# Combustion in Air Breathing Aero Engines

**Dr. Swetaprovo Chaudhuri**
**Department of Aerospace Engineering**
**Indian Institute of Science, Bangalore**

## Lecture – 24
## Laminar Premixed Flames III

Welcome back. So, in the last class, we have seen that how we can using the conservation equation of mass momentum and energy, we can connect the downstream that is the burn state of the gas with respect to the un burn state that is upstream state of the gas whereas, with reference which we are defining upstream and downstream with reference to the chemical wave which separates the upstream and downstream states.

And we are talking about essentially premixed combustion, and we are states found that there can be 2 states in which this chemical wave can propagate and one is the state in which it propagates at the subsonic speed, and another is a state at which it propagates into a supersonic speed, and when it propagates at a supersonic speed we call it a detonation and we have pressure rise, density rise across the wave and when it propagates at a subsonic state we have pressure drop density drop across the wave.

So, for all practical purposes all modern engines use deflagration or a flames or the non premixed flames or premixed flames and so far we have not used detonation as such except there are some developments like pulse detonation engine or a rotating detonation engines, but these are very much under development and that is not been practically realised here yet. But there are still lot of interest in the topic because it allows you to increase the pressure downstream of the wave whereas, across the deflagration you have pressure drops and that leads to the loss of efficiency for a combustion engine ok in the whole thermo dynamics circle we do some efficiency for that.

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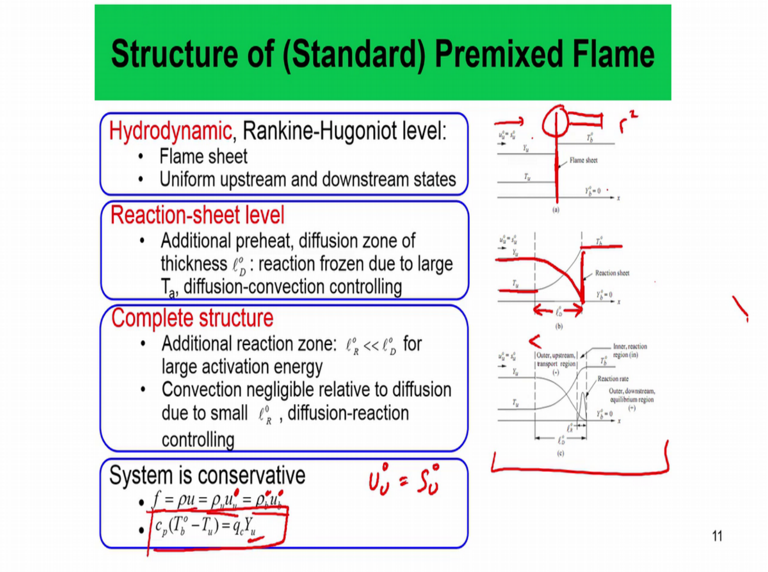

## Structure of (Standard) Premixed Flame

**Hydrodynamic, Rankine-Hugoniot level:**
- Flame sheet
- Uniform upstream and downstream states

**Reaction-sheet level**
- Additional preheat, diffusion zone of thickness $\ell_D^o$: reaction frozen due to large $T_a$, diffusion-convection controlling

**Complete structure**
- Additional reaction zone: $\ell_R^o << \ell_D^o$ for large activation energy
- Convection negligible relative to diffusion due to small $\ell_R^0$, diffusion-reaction controlling

System is conservative
- $f = \rho u = \rho_u u_u^o = \rho_b u_b^o$
- $\boxed{c_p(T_b^o - T_u) = q_c Y_u}$

$U_U^o = S_U^o$

So, now, we talk about the deflagration wave and that is essentially a laminar premixed flame. So, we have seen that in a hydro dynamic, we can have a very very different descriptions or descriptions of the premixed flame at different levels. So, we can look into hydro dynamical level or the Rankin Hugoniot level where essentially we have seen we consider essentially the whole flame the whole structure to be a sheet, but we do not really resolve the structure of the sheet what happens inside the flame or how does chemical reactions happen we do not care.

We are only happy with the uniform upstream and downstream states, where we have seen that this our flame and this is our flame and this is the downstream state, and this is upstream state and. So, we have upstream we have Y u that is mass fraction of the fuel un burn and this is the mass fraction of the fuel burn which is equal to 0 and this of course, if it is a stationary wave that is if the wave is not propagating with respect to our coordinate system then of course, it invariably implies that there is a flow into the wave which is that is un burnt reactants of flowing into the wave at a velocity of U u 0 which is equal to the S u 0 or the burning speed or the burning velocity of the flame speed of the of this flame.

So, this is the flame sheet limit. So, it means that that we are viewing the flame for a very very far distance where you are not really looking into what happens inside the sheet, but then if you zoom in if you put a magnifying glass and zoom into this sheet, and what you see what happens is that the that sheet essentially has a finite thickness, it has diffusion zone or a pre heat zone where the temperature actually increases. So, if this a un burn gas temperature this a un burn gas temperature. So, there is a region or a temperature sharply increases, and this diffusion zone is characterised by this thing which is this l D 0 and the reaction zone is essentially frozen due to large temperature T a and the we again consider this reaction sheet limit where the where essentially your because the reaction zones are happen in a very very small distance because of the large energy activation energy that we have already seen.

But there is also mass diffusion zone where this species does not change abruptly, which is not possible in reality because everything has to change in a gradual manner, it only depends in a persuasion of (Refer Time: 04:02). So, when you zoom into this in this flame sheet you see this, you see this pre heat zone and the diffusion zone, but you still your we our magnifying our zooming capacity is not enough to look into the reaction sheet. But if you zoom in further you will see that there is also a reaction sheet which is also happens over a small and a finite distance or a finite length.

So, this additional reaction zone this l R which is this thickness that is at the temperature increases like this, that as species decreases like this and this is the reaction where the reaction is speaking and that happens over a length of l R 0, and l R which because its standard conditions we are considering and this is the l D 0.

So, this is the flame structure the one d flame structure in the x direction of course, in a gas turbine engine you can have a flame like this, that is these are the (Refer Time: 04:57) you can have a flame like this in a actual gas turbine combustor. So, basically then in that if you this is our x. So, or this our x direction. So, here what we will see that whatever happens here in this thing is essentially is happens like this. So, the temperature increases like this the species decreases like this. So, this we need to understand this flame structure to essentially know what happens actually in a how the flame structure in actual engine behaves.

So, this idealise version is very very important, but once again to understand that the this thing is essentially an element, this flame structure that we see here is can be considered as a building block or an element of the complicated flame structure that happens in a actual engine. Just like the chamber flame is actually element of the complicated flame structure that can happen in some other non premixed combustors.

So, this type of understanding is very very important, but now itself even going into the details we can now understand that because activation energy is very large your l R 0 must be very very larger than this then l D 0. So, this is one important thing. So, I have a thing is that system is of course, conservative and your f that is the burning flux or the mass flux rho u is equal to your rho U u 0, that is upstream velocity and the rho b u b 0 that 0 comes because it is a reference conditions, this is also the reference conditions this is a plan under laminar flame speed this is U u 0 well that we will see, because if the if in a state in the we are at a it is a scored n transformation. So, when the in a you know if our curve if our flame is steady with respect to our coordinate system then it means that the flow is coming the where the burn mixture is the un burn mixture is approaching at a velocity, U u 0 whereas, if the un burn mixture was our reference flame whereas, that to the un burn mixture that is un burn mixture was fixed then of course, the flame would have propagated into the un-burn mixture the same velocity we use here, but then we will call it s l 0.

