Combustion in Air Breathing Aero Engines
Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

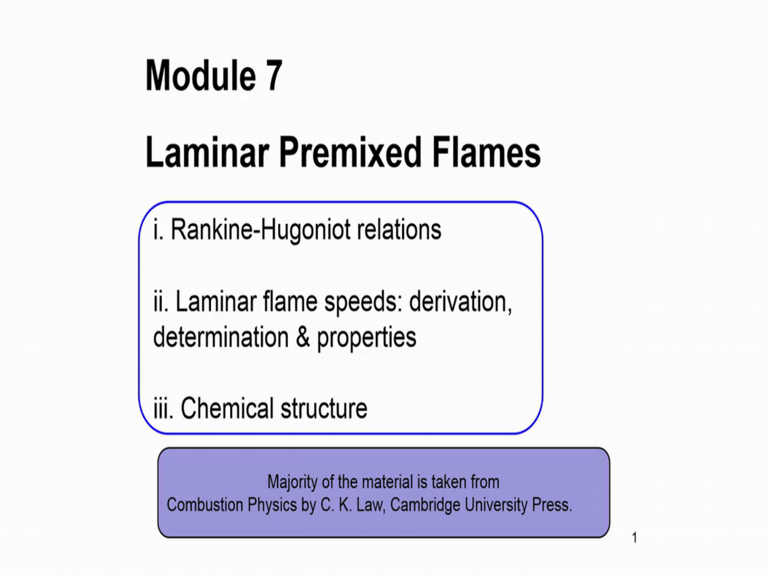
Lecture – 22
Laminar Premixed Flames I

Hello friends, welcome back to this course on combustion in air breathing aero engines. So, as you have seen that we have covered quite a bit of ground so far we have talked about kinetics we have talked about chemical equilibrium we have talked about kinetics oxidisation mechanism of fuels, then we have looked into transport phenomena and then looked into the governing equations; and after that we went to basically on the formulations for non premixed flames and then we looked into a very ideal kind of a non premixed flame which is a chambered flame, and then we looked into the droplet evaporation and droplet combustion which is also an example of a non premixed combustion.

Now then of course, we have talked we have said that in nature or in by design fuel and oxidisers are separate and then of course, the when they meet because of diffusion and because of the continuous consumption of this oxidiser and the fuel by the flame then a non premixed flame is actually formed. The non premixed flame the issue is that in the non premixed flames the fuel and the oxidiser are not mixed at the molecular level a priori and they only meet at the flame location itself, and where they basically burn in stoichiometric proportions and as a result the flame temperature that is formed is adiabatic flame temperature and which is also the reason why non premixed flames produces lot of pollutions in terms of nox, soot, etcetera.

So, those we have discussed about non premixed flame we have discussed plenty about droplet about condensed matter evaporation and then into droplet evaporation and droplet combustion also, but as you know that there is another variant of flames another in another mode combustion can happen of course, and that is in premixed mode. So, in the premixed mode the fuel and the oxidiser they are mixed with the molecular level, they are perfectly mixed before combustion happens; and this is different from the non premixed combustion there are quite a few differences with respect to non premixed combustion, but you will see that there are also quite a few similarities that is both essentially will see that there is another in flames that is premixed flames that we will talk about, those are also very strongly controlled by diffusion. So, it is to be borne in mind that both the non premixed flame as well as the premixed flame they both are in both processes diffusion plays a very important role and we need to understand that very well and then.

(Refer Slide Time: 02:58)



Module 7

Laminar Premixed Flames

i. Rankine-Hugoniot relations

ii. Laminar flame speeds: derivation, determination & properties

iii. Chemical structure

Majority of the material is taken from
Combustion Physics by C. K. Law, Cambridge University Press.

So, next we go into this formal into this course into this in today’s class where we will talk about laminar premixed flames, and here we will see that these are the topics that we will cover. Will cover this Rankin Hugoniot relations and you will see that these basically talk about the how the thermo dynamic properties in the burnt and un burnt gases can be connected by this sort of relations and very interesting properties of this of the different types of the modes of premixed combustion that can happen emerges from this kind of analysis, then we will go into the laminar flame speeds which is a most important property of a premixed flame and we will go into different levels of derivation where we will do a phenomena logical analysis just to give an idea about what are the processes that are important, and then we will go into the chemical structure of the flames.

