little clearly. So, this is the fluctuating strain rate tensor.

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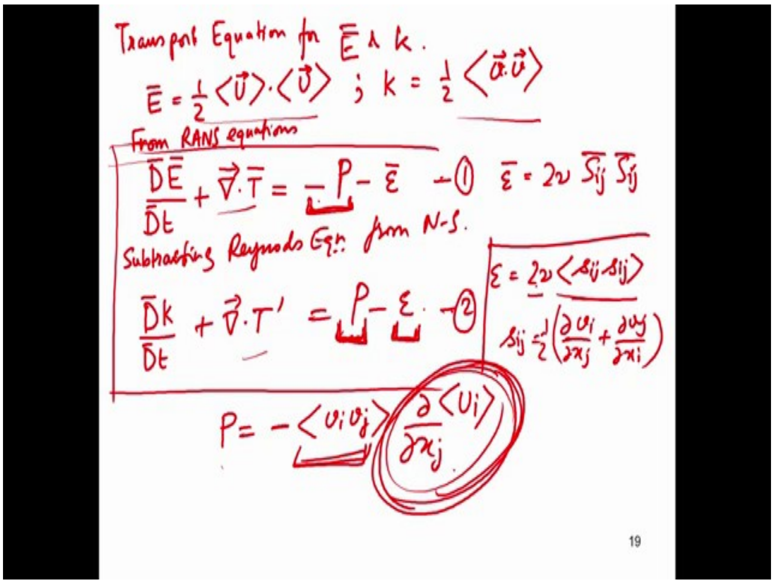

So, next we go on to the transport equation of the kinetic energy of the mean flow. So, we can show that, so, this is we will write down the transport even if you do not really pay attention on understand the previous just this mean of this evolution equation of the mean of the of the of the kinetic energy of the evolution equation of the mean kinetic energy please pay attention to this to this; this whole slide this is very very important.

So, will write down the transport equation for e bar and k whereas, e bar is the whereas, e bar if you remember is equal to half of the mean half of the average velocity dot average velocity vector and k is the half of fluctuating velocity vectors averaged after their products because individually they will be 0. So, if the write down the transport equation for these 2 things we get. So, d bar this is the average this is the convective derivative where the convection is done by the average velocity.

This d bar d bar t material average convective derivative of e bar that is of the kinetic energy the mean flow plus the divergence of the t bar that is the transport is equal to minus P minus epsilon I will show what we are now and also by subtracting this we can get from the Reynolds RANS equation essentially. So, this can be obtained from RANS equations and subtracting Reynolds equation from Navier stokes.

And then multiplying with U j one gets d bar k dt that is the same convective derivative of k, but the convective velocities average velocity plus divergence of d fluctuating is equal to P minus epsilon. So, what are this P minus epsilon this P and epsilon this epsilon as you have seen is epsilon is nothing, but epsilon is nothing, but twice nu fluctuating strain rate tensor.

And whereas, this epsilon bar is nothing, but twice nu S ij bar and S ij bar whereas, this mean strain rate mean strain rate times means strain rate and. So, this is essentially the production term what is production, but before going into that you see that in this set of equations ok.

In these set of equations these are very similar quantities in them of course, this is d bar d bar is there d bar d bar is there if you just compare these 2 this equation and this equation you see this is there of course,, but this is an equation of transport equate this is the transport equation for e bar that it is; this is a transport equation e bar of the kinetic energy of the mean flow this is the transport equation of the turbulent kinetic energy this is the kinetic energy of the mean flow this is the turbulent kinetic energy right.

And then both have transport terms transport terms i will come to this later, but you see here the P; P appears minus, so, whereas, in this case the P appears plus. So, if this P is actually always mostly always a positive quantity. So, the thing is that what serves as a sink P this minus of P serves as a sink in this current transport equation for the kinetic energy of the mean flow and this serves as a source.

