Combustion in Air Breathing Aero Engines
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Lecture - 21
Laminar Non-Premixed Flames IV

Welcome back, we going to look into droplet vaporization and then subsequently droplet combustion. Now, as I told in previous classes, this is very very important. Why, because in you know all aero engine combustors essentially use liquid fuels like kerosene different variance of jet fuels and even the gas turbines, ramjets, scramjets everybody all of these engines and even after to one wants to use jet fuels different variants of fuels in some form. And then this has to be burn, of course that is because of the large energy density. Now, how do you burn it? You cannot have direct combustion of the liquid itself, so the liquid must evaporate and then it must be burned.

Now, at the same you want the liquid to evaporate quickly also because you want fast burning of a lot of fuel in lot of air. So, to quickly evaporate, if you have a pool like that, the surface area of that pool is not very large compare to its total volume. So, to increase the surface to volume ratio, it is best to have this droplets (Refer Time: 01:24) small droplets, the smaller the droplet the better is the surface to volume ratio. You can understand clearly it is like s by v is 4 pi r square divided by fourth of pi r cube. So, it is proportional to 1 by r. So, smaller is smaller is radius the larger is the surface to volume ratio.

So, one needs to have a very small droplets formed. So, and this can be created by a spray, but once this spray is created, once is droplet is formed, the question is that how does this droplets evaporate in a given hot environment. Of course, in set a gas turbine in the temperature is high of the order of over 700 Kelvin, so using in scramjet, the static temperature is pretty high. Of course, once again that comes out thermo dynamic reasons the same reason by the pressure is high the temperature must be high because only when you at heat in this kind of environments, you can at high pressure and you can basically extract useful work.

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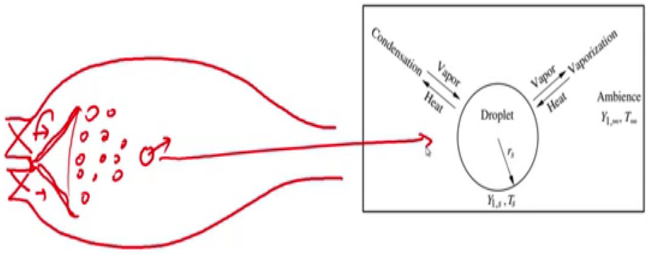
d^2-Law of Droplet Vaporization



So, combustion happens in the air breathing engines in a high pressure and the incoming air flow is also at high temperature. So, basically you are injecting a spray like this. You are injecting, you are essentially injecting if it is a say air craft engine, if this is your say your combustor, this is the typical cross section of an annular combustor and this is the solars and this is your injectors. So, your liquid is injected like this, the spray is form like this, but the air comes from like this, and it is soling and the spray comes along like this. And then these sprays will break up, and these droplets will form, different droplets of different sizes will form. And each of these droplets will essentially can be will undergo evaporation. And the method by which these droplets will undergo evaporation, the mechanism and the heat and mass transfer associated with it that will take up in this class.

So, here you have a single droplet and with this is now my interface is a liquid air interface which is the droplet radius and we can say that at the droplet at this just above the droplet radius my species mass fraction is Y 1 s and the temperature is T s. And of course, the droplet is undergoing evaporation in some cases, it can also undergo condensation. So, when it is undergoing vaporisation is this Y 1 s will diffuse out and it will go out, and this hot ambient has to supply heat supply the latent heat of evaporation.

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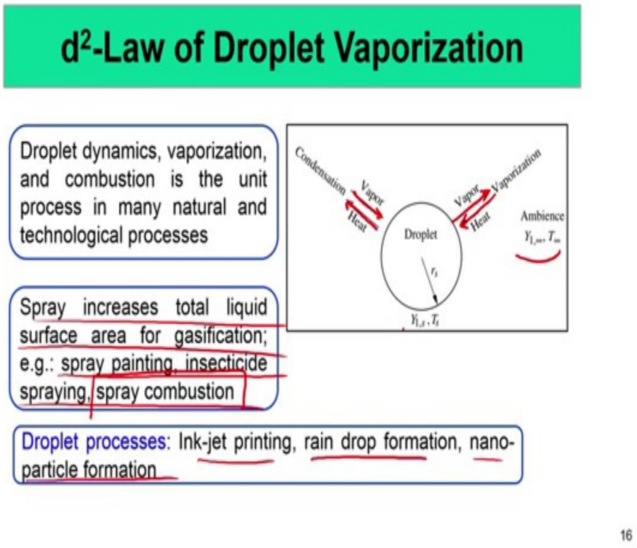

d²-Law of Droplet Vaporization

Droplet dynamics, vaporization, and combustion is the unit process in many natural and technological processes

Spray increases total liquid surface area for gasification; e.g.: spray painting, insecticide spraying, spray combustion

Droplet processes: Ink-jet printing, rain drop formation, nano-particle formation

While it is condensing, the vapour has to essentially going in and the heat of this essentially going out right, so that is the thing. But in our case in our for our interest we will basically looking to the droplet evaporation and the ambient is at the species mass fraction is Y 1 infinity and its temperature is T infinity. As you said the droplet dynamics vaporization combustion is the unit process in many natural and technological processes. It is not only this thing in terms of sprayed, drawing in terms of like cooling the all this droplet evaporation is a very important phenomena that needs to be understood.

So, spray why we do not have a spray because spray increases the total surface liquid surface area for gasification. For example, spray painting, insecticide spraying and spray combustion, which is the process of interest in our in air breathing combustion engine, air breathing aero engine. So, the droplet processes and all this things like ink jet printing, rain drop formation, nano particle formation and their different things which are involved in droplet process.