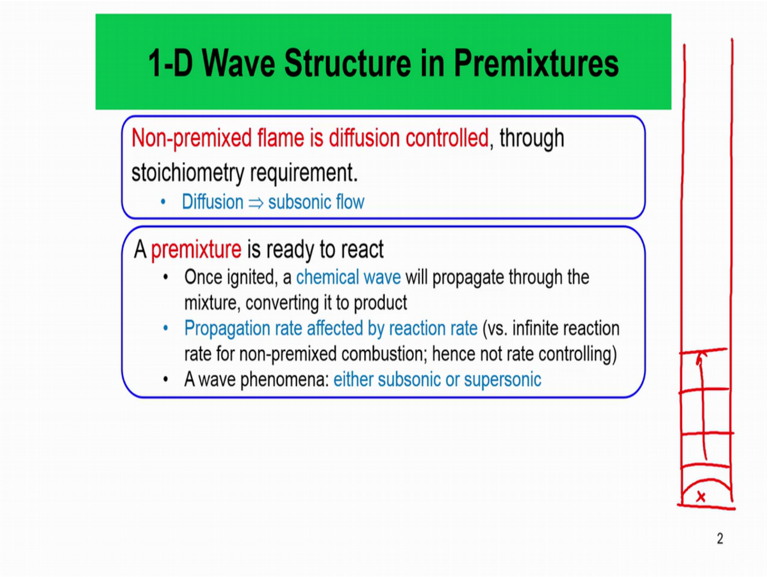
So, premixed flames as you see that though it is has we have discussed previously that that when we compare we almost compare premixed flames with non premixed flames in terms of the properties and the advantages they have, and we will see that unlike non premixed flames in the premixed flames one can readily control the temperature by just controlling the equivalence ratio, and that gives an advantage becomes this allows you to have low temperature not very low temperature, but still lower than the non premixed flame temperatures one can achieve, and one can design the combustion process accordingly.

But then there are other challenges with the premixed flame because you will see later that the premixed flame is essentially a wave phenomena and so you need to basically control the wave and to do some design in the combustor to have it is proper to basically stabilise the wave. So, there are very interesting features that emerges from the premixed flames. So, sometimes to control over to have a control nowadays in aero proportion engine, many of the engines try to go towards the more and more premixed mode from the non premixed mode because as I have said earlier that the premixed mode allows you have to have better control over the over by over to lower essentially the emissions nox emissions, soot emission these can be much lower in case of premixed combustion and with respect to those un non premixed combustion. So, for that it is very important to know premixed combustion and as I said that even modern gas turbine engines, but previously previous generation gas turbine engines where very strongly non premixed.

So, but the modern generation gas turbine engines for example, the GENX engine that powers a blowing 787 as you know that those use this 2 in annular premixed swirl combustors. So, this combustion happens in a partially premixed mode. So, the idea is that from non premixed you are trying to go to move towards premixed to basically have a control over the emissions. So, it is important to first understand that why basically you can have a control over the emissions when you do premixed combustion. So, we will take those things in a non in this class and we will also do appropriate comparisons with the non premixed flames to basically understand it.

But of course, it must be borne in mind that this in a combust and in actual combustor it is not a laminar premixed flames that that you encountered these are very strongly trouble in premixed flames that we encountered, but as you know that to understand something very complex you need to understand the basics first. So, that is why we will go deep into this discussions on non pre laminar premixed flames, because only when you understand laminar premixed flames we will be in a position to understand trouble of premixed flames and which is what happens inside modern or post modern combustors that we that we encounter or we will continue to encounter in some few years to come.

(Refer Slide Time: 06:31)



1-D Wave Structure in Premixtures

Non-premixed flame is diffusion controlled, through stoichiometry requirement.

- Diffusion ⇒ subsonic flow

A premixture is ready to react

- Once ignited, a chemical wave will propagate through the mixture, converting it to product
- Propagation rate affected by reaction rate (vs. infinite reaction rate for non-premixed combustion; hence not rate controlling)
- A wave phenomena: either subsonic or supersonic

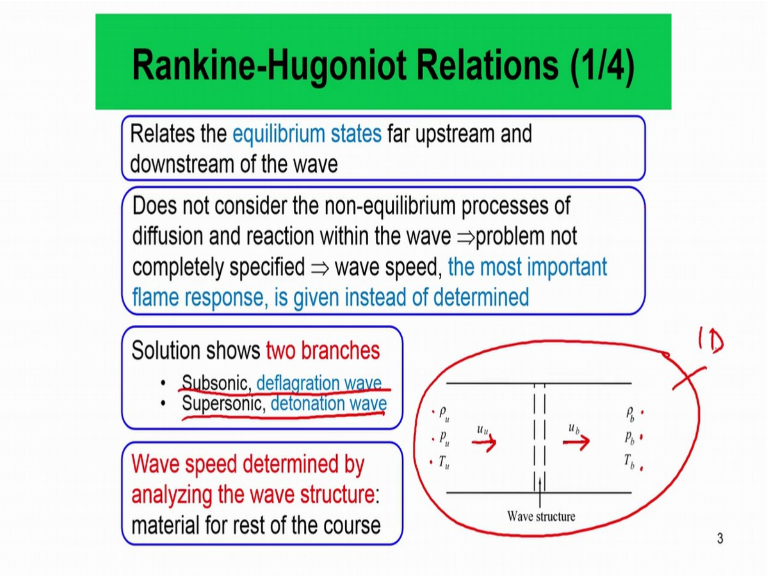
So, the as I said that here we will discuss the one d wave structures of Pre-mixtures. So, as we have seen that non premixed flame is basically diffusion controlled because we can assume that the reaction is much much faster than the diffusion, and it is of course diffusion control and we have seen that the flame stabilises in the location where it is essentially in a fuel and the oxidiser are essentially in the stoichiometric proportions and the temperature achieved is essentially at a is the adiabatic flame temperature for the corresponding stoichiometric mixture.

