Combustion in Air Breathing Aero Engines

Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture – 26
Laminar Premixed Flames V

So hello welcome back. So, as I said in the previous class that we will take up the Zeldovich Frank Kamenetskli analysis in this class, it is a basically a derivation I do not want to show it on slides I want to do the derivation for you and so please pay attention. This derivation there are two purpose if the first purpose is that in once again consider this laminar premix laminar planner laminar premix flame, that is stabilized at x equal to 0 in an w in finite domain that is x goes in the left hand side x goes up to minus infinity on the right hand side x goes up to plus infinity and of course, its laminar flame and there are no and it is in a if the flow is uniform and there is no non uniformity in the flow. So, that is no stretch on the on the flame also.

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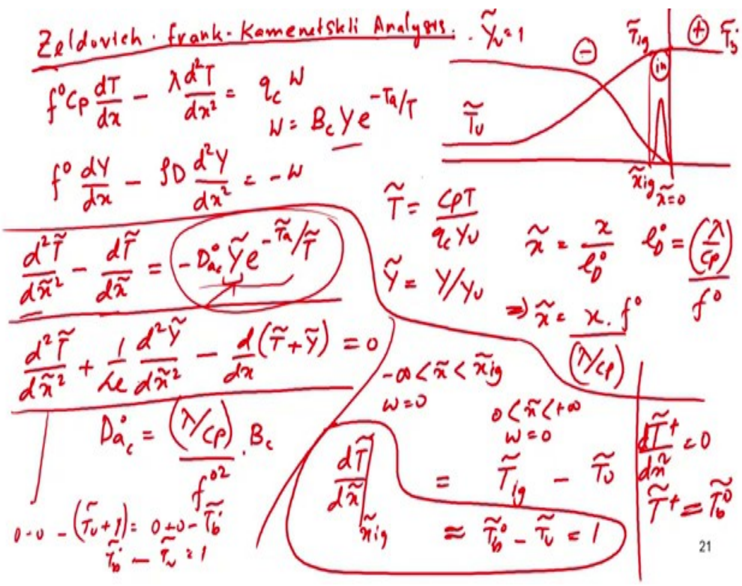

So, this is the Zeldovich analysis that we will do. So, once again this is our structure of the flame x equal to 0 that is the end of your reaction zone, also we will draw the reaction zone later. So, the temperature varies like this that is not linear because of the presence of both combustion convection and diffusion, this is Tb 0 which is the other very flame temperature of course, there is no heat loss, and we will call this as xig that is this temperature and this temperature to be essentially your Tig and from this xig to this part is essentially minus you will call this minus, we will call x greater than 0 to be plus. So, x less then xig will refer will refer it with this superscript minus anything greater than xig will refer it with plus and this inner part will refer it as inner layer in. Similarly there will be a our species profile this is Yu this is Tu and if you normalize Yu tilde is equal to one and this is Tu tilde.

So, everything can be normalized like this xig equals be normalized. So, this if you remember the equations is started with where the this way f 0 convective term minus diffusion term, w is essentially thus a species consumption rate and the diffusion on the species equation convection term minus diffusion term this is not omega this is w al right. Now when we normalize it by saying T tilde is equal to CPT by qc Yu normalized by Y tilde is equal to Y by Yu and x tilde is equal to x by lD 0 which is lD 0 is the diffusion zone thickness lambda by CP by f 0, this implies x tilde is equal to your x divided by lambda by CP 0 ok.

Now then if you normalize this things that the equations that we will get are in much thiner. So, the pay normalization is what the pay in and it becomes much cleaner in the sense that you clearly see this is the diffusion term this is the re convection term and that is given by minus Dac 0 Y tilde, and then you normalize the diffusion equation species diffusion equation and the add it to the normalized temperature equation and you will get ok.

So, once again remember we are just considering the diffusion species a diffusion is a species equation for the deficient species, because we then the other that is the abundant species can be assumed to be constant, and that is why in the reaction that only the diffusion species come in all right. So, now, with this we can proceed and first we what we will do is that we will integrate this equation in the in three different zones, we will integrate the equation in this minus zone, we will integrate the equation in the plus zone, and we will integrate this equations in the inner zone. And then we will see we there is one problem that we have to face is that this temperature equation while integration it cannot be integrated directly, because when you integrate in the inner zone where this reaction rate is dominant then you have to tackle basically this Y tilde ok.

