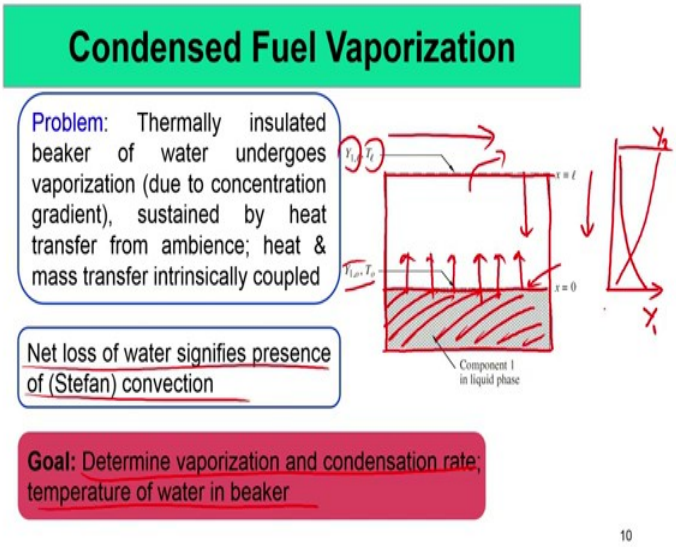
Combustion in Air Breathing Aero Engines
Dr. Swetaprovo Chaudhuri
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture – 20
Laminar Non-Premixed Flames III

Welcome back. So, next we are going to look into this thing called condensed fuel vaporization. Well as I have told you that in gas turbine engine or you know gas turbine engines combustion is so very important it is indispensable because of the fact that you can burn liquid fuels and the heat that is generated of that you can use it for propulsion or for power generation. So, as a result of that understanding how the fuel vaporizes because in air breathing engines, in air breathing engines combustion happens in the gas phase.

So, for that understanding how the fuel vaporizes is very important because only after vaporization happens you can have mixing that is the we can have the fuel and air mix at some point and then you can have a flame. So, a vaporisation of the condensed fuel is a very key process in understanding combustion of liquid fuels or combustion in air breathing aero engines. So, the very we will start with a one-dimensional problem, one-dimensional problem in Cartesian coordinates.

(Refer Slide Time: 01:45)



Condensed Fuel Vaporization

Problem: Thermally insulated beaker of water undergoes vaporization (due to concentration gradient), sustained by heat transfer from ambience; heat & mass transfer intrinsically coupled

Net loss of water signifies presence of (Stefan) convection

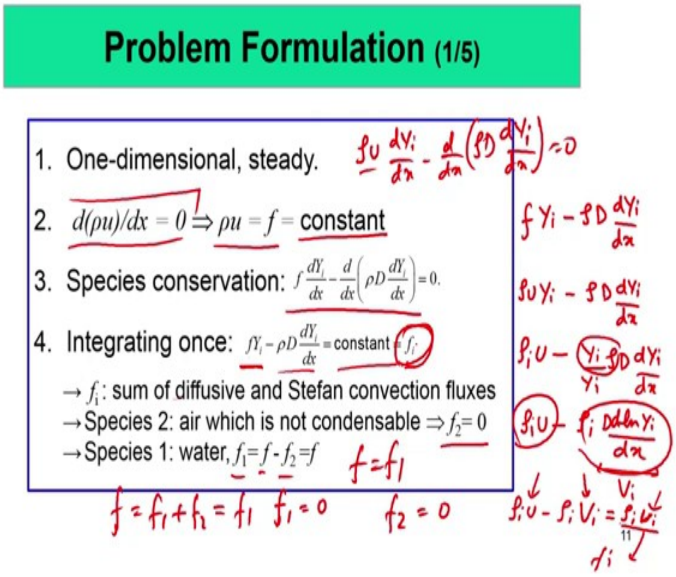
Goal: Determine vaporization and condensation rate; temperature of water in beaker

And we will consider a thermally insulated beaker; let us say this is the thermally insulated beaker. And this is one-dimensional we are solving the problem only in this direction which is the x-direction from x is from here. We will solve only in the gas phase actually. So, this contains this beaker contains this liquid, which can be say water and this is the interface between air and water. And on top here, there is a breeze which helps keep the mass fraction of the water vapour because water vapour is continuously form from here, which gives the mass fraction of the water vapour here fixed. So, that mass fraction that is kept fixed is Y 1 l, and its temperature is also get fixed by this convective flow.

