# Combustion in Air Breathing Aero Engines

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## Lecture - 25
## Laminar Premixed Flames IV

So, hello, welcome back, so, as you know we were talking about laminar premixed flames, and in that regard we were discussing the we were discussing at first about the laminar premixed flame as a wave phenomena as a chemical wave phenomena that propagates into the un-burn fuel air mixture, and then we were considering how to connect the downstream and the upstream properties of this wave, and then we did this all this Rayleigh lines we got this Hugoniot curves and we found that at the intersection points which are given by basically Rankin Hugoniot conditions as well as Chapman Jouguet conditions what kind of properties we can expect.

But then we found out that this description except for the Chapman Jouguet point that is a tangency point in incomplete because it does not take into account the structure of the wave, and as a result because the wave structure is not resolved it does not tell you about the wave speed. So, the wave structure and the wave speed these are very fundamental parameters that we must know as such the wave speed is essentially the laminar flame speed and to know that you basically need to consider the wave structure and as a result we need to understand the structure and the analysis of laminar premixed flames which are also called the deflagration waves because they are propagating at sun sonic speeds alright.

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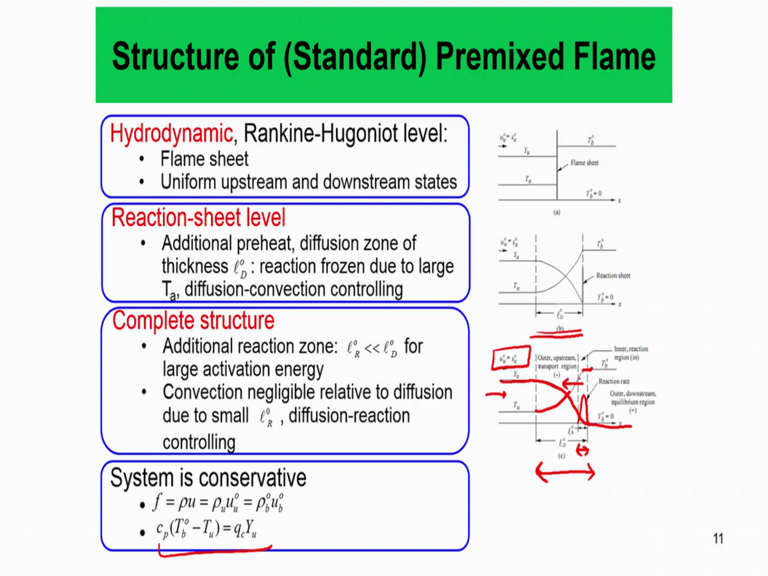

## Structure of (Standard) Premixed Flame

**Hydrodynamic, Rankine-Hugoniot level:**
- Flame sheet
- Uniform upstream and downstream states

**Reaction-sheet level**
- Additional preheat, diffusion zone of thickness $\ell_D^o$ : reaction frozen due to large $T_a$, diffusion-convection controlling

**Complete structure**
- Additional reaction zone: $\ell_R^o << \ell_D^o$ for large activation energy
- Convection negligible relative to diffusion due to small $\ell_R^o$ , diffusion-reaction controlling

**System is conservative**
- $f = \rho u = \rho_u u_u^o = \rho_b^o u_b^o$
- $c_p(T_b^o - T_u) = q_c Y_u$

So, now if you go into that we were discussing these things that in the in the hydro dynamic limit that is of the Rankin Hugoniot level we consider a flame sheet, and the n uniform upstream and downstream states and then this was step separate way of flame sheet and at the reaction sheet level we consider that the flame had a pre heat zone that is this is the pre heat zone which is a finite thickness where the temperature increases, but still the reaction zone is very very narrow. And the reaction zone is narrow because it is it arises from the fact that your activation temperature or the activation energy is very large and that is the result to why that is the reason why your reaction zone is narrow and that is which justifies this assumption of a reaction zone being considered as a reaction sheet and we consider the temperature increase happening over finite region.

And we considered that this frozen this part is the reaction zone is essentially for this part they does not have any reaction because it is basically diffusion and convection controlling, and then we go into the complete structure at least in the analysis at least for analysis where we consider that the reaction zone as a much much smaller than the diffusion zone and it has become once again because of large activation energy, and then the convection in this zone is relatively negligible compared to the compared to basically this process is convective diffusive and we consider this process to be diffusive reactive.

So, in this structure that is you have a flame here, you have a flame sitting this is your flame. So, the flame is essentially stationary with respect to the reference flame so; that means, that the fuel air mixture must be approaching the flame at a speed equal to local flame speed, and that is the planar laminar flame speed in this case and that the therefore, which gives rise to this condition that is U u 0 that is un burn velocity must be equal to the laminar flame planar laminar speed.

And then as it approaches the temperature should increase because the temperature should increase because of the; because you have heat release in this. So, as a fuel air mixture approaches it is going to be burn and that as we said because the activation energy is very large, the reaction zone of the heat release zone is confined in a very small region and once heat release happens then this heat is basically conducted upstream. So, that is how the flame structure is maintained that is. So, the fuel air mixture is coming from here and then it is heat is being released here in this reaction zone, and this heat gives rise to the high temperature because it is essentially control conversion of enthalpy of formation to enthalpy a sensible enthalpy as a result of which the temperature rises and once the temperature rises it is the heat is being diffused upstream to the lower temperature region and that causes this fuel emission to heat up right.

And once again the fuel air mixture the fuel mass reaction is essentially consumed at the reaction zone and that cause it decrease, and this continuous supply is happening because there is no fuel at the downstream. So, then what we discussed is that then the because of the large activation energy your reaction zone is confined in a very narrow region whereas, your pre heat zone is confined in a much larger region and then we can find out what is the relative thickness of the of the reaction zone with respect to the pre heat zone.

And of course, this is conservative and as a result of that your we can arrive at this equation that is the that is the burner gas temperature is essentially equal to adiabatic gas adiabatic flame temperature, but then it is adiabatic flame temperature at the given stoichiometric that is where we deviated from a non premixed flame where the temperature is essentially the adiabatic flame temperature the stoichiometric adiabatic flame temperature whereas, that in a premixed flame the temperature is essentially or your adiabatic flame temperature at a given field.