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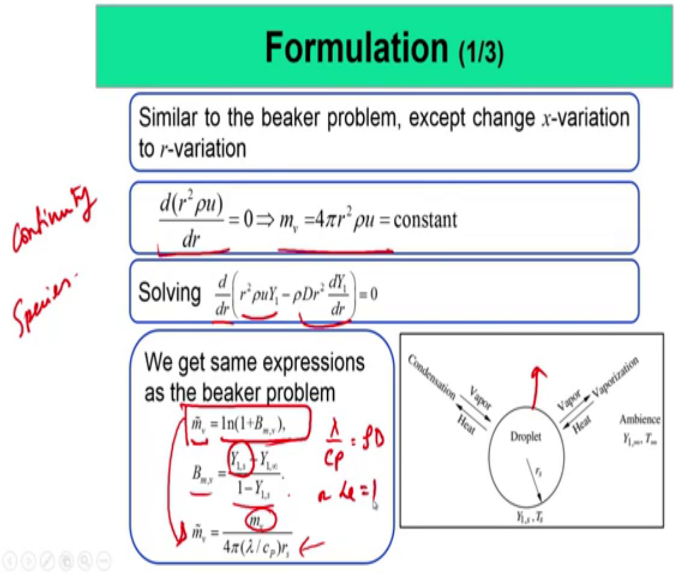

Formulation (1/3)

Similar to the beaker problem, except change x-variation to r-variation

Continuity

$$\frac{d(r^2 \rho u)}{dr} = 0 \Rightarrow m_v = 4\pi r^2 \rho u = \text{constant}$$

Species.

Solving $\frac{d}{dr}\left(r^2 \rho u Y_1 - \rho D r^2 \frac{dY_1}{dr}\right) = 0$

We get same expressions as the beaker problem

$$\boxed{\tilde{m}_v = \ln(1 + B_{m,v}),}$$

$$B_{m,v} = \frac{Y_{1,s} - Y_{1,\infty}}{1 - Y_{1,s}}.$$

$$\tilde{m}_v = \frac{m_v}{4\pi(\lambda / c_p) r_s}$$

$\frac{\lambda}{cp} = \rho D$

a $Le = 1$

So, similar to the beaker problem, this we will consider the droplet evaporation problem except that in this case we do not choose Cartesian coordinates, we will choose spherical coordinates. But it is one-dimensional because essentially we will consider the symmetry we will consider the droplet asymmetric or we will consider the droplet symmetric in the azimuthal in the z-th plane in that is in the r theta coordinates. And we will only have r variation. So, this becomes a one-dimensional problem.

So, if it is a one-dimensional problem by applying continuity by applying continuity we get d d r of r square of rho u is equal to 0. And this gives a mass flow rate m v of this vapour to be essentially 4 pi r square rho u. We can say that this is one can ask that where they are two phases, one is the droplet phase, one is the air phase why are we considering now mass flux the vapour phase. This is because with the only this droplet is evaporating its going out, and then there is no condensation of the air that is happening. As a result of this we can say m v is equal to 4 pi r square times rho u which is equal to constant.

And then we can solve the species diffusion and the species conservation equation. Once again you get this as the we essentially this is the equation, this comes from the convective velocity this comes from the diffusible velocity, and these two will give you the species velocity, and we can get integrate it and just as we did it in previous class. So, the beaker problem and we will get the same thing, but here we have just uses non-dimensional mass in m v, non-dimensional mass for the that is m v tilde and that is given by ln 1 plus B m,v exactly similar formula will get. Where is B m,v is the mass transfer number which is equal to 1 Y 1 by 1 1 s minus 1 Y 1 infinity.

That is once again the potential for driving the evaporation in the droplet and that is essentially the difference between the mass fraction and the surface and the mass fraction at the infinity and that is this is normalized by 1 minus Y mass fraction of the surface. And of course, the normalization of the mass flow rate that we have done is essentially is equal to m v tilde is equal to m v which is the actual mass flow rate divided by 4 pi lambda by c p r s. Now, please pay attention to this particular normalization because this will turn out hanky and we do the d square law of droplet evaporation.

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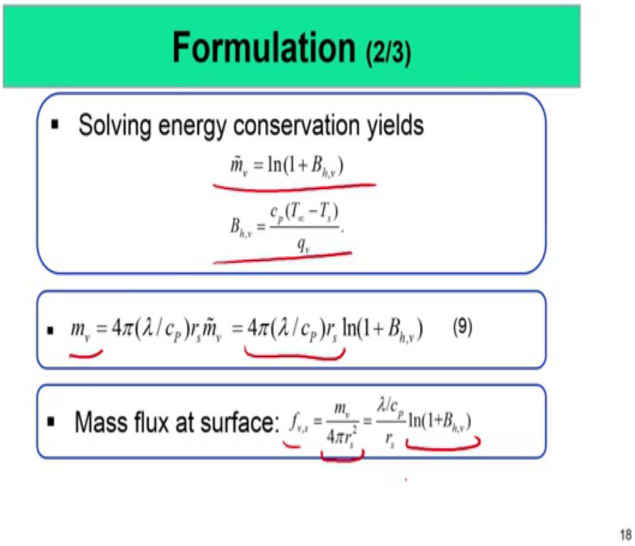

Formulation (2/3)

- Solving energy conservation yields

$$\underline{\tilde{m}_v = \ln(1 + B_{h,v})}$$

$$\underline{B_{h,v} = \frac{c_p(T_\infty - T_s)}{q_v}.}$$

- $\underline{m_v} = 4\pi(\lambda / c_p) r_s \tilde{m}_v = \underline{4\pi(\lambda / c_p) r_s} \ln(1 + B_{h,v})$ (9)

