

**Combustion in Air Breathing Aero Engines**  
**Dr. Swetaprovo Chaudhuri**  
**Department of Aerospace Engineering**  
**Indian Institute of Science, Bangalore**

**Lecture - 31**  
**Introduction to turbulent flows**

Welcome back. So, in these in this course, we have of course, covered many things mainly at a fundamental level, and we have covered kinetics different mechanisms by which reaction happens for different kind of fuel and mixtures. We have covered different kinds of flames non premix flames flame premix flames their ignition extinction behavior, but all of those are at laminar level. So, we really have not brought flow into the picture, but as you know that combustion a large part of combustion is also flows it says the combustion is essentially chemically reacting flows with large heat release.

So, as an especially in an engine, flow is a the type of the flow the nature of the flow has a very important effect on the combustion behavior the most important thing most charact most striking feature of the flow apart from the different kind of patterns etcetera that is formed is that in an engine the flow is invariably turbulent. Why because you see engines operate by the principle aero engines operate by the principle of like thermodynamic cycles say most important among them is the brayton cycle.

Now, in the brayton cycle, if you see that we I mean before you extract work the heat addition process is essentially a constant pressure heat addition process which happens at high pressure. So, a combustor in an engine is invariably at much higher pressure relative to the surrounding ambient pressure. Now that pressure can be different for different engines for example, in a gas turbine engine the pressure can be 30 40 barre in a or 30 40 atmosphere in a in a scramjet engine. It can be of the order of 1 2 2 3 4 5 atmosphere in an afterburner it can also be little low pressure, but the fact of the matter is that in all this cases the engines operate at very high flow rates and there at high pressure and the resulting effect is that in engines, in gas turbine engine in scramjet engine in afterburners in ramjet engines all aero engines the flow is intensely turbulent intensely turbulent the flow is intensely turbulent please keep this in mind an engine flow is intensely turbulent.

So, the combustion process associated combustion process that happens is also extremely is a very strongly turbulent combustion process that happens anyway. Now, what does

turbulence due to combustion that is a very difficult question because turbulence itself is a complex process and combustion also all itself is a complex process. So, when they meet the resulting the resulting process that emerges of turbulent combustion is actually very complex, but for that we need to understand to understand basically the flows in an engine we need to understand turbulence at a very good level we need to understand combustion at very good level.

And also then after that we have to understand how turbulent combustion behaves. So, so far we have basically developed an understanding of combustion. Now we will develop an understanding of turbulence then we go to turbulent combustion. And at that point at that point will be in a very good situation and come take up actual processes that happens in a different kind of aero engines.

So, then is how the course is structured. So, we slowly build up complexity we slowly understand laminar combustion process, we slowly understand turbulent combustion process. So, we then we understand how they interact and how they and what is thus resulting interaction that emerge. And then we understand how it behaves in an actual engine. So, this is the this is the whole story about this course.

So, you see that in engines in aero engines the turbulence Reynolds number which we can define as essentially  $u' \text{ times } l$  at  $u'$  prime means  $u$  rms, say  $u'$  prime rms rather the rms of the fluctuating component of velocity, times the characteristic length scale which is can be the diameter of the engine or divided by the kinematic viscosity that is of the order of say 50000 in a gas turbine engine it is of the order of 10000 to 5000 in an after (Refer Time: 04:48) scramjet engine, but this is turbulence Reynolds number, this is not just normal Reynolds number.

So, inferable all flows are very turbulent. And we need to understand turbulence as well as combustion which we have understood combustion at already at a good level, we have not talked about turbulence. So, in this class will talk about turbulence and then we will utilize this concepts to go into turbulent combustion and different modeling approaches. So, then this we come to module 9 where we talk about combustion in turbulent flows.

