

Combustion in Air Breathing Aero Engines
Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture - 30
Limit Phenomena II

Hello, welcome back. So, in the last class as we have seen as you have seen that we discussed the limit phenomena of a homogeneous mixtures, in the sense that we were talking about under what condition given system of a given mixture a homogeneous mixture can ignite. Then what time it would take to ignite, and first we did the analysis without a heat loss. Then we did the analysis with heat loss with different complexities that is if we did not we considered that the fuel mass fraction changes very little in the initial time.

And then we go went into incorporate those complexities also where we were discussing with a for a flow reactor where basically we found this conditions were basically the heat generation balance is the heat loss. And which will which we have shown that one of them will correspond to the ignition. And watching one of those will correspond to the extinction. And we could use this s curve analysis to basically explain that.

So, those so, we did both transient analysis and we both did both as well as steady state analysis and we developed the concepts of ignition essentially. And we discuss the concept of extinction also. But here we will so, but. So far in none of those analysis we considered any flame as such. So, as you know by now that the difference between homogeneous mixture combustion and that between a flame is that the flame not only incorporates reactions. It also incorporates transport so, in corporate this diffusion and that is that is what makes flame different from homogeneous combustion. And where there are no temperature or species gradients, in a flame you have got very strong temperature and species gradients.

So, those temperature and species gradients complex add complexity to this kind of extinction analysis, but that is more practical, because in a gas turbine engine or in a scramjet engine what do you have is essentially the flames. So, to understand whether the flame that you have producing creating is whether that can be survive or not, we need to understand how extinction happens inside a real flame. Once again as I tell you that

that of course, the combustion that actually happens in a real gas turbine engine, the combustion that happens in a gas turbine engine or in a scramjet engine or in any engine as such, it happens in strongly turbulent flows at high pressure. But of course, understanding that in a for theoretical point of view it is very complex, which we will reach there will of course, go there and develop suitable models for analysis.

But then we consider that be where to understand that first we need to understand the unit building block of such complex flames. And to that end basically we are taking up the analysis of laminar flames. So, once again here will go to this laminar premixed flame analysis, similar to what we have done before, but previously we have done it from an adiabatic planar lamina premixed flame. In a in a double infinite domain where you remember we defined we obtained this expression for the burning flux of $f_0 f_{naught}$ square which was proportional to if you remember it was proportional to λ by C_p times B_c Lewis number this, e to the power of minus r_n is number divided by $Zeld$ which square ok.

So, that we have derived, but that was for an adiabatic condition, where there was no heat loss. Now what we will do is that, we will consider the same premix flame this lamina planar lamina premixed flame in a double in finite domain. But here we will consider the effect of heat loss and we will show that how by considering the effect of heat loss we a flame can essentially extinguish. So, it is an intuitive to understand that is once again, we will have we will see that how increasing heat loss can lead to the point of extinction, and the by doing the phenomenological analysis will derive the point of extinction and the burning flux associated with that extinction point. So, this is the analysis here. We will consider the premixed flame through extinction the planar lamina premix flame in that double in finite domain extinguishing through heat loss.

(Refer Slide Time: 04:20)

Premixed Flame Extinction (Through Heat Loss) (1/3)

The standard flame, being adiabatic, does not exhibit any extinction behavior, i.e. finite f^0 for finite Y_u .

f in terms of f^0
burning heat loss flux under.

11

So, that is so, here is the analysis. So now, as we have seen before that the standard flame which is the planar lamina premixed flame in an adiabatic condition, that being adiabatic does not exhibit any extinction behavior as such. So, for finite Y_u that is the unburned fuel mass fraction, if it is a limp premix flame, you have a finite f^0 which expression I already told you.

