

Optimal Control, Guidance and Estimation

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Lecture No. # 32

Integrated Estimation, Guidance & Control - II

Hello everyone now we will continue with our lecture series on this Optimal Control Guidance and Estimation course (Refer Slide Time: 00:25). Last lecture, we started this recent ideas, of Integrated Guidance and Control thing like that. And here will we will also talk about estimation this particular lecture. And we will continue further this development with first integrated estimation guidance and then, followed by integrated estimation guidance and control as well. So, let us see and we will not able to cover everything, but we will cover I mean what we are come of with **with** last couple of years and details and that actually really **alright**.

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Motivation

- To fuse the estimation, guidance and control loops at various levels
- Benefits arise because integrated designs are capable of retaining and exploiting the synergy between various subsystems
- Integrated design approaches proposed in literature can be broadly classified into three groups:
 - Integrated guidance and control (IGC)
 - Integrated estimation and guidance (IEG)
 - Integrated estimation guidance and control (IEGC)

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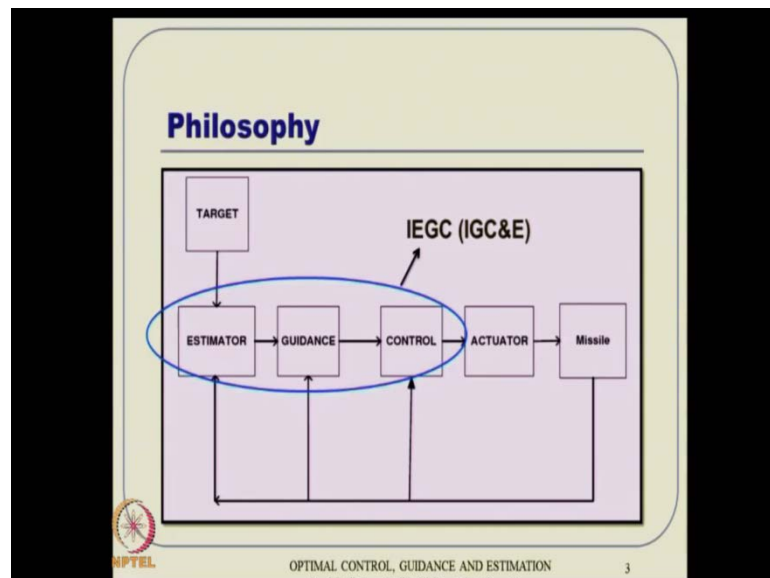
So, the motivation turns out to be something like this; obviously, the interest is to fuse the estimation guidance and control loops at various levels. And typically the benefits

arise because integrated designs are capable of retaining and exploiting this synergy between various subsystems.

And then, integrated approaches can be categorised into various categories first things and can be something like integrated guidance and control. And then integrated estimation and guidance and integrated estimation guidance and control as well. Now, what we turns out that this particular, when topic when we will talk about integrated estimation and all that, it has lot more impact, when we will talk about estimation of exogenous input for control design.

Other wards something like inertial guidance, where the target information is a estimated not really recovered from the **from the** (()) own instrument system actually. They are typically good, what **what** is not good is exogenous input that we collect from around sense us and how can we actually fuse that into the guidance and control design loops. So, that the efficiency of performance can be must rate actually that is what we are interested in.

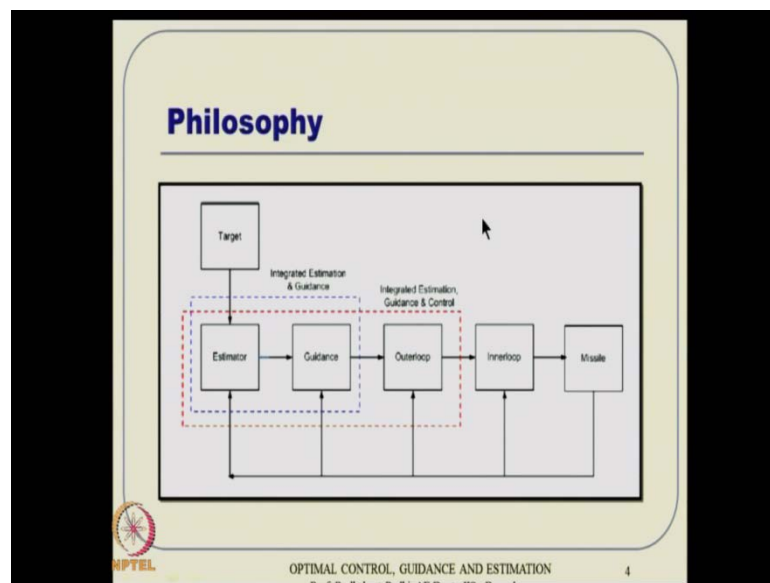
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So, the philosophy as we saw that in the **in the** last class, what we did is typically this one guidance and control. What suppose somebody wants to do this way estimation and guidance, then that least to this **this** concept of integrated estimation guidance.

And also summary you can think about putting something in together, then it **it** is integrated estimation guidance and control. And some people would like to call it as Integrated Guidance and Control and Estimation, IGC and E, so anyway. So, the concept is either we are talk about I EG here or we are talk about I E G C here. So, both of that we will talk in **in** this lecture a some sort of little overview of what is our own way of two things actually.

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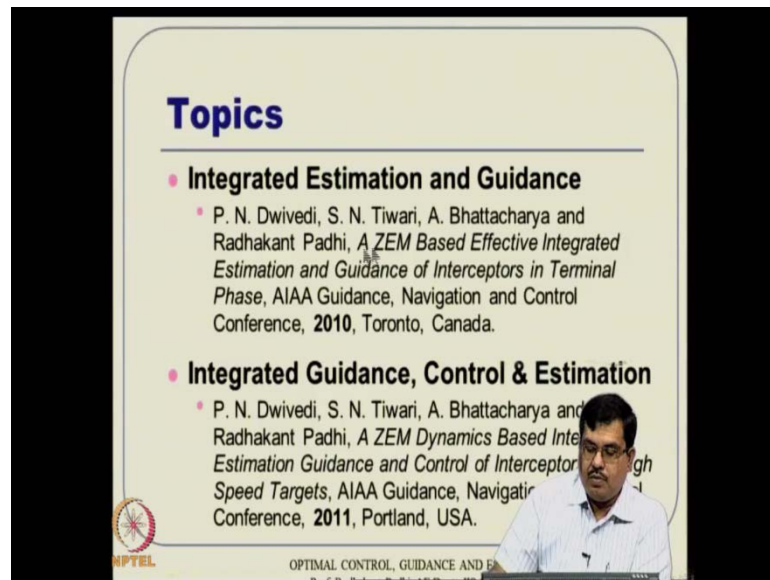


And this is different representation again this **this** picture slightly confusing (Refer Slide Time: 03:01) what **what** we are typically proposing here is something little more elaborately something like this. And remember the **the** control when we will talk about here, this particular block is actually constitute of two loops, outer loop, inner loop. Outer loop is something that you take lateral acceleration commands from guidance loop and try to generate the corresponding body rates especially the **(())** rates.

And then around instrumentation and think like that pitch once you available go through inner loop generated the control soft inflection. And then pass it through the actual dynamics and then similar back and all that actually I mean **I mean** this particular output what we are talk about can be define into the **the** can be decompose into this **this** four control surface deflections delta 1 to delta 4 and then you find it find it to the inertial actually then **(())**.