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**Module 9**

**Combustion in Turbulent Flows**

- i. Introduction to Turbulence
- ii. Reynolds Averaging
- iii. Turbulent Kinetic Energy
- iv. Closure Problem and Models

Majority of the material is taken from

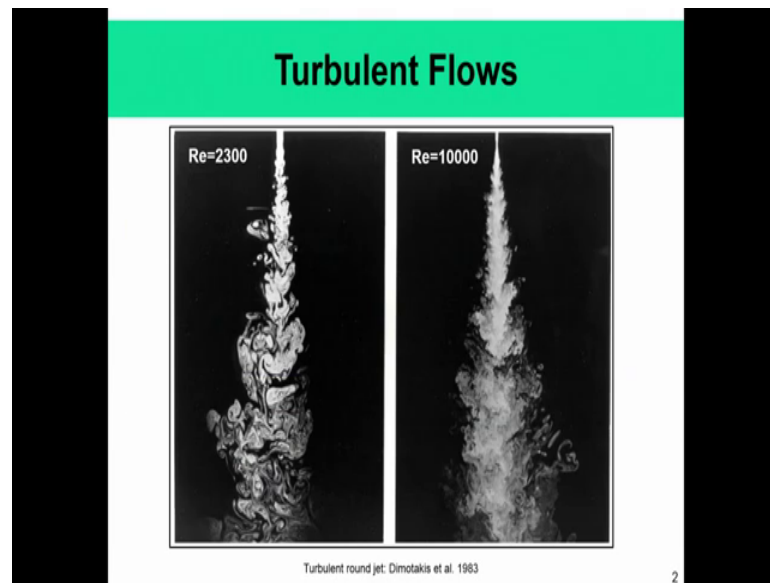
1. Turbulent Flows by S. B. Pope, Cambridge University Press.
2. Turbulent Combustion by N. Peters, Cambridge University Press.

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So, as it you see that this is organized in this manner that combustion in turbulent flows will first go into introduction to turbulence will go into Reynolds averaging will go into like turbulent kinetic energy, and then we go into different closure problems and models and majority of this material. Now we go to a different book will mainly follow like pope popes Steven popes book published by Cambridge university press and will consider also turbulent combustion the book on turbulent combustion by Norbert peters which is also published by Cambridge university press.

So, for these parts from these part onwards until we go to engines will essentially follow these books. So, turbulent smart will mainly follow popes book and combustion turbulent combustion part will follow peterss book. So, that is how it will work.

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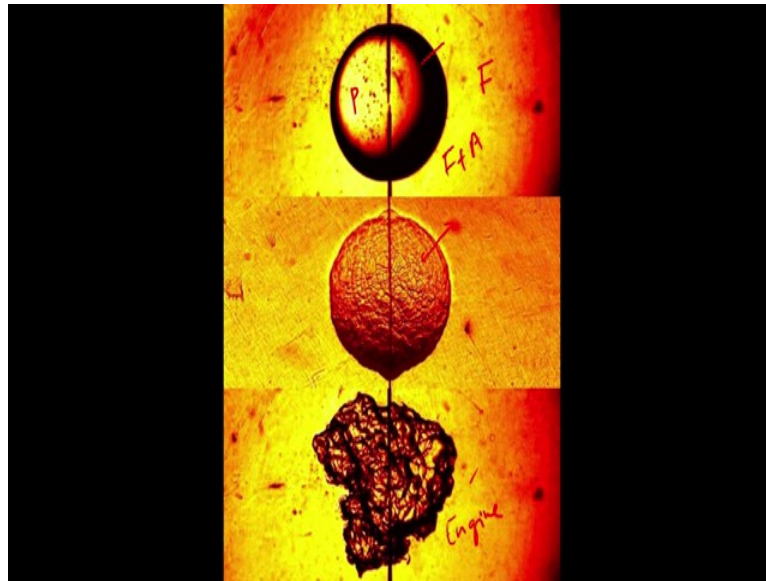


Now, turbulent flows of course, turbulent flows are ubiquitous in nature we can find turbulent flows everywhere say the smoke coming out from are chimney a large chimney is in variably turbulent and the most important characteristics of the turbulent flow that we find is that it is strongly dependent on Reynolds number at least in the in visually will see what that actually means. So, we see that if we see this turbulent flow the structure we see that the overall, if we consider these 2 jets at essentially at 2 different Reynolds number.