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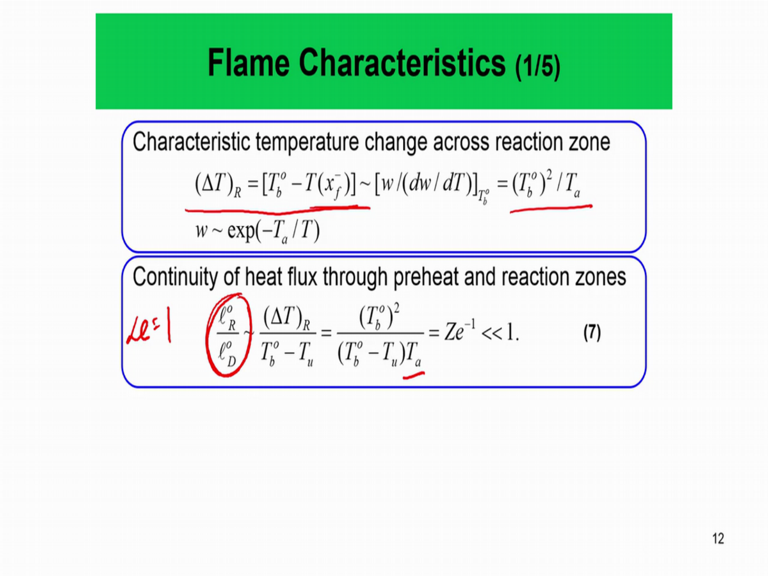

## Flame Characteristics (1/5)

Characteristic temperature change across reaction zone

$$\underline{(\Delta T)_R = [T_b^o - T(x_f^-)]} \sim [w/(dw/dT)]_{T_b^o} = \underline{(T_b^o)^2 / T_a}$$

$$w \sim \exp(-T_a/T)$$

Continuity of heat flux through preheat and reaction zones

$Le = 1$

$$\frac{\ell_R^o}{\ell_D^o} \sim \frac{(\Delta T)_R}{T_b^o - T_u} = \frac{(T_b^o)^2}{(T_b^o - T_u)\underline{T_a}} = Ze^{-1} << 1. \qquad (7)$$

So, now we estimated at this we estimated at these things that is what could be the to estimate what should be the relative thickness of the reaction zone with respect to the pre heat zone, we found out the temperature across the temperature between the between that is at the adiabatic flame temperature of the burner gas minus the temperature at the start of the reaction zone, which is T x of minus and that is given by this the T b 0 square divided by T activation energy.

And then we found out that the ratio of the reaction zone thickness to the to the diffusion zone thickness is of course, here we do not distribute discriminate preheat mass diffusion and molecular and thermal diffusion, we assume that the Lewis number is essentially equal to one and under that assumption that is the Lewis number is equal to one we arrive at the assumption we arrive at the analysis that the arrive at the conclusion that the reaction zone to preheat zone thickness is given by 1 by Zeldovich number, and of course, Zeldovich number is a very large quantity because of the fact that it is the activation energy is adiabatic the activation temperature is very large adiabatic; as a result the l R by l D 0 is much longer than 1.

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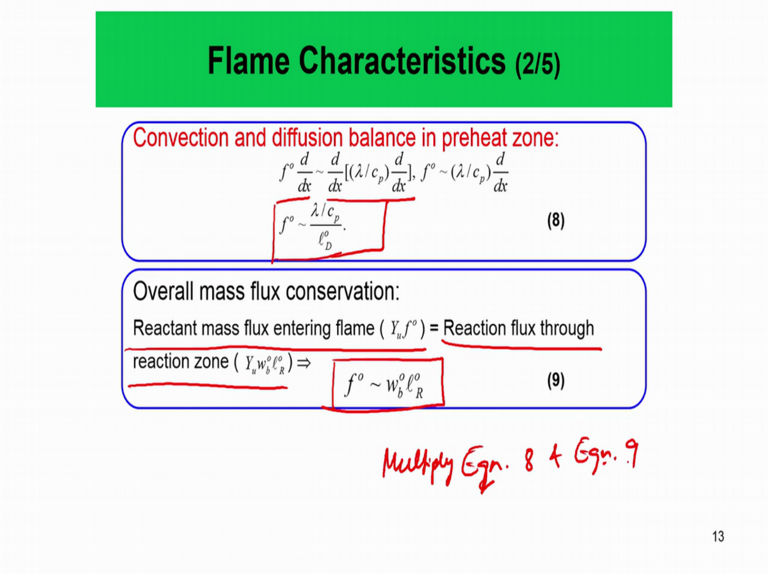

## Flame Characteristics (2/5)

Convection and diffusion balance in preheat zone:

$$f^o \frac{d}{dx} \sim \frac{d}{dx}[(\lambda / c_p)\frac{d}{dx}],\ f^o \sim (\lambda / c_p)\frac{d}{dx}$$

$$\boxed{f^o \sim \frac{\lambda / c_p}{\ell_D^o}.} \qquad (8)$$

Overall mass flux conservation:

Reactant mass flux entering flame ( $Y_u f^o$ ) = Reaction flux through reaction zone ( $Y_u w_b^o \ell_R^o$ ) ⇒

$$\boxed{f^o \sim w_b^o \ell_R^o} \qquad (9)$$

Multiply Eqn. 8 + Eqn. 9

So, this is the thing and then very importantly we wanted to analyse that is the burning flux can be at can be arrive at a qualitative measure of the burning flux and for that we said that you see in the flame structure ahead of the reaction zone in the preheat zone, there is no reaction happening. So, essentially it is a balance of convection and diffusion. So, if we can balance these 2 things. So, it is this is a convective side this is the diffusive side, and then if you can balance we get f 0 is nothing, but lambda by C p times l D 0 and the once again we can balance between the reactant mass flux entering the flame which is given by Y u f 0, and the reaction mass flux that is the entering the flux. So, the reaction zone which is given by Y u times reaction rate at w b 0 times l R 0 and then this is given by f 0 is equal to w b 0 times l R 0.

And if we then multiply these 2 things that is sorry this this we multiply this that is equation if you multiply equation 8 and multiply equation 8 and equation 9 we get f 0 square essentially is equal to lambda by C p times w b 0.

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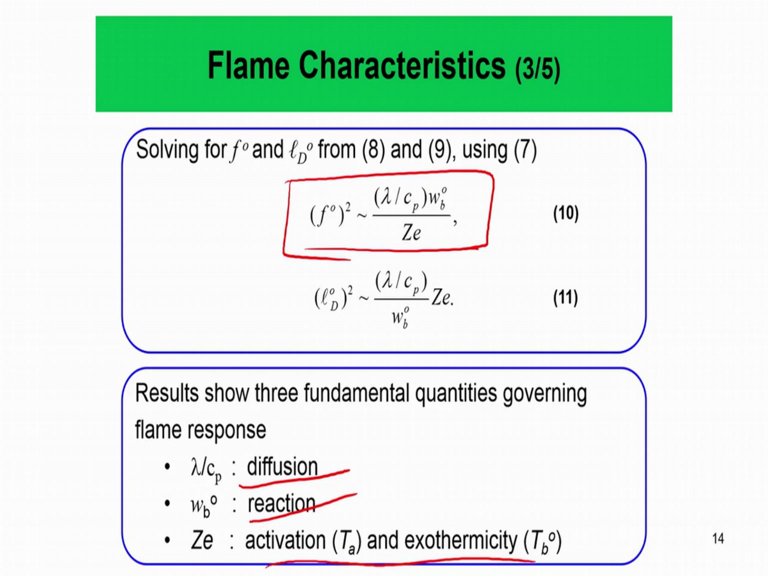

## Flame Characteristics (3/5)

Solving for $f^o$ and $\ell_D^o$ from (8) and (9), using (7)

$$\boxed{(f^o)^2 \sim \frac{(\lambda/c_p)w_b^o}{Ze},} \qquad (10)$$

$$(\ell_D^o)^2 \sim \frac{(\lambda/c_p)}{w_b^o} Ze. \qquad (11)$$

Results show three fundamental quantities governing flame response

- $\lambda/c_p$ : diffusion
- $w_b^o$ : reaction
- $Ze$ : activation ($T_a$) and exothermicity ($T_b^o$)

That is a reaction rate at the burner gas temperature this reaction rate is essentially not there in mean is essentially the is we will see later it is essentially the fuel consumption rate at the burner gas temperature at divide by Zeldovich number.