- Mass flux at surface: $\underline{f_{v,s}} = \frac{m_v}{\underline{4\pi r_s^2}} = \frac{\lambda / c_p}{r_s} \underline{\ln(1 + B_{h,v})}$

Now, once again this is not enough because you do not know what you are Y 1 s is. So, this is this even if you console for m v tilde, you do not know because this is unsolved for why we do not know what Y 1 s is and that for to know what Y 1 s is you need the energy conservation equation. This is reasonable because you see that you cannot really solve the full problem just by the species diffusion because ultimately the droplet is evaporating because somebody is supplying is the latentinty vaporization, and the latentity vaporization is being supplied by the ambient. So, you have to consider the conduction and the convection process is that are involved to consider this latentity to close this problem and that is why you need to and that shows up in the fact that you are Y 1 s that is mass fraction of the surface is not known. And that is why you need to solve this energy conservation relation and that is what it this tells m v tilde is nothing but 1 plus B h, v 1 plus this the heat transfer number and the heat transfer number is c p t infinity minus t s.

Now, one thing to notice that in both class of the heat transfer number and mass transfer number in both the beaker problem as well as in the droplet problem they do not change that is they are independent of geometry and as a result they are universal that is why they are basically universal numbers. So, if you once again write in the dimensional form you get this similar type of relation, but of course here it is changes a little bit, it is a 4 point pi times lambda by c p by times r s divided by ln 1 plus B h, v. And then the mass flux of the surface then we can divide the mass flux from by the mass flow rate divide by the droplet surface area from at the evaporation taking place. And that once again comes back to the same thing 4 lambda y c p r s in with the beaker problem it was l times ln 1 plus B h, v.

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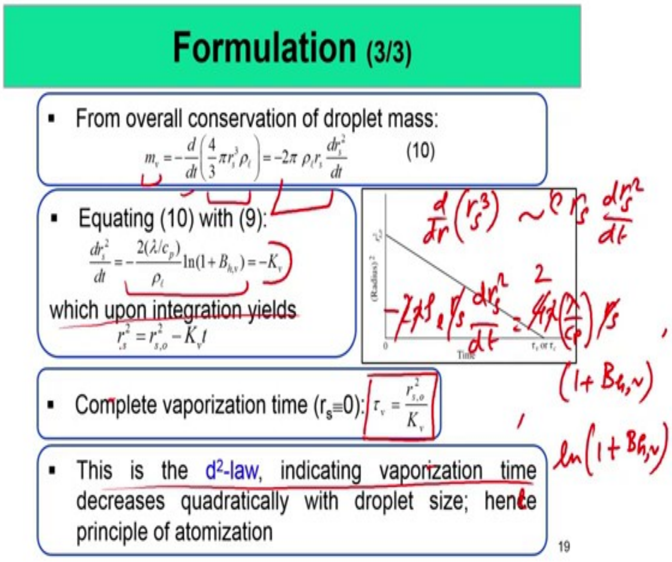

Formulation (3/3)

- From overall conservation of droplet mass:

$$m_v = -\frac{d}{dt}\left(\frac{4}{3}\pi r_s^3 \rho_l\right) = -2\pi \rho_l r_s \frac{dr_s^2}{dt} \qquad (10)$$

- Equating (10) with (9):

$$\frac{dr_s^2}{dt} = -\frac{2(\lambda/c_p)}{\rho_l}\ln(1+B_{h,v}) = -K_v$$

which upon integration yields

$$r_s^2 = r_{s,o}^2 - K_v t$$

- Complete vaporization time (r_s=0): $\tau_v = \frac{r_{s,o}^2}{K_v}$

- This is the d²-law, indicating vaporization time decreases quadratically with droplet size; hence principle of atomization

$$\frac{d}{dr}(r_s^3) \sim r_s \frac{dr_s^2}{dt}$$

$$-2\pi\rho_l r_s \frac{dr_s^2}{dt} = 4\pi\left(\frac{\lambda}{c_p}\right) r_s,$$

$(1+B_{h,v})$

$\ln(1+B_{h,v})$

Now, what we can do is that this is so if we have the when this is also is equal to rho d by as you see here. And this is of course, one important thing to notice that here we get even in this when we do the species conservation the equation of mass flow rate that we get is m v divide by lambda y c p we can apply it here. Because we assume that lambda by c p is essentially is equal to rho d or in other words lose number is equal to 1. So, because of that assumption we can assume that these two things are same that is lambda by c p is essentially is equal to rho d.

And from now from the overall conservation of the droplet mass, now what do we mean by the what is that that of course, you have the so go back little bit now what I mean is that of course, you have the you have the pool droplet. And now this droplet is giving out this vapour with that is essentially the droplet liquid phase of the droplet is converting to a to this vapour phase. And now while that is happening since the liquid phase is being converted into the vapour phase the droplet has to stink, because the liquid mass of the droplet is basically becoming vapour and that is going outside.

So, this mass of the droplet has to reduce and the volume of the droplet also has to reduce so that means this is giving out this droplet is giving out this vapour flux. And for that it is this droplet has two stink of course, this happens in such a manner that we can consider this while we do this analysis we can when we do the analysis in a gas phase we can consider it to be steady. But now we can essentially write that the m v that is the we can equate that is the rate at which the vapour is living the droplet at the surface that is the mass flow rate of this vapour at the surface that is visible to the rate of change of the mass of the droplet itself and that is the rate of change of the mass of the droplet at any particular time is given by fourth-third pi r cube time rho l. And that is the rate of change of that of course, the minus sign comes because the droplet stinking.

So, then we can write it in this form, but we see clearly here we have just announce small mathematical manipulation that is mathematical or jugglery that is this 4 pi times rho l we have just written this thing that is d by d r of r s cube. We have written this in terms of r s times d r s square d t. Of course, there is a constant involved so that we have written. And why we have written that does will become apparent in a very short amount of time.