So, how do you tackle Y tilde. So, something has to emerge from this other equation that is a coupling function equation so that you can write this temperature equation in a purely temperature form. So, we get you have to get read of the Y tilde in some other way, but before that if we integrate across the at the minus and this plus you see the most important thing is that we can conveniently set in these right hand side in this temperature equation to be 0. So, that is the advantage because that in this regions in the entire minus. So, from x equal to infinity to my xig there is no reaction taking place.

So, with the as I said before that there is a consequence of the fact that your activation energy is very large, and as a result of the activation energy is large your temperature your reaction is only activated at very high temperature right. So, there is no reaction taking place at in this minus zone there is no reaction taking place at plus zone. So, in these regions if we integrate that is if we integrate between minus infinity x to xig tilde then your right hand side w is equal to 0, similarly if you integrate between xig x similar between if you integrate between 0 to x tilde to x simple of plus infinity then also your w is equal to 0 all right.

So, if you do that what you get what you get is the following that is if we just integrate up to this boundary. So, in integrating form x equal to minus infinity. So, this is of course, equal to 0 because at x equal to minus infinity that is no temperature gradient. So, then this become equation becomes is equal T tilde minus Tu tilde now see the t tilde is very very close to your Tb 0. So, in many cases we can do the approximation of course, depending on the case we can do the approximation that this is equal to; because this difference T minus Tu is so large, we can easily replace the error generated by replacing T with Tb 0 is equal to very small.

So, we can generate with this and this Tb 0 minus Tu tilde as you have seen previously when we do the analysis that is equal to one. So, just refer to the previous class that your when we did this integration your this Tb 0 minus Tu tilde is equal to 1 and that came from this equation. So, when we just to integrate this equation from minus infinity to plus infinity what we got was that this will go went out this went out, and we got minus Tu tilde plus Yu tilde which is equal to 1 is equal to on the right hand side once again this went out this went out. So, we got 0 plus 0 minus Tb tilde ok.

So, as a result of that we got Tb tilde minus Tu tilde is equal to 1. So, that is the reason. So, now, once we have got that we let us stored this data they stored this in own memory and we will come back to this as soon after.

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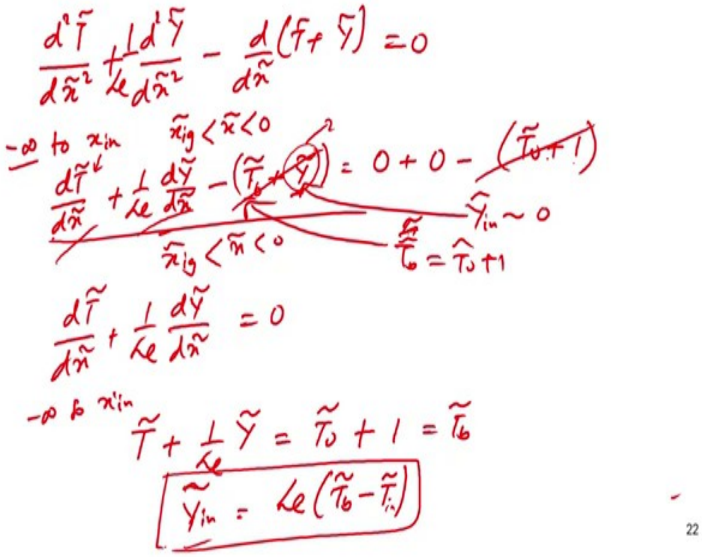

Similarly, on the right hand side if you do this just on the right hand side if you do this integration on the right hand side you will find integrating dT tilde dx tilde that if there is nothing was should do on the right hand side that is equal to 0. So, that is only condition that you have and that your t everywhere your Tu tilde plus is equal to your Tb tilde is 0 ok.

So, that is the that is all that you can have on the right hand side that is on the plus side. So, we will just remember this and not we will just store this, I will actually let us write it keep it stored in the previous class in the previous slide. So, your dT tilde plus dx tilde is equal to 0 and their T tilde plus is equal to is equal to your Tb 0 tilde. So, these are the things that you have all right.

So, now let us focus on the inner zone, and this is the most important thing because here you see that you this this is challenging because in the inner zone you have your temperature equation d 2 T tilde at you have the temperature equation like this d 2T tilde dx is square tilde minus d tilde dx tilde is equal to minus Dac 0 Y tilde e to the power of minus ta tilde by T tilde. So, the pervasive problem is that yes of course, this is challenging because there is a non-linear term e to the power of minus ta every over tTand this is even more challenging because this is you see here this thing is coming all right.