We see that the structure overall structure is not very different is as the jet is essentially diverging. So, this jet is diverging like this. This jet diverging like this and it is not very different on the overall thing, but what is different is that the scale the smallest scales of the jet are very different at different Reynolds number. So, you see that here of course, this different kind of structures will form, but the smallest scale of this structure is say of this scale whereas, the smallest scale is that it is very small here we cannot when showed by this thing.

So, we see that has Reynolds number increases from 2300 to 100 to about 10000. The small scales of this flow becomes smaller and smaller. And that is such you learn universal property of turbulence as you increase the Reynolds number, the smallest scales becomes smaller and smaller it develops more and more fine scale structures.

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So, this is one of the most important things. And then we talk about combustion. So, these image this very well visually attractive image shows the different tears of combustion. So, the first one is essentially all 3 are essentially expanding flames which are ignited by a pair of electrodes at a center of a volume which is filled the by fuel air mixture. So, you have essentially a big volume inside a inside a ball. And then you at then you fill up that volume with fuel air mixture. And then you ignite it with the spark like something like what happens in a engine. And then of course, flame will propagate it will consume the surrounding fuel air mixture and it will convert them into products. So, this is essentially fuel, this is essentially fuel plus air mixture and this is product the inside of this flame, but.

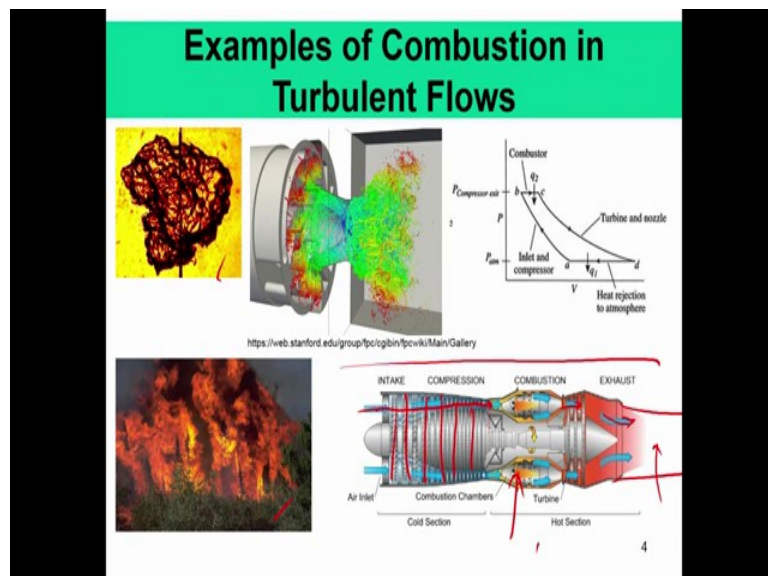
So, this is happens in time. So, you see that this shows the different structures of a flame, that we can expect this is what happens in a laminar flame when there is no difference in flow. This is when there is you have this pressure is very high and the flame develops this intrinsic instabilities, this is call the darrieus landau instability which will not cover in this course. And this is when the surrounding flow is turbulents. Of course, you cannot see turbulence because you are only visualizing this flame by this (Refer Time: 09:23) imaging which gives you the density gradients all though second which essentially gives you the density gradients.

So, but you see that when you have turbulence this nice symmetric shape of the flame is lost and it develops this different structures on these things. And of course, as you can think that just like the jet if you increase Reynolds number in this case also in this turbulent premix flame the size of the structures will become smaller and smaller. So, this is also a very striking part. So, here we see that how exactly turbulence disturbs the flames front to the to create this different kind of structures on the flame.