But now what we have is that, instead this was the adiabatic flame which we initially had, this is the adiabatic flame temperature essentially this temperature. And now instead of that we have a flame in which there is some heat loss. So, whenever there is a heat loss of course, the temperature will deviate from that adiabatic flame. And we will estimate the different modes of deviation, how much it will? How much the temperature or how much heat loss can happen through the preheat zone? That is this zone how much of heat loss can happen to the reaction through the reaction zone? And how much of heat loss can happen through the final product zone? And will something happen see how we can obtain an expression for F , but this f is not f^0 because the f^0 represents this idealized standard condition where there is no heat loss.

So, now we will find out f in terms of f^0 . So, that is the idea here. So, f corresponds to the situation where there is the burning flux under heat loss. So, this is the situation that we will consider. So, that is to that end once again will proceed with this analysis here.

(Refer Slide Time: 06:03)

Premixed Flame Extinction (Through Heat Loss) (1/3)

The standard flame, being adiabatic, does not exhibit any extinction behavior, i.e. finite f_0 for finite Y_u .

Heat loss lowers flame temperature from T_{ad} , leading to abrupt extinction, at finite Y_u . System becomes non-conservative

$f_0^2 \sim T_{ad}^4 (e^{-T_a/T_{ad}})$
 $f_0^2 \sim T_f^4 (e^{-T_a/T_f})$

11

And so, what it is intuitive to understand that heat loss what it does essentially is that, heat loss lowers the flame temperature from the adiabatic flame temperature. And that can essentially leads to abrupt extinction.

Now, why abrupt? Because it is an because as we have seen that the temperature the burning flunks essentially is an Arrhenius depends on burn gas temperature. You see that of course, it has a rubber band it has (Refer Time: 06:24) or dependence on that also So, if you remember that we obtain that f_0^2 is essentially T_{ad} to the power of 4 times e to the power of minus T_a activation by T_{ad} . So, it has got Arrhenius dependence as well as power dependence.

So, this Arrhenius dependence is actually very sensitive. So, if you part of the flame temperature little bit your f_0^2 will be part of it; and because this was there for f_0 . So, by that analogy we can expect that the actual f_0 will also be something like T_f to the power of 4 times T_a by T_f whereas, this T_f is essentially this T_f . That is the if the flame is undergoing heat loss then the burnt are the maximum temperature that it will reach is given by T_f . Whereas, if there is no heat loss then the flame temperature the maximum temperature it will reach is this adiabatic flame temperature.

So, we will expect this same analogy and we will show that this essentially will be true. And So, that is the then the question would be that you see that here we have an

Arrhenius dependence of course, T_{ad} is fixed it does not change, but T_f can vary that is if you have more and more heat loss, your T_f will continue to decrease and this will again essentially decrease $f_{naught\ square}$. So, at some point there will be an abrupt extinction because this is a very strong Arrhenius dependence.

So, we will do this analysis in a phenomenological manner. One can do this analysis in a more detailed Frank-Kamenetski analysis with heat loss also what we will now do that we can take a refer to the CK loss book to do that to check that analysis. But you see this is a practical condition, because under no engine in no engine you have a flame which is aromatic. Most you have some heat loss here and there somewhere.

(Refer Slide Time: 08:10)

Premixed Flame Extinction (Through Heat Loss) (1/3)

The **standard flame**, being **adiabatic**, does not exhibit any extinction behavior, i.e. finite f^o for finite Y_u .

Heat loss lowers flame temperature from T_{ad} , leading to abrupt extinction, at finite Y_u . System becomes **non-conservative**

Radiation from flame is an inherent heat loss mechanism

If loss occurs in the preheat zone, and with $L(T)$ being a volumetric heat loss rate, then heat loss flux is

$$q^* = \int_0^{x^*} L dx \approx \ell_D L \approx \frac{\lambda c_p}{f} L$$

$q^* = \frac{(\lambda c_p) \cdot L}{f}$

So, it is important to understand how a flame behaves under the under the heat loss and of course, the system becomes a non conservative because you have heat loss. So, radiation how so, if you ask that question that, how can it have heat loss well radiation is of course, a very inherent heat loss mechanism; which is present in small or biggest extent depending on the type of the flame. So now, we will see that, that will find out the heat loss in different parts, of the flames.

