Combustion in Air Breathing Aero Engines Dr. Swetaprovo Chaudhuri Department of Aerospace Engineering Indian Institute of Science, Bangalore

Lecture - 19 Laminar Non-Premixed Flames II

So, with this in mind we can now actually go in with this recapitulation of this with this recollection, we can actually just go into this one dimensional chambered flame of this coupling function formulation.

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So, then this is the problem derivation is your problem definition as you have seen that on the left hand say one dimensional flame, and on the x axis we plot distance and on the y axis we plot both the fuel mass fraction as well as temperature of course, these are different axis properly axis properly has to be chosen, and then on the left hand side you have the fuel region, but it is not pure fuel on the right hand side if the oxidiser region, but it is not pure oxidiser. So, then the mass fraction of this thing essentially becomes this is the mass fraction on the left hand side which is given by Y F 0 and this is the mass fraction on the right hand side for the mass fraction of fuel on the right left hand side which is given by Y F 0 and this is the mass fraction of oxidiser on the right hand side which is given by Y o l and then of course, then I can define the boundary conditions that at x equal to 0 your Y F is essentially is equal to Y F 0 and your Y 0 is equal to 0 because

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this is the boundary that is at x equal to 0 that is on the left hand side your Y F is equal nothing but Y F 0.

On the right hand side, but on the left hand side on this boundary they have no oxidiser because the entire oxidiser has been consumed any time of course, any point beyond x f your oxidiser has been consumed. So, on the left hand side your oxidiser is equal to 0 now product oxidiser is equal to also 0, and your temperature on left hand boundary is equal to T 0. So, on the right hand side similarly you have Y F is equal to 0 because the entire fuel has been consumed after the reaction sheet and, but your y oxidiser is nothing but Y 0 l that is once again this is not equal to 1, because there can be some nitrogen mixed with the it can be air you have nitrogen mixed with oxidiser we are talking in regionals. So, on then left on.

So, your product is equal to 0 and your T is essentially equal to T l on the right hand side all right. So, with this we can do a coupling function for formulation of course, you see that there is no flow. So, we do not do any have to involve there is an option that your convection is less in power than diffusion, though on the straight away it is a and also of course, it is a steady flame. So, essentially your unsteady term the time the dou dou t term goes away, your dou dou your divergence of rho u Y i rho u h s this traverse type of terms go away and you are essentially left with the balance of diffusion and reaction.

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$$\nabla \left(\frac{\lambda}{c_{p}} \nabla h^{2}\right) = \sum_{i=1}^{N} h_{i}^{i} W_{i}$$

$$T \left(\frac{1}{c_{p}} \nabla h^{2}\right) = -W_{i}$$

$$T \left(\frac{1}{c_{p}} \nabla h^{2}\right) = -V_{i}$$

$$T \left(\frac{1}{c_{p}} \nabla h^{2}\right) = -V_{i$$

And then of course, we can do this what we just did in this this thing and we arrive at this coupling function formulation function for apply immediately we can apply this formulation in this case and we have this Laplacian is also now dissolved into is simplified into this term one dimensional dou 2 B i dou x square is equal to 0 and your beta i essentially now can be straight away solved into in two integrations as is a linear function of x.

So, beta i is nothing but c 1 i is a constant and c 2 i, but of course, beta i can be like there can be i is for fuel they can be because your beta i is nothing but beta i is nothing but your T tilde plus Y i tilde or by the way you are also showing that the Lewis number is equal to 1 here. So, is this denominator of Y i tilde goes away. So, this essentially T tilde plus Y i tilde now then for beta we can have a beta F is equal to T tilde plus Y F tilde we can have a beta o as that is this is for the fuel this is for the oxidiser, we can have T tilde plus Y o tilde ok.

So, there can be two betas essentially and then of course, we have to use that because we want to invoke these boundary conditions. So, this coupling formulation what we have here we can this is essentially contains the information about the temperature within the system it also contains information about the mass fraction within the system, mass fraction within this domain and we can now what we to move further we need to basically apply the boundary conditions here, and we need to apply these boundary conditions to obtain these constants.