So, essentially U u 0 is equal to S u 0 this un-burn gas thing un burn flame speed that you see. One other thing is that this is very important this comes from the energy conservation the C p T b 0 minus T u 0 is equal to q c times Y u ok that is this is a mass fraction and this if you just integrate across you will get this, and this gives you the flame what flame temperature would be. This is essentially the adiabatic flame temperature for the stoichiometric not for the stoichiometric, but for the equivalence ratio of the mixture that is being considered.

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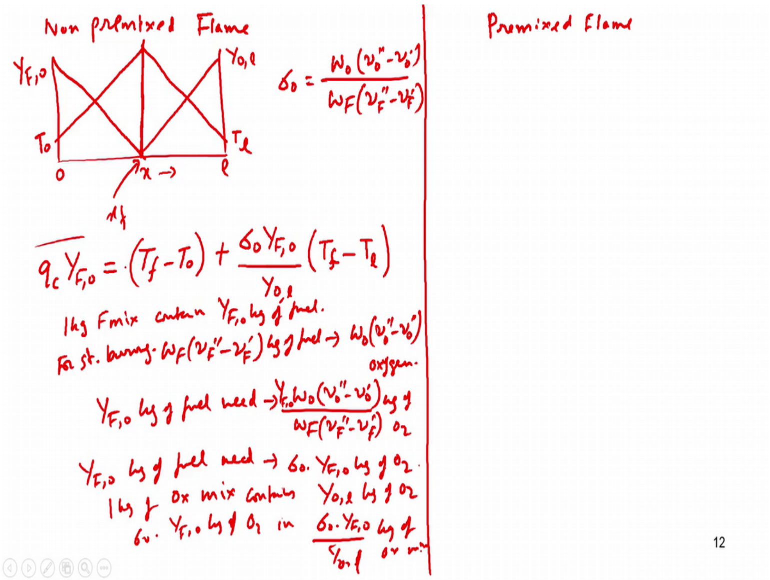

Non premixed Flame

$$\sigma_0 = \frac{W_O(\nu_O''-\nu_O')}{W_F(\nu_F''-\nu_F')}$$

$$\overline{q_c}\, Y_{F,0} = (T_f - T_0) + \frac{\sigma_0 Y_{F,0}}{Y_{O,l}}(T_f - T_l)$$

1kg Fmix contain $Y_{F,0}$ kg of fuel.

For st. burning. $W_F(\nu_F''-\nu_F')$ kg of fuel $\rightarrow$ $W_O(\nu_O''-\nu_O')$ oxygen.

$Y_{F,0}$ kg of fuel need $\rightarrow \frac{Y_{F,0} W_O(\nu_O''-\nu_O')}{W_F(\nu_F''-\nu_F')}$ kg of $O_2$

$Y_{F,0}$ kg of fuel need $\rightarrow \sigma_0 . Y_{F,0}$ kg of $O_2$.

1kg of ox mix contains $Y_{O,l}$ kg of $O_2$

$\sigma_0 . Y_{F,0}$ kg of $O_2$ in $\frac{\sigma_0 . Y_{F,0}}{Y_{O,l}}$ kg of ox mix

Premixed Flame

So, then we need to just understand the difference between the temperature for the non premixed flame and the premixed flame. So, if you consider the non premixed flames here we in this thing will on the let hand side will consider a non premixed flame, and here we will consider a premixed flame. So, the non premixed flame was like this on this maybe a one d both or one d variations. So, this is 0, this is l, this is x. So, the flame was somewhere here. So, this was your temperature T 0 T l this was Y F 0, this is just the previous thing that we did nothing can survive no fuel this is a fuel mass fraction on the left hand side, because it can contain some inert nitrogen also and no fuel can survive beyond the flame this is the flame location x f, and this was your Y o l mass fraction of the oxidiser on the l on the right hand side and these things were linear because it was just diffusion controlled.

And we saw that this was just by coupling function this becomes solution of the Laplace equation delta 2 beta i del x square is equal to 0, and the temperature formula that we obtained was this, and so what happen was that. So, this was the heat release and. So, by and this is the fuel mass fraction. So, if we consider one k g of fuel mixture. So, this was amount of heat release. So, this one k g of fuel mixture was used for heating up one k g of this one heat release from this one k g of the fuel mixture which contained Y F 0 k g of fuel was used for heating up one k g of fuel mixture and this we found what was the exactly the stoichiometric amount of oxidiser, exactly the amount of oxidiser required for stoichiometrically burning the one k g fuel mixture or Y F 0 k g of fuel mixture this was the thing right.

So, this is what we had obtained exactly in the previous class. So, as a result of this T f what we have obtained was the adiabatic stoichiometric adiabatic flame temperature for this for the system. So, now, if you say that Y F 0 is equal to 1. So, just to show you back this is how we obtain this if we just go back to the previous class in module 6 this was the thing that we obtained.

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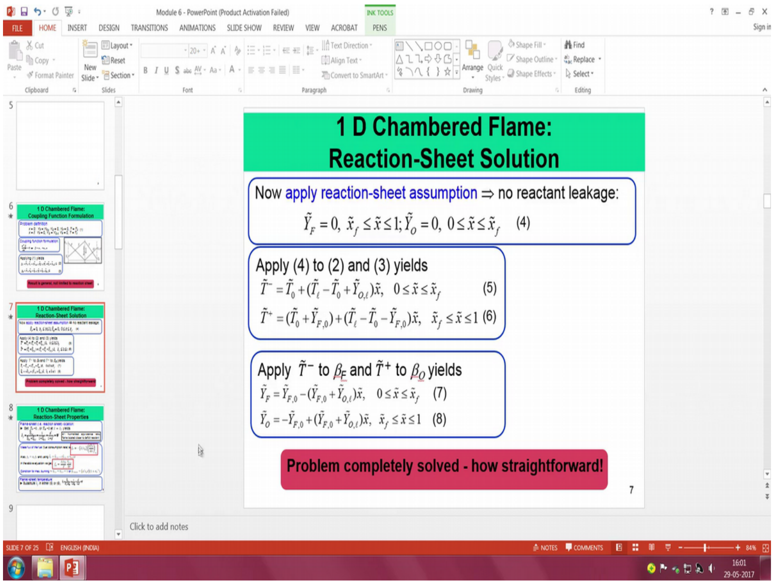

## 1 D Chambered Flame: Reaction-Sheet Solution

Now apply reaction-sheet assumption ⇒ no reactant leakage:

$$\tilde{Y}_F = 0,\ \tilde{x}_f \le \tilde{x} \le 1; \tilde{Y}_O = 0,\ 0 \le \tilde{x} \le \tilde{x}_f \quad (4)$$

Apply (4) to (2) and (3) yields

$$\tilde{T}^- = \tilde{T}_0 + (\tilde{T}_\ell - \tilde{T}_0 + \tilde{Y}_{O,\ell})\tilde{x},\quad 0 \le \tilde{x} \le \tilde{x}_f \quad (5)$$

$$\tilde{T}^+ = (\tilde{T}_0 + \tilde{Y}_{F,0}) + (\tilde{T}_\ell - \tilde{T}_0 - \tilde{Y}_{F,0})\tilde{x},\quad \tilde{x}_f \le \tilde{x} \le 1 \quad (6)$$

Apply $\tilde{T}^-$ to $\beta_F$ and $\tilde{T}^+$ to $\beta_O$ yields

$$\tilde{Y}_F = \tilde{Y}_{F,0} - (\tilde{Y}_{F,0} + \tilde{Y}_{O,\ell})\tilde{x},\quad 0 \le \tilde{x} \le \tilde{x}_f \quad (7)$$

$$\tilde{Y}_O = -\tilde{Y}_{F,0} + (\tilde{Y}_{F,0} + \tilde{Y}_{O,\ell})\tilde{x},\quad \tilde{x}_f \le \tilde{x} \le 1 \quad (8)$$

**Problem completely solved - how straightforward!**

So, this was the equi-temperature equation that we had, and then we substituted the substituted this thing into the x f that is here, that is the x f that we obtained here and then we this is the temperature value that we the temperature formula that we that we got.