So, this makes these different scales of different or different orders or the structures at different scales and interaction and the flame propagation at depending on these different structures makes understanding turbulent combustion very difficult, but this is what is happening in an engine in an aero engine a flame is looks something like this, it does not look like this it does not look like this it looks something like this.

So, this is what is happening in an engine. And we need to we need to understand this. And for that essentially we can consider at the first level we can consider that the turbulence disturbs the flame structure, but by causing perturbations at different length scales, but it is more than that turbulence actually can go inside the flame structure to disturb it also, but will come to that in different types.

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Now, here are some examples of combustion in turbulent flows. Of course, you have seen that this is what happens in an engine this is what happens in this volume. So, this is you see that you see where this is essentially the kind of a gas turbine combustor, where

you have this swirling flow and this the you see the different kind of turbulent structures coming out. And you can have been nature also you can have turbulent in combustion in form of wildfires. And in an engine of course, you see that turbulent combustion is happening inside the combustor so this is a aero gas turbine engine. And this is it is intake which is followed by the different stages of compression compressors and then this compressed air this is this is the track of the compressed air well compressor air compressed air. And it then it enters into this combustor and then it goes into the turbine and then it goes into the exhaust.

So, the combustion have then happened here. And then if you have an afterburner here it can also happen here. So, in this case there is no afterburner. Of course, So, this is the thing. So, this is the. So, the entire combustor might look actually small, but the actually the thing is that this is the point vary of the highest pressure. So, this is the point where you have the where you have the high pressure. So, of course, when the when the flow is compressed it occupies a very small amount of area. So, that is why it are those occupies a small volume. So, that is why it is small. And so these are the different examples of turbulent flows inner in and in different engines.

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**Introduction to Turbulence (1/4)**

Heisenberg is reported to have said, *"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."*

Nearly all macroscopic flows, occurring in nature or in engineering devices are turbulent

Turbulence may be simultaneously **problematic** and **beneficial**

- Turbulence leads to increased drag on surfaces; loss in efficiency
- Turbulence enhances mixing in air-breathing combustion engines

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Now, once again go make turbulence of course, turbulence is a complex thing. And again Heisenberg you know that is famous for this uncertain to principle and for when asked about turbulence he said that when I meet god I am going to ask him 2 questions. So,

why relativity and why turbulence I really believe he will answer have an answer for the first that is he will and have an answer for relativity, but is no answer for turbulence.

So, turbulence still is the most important unresolved problem in classical physics and it still continues to be like that, but the things that of course, is very beautiful it has got this numerous, but beauty is beauty is in terms of understanding mathematics physics etcetera. The dynamics is very beautiful and complex, but when you have combustion then it makes even more 7 more complex object to look at, but the thing is that this is what is happening in engine. So, we need to understand it.

So, now search almost nearly all macroscopic flows occurring in nature and in general devices are turbulent and the turbulence may be simultaneously problematic and beneficial. For example, you know the turbulence leads to increase drag on surfaces it leads to loss in efficiency, but at the same time by the same reason it also has a higher heat transfer rates. So, which enhances heat transfer and of course, in an engine turbulence enhances mixing in air breathing engines and of course, you want mixing is very important because after you inject the fuel, if you does not inject in a pre mixed data already, it we after you inject the fuel you need to have a quick mixing. So, that combustion can happen. So, that turbulence can enhance that and it enhance is mixing in air breathing engines.