But that is this convective flow is not a part of our problem our problem is only focused into this region so, between x equal to 0 and x equal to l. And here at the just some of the interface the fuel vapour mass fraction is Y 1 0, and the temperature that the point is T 0. So, now, as the fuel as this liquid which is water or a fuel which is reach vaporises from here that and you know that the liquid meniscus will come down and of course, the liquid is vaporizing here and it is going out here. So, these there is a continuous flow, there is a bulk flow of liquid of fluid that is happening, and as a result because there is no convection inside, there is no special gradient inside this thing, the only thing that drives is essentially this diffusion flux.

So, this diffusion of this of the liquid is essentially driving the convection and that is called the Stefan flow. There is the net loss of a water signifies the presence of Stefan convection. So, this is the thermally insulated beaker of water is this undergoing vaporization, this is undergoing the vaporization from here. And of course, for vaporization you need the latent heat of vaporization and that is supplied by the ambient. So, essentially heat is defusing from this direction from the top to bottom and the fluid is and the will water vaporising diffusing from bottom to top. So, as a result of this thing, you will see that the heat and mass transfer of this problem will be integrally coupled. So, this is the problem at hand that we have.

(Refer Slide Time: 04:31)



Problem Formulation (1/5)

1. One-dimensional, steady.
2. $d(\rho u)/dx = 0 \Rightarrow \rho u = f =$ constant
3. Species conservation: $f\frac{dY_i}{dx} - \frac{d}{dx}\left(\rho D\frac{dY_i}{dx}\right) = 0.$
4. Integrating once: $fY_i - \rho D\frac{dY_i}{dx} = \text{constant} = f_i$
   - → f_i: sum of diffusive and Stefan convection fluxes
   - → Species 2: air which is not condensable $\Rightarrow f_2 = 0$
   - → Species 1: water, $f_1 = f - f_2 = f$

$\rho u \frac{dY_i}{dx} - \frac{d}{dx}\left(\rho D \frac{dY_i}{dx}\right) = 0$

$f Y_i - \rho D \frac{dY_i}{dx}$

$\rho u Y_i - \rho D \frac{dY_i}{dx}$

$\rho_i u - \frac{Y_i \rho D}{Y_i} \frac{dY_i}{dx}$

$\rho_i u - \rho_i \frac{D\, d\ln Y_i}{dx}$

$\rho_i u - \rho_i V_i = \rho_i v_i$

f_i

$f = f_1$

$f = f_1 + f_2 = f_1 \quad f_1 = 0 \qquad f_2 = 0$

11

So, then the goal is to determine the vaporization and the condensation vaporization and condensation rate and the temperature of the water in the beaker. So, of course this is the steady problem, there is no net change of the species profiles inside this beaker there is happening. And it is a one-dimensional in this steady problem. And also you see that by that if we apply the continuity equation, we get this d rho u d x is equal to 0, and then it means that rho u is equal to this the flux the mass flux is essentially is equal to constant.

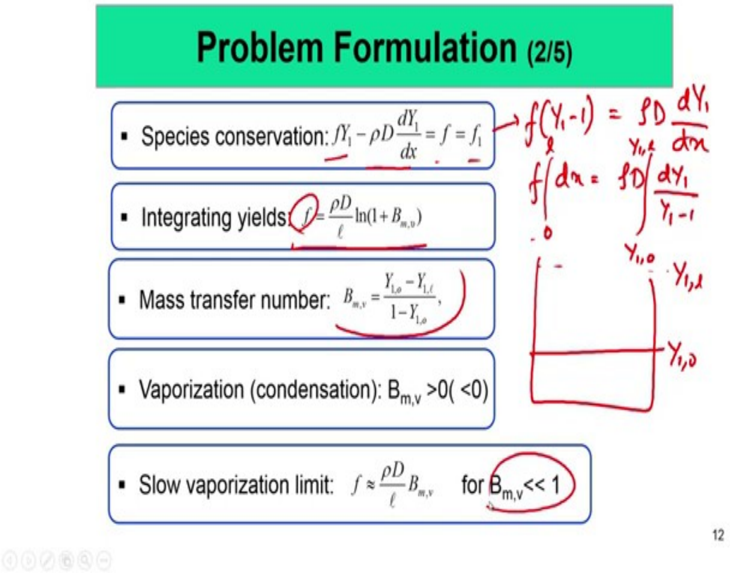
Now, we can also apply species conservation for I is equal to you can be i equal to 1 can be the vapour water vapour phase; and i is equal to 2 can be the air phase off course you have to see the there is also air here. Because if there is a species profile which is say this is the species profile direction of Y 1 which is going down like this. Of course, there has to be air which has to be like this because the mass fraction this is Y 2 has to be go down like this because the sum of Y 1 plus Y 2 is equal to 1. So, for that if you consider the species conservation this is f times d Y i d x minus d d x of rho d d Y i d x equal to 0. Of course, f is equal to rho u because actual formula is like this that rho u d Y i d x minus d d x of rho d times d Y i d x equal to 0, so that is the thing.