So, this is the. So, we arrive at the conclusion that this is essentially this f 0 is essentially controlled by diffusion lambda by C p reaction w by 0 and activation temperature activation energy and exothermicity which are given by Zeldovich number which contains both activation temperature as well as exothermicity. So, then we discussed that this is qualitative and see this is qualitative we just arrive at like as an order of (Refer Time: 08:56) to essentially analysis by a (Refer Time: 08:57) analysis then we have to do more detailed analysis where we have to check that how accurate is this conclusion that f 0 square is equal to lambda by C p times w b 0 divided by Zeldovich number.

So, to do that basically we have to do a more detailed analysis and that we will see later in the following.

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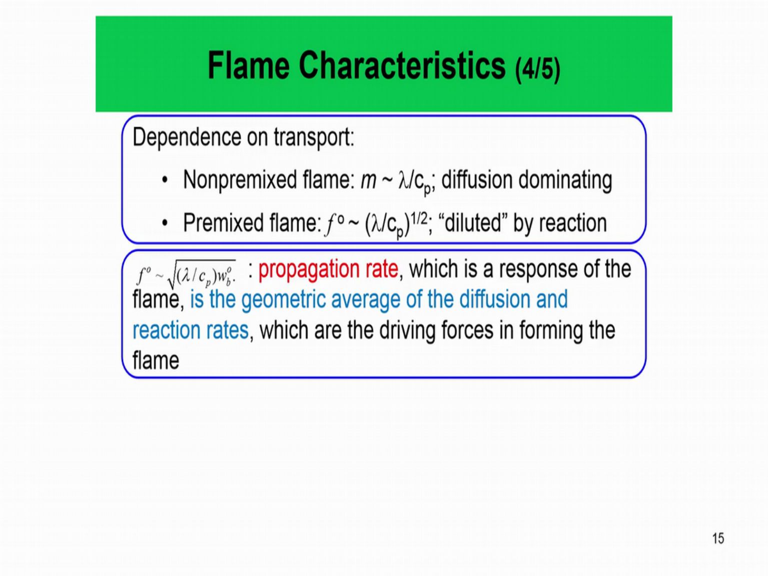

## Flame Characteristics (4/5)

Dependence on transport:

- Nonpremixed flame: $m \sim \lambda/c_p$; diffusion dominating
- Premixed flame: $f^o \sim (\lambda/c_p)^{1/2}$; "diluted" by reaction

$f^o \sim \sqrt{(\lambda/c_p) w_b^o}$. : propagation rate, which is a response of the flame, is the geometric average of the diffusion and reaction rates, which are the driving forces in forming the flame

Before that we are once again contrast be doing non premixed flames and premixed flames, and we said found that the burning flux of the non premixed flames proportional to lambda by C p which says it is diffusion terminative whereas, for a premixed flame you see that this is f 0 is equal to lambda by C p to the power of half and it is essentially diluted by reaction and that is the a thing that we get here.

So, f 0 is essentially lambda by C p to the power of half times reaction (Refer Time: 09:50) to the power of half, which we can see also that the burning flux is essentially is a unit mean of your thermal diffusivity, where at the density is removed lambda by C p times reaction rate, but it means that both the diffusion as well as reaction are equally important role of premixed flame whereas, for a diffusion flame or a non premixed flame you are essentially decontrolled by diffusion.

So, this was a thing that that f 0 while f 0 is equal to lambda by C p times reaction rate is a propagation rate, and the burning flux which is a which is a response of the flame which is the most important property of flame is a unitary diversion of diffusion and reaction rates and these are the driving forces informing the flame alright.

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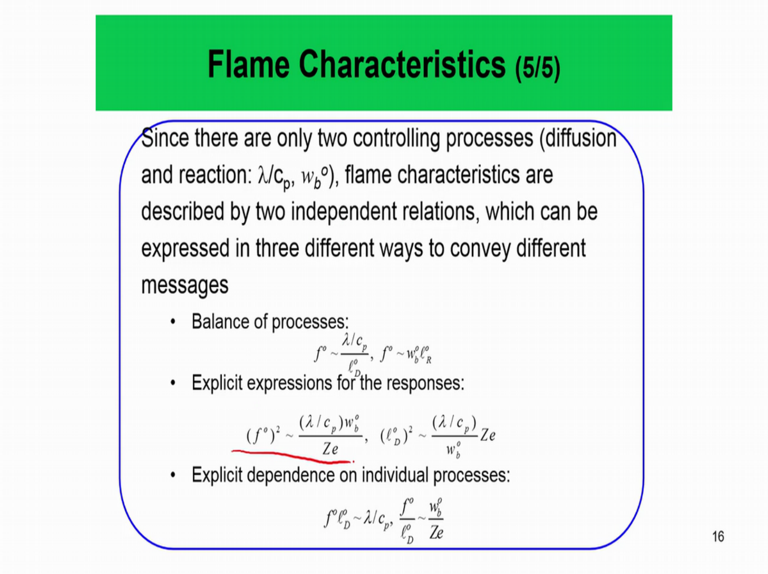

## Flame Characteristics (5/5)

Since there are only two controlling processes (diffusion and reaction: $\lambda/c_p$, $w_b^o$), flame characteristics are described by two independent relations, which can be expressed in three different ways to convey different messages

- Balance of processes:

$$f^o \sim \frac{\lambda/c_p}{\ell_D^o},\ f^o \sim w_b^o \ell_R^o$$

- Explicit expressions for the responses:

$$\underline{(f^o)^2 \sim \frac{(\lambda/c_p)w_b^o}{Ze}},\ (\ell_D^o)^2 \sim \frac{(\lambda/c_p)}{w_b^o}Ze$$

- Explicit dependence on individual processes:

$$f^o \ell_D^o \sim \lambda/c_p,\ \frac{f^o}{\ell_D^o} \sim \frac{w_b^o}{Ze}$$

And then we these are the summary that we have obtained that this f 0 square is equal to lambda by C p times w b 0 divide by Zeldovich number.