Now,. So, essentially it is a non premixed flame is essentially is purely controlled by diffusion and it is essentially it happens in subsonic flows. Now pre-mixture when you have a pre-mixture that usually you fuel mix and the fuel and the oxidiser. So, when you mix methane with air or when you mix hydrogen with air. So, it can then readily react all right now once you ignite at a point then basically what happens is that a chemical wave which is essentially the flame or which is essentially I was you will see later this either a deflagration or a detonation depending on it is speed, a chemical wave is formed and that propagates through the mixture converting into product. So, if you have a tube like if you have a long tube if you have a tube like this or if you have a tube long tube like this and this is filled with this is entire tube is closed here and this entire tube is filled with fuel air mixture and then you ignite at this point. So, then immediately yours flame will be formed and it can be of course, in difference shapes, but anyways in ideal version it would be a laminar flame in form, and this will propagate onto this until the entire mixture is consumed.

So, essentially it is a wave structure that is propagating through this entire tube and so as we will see that one of the basic description of a premixed flame is basically it is wave phenomena. So, the propagation rate unlike in the as we have seen previously in the non premixed flame the propagation the burning flux of or burning rate or the burning flux f that was nox that is proportional to basically the thermal diffusivity. So, it is essentially was purely diffusion control, but here you will see that the reaction rate plays a very important role, alright.

So, here it is you will see that basically it will be it will very emerge very clearly that in premixed flame both the diffusion and the reaction processes are equally important. So, when the reaction rate becomes very important then of course, you cannot assume that it is infinite with comparison to the diffusions, and then you have to basically consider the detailed reaction rates you might have to consider detailed reaction rates of course, a lot can be understood by considering the one step reaction rates also, and this is because it is a wave phenomena it can either be subsonic or supersonic; and this subsonic state will be essentially called deflagration and the supersonic state will be called detonation.

(Refer Slide Time: 09:31)



Rankine-Hugoniot Relations (1/4)

Relates the equilibrium states far upstream and downstream of the wave

Does not consider the non-equilibrium processes of diffusion and reaction within the wave ⇒problem not completely specified ⇒ wave speed, the most important flame response, is given instead of determined

Solution shows two branches

- Subsonic, deflagration wave
- Supersonic, detonation wave

Wave speed determined by analyzing the wave structure: material for rest of the course

Now, first you will in this we will go into this first description that is a Rankin Hugoniot relations, and what this does is basically you have in the you have a if you have a premixed flame, the flame as you as you know because of the large activation energy is confined into a very very thin region. So, upstream of the flame and downstream of the flame these are there is no reaction happening because reaction is confined in a very thin region. So, upstream of the flame is of course, reactants that is fuel and the oxidiser mixture, the downstream of the flame is of course the burnt products and both are essentially in the down is in chemical is in is in equilibrium state and so, we can assume that equilibrium state exists in the form of upstream and downstream of the wave and so, we will not consider in the first level description in this Rankin Hugoniot analysis.

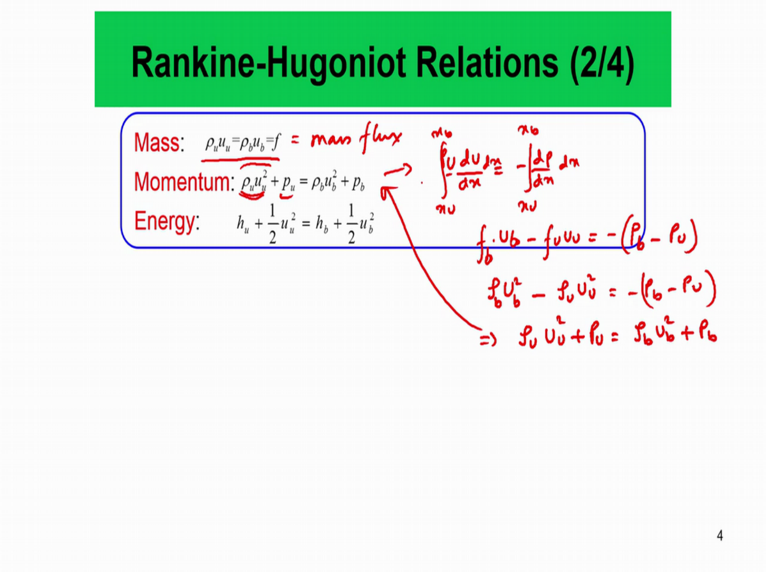
So, we will not consider this we will not consider this non equilibrium process of diffusion reaction within the wave, and as a result of that we will see that the wave speed will not be specified, but then we will just try to equate try to somehow relate the downstream state of the wave with respect to the (Refer Time: 10:41) is downstream state of the wave and vice versa. And of course, that will not give unless it is a very special case as we will see that the wave speed will not be specified. So, you still to close the problem you have to specify the wave speed, but then without that as long as that is specified we can relate we will see that we can relate the downstream state and the upstream state of the wave in a very comprehensive manner just by using the very basic governing equations of mass conservation momentum conservation and energy conservation and where we do not have to consider details of the chemical kinetics, because we are not essentially interested into the wave structure.