So, you have a Yu tilde that is a different variables we have to solve it in couple manner this is very difficult. So, we have to get rid of that sometime. So, for that to get rid of this what we will do is that we will consider this other equation that is there that is a coupled equation that is d 2T tilde. So, if you are not if you feel uncomfortable about this equations I suggest you just go back and derive this equation yourself, from the one dimensional energy from the simple diffusion control system equations you derive your one dimensional equation will go through the dedication once again and then you normalize it yourself, and then you come back and do this derivation again you can just pause this video right. So, I suggest you before you if you are not comfortable with this equation you should just take a pause and go back to this study division and then come back and then go through the analysis. Because this to comprehend this whole analysis the derivation is varying the derive the big understanding this the in different terms of this equation is very important and that understanding can only arise if you have the you are some derived equation ok.

So, this is once again the coupled equation that you get d 2 T tilde dx square minus 1 by sorry this is a there is a 1 by Lewis number here minus d d x tilde t tilde Y tilde is equal to 0. Now let us integrate this guy from minus infinity to any point inside x in. So, what we get is the following that is we get d tilde dT tilde dx tilde in minus or will not call in we will just remove the in and we assume that this T tilde that we get now is essentially anything is mentioned is within this within xig and x and 0. So, this is only in the reaction zone.

So, now we in integrating within the reaction zone of course, this is equal to 0 because at at x equal to minus infinity this is 0 plus again this is a at x equal to minus infinity this is 0, minus Tu tilde plus 1 right. So, this is the thing this Y tilde is equal to 1. Now at anything between and see if you remember the flame structure. So, now, this left hand side is between is between xig minus x tilde and 0. So, it is basically within this part. So, what have we got in this part you have got T tilde Y tilde.

Now, we can assume that T tilde is essentially compared to the other term this thing is essentially your Tb tilde or we can call it like T in tilde which is essentially close to you

Tu tilde in Tu Tb tilde and this part is essentially your Y in and Y in is essentially very small compared to the other term. So, it is almost equal to 0 when compared to all these terms and this is equal to Tb tilde and Tb tilde is equal to Tu tilde plus 1. So, xig in is essentially do you see this goes to 0 and this this Tb tilde is essentially will cancel with this thing. So, all these things essentially cancel out.

So, what we are left with is dT tilde dx tilde plus 1 by Lewis number dy tilde dx tilde, but please remember this holds only within the reaction zone it is not entire the thing it is not in the enter any part of the flame its only when you have left hand side in the reaction zone you can do this thing. So, it that is why the integration thing is important where we are integrating it from so, du tilde dT tilde dx tilde plus 1 by Lewis number dy tilde dx tilde is equal to 0 and what you get by integrating is essentially now if you integrate this equation from minus from x ig where. So, if you integrate this again actually from say minus infinity to two to xig x in. So, the what we did before we get t tilde plus 1 by Lewis number Y tilde Y in is equal to your t tilde is a Tu tilde plus 1 ok.

So, this Tu tilde plus 1 is nothing, but Tb tilde. So, we get essentially is Y in is equal to Tb tilde minus essentially is equal to your Lewis number times Tb tilde minus T tilde. So, as you saw that here we considered here this to be almost equal to 0, but when you considered with respect to other terms, but in general if this is small because you see inside in this is essentially this Tb in is Tb tilde is almost equal to t in tilde. So, in ig Y in is very very small this is itself a small quantity, but itself its I mean we can by doing this we can estimate how small it is that is the point here, and it is not 0 because if it was 0 then there would not have been rather than there would be no right hand side in this temperature equation.

So, it is not 0 actually, but it is what we have done here is that we are not considered a 0 we have just considered that this equation this term is essentially smaller than this terms. So, this term is essentially much smaller than this term, and this term which is a assumption. So, then we can write this Y in tilde essentially is equal to a Lewis number times Tb tilde minus T in tilde. So, this is a very important thing because now if you go back to this if you go back to our energy equation ok.