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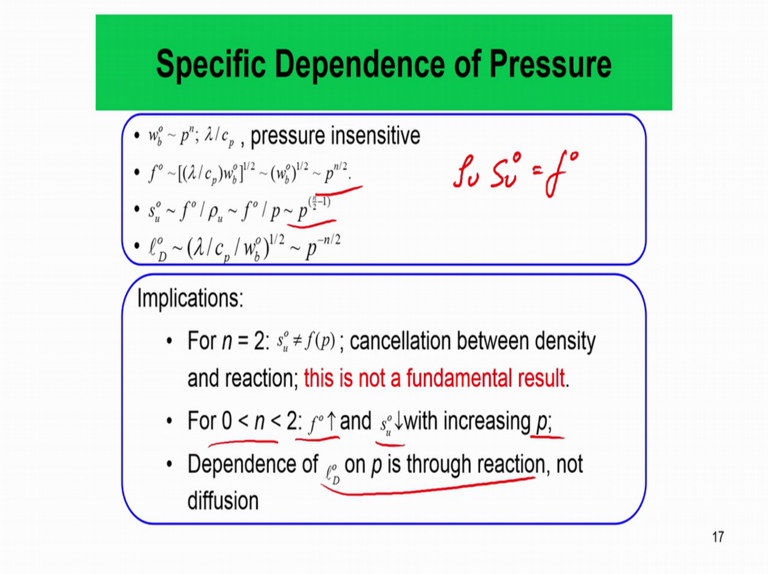

## Specific Dependence of Pressure

- $w_b^o \sim p^n;\ \lambda/c_p$ , pressure insensitive
- $f^o \sim [(\lambda/c_p)w_b^o]^{1/2} \sim (w_b^o)^{1/2} \sim \underline{p^{n/2}}.$
- $s_u^o \sim f^o/\rho_u \sim f^o/p \sim \underline{p^{(\frac{n}{2}-1)}}$
- $\ell_D^o \sim (\lambda/c_p/w_b^o)^{1/2} \sim p^{-n/2}$

$\rho_u S_u^0 = f^0$

Implications:

- For $n = 2$: $s_u^o \neq f(p)$; cancellation between density and reaction; this is not a fundamental result.
- For $\underline{0 < n < 2}$: $\underline{f^o \uparrow}$ and $\underline{s_u^o \downarrow}$ with increasing $\underline{p}$;
- Dependence of $\ell_D^o$ on $p$ is through reaction, not diffusion

And then we looked into the pressure sensitivity and we found that the reaction rate can be said that it is proportional to pressure to the power of n, if n is the order of the reaction then we can show f 0 is essentially pressure to the power of n by 2 and consequently you can say that because of the density dependence of S u wise that is S u 0, S u 0 is equal to nothing, but your f 0 your f 0 u times S u 0 is essentially nothing, but your f 0 the 0 basically comes because it is essentially standard parameters this flame that we are discussing is essentially a standard laminar premixed flame it is a planar laminar premixed flames without any flow non uniformities and that is why this 0 comes at standard parameters. So, now, it is an adiabatic condition rate in a w infinite domain or the flame.

So, these are these are the things and these are things that if f 0 that for pressure increasing your f 0 definitely increases what S u to 0 might go down if n is negative and 0 and 2, and the dependence of l beyond pressure wise more fundamental that is reaction that is a flame thickness decreases in pressure, because of reaction not due to diffusion. So, that is a thing.

So, that is that is one of a very interesting thing that reaction rate actually causes the diffusion zone thickness to decrease and not through diffusion. So, that is one of interesting thing that you should take a note about.

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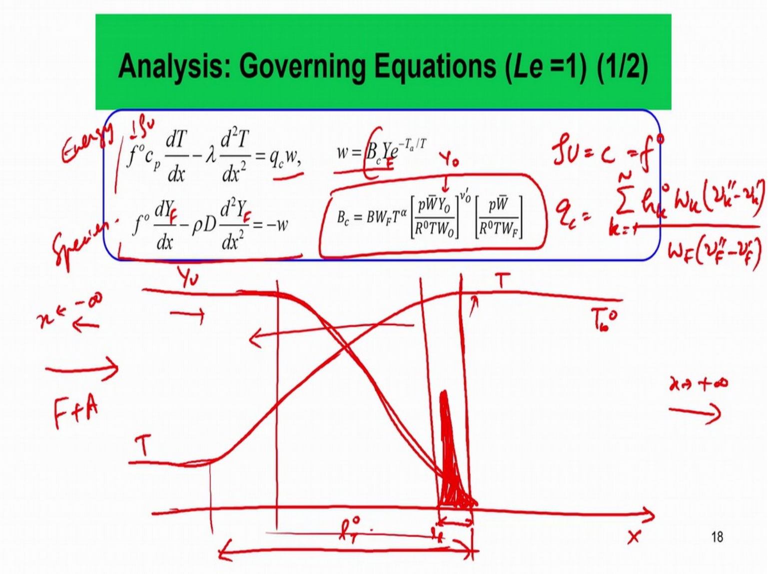

Analysis: Governing Equations (*Le* =1) (1/2)

$$f^o c_p \frac{dT}{dx} - \lambda \frac{d^2T}{dx^2} = q_c w, \qquad w = B_c Y e^{-T_a/T}$$

$$f^o \frac{dY}{dx} - \rho D \frac{d^2Y}{dx^2} = -w \qquad B_c = B W_F T^{\alpha}\left[\frac{p\bar{W}Y_O}{R^0 T W_O}\right]^{\nu'_O}\left[\frac{p\bar{W}}{R^0 T W_F}\right]$$

So, next we go onto the analysis. So, once again now our analysis is basically on the same thing that is what we are considering. So, far that is if this; our domain is a one dimensional flame. So, if you have a flame here, and this fuel air mixture is coming and the flame is essentially stationary this is the basically the final boundary of the flames. So, where you have the reaction zone like this.

Now, where you have the reaction zone like this and your diffusion temperature zone like this your species diffusion zone like this and. So, then this your essentially equal to your l D or rather l T 0 because this is a thermal flame thickness and this we can say that the is essentially the mass diffusion zone thickness and, but this is definitely the reaction zone thickness that we have and this is T b 0.

So, this is basically as you see is a double infinite domain this goes up to infinity plus infinity this goes up to essentially equal to minus infinity. So, here x goes up to minus infinity here x goes up to plus infinity and these 2 directions, and this is your temperature this is your temperature and this is your fuel mass fraction. So, once again to repeat your fuel un-burn fuel air mixture is approaching the flame from the left and as because of flame is stationary, it is coming in this and then here the reaction zone is happening the reaction is consuming this fuel that is the fuel mixture that is being supplied, and it is consumed here as a result of which it is a the heat resell being released and because the heat release the temperature has reason and that is the enthalpy of formation has been comforted to enthalpy of there is a sensible enthalpy.

And once it becomes hot then heat must be conducted upstream and which pre heats this premixed fuel air mixture that is coming, and that gives rise to this broader flame thickness**,** but if you see typically the flame is very small actually it is say when it goes this l T 0 itself goes from about 0.1 millimetre to 1 millimetre for a practical flames at a positive pressure, and as you increase the pressure that is in a aero gas turbine engine if you have a premixed flame this flame thickness should be have not small.