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So, applying these boundary conditions what we get is essentially that, we get that beta f is essentially T tilde plus Y F tilde, and Y F tilde that is our beta we assumed that the beta one is equal to essentially beta fuel, and that is given by T tilde plus Y F tilde and then if you apply this sort of things in here we get this and we get Y F tilde 0 plus T 0 tilde plus T tilde minus T 0 tilde minus Y F 0 x tilde of course, it is to be linear in x. Similarly, with applying this we get this boundary condition. You should this simple this on your own and this will give you the confidence to basically solve any kind of flames later.

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So, this is what we have arrived and you see that this result is in general we are not limited to a reaction sheet why, because I mean we are not invoked any assumptions about the how large the chemist reaction rate is you can comparison to diffusion etcetera ok. (Refer Slide Time: 06:12)



Now, we will come where the reaction sheet approximations will be incorporated. So, this is where we apply the reaction sheet approximation, that is we assume that that is if this was our flame, this is the left boundary Y F 0 this is our right boundary Y of Y o l and this is our T 0 this this is t and this is our T l and. So, this is we assume that this Y F goes to 0 are the flame location, this is 0. Similarly, we say that Y 0 all Y 0 goes to y o goes to 0 at the flame location. So, in this region there is no fuel and in this region there is no oxidiser. So, this is only possible when the reaction sheet representation is incorporated that is when we say that the reaction rates are much much much infinitely fast when compared to diffusion. So that there is no reactant leakage and as a result of that your these things is 0.

So, as to put it informally we said that Y F tilde is equal to 0 between x f tilde when x lies between x f tilde x and 1. So, of course, we can non-dimensionally as x is equal to x by l. So, x which is x tilde which is one when x is when x equal to l and of course, in these regions since oxidiser is equal to 0 your mass fraction of the oxidiser is equal to 0 when between 0 and x when x is between 0 and x f tilde. Now, then you can apply all these equations and get the basically the you can apply this, we can to this using this this and this we can essentially obtain the temperature, because your y is get eliminated in this sides and then you obtain the temperature on these two sides as this and this that is the temperature in this side is given by this this equation and the temperature on this side

is given by temperature this equation, but the most important thing to note is that the temperature is linear in x T tilde is linear in n x tilde.

Now, that is why the where does this come from this comes from the fact that this is a this is a essentially a diffusion problem and the is essentially we are solving for the Laplace equation and we are applying for the boundary conditions essentially we find that the beta i is essentially linear in x, that is where it comes from because the solution of Laplace equation when you have boundary conditions like this, it gives you linear solutions.

Then of course, we can also when we incorporate this temperature equation back into the beta i equation, we also get equations for this Y F tilde and Y 0 tilde and those are also given by linear in x and this is essentially the profile we get that this decreases like this as a Y 0 decreases like this and the temperature increases like this and decreases like this. So, what is what is one point is clear here that this this in a whole domain the maximum temperature point is that t is at t equal to x f tilde. So, this is that point where the temperature is maximum and, but then that is completely solves the problem. So, you see how straight forward it is that just by this whole coupling function this this problem contains all the complexities.

It contain the reaction rate etcetera, but we just converted removed those things by just considering the fact that your coupling function formulations and by as a constituents Lewis number is equal to 1, and using that we just solve this whole thing in just one step by forming a Laplace equation out of this complicated diffusive reactive equations. So, that is the strength of coupling function formulations yes it requires some time to properly normalise them properly stoichiometric weight them etcetera, but once those things those parts are done it we can just get the solution in one step and that is the power of normalisation also.

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So, this is strongly encouraged that you solve these things in a in this kind of in this manner instead of going through the full dimensional form or going through the full diffusive reactive reactions.

Now, the interesting thing is that what needs to be noted is that, that if we set that Y F tilde is equal to 0 and or Y tilde is equal to 0 then we can using these two things we can find out what is the location of the flame, that is what is x f tilde. So, that gives you basically x f tilde and we find that x f tilde is just nothing but this Y F 0 tilde divided by Y F 0 tilde plus 1 Y 0 l. Now this is equal to 1. So, this becomes essentially 1 by 1 plus Y o l tilde and which is essentially given by the phi star that is equivalence ratios normalised equivalence ratio is a equivalence ratio star divided by 1 plus phi star and which is essentially the normalised equivalence ratio. So, essentially the flame is stabilised at a the normalised the x by l, x f by l is essentially phi star which is it is the normalised equivalence ratio of the fuel mixture that we have ok.