So, we say that that this is the heat loss parameter, it is the essentially this $L T$, this capital $L T$ will represent this heat loss in the preheat zone with this capital $L T$. L is the heat loss parameter T is the temperature. So, L is a function of temperature and this is essentially a volumetric heat loss heat loss rate, so then the total heat loss if this is a

volumetric heat loss rate. So, then the total heat loss flux that is heat loss per unit area because of course, we have to consider a unit area then q_{minus} , there is a this region is minus this region is plus if we say stick to the previous convention.

While say that this is essentially 0 to l_D that is integrate from and this is the part from 0 to this is the integration domain. l_D is the flame thickness if you remember, is the heat loss volumetric heat loss times dx . And that we can say that if we assume that this essentially is constant this heat loss rate just for the first cut analysis, we can say that this is l_D times L . Now what is l_D if you remember l_D is essentially $\lambda / C_p \times f$. And it can be shown that this $\lambda / l_D \times f$ is whereas, is essentially equal to λ / C_p where, λ is the thermal conductivity C_p is the constant pressure specific it ok.

Now, this $l_D \times f$ this l_D is neither $l_D = 0$, this f is neither $f = 0$. So, this l_D is essentially the flame thickness when the flame is undergoing heat loss and f is the burning flux for the flame when it is undergoing heat loss. So, this is not equal to $f = 0$ this is not equal to $l_D = 0$. Now you see that why this fact that we can expect that the flame will undergo a rapid extinction can undergo a rapid extinction, under some amount of heat loss then we understood from this analysis itself.

How? It can be understood because of the fact that, that that you suppose when your heat loss rate increases. What happens when your heat loss rate increases? When your heat loss rate increases, then the of course the total heat loss increases, q_{minus} increases right, q_{minus} increases. When q_{minus} increase, what happens? When q_{minus} increase of what is the temperature will come down and as we know that the heat flux the burning flux is essentially a very strong function of temperature.

So, when heat flux heat loss increases and the temperature comes down, then this f this also will come down. When f comes down what happens is that because $l_D \times f$ is equal to λ / C_p . Then l_D will increase, that is in to the flame thickness will increase. So, when the flame thickness increases now the heat loss happens over a much larger volume and this will again go and feedback to q_{minus} . So, you see that there is a non-linear there is of kind of a feedback mechanism when you, when there is a little bit of heat loss it again it is just not a straight direct when the heat loss L_T increases it is not a straight direct go goes into effect the total heat loss.

It of course, directly affects the heat loss, but then the feedback mechanism again goes back and to reduce the q minus further, and that is the reason why you have a rapid extinction when you have some amount of heat loss. So, from that we can expect that there is an if there is an epsilon order of change in temperature. There will be actually an order of one change in heat burning flux. So, that is the reason we will see that also.

(Refer Slide Time: 11:49)

**Premixed Flame Extinction
(through Heat Loss) (2/3)**

Overall energy conservation

$$\dot{m}c_p(T_{ad} - T_f) = \dot{m}c_p(T_f - T_{ad}) + \frac{\lambda}{f} \frac{c_p L}{T_{ad}} \quad (7)$$

$$T_f = T_{ad} \left(1 - \frac{\lambda/c_p^2}{f^2 T_{ad}} L\right) = T_{ad} (1 - L' \tilde{f}^2) \quad (8)$$

$$L' = \frac{\lambda/c_p^2}{(f^0)^2 T_{ad}} L; \tilde{f} = f/f^0 \quad (9)$$

In analogy to standard flame result:

$$(f^0)^2 = \frac{(\lambda/c_p) w^0}{Z e}; w^0 = \exp(-E_a/R^0 T_{ad}) \quad (10)$$

we can write $(f)^2 = \frac{(\lambda/c_p) w}{Z e}; w = \exp(-E_a/R^0 T_f) \quad (11)$