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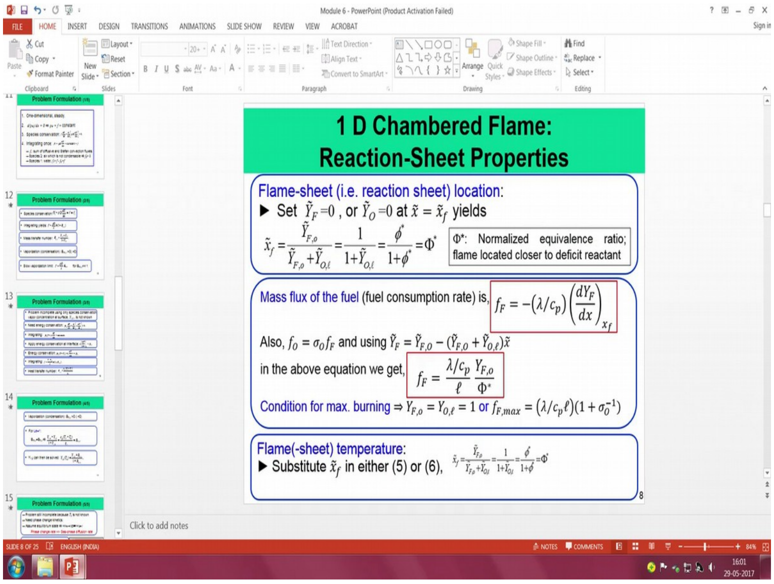

1 D Chambered Flame:
Reaction-Sheet Properties

Flame-sheet (i.e. reaction sheet) location:

- Set $\tilde{Y}_F=0$ , or $\tilde{Y}_O=0$ at $\tilde{x}=\tilde{x}_f$ yields

$$\tilde{x}_f=\frac{\tilde{Y}_{F,o}}{\tilde{Y}_{F,o}+\tilde{Y}_{O,\ell}}=\frac{1}{1+\tilde{Y}_{O,\ell}}=\frac{\phi^*}{1+\phi^*}=\Phi^*$$

$\Phi^*$: Normalized equivalence ratio; flame located closer to deficit reactant

Mass flux of the fuel (fuel consumption rate) is, $\boxed{f_F=-(\lambda/c_p)\left(\frac{dY_F}{dx}\right)_{x_f}}$

Also, $f_O=\sigma_O f_F$ and using $\tilde{Y}_F=\tilde{Y}_{F,O}-(\tilde{Y}_{F,O}+\tilde{Y}_{O,\ell})\tilde{x}$

in the above equation we get, $\boxed{f_F=\frac{\lambda/c_p}{\ell}\frac{Y_{F,o}}{\Phi^*}}$

Condition for max. burning $\Rightarrow Y_{F,o}=Y_{O,\ell}=1$ or $f_{F,max}=(\lambda/c_p\ell)(1+\sigma_O^{-1})$

Flame(-sheet) temperature:

- Substitute $\tilde{x}_f$ in either (5) or (6), $\tilde{x}_f=\frac{\tilde{Y}_{F,o}}{\tilde{Y}_{F,o}+\tilde{Y}_{O,\ell}}=\frac{1}{1+\tilde{Y}_{O,\ell}}=\frac{\phi^*}{1+\phi^*}=\Phi^*$

8

So, and this thing was the exactly the amount of the air this is we can we obtained in this manner that is I just go little briefly over this because this is. So, important that is one k g of fuel air mixture contains fuel mixture contains Y F 0 f of fuel right.

Now, for stoichiometric burning now whereas, this sigma 0 was nothing, but w 0 times nu 0 double dash minus nu 0 dashed divided by w f times nu f double dash minus nu f dashed. So, for stoichiometric burning w 0 1 k g for stoichiometric burning one k g or stoichiometric burning sorry this is w F times nu F double dash minus nu F dashed k g of fuel needs this w 0 times nu 0 double dash minus nu 0 double dashed oxidiser oxygen right. So, Y F 0 k g of fuel needs w 0 times nu 0 double dash minus nu 0 dashed divided by w F times nu F double dashed minus nu F dashed k g of oxygen times w 0 F 0, right.

So, this is nothing, but. So, for stoichiometric burning Y F 0 k g of fuel will get sigma 0 times Y F 0 k g of oxygen right, but one k g of air or oxidiser mixture mix contains y 0 l k g of oxygen. So, sigma 0 times Y F 0 k g of o 2 can be found in sigma 0 Y F 0 divided by y o l k g of oxygen mixture, alright.

So, as a result this thing was exactly the amount of oxidiser mixture that is air required for stoichiometric burning. So, this you see the thing is that if you just review all these things, the thing which comes out is that these heat release these heat release goes onto to support onto heating up one k g of fuel mixture and exactly the amount of oxidiser mixture required for stoichiometric burning of this fuel mixture. So, the burning that is happening is stoichiometric.

So, when the burning that is happening is stoichiometric, then the flame temperature reached is nothing, but the adiabatic flame temperature. So, here the most common situation is that the fuel does not contain nitrogen say it is pure fuel.

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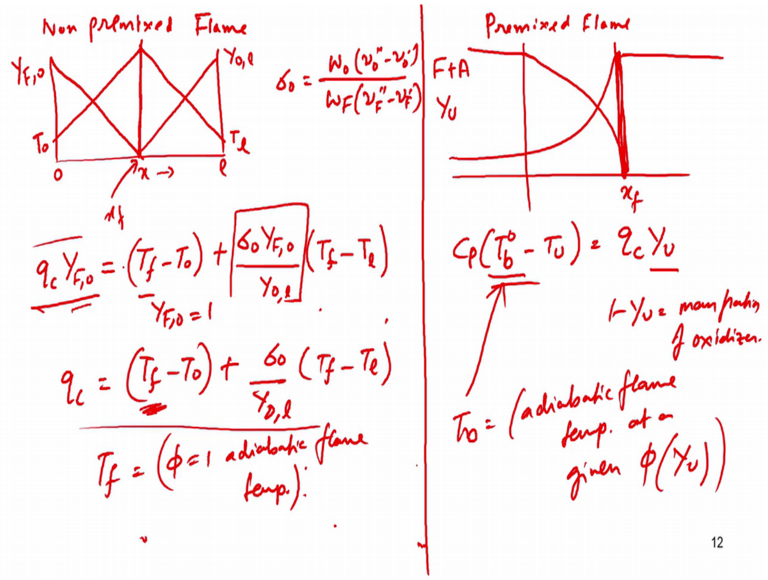

$$\sigma_0 = \frac{W_O(\nu_O'' - \nu_O')}{W_F(\nu_F'' - \nu_F')}$$

$$q_c Y_{F,0} = (T_f - T_0) + \boxed{\frac{\sigma_0 Y_{F,0}}{Y_{O,l}}}(T_f - T_l)$$

$Y_{F,0} = 1$

$$q_c = (T_f - T_0) + \frac{\sigma_0}{Y_{O,l}}(T_f - T_l)$$

$T_f$ = ($\phi = 1$ adiabatic flame temp.)

$$C_p(T_b^0 - T_U) = q_c Y_U$$

$1 - Y_U$ = mass fraction of oxidizer.