So, now once you equate this thing that is minus 2 pi rho l times r s d r s square d t is equal to this thing that is this guy which is nothing but 4 pi lambda by c p r s times ln of 1 plus b v that is 4 pi lambda by c p times r s times l n 1 plus B h, v. Once you do that once as one you do that then few things cancel on both sides and you see this r s is r s cancels. So, we get is essentially d r s square d t is equal to 2 times lambda by c p divide by rho l times ln of 1 plus B h, v.

And of course, if you integrate this and we call this as the this droplet vaporization constant which is k because you see this are all constant for this particular problem. And then each of these upon in integration yields this, this thing that is r s square is equal to r s 0 that is this radius square at any time t is equal to the r s radius r s the droplet at time t 0 that is the initial time. And that is given by this k v t there is the droplet vaporization constant k v times this time t.

So, essentially if we set this r s to be equal to 0, sorry this if you can set that is the time it will complete vaporization when this essentially should be goes to 0, so we may get r s square by k v is equal to the time of a vaporization. So, this is the very reviling thing that is the total time for vaporization goes as r r s square or essentially it is a d square that is the diameter square or the radius square. So, this is the d square law indicating that the vaporization time decreases quadratically with the droplet size hence the principle of atomization. So, this is the thing that the times scale or the time for the complete vaporization is essentially equal to the radius square the initial radius of the droplet square divide by the k v the droplet vaporization constant. So, large the time changes quadratically, so a larger droplet more time small droplet less is the time.

So, if you can need a spray which atomizes into very, very small droplet then it means that spray will essentially become the fuel vapour vapour cloud in a very short amount of time. So, that is what is required. So, that you can proper mixing and we can have proper combustion with less suite etcetera. So, this is the thing that we wanted to discuss. Now, then we will finally, the topic is the d square law droplet burning so far we are discussed droplet vaporization. Now, once we have the vaporization we will apply the principles of the non-premixed flame the previous class to understand the droplet vaporization.

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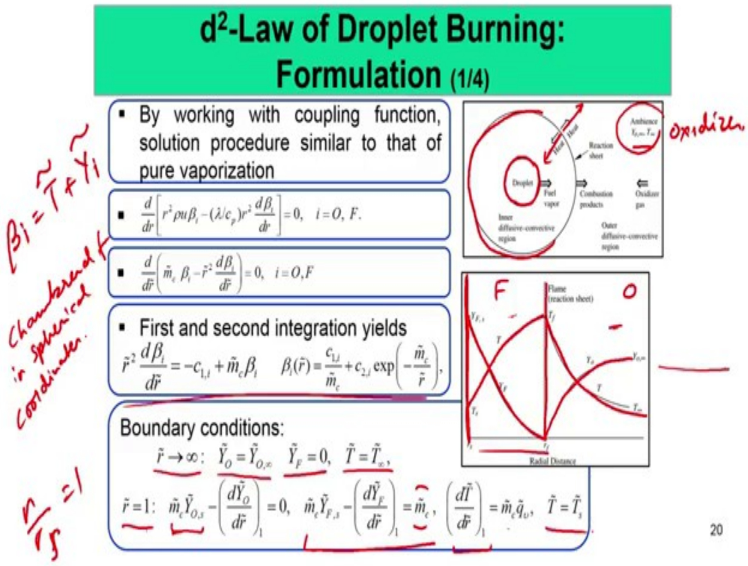

d²-Law of Droplet Burning: Formulation (1/4)

- By working with coupling function, solution procedure similar to that of pure vaporization
- $\frac{d}{dr}\left[r^2 \rho u \beta_i - (\lambda/c_p) r^2 \frac{d\beta_i}{dr}\right] = 0, \quad i = O, F.$
- $\frac{d}{d\tilde{r}}\left(\tilde{m}_c \beta_i - \tilde{r}^2 \frac{d\beta_i}{d\tilde{r}}\right) = 0, \quad i = O, F$
- First and second integration yields

$$\tilde{r}^2 \frac{d\beta_i}{d\tilde{r}} = -c_{1,i} + \tilde{m}_c \beta_i \qquad \beta_i(\tilde{r}) = \frac{c_{1,i}}{\tilde{m}_c} + c_{2,i} \exp\left(-\frac{\tilde{m}_c}{\tilde{r}}\right),$$

Boundary conditions:

$$\tilde{r} \to \infty: \quad \tilde{Y}_O = \tilde{Y}_{O,\infty} \quad \tilde{Y}_F = 0, \quad \tilde{T} = \tilde{T}_\infty,$$

$$\tilde{r} = 1: \quad \tilde{m}_c \tilde{Y}_{O,s} - \left(\frac{d\tilde{Y}_O}{d\tilde{r}}\right)_1 = 0, \quad \tilde{m}_c \tilde{Y}_{F,s} - \left(\frac{d\tilde{Y}_F}{d\tilde{r}}\right)_1 = \tilde{m}_c, \quad \left(\frac{d\tilde{T}}{d\tilde{r}}\right)_1 = \tilde{m}_c \tilde{q}_v, \quad \tilde{T} = \tilde{T}_s$$

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$\beta_i = \tilde{T} + \tilde{Y}_i$

Chambered f in spherical coordinates.

$\frac{r}{r_s} = 1$

oxidizer

So, now, here we have the droplet, so now instead of and here you have the flame. So, the flame is that because here you essentially can have oxidizers in the ambient as you see here this contains the y 0 infinity t infinity. So, this contains the oxidizer is in the ambient and this droplet evaporation will droplet contains, basically the droplet of fuel. So, we will this droplet will provide the fuel vapour, this contains oxidisers. So, the fuel vapour in the oxidizer meet and if you have a flame at this location at a certain stand of distance at a certain radius that radius will given by this r f flame radius.