The wave structure is essentially controlled by kinetics and diffusion processes, but whatever happens far upstream and far downstream as you know the far downstream rates is essentially in equilibrium state, and that is given by equilibrium relation. So, you do not need chemical kinetics to basically know that. So, you will see that these solutions of this kind of an analysis will give you 2 branches, one will give you a subsonic deflagration and wave and another will give you a; this is a subsonic deflagration and it another will give you a sub supersonic detonation wave.

The wave speed will be as I said that will be analysing will be obtained by analysing the wave structure and that is basically wave speed means the flame speed and that will come later. So, for now we will not consider flame speed and we will consider these very idealised description that is from here. So, what does it say? It says that that if we consider this premixed flame or this wave which we considered to be in a stoichiometric state with respect to the our say our laboratory coordinates then that if that is so, then of course, it has to be to sustain that structure in a steady state of course, it has to be fed by premixed reactants and this premixed reactants are being fed into the variable at a velocity u u, and the burnt products and the and the premixed reactants of course, are converted into burnt products in this wave structure which we are not specifying here and this the velocity of the burnt gas is essentially U b.

So, similarly the properties upstream of this wave is given by rho u P u and T u to subscript u means un burnt whereas, properties downstream that is in the burnt state is given by the rho b P b and T b. So, these are the different states of the upstream state and the downstream state or the un burnt state and the burnt state of the wave, and what our objective in this analysis would be to have relation between them and of between the upstream step and the downstream step and we will not consider the wave structure. But as I have said that the problem will not be complete without specifying the wave velocity and for that needs will be put in as an external parameter all right which should be specified by something else, which we will not consider here we will consider that is to be given for this problem. So, what are the how do we arrive at this how do we relate the upstream and the downstream states which are in equilibrium of this wave.

(Refer Slide Time: 13:34)



Rankine-Hugoniot Relations (2/4)

Mass: $\underline{\rho_u u_u = \rho_b u_b = f}$ = mass flux

Momentum: $\rho_u u_u^2 + p_u = \rho_b u_b^2 + p_b$ $\rightarrow$ $\int_{x_u}^{x_b} \rho u \frac{du}{dx} dx = -\int_{x_u}^{x_b} \frac{dp}{dx} dx$

Energy: $h_u + \frac{1}{2}u_u^2 = h_b + \frac{1}{2}u_b^2$

$\rho_b \cdot u_b - \rho_u u_u = -(p_b - p_u)$

$\rho_b u_b^2 - \rho_u u_u^2 = -(p_b - p_u)$

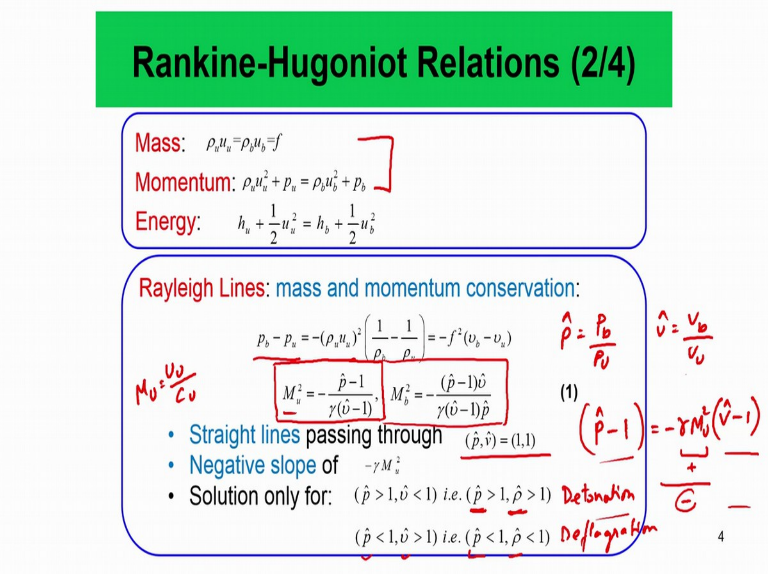
$\Rightarrow \rho_u u_u^2 + p_u = \rho_b u_b^2 + p_b$

For that of course, as you know in any problem fluid mechanics essentially reacting fluid mechanics though we are not considering that reactions here. So, here where there some change of state happening in some there is a reactant going into products. So, of course, mass is conserved. So, then if you write the continuity equation and integrate that from upstream to downstream, this is what you will find that rho u u u times rho u u u is equal to that is the density of upstream times velocity of upstream is equal to density downstream or the density of the burnt state times the velocity of the burnt state and this is equal to f which is the mass flux it is not the mass fluid it is the mass flux and we will also calls call this as a burning flux because this is the flux which is being burnt and converted into products.