$T_{b0}$ = (adiabatic flame temp. at a given $\phi(Y_U)$)

So, here this q c is equal again it means your Y F 0 is equal to one if it is pure fuel that is that is no oxidiser, but of course, the air will contain some nitrogen. So, then it means is essentially q dc is equal to T f minus T 0 plus sigma 0 by Y o l times T f minus T l right.

So, this temperature thus obtained is the phi equal to one adiabatic flame temperature, it is always at the peak this flame temperature. On the other hand if you consider the premixed flame that is this structure this is your flame sheet this was also your flame sheet where your x f here is. So, here your fuel and oxygen mixture are premixed right and this your fuel mixture is Y u, 1 minus Y u is equal to your essentially your air mixture as oxidiser mixture yes and your species is reducing like this here is your flame sheet thin. So, here the temperature thus reached is C p times T f minus T u or we will say call it T b minus T u is equal to q c Y u. Now you see here on this side your Y F 0 you could set it to one whenever there is no nitrogen in the fuel which is the most slightly the case right. So, then fuel mass fraction is equal to one its pure fuel that is going out.

So, when a pure fuel the flame temperature is the phi equal to one adiabatic flame temperature, but this Y u here you see it can never be equal to one the reason the reason is that 1 minus Y u is equal to your mass fraction of the oxidiser or the air, Y u can never be own because this is mixed with the oxidiser. So, it is always less than one as a result this temperature is still the adiabatic flame temperature because there is no heat loss, but this is the adiabatic flame temperature at a given phi which is the function of your Y u.

So, depending on the stoichiometric depending on the equivalence ratio now you can essentially control your T b or T b 0. So, that is the difference here, but even is that here you cannot control T f, T f is always a stoichiometric adiabatic flame temperature as long it is a pure fuel on the other hand here just by changing the stoichiometry if you just put in less fuel or and more air you can control the flame temperature. So, that is why premixed flames are very attractive in any engineering things it allows you to engineer the flow inside or the it allows you to engineer the product composition and the product temperature also that also the composition because you see that temperature is very very important you have seen that when you is considered this thermal nox production of a Zeldovich mechanism, the this n 2 could be split up only at very high temperature right.

So, first or vice versa whenever there is high temperature this n 2 can be split up by an oxygen right. So, you always want to reduce emissions unless there is other some other phasing needs you always want to combustion to happen at a temperature about 500 about 1600, 1700 Kelvin. So, that you can readily with this premixed flame where you just control the equivalence ratio and then you give the get the low temperature.

Of course there are things associated with it like that swapped challenge conflict flame stability thermal (Refer Time: 20:34) constellations that we have to tell, but here unlike in a non premixed flame where you cannot control the adiabatic flame you control the flame temperature as long as your fuel is pure here because its mixed with air if you give in less fuel you get you get less temperature you can appropriately control the flame temperature and it and it can also control the things like emissions etcetera.

So, that is the advantage.

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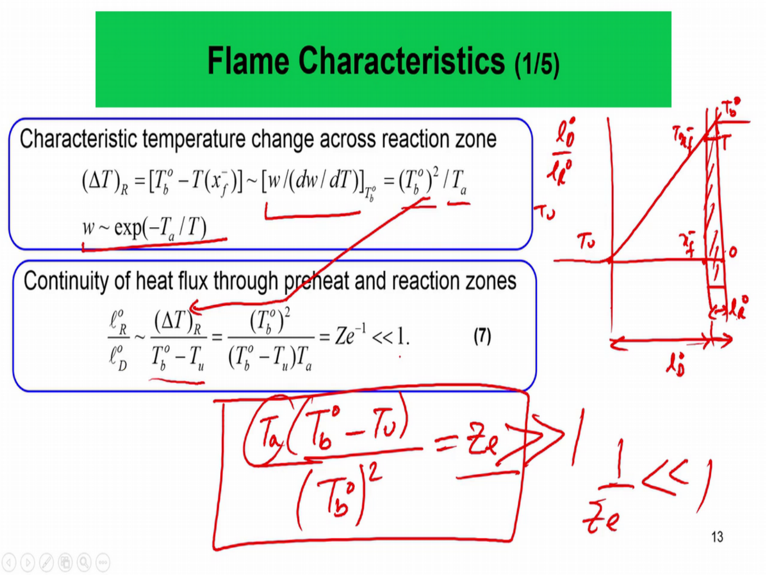

## Flame Characteristics (1/5)

Characteristic temperature change across reaction zone

$$(\Delta T)_R = [T_b^o - T(x_f^-)] \sim [w/(dw/dT)]_{T_b^o} = (T_b^o)^2 / T_a$$

$$w \sim \exp(-T_a/T)$$

Continuity of heat flux through preheat and reaction zones

$$\frac{\ell_R^o}{\ell_D^o} \sim \frac{(\Delta T)_R}{T_b^o - T_u} = \frac{(T_b^o)^2}{(T_b^o - T_u)T_a} = Ze^{-1} << 1. \qquad (7)$$

$$\boxed{\frac{T_a(T_b^o - T_u)}{(T_b^o)^2} = Ze} >> 1 \qquad \frac{1}{Ze} << 1$$

So, then we go into the flame characteristics, that is how much one very important characteristics to note is that if you were considering this. So, if you were considering the premixed flame structure where the reaction zone was like this, this is your temperature T b 0 this is your T u this is your l R and this is your l D, l R 0 l D 0. So, if you want to find out what is the relation between l D 0 and l R 0, we can we want to find out this thing l D 0 by l R 0.

So, then we have to find out basically l R 0 what it is it what is it? L R 0 is or we go into simpler description that is when the flame sheet limit that is this description where we consider there is a this finite thickness, but we do not resolve the structure. So, this is your basically then your; is l D 0 this is your l R 0. So, then by similarity of the triangles here we can say that this is your T b 0 and this is your T x f minus that is just and this is basically your reaction zone.

And. So, this is your T x f minus x f minus is just back to the flame, and this is your say 0 and. So, you see this is the temperature that we are talking about. So, the temperature reaches after combustion which is a one gas flame temperature which is the adiabatic lame temperature at that given equivalence ratio, and this there is small thickness which is the reaction zone thickness l R 0, and just before the reaction zone thickness the temperature is slightly smaller than T b 0. So, we need to find out to find out what is the ratio of this l R 0 by l D 0 that is the reaction zone thickness to the to the diffusion zone thickness or the preheat zone thickness, we need to find out what is the we will apply the similarity of triangles that we what we will do is that we will do something like a T b 0 minus T x f minus divided by l R 0, that is this difference by this ratio is essentially equal to T b 0 minus T u where T u is here T u divided by l D 0.