That **that** part is a where it is, what we are talking here is either this part, either estimator and guidance together or this part, where the outer loop of the **of the** control is also integrated. That means, we directly generated this **this** body rates from after doing estimation actually. So, that kind of things can **can** come in basically that way **alright**

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Topics

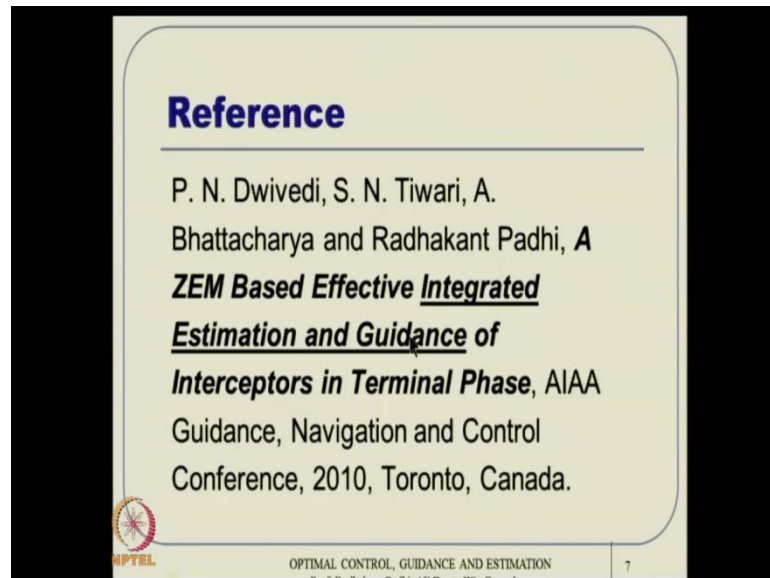
- **Integrated Estimation and Guidance**
 - P. N. Dwivedi, S. N. Tiwari, A. Bhattacharya and Radhakant Padhi, *A ZEM Based Effective Integrated Estimation and Guidance of Interceptors in Terminal Phase*, AIAA Guidance, Navigation and Control Conference, **2010**, Toronto, Canada.
- **Integrated Guidance, Control & Estimation**
 - P. N. Dwivedi, S. N. Tiwari, A. Bhattacharya and Radhakant Padhi, *A ZEM Dynamics Based Integrated Estimation Guidance and Control of Interceptor against High Speed Targets*, AIAA Guidance, Navigation and Control Conference, **2011**, Portland, USA.

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That topics that your covering here is **is** taken from 2 and 4 around papers, first thing is what we presented in 2010 a double agency conference. And subsequently in 2011, what we presented in talk **talk** next actually. This two, two paper are typically the material that, I mean typically contain the material that I am going to talk today actually.

So, those of are interested you can see this paper from what it is actually. **Alright** the first thing is as this first paper how it is talks about a ZEM miss based effective integrated estimation and guidance of interceptors in terminal phase. So, these are details of the other work actually.

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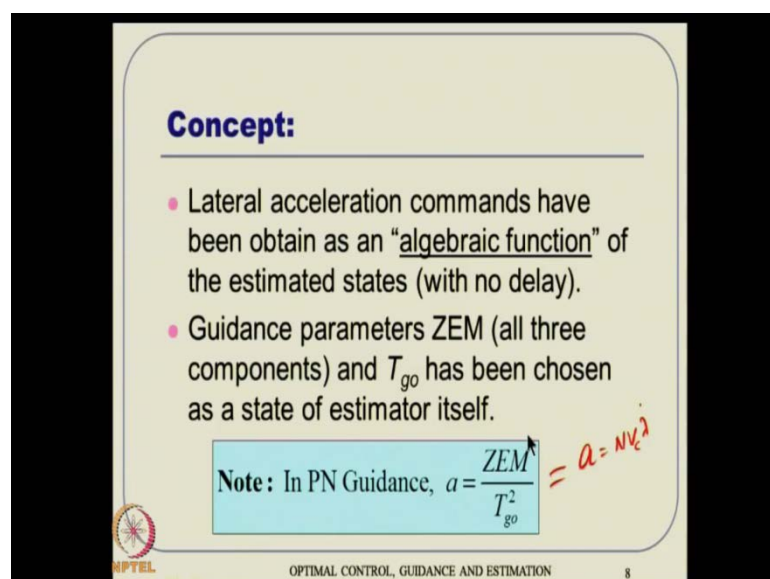
Reference

P. N. Dwivedi, S. N. Tiwari, A. Bhattacharya and Radhakant Padhi, ***ZEM Based Effective Integrated Estimation and Guidance of Interceptors in Terminal Phase***, AIAA Guidance, Navigation and Control Conference, 2010, Toronto, Canada.

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So, again the reference in detail is something here, so again we are talking about something like integrated estimation and guidance. we are not typically bothered about control part as it is; however, the simulation things that we are going to, so also talks about control in the loop. And the **and the** it is not necessary I mean that result is you see here or not from point mass simulation, whatever is I mean even if it is something like this (Refer Slide Time: 05:33) this part is also included as part of the simulation all that what you see here actually. Then what you see what you present results is **is** the full system of simulation result essentially.

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Concept:

- Lateral acceleration commands have been obtain as an “algebraic function” of the estimated states (with no delay).
- Guidance parameters ZEM (all three components) and T_{go} has been chosen as a state of estimator itself.

Note: In PN Guidance, $a = \frac{ZEM}{T_{go}^2} = a = NV_c^2$

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So, let us restart how do you do it and think like that, the concept is something like this first is lateral acceleration commands have been obtain as an algebraic function of the estimated states; that means, there is typically no delay. So, estimation certain states and then, directly computed the **the** lateral acceleration required for the inertial as some sort of a direct algebraic function actually.

How can you do that, because this is a kind of motivated from this **this** guidance, so PN guidance can be **can be** represented, what we know is something like this PN guidance is typically implemented something like a lateral acceleration something like negation constraint. And then v either V_N or V_c closing velocity times λ dot, that is lateral acceleration. But, this is also equivalent to something like this **a which is some** which is called as a this ZEM is called is something like 0 effort miss and then, that time $2g$ square.

So, it can be actually shown that this same expression can be return something like this. So, then the idea is some of if you can estimate this **this** ZEM as well as T go, then we got it actually we got lateral acceleration. So, how can you do that **that** is the whole idea there and this requires lot of algebra in the implementation in the frame of copy cafe and think like that, so let us see how **how** do you that actually.

Essentially in the state estimation this **this** ZEM and T go's are actually state and then using state estimation concept that you have studied in **in (())** filtering and all that we estimate. And then directly compute in some sort of formula like this, how we will talk in we will talk those details as we go long actually.

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Summary of Extended KF: Continuous-Discrete Formulation	
Model	$\dot{X}(t) = f(X, U, t) + G(t)W(t)$ $Y = h(X_k) + V_k$
Initialization	$\hat{X}^-(t_0) = X_0$ $P_0^- = E [\tilde{X}^-(t_0) \tilde{X}^{-T}(t_0)]$
Gain Computation	$K_{e_k}(t) = P_k^- C_k^{-T} [C_k^- P_k^- C_k^{-T} + R]^{-1}$ <p>where, $C_k^- = \left[\frac{\partial h}{\partial X} \right]_{\hat{X}_k^-}$</p>

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So, just a little bit summary of our KF and all the three results that you see here in the fame of copy cafe but, someone wants implement you KF you can also do that. But, the concept remains same, we have this the state equation, where this I mean non-linear equation state equation I mean this corrupted by noise. And then output equation a corrupted by noise as well both of this non-linear equations. And then typically proceed like this, you start with some initialization of state as well as covariant matrix, covariance matrix and then compute the again.