So, instead of rho u we have put f because that is the constant. And then if you integrate it once we get this thing there is a f Y i minus rho d Y i d x is equal to constant and that is equal to f i. Why f i that is an interesting thing; for that you have to understand that what are those that is why I have written this what are these two things. So, we can say that this guy we have to not this guy, we have to look into this guy that is what is f Y i minus rho d d Y i d x what is this thing, or we can write this f as rho u times Y i minus rho d d Y i d x.

Now this guy rho U Y i U Y i this is you see rho times Y i, I can write this as rho I times u all right minus we can write this as Y i by Y i times rho d d Y i d x. So, rho I times u minus you can just combine these two things and we can write it as rho i times D d ln Y i d x you know what is this guy, this is nothing but the species deficient velocity. So, rho i times U minus rho i times v what is this that is nothing, but rho i times U i. So, this U i this rho i times U i is nothing but f i there is a flux of the species i. This is the bulk velocity this is the species, this is the species diffusion velocity and this is the species velocity U i or V i, this was v i. So, this rho i times U i is nothing but the species flux of a f i, so that is why this you get this when you take the sum of these two things this constant that you get is nothing but the species flux which is f i. So, f i is essentially the sum of the diffusive and the Stefan convective fluxes. So, this is the Stefan convective flux and this is the diffusive flux. So, this is the sum of these 2.

But as a as we know that if we talk about f i for if f 1 is not equal to 0, because that is the species this is the water vapour that is diffusing that is the motion of the; flux of the of the water vapour. But if 2 that is not moving that is stagnant that is not moving because that is condensable at the interface and that is stagnant and as a result of that f 2 is equal to 0. So, the species one is essentially water is given by f 1 is equal to f minus f 2 because f 1 plus f is nothing but f 1 plus f 2; and this f is f 2 f it is equal to 0 then it becomes f 1. So, f is essentially is equal to f 1. So, we can replace this f i if it is 1, we can replace this f 1 with f.

(Refer Slide Time: 10:07)



Problem Formulation (2/5)

- Species conservation: $fY_1 - \rho D\frac{dY_1}{dx} = f = f_1$
- Integrating yields: $f = \frac{\rho D}{\ell}\ln(1+B_{m,v})$
- Mass transfer number: $B_{m,v} = \frac{Y_{1,o} - Y_{1,\ell}}{1-Y_{1,o}}$,
- Vaporization (condensation): $B_{m,v}$ >0(<0)
- Slow vaporization limit: $f \approx \frac{\rho D}{\ell}B_{m,v}$ for $B_{m,v} << 1$

$$f(Y_1-1) = \rho D\frac{dY_1}{dx}$$

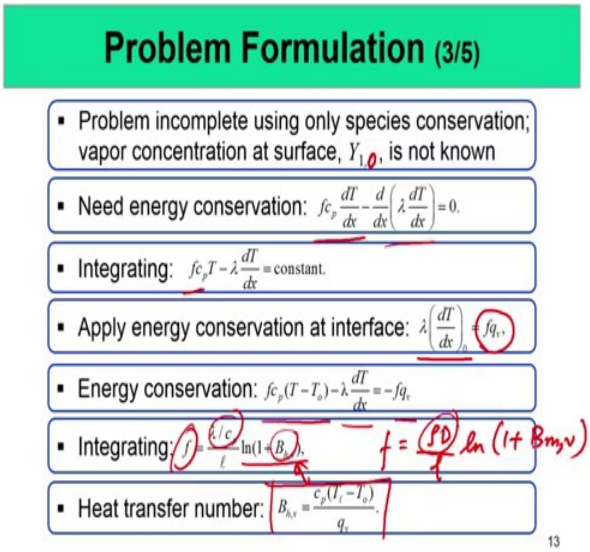
$$\int_0^\ell f\,dx = \rho D\int_{Y_{1,0}}^{Y_{1,\ell}}\frac{dY_1}{Y_1-1}$$