So, that is the thing and we can also find out by because we know the gradients we can also find out what is the fuel mass fuel consumption rate, and that is given by this thing that is minus lambda by C p is equal to d Y F d f you may surprise this essentially what we have done is that we have just replaced the rho d, d y by d f with lambda by c p because our assumption Lewis number is equal to 1. So, that is why we can do this switching between rho d and lambda by C p. So, that is what we have obtained and then we can also show what is the criteria of mass maximum burning when Y F 0 is obvious is that when Y F 0 is equal to and Y is equal to 1 well that is when you have pure fuel and pure oxidiser then you have maximum burning and the final thing is that how to obtain the flame sheet temperature.

Now, since we know the x f star location we can substitute this in the temperature equation and what we get is essentially this form that is if we substitute this x f tilde in either these two this this one that you have obtained as in either these two equations, 5 or 6 which was the temperature equations we could easily get recovered the temperature that is this was the x f tilde equation that we have got.

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And then what we can get is that we will get this this equation that is t actually I will just write in the next sheet this is that is if you this needs some discussion that how to get the temperature substitute in temperature equations, we get T f tilde minus T 0 tilde that is what I want to say is that if you substitute this thing, in these equations that is in the temperature equations, where you see that Ts minus this this in this part T minus is essentially T 0 tilde plus T l tilde minus T 0 tilde plus y o l tilde times x tilde, that if you instead of x tilde if you substitute x f here ok.

X f is a function of what 1 by 1 plus Y o l tilde similarly that in this thing if you substitute x f here, which and then the similar things and then you if you substitute the other values of this Y F Y 0 etcetera what you will get finally, is this form n that is T f

minus T 0 tilde divided Y F 0 tilde plus T f tilde minus T 0 tilde divided by Y 0 l tilde is equal to 1 all right. Now what are all these things? Now Y F tilde is equal to Y F divided by Y F 0.

Similarly Y 0 tilde is equal to Y 0 divided by sigma 0 Y F 0 and q c is equal to summation y is equal to 1 2. So, and essentially this sigma 0 is given by w 0 times sigma 0 double dash minus sigma 0 dash divided by w F Y F double dash minus Y F dashed. If you now substitute all these things here what you will get is that you T f tilde by the way the T f t f tilde is given by C p T divided by Y F 0 q c, and if you substitute all these things goes if you substitute here.

So, essentially what we have done is that we have substituted except till did the equation in the T tilde equation right and then all these normalisation this weighting functions if you substitute back here what you get is essentially C p by Y F 0 q c T f minus T 0 divided by Y F 0 is actually equal to 1, but anyways plus sigma 0 divided by Y 0 l times T f minus T 0 is equal to 1 you can just do the normalisation like this. Now, of course these two cancels and this is q c. If you now we can write this as then this implies T f minus T 0 plus sigma 0 times Y F 0 by Y 0 l times T f sorry this is this is T l tilde is equal to q c Y F by c p this is what you get let me just put it here itself.

So, this gives you the flame temperature that is what is the temperature at x f that is T at x f is equal to T f, and that is given by this equation there is a temperature at x f is given by T f minus T 0 plus sigma 0 times Y F 0 divided by Y o l, times T f minus T l plus q c 1 by f by C p. Now what does this mean, what does this significance? Now to understand the significance the right hand side you see what you have? On the right hand side you have q c that is a chemical heat release and which is given by this times Y F, that is it is the heat release that is you get by burning Y F k g of fuel all right.

Now what does that do, where does Y F come from? Y F come from the fact that that in the mixture that we have considered that the fuel mixture on the left hand side 1 k f of fuel mix, mix is a short form mixture you can write contains Y F k g fuel at rate. So, that is where Y F comes from. So, basically this amount of heat release you can get by burning is in terms of basically one k f of fuel 1 k g of fuel mixture.

So, q c times Y F is this thing. So, 1 k g of fuel contains Y F k g of 1 k g of fuel mix consider the what fuel mixture contains Y F k g of fuel now similarly for stoichiometric

burning what you need for stoich burn w F times nu f double dash minus nu f dash k g fuel needs w 0 times nu 0 double dash minus nu 0 dashed k g oxidiser all right now. So, for stoichiometric burning this amount of k g of fuel pure fuel needs this k g of oxidiser. Then it means that 1 k g of fuel needs w 0 times nu 0 double dash minus nu 0 dashed divided by w F times nu f times double dash minus nu f dashed k g oxidiser and if we just replace this with Y F then this needs Y F k f of oxidiser ok.