And then of course, the momentum so, this is your; this is the momentum the axial it is one once again this is you see that this analysis is in purely in ID. So, we can basically if we what we can do is that we can take the complicated momentum equations and then we can just integrate it around. So, then this is what you will get. So, this is what you get by integrating the acceleration terms and this is what you get by integrating the pressure gradient terms around the one dimensions. So, this is you get rho u times U u square plus P u is equal to rho b times u b square plus p b. So, this is what you get we can just simply show this by considering the one d momentum equation of course, in a steady state that is if this was the momentum equation if you remembered is equal to minus d p d x. So, if you integrate between x u and x b d x and you will get is essentially and of course, the rho is here and you will get and we can just write rho u to be essentially f. So, f is constant. So, it comes out. So, f times u. So, f U u u minus or f b u b minus f U u u is equal to minus P b minus P u.

So, f p is nothing, but rho u u u. So, you find that or we can write it as rho b u b. So, you find that rho b u b square minus rho u u u square is equal to minus P b minus P u and this implies rho u u u square plus P u is equal to rho b u b square plus p b. So, this is exactly what you get. So, we will not o course we here we are this is just to show that if you consider the one d equations and the differential form in the p ds you convert the 2 basically you convert the 2 ds because there is no time variation there is no variations y and z. So, it is only an x variation and then you can just write down these things which is the momentum equation in the integrated the momentum equation in the x directions.

(Refer Slide Time: 17:10)



Rankine-Hugoniot Relations (2/4)

Mass: $\rho_u u_u = \rho_b u_b = f$

Momentum: $\rho_u u_u^2 + p_u = \rho_b u_b^2 + p_b$

Energy: $h_u + \frac{1}{2}u_u^2 = h_b + \frac{1}{2}u_b^2$

Rayleigh Lines: mass and momentum conservation:

$$p_b - p_u = -(\rho_u u_u)^2\left(\frac{1}{\rho_b} - \frac{1}{\rho_u}\right) = -f^2(\upsilon_b - \upsilon_u)$$

$$\boxed{M_u^2 = -\frac{\hat{p}-1}{\gamma(\hat{\upsilon}-1)}}, \boxed{M_b^2 = -\frac{(\hat{p}-1)\hat{\upsilon}}{\gamma(\hat{\upsilon}-1)\hat{p}}} \quad (1)$$

- Straight lines passing through $(\hat{p},\hat{v}) = (1,1)$
- Negative slope of $-\gamma M_u^2$
- Solution only for: $(\hat{p} > 1, \hat{\upsilon} < 1)$ *i.e.* $(\hat{p} > 1, \hat{\rho} > 1)$ Detonation

$(\hat{p} < 1, \hat{\upsilon} > 1)$ *i.e.* $(\hat{p} < 1, \hat{\rho} < 1)$ Deflagration

$M_U = \frac{U_U}{C_U}$

$\hat{P} = \frac{P_b}{P_U}$ $\hat{v} = \frac{V_b}{V_U}$

$$(\hat{P}-1) = -\underbrace{\gamma M_U^2}_{+}(\hat{V}-1)$$

$\ominus$

So, this is it and then the energy equation is of course, you have the enthalpy the h u and then you have which is the inter energy plus p y p p p p v where v is a special column here. So, and then you have the kinetic energy term and then you have the burnt gas enthalpy and then you have the burnt gas velocity.

Of course you have to remember that this h u and h b are total enthalpies and even though there is reaction happening the total enthalpy is comes out because what happens; the enthalpy of formation is converted to sensible enthalpy as combustion happen. So, this individually this contains the enthalpy of formation and the enthalpy of and the sensible enthalpy. So, now, you can get by if you use these 2 things if you use the mass and the momentum equations together. So, if you combine then the then you basically get something like this that is P b minus P u is equal to if you just consider the momentum equations this is just giving by if you just take the rho u times U u square and then you of course, have left over rho b and rho u in the denominator, but then using the mass conservation equation you can substitute rho u u u is equal to of course, this is also obtained by mass conservation equation.

So, and you can of course, substitute this and you get v v minus v u which are the specific volumes. So, these are the specific volumes and should not be considered with velocity and then of course, if you write that that sound speed is equal to root over gamma r t and then you do this manipulations you will find that M u square. So, M u square is nothing, but M u is nothing, but the mach number or the un burnt gas which is nothing, but U u times c and this U u times C u and this m b is nothing, but U u times C b that is ratio of the velocity to the acrostic velocity to the acrostic speed and then you can write this equation. So, this gives you when you combine the mass and the momentum it is gives you a very interesting thing.

So, it gives you that M u square that is the mach number square is essentially equal to p cap whereas, P cap is nothing, but, P by P u and v cap is nothing, but v by v u that is with respect to un burnt gas and of course, here the oh sorry P cap is not P by P u, P cap is essentially the ratio of P cap is equal to P u by P b and v cap is essentially v p cap is equal to P b by P u and v cap is equal to v b by v u. So, p cap is essentially the ratio of the gas of the pressure or the burnt gas with respect to the pressure the un burnt gas whereas, v cap is the ratio of the specific volume in the burnt gas with respect to the specific volume in the un burnt gas and similarly the whereas, this M u is nothing, but U u is equal to nothing, but M u is equal to U u by C u which is the acrostic velocity c u is equal to root over gamma t.