Using (8) in w

$$w \sim \exp[-(E_a/R^0)/(1 - L' \tilde{f}^2)] = \exp(-E_a/R^0) \exp(-\tilde{L}'/\tilde{f}^2) \quad (12)$$

$$\tilde{L}' = (E_a/R^0) L'$$

12

So, now it can be shown that your q plus by it considering conserved by considering basically the where the convective loss, we can also show that that will also be essentially lambda by C p by f times l. So, both in q minus and q plus both will be equal to the same expression. Now we considered that overall energy conservation. Now in suppose the condition that when if there was no heat loss, if L was equal to 0, if there was no heat loss then there then the system would have been adiabatic, then the overall flame temperature adiabatic flame temperature.

So, we can say that this if there was no heat loss that is, we can say that this $f c_p T_{ad} - \dot{m} c_p T_f$ is essentially $f c_p T_f - \dot{m} c_p T_{ad}$ plus this heat loss rate. So, this is the current temperature that has reached and this is the loss. So, by energy conservation if we add these 2 we would have reached the a adiabatic flame temperature. So, then we can write that when T we can write T f essentially is equal to in this form. Essentially this is a T ad minus times one minus L dash times f tilde square whereas, f tilde is nothing but your f by f 0. And we can this L prime is nothing but a heat loss and normalize heat loss

parameter which is essentially given by this thing. And this f tilde of course, equal to f by f_0 .

And now if you just put this thing here. If you will go back into the standard flame result, that is f naught square is equal to λ by C_p times w naught by Zeldi which number, and this thing was this w naught equal to e to the power of minus ea by rT_0 . Now if we now replace this thing then we say that if we say now say that are the similar dependency will hold also when there is a some amount of heat loss.

So, we will see f square is equal to λ by C_p times w , but not w_0 by Zeldi which number. Whereas, now w is essentially represent by to minus ea by rT_0 times T_f . You see that is the so that temperature. So, we say that the burning flux only responds to that this responding to the temperature, because that is the most important factor that controls the burning fluxes. So, we say that the essentially the same functional dependence holds and we only change the temperature. So, then we can write essentially this w in terms of these heat loss parameters and these things in a manner that this is essentially is equal to this. You can just do the analysis it is not it is not a little complicated it is quite straightforward.

And then where as this L tilde is essentially equal to L tilde is essentially equal to your this your this is the your quantity here ok.

(Refer Slide Time: 14:36)

Premixed Flame Extinction
 (through Heat Loss) (3/3)

C/B using D:

- $\tilde{L} \equiv 0, \tilde{f} = 1, f = f^0$
- from which $\tilde{f}^2 \ln \tilde{f}^2 = -\tilde{L}$. (13)

(13) is the generalized equation governing flame propagation with loss.

- For $\tilde{L} \equiv 0, \tilde{f} = 1, f = f^0$
- Extinction, turning point: $\left(\frac{d\tilde{L}}{d\tilde{f}^2}\right)_{ex} = 0$
- Solving: $\tilde{L}_E = e^{-1}, \tilde{f}_E = e^{-1/2}$

$\tilde{L}_E = e^{-1}$

13

So, with this you can estimate that, that we can find out that this normalized burning flux \tilde{f}^2 will be essentially is equal to $\tilde{f}^2 \log \tilde{f}^2$ will be essentially is equal to $-\tilde{L}$. Whereas, \tilde{L} is nothing but this quantity $e^{-\tilde{L}}$ to the $e^{-\tilde{L}}$ times L' whereas, L' is given by this.