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**Introduction to Turbulence (2/4)**

Turbulence is NOT easy to define precisely. However, it exhibits following generic characteristics:

- **Fluctuations** – make turbulent flows appear irregular, chaotic, and unpredictable even though turbulence is not entirely random
- **Nonlinearity** – flows change state drastically beyond critical numbers
- **Vorticity** – range of eddy sizes; range increases with Reynolds number,  $Re$
- **Dissipation** – energy at small scales is converted to heat due to viscosity; steady turbulence needs continuous supply of energy
- **Diffusivity** – rapid rates of mixing and diffusion of species, momentum, and heat

Flows that seem random, such as wind-driven ocean-surface waves are not turbulent

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So, how do define turbulence is not easy to define precisely; however, it exhibit is the following general characteristics. Now what of that first most important characteristic are fluctuations? Thus these fluctuations make turbulent flows look irregular chaotic and unpredictable even though turbulence is not entirely random. It is not if you see if you put if you say, for example, take a velocity probe a hot wire anemometer and stick it into a turbulent jet you will get signals which are fluctuating, but it is not noise because noise, if it is not grant a random noise a 0 correlation that one point to the next point there is no correlation among them were as in turbulence, it is there is a finite amount of correlation between the velocity at one point at first data point and the velocity of the second data point of course, it changes. So, the correlation is not in fine and it changes, but at the same time it is not exactly noise.

Then you have non-linearity the flow stream state drastically beyond the critical numbers. That is the in a if you consider a pipe flow the Reynolds number is about say beyond below the critical Reynolds number the flow behaves like one the after. The critical Reynolds number the flow behave like another vorticity you have vorticity is very important in turbulent flows, it is very much rotational and range of eddy sizes increases and the range increases in an ensemble. So, if you go back to the to this to this picture you see that this the range of eddy sizes increases is Reynolds somewhat the meaning is that the largest size eddies is problem of this scale.

So, which is also the same here it will be of the order of this scale, but here the smallest size eddies of the order of this scale which is say, this whereas, in this one it is very small can be of the order of even can be even smaller than this. So, that is that is what we are we are talking about. So, as Reynolds some increases the range of these, eddy sizes the range of this rotational coherent structures essentially increases with the increasing if the increasing Reynolds number. So, that is a very important and striking feature of turbulence.

dissipation that the energy at small scales, that you will see later that there is a continuous cascade of turbulent kinetic energy our different scales and the ultimately it is converted to thermal energy and it is converted to thermal energy by the effect of viscosity. Or so thus to maintain turbulence you need to have a continuous supply of energy. And will see that how even the flow itself can provide as suppliers of this

turbulent energy turbulent kinetic energy and then you have diffusivity. So, you have rapid rates of mixing and diffusion of the species momentum and heat.

So, some people can think that that you can you can think that there is an enhanced amount of diffusivity in this enhancer some on enhanced diffusivity, in this when there is turbulence and as such that. So, there are some flows that might seem random such as wind driven ocean, but that is the surface the surface waves are not essentially turbulent it can be turbulent on the certain conditions, but just when you see some fluctuations and random that does not mean that that is turbulence or turbulence has it is own characteristics which will show now.

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**Introduction to Turbulence (3/4)**

- If the governing Navier-Stokes equations are deterministic, why are the solutions (i.e. turbulent flows) appear random?
- Turbulent flows are **acutely sensitive to perturbations** in initial condition, boundary conditions, and material properties
- With proper design and careful efforts these **perturbations can be reduced, but they cannot be eliminated**
- Perturbations are present even in laminar flows, but at high  $Re$  of turbulent flows, the flow field is extremely sensitive to small changes
- Sensitivity and critical numbers of NS equations can be explained using a much simple set of deterministic equation – Lorenz system

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So, one thing is that of course, where does turbulence come from turbulence comes from the momentum equation right. The Navier Stokes equation the reason is that the Navier Stokes equations are non-linear, because it is you see the convective acceleration term which is that we dot divergence on  $v$  vector, that is there is a non-linear getting  $v$ . So, this turbulence essentially stems from that term itself. So, it is that non-linearity that makes turbulence non-linear.

Now, the question is that if the governing Navier Stokes equations are deterministic, why is it solutions that is turbulent flow appear random the reason is that that just like any other non-linear systems. Of course, turbulent is one various important non-linear

systems the turbulent flows are actually sensitive to perturbations in initial condition boundary condition and material properties. And of course, that is what clears this.