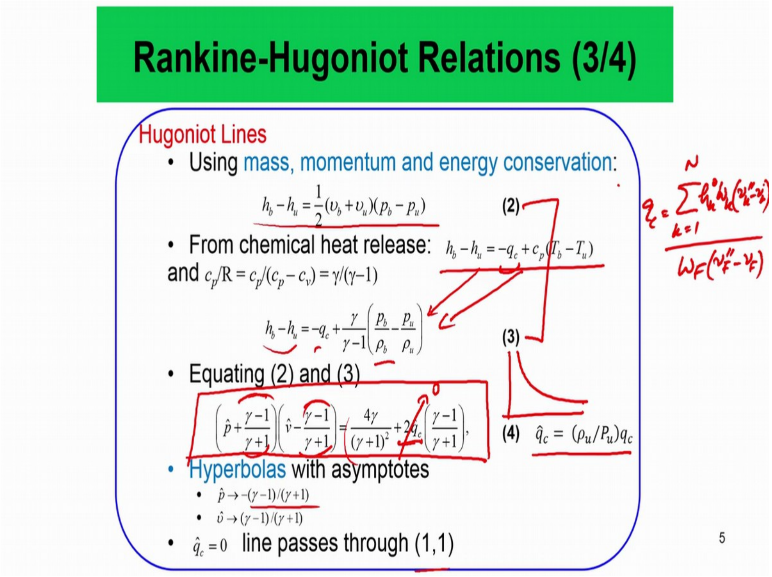
So, similarly, you can write both in terms of the of in terms of like. So, here is what we wanted that is we wanted to relate the ratio the burnt gas state to the un burnt we wanted to relate the this 2 states that is a burnt gas burnt gas state to the un burnt gas state in terms of the thermo dynamics properties of the burnt gas burnt gas state to the thermo dynamics properties of the un burnt gas state. So, this is the ratio of the burnt gas pressure to the un burnt gas pressure minus 1 divided by gamma times the specific volume of the burnt gas with respect to the specific volume of the un burnt gas minus 1 is giving by M u square, similarly this is also given by M b square. So, what does this tell you? This tells you a very interesting thing that of course, it is a essentially straight lines which pass through 1 1 where p cap is equal to 1 v cap is equal to 1 and you see this M u square is a essentially always a positive quantity because it is a squared.

So, minus gamma m square is essentially the slope of this straight lines p cap minus 1 and v cap minus 1. So, because this is always positive this slope of this lines are always negative or also we have to see that another thing is that because we can write P cap minus 1 is equal to minus gamma M u square v cap minus 1, as I said that p cap is the ratio of P b by P u v cap is the ratio of v b by v u and as such as a result of because p cap minus 1 minus is equal to minus gamma u square v cap minus 1 and this is essentially always positive. So, hence this thing is always negative.

So, it means that when this is positive this has to be negative or when this is negative this has to be positive so; that means, that if p cap is greater than 1 v cap has to be less than 1 that means, that when v cap and v cap is essentially the (Refer Time: 22:30) volume which is inverse of density. So, it means that when p cap is greater than 1 the density ratio is also greater than 1 that is when the pressure ratio between the burnt and un burnt gas is greater than 1, the density ratio is also greater than 1 and you will see that this corresponds to later to the detonation stage, where the pressure and density both increase by crossing the wave. I am not crossing the; I am not calling it a flame because detonation is not essentially a flame a flame is essentially subsonic structure.

So, crossing that chemical wave your peak pressure ratio and density ratio both increase whereas, in this case you see p cap is less than 1 v cap is greater than one; that means, pressure ratio is less than 1 density ratio is less than 1 and that is your deflagration state where both the pressure and the density drops by crossing the chemical wave.

(Refer Slide Time: 23:27)



Rankine-Hugoniot Relations (3/4)

Hugoniot Lines

- Using mass, momentum and energy conservation:

$$h_b - h_u = \frac{1}{2}(\upsilon_b + \upsilon_u)(p_b - p_u) \qquad (2)$$

- From chemical heat release: $h_b - h_u = -q_c + c_p(T_b - T_u)$
and $c_p/R = c_p/(c_p - c_v) = \gamma/(\gamma-1)$

$$h_b - h_u = -q_c + \frac{\gamma}{\gamma-1}\left(\frac{p_b}{\rho_b} - \frac{p_u}{\rho_u}\right) \qquad (3)$$

- Equating (2) and (3)

$$\left(\hat{p} + \frac{\gamma-1}{\gamma+1}\right)\left(\hat{\upsilon} - \frac{\gamma-1}{\gamma+1}\right) = \frac{4\gamma}{(\gamma+1)^2} + 2\hat{q}_c\left(\frac{\gamma-1}{\gamma+1}\right), \qquad (4) \quad \hat{q}_c = (\rho_u/P_u)q_c$$