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Summary of Extended KF: Continuous-Discrete Formulation	
Updation	$\hat{X}_i^+ = \hat{X}_i^- + K_{e_i} [Y_i - h(\hat{X}_i^-)]$ $P_i^- = (I - K_{e_i} C_i) P_i^- (I - K_{e_i} C_i)^T + K_{e_i} R_i K_{e_i}^T$ <p style="text-align: center;">(preferable)</p> $= (I - K_{e_i} C_i) P_i^- \quad (\text{not preferable})$
Propagation	$\dot{\hat{X}}(t) = f(\hat{X}, U, t); \quad \hat{X}_k^+ \rightarrow \hat{X}_{k+1}^-$ $\dot{P}(t) = AP + PA^T + GQG^T; \quad \hat{P}_k^+ \rightarrow \hat{P}_{k+1}^-$ <p>where $A(t) = \left[\frac{\partial f}{\partial X} \right]_{\hat{X}(t)}$</p>

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And then follow with the, I mean this **update the states actually** update the states as well as covariance matrix, then you propagate the state and covariance matrix. So, this update happens in **in** discrete time and in propagation happens in continuous time all the details we have seen before.

So the, I mean all this techniques are available, so what matters in particular problem is how do you formulate the problem write here. Once this is available rest of the things we can have the steering process and then **then** our (()) experience can coming and think like that. But, given a problem how do you actually formulate this to begin with and then go head solve it actually.

So, that is what you are interested, so essentially we will **we will** not talk too much implementation details of kalman filtering we are already done that. So, what we really want to see is how do we come of this two **this to** equations state equation and output equation. And this particular given problem rest of things we are typically I mean I can if you long I can say that mechanical actually **alright**, so this is how it is.

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Problem Formulation

State, Measurement & Desired Output Equations

$$\dot{X} = f(X) + G(X)U$$

$$Y = h(X)$$

$$Z = g(X)$$

States variables are

$$X = [ZEM_x, ZEM_y, ZEM_z, \Delta V_x, \Delta V_y, \Delta V_z, a_{tx}, a_{ty}, a_{tz}, T_{y0}]^T$$

The measurement variables are the direct output of seeker (range, range rate, gimbal angles and gimbal angles rates) given as

$$Y = [r_m, \dot{r}_m, \phi_{ym}, \phi_{zm}, \omega_{yym}, \omega_{zzm}]^T$$

Output variables are

$$Z = [a_{x_s}, a_{y_s}, a_{z_s}]^T$$

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So, let us a try to see that, so what you are interested is; obviously, in the **in the** implementation if you see in kalman filtering this w **(())** use it actually. So, the covariance of W, W transpose expected value W W transpose think like that like q r and think like that typically part of the design. But, W and V the inertial are not the part of

design. So, need not come of with this part of hit is part of the formulation, implicitly assume these are their basically that way.

What you will mean is the how you write the state equation and output equation and then we can **we can** mathematically write something like that and then start guessing this inertial condition as well as covariance matrix and **and** carry out further algebra basically **alright**.

So, what you are interested in is something like this \dot{x} is **f x** f of x plus g of x times u this dynamics happens to be kind of control f fine. So, we can write this way really does not matter what we write, what if you write it already like this and control computation becomes slightly 0 and all that actually. So, we are not too much consent about that this particular problem **is a** is essentially estimation problem. And this is your sensor output, so these two way to write, but also remember this Z and which **which** typically represents the desired output then nothing but, the inertial lateral acceleration.

And this lateral acceleration are also computed is in algebraic function of the estimated state actually, that is the whole duty here **alright**. So, state variables are what taking as a this components of the ZEM along x y z component and then the relative velocity along x y z as well as the target acceleration along x y z and time to go T go.

So, we have three coming from here, three coming from here, three coming from here (Refer Slide Time: 11:09) and all, so essentially it is a ten state estimation problem actually. And these are not that easy numerically essentially, but normally we use to do that, because we **we** want to derive additional benefits and think like that actually. So, we can escape from there. And what are the measurement thing available to us typically these are range and we are assuming the seeker, because terminal phase problem, distance between inertial surface between inertial target is not very much.

So, we assume that these are seeker estimation and all that, the seeker are also available which can be reverse range **range** rate as well as this **this** gimbal angles. And they write actually. So, this **this** gimbal angles are something like the seeker frame the compare I mean whatever angle it makes with respect to body frame actually. We will **we will** give some diagrams also the output variables are nothing but, lateral acceleration in x y z component actually.

But ultimately remember we want output I mean this **this** lateral acceleration in **in** body frame like that that is what the aero dynamics will coming into picture and then try to realization. Where is the all this formulation that we are talk about ZEM **Z E M** components and then velocity components everything else is typically in the inertial frame.

So, inertial frame the **the** estimation goes on and then using this transformation matrix and all and more of that seeker in the flight dynamics lectures and all. So, using those transformation matrixes, we will able to converted to the body frame and think like that. So, remember these are our states, these are our measure outputs and these are our desired output, which **which** we want predict into the auto control design loop actually.

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ZEM-Dynamics in 1-D & Time-to-go Dynamics

ZEM in y-direction

$$ZEM_y = \Delta y + \Delta V_y T_{go}$$

$$\frac{d}{dt}(ZEM_y) = \Delta \dot{y} + \Delta V_y T_{go} + \Delta V_y T_{go}$$

$$\Delta V_y = a_T - a_M$$

Time-to-go (T_{go})

$$T_{go} = -(\Delta R / \Delta \dot{R})$$

$$\dot{T}_{go} = -\left(\frac{\Delta \dot{R} \Delta R - \Delta R \Delta \ddot{R}}{\Delta \dot{R}^2}\right) = -\left(1 - \frac{\Delta R \Delta \ddot{R}}{\Delta \dot{R}^2}\right)$$

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So, at go we want this dynamics, so some basics concept first this is a inertial here this is a target here, is just 2-D picture sort of thing. The reference line and there is something like LOS angle and these are typically inertial guidance diagram sort of thing. So, the Z E M along this y direction if you see this, so very simple it is talks about the inertial supuration delta Y plus delta V times T go. Whatever delta V y is define is something like V T minus V M, so and that is what is assume to be kind of constraint here if you assume that then Z E M in y is something like this actually.

So, when you put it work here ZEM then the d y d t ZEM, then it turns out to be something like this actually. Now, so there are different levels of kind of computation

that you can propose for example, if you somebody wants I mean to incorporate this ΔV by dot also, has will be plus this half times ΔV y dot times $T V$ square and all that actually.

So, that is a depending on the **on the** formulation essentially **essentially** it come from our I mean little bit dynamic physics that we were of that distance is a nothing but, I mean this inertial distance plus V times T plus half T square sort of thing actually. So, if you **if you see** that then Z E M y is Δy plus ΔV a times T go, here we are assuming the ΔV is not really very constraint but it is projected sort of thing. So, every time it is update the there is **there is** V y change ΔV by dot is accounted here of course, what as per as a Z E M component a I mean computation is a constraint that here assuming ΔV y is **is** constraint actually.

So, is a little bit discrepancy that is how it subject our get the how the formulate the problem actually. And I will simplicity this necessary to have the output formulation in a good way output equation actually will **will** see that in a couple of lecture later. Anyway coming back to that, this is what it is d by $d t$ of that is derivative of this Δy dot plus derivative of the whole thing which is talks about ΔV y dot plus t go plus ΔV y times T go dot.

And where ΔV y dot is nothing but, target lateral acceleration in the y direction minus inertial acceleration y direction, so this **this** can be computed that way. Now, whatever time to go t go is very crew way it is represented ΔR by ΔR dot dot, so T go is minus of that really. So, then T go dot it is very easy to see that just **d** $d y$ by $d t$ go. So, computed this way this times the ΔR dot times derivative that minus ΔR time double derivative of that to by ΔR square sort of thing.

And these two we will cancel out this **this** is a essentially negative of 1 minus system actually. So, T go dot we derive this way and ΔV y dot is derive that way. And **and** ZEM y d by $d t$ that way of that way and this what would see here, this y direction can be extended to x and to z components also basically.