So, which means that there should be your grade resolution to resolve the flame thickness should be very very small you should at least have 20 grade points or 15 to 20 grade points in this flame thickness to accurately to do a simulation. Now how do we modern this problem we this is the one dimensional equation. So, this all the energy equation that we derived we do not need to consider the momentum equation because the momentum equation essentially boils down to your u d u, d x though u d u, d x is equal to minus T p d x and we consider the that for order one acceleration your pressure gradient should be order of for your should be of the order of mach number squared and here we are considering about subsonic flames.

So, your pressure gradient is really small. So, we do not need to bother about the momentum equation and that mass flux rho u is essentially that is constant everywhere because of the continuity equation. So, that is that the momentum and the continuity equation boils down to rho u is equal to f 0 is equal to constant. So, rho u is equal to constant is equal to f 0, and that is why here essentially we had rho u and we can consider the energy equation directly. So, you get rho u times C p d T d x minus lambda thermal conductivity times d 2 T d x square is equal to q c w, q c is a chemical heat released at normalised which was given by divided by reference fuel.

So, this is when we write it is terms of this thing and this is essentially the heat release rate that we get and this is the species equation this is energy equation that we have that is f 0 times d Y d x minus rho d times d 2 Y d x square is equal to minus w whereas, this is the this is the species say we call this as Y F essentially or Y when the this Y can be written in this form that is equal to essentially b c times Y times e to the power minus T a by T.

Now, we can there is point here if you can question that you see at least for the reaction to take place in the fuel and the oxidiser. So, there should be Y f and Y o. So, this Y is essentially Y f; that means, this is also Y f, but here we are considering basically if we consider conditions which are far away from this stoichiometry that is it is either lean fuel lean or fuel say it is a fuel lean condition then the reaction essentially depends on the Y f and the Y o can be considered to essentially constant for this parameter, and this Y o constant can be absorbed within this B c and you see that the B c is given by this form.

So, this is then what we have as such with an Y os can be assumed in the we see here you see that there is a this is a Y o determinants. So, this, but this is only valid this kind of assumption is only valid because you see this your it means that your Y u would be consumed at a great level it will go from Y u is equal to some small point 2 or point something 2 completely near about 0 or it will change drastically whereas, your Y o if you plot it will not change as much as you have a much lesser change we will if it is fuel lean condition of course, if it is a fuel rich condition then the diffusion species will become Y f and then you will have to consider the fuel mass fraction of Y o.

So, by that way we can only consider the temperature degradation of only one species equation, by assuming that the reaction rate is essentially controlled by the mass fraction of the diffusion species the whereas, where the abundant species that is a Y o in this case is does not change too much. As a result we do not need to explicitly consider it is dependence and this can be absorbed within the pre exponential factor which is the modified pre exponential factor which is B c itself.

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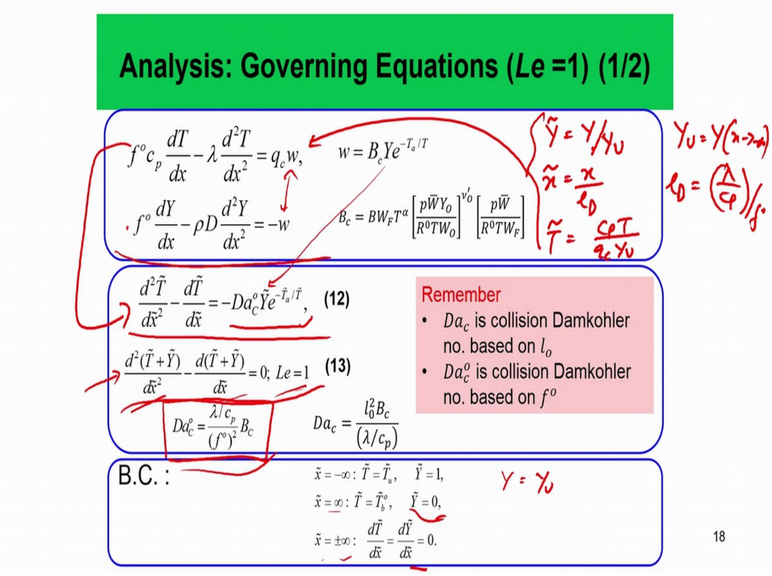

## Analysis: Governing Equations (*Le* =1) (1/2)

$$f^o c_p \frac{dT}{dx} - \lambda \frac{d^2T}{dx^2} = q_c w, \qquad w = B_c Y e^{-T_a/T}$$

$$f^o \frac{dY}{dx} - \rho D \frac{d^2Y}{dx^2} = -w \qquad B_c = B W_F T^{\alpha} \left[\frac{p\bar{W}Y_O}{R^0 T W_O}\right]^{\nu'_O} \left[\frac{p\bar{W}}{R^0 T W_F}\right]$$

$\tilde{Y} = Y/Y_U$

$\tilde{x} = \frac{x}{l_D}$

$\tilde{T} = \frac{C_p T}{q_c Y_U}$

$Y_U = Y(x \to -\infty)$

$l_D = \left(\frac{\lambda}{C_p}\right)/f^o$

$$\frac{d^2\tilde{T}}{d\tilde{x}^2} - \frac{d\tilde{T}}{d\tilde{x}} = -Da_C^o \tilde{Y} e^{-\tilde{T}_a/\tilde{T}}, \quad (12)$$

$$\frac{d^2(\tilde{T}+\tilde{Y})}{d\tilde{x}^2} - \frac{d(\tilde{T}+\tilde{Y})}{d\tilde{x}} = 0; \; Le = 1 \quad (13)$$

$$\boxed{Da_C^o = \frac{\lambda/c_p}{(f^o)^2} B_C} \qquad Da_C = \frac{l_0^2 B_C}{(\lambda/c_p)}$$

Remember
- $Da_c$ is collision Damkohler no. based on $l_o$
- $Da_c^o$ is collision Damkohler no. based on $f^o$

B.C. :

$\tilde{x} = -\infty: \tilde{T} = \tilde{T}_u, \quad \tilde{Y} = 1,$

$\tilde{x} = \infty: \tilde{T} = \tilde{T}_b^o, \quad \tilde{Y} = 0,$

$\tilde{x} = \pm\infty: \frac{d\tilde{T}}{d\tilde{x}} = \frac{d\tilde{Y}}{d\tilde{x}} = 0.$

$Y = Y_U$

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So, now then lets erase this things now then we can basically normalise it this this things and the normalisation parameters these are the normalisation parameters that is we can write Y tilde is equal to Y by Y Y u once again this normalisation is done because this then the equations become much much simpler it get rids of all the different constants and it will become much more revealing we can look into the different process these are being they are being active without being distracted by your different numerous constants.