So, then we get f Y 1 rho D d Y 1 d x is equal to f is equal to f 1. And because this is f we get essentially f of Y i minus 1, we can write this equation is this rho D d Y 1 d x and we can write this as f d x is equal to rho D d Y 1 by Y 1 minus 1. And then we can integrate this and we will end up with this, integrate between 0 to l essentially and this is between integrate this between Y 1, 0 to Y 1, l, and will get with this will get this f to be rho d by l lines long of 1 plus B m, v. Where m, b is the mass transfer number, which is nothing, but this quantity. So, this essentially is what you see this mass transfer number is nothing but the difference between the species mass fraction at the boundaries that is this is nothing but this is the Y 1, 0 and this is where you have Y 1, l. So, this is the difference between this divided by Y 1 minus Y 1, 0, so that is the thing.

So, it says you see that this flux this net mass flux has a logarithmic dependent on this mass fraction mass transfer number, where is B m, v is given by 1 minus Y 1, 0 minus Y 1, l divided by 1 minus Y 1, 0. And of course, you have vaporisation when this is positive, and we have condensation will this is negative. And of course, you see that when you have negative this Y 1, l should be greater than Y 1, 0. So, if the ambient mass fraction of the species is 1 is greater than the mass fraction at just some of the interface of vapour just some interface then you have condensation.

And of course, if it is slow vaporization that if B m, v is small much, much less than one then you can approximately the logarithm to be linear and you get rho d log l times log B m, v, but of course, you see that when this become a large you have a logarithmic dependents with become which tells you that the f essentially slows down with increasing B m, v and it just not directly proportional to B m, v. This proportional to only holds, when this if it is very small.

(Refer Slide Time: 12:36)



Problem Formulation (3/5)

- Problem incomplete using only species conservation; vapor concentration at surface, $Y_{1,0}$, is not known
- Need energy conservation: $fc_p\frac{dT}{dx}-\frac{d}{dx}\left(\lambda\frac{dT}{dx}\right)=0.$
- Integrating: $fc_pT-\lambda\frac{dT}{dx}=\text{constant}.$
- Apply energy conservation at interface: $\lambda\left(\frac{dT}{dx}\right)_0=fq_v,$
- Energy conservation: $fc_p(T-T_0)-\lambda\frac{dT}{dx}=-fq_v$
- Integrating: $f=\frac{\lambda/c_p}{\ell}\ln(1+B_{h,v}),$ $f=\frac{\rho D}{\ell}\ln(1+B_{m,v})$
- Heat transfer number: $B_{h,v}=\frac{c_p(T_\ell-T_0)}{q_v}.$

13

Now, you might think that the problem is completed solved, but it is not. Because you see that in this flux in this vaporization in this mass flux f, we do not know B m, v because we do not know what is our Y 1, we do not know what is our Y 1, 0 essentially. So, this is Y 1, 0 that is problem is in complete because you only species conservation or vapour conservation at the surface that is Y 1, 0 is not known. So, we need to know Y 1, 0, essentially we need the energy conservation. And that is the energy conservation is nothing but rho s rho u c p rho u C p dT d x minus d t d x of lambda d T d x of course, you do not have right hand side in either of this case. Because there is no reaction by neither in the species conversation equation you have in right hand side either in this one you have right hand side because there is reaction. And what we if we integrate this; what you get is essentially f C p t minus lambda d T d x is essentially equal to constant.