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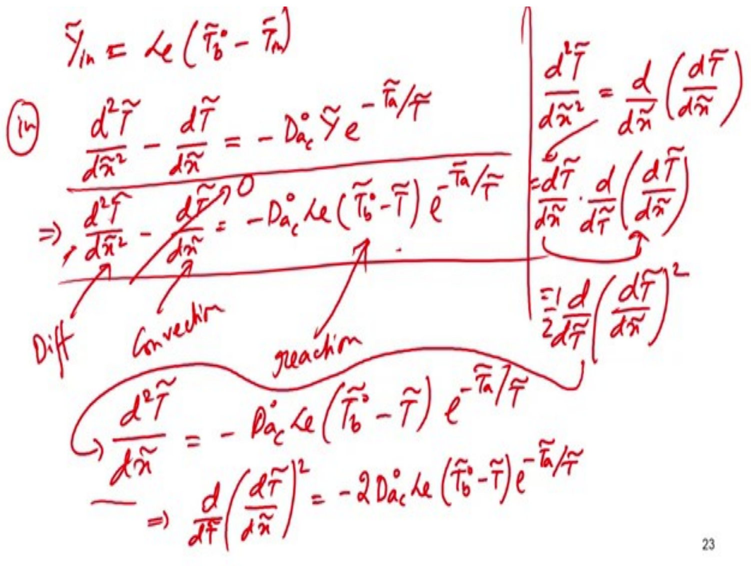

So, what we will get now we can substitute this we have got Y tilde in is equal to Lewis number times Tb tilde minus Tu T tilde at the inner zone T in tilde. So, now, we can substitute this thing inside our energy equation inside the inner zone. So, integrating in the inner region. So, everything is own is only within the inner region so that must be kept in mind. So, this was the equation we have replace the in because it is only with in the inner zone it is employed.

So, you see now this equation does not have any y. So, now, it is exact its explicit in t tilde and now we can do much more things with it is we can integrate it. Now one more thing is that you see within the inner zone that now this is basically comes from diffusion this if we can understand with this comes from convection and this comes from reaction. Now in the preheat zone we saw it is essentially a balance of your diffusion and convection and essentially in the reaction zone what we can do now is that because the temperature gradient is so, steep we can consider only the higher order temperature gradients actually the first order temperature gradient is not so, steep here as you can see from this.

So, but that second order temperature gradient is large so, and this can be shown by integration also that if you just do the integration you will see that this will give you the dT a plus dx tilde minus dT ig dx tilde plus minus this will give you Tb 0 tilde minus Tig tilde and then this is essentially is equal to minus Dac Lewis integral of this things right at dx tilde. So, this is very small because TbT you can Tb b 0 is very small which again suggests that this convection the convection process inside the flame inside the inner layer of the reaction zone is very small is small only in the in the inner layer ok.

So, that is why we can neglect this things and we will only consider this convection is essentially will be neglected and will only in the inner layer will only consider the balance between your diffusion and your and the reaction. So, this part will go to 0 now we will just a neglect it with respect to this term, this term this part is actually never 0 because obvious reasons and me what we will considered to be small with respect to this and this things.

So, now if you consider that what we get is. So, now, this also integration is difficult. So, let us do a small mathematically jugglery this is it right. So, we can write this as right this is exactly equal to this and then we can put this inside this. So, then we can substitute this whole thing here, and what we get is two comes from here now we are ready for integration.

So, if we integrate what we will find is that.

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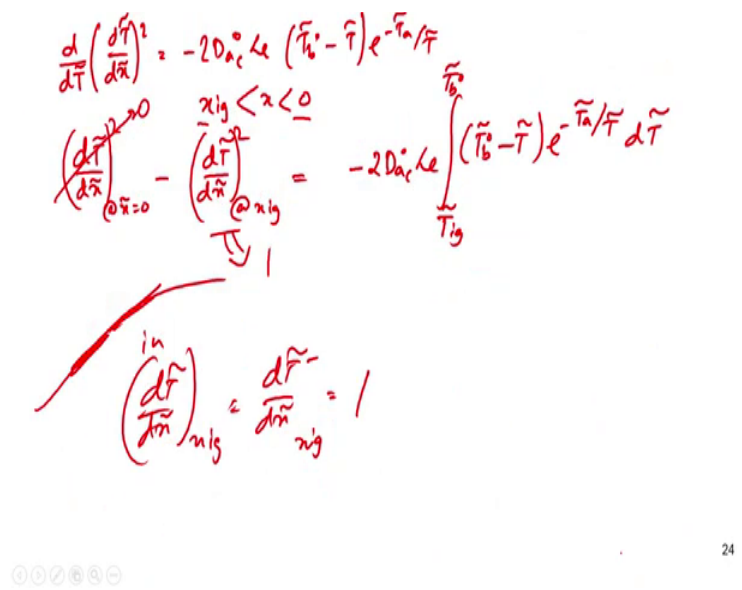

So, in this is the equation that we got then if we integrate this between xig and 0 from here to here if we integrate form xig from Tig tilde. Now if you remember this guy this act not yeah at x equal to 0 this is essentially equal to the right hand side this plus. So, this is essentially equal to by matching conditions your dT tilde dx tilde square at plus which is equal to 0. So, in the plus side. So, essentially we can get rid of this thing this goes to 0, but of course, this does not this does not go to 0. So, this goes to 0 and we can write.