So, this flame will provide will basically conduct heat in will provide heat conduction in both the droplet as well as the ambient. The droplet the heat that it provides to the droplet will essentially help in sustaining the evaporation, because it will this flame is now going to provide the latent heat of evaporation require for the droplet to convert from this liquid phase to the gas phase.

So, this will work with the coupling function formulation and the solution proceed will be similar to the rough vaporization. So, if you look into this, this will be the structure of the flame So, here you have the y once again the s means the surface of the droplet here is the y f s will continuously non decay because decay much faster, because it will be consumed completely at the flame. The temperature will increases from the droplet surface to the flame radius because of course the flame is conducting heat and then of course, it will again decay on this sides because of conduction. So, this side is the fuel side this is the oxidizer side and we assume that by the reaction sheet there is no fuel leakage on this side there is no oxidizer leakage on this side no oxidizer and there is no fuel in this side. So, this will give you also a profile something like this.

So, now we will just do straight to the coupling function formulation, but you should do it derive this by yourself also. And by the coupling function formulation is just becomes very simple, if you remember the coupling function it is essentially beta i is equal to T tilde plus Y i tilde T tilde. Once again we apply the least number equal to 1, and t tilde is essentially this non-asymmetrically c p t and Y i tilde is asymmetrically get it Y f by Y f 0 as asymmetrically mass fraction. And once you do the formulation if you remember just you when we write it in the spherical coordinates we just get this form.

And now once again we can just simply integrating the previous just as we did, but before that we can just write it in a normalised form that is you can write it as r by r s. And then if we integrate we get these forms once again. And now we have to apply the boundary condition it is exactly the similar way as we did it for the chambered flame it is like the exactly like the chambered flame in, but just in spherical coordinates. So, in the boundary conditions that we have to apply and the boundary conditions are essentially at r is equal to infinity that is here your Y 0 is equal to Y 0 infinity which is fixed the say if it is air we have Y 0 is fixed by the fact that the volume percentage is about 21 percent of oxygen. So, this is that if you convert that into mass fraction this is the value that you get some 0.20 something excreta. Y F 0 should be equal to 0, Y f tilde because there is no reactant of the there is no fuel leakage to the flame and t tilde is equal to t infinity.

Now, similarly at tilde is equal to 1 that is at the r by r s is equal to 1, this is the boundary there then that is at this point there you basically this says that in the m c there is mass rate of that is normalised mass rate of evaporation which is also the mass rate of mass flow rate of combustion the burning rate is symptoms Y O, s if you apply it for the oxidizer that is this exactly this oxidizer flow is exactly balanced by the diffusion. And there is no net flow of the oxidizer here because there is no oxidizer condensation into the droplet that is happening.

Then we will see that we will look into this into the fuel, but here there is there is on the right hand side you have the fuel vapour flux so that is essentially is equal to the m c itself before as we have shown. Because it is a balance between your bulb velocity and the diffusion molecular diffusion velocity and which is essentially the species velocity times the its velocity density and all this things it becomes essentially the mass flux of this species or the total flux of the mass flux that is supporting the combustion that is being burned. And of course, we assumed that the heat the flux is for evaporation is supplied by the by conduction and that is why d t d r is equal to m c times q v and your T is equal to the T temperature of the surface.

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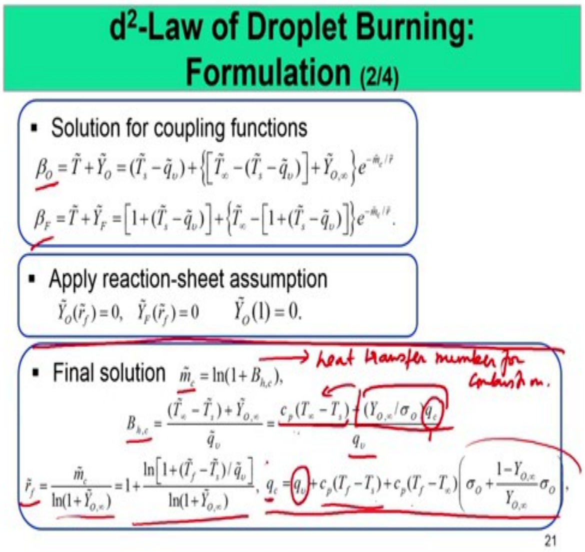

d^2-Law of Droplet Burning: Formulation (2/4)

- Solution for coupling functions

$$\beta_O = \tilde{T} + \tilde{Y}_O = (\tilde{T}_s - \tilde{q}_v) + \left\{\left[\tilde{T}_\infty - (\tilde{T}_s - \tilde{q}_v)\right] + \tilde{Y}_{O,\infty}\right\} e^{-\tilde{m}_c/\tilde{r}}$$

$$\beta_F = \tilde{T} + \tilde{Y}_F = \left[1 + (\tilde{T}_s - \tilde{q}_v)\right] + \left\{\tilde{T}_\infty - \left[1 + (\tilde{T}_s - \tilde{q}_v)\right]\right\} e^{-\tilde{m}_c/\tilde{r}}.$$

- Apply reaction-sheet assumption

$$\tilde{Y}_O(\tilde{r}_f) = 0, \quad \tilde{Y}_F(\tilde{r}_f) = 0 \qquad \tilde{Y}_O(1) = 0.$$

- Final solution $\tilde{m}_c = \ln(1 + B_{h,c})$,

heat transfer number for combustion.