Even though this considers a little bit of diffusion zone strictly speaking the diffusion zone is here, but since l R 0 is small than l D 0 this equation holds. So, for that we need to find out this thing that is T b 0 minus T x f. So, how do you find that out we find that out by saying that T b minus T x f is essentially proportional to w by divided by d w d T at T b 0 and that is given by whereas, if assume if we say that this w is proportional to of course, there are other things also that is w is proportional to int to the power of minus T a by t.

We will consider that we will say that we just take this derivatives and it find that its essentially becomes ratio of T b square divided by T a. So, that is T b square is a burn gas flame temperature and T a is the activation temperature. So, you can just find this out and then we apply this thing that is l R 0 by l D 0 is essentially delta T across the reaction zone divided by delta T b minus T u and then this we substitute this here substitute this here and we get l R 0 by l D 0 is essentially equal to T b 0 square divided by T b 0 minus T u by times T a whereas, this thing is a very important quantity and that is called the Zeldovich number.

So, this guy is called the Zeldovich number and it is a very very important quantity, and this Zeldovich number is always very large you see that in the bottom you have a very large quantity which is T b 0 square on the upstairs there is and the numerator you have T b 0 minus T u which is proportional to T b 0, but since T a is much much larger than T b 0. You see activation temperature is much larger than T b 0 that is the burn gas flame temperature which comes from the kinetic considerations that why your activation number should be large because that is the only a small fraction of molecule should be excited to start the reaction.

So, this T a is very very large and they should have a very large energy, and as a result of that the Zeldovich number is very very large or as whereas, inverse of this is much much greater than one whereas, one by Zeldovich number should be much much less than one. So, essentially we see that l R 0 by l D 0 which is equal to 1 by Zeldovich number that should be much much smaller than one. So, this is the deal this is the consideration that l l R 0 by l D 0 should be much smaller than one.

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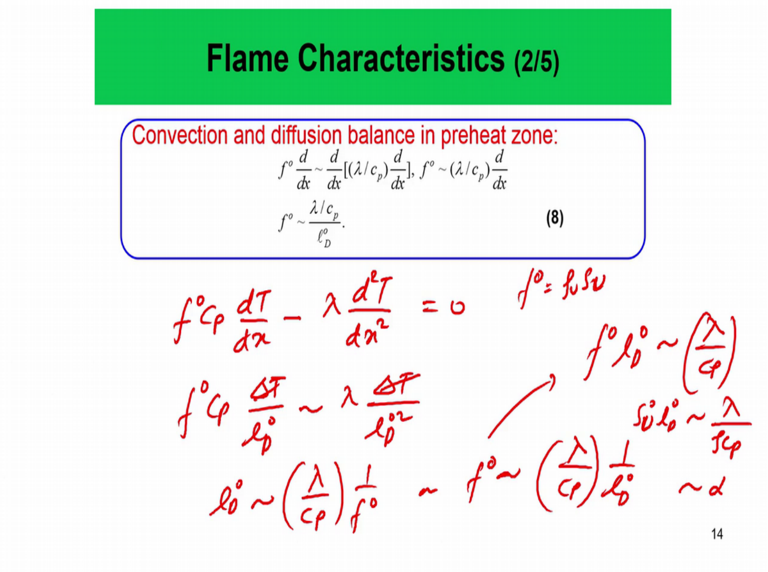

## Flame Characteristics (2/5)

Convection and diffusion balance in preheat zone:

$$f^o \frac{d}{dx} \sim \frac{d}{dx}[(\lambda / c_p)\frac{d}{dx}],\ f^o \sim (\lambda / c_p)\frac{d}{dx}$$

$$f^o \sim \frac{\lambda / c_p}{\ell_D^o}. \qquad (8)$$

$$f^o c_p \frac{dT}{dx} - \lambda \frac{d^2T}{dx^2} = 0 \qquad f^o = f_u S_u$$

$$f^o c_p \frac{\Delta T}{l_D^o} \sim \lambda \frac{\Delta T}{l_D^{o2}}$$

$$f^o l_D^o \sim \left(\frac{\lambda}{c_p}\right)$$

$$S_u^o l_D^o \sim \frac{\lambda}{\rho c_p}$$

$$l_D^o \sim \left(\frac{\lambda}{c_p}\right)\frac{1}{f^o} \sim f^o \sim \left(\frac{\lambda}{c_p}\right)\frac{1}{l_D^o} \qquad \sim \alpha$$

14

Now, we can go into balance the of the diffusion and the and the and the convection and the diffusion flux balance in the preheat zone of course, you see that in this structure that in this zone in this zone of l D 0 there is no reaction this w equal to 0, or in this part that is in this region this is how the temperature increases. So, in this region we can consider the; your reaction rate the species independent reaction rate or the species production rate that is equal to 0.

So, that is equal to 0 and as a if that is so, then essentially your just this just like the previous relations that we used while deriving the coupling functions that when the flame a flame essentially means reactions, but because this Zeldovich number is very because the not the Zeldovich number yeah because the Arrhenius number is or the or the activation temperature is very high as a consequence of that your Zeldovich number is very large and as a result your reaction zone thickness that is the l R 0 where the reaction happens is much much smaller than your preheat zone thickness. So, in lot of the flame actually there is no on this assumption that is no reaction happening its only balance between diffusion and convection?

So, if there is balance between diffusion and convection, we can just consider this equation that is say anything we consider the temperature equation the one dimension temperature equation which is f 0 times C p d T d x minus lambda d 2 T d x square is equal to 0 right. So, then it means that then if we do an order of magnitude analysis we say that this is represent by f 0 C p where this happens is essentially delta T, which is equal to T b minus T u divided by l D 0 that is the diffusion length and that should be proportional to of the same order of magnitude as lambda times delta T divided by l D 0 square.

Then this cancels and your l D 0 is nothing, but lambda by C p times one by f 0 or or your f 0 is nothing, but lambda by C p times 1 by l D 0. So, this a very very important relation you see or you see that or we say that l f 0 times l D 0 is essentially proportional to lambda by C p which is your thermal diffusivity or the specific gate or of course, f 0 is essentially is equal to your rho u times s l or you can also right that s l 0 times l D 0 or S u 0 times l D 0 is proportional to equal lambda by rho C p essentially is equal to thermal diffusivity.

So, S u times l D 0 is equal to constant is equal to thermal diffusivity, and that arrives just from the fact that in the preheat zone it is just a balance between your convection and diffusion and there is no reaction at that place. So, that is your consideration that with that we can arrive at this now of course, we have to whatever flux goes in whatever f 0 goes in through the through this convection diffusive convective diffusive zone, that must also be reacted.

So, if you remember the dimensions of the reaction rate and then we can also relate your f 0 essentially to this thing that is the W b 0 the species production rate at the burn gas temperature times l R 0. So, that is how you basically dimensionally you connect with f 0 with the reaction rate. So, the assumption is the reaction rate mass flux entering through the flame that is y 0 times f u which is equal to the reaction flux through the reaction zone which is y 0 times W b 0 because that is your there is a species consumption rate times l R 0.