So, from now I will focus only on the RHS because as we will see that because one we can also see that show that this by matching conditions that is by because this slope has to match this slope. So, at xig what is happening is that the temperature is increasing like this right. So, this is if it is xig. So, this slope because of the because of this continuity this and the smoothness this this slope has to manage this slope. So, essentially this on the this is the inner side and this is the minus side.

So, at this point this dT dx tilde xig is equal to your dT tilde dx tilde minus at xig right and this we obtained to be essentially is equal to 1. So, this is becomes essentially equal to 1 and if you this came from this fact that if you remember that your dT tilde dx at xig is equal to indeed, but this was a integrated for the minus sign at Tb 0 minus Tu 0 is equal to 1. So, essentially this is one. So, we can we will just focus on this right hand side from now because the left hand side is equal to minus 1.

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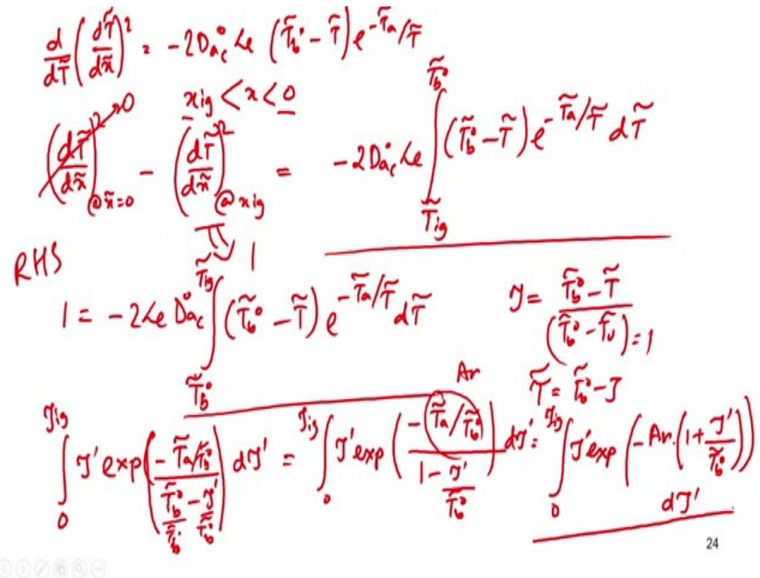

So, if you just remove the minus sign and put it here then it will get is essentially this one is equal to your minus 2 Lewis number times Damkohler number 0 times T b 0 tilde to

Tig tilde times Tb 0 tilde minus T tilde into the power of minus Ta tilde by T tilde dT tilde ok.

Now, let us consider we will just focus only on this right hand side now let us introduce another change of variables where tau is equal to Tb 0 tilde minus T tilde. And using we can also write it as like Tb 0 tilde minus Tu tilde because this this guy is equal to one. So, that is a good thing about normalization it becomes much simpler. So, essentially then it becomes then this becomes is equal to we will just consider this we figure is consider this integral then this becomes Tb 0 then this is become 0, and this T becomes Tb 0 tilde minus T tilde times tau or tau dash, what we will just represent this this guy as tau ig Tb is tau dash minus e to the power of minus Tig.

So, if we substitute this now t tilde is nothing, but Tb 0 tilde minus tau. So, we can write as exponential of minus ta tilde divided by Tb 0 tilde minus tau. So, this is the thing d tau dashed tau dash and then we can write if you know the to bring this exponential downstairs actually we can should write it like this form, and then it this becomes if you normalize both sides by Tb 0 tilde, Tb 0 tilde and Tb 0 tilde what you get is 0 to tau ig tau dashed exponential of minus; obviously, 1Tb 0 minus tau tilde whereas, this is a small quantity because tau tilde is close to be 0 tilde. So, tau tilde is will definitely much much smaller than Tb 0 tilde, and then if this is a small quantity we can write this as and this is you see if you recognize is Arrhenius number, this obtained from the binomial expansion ok.