Now, what is this? This is nothing but Y F sigma 0 where sigma 0 is nothing but this. So, now, Y F k g of fuel was contained in 1 k g of fuel mixture therefore, 1 k g of fuel mix needs Y F times sigma 0 k g oxidiser. Now Y 0 k g oxidiser you get in 1 k g of oxidiser mixture right because on the right hand side you have basically Y 0 is a fuel mass fraction of the oxidiser. So, this amount of oxidiser Y 0 times Y F times sigma 0 k g oxidiser, which is needed for stoichiometric burning in sigma 0, Y F by Y 0 k g of oxidiser mix. So, what you get is that for stoichiometric burning 1 k g of fuel mix needs sigma 0 times Y F by Y 0 k g of oxidiser mixture, this is very very important to understand.

So, just please go over this once again that is what we have done is that 1 k g of fuel mixture contains Y F k g of fuel all right on the left hand side, now for stoichiometric burning w F times this k g of fuel needs this k g of oxidiser. Therefore, Y F k g of fuel needs this k g of oxidiser which is nothing but sigma Y F times sigma 0. Now you find Y 0 k g of oxidiser, because you see on the right hand side. Now Y 0 times sigma Y 0 k g of oxidiser in 1 k g of oxidiser mixture. Therefore, this amount of where will you find this amount of oxidiser mixture you will finds Y F times sigma 0 k g of oxidiser mixture in sigma 0 times Y F by Y 0 k g of oxidiser mixture all right for stoichiometric burning 1 k g of fuel air mixture, essentially needs sigma 0 times Y F by Y 0 k g of oxidiser mixture because on the left hand side and the right hand side both contains inverse that is the whole point ok.

So, for stoichiometric burning 1 k g of fuel mixture contains sigma 0 times Y F by Y 0 k g of oxidiser mixture, now going back into this what is this doing. This is considering per k g of fuel mixture. So, if you have 1 k g of fuel mixture this is the amount of heat is being released which is the standard heat release in 1 k g of mixture u c times Y F and that is heat doing what that is heating up 1 k g of the fuel mixture from T 0 to T f, and it is also heating up exactly the amount that is required for stoichiometric burning exactly

the amount of oxidiser that is oxidiser mixture that is heating required for stoichiometric burning.

So, these heat release it is doing the job of heating up 1 k g of fuel mixture and this also heating up the exact amount of oxidiser mixture for stoichiometric burning. So, then what is the flame temperature? Exact then at the then it says that at the flame what is happening is stoichiometric burning is happening because of the flame your temperature is raised from T 0 to T f and T l to T f and the this amount of heat is released is heating up one k f of fuel air mixture not 1 k g of fuel air mixture 1 k g of fuel and inert mixture, 1 k g of fuel mixture and the exact amount of oxidiser mixture which is required for stoichiometric burning.

So, the burning that is happening in the flame is essentially stoichiometric burning and the flame temperature corresponding flame temperature is essentially the adiabatic flame temperature all right. So, this proves that the flame temperature is a non-premixed flame is essentially the adiabatic flame temperature and this is the one of the most important hallmarks of the of the of a non-premixed flame, that the temperature unless there is no reactant leakage that is you are the flame temperature is strongly the is exactly the adiabatic flame temperature you have no control over it, because the mixing at flame is always at the stoichiometric is at a stoichiometric proportions. Then the flame itself stabilises at a location where you are, it is exactly of the fuel air mixture at a stoichiometric proportion.

So, the temperature the mixing is the burning air stoichiometric and the temperature the such thus reached is also the adiabatic flame temperature. So, that you cannot control the temperature and because you cannot control the temperature you cannot control the emissions and also you have soot and lot of nox, because the temperature can be so high. So, this is one thing one very important thing about the non-premixed flame that the temperature reached is always the adiabatic flame temperature and you can clearly show that. So, find it from this equation that the flame temperature reached is the why the flame temperature reaches the adiabatic flame temperature.

But I suggest you go through the book also, and you go through this derivation once again to convince yourself that this is the case all right. So, that is for this class and we will meet again to discuss the Stefan flow that is liquid evaporation and then droplet evaporation and condensation.

Thank you.