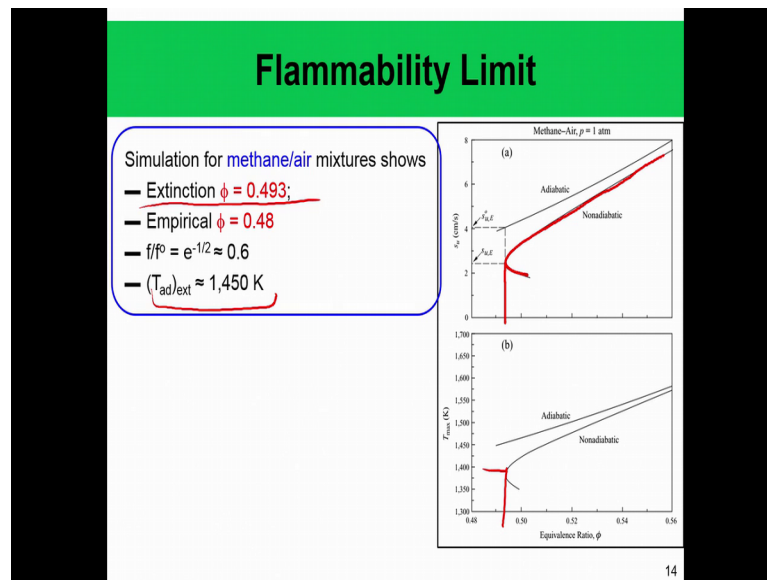
So, I will not go into the detailed analysis you can you should do it actually on your own. And but this shows the that just by considering the burning flux dependence on temperature, we can estimate this heat flux ratio, the we can estimate the burning flux ratio that is this \tilde{f} is essentially \tilde{f} is nothing but f/f_0 . We can estimate that ratio and we can find out the functional dependence and this is essentially non-linear dependence comes that is $\tilde{f}^2 \log \tilde{f}^2$ is equal to $-\tilde{L}$ ok.

So, this is the situation So, we can end if we plot now this thing we see that at of course, if we plot this whole quantity that is \tilde{f} as a function of this normalized heat loss parameter \tilde{L} . So, we see that when \tilde{L} is 0 then \tilde{f} is equal to 1. So, which is correct now as we increase \tilde{L} we see that this \tilde{f} slowly decreases. That is the ratio becomes smaller and 1, smaller than 1, smaller than 1, and then at this point it has a basically as a turning point.

So, then this turning point is essentially the extinction point, because it can be shown then the little this point is essential unstable. So, the flame essentially is essentially exists up to this point and that is at a at a value of this $\tilde{L} \approx 1$ by ϵ . So, at and that that value this \tilde{f} is essentially given by this thing. So, this clearly shows that the extinction point is given by this thing, and this is that that is when you have a when you have a heat loss when you have increasing heat loss, then extinction happens rather suddenly. Because you see here if you look into this region here it jumps it, it goes instead initially gradually and then it suddenly jumps like that before it turns.

So, this is the non-linear behavior that we are talking about. And this is corresponds to essentially the extinction. So, this analysis clearly shows that that with increasing a heat loss you will get extinction. And you will essentially get is heat extinction for all flames . And that is intuitively to understand, but this gives a very beautiful expression this $\tilde{f}^2 \log \tilde{f}^2$ is equal to $-\tilde{L}$ and from that which you can calculate this analytical calculate the extinction point.

(Refer Slide Time: 17:17)



So, this is an important analysis that we have done, finally we will end this topic with this flammability limit this is the limit at which that which mixtures can essentially can propagate. As a premix as a premix flame of course, if the system is particularly perfectly adiabatic which is not a practical scenario.

So, then the then there is very difficult to or rather impossible to find an extinction limit, but if there is some small amount of radiation then the flame this shows a this of this calculations of this one dimensional premix flame, with the radiation shows this kind of non adiabatic behavior where it has a extinction point, and that is happens and that we can quantify the flammability limit. And for that we can show that this extinction happens for methane air mixture from calculations we see that it happens at 0.493 whereas, for empirical it happens at essentially 0.48 and this adiabatic flame temperature at extinction is essentially at form 450 Kelvin which is quite small with respect to them stoichiometric adiabatic different temperature.

So, with that we come to an end for this part of these limit phenomena. And we essentially come to an end for all our lamina flame analysis. And then we will move on to turbulence and turbulent flame analysis, so till then goodbye.