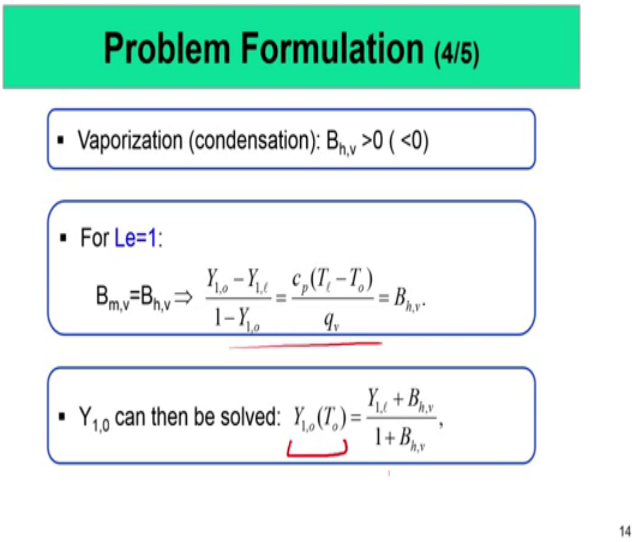
And then you have to what have to do is apply energy conservation at the interface when you say that the energy that goes for the vaporization that is supplied purely by conduction. So, as a result of that this lambda d T d x equal to 0 is essentially the mass flux that is evaporating times latent heat of vaporization that is q v. And using that you can apply the energy conservation and we get f C p t minus T 0 minus lambda d T d x is equal to f q v. And then if you integrate we will get a we find that f is equal to lambda by C p times by l and the distance between the free stream between the outside and the and the surface and that is times once again ln 1 plus B h, v, but this B h, v is nothing but the heat transfer number and its C p t l minus T 0 minus q v.

This once again shows that you get exactly the similar things because in the previously we got f is equal to rho d by l as you see here rho d by l times ln of 1 plus B m, v Y l times ln 1 plus B m, v. So, this and this are exactly very similar form, because the nature of the equations is essentially similar. Of course, you see that this rho lambda C p is and this is for the characteristics of C p t for the heat transfer this c p T this diffusive for mass from of course, we can assume these numbers to be one and this two can be same, and this is not required now. But this is the heat transfer number that we have obtain that goes into here.

So, the heat transfer number is nothing but B h, v is equal to c p times T l minus T 0 by q v, but this heat transfer number is mass transfer number we can think of these the driving potential that drives the species flux of the heat transfer actually. So, larger this B m, v the more is the heat which is more is the mass flux f of more is the evaporation is essentially the mass flux of water because there is no mass flux of air that is convicting. So, this f is the essentially contains only the mass flux of water, so it is essentially the evaporation flux. So, this evaporation flux is larger for larger b either m by either m v that is mass transfer number of the heat transfer number. So, this acts as essentially potential and this potential you see depends on the difference of the temperature between the free stream and the surface or between the between the in terms of the temperature difference or in terms of the mass fraction difference.

So, larger the mass fraction, larger is the evaporation rate, larger is temperature difference, larger is the evaporation. And of course, you have B h, v when this is larger you have condensation when this is positive that is when the temperature difference between the outside and the d zero is positive, you have you have potential; and when is negative, we have condensation. Then we can if you say (Refer Time: 16:40) equal to one then of course, you see that these two becomes then these becomes same. And of course, these two f are same and this both are represents the mass flux the or the vapour you know mass flux of water vapour that is escaping the interface and then this log becomes equated and for this to be equated. Of course, this inside this mass transfer number and the heat transfer number has to be same.

(Refer Slide Time: 17:06)



Problem Formulation (4/5)

- Vaporization (condensation): $B_{h,v} > 0$ (<0)

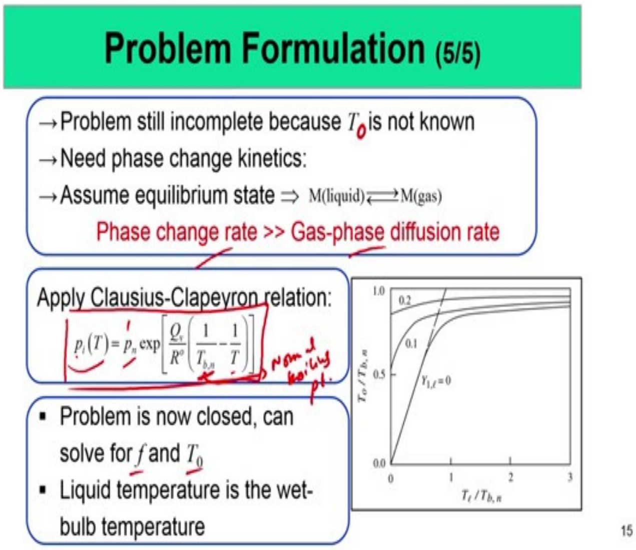
- For Le=1:

$$B_{m,v} = B_{h,v} \Rightarrow \underline{\frac{Y_{1,o} - Y_{1,\ell}}{1 - Y_{1,o}} = \frac{c_p (T_\ell - T_o)}{q_v}} = B_{h,v}.$$

- $Y_{1,0}$ can then be solved: $\underbrace{Y_{1,o}(T_o)} = \frac{Y_{1,\ell} + B_{h,v}}{1 + B_{h,v}},$

And we get this that is relation for the heat transfer number equation and this to be same. And using this relation essentially you can solve for Y 1, 0 which was actually holding or to describe the f because f contained the mass transfer number and the mass transfer number contained Y 1, 0. So, Y 1, 0 as you see is essentially is a function of T 0 and that is equal to Y 1, l plus B h, v divided by 1 plus B h, v.