So, if the proper design careful affects these perturbations can be reduced, but they cannot be eliminated and perturbations are even present in laminar flows, but at high announced number of turbulent flows the flow field is extremely sensitive to small changes. So, one small change can result. So, you have a same initial condition suppose you have velocity initial velocity at the inlet is  $v$  is equal 10 meters per second. In one case an another case is a set 10.2 meters per second. Then this small difference of the velocity inlet velocity can actually cause in a different kind of turbulent structures.

Statistical properties we not be very different, that is the average properties may not be very different, but the instantaneous properties can be widely different. And that is very difficult that is very natural because you know in a when flow interesting to an engines there is you have absolutely no control whether the velocity will be say 10.1 meters per second a particular at a velocity at a particular point will be 10.1 meters per second the 10.2 meters per second so, but the flow itself is turbulent.

So, the thing is that this can be appreciated by the why if we consider this Lorenz system.

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Introduction to Turbulence (4/4)

Governing equation for Lorenz system:

$$\begin{aligned}\dot{x} &= \sigma(y - x), \\ \dot{y} &= \rho x - y - xz, \\ \dot{z} &= -\beta z + xy\end{aligned}$$

For  $\sigma = 10, \beta = \frac{8}{3}$ , and  $\rho = 28$ :

a)  $[x(0), y(0), z(0)] = [0.1, 0.1, 0.1]$   
b)  $[x(0), y(0), z(0)] = [0.1000001, 0.1, 0.1]$

For  $\sigma = 10$  and  $\beta = \frac{8}{3}$ :

- If  $\rho < \rho^*$  ( $\approx 24.74$ ), stable evolution of variables
- If  $\rho > \rho^*$ , extreme sensitivity to initial conditions

That is that this the effect of turbulence being like chaotic like this is because of the fact that turbulence the governing equation is non-linear partial differential equation. So, if

you consider this non-linear equations like this, that if you consider this a Lorenz system that is where is given by  $\dot{x}$   $\dot{y}$  and  $\dot{z}$  where as this is a couple set of systems were which deficit still depends on  $y$  and  $x$  etcetera.

So, in this case you see that the solution for a given  $\sigma$  which are essentially constant  $\beta$  and  $\rho$ . If you have this current set of initial conditions  $x_0$  at time  $t$  equal to 0 you basically then solve it over time this Lorenz system, you see that at  $x$  with a  $\bar{x}$  initial conditions of  $x$  at time  $t$  equal to 0  $y$  at time  $t$  equal to 0  $z$  at time  $t$  equal to 0 if this is a point one point one point one then we get one set of solutions.

Now, if you change it to this 1 2 3 4 5 6 7th place of decimal of this thing. And you get a solution which is initially it stays same as you see up to this point, it is same, but then this difference starts happening. So, then if you blot this difference that is this is the  $x(t)$  and  $x'(t)$  the only difference between this to the solutions is the fact that this the second one the initial condition of  $x$  has a has the difference of the 7th place of decimal in this thing.

So, you see that the difference builds up and these differences of the order of the actual solution. So, this after some time there is no resemblance between at least instantaneously there is no resemblance between these 2 solutions. So, you see that this type of coupled equations, when you have differential system equations governed by differential equations when you have these kinds of things any if small perturbation in the initial conditions can result in different solutions.

So, that is why the whole purpose of turbulence is not to predict instantaneous structures arranged in 10 years velocities at a particular point rather we will predictor we are interested in predicting the statistics.