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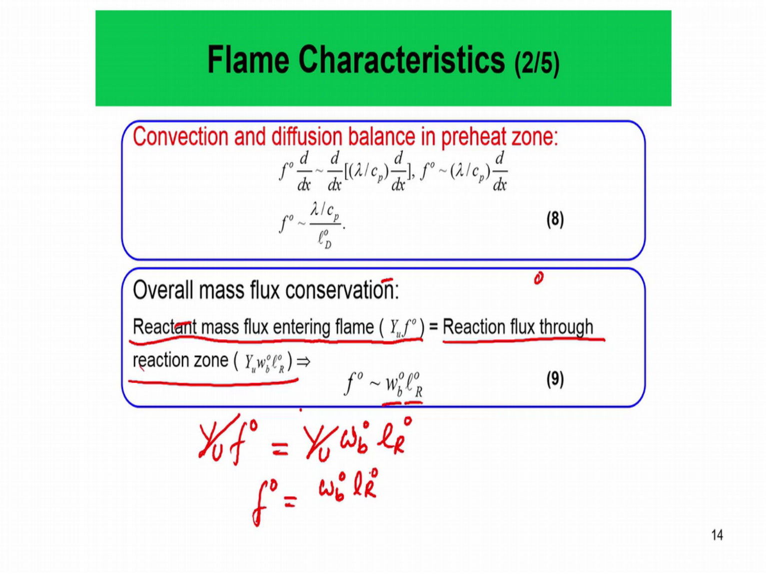

## Flame Characteristics (2/5)

Convection and diffusion balance in preheat zone:

$$f^o \frac{d}{dx} \sim \frac{d}{dx}\left[(\lambda/c_p)\frac{d}{dx}\right],\ f^o \sim (\lambda/c_p)\frac{d}{dx}$$

$$f^o \sim \frac{\lambda/c_p}{\ell_D^o}. \qquad \textbf{(8)}$$

Overall mass flux conservation:

Reactant mass flux entering flame ( $Y_u f^o$ ) = Reaction flux through reaction zone ( $Y_u w_b^o \ell_R^o$ ) ⇒

$$f^o \sim w_b^o \ell_R^o \qquad \textbf{(9)}$$

$$\cancel{Y_U} f^o = \cancel{Y_U}\, \omega_b^o\, l_R^o$$

$$f^o = \omega_b^o\, l_R^o$$

So, when you equate these 2 things that is Y u times f u 0 is equal to Y u times W b 0 times l R 0 u this; this cancels and your f u 0 is nothing, but W b 0 times l R 0.

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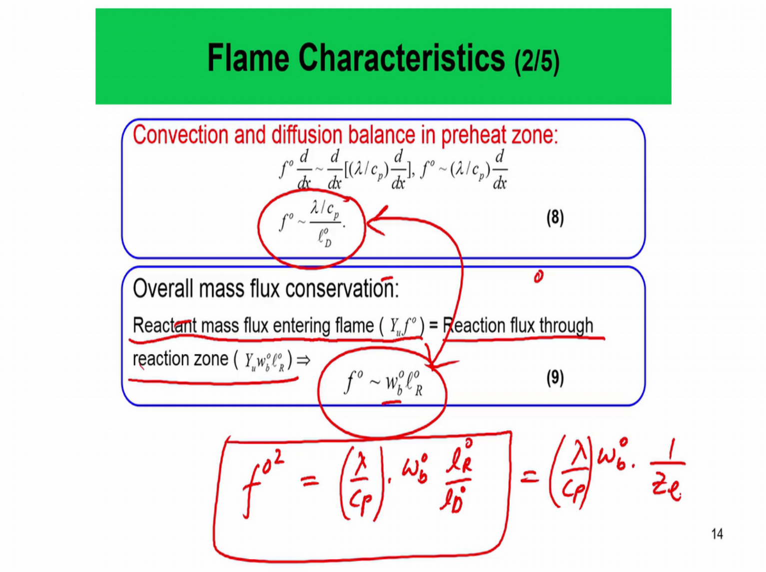

## Flame Characteristics (2/5)

Convection and diffusion balance in preheat zone:

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$$f^o \sim w_b^o \ell_R^o \qquad \textbf{(9)}$$

$$\boxed{f^{o2} = \left(\frac{\lambda}{C_P}\right) \cdot \omega_b^o\, \frac{l_R^o}{l_D^o}} = \left(\frac{\lambda}{C_P}\right)\omega_b^o \cdot \frac{1}{Ze}$$

So, now of course, what you can do is that you can relate this guy with this guy what you can do is that you can just multiply this 2 things, and you will get f 0 squared is equal to lambda by C p times W b 0 that is the reaction or the species production consumption rate the burn gas temperature times l R 0 by l D 0. This is a beautiful relationship that is emerges by very very simple consideration just by considering the species flux just by assuming that the species flux that travels through the preheat zone that is the convective diffusive zone is also has to be consumed in the reaction zone.

So, from that thing if this emerges this relationship emerges, and you see what it is this is nothing, but lambda by C p there is a thermal diffusivity of without density hence w 0 and this is what l R 0 by l D 0 is nothing, but one by Zeldovich number.

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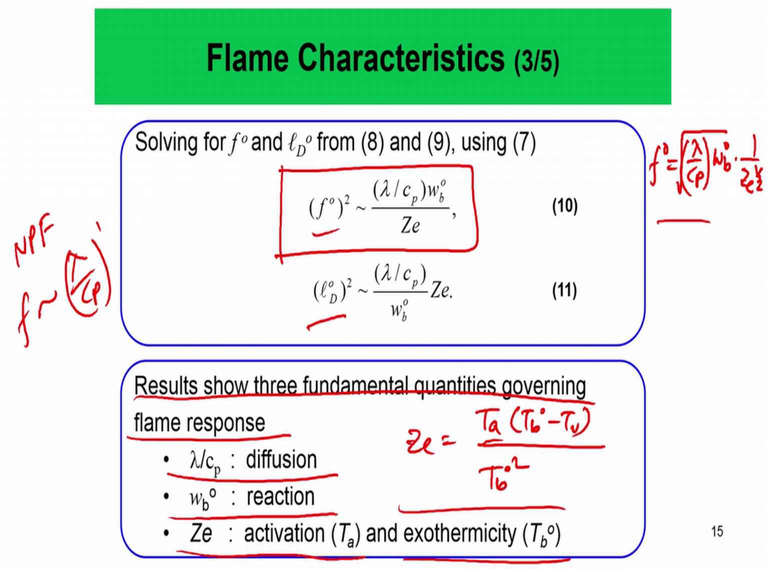

## Flame Characteristics (3/5)

Solving for $f^o$ and $\ell_D^o$ from (8) and (9), using (7)

$$\boxed{(f^o)^2 \sim \frac{(\lambda / c_p) w_b^o}{Ze},} \qquad (10)$$

$$(\ell_D^o)^2 \sim \frac{(\lambda / c_p)}{w_b^o} Ze. \qquad (11)$$

Results show three fundamental quantities governing flame response

- $\lambda/c_p$ : diffusion
- $w_b^o$ : reaction
- $Ze$ : activation ($T_a$) and exothermicity ($T_b^o$)

NPF

$f \sim (\lambda/C_p)$

$f^0 = \sqrt{\left(\frac{\lambda}{C_p}\right) w_b^0} \cdot \frac{1}{Ze^{1/2}}$

$Ze = \frac{T_a (T_b^0 - T_u)}{T_b^{0\,2}}$

So, then this thing emerges that your f 0 square is that is the burn that is the flux the burning flux squared is essentially lambda by C p times W b 0 z is Zeldovich number, or your you can write it like this your f 0 is essentially square root of lambda by C p times W b 0 and by times Zeldovich number to the power of one half.