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System Dynamics in 3-D

ZEM_x	$\Delta V_x + (a_{tx} - a_{tx}) T_{sp} + \Delta V_x \dot{T}_{sp}$
ZEM_y	$\Delta V_y + (a_{ty} - a_{ty}) T_{sp} + \Delta V_y \dot{T}_{sp}$
ZEM_z	$\Delta V_z + (a_{tz} - a_{tz}) T_{sp} + \Delta V_z \dot{T}_{sp}$
$\Delta \dot{V}_x$	$a_{tx} - a_{tx}$
$\Delta \dot{V}_y$	$a_{ty} - a_{ty}$
$\Delta \dot{V}_z$	$a_{tz} - a_{tz}$
a_{tx}	$-\frac{\Delta x}{r}$
a_{ty}	$-\frac{\Delta y}{r}$
a_{tz}	$-\frac{\Delta z}{r}$
\dot{T}_{sp}	$-1 + (\Delta R \dot{\Delta R}) / \Delta R^2$

$ZEM = \Delta V + \Delta V \dot{T}_{sp}$

$\frac{d}{dt}(ZEM) = \Delta \dot{V} + \Delta V \dot{T}_{sp} + \Delta V \dot{T}_{sp}$

where,

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$\Delta \dot{R} = (\Delta x \dot{\Delta x} + \Delta y \dot{\Delta y} + \Delta z \dot{\Delta z}) / \Delta R$$

$$\Delta \ddot{R} = (\Delta R \cdot [\Delta x \dot{\Delta x} + \Delta y \dot{\Delta y} + \Delta z \dot{\Delta z}] - [\Delta x \dot{\Delta x} + \Delta y \dot{\Delta y} + \Delta z \dot{\Delta z}] \cdot \Delta \dot{R}) / \Delta R^2$$

Inertial Components

So, taken together we can write it something like this **this** Z E M x direction very similar expression what we because you take this one and here you put it back; that is what d by d t y. So, similar Z E M x something like this Z E M y and Z E M dot y, Z E M x, Z E M x dot this turns out to be like this, very similar to what this kind of expression actually.

Where delta V x dot delta V y dot is **is is** computed something like this, so these three comes from like that and then T go dot derive this way. So, that comes like that where as a t x dot a t y dot and all that there all also coming from some sort of dynamics actually. The a t x dot is a negative of a t x of tau is a first setter table dynamics that you are incomplete of the target isolated component. In other wards it assumes that target can momentarily proof some isolation, but eventually I mean if it is state's there, then the target essentially eventually going to 0 actually. It cannot continuously keep on axial rating the vehicle capability may not be their actually **alright**.

So, **this is a** this is the formulation this is the frame work of state equation, now this delta I mean this delta R and delta R dot can be computed that is also basically. Now, what about output equations that is the more important and also remember these are typically inertial components actually, inertial frame components, we do not to forget that actually.

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Measurement Equation (in LOS frame)

Using $\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} Z_r - \Delta z_r T_{rp} \\ Z_t - \Delta z_t T_{tp} \\ Z_b - \Delta z_b T_{bp} \end{bmatrix}$

$$\begin{bmatrix} r_l \\ \dot{r}_l \\ \lambda_a \\ \dot{\lambda}_a \\ \lambda_e \\ \dot{\lambda}_e \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \\ \frac{\Delta x \dot{\Delta x} + \Delta y \dot{\Delta y} + \Delta z \dot{\Delta z}}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}} \\ \tan^{-1} \frac{\Delta y}{\Delta z} \\ \frac{(\Delta z \dot{\Delta y} - \Delta y \dot{\Delta z})}{(\Delta y^2 + \Delta z^2)} \\ \tan^{-1} \frac{\Delta x}{\Delta z} \\ \frac{\Delta x (\Delta y^2 + \Delta z^2) - \Delta y (\Delta x \dot{\Delta y} + \Delta z \dot{\Delta z})}{(\Delta x^2 + \Delta y^2 + \Delta z^2) (\sqrt{\Delta y^2 + \Delta z^2})} \end{bmatrix}$$

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Now, output equation ultimately remember that we want this outputs are given in the form of range, range rate, gimbal angles and their rates. But, that to know this we also have **have** to find out this other variables what I mean actually especially this quantities lambda dot lambda e dot and all that.

So, this is lambda, lambda e definition sort of thing and there rates will **will** need in **in** formulation here. So, we need to find out this LOS angles essentially and there rates. So, the r_l and \dot{r}_l LOS separation and there rate of change can we directly computed like that. Because, that is that is very easy actually these two first component r_l is square root of this part and \dot{r}_l dot is a derivative of that actually, this separate back here.

Now, coming to the lambda a, again lambda e, lambda a is this lambda is this **this** angle. So, this angle can be directly represented something like any mores delta y by delta z from these two components actually. If you formulate this **this** form, then lambda is nothing but, delta y you have to see some way like that this parallel to a x actually anyway.

So, delta y divided by delta z if you **if you** put a well let me put the back if we put it something like this parallel to this turns out to be parallel that anyway. So, this **this** **10 10** lambda a is delta y by delta x. So, you can computed that way and then lambda dot derivative dot. Similarly, lambda e can taken it in different frame this one and this is projection and then in that projection is this angle appears like that.

So, λ is Δx divided by square root of Δy square plus Δx square and then $\dot{\lambda}$ you can **you can** take derivative of that write it actually. So, these components are available especially these two will directly go to our formulation, but these **these** four components especially $\dot{\lambda}$ and $\dot{\lambda}_e$.

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Measurement Equation (in Seeker Gimbal Frame)

$$r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 + \eta_1}$$

$$\begin{bmatrix} \dot{r}_m \\ \dot{\phi}_{ym} \\ \dot{\phi}_{zm} \end{bmatrix} = \begin{bmatrix} \frac{\Delta x \Delta \dot{V}_x + \Delta y \Delta \dot{V}_y + \Delta z \Delta \dot{V}_z}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}} \\ \tan^{-1} \left(\frac{n}{m} \right) \\ \tan^{-1} \left(\frac{m}{\sqrt{l^2 + n^2}} \right) \end{bmatrix} + \begin{bmatrix} \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}$$

Unit LOS vector in body frame (l, m, n) is

$$\begin{bmatrix} l \\ m \\ n \end{bmatrix} = C_1^T C_1^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \phi_y \cos \phi_z \\ \sin \phi_z \\ -\cos \phi_y \sin \phi_z \end{bmatrix}$$

$$\begin{bmatrix} \dot{\omega}_{ygm} \\ \dot{\omega}_{zgm} \end{bmatrix} = C_1^T C_1^T C_1^T \begin{bmatrix} -\dot{\lambda}_e \sin \lambda_e \\ \dot{\lambda}_e \cos \lambda_e \end{bmatrix} + \begin{bmatrix} \eta_5 \\ \eta_6 \end{bmatrix}$$

DCM matrix of LOS to gimbal frame

Where

- r_m = range
- ϕ_{ym} = gimbal angle along y
- ϕ_{zm} = gimbal angle along z
- $\dot{\omega}_{ygm}$ = gimbal angle rate along y
- $\dot{\omega}_{zgm}$ = gimbal angle rate along z

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We need that in formulating this I mean this gimbal's angle rates actually here. **Alright**, so these are components that remember all this Δ by happens in the inertial component and we need to find it out from the state vector actually. So, this is where this equation comes handy here. (Refer Slide Time: 20:34), so if Δx is computed reversely as ZEM minus ΔV times T go.

So, this Δx to Δz can be first computed that way and then there is used here and that form. And then you have this **this** two dots $\dot{\lambda}$ and $\dot{\lambda}_e$ which will be required in the next slide in the actually. Also remember this **this** measurements are typically available in seeker gimbal's frame and LOS frame information is obtained through a series of transformations actually will we will see that actually.