So, production serves as a source in the transport equation for the turbulent kinetic energy. So, what serves as a sink in the first equation this equation serves as a source in equation 2. So, what is this thing that is serving as a sink and serving as a source? So, let us look into these this production term is very interesting its essentially minus U i times U j fluctuating covariance of that Reynolds stress times duo d d partial derivative of U i this is the mean velocity gradient.

So, essentially, so, now, you see what. So, this serves this quantity serves as a sink in the first equation and it is also the source in the second equation. So, what is this thing its of course, has got Reynolds stresses and it is mainly the mean velocity gradient. So, basically this thing that this is the mechanism of turbulence that this means velocity gradients is sucking or reducing the turbulent the energy of the mean flow the kinetic energy of the mean flow is reduced by this term and that is converted into turbulent kinetic energy by the same term.

So, turbulent kinetic energy is essentially produced at the cost of the kinetic energy of the mean flow and this production is happening through the action of the mean velocity gradient or mean shear and also the Reynolds stress terms. So, this is the very very important concept of turbulence. So, any turbulence flow where you have a velocity gradient that is if in contact with some solid body of course, it will develop mean velocity gradient.

So, suppose in a pipe flow where does it because of the mostly boundary condition your velocity you will your velocity at the wall will become will be the tangential velocity at the wall will become 0 whereas, the center line velocity is large. So, that develops a mean velocity gradients. So, as soon as there is a mean velocity gradient that essentially reduces the kinetic energy of the mean flow and it produces turbulent kinetic energy.

So, but for that there has to be a mean velocity gradient and. So, here this is the mechanism by which turbulent kinetic energy is produced by this production term. So, this is very very important concept of turbulence. So, this is how turbulent kinetic energy is produced it acts as a sink in the transport equation for the kinetic energy for mean velocity. So, the transport equation for e bar capital e bar which is the kinetic energy of the mean flow this half U i U average dot average dot U average.

So, this is acting as a sink for this equation. So, basically this what it is happening is that once again to reiterate kinetic energy of the mean flow that is e bar is reduced and it is sucked by this production term and it is acts as a source in for turbulent kinetic energy. So, mean velocity kinetic energy the mean flow is reduced and turbulent kinetic energy is produced and that is done by this mean velocity gradients d U i dxj d mean U i d xj times the Reynolds stress terms.

And of course, this is the source of the turbulent kinetic energy, but then if there is a source there has to be a sink and that sink is essentially dissipation. So, this is the full this is the full thing. So, this dissipation is essentially the twice nu times mean of S ij times fluctuating strain rates where the fluctuating strain rates are nothing, but essentially d half of d U i d xj plus d U j d xi.

So, this is the dissipation. So, the production is happening through the mean velocity gradients and the dissipation is happening through this fluctuating strain rates times the viscosity. So, now, also another thing is that the mean velocity gradients of course, that is the large scale. So, if you have a jet. So, thus the mean velocity gradient is essentially persists over the entire width of the jet. So, this is a large scale phenomena.

So, it is of course, when it taking away energy from the kinetic energy the mean flow that is a large scale process because it happens to this one, but the dissipation rate this is the turbulent kinetic energy dissipation rate which is given by this; this you will see that this happens to this fluctuating strain rates and this fluctuating strain rates are essentially dominant at the small scales.

So, this itself tells you why turbulence is essentially produced at the large scale and it is dissipated at the small scales, but this is the this whole purpose of this exercise is essentially come at these 2 equations which shows you clearly how turbulence is essentially produced by taking away that kinetic energy of the mean flow and its and its acts as a source for the turbulent kinetic air for the transport equation for the turbulent kinetic energy. And then it goes to several processes and then it is dissipated into as thermal energy by this turbulent kinetic energy dissipation rate which is epsilon.

So, with this we have developed this basic framework by which the mechanics of turbulence happen, but then this tells you this kinetic energy is taken from the mean flow, and it is taken by the velocity gradients. And you have just said loosely that it is dissipated at the small scales by this quantity turbulent dissipation rate, but exactly how does this happen, what is the mechanics and that is the very big mystery. And this concept of turbulence and this is essentially the whole thing that we will derive. That we will discuss in the next class.

Until then, thank you very much.