(Refer Slide Time: 17:42)



Problem Formulation (5/5)

→Problem still incomplete because T_0 is not known

→Need phase change kinetics:

→Assume equilibrium state ⇒ $\text{M(liquid)} \rightleftarrows \text{M(gas)}$

Phase change rate >> Gas-phase diffusion rate

Apply Clausius-Clapeyron relation:

$$\boxed{\underline{p_i(T)} = \underline{p_n} \exp\left[\frac{Q_v}{R^o}\left(\frac{1}{T_{b,n}} - \frac{1}{T}\right)\right]}$$

Normal boiling pt.

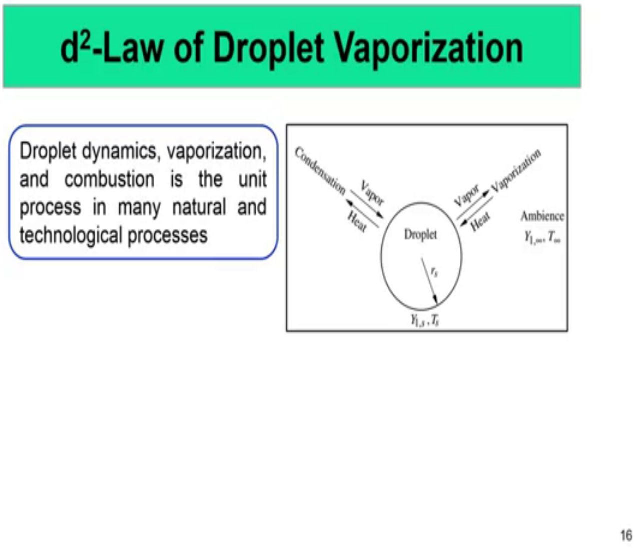
- Problem is now closed, can solve for $\underline{f}$ and $\underline{T_0}$
- Liquid temperature is the wet-bulb temperature

And this is still not solved; it is still not solved because this T 0 that is the temperature at the surface is not known. And for that we need phase change kinetics. And for that we have to assume the equilibrium state and where we assumed this species the same species goes from a liquid phase to a gas phase and of course, the phase change rate is mass much much faster than the gas phase diffusion rate. And then we can apply the thermo dynamic relation which is called the Clausius-Clapeyron relation which says that the partial at that reference temperature T is equal to the reference pressure which can be assumed to be 1 e to the power of Q v that is the that is the latent heat of vaporization divide by R 0 times 1 minus T b, n minus 1 by T. And this is once again this can be a difference temperature, whereas this gives the relation between p, i and T that is the partial temperature when there is a phase change.

And we can assume if this is 1 bar this becomes the essentially the normal boiling point. And then you can convert this partial pressure into essentially mole fraction and mole fraction you can get mass fraction. So, then using these two relations simultaneously, this one and this one can iteratively solved for, and find f and T 0 together. And the liquid temperature so found is the wet bulb temperature. So, this essentially solves our problem of estimating the evaporation rate or evaporation flux from a liquid pool in which what the top of which is the continuously removal of the vapour flux that is being that is coming out of the liquid pool.

So, this is very important and we can now in a position to tackle the more practical problem of droplet evaporation where the droplet contains essentially continue around water or contain a liquid fuel. And then for the liquid fuel case we can use this further to essentially gain find out what is the how does the droplet bound, and then we will what will do is that field use these ideas that we have just developed here, use these ideas to essentially describe the droplet evaporation rate. And then once you have the droplet evaporation rate we can use the previous thing long previous coupling function ideas and we will essentially couple these two things that is this evaporation analysis and the non premixed flame coupling function analysis. And using these two we can describe the droplet combustion so that we will be doing in the next class.

(Refer Slide Time: 21:04)



d^2-Law of Droplet Vaporization

Droplet dynamics, vaporization, and combustion is the unit process in many natural and technological processes

So, droplet vaporization and this is the d square-law droplet vaporisation that will do in the next class.

So, thank you very much.