So, these this is what range we have this and we have assume here that range is corrupted by some noise and then range rate gimbal's angles are also corrupted by some noise and this gimbal's angles are essentially this angles now **now** this assume this inertial body x is actually.

So, with respect to the body x is what this LOS vector means these are what we saw with respect inertial exercise this LOS what it makes with respect to inner cell exercise. And here what you tell this LOS and the seeker is typically alien towards the aliens all the time that is what another implication of that. And then we tell this x is what we are seeing here is nothing but, this ϕ and γ sort of thing these are I mean ϕ x ϕ by y typically different people define that way.

So, this **this** angle ϕ 1 by ϕ z well let me put it that way, so this some small mistakes here probably this is ϕ y this is ϕ z actually. That anyway **this this** these angles are represented something like this and then this gimbals angles are represented like that. Now, you can see that this transformation matrix involve, this c 1 2 this LOS to inertial and then inertial body actually.

So, what you **what you** getting here this LOS vector in a LOS frame really, so LOS vector and LOS frame this is 1 0 0 that one is converted back two inertial frame is in this **this** matrix. And then inertial frame to **(())** these are typically available with us actually, this **this** angle information and all. Then this can be represented I mean expression as a function of that and then that can be substitute here basically.

Alright this all, this things happen these are little bit complex looking algebra but, what especially is effectible algebra it is not mass of inertial. **Alright**, so our formulation final talks range, range rate, gimbals angles and then there rates these are corrupted by noise everywhere actually.

So, now we got this **output formulation**, output equation formulation for the information that are available essentially we got what you wanted x dot equal to f of x u and y equal is a function of x actually. So, now, we are ready to implement our kalman filter use it and then finally, got this information actually **alright**.

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Unit LOS Vector in Body Frame & Transformation Matrices

$$\begin{bmatrix} l \\ m \\ n \end{bmatrix} = T_i^b = C_i^b C_i^e = (C_i^b)^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \phi_y \cos \phi_z \\ \sin \phi_z \\ -\cos \phi_z \sin \phi_y \end{bmatrix}$$

where,

$$C_i^e = \begin{bmatrix} \sin \lambda_e & \cos \lambda_e \sin \lambda_a & \cos \lambda_e \cos \lambda_a \\ 0 & \cos \lambda_a & -\sin \lambda_a \\ -\cos \lambda_e & \sin \lambda_e \sin \lambda_a & \sin \lambda_e \cos \lambda_a \end{bmatrix}$$

$$C_i^e = \begin{bmatrix} (q_1^2 + q_2^2 - q_3^2 - q_4^2) & 2(q_1 q_2 + q_3 q_4) & 2(q_1 q_3 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & (q_1^2 - q_2^2 + q_3^2 - q_4^2) & 2(q_2 q_3 + q_1 q_4) \\ 2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & (q_1^2 - q_2^2 + q_3^2 + q_4^2) \end{bmatrix}$$

$$C_i^b = \begin{bmatrix} \cos \phi_y \cos \phi_z & \sin \phi_z & -\cos \phi_z \sin \phi_y \\ -\cos \phi_z \sin \phi_y & \cos \phi_z & \sin \phi_y \sin \phi_z \\ \sin \phi_y & 0 & \cos \phi_y \end{bmatrix}$$

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Little **little** our that l m n are represented something like this, where this inertial I mean inertial LOS and inertial body and body to gimbals these are all computed that way actually. Sometime this LOS angles with respects to inertial frame, these are angles with respect to inertial, I mean these are this angles lambda e and lambda a are angles that LOS frame with respect to inertial frame. And then these are the angles that the LOS makes with respects to body frame and these are the spontaneous components essentially contain angles that contain relationship between inertial frame to body frame actually. So, these things will be really used in the formulation sort of thing.

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Output Equation: Guidance Commands

Body Components

$\begin{bmatrix} a_{x_c} \\ a_{y_c} \\ a_{z_c} \end{bmatrix} = C_i^b \begin{bmatrix} N ZEM_x \\ N ZEM_y \\ N ZEM_z \end{bmatrix}$

Inertial Components

where C_i^b is the DCM matrix of inertial to body.
Here $N = 3$ is the navigation constant.

Note: (i) No lag between Estimation & Guidance!
(ii) ~~Lateral acceleration along x-axis cannot be realized.~~

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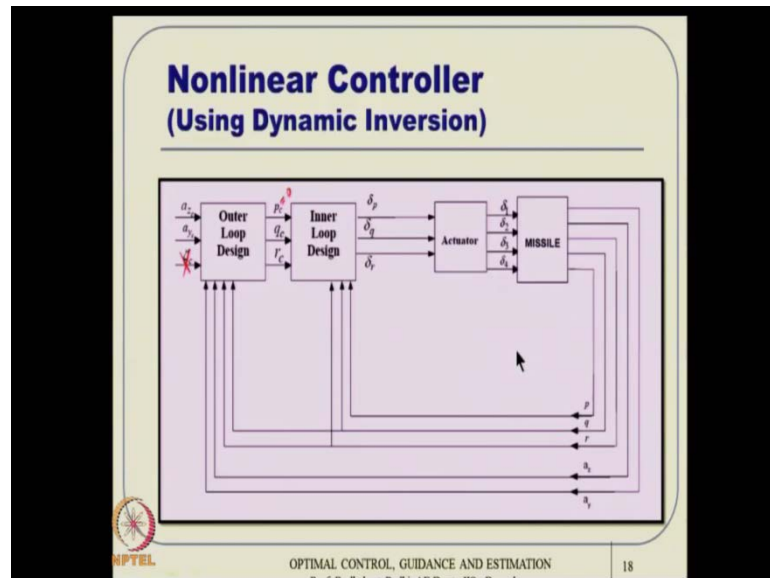
Finally, what you getting what **what** you get is going back all to the first slide, this one ZEM by T square go (Refer Slide Time: 24:33). We got some sort of estimated value of ZEM T go, so we want go back and see what we can do. Now, this one same thing remembers ZEM x y z components are computed in the inertial frame. So, if you simply put this thing that happens to be inertial component, now what we really need is body component. Finally, again use this inertial body transformation to convert this **this** components to body component.

And also remember in the terminal phase typically no lateral acceleration along x axis well not typically lateral acceleration and no lateral acceleration along x axis. So, this ward lateral probably not required because y called z lateral acceleration and is called axial acceleration here.

But, typically you can say that later acceleration along x axis cannot be realized. So, we ignore that and is other leave that fact that as long as T go estimation is good little it is also take care of that. And then the keep on note here is no log between estimation and guidance. We do all the several algebra and pass it through think like that the only motivation then finally, I do not have to realized in loops synthetics sort of thing, I want to avoid that.

So, this is what is done here, we tell body components can be realized something like this actually. **Alright**, so then this **y** a y c and a z c can be given to the **the** sort of file at logic basically.

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And that is also done here actually in other words somebody can stop here and then take only 3 D point mass module and start simulating think like that. That may be first validation of concept, what we are really talking about really validation of (()) and think like that. We have to pass it through the really auto pilate synthetic loop as well and see what is form of actually. And this is what done here it details I will not talk, I think one of the previous lecture see also talk about little bit about that.

We have this lateral acceleration coming this accelary role angle sort of thing command also coming in other words \dot{c} is nothing but, I mean \dot{z} is nothing but p . And what you are looking for this equation sort of thing is I mean this essentially we will not very, so much in that what telling is $p c$ can be, it need not coming here also for simply sit this may not be fail what here you can assume that $p c = 0$.