- Hyperbolas with asymptotes
  - $\hat{p} \to -(\gamma-1)/(\gamma+1)$
  - $\hat{\upsilon} \to (\gamma-1)/(\gamma+1)$
- $\hat{q}_c = 0$ line passes through (1,1)

$$q_c = \frac{\sum_{k=1}^{N} h_k^0 W_k(\nu_k'' - \nu_k')}{W_F(\nu_F'' - \nu_F')}$$

So, now we can go into another set of lines because you see we have used we have three conservation equations we are the conservation equation of mass momentum and energy. So, using the mass and the momentum conservation equation we have arrived that this Rayleigh lines, which are essentially straight lines passing straight lines of where the variables are essentially p cap and v cap that is variables are pressure ratio and the specific volume ratio.

So, these are essentially the Rayleigh lines are essentially straight lines that pass through p cap equal to 1 v cap equal to 1 and the slope of them are given by of this set of lines are giving by gamma M u square. Where M u square is essentially the rho is essentially the mach number of the un burnt gas which is also can be ratio taken as a mach number of the flame speed because your rho u u is we will see that is nothing, but the flame speed so, but this is not specified we do not know what M u is or what U u should be for the flame to be stabilised that is why it is essentially the flame speed and, but we found that the interestingly something more we found is that that when p cap is greater than 1, v cap must be less than 1 or when p cap is when pressure ratio is greater than 1, the density ratio also must be greater than 1 and when the pressure ratio is less than 1 the density ratio also less than 1 and we have stated that. The first condition corresponds to your detonation state and the second condition corresponds to your deflagration state.

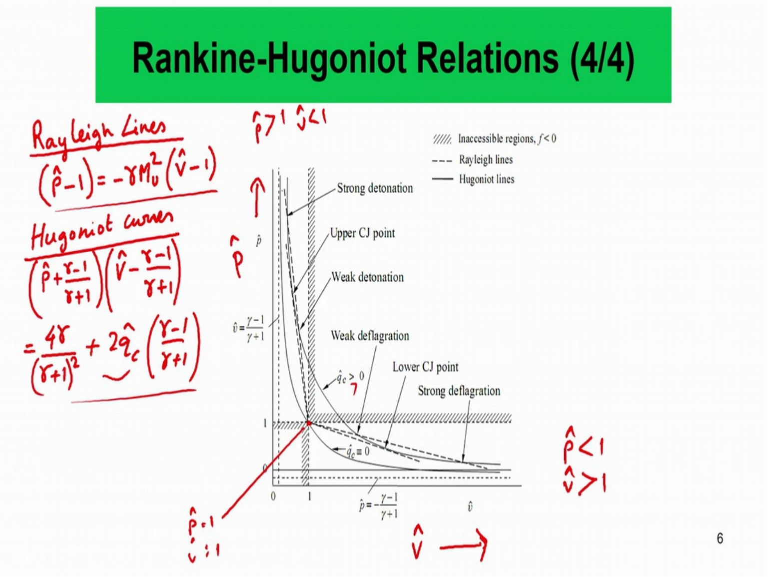
But then as we have not used another equation that is so far we have not used the energy equation. So, we can also use the energy equation and along or in combination with the mass momentum and energy equation with when the mass the momentum equations and that is what we will do now. So, if you do that you find that if we use the mass momentum energy equations, we come up with the Hugoniot lines which is this that is the enthalpy difference between the burnt and un burnt gas also (Refer Time: 25:19) total enthalpy difference is nothing, but half times v b plus v u divided times P b minus P u. Now of course, this since because these are the total enthalpy differences now we can write them as we can write h b essentially equal to h b 0 plus h b s minus h u 0 minus h u s and so, once you do that you can essentially see you know that the heat of combustion is essentially the enthalpy of formation of the products minus the reactants and that is when from that you basically get the heat of this heat release the chemical heat release. So, which you have the definition of which we learnt previously that is it is essentially the h 0 k times w k times nu k double dash minus nu k dash divided by your with the reference species that is w f times, molecule where w f being the molecule of the times nu double dash minus nu f nu f double dash minus nu f dash.

So, that was your chemical heat release or. So, that is lets write it down here, this is essentially summation k is equal to 1 to n, h k 0 w k nu k double dash minus nu k divided by w F times nu F double dashed minus nu F. So, this was what we defined as chemical heat release and the of course, we can relate the sensible enthalpy to the temperature using the specific heat ratio that is c p once you do that we can write this. So, once you combine these 2 things we can write that when from here itself we can write that h b minus h u is equal to q c times gamma by gamma minus 1 times P b this is of course, by P b rho b is essentially r t b minus P b minus where rho u is equal to r t u. So, from that using the ideal gas state we can go from directly from here to here, and then you creating these 2 things together we arrive at this Hugoniot lines which are nothing, but this things that is the look at this very carefully I am not going to the derivation, but the formula needs to be examined very very carefully these are very important.