So, here you get plenty of information about premixed flames and also you can of course, find out l D 0 is equal to lambda by C p times W b 0 times Zeldovich number. But these thing is as I said that the burning flux or the burning velocity is the most important property of a premixed flame and from this very simple consideration itself you can get a plenty information one has to always remember this that is f 0 square f 0 is essentially equal to square root of lambda by C p times W b 0 times Zeldovich number that is a thing which means that you see it this f 0 is essentially the geometric mean of lambda by C p and W b 0.

So, it means that in a premixed flame your lambda by C p and W b 0 both are equally important both diffusion and reaction are very very important, and that is why the premixed flame is much more sensitive to chemical reaction mechanism and everything whereas, in a diffusion flame which is basically diffusion control as such you have seen that your f 0 is essentially proportional to lambda by C p in a non premixed flame your f 0 or f in a chamber flame you have seen that was inverse proportional to lambda by C p times that the velocity the species gradient etcetera.

But here it has this has r to the power of 1, here it is 2 to the power of half. So, of course, here the diffusion importance of diffusion has gone down at the cost of chemical reactions, but this thing has to be always remembered; this is the first order calculation a very basic calculation which use this thing this analysis, but is very very important. We will go into more sophisticated analysis later to arrive at this again, but this itself gives a lot of information it says that f 0 square that is a burning flux square, burning flux is essentially proportional to is proportional to the square root of the thermal diffusivity without the density and this is also proportional to the square root of the reaction rate at the burn gas temperature, and also is proportional inversely proportional to the Zeldovich number 2 the power of half.

So, all these things is a very very important information and which is now this f 0 is now used to close actually the Rankin Hugoniot relation where you see m u was not specified. So, this essentially this results to see fundamental quantities governing the flame response lambda by C p is a measure of diffusion W b 0 is a measure of reaction and Zeldovich number is a measure of activation because Zeldovich number you see what it was this is essentially T a times T b minus T u divided by T b 0 squared.

So, this is the measure of essentially the activation energy and the exothermicity T b 0. So, these 3 things show how the premixed flame actually responds and as such the response of premixed flames is more rich in terms of response; there are non premixed flames because of these things.

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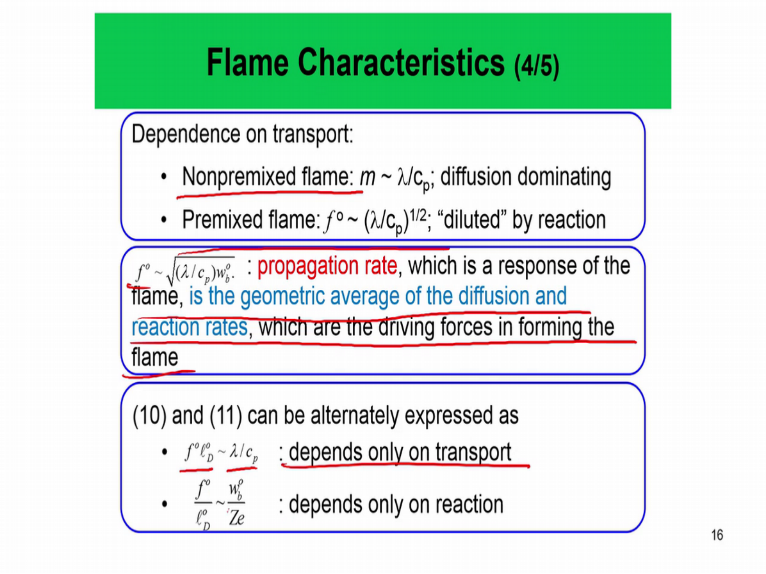

## Flame Characteristics (4/5)

Dependence on transport:

- Nonpremixed flame: $m \sim \lambda/c_p$; diffusion dominating
- Premixed flame: $f^o \sim (\lambda/c_p)^{1/2}$; "diluted" by reaction

$f^o \sim \sqrt{(\lambda/c_p)w_b^o}$. : propagation rate, which is a response of the flame, is the geometric average of the diffusion and reaction rates, which are the driving forces in forming the flame

(10) and (11) can be alternately expressed as

- $f^o \ell_D^o \sim \lambda/c_p$ : depends only on transport
- $\dfrac{f^o}{\ell_D^o} \sim \dfrac{w_b^o}{Ze}$ : depends only on reaction

So, as we have seen that here your non premixed flame the burning flux the was proportional to lambda by C p, which says it was diffusion dominating preheat diffusion and here we have premixed flame your f 0 is equal to lambda by C p to the power of half and where because your reaction is also equally important is also proportional to the power of half.

So, as a result what you have seen is that f 0 is essentially proportional to square root of our lambda by C p times W b 0 or the propagation rate is a response of the flame which is the geometric average of the diffusion and reaction rates which are the driving forces in the forming the flame. And here of course, we can say that f 0 times l D 0 is equal to lambda by C p which depends only on transport whereas, f 0 by l D 0 is equal to W b 0 times Zeldovich number which depends only on reaction.

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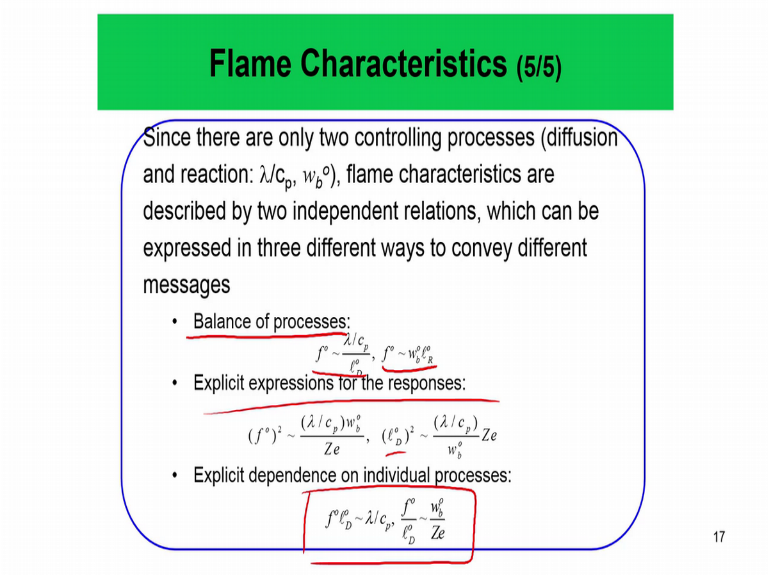

## Flame Characteristics (5/5)

Since there are only two controlling processes (diffusion and reaction: $\lambda/c_p$, $w_b^o$), flame characteristics are described by two independent relations, which can be expressed in three different ways to convey different messages

- Balance of processes:

$$f^o \sim \frac{\lambda/c_p}{\ell_D^o}, \quad f^o \sim w_b^o \ell_R^o$$

- Explicit expressions for the responses:

$$(f^o)^2 \sim \frac{(\lambda/c_p)w_b^o}{Ze}, \quad (\ell_D^o)^2 \sim \frac{(\lambda/c_p)}{w_b^o}Ze$$

- Explicit dependence on individual processes:

$$\boxed{f^o \ell_D^o \sim \lambda/c_p, \quad \frac{f^o}{\ell_D^o} \sim \frac{w_b^o}{Ze}}$$

So, to end this thing discussion that is flame characteristics as we have seen that there are only 2 controlling processes that is diffusion and reactions and the flame characteristics are described by independent relations which can be expressed in 3 different ways to convey different messages. So, if we just consider the balance of the processes that is balance of the convective and the diffusive terms convective and the diffusive fluxes we obtain this 0 is equal to lambda by C p divide by l D 0.