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**Probabilistic Description (1/3)**

While neither deterministic nor statistical analyses have been able to conquer details of turbulence, **statistics do offer ways of useful tools** for understanding turbulence

**Reynolds decomposition:**  
Turbulent field quantity = Mean (1<sup>st</sup> moment) + Fluctuation  
$$U = \langle U \rangle + u$$

Averaging (or defining) mean depends on the nature of turbulent flow:

- **Temporal average:** when  $U$  is stationary in time
- **Spatial average:** when  $U$  is homogeneous in space
- **Ensemble average:** arithmetic average over independent realizations under identical conditions

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So, that is why while neither the deterministic nor statistical analyses have been able to conquer details of turbulence, the statistics do offer useful tools for understanding turbulence. So, the whole purpose is to understand the different statistics of turbulent flows either for turbulent non reacting flows or for turbulent combustion etcetera. So, we will be interested in understanding the statistics of these turbulent flows.

Now the most important thing, we will go here where to understand about statistics is that the first point to notice that we will call about we will talk about Reynolds decomposition. So, if you have a turbulent field quantity which can be anything velocity temperature pressure etcetera species etcetera. We can decompose it into mean and fluctuation. So,  $U$  is essentially equal to  $\langle U \rangle + u$ , which is the mean and fluctuation.

So, we will come to this what actually these are these are and how to get this we will discuss this in this class. So, now, if you see this thing that averaging of the mean depends on the nature of the turbulence. And we will talk about averaging, but what particular averaging we are talking about depends on the particular application at hand so we will consider a temporal average when say  $U$  is stationary in time, we will consider spatial average when  $U$  is homogeneous in space, but this we basically consider a generalized averaging which is ensemble average when arithmetic average over which is essentially an arithmetic average over independent realizations and their

identical conditions. So, essentially you perform the  $x$  between different times and for each experiment you measure some particular quantity and that essentially becomes ensemble average.

So, it is not exactly temporal average a spatial average, but of course, this can correspond to those things under particular situations. So, we will go into Reynolds average Navier Stokes equation and 2 for that to develop that understanding we will basically have to develop some statistical parameters, which will come in soon in this course also. So, I hope that you have some basic introduction to statistics, but I will just for the sake of completeness I will just recapitulate some of them here.

So, first in this introduction to turbulence we will not consider density variation, though the purpose of this course is to understand turbulent combustion, but just to introduce these concepts in a similar in a simpler manner. We will first just do we will consider at the Navier Stokes equation of the continuity equation or other equations without considering any combustion. But eventually at the later part of these lectures we will introduce a density variation and we will consider the full equations with complexor complexity.

So, first we will just start with equations continuity equations and momentum equations for non reacting isothermal constant density turbulent flows. So, that is the thing. So, essentially, first we will consider non reacting flows without density change.

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**Reynolds-Averaged Navier-Stokes Equations**

*Non-reacting flows without density change.*

Momentum equation:- 
$$\rho \frac{DU_j}{Dt} = \frac{\partial \tau_{ij}}{\partial x_i}$$

$$\tau_{ij} = -p \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\Rightarrow \rho \left[ \frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} \right] = - \frac{\partial \tau_{ij}}{\partial x_i}$$

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So, the momentum equation or the Navier-Stokes equation under certain such condition becomes,  $\rho \frac{D u}{D t}$ , this is material derivative  $\rho$  times density times  $\frac{D u}{D t}$  which is the full material derivative is essentially the partial of the stress tensor around  $x$  direction.

Whereas, this  $\tau_{ij}$  is given by this is actually  $\tau_{ij}$ . So, this is actually  $\frac{D \tau_{ij}}{D t}$  by  $\frac{D u}{D t}$  or as this  $\tau_{ij}$ , is given by  $-\mu \nabla^2 u_{ij}$   $\mu$  is the pressure  $\delta_{ij}$  is Kronecker delta function. So, I introduce the tensor notations here,  $\mu \nabla^2 u_{ij}$  plus  $\tau_{ij}$  repeated mean this is means addition. For example, in this case; so it is addition over also. So, this is actually this implies. So, this is the Navier-Stokes equation, now let us define the concept of probability.