So, Y u is essentially Y by Y u that is the Y divided by Y un burn, that is Y u is equal to Y at x equal to minus infinity that is at the left boundary, your other thing is x tilde is nothing, but x by l D l D is whereas, your l D is nothing, but lambda by C p times f 0 and your T tilde is equal to C p T by q c Y u and with this normalisation if you apply here if you apply into this these normalisations if you apply here, we get this relation that is this relation becomes this and that is d 2 T d square d 2 T tilde d x tilde square minus d T tilde d x tilde is equal to minus D a c, this is a collision Damkohler number which is defined like this, that is lambda by C p divided by f 0 square times B c times Y tilde the

Y tilde comes from this times c T minus T a tilde T activation energy tilde divided by T tilde.

And then you can normalise the Y tilde also in a similar manner and then if you apply the then if you add these 2 together what you get is essentially this equation this equation, d 2 T tilde minus plus Y tilde this is once again like a coupling function or de coupling function and this is like a species diffusion term this is a convective term is equal to Lewis number then this is only true for Lewis number equal to one and this as you see that, this when you apply this when you do the result of the normalisation as we have seen previously in coupling function also is that this allows us to combine this species and the temperature equation very carefully and very intelligently we can be able to get rid of this fairly non-linear and complex reaction rate terms.

But of course, the reaction rate has to be as to be included somewhere. So, it is including the temperature equation. So, you see this one dimensional steady energy equation is written in this form in the in the normalised form and this one dimensional steady species equation is written plus the energy reaction combine is written in this form and whereas, this is your Damkohler number. And the boundary conditions are at x is equal to minus infinity your T u tilde that is your un burn gas temperature normalised is equal to T is your T tilde is equal to your T u tilde, and your Y u Y tilde is equal to 1 because Y is equal to Y u. So, when you normalise you divide this to get rid of 1 of course, at x equal to plus infinity that is on the right hand side you have your T tilde is equal to T b tilde as 0 and your Y is everything is consumed because if fuel is leaned and your Y tilde is equal to 0 and at x is equal to plus minus infinity of course, both are plus infinity to minus infinity both the gradients, that is there no temperature gradient in either that is no temperature that is no species gradient and both goes to 0.

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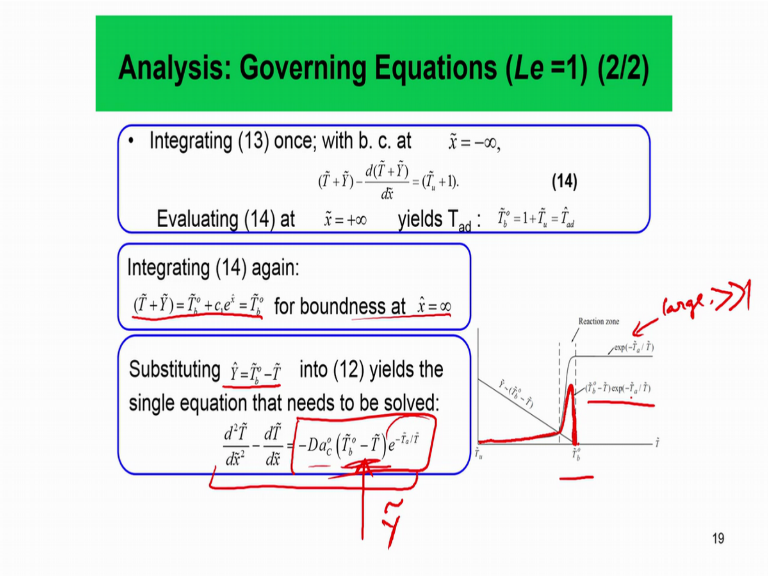

## Analysis: Governing Equations (*Le* =1) (2/2)

- Integrating (13) once; with b. c. at $\tilde{x} = -\infty$,

$$(\tilde{T}+\tilde{Y}) - \frac{d(\tilde{T}+\tilde{Y})}{d\tilde{x}} = (\tilde{T}_u + 1). \qquad \textbf{(14)}$$

Evaluating (14) at $\tilde{x} = +\infty$ yields T$_{ad}$ : $\tilde{T}_b^o = 1 + \tilde{T}_u = \hat{T}_{ad}$

Integrating (14) again:

$(\tilde{T}+\tilde{Y}) = \tilde{T}_b^o + c_1 e^{\hat{x}} = \tilde{T}_b^o$ for boundness at $\hat{x} = \infty$

Substituting $\hat{Y} = \tilde{T}_b^o - \tilde{T}$ into (12) yields the single equation that needs to be solved:

$$\frac{d^2\tilde{T}}{d\tilde{x}^2} - \frac{d\tilde{T}}{d\tilde{x}} = \boxed{-Da_C^o\left(\tilde{T}_b^o - \tilde{T}\right)e^{-\tilde{T}_a/\tilde{T}}}$$

$\tilde{Y}$

So, now if you integrate this equation this is a first equation we will consider this is a equation number thirteen if you integrate this equation you get that T tilde plus Y tilde is equal to this one, which can be shown in this manner I can just write down. So, this is the 2 equations that you have.

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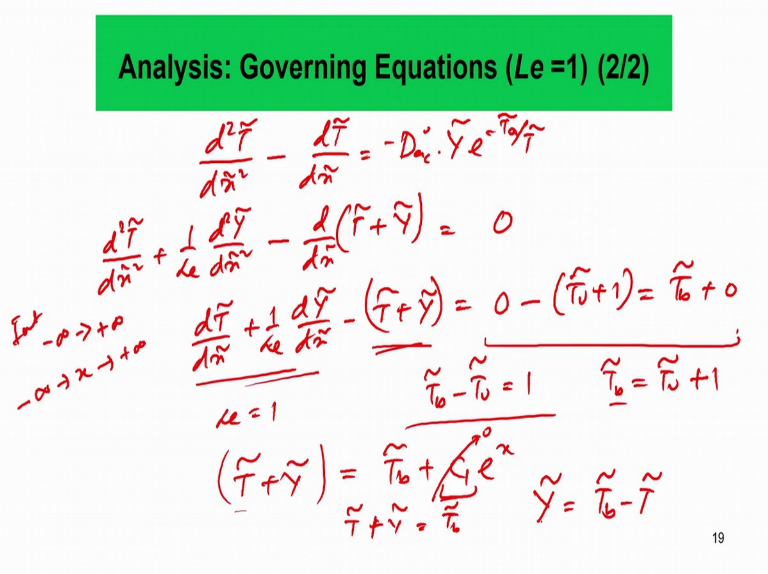

## Analysis: Governing Equations (*Le* =1) (2/2)

$$\frac{d^2\tilde{T}}{d\tilde{x}^2} - \frac{d\tilde{T}}{d\tilde{x}} = -Da_c^o \cdot \tilde{Y} e^{-\tilde{T}_a/\tilde{T}}$$

$$\frac{d^2\tilde{T}}{d\tilde{x}^2} + \frac{1}{Le}\frac{d^2\tilde{Y}}{d\tilde{x}^2} - \frac{d}{d\tilde{x}}(\tilde{T}+\tilde{Y}) = 0$$