So, we see that the pressure ratio p cap plus gamma minus 1 by gamma plus 1, times v cap that is a that is a special volume ratio minus gamma minus 1 by gamma plus 1 is essentially 4 gamma plus 1 plus q c cap which is this guy and q c cap means of course, this thing where rho u by P u times q c times gamma minus 1 by gamma. So, what this is this is essentially we can write in a hyperbolic form that is hyperbolas are essentially you see x y is equal to constant. So, it is essentially in that form so, with asymptotes, you see this these things p cap plus this is. So, it will generate lines like this as you know that whereas, these asymptotes are given by this things. So, the asymptotes here are essentially this one the asymptote of p cap is essentially gamma minus 1 by gamma plus 1 that is this part whereas, asymptote of v cap is essentially gamma minus 1 by gamma plus 1 this is this thing and now the thing is that we can also this equation of course, we have derived for combustion where we have essentially equated the burnt state with the un burnt state of combustion, but we can also have a we can also reduce to a state where there is no combustion. So, what we have to do is that we have just have to set this be equal to 0.

So, when we equal to set this equal to 0 we find that this then this lines passes to 1 1. So, if you remember that 11 was also the line through which the Rayleigh lines pass. So, when you have do not have any combustion. So, these 2 essentially intersect at these lines. So, we will see also they could intersect somewhere else also.

(Refer Slide Time: 29:06)



Rankine-Hugoniot Relations (4/4)

Rayleigh Lines

$(\hat{p}-1) = -\gamma M_u^2 (\hat{v}-1)$

Hugoniot curves

$\left(\hat{p}+\frac{\gamma-1}{\gamma+1}\right)\left(\hat{v}-\frac{\gamma-1}{\gamma+1}\right) = \frac{4\gamma}{(\gamma+1)^2} + 2\underbrace{\hat{q}_c}\left(\frac{\gamma-1}{\gamma+1}\right)$

So, this is what you get by combining the Rankin Hugoniot and the Rayleigh lines So, if you remember the Rayleigh lines was given by this p cap minus 1 is equal to gamma M u square times v cap minus 1. So, this is the Rayleigh line we will just only use the M u whereas, the Hugoniot lines or Hugoniot curves are given by p cap plus gamma minus 1 oh sorry it is not visible in that part. So, p cap plus gamma minus 1 by gamma plus 1 times v cap minus gamma minus 1 by gamma minus plus 1 is equal to 4 gamma by gamma plus 1 whole square plus 2 q c cap normalised chemical heat release times gamma minus 1 by gamma plus 1. So, we have seen that this Rayleigh lines basically are arrive from mass and momentum.

Whereas these lines essentially arise from mass momentum and energy conservation. The difference is that these as you see that these are essentially straight lines whereas, these are essentially hyperbolas. So, these lines that you see in this thing if you plot these lines in the p cap v cap diagram. So, this axis is essentially p cap, and this axis is essentially v cap this is a very very important and fundamental diagram in combustion. So, we have to really go into this, v cap this is the v cap axis this is the p cap and this is the 1 1 point, this is the point where p cap is equal to 1 and v cap is equal to 1. Now these lines if you plot this Rayleigh lines these are these lines and of course, it depends on the slope. So, as the M u changes you do not know what is the M u is right now. So, you can have different sorts of these lines for different m us like this, but all of them as to meet up p cap v cap is equal to 1 or also here. So, this these set as you see corresponds to large this is this part corresponds to p cap greater than 1 and v cap less than 1 whereas, you can also have another part as you see this part corresponds to p cap less than 1 and v cap greater than 1.

So, as you will see that when p cap greater than 1 that is when the pressure rises, when pressure rises in the combustion gases pressure rise in crossing the flame we will call that detonation and pressure drops across a flame we will call that deflagration. So, this part essentially corresponds to the deflagration and this part essentially corresponds to the detonation of course, there is nothing that is possible because of this slope because the negative nature of the slope. If it something comes in this part then that will have a gamma M u square to be positive and that is not possible. So, everything has a negative slope on these lines all right. So, now these are the straight lines and then you all can also plot these Hugoniot curves. So, which are basically these rectangular hyperbolas which are these lines, and here you see that the parameter that changes we do not have any mach number square, but we have a q c. So, this is the point where q c equal to 0 then it intersects through the passes through the 1 1 curve, but then as you increase q c this curve shifts in this side all right. Now the of course, as you have seen that the previous one the Rayleigh lines are given by mass and momentum conservation whereas, the Hugoniot curves are essentially given by are essentially given by your mass and energy conservation.

So, of course, all three should be conserving simultaneously and these 2 equations should hold simultaneously. So, that where these the points to add these equations are simultaneously hold, that is the point of intersection of the straight lines that is the Rayleigh lines with this hyperbolas that is the Hugoniot lines, those are basically have an acceptable states of combustion that we can have. So, that is the thing. So, we need to find out these points of intersections of these straight lines and these hyperbolas.