$$B_{h,c} = \frac{(\tilde{T}_\infty - \tilde{T}_s) + \tilde{Y}_{O,\infty}}{\tilde{q}_v} = \frac{c_p(T_\infty - T_s) + (Y_{O,\infty}/\sigma_O) q_c}{q_v}$$

$$\tilde{r}_f = \frac{\tilde{m}_c}{\ln(1 + \tilde{Y}_{O,\infty})} = 1 + \frac{\ln\left[1 + (\tilde{T}_f - \tilde{T}_s)/\tilde{q}_v\right]}{\ln(1 + \tilde{Y}_{O,\infty})}, \quad q_c = q_v + c_p(T_f - T_s) + c_p(T_f - T_\infty)\left(\sigma_O + \frac{1 - Y_{O,\infty}}{Y_{O,\infty}}\sigma_O\right).$$

Now, then we can solve for this coupling functions we can solve for beta 0 and beta f because your beta i is as we discussed beta i is nothing but your T tilde plus Y i tilde beta i can be it can be fuel can be oxidizer. So, we solve for beta oxidizer we solve for beta fuel and then we apply the reaction sheet assumption that is on the left hand side on that is at a smaller radius. Here there is no oxidiser, there is no fuel and as well as you do that we get the final solution where again you get the exactly similar form. So, these are the final solutions pay attention to this. So, we get this similar thing that is m c tilde is essentially equal to your ln of 1 plus B h, c that is once again B h, c is if you remember is the heat transfer number for combustion.

So, this is the heat transfer number for combustion previously was B h, v which is heat transfer number for evaporation. We will see exactly now, what is the difference is the difference is essentially you see that here this B h, c involves of course, this c p T infinity minus T s which was also there in the heat transfer number for combustion area for evaporation, but now we have this part that is here you have heat transfer number contains a very important this chemical heat reduce rate term which is the q c. And this q c actually is now dominant over this guy. Now, the heat transfer is not actually coming from ambient of course, there is some heat transfer from the ambient, but now bulk of the heat transfer that causes the droplet evaporation and then the combustion of this vapour is essentially this provided by q c. And this is normalized by the q v that is a latent heat of vaporization well.

So, this q c is now this term is essentially much, much bigger than this term. So, the mainly the droplet evaporation or the droplet vaporization is happening through the heat transfer through the heat of that is supplied by the flame that is situated at the stand of distance of r f. So, then that can be estimated this r f is an important parameter what is the radius of the flame in fuel given in size of the droplet and that is estimated by this once again from this reaction sheet assumptions.

And then we can get this by this formula. And once again we can get this temperature in an implicate form where we show that this q c that is this that is a heat of combustion is now essentially this same thing that is 1 k g of s 1 of course, 1 k g of now this is there is no fuel mixture because it is now purely droplet. So, so 1 k g of fuel now when you burn it in a essentially gives you rises the temperature from T s to T f. And it rises the temperature T infinity to T f of course; these are the stash metric balances we show that the burring parameter, but then it also is also used for latent heat of evaporation. So, the same temperature by the same argument it is essentially the automotive flame temperature, but with the consideration for latent heat f evaporation.

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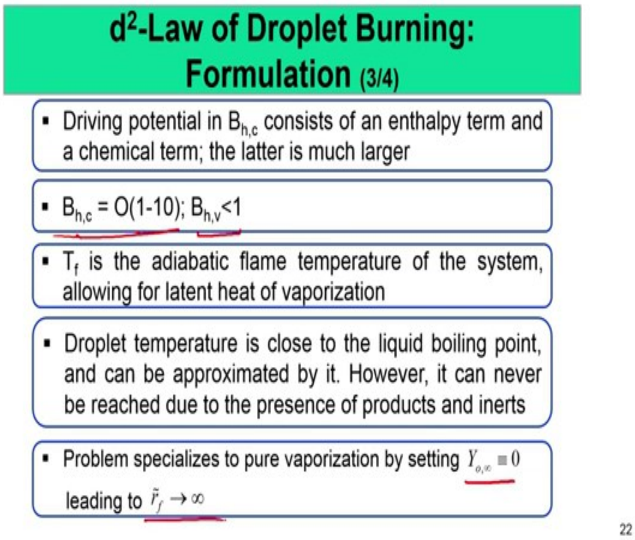

d^2-Law of Droplet Burning: Formulation (3/4)

- Driving potential in $B_{h,c}$ consists of an enthalpy term and a chemical term; the latter is much larger
- $B_{h,c} = O(1\text{-}10)$; $B_{h,v}<1$
- T_f is the adiabatic flame temperature of the system, allowing for latent heat of vaporization
- Droplet temperature is close to the liquid boiling point, and can be approximated by it. However, it can never be reached due to the presence of products and inerts
- Problem specializes to pure vaporization by setting $Y_{o,\infty} \equiv 0$ leading to $\tilde{r}_f \to \infty$

So, in this case the driving potential is B h, c as we discussed before that is for the driving droplet evaporation or the droplet vaporisations. And driving potential is B h, c it consist of an enthalpy term which is this guy as we discussed. This is much larger than the enthalpy term and the chemical term, enthalpy term is actually this one. And this is the chemical term.

So, chemical term is actually much bigger than the enthalpy term and we shows that in the case of droplet combustion of course, flame around then this flame gives much more heat and as a result of droplet evaporation is much faster, so in terms of normal droplet vaporization. And then as a result this B h, c is the order of 1 to 10 whereas, this heat transfer number that the heat transfer number for combustion is are order 1 to 10, whereas from the heat transfer number vaporization is less than 1. So, it is a order of magnitude difference is there.

And T f is the adiabatic flame temperature of the system allowing latent heat of vaporization as we discussed. The droplet temperature is close to the liquid volume boiling point and can be the approximated and it can be never reached the presence of products and inerts. And we can fold this problem back to the vaporization problem if we said y zero infinity that is equal to 0 and then of course, it will lead to the r f to be equal to t infinity.