Int $-\infty \to +\infty$

$-\infty \to x \to +\infty$

$$\underline{\frac{d\tilde{T}}{d\tilde{x}} + \frac{1}{Le}\frac{d\tilde{Y}}{d\tilde{x}}} - \underline{(\tilde{T}+\tilde{Y})} = \underbrace{0 - (\tilde{T}_u + 1) = \tilde{T}_b + 0}$$

$Le = 1$

$\underline{\tilde{T}_b - \tilde{T}_u = 1}$ $\quad \tilde{T}_b = \tilde{T}_u + 1$

$$(\tilde{T}+\tilde{Y}) = \tilde{T}_b + \cancel{c_1}^{\,0} e^x$$

$\tilde{T} + \tilde{Y} = \tilde{T}_b$ $\quad \tilde{Y} = \tilde{T}_b - \tilde{T}$

D 2 tilde d x square tilde minus d tilde d x tilde is equal to D a c 0 times Y tilde times e to the power of minus T a tilde Y T tilde right. So, this is the first equation you get and the second d equation is this form that is d 2 T tilde d x tilde square plus 1 by Lewis number d 2 Y tilde d x tilde square actually this is for Lewis number 1 equal to 1, but you do not need to set Lewis number one right now you can just get this equation still minus d d x tilde you should do this on your own also and that will give you the confidence to tackle these kind of problems is equal to 0.

Now, if you integrate this equation between it seems that there is no right hand side you can integrate this equation minus infinity plus infinity integrate. So, you get d T tilde d x tilde at plus infinity which is plus 1 by Lewis number d or if you just to say integrate from minus infinity to x 2 plus infinity on the 2 sides, I will just this is on the when you integrate up to x this is what you get d x tilde minus T tilde plus Y tilde. Now if you set these 2 what happens in minus infinity this of course, goes to 0 both minus T u tilde plus 1 is equal to, but on the right hand side of course, again this goes to 0. So, on the right hand side you get T b tilde plus 0.

So, if you just look into these things what you get is T b tilde minus T u tilde is equal to 1. So, this is a very interesting thing; that means, your burner gas temperature normalise is essentially your un burn gas temperature normalise plus 1 of course, it is a normalised quantities, but then this is thresholds order. So, there is a order one difference between T b tilde and T u tilde and of course, that is because of the heat release that is a heat of combustion that is happening. So, this is the thing and now if you set Lewis number to be equal to 1, we can integrate this further and we can write that your T tilde plus Y tilde essentially there will be a logarithmic dependence is essentially is equal to T b tilde plus C 1 times e to the power of x.

Now, of course, this this T tilde plus Y tilde cannot blow to infinity. So, if they sees either it will go to either minus infinity or plus tend to infinity sees a non 0 quantity. So, this must be a 0. So, then it means T tilde plus Y tilde is equal to T b tilde and then it means that your Y tilde at any point is equal to T b tilde minus T u T tilde at any point inside the flame. So, this is also very revealing and of course, as you see that this falls on to the fact that if you put T u tilde here it will becomes Y u tilde which is equal to 1 and if you T b tilde here then it will become 0. So, it satisfies about boundary conditions quite efficiently.

So, once again if you just go back to this analysis and this is what we are what we want to show here that this is how you do it and this T b tilde is essentially your T adiabatic temperature. So, that we got and boundary is of course, this has to be bounded. So, at you get this kind of an equation that and if you integrate you get this Y tilde is equal to T b tilde minus T tilde and then if you substitute that into the first equation in equation number 12; you see if you remember that the equation number 12 still had that had this thing that is this equation had this this is a diffusion this is a convection term d 2 T tilde d x square minus d tilde d x tilde.

And you see on the right hand side the complexity not only arises in combustion on this right hand side the combustion there is a source term problem arises not only due to the fact that it is a non-linear that is non-linear or (Refer Time: 26:21) type dependence e to the power of minus T tilde by T tilde T tilde by T tilde, but also the fact that these equations becomes couple through the fact that it by the law of mass action this Y tilde arises. So, this becomes it becomes the couple set of equations which is even more difficult to solve.

So, now once we know that Y tilde is equal to T b tilde 0 T b 0 tilde minus T tilde, you can immediately substitute this into that and then this becomes is becomes fuelled in T and. So, d 2 tilde d x square, minus d 2 d tilde d x tilde is equal to d minus D a c tends T b tilde 0 minus T tilde times e to the power minus T by T tilde. And if you plot this if you plot this equation this is if you plot this just this right hand side in as a function of temperature this is what you will get, that the if you just do not have this part which this this part essentially comes from your Y tilde.

So, if you just plot this part. So, you see as T u increases to T, T bs. So, this one increases and it rapidly rises; and this rapid rise is once again due to the fact that your T a this is large this is a large quantity much much greater than this is much much greater than one, T a tilde by T tilde and that is why it rises like that, but then if you plot in the Y dependence, you see the Y dependence is linear in Y in T tilde and that decreases linearly with t.

So, this decodes down like this. So, then if you couple these 2 things of course, when it goes to 0 at T b tilde then this also must come down to 0 and then the reaction zone which is a right hand side becomes a structure like this. So, you see temperature space it gives a willing thing that is in this this function that that is your reaction or the consumption the; this reaction rate of the of a given species or the reaction rate the species which had been independent reaction rate they are just coupled by a (Refer Time: 28:017) coefficient.

So, these 2 things are essentially becomes just a coupling between these 2 terms this T b tilde and T tilde, and which shows it rises rapidly because of large activation energy, but then it goes down, because your species has been consumed because of the reactions right. So, this tells you even in temperature space why this reaction the right hand side must be a very very sharp the very thin and the sharply rising function. So, this is very important.

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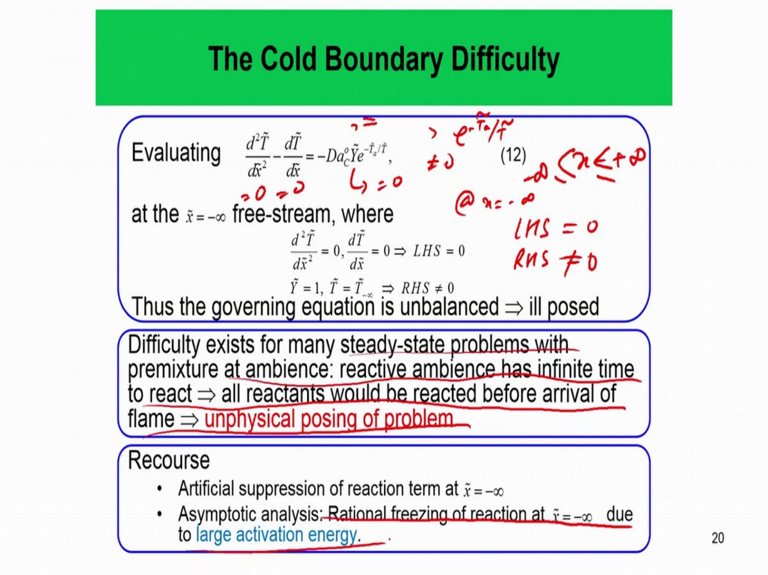