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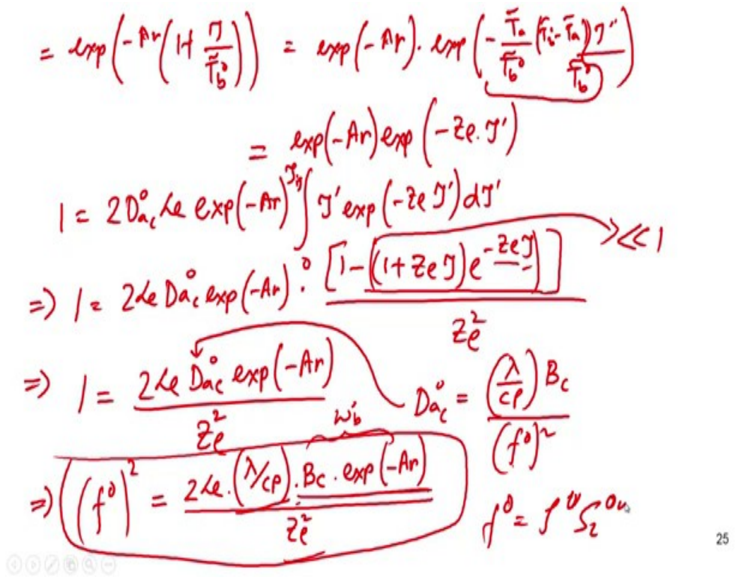

Simplified into this is what we can write this is what we had exponential of minus Arrhenius number the times 1 plus tau and this can be essentially written as exponential of minus Arrhenius number times exponential of minus tau a tilde by Tb 0 tilde times times tau by Tb 0 tilde.

Now this is one of course, and we can replace the one by Tb 0 minus Ta and then you see this becomes exponential of minus Arrhenius number times exponential of this whole thing is nothing, but Zeldovich number minus Zeldovich number times tau tilde . So, now, if you substitute this whole thing in this integral what we get is, the whole thing becomes one is equal to integral sorry this two times Lewis number times damkohler number integral 0 to tau ig is essentially equal to tau dashed times exponential of and this also can actually go out. Now this is only within the inner layer just remember that thing ok.

So, this is the integral that we get. So, now, if we integrate this further what you get is in this integral becomes there is a Zendovich number square in the downstairs times 1 minus 1 plus is Zeldovich number times tau e to the power of minus Zeldovich number tau that is the integral. Now of course, we can say that what you have is now we see that the Zeldovich number appears on the in the numerator of the exponential of course, it is multiplied with tau.

So, it is not obvious that it will go to 0, but if Zeldovich number is so, large that is different can overcome for this in overcome this tau then essentially this whole thing becomes essentially 0, that is where p only because of the fact that you are this is an exponential in the it is exponentially in minus Zeldovich number so, then this whole thing becomes 1. So, we will consider that the problem is post such a way that this whole thing goes to goes to 0 or its much smaller than much much smaller than one. So, then this becomes two times Lewis number times Dac is a times exponential of minus Arrhenius number by Zeldovich number.

Now, this closes the problem why because if you have to remember this definition of of essentially the Dac the collision the Damkohler number and that contains the collision Damkohler number and that contains lambda by CP times Bc by f 0 square now let us substitute it here. So, if you substitute this into this you immediately see the f 0 square comes in the denominator. So, we can take it on the left hand side. So, then it implies f 0 square this is Zeldovich number square two times Lewis number times ok.

Now, this is this Bc times exponential of minus Arrhenius numbers number is essentially your wb 0 right that is a reaction that measured at the near the bond gas temperature. So, so, but. So, this equation then it is becomes very very similar to the scaling equation that we obtain previously, but of course, you see that there is a Zeldovich number squared dependent in contrast to the previous thing where we obtained that where we obtained this like and with there was a Zeldovich number to the one dependent.