So, you can **you can** also do that basically, so ultimately what is need is role rate stabilization role rate is go to 0; angle of role which what particular angle it is stabilization it does not really matter that much actually. **Alright this is** this can be done this way also then you convert it outer loop and then go the inner loop control and then there is a some sort of a fin deflation logic you put it back. And then δ_1 to that actuator thing passing through with **with** rate limits and position limits also.

And finally, whatever numbers you get it come one back and then kind of simulate back it actually. This details are write and here, so just pass simplicity we given in this way sort of thing.

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**OUTER LOOP:
Command Transfer**

$$\begin{bmatrix} \dot{a}_{y_c} \\ \dot{a}_{z_c} \end{bmatrix} = \begin{bmatrix} K(a_{y_c} - a_y) \\ K(a_{z_c} - a_z) \end{bmatrix}$$

Handwritten notes: $a_x = a_x(\alpha)$, $\dot{a}_x = \left(\frac{\partial a_x}{\partial \alpha}\right) \dot{\alpha}$, $\dot{\alpha} = \left(\frac{\partial a_z}{\partial \alpha}\right)^{-1} \dot{a}_z$

$$\begin{bmatrix} \dot{\alpha}_c \\ \dot{\beta}_c \end{bmatrix} = \begin{bmatrix} a_{z_c} \frac{\partial \alpha}{\partial a_z} \\ a_{y_c} \frac{\partial \beta}{\partial a_y} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial \alpha}{\partial a_z} \\ \frac{\partial \beta}{\partial a_y} \end{bmatrix} = \begin{bmatrix} \frac{m}{QSC_{L\alpha}} \\ \frac{m}{QSC_{V\beta}} \end{bmatrix}$$

$$\begin{bmatrix} p_c \\ q_c \\ r_c \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\alpha}_c + \frac{1}{\cos \beta} \left[p \cos \alpha + r \sin \alpha \right] \sin \beta + \frac{1}{mV_c M} (-0.5 \rho V^2 SC_L - mg \cos \gamma \cos \mu) \\ -\frac{1}{\cos \alpha} (\dot{\beta}_c - p \sin \alpha - \frac{1}{mV_c M} (0.5 \rho V^2 SC_Y + mg \cos \gamma \sin)) \end{bmatrix}$$

This body rate command is going to inner loop

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Alright so, the output kalman transpose is when we do it here, it done this typically this way this a y c dot and a z c dot are computed like this first order dynamic sort of thing. And then you assume that z c is a function of alpha only, so as as dot is something like this this comes from this a z is assume to be a function of alpha only. So, as a dot is typically a del a z by del alpha into alpha dot. So, when we compute my alpha dot, this is the we can reverse compute that way this is nothing but, z dot multiplied by del alpha 1 by essentially a z dot into 1 by del a z by del alpha. So this can be denoted by (()) actually assuming quantity.

So, this is how it is computed, this is what you see here in the alpha c command actually. Alright this is how it is alpha c command you get it that way beta c is also equivalent that way, y is clearly function of for beta using there and then get it that way. And then you get p c is 0 that is what p c process 0, q c and r c can be computed from alpha c dot in beta c dot that way, details are we will not too much here actually.

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INNER LOOP: Body Rate Tracking

$$\begin{bmatrix} \dot{p}_c \\ \dot{q}_c \\ \dot{r}_c \end{bmatrix} = \begin{bmatrix} -2\xi\omega_p p + \omega_p^2 \int (p - p_c) \\ -2\xi\omega_q q + \omega_q^2 \int (q - q_c) \\ -2\xi\omega_r r + \omega_r^2 \int (r - r_c) \end{bmatrix}$$

$$\begin{bmatrix} C_{l_c} \\ C_{m_c} \\ C_{n_c} \end{bmatrix} = \begin{bmatrix} \frac{(p_c - (I_x - I_z) r_{eq}) I_x}{0.5\rho V^2 S d} \\ \frac{(q_c - (I_x - I_z) r_{eq}) I_y}{0.5\rho V^2 S d} \\ \frac{(r_c - (I_x - I_z) p_{eq}) I_x}{0.5\rho V^2 S d} \end{bmatrix}$$

$$\begin{bmatrix} \delta_p \\ \delta_q \\ \delta_r \end{bmatrix} = \begin{bmatrix} \frac{C_{l_c} - \frac{C_{l_c} \dot{p}_c}{\omega_p}}{C_{m_c} - C_{m_q} q - C_{m_a} \alpha} \\ \frac{C_{l_c} \dot{p}_c}{C_{m_c} - C_{m_q} q - C_{m_a} \alpha} \\ \frac{C_{n_c} - C_{n_r} r - C_{n_\beta} \beta}{C_{n_c} - C_{n_r} r - C_{n_\beta} \beta} \end{bmatrix}$$

This commands are decomposed to fin deflection commands and fed to the actuators.
 (passed through second-order dynamics, rate and magnitude bounds, and then combined
 back to feed into the 6-DOF dynamics for realistic simulations)

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Then once you have this **this** commanded body rates and you can go back and put this kind of dynamics now and then realized control surface deflection actually. And this components are decomposed to fin deflection commands and then fed to the actuators, if is really inertial flight you stop there you fed actuator, actuator response and then you go and flight actually. And simulation it is also passed through second order dynamics to mimic the second order dynamics.

And then it is also pass through rate and magnitude bounds and then combined back to fed into this form again that will be the realized deltas. These are **these are** commanded deltas once you take it through the that realized deltas and then those of the values that will effect to the system dynamics for simulation actually. So, this is how the simulation carried out.

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Simulation Results

Table 1. Seeker (one σ) measurement noise

$r(m)$	$\dot{r}(m/s)$	$\phi_y(deg)$	$\phi_z(deg)$	$\omega_{oy}m(deg/sec)$	$\omega_{oz}m(deg/sec)$
15	5	0.3	0.3	0.03	0.03

Table 2. Tentative Filter tuning parameter(R_h)

ZEM_X	ZEM_Y	ZEM_Z	ΔV_x	ΔV_y	ΔV_z	a_{tx}	a_{ty}	a_{tz}	T_{go}
1×10^5	1×10^5	1×10^3	200	200	200	1×10^5	1×10^5	1×10^5	0.0008

Table 3. Tentative Filter tuning parameter(Q)

ZEM_X	ZEM_Y	ZEM_Z	ΔV_x	ΔV_y	ΔV_z	a_{tx}	a_{ty}	a_{tz}	T_{go}
50	50	50	0.5	0.5	0.5	1.0	1.0	1.0	0.00001

The R matrix is chosen as square of $1\text{-}\sigma$ measurement noise as a diagonal element

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Now, coming to some results sort of thing these are simulation results r \dot{r} ϕ ϕ_z think like that. Some of these numbers are available in the **in the** paper also, see some of that actually if you really want to have an idea work kind of numbers we want to use all that.

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RESULTS

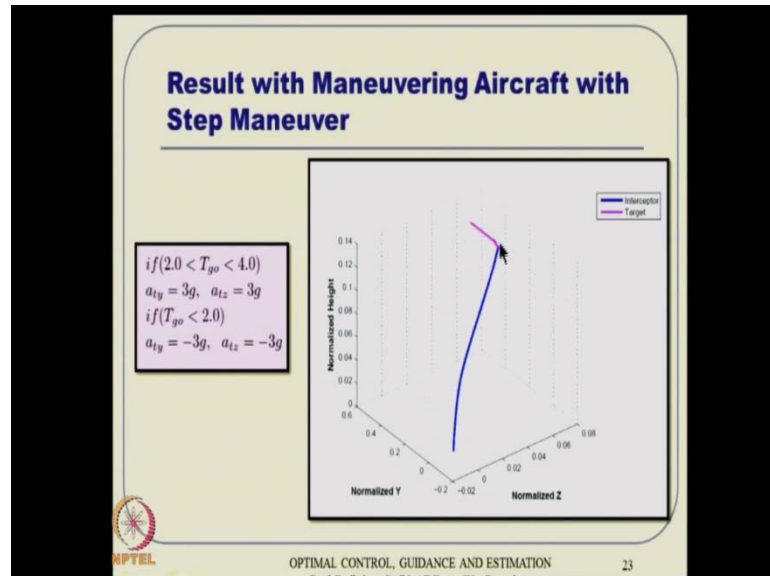
1. **Maneuvering Aircraft**
 - Step Maneuver
 - Sinusoidal Maneuver
2. **Ballistic Target**

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A result are typically done in three cases, one is the target doing step maneuver and little bit surface to send also first some sort of step. And then up to sometime reference step

and sort of thing and then we have a sinusoidal maneuver and then finally, some sort of ballistic target actually.