## The Cold Boundary Difficulty

Evaluating $\frac{d^2\tilde{T}}{d\tilde{x}^2}-\frac{d\tilde{T}}{d\tilde{x}}=-Da_C^o\tilde{Y}e^{-\tilde{T}_a/\tilde{T}}$, (12)

at the $\tilde{x}=-\infty$ free-stream, where

$$\frac{d^2\tilde{T}}{d\tilde{x}^2}=0,\frac{d\tilde{T}}{d\tilde{x}}=0\Rightarrow LHS=0$$

$$\tilde{Y}=1,\ \tilde{T}=\tilde{T}_{-\infty}\ \Rightarrow RHS\neq 0$$

Thus the governing equation is unbalanced ⇒ ill posed

Difficulty exists for many steady-state problems with premixture at ambience: reactive ambience has infinite time to react ⇒ all reactants would be reacted before arrival of flame ⇒ unphysical posing of problem

Recourse

- Artificial suppression of reaction term at $\tilde{x}=-\infty$
- Asymptotic analysis: Rational freezing of reaction at $\tilde{x}=-\infty$ due to large activation energy.

$=0$ $=0$ $=0$ $\neq 0$ $e^{-\tilde{T}_a/\tilde{T}}$

$-\infty \leq x \leq +\infty$

@ $x=-\infty$

$LHS=0$

$RHS\neq 0$

And this is also a something a de coupling or a de coupling function that we used. Now there is a something called very important in combustion in premixed flames and the cold boundary difficulty; now what is that. So, as we equation that we got here that is basically the energy equation that we have got is essentially d 2 T tilde d x tilde square and as d tilde d x tilde is equal to minus d x is 0, d x refer conditions tends for 5 tilde times e to the power of minus T by t.

Now, at x equal to minus infinity of course, it should be satisfied this equation is satisfies between x equal to between equal to minus infinity to plus infinity right. So, at minus infinity you see that this term is equal to 0; this term is equal to 0. So, but then the problem is that because of course, there cannot be any gradient tens to minus infinity, but you see that on the right hand side definitely this is non 0, this is not equal to 0 this is not equal to 0 and this quantity is essentially e to the power of minus T a by T tilde it is small because your T tilde is much much larger than T tilde, but it still not exactly equal to 0.

So, on the left hand side you have exactly 0 quantities, but on the right hand side you have non 0 quantities. So, essentially this is a problem that it is the (Refer Time: 29:57) is served that this on the left hand side is equal to. So, the problem is that the left hand side of this equation at x equal to minus infinity your LHS is equal to 0, but your RHS is not equal to 0 so; that means, there is a problem and the problem arises because of the fact that this reactions even though this temperature is small the theoretically the even the theoretically it suggests that there has to be (Refer Time: 30:25) dependence there is a some reaction going on, but which is not true.

And the thing is that, but the problem is that at x equal to at x equal to minus the problem is that even if some small reaction is going on, if the convection is starting from x equal to minus infinity. So, it means that it has infinite temperature arrives at the flame location. So, infinite amount of time there will be an small reaction read then entire field will be consumed by the time it arrives at the at the flame location, which is really not physical.

So, on the right hand side we do not have a such problem because on the right hand side you see this problem is alleviated by the fact that it is solved by the fact that yes this is equal to 0 this is equal to 0, but your this is not equal to 0 from the. So, at the hot boundary there is much of problem as such on the left hand side or left boundary what people does that they just set this reaction that to be 0 until and unless some particular temperature or this ignition temperature is a rich that is how this problem is solved.

So, this difficulty exists for many steady state problems with pre mixture of ambience. So, that is reactive ambience is infinite time to react. So, all the reactions should be reactants should be reacted before the arrival of the flame, and which reaches to the unphysical posing of the problem. So, recourse is that the artificially suppressed of the reaction term at the reaction term at x equal to minus infinity and in a synthetic analysis the rational freezing of the reaction at x equal to minus infinity happens due to the large activation of energy.

So, this is the thing and then we will what we will move on to is the fact that we searched that we arrived at this thing that is f 0.

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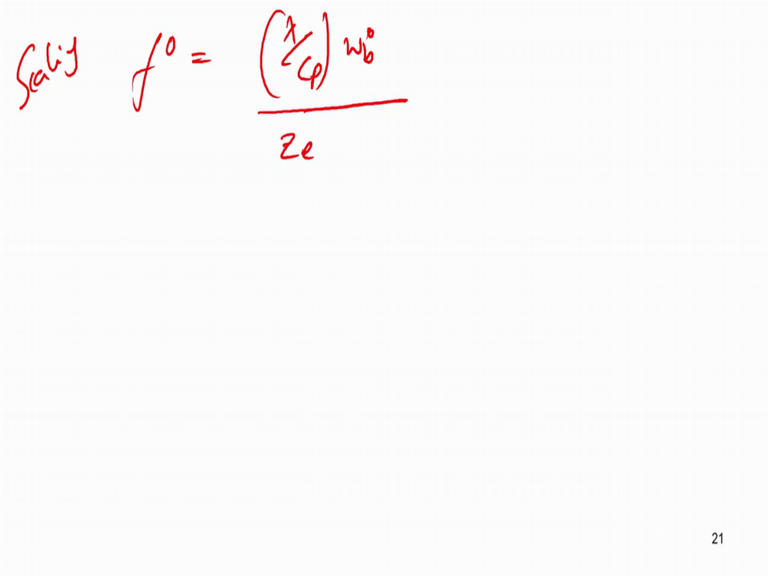

Scaling $f^0 = \frac{\left(\frac{\lambda}{C_p}\right) w_b^0}{Ze}$

By just scaling analysis we arrive at this at this argument that f 0 is equal to lambda by C p times times w b is 0 that is that is a species (Refer Time: 32:08) or the reaction rate at reaction rate un burn gas temperature burn gas temperature at very high temperature and then of course, then we arrived that the fact that the we got it this thing that is it was divided by zeldovich number right. So, this was a thing that we arrived now we will see that does this really is true. So, this can be (Refer Time: 32:37) to the fact that when you do a boundary layer unless it is an feed mechanics you do first scaling analysis, but then you do a more detailed analysis to see whether that is really true or not.

This say for example, you get the boundary thickness to be x by round somewhat to the power of x by square root of then all sum alright. So, this you have to see that whether there is your delta your boundary thickness delta is x is equal to x by r e x square root of r e x right. So, eventually the delta grows x to the power of half. So, that this result that result you arrive from scaling analysis, but then you still do a detailed analysis to ensure to check whether that is scaling analysis result that you have got is to that it is correct or not.

So, similarly here we will do a more detailed analysis which is called the frank Kaminsky or Zeldovich analysis, which was done by frank Kaminsky and Zeldovich 2 basically determine this burning flux which I have said is the most important thing in premixed flames. So, we will next embark to this embark on this Frank Kaminsky

Zeldovich analysis to arrive at a very rigorously, we arrive at the burning flux for a laminar premixed flames. So, that will be taken up in the next class.

Thank you.