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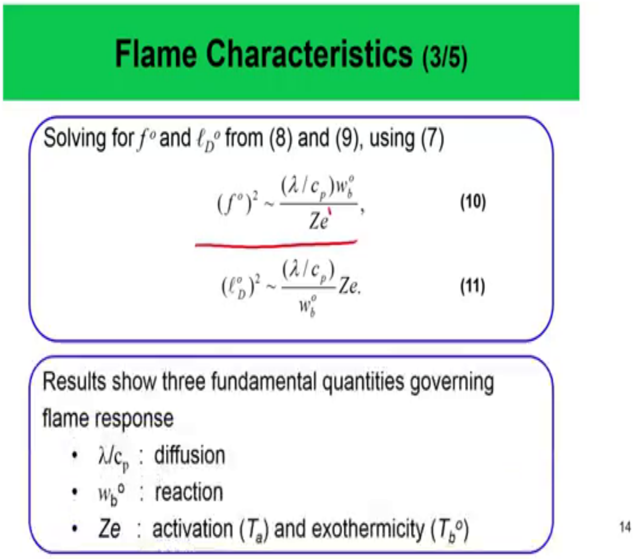

So, of course, when you do this. So, here actually the two Zeldovich number comes from one comes from the reaction rate and the other comes from the fact that y you another Zeldovich number essentially comes from the Y in is essentially is proportional to one by zeldavich number. So, of course, we did the scaling analysis you did not consider that right so, that is where this comes from. So, that is where this analysis if you see its much more appealing and this gives a very nice expression of and shows you what this thing depends on, and what they have 0 depend on essentially, but this basic nature of the fact that f 0 is essentially a geometric mean of your lambda by CP and the reaction rate that is still maintained here and so, it is indeed that you that that inside was indeed correct, but this is the complete expression.

So, as you. So, know that f 0 is essentially rho u times rho u times SL u that is SL SL 0 or SL u that is the flame speed on the bound side. So, this essentially gives you the explicit expression for the flame speed and which can reach your derived. So, you I argue to go through this derivation once more, and be familiar with the detailed analysis of it. So, essentially what we have done here if we summarize if we consider the frank Kamenetskii analysis.

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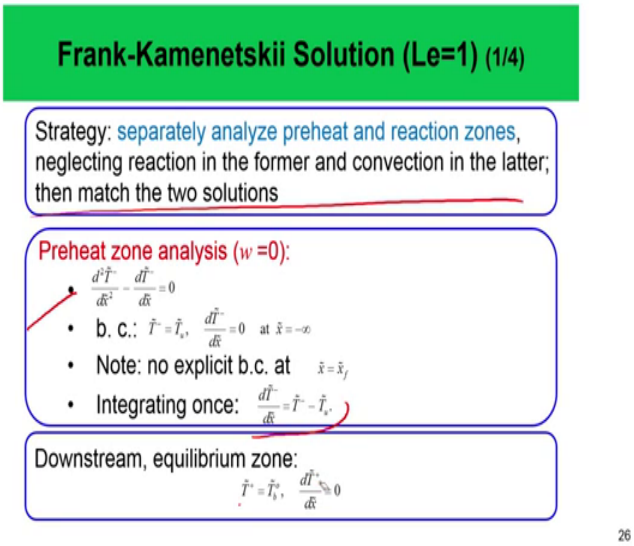

We have essentially separated the analyze the separately analyze the preheat on the reaction zone and we have neglected the reaction in the former reaction in the former zone that is in the preheat zone and the convection in the latter and then we have match the solutions.

So, preheat zone analysis was this that we consider the w to be equal to 0, and this we integrated once and we found this and if the because t plus is and in the equilibrium zone of course, your downstream equilibrium zone.

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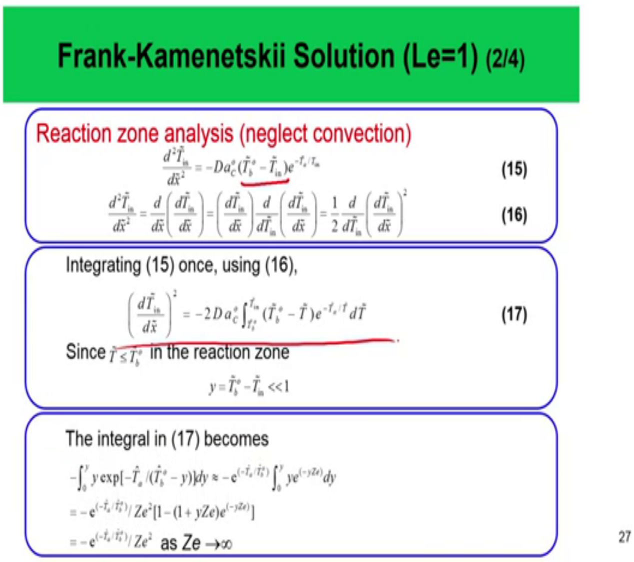

That this is TDT dT dx tilde is equal to 0 of course, then we did the reactions zone analysis where essentially we substitute at the y into Tb this Tb 0 minus t in and then we integrate it in this form and then we found that now that dT this is equal to this dT in dx square is essentially be at x x at x at xig this essentially equal to one.