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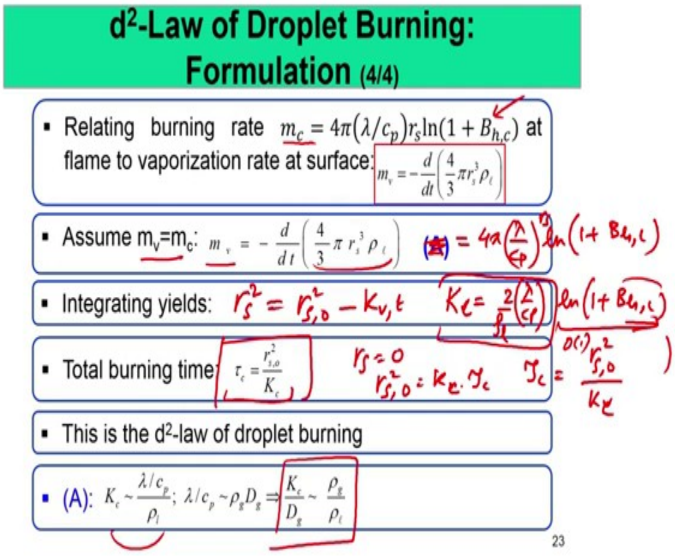

d^2-Law of Droplet Burning: Formulation (4/4)

- Relating burning rate $m_c = 4\pi(\lambda/c_p) r_s \ln(1 + B_{h,c})$ at flame to vaporization rate at surface: $m_v = -\frac{d}{dt}\left(\frac{4}{3}\pi r_s^3 \rho_\ell\right)$
- Assume m_v=m_c: $m_v = -\frac{d}{dt}\left(\frac{4}{3}\pi r_s^3 \rho_\ell\right)$ $= 4\pi\left(\frac{\lambda}{c_p}\right) \ln(1 + B_{h,c})$
- Integrating yields: $r_s^2 = r_{s,0}^2 - K_{v,}t$ $K_c = \frac{2}{\rho_l}\left(\frac{\lambda}{c_p}\right)\ln(1 + B_{h,c})$ $O(1)$
- Total burning time: $\tau_c = \frac{r_{s,0}^2}{K_c}$ $r_s = 0$ $r_{s,0}^2 = K_c \cdot \tau_c$ $\tau_c = \frac{r_{s,0}^2}{K_c}$
- This is the d^2-law of droplet burning
- (A): $K_c \sim \frac{\lambda/c_p}{\rho_l}$; $\lambda/c_p \sim \rho_g D_g \Rightarrow \frac{K_c}{D_g} \sim \frac{\rho_g}{\rho_\ell}$

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So, this using this once again we can relate the burning rate that is there is m c is equal to 4 pi times lambda by c p r s times of ln 1 plus B h, c droplet heat transfer number combustion at the flame to the vaporization rate that is at the surface. So, essentially the droplet once again is stinking because of it is losing the mass because the phase change of the liquid to vapour phase change and then this vapour goes on to meet the oxidizer and then it burns. So, once again if you do the same thing, we can find out this if we set m v equal to m c, it is not assumption its setting that if you set m v equal to m c and whereas, m v is given by minus d t of pi r cube l. There is rate of change of the mass of the droplet which is the liquid and this integration will essentially will give you will there is some issue here.

So, what this integration will this just if you just can go back here just remove this, this is some problem with this. And then this integration will essentially this we are essentially what we are doing here is that here setting this m v to b is equal to m v equal to m c setting means that this guy minus d d t of four-third pi r cube l to essentially 4 pi times lambda by c p times ln of 1 plus B h, c.

So, then we can we essentially and the as we know that we wrote this as r s times d r s square d t. So, then we can write f m v is equal to minus 2 pi rho l times r s times d r s square d t and that is equal to 4 pi lambda by c p ln of 1 plus B h, c. And then this if we integrate this then what we find is that then if you just cancel these two things pi by 2 and the we get d r s square d t is equal to lambda by there is r s also here, yes. So, this r s and this r s cancels, lambda by c p times 1 by rho l minus ln of 1 plus B h, c.

So, now this is essentially constant and if it is constant we can write this as once again the vaporization is exactly same as the droplet evaporation constant. But of course, you note that this droplet heat transfer number this is due to combustion the previous one was d 2 vaporization. And then if you integrate this thing r s with the r s square d t is equal to k v then what this yields is nothing but then this yields this r s square is equal to r s 0 square minus k v t; whereas k v is given by 2 times lambda by c p by rho l that is the density of the liquid ln of 1 plus B h, c.

So, now if we set once again that the total burning time to estimate we set r s is equal to 0 and then we get r s square is equal to k v times tau c the therefore tau c is equal to r s 0 that is the initial radius by k v. So, this is what we get that is the total burning time or we write it as k c because this involves this here droplet because this involves this droplet in heat transfer number for combustion. So, then this is the total burning time then we get it as this is the thing whereas, we can write this as equal to k c because it involves k c. So, this as we get the total burning time tau c is equal to r s square the initial radius square divide by k c. So, once again you see that the total time is directly proportional to the radius square of the initial of the initial radius square of the droplet of the initial diameter square of the droplet it depends quadratically, so that is the thing.

Now, of course, the if you see that the formula for k c is essentially lambda by c p times rho l ln of 1 ln of B h, c. If this is the order of one if then we can just write this as we can say that k c is directly proportional to this if this guy is order one because this guys order 1 to 10, so ln of 1 plus B h, c will of order of the order of one essentially. So, we can say that k c is essentially 2 times lambda by c p by rho l then we can set it as rho g times t g is equal to this and we can estimate this have an estimate of k c in this form.