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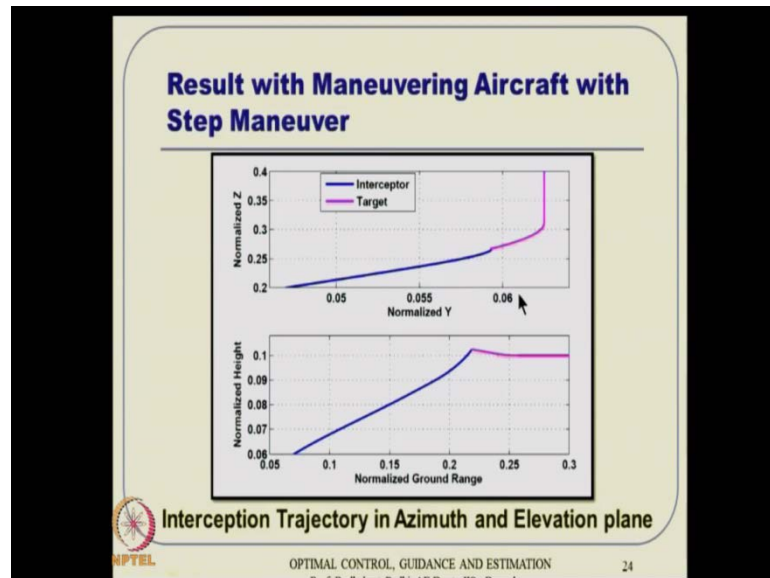
So, this is what the command is given to the target, assume that first final T go is 4 seconds at time to go is essentially 4 seconds available. So, within 2 to 4 this **this** commands are given 3 g sort of thing. And if you less than 2 **less than 2** second 2 hours the very end, target does exactly opposite actually. It kind of writes to surprise **surprise** the inertial sort of thing and these are very common inverse done by aircraft also.

So, if you do that, this is the interceptor and this is **this is** target trajectory and this is the interceptor and does not feel like a any anything that matters again the target surprise I mean surprise inverse actually. As long this some T go I mean; obviously, the very, very final moment if somebody does not I mean very close to that T T go 0 somebody does quick surprise maneuver probably that mean difficult to do.

But, you have something like it 2 seconds T go it is **is** not small, I mean it is not big it is also very small in some sense but, still we have to T go 2 seconds to go basically. Whether you **you** are estimation guidance and control response to that are not; remember all if you **if you** look it one of my previous lecture it also talks about, this problems I mean this formulations are very critical when time to go unless actually.

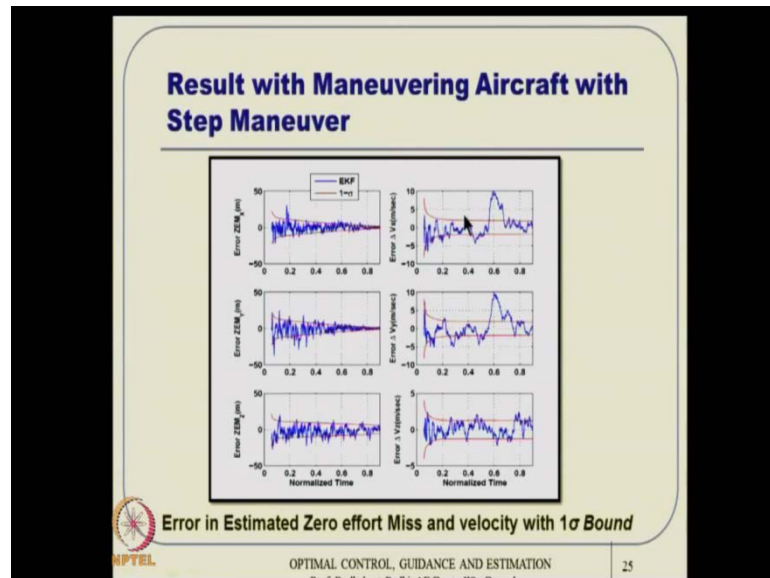
So, from here onwards T go less than 0 you can think sink of that is new problem and in this new problem that time to go really 2 seconds in this very small. So, whether it can be really do deserve the picture tells that it can be do the job actually.

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And then **this** these are the pictures in a z and y and these are pictures in z and x axis actually. So, the same things decompose into two image plane sort of thing, two image plane sort of thing and this is done that way you can see, that put towards the end here as well as the here, there meeting is very close actually that is why the mystery sense was very less.

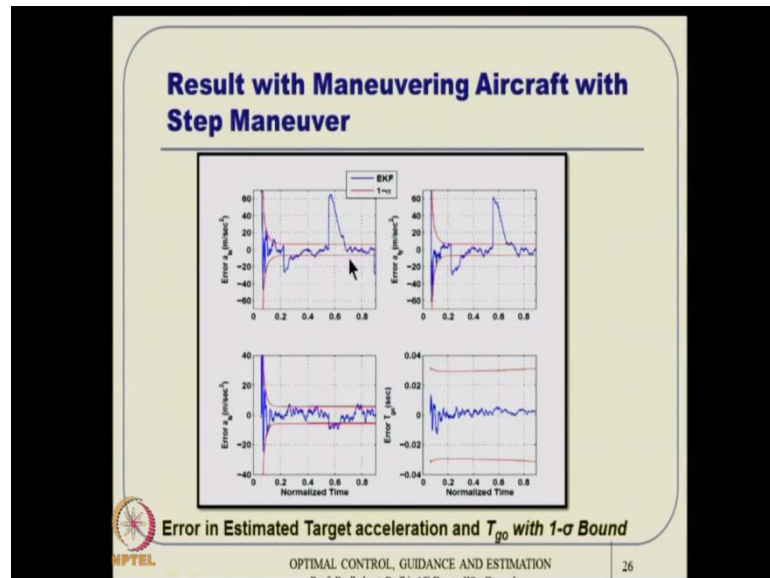
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And also we want to see the they told in kalman filter class is consistent c 1 think like that. So, we have to put this consistent check at least, so this sigma bounds sort of thing. So, you have this 1 sigma taken from p matrix and these are the bounds plus or minus square root of 1 sigma. And then ultimately the estimated values comes within that bound. So, that is the validation check these **these** ZEM's are computed very good that sense and then you can see this **this** r r delta y by delta z their also coming with the bound, momentarily they are going out that is the point where things are the target is surprise more actually.