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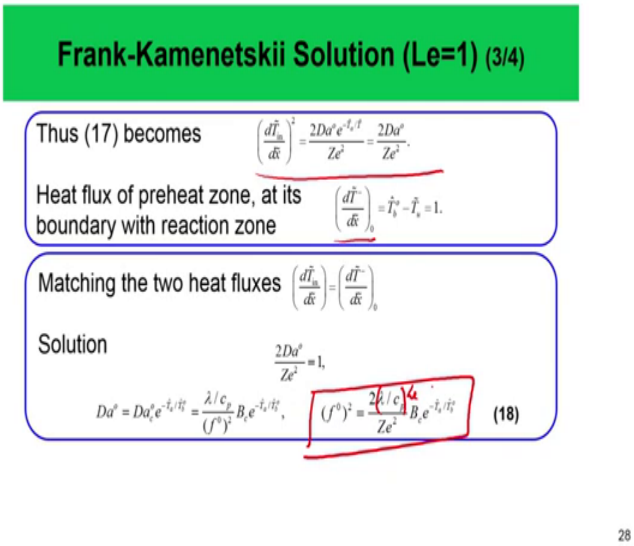

And then we matched it and then we find this expression this very nice expression of f 0 square is equal to, but this is this is what is in the slides essentially for Lewis number one, but we have not done it in the analysis we have considered Lewis number to be not equal to one. So, it appears in the numerator if it is not so.

So, it appears this is a beautiful relation that we get the f 0 square there is a burning flux squared is equal to essentially 2 times lambda by CP by Zeldovich number squared times bc times e to the power of minus a tilde by Tb tilde.

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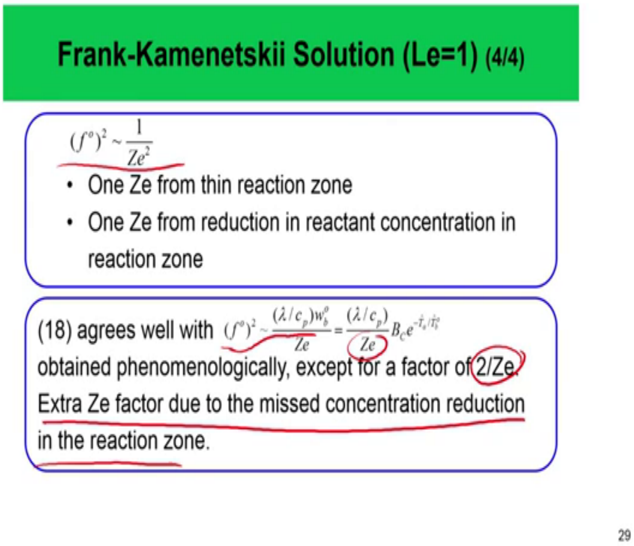

So, you see that here f 0 is essentially proportional to 1 by Zeldovich number square. So, one is a there Zeldovich number comes from the thin reaction zone of the that analysis on the other forms. So, on the reduction in the reactant consider in the concentration in the reaction zone and you see that this ig means well with this previous scaling analysis, but you see that you did not get the Zeldovich number squared you only get our Zeldovich number one, and also we did not get the factor of two we it was correct except for the factor of two by Zeldovich number.

So, extra Zeldovich number we missed due to the concentration of due to the concentration reduction in the reaction zone essentially in the in the reaction zone the concentration goes most small value Yig or small Yin, but it then goes to 0. So, there is a reduction it is not constant so, that another Zeldovich number constant. So, I will stop here and then in the next class we will go into how to measure basically the flame speeds, and how to basically find out on what parameters the and basically understand on what parameters is of flames we depend on flames speed is a very very important quantity its basically this quantity determines whether you can have a flames stabilized in a natural gas combustor, you know that you have a flame stabilized in a premixed combustion in aero gas turbine engine whether you can have a flame stabilized in a planer whenever there is a premix combustion happening and the flame is to be stabilized.

So, this determines a not in the flame speed of course, you can stabilize flames which are much faster than the flames speeds, but that relies on something else which will come later, but the property the inherent property of the flame that governs flames stabilization is essentially flames speed. So, you go over this in details you do the analysis on your own hands and we comfortable with it and this gives you a lot of confidence in premix combustion so.

Thank you very much and see you next class.