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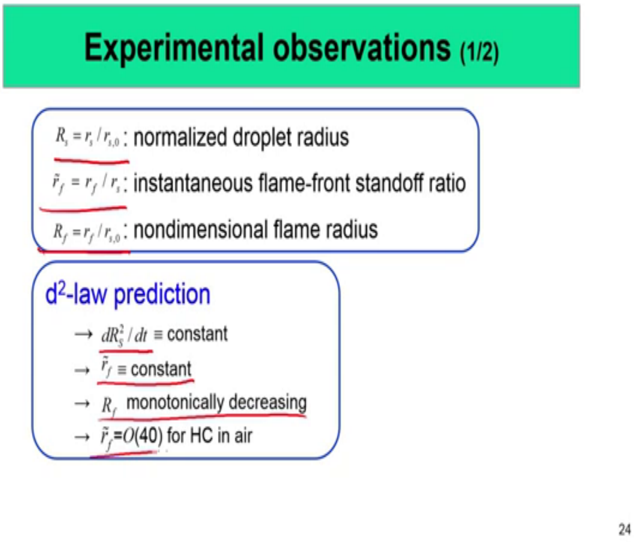

Experimental observations (1/2)

$R_s = r_s / r_{s,0}$: normalized droplet radius

$\tilde{r}_f = r_f / r_s$: instantaneous flame-front standoff ratio

$R_f = r_f / r_{s,0}$: nondimensional flame radius

d^2-law prediction

→ $dR_S^2 / dt \equiv$ constant

→ $\tilde{r}_f \equiv$ constant

→ R_f monotonically decreasing

→ $\tilde{r}_f = O(40)$ for HC in air

So, just if we now find out the experimental observations if we define this as normalized radius r s by the initial radius r f is the flame radius by the instance constant of ratio. And this is the non-dimensional flame radius. And d square law prediction say that d r s square d t should be is equal to constant then small r f tilde should be equal to constant and capital R f should be monotonically decreasing small r f should be order of one carbon in air.

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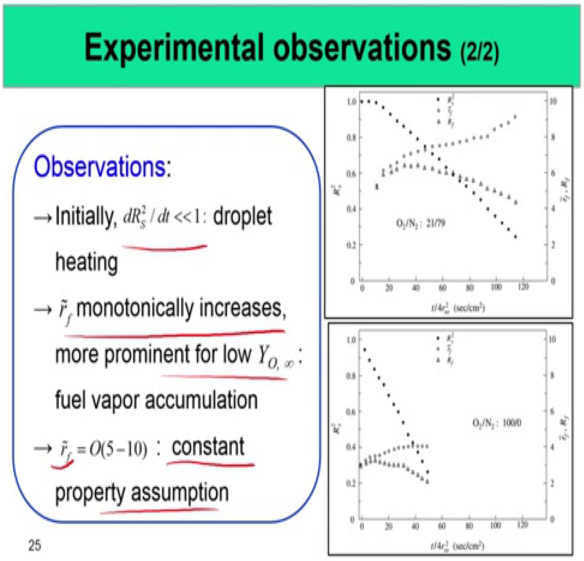

Experimental observations (2/2)

Observations:

→ Initially, $dR_S^2 / dt << 1$: droplet heating

→ $\tilde{r}_f$ monotonically increases, more prominent for low $Y_{O,\infty}$: fuel vapor accumulation

→ $\tilde{r}_f = O(5-10)$: constant property assumption

And actually we find that these laws are reasonably some of them are obeyed and we find that d R s square d t is must less than one r f tilde actually monotonically increases and it is more prominent for low Y 0 tilde for fuel vapour accumulation. And this f f tilde this normalized flames standoff ratio flame standoff radius is essentially 5 to 10 which is arises from which can be expressed from the constant property assumption.

So, with this we close today's class. So, we have covered lot of grounds in the form of we have studied lot of about non-premixed flames, we use the coupling function formulation to see how today's and previous days class. We have combined them to see how we have basically using the coupling function formulation one can explain the structure of non-premixed flame, one can obtain the burning flux one can obtain the species profiles one can obtain the temperature profiles. And most importantly we can find out that the flame temperature becomes essentially the adaptive temperature for non premixed flame for the given fuel layer mixture actually fuel flame mixture of the left hand side we have shown the right hand side.

We have gone into look at standard evaporation of a liquid pool and then found out on what parameters does this evaporations flux depend on we have introduce the concept of droplet not droplet but heat transfer number mass transfer number which merges actually the driving potential for the evaporation flux. And larger this droplet mass transfer number larger is droplet heat transfer number larger the evaporation fluxes are and then we have also seen how both the species energy equation and required actually that is why this two numbers come to describe complete the evaporation. And then sometimes to we are also need the thermodynamics the Clausius Clapeyron relation to find relation between your species mass fraction or species mole fraction with the corresponding temperature.

And then using these things and this concept we have gone to describe droplet evaporation, we have found that the droplet evaporation has the time taken for a droplet to completely evaporate is essentially r s that is the radius of the droplet. We need radius of the initial the radius of the droplet of the initial times squared divided by the constant called by the droplet vaporization constant. We have seen that when you consider a flame droplet combustion that is when if this liquid droplet is all considered full with fuel vapour. Once this evaporates at this vapour meet the oxidizer and can have a flame and then we have found that the in this case the flame is actually leading source for the heat required for droplet evaporation. And then also we found that the time take for the droplet to burn is actually also directly proportional to the r s square by k c, whereas k c here is the droplet combustion constant.

So, with that we just will close today's class and then we will move onto non-premixed flames.

Thank you very much.