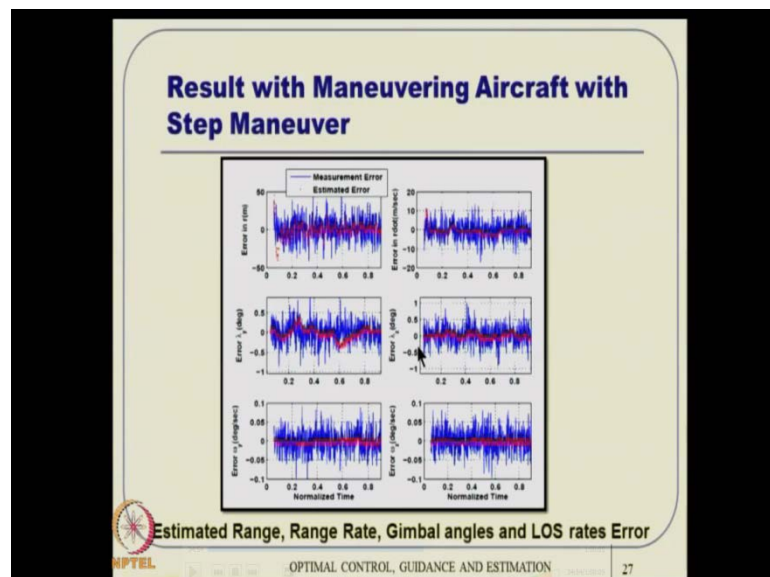
But, again within very small time comes back then your estimation is very good and then followed by response is also good. So, able to fletch the target actually. This **this** times are typically given in normalizes time we needs and then I mean, the we do not want reveal the exact time a simulation think like that. So, very I mean actuators practice whatever actually. So, do not go back the numbers in the time here, they are not absolute quantities actually.

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Alright, so this is the **this is the** point here, then also are other variables all you can see that target acceleration, here it does surprise maneuver. But, very quickly there are comes down; that means, we are able to snow, what the target is doing very quickly actually.

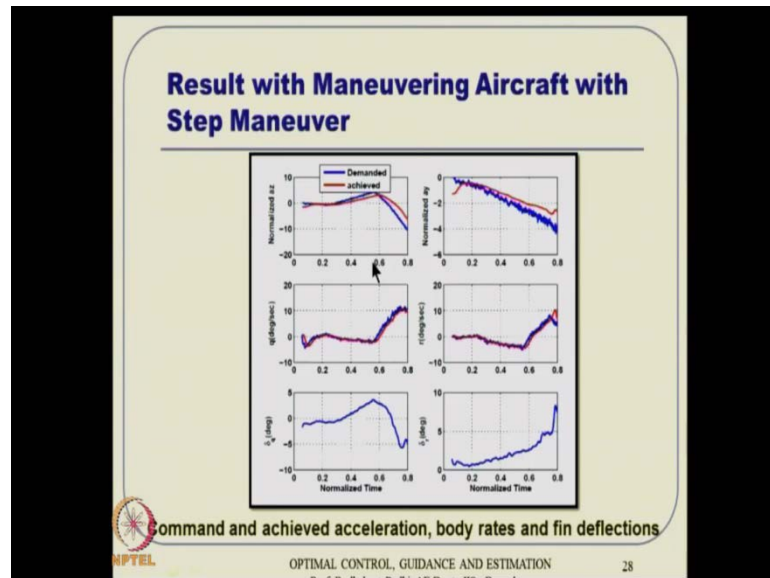
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And you can also see this measurement are inverse estimated error and think like that. So, that is another sort of test actually a kind of white test and all that actually. So, result all thing are whatever y minus the $c x$, if you remember that **that** has to be white actually.

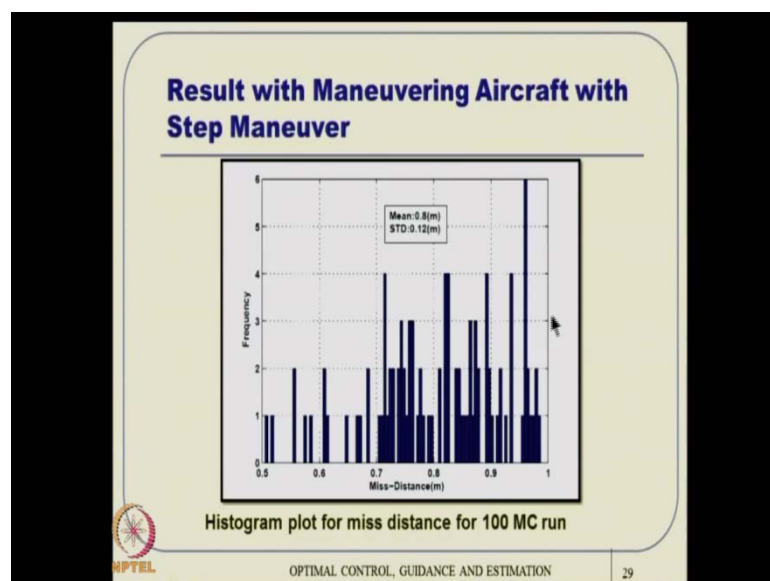
So, if you flat it should feel that took it blue line has to be some sort of white noise estimated one should be some sort of smooth curve actually; that is what is see here that validates one more thing actually.

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Now, these are the lateral acceleration, then three components and then fin deflection delta q and delta r there are bounds actually. The bounds are plus minus 30 degrees straggly there are within bounds all is able to fletch the target in a good sense.

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Now, the finally, the something like a metrical simulation; that means, radium simulation take a radium initial condition of the machine radium target initial condition think like that. And then do repeated simulation and then finally, come of some inner dynamic simulation. So, this is done for 100 runs like that, then the mean standard deviation out of the 100 text case simulation it happens to be something like this, mean is 0.8 and then this standard deviation is 0.12.

So, even if you take mu plus 3 sigma is sort of error finally. So, this happens to be 3 sigma is 0.36 **0.36** plus 0.8 is something like point well 0.8, .36 frame 2, I mean 1.1 sort of thing the 1.16 sort of thing. So, 1.1, 1.2 meters level of accuracy we are getting; that means, almost I mean if it classical aircraft target it is actually hit to actually, will be able to hit the target.

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Result with Maneuvering Aircraft with Step Maneuver

Results of 100 MC runs with perturbation cases

Case	Thrust	CN	CN _i	C _m	Mean miss distance(m)	Standard deviation miss distance(m)
1	Nominal	Nominal	Nominal	Nominal	0.8	0.12
2	+10%	+10%	+10%	+10%	1.3	0.6
3	-10%	-10%	-10%	-10%	0.7	0.5
4	-10%	+10%	+10%	+10%	0.6	0.4
5	-10%	-10%	-10%	-10%	0.8	0.4

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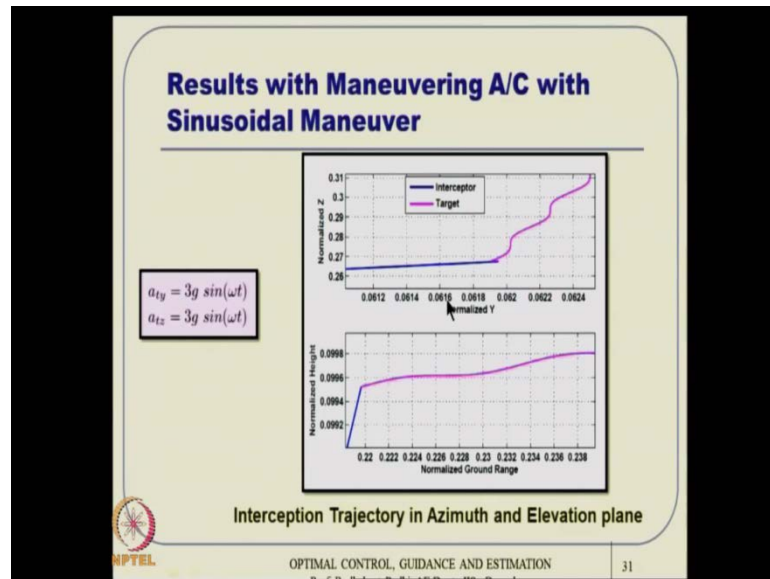
Alright, so this is what it is, then this we will also done something like radium perturbation of this **this** aero dynamic parameter actually. So, aero dynamic parameters are