Whatever is flowing this flux has to consumed if the if there is complete combustion and of course, which is a case and f 0 is essentially W b 0 times l R 0, l R is the reaction zone thickness and then we can combine these 2 to find out explicit relation for the responses whereas, f 0 square is equal to lambda by C p times W b 0 times by Zeldovich number, and this l D 0 square is essentially lambda by C p divided by W b 0 times Zeldovich number and when we do the explicit relationships we can find this sort of things by considering only the diffusive thing diffusive properties of the reactive properties f 0 times l D 0 is only dependent on diffusion whereas, f 0 by l D 0 is only dependent on reaction.

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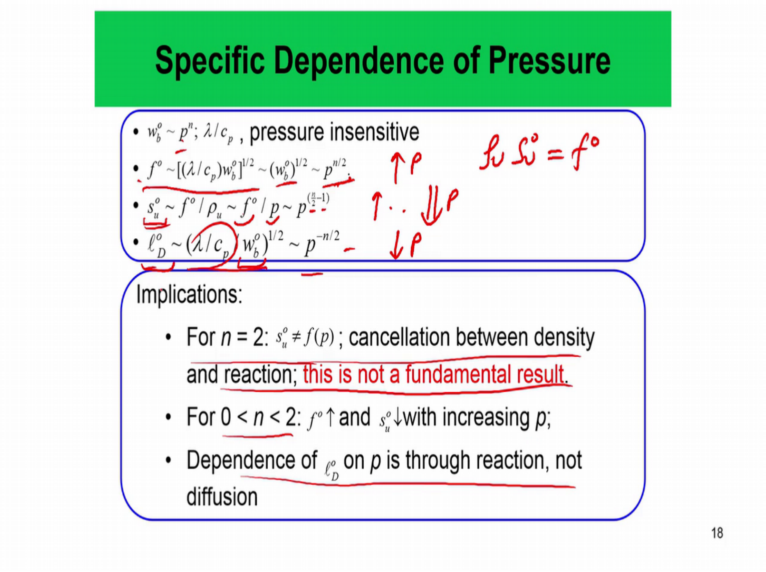

## Specific Dependence of Pressure

- $w_b^o \sim p^n;\ \lambda / c_p$ , pressure insensitive
- $f^o \sim [(\lambda / c_p) w_b^o]^{1/2} \sim (w_b^o)^{1/2} \sim p^{n/2}$ $\uparrow p$
- $s_u^o \sim f^o / \rho_u \sim f^o / p \sim p^{(\frac{n}{2}-1)}$ $\uparrow \cdot \cdot \Downarrow p$
- $\ell_D^o \sim (\lambda / c_p) / w_b^o)^{1/2} \sim p^{-n/2}$ $\downarrow p$

$\rho_u S_u^o = f^o$

Implications:

- For $n = 2$: $s_u^o \neq f(p)$ ; cancellation between density and reaction; this is not a fundamental result.
- For $0 < n < 2$: $f^o \uparrow$ and $s_u^o \downarrow$with increasing $p$;
- Dependence of $\ell_D^o$ on $p$ is through reaction, not diffusion

18

So, then the thing comes that how does your f 0 or s l depend on pressure, now we know that your W b 0 because it is a law of mass action whereby because W b 0 is essentially given by gamma by law of mass action that is the reaction rate should proportional to the product of the concentrations of the raised to the to the raise to the power of the stoichiometric exponents, and we can also alternatively write it as this concentration is the essentially proportional to the pressure. So, we can write it as W b 0 is essentially to the power of is proportional to P to the power n where n is the reaction order.

Now, we have already known that f 0 is equal to lambda by C p times W b 0 to the power of half whereas, W b 0 is essentially p to the power of n. So, this gives you a p to the power of n by 2 dependents. So, f 0 is essentially p to the power n by 2, now S u 0 which is the burning speed or the or the flame speed of the burning velocity S u 0 is proportional to is essentially this rho u times S u 0 is equal to f 0 that is a flux.

So, this is equal to this. So, is essentially becomes once again P is equal to rho r t. So, your P rho u is essentially proportional to pressure that is the upstream density where the there is no temperature variation that is we can do this, so, f 0 by P, so, then this also gives you a. So, this part gives you a P to the power of n by 2 dependents and this parts gives you a p to the power of n minus 1 dependent. So, this you get P to the power of n by 2 minus 1 dependents and then of course, your since l D 0 and f 0 are connected we can connect these 2 also and that gives you a P to the power of n minus n by 2 dependents.

So, you see that burning flux always increases with pressure as long as n is positive which is a guess. So, burning flux always increases with pressure, but you see that there is in the burning flux you also have density build to into it. So, it also increases because you have lot of because your gas is more dense. So, it is more mass is consumed more mass flux is consumed that is why it is dependent on pressure. When you take off the dependence when you take up the density this burning flux as a n by 2 minus 1. So, whenever n is only greater than greater than 2 then it becomes when n is equal to 2 it becomes pressure insensitive, and if in our cases when n is equal to 1 then it actually decreases with pressure as we will see later that S u actually decreases pressure whereas, f 0 is actually increases with pressure.

Whereas a flame thickness that is l D 0 that has a clear negative dependence, because omega b appears in the denominator in this case, so, in that case we see that P n by P is actually. So, f 0 is which goes up with pressure this can go up or most, but mostly goes down with pressure whereas, this definitely goes down with pressure; l D 0 that is the flame thickness goes on if pressure is. So, higher the pressure is, but the flame thickness becomes smaller.

So, this gives you an idea about the challenge in computations also, because you see gas turbine engines or other engines aero engines operator high pressure because when you only you add heat at high pressure you can get (Refer Time: 39:53) right. So, now, of course, at high pressure you see that your l D 0 reduces with pressure. So, if you do want to do high pressure simulation your flame thickness becomes very small. So, you need more and more grid points or if you are an experimental is doing optical diagnostics you need more and more camera resolution basically image the flame thickness or solve for the flame thickness. So, this gives you a very challenging task to modern flames at high pressure.

So, as we have seen that 2 implications we have already discussed that for n equal to 2 f is not function of pressure and this is a cancellation between density and reaction, but this is not a fundamental result for n equal to between 0 and 2 f increases pressure, but S u decreases with pressure which is mostly the case and whereas, l dependence of l D on pressure is through reaction, but nor through diffusion as a result of that because lambda by C p does not depend on pressure. So, as a result of this your l D 0 invariably decreases with pressure.

So, with that we will end this class and then we will go into analysis of the governing equations to derive the flame speed in a more rigors manner, here it is done in a more phenomena logically. Now, one can ask that why should we do it more (Refer Time: 41:02) because then when you do things in a reverse way in a mathematical detailed manner, then all the properties of the flames also becomes much more clearer and it will tell you also whether this analysis that we have done whether that is correct or not and but to gain more understanding one needs to do a detailed rigorous analysis which we will do in next class using the frank Kaminsky analysis for that until then.

Thank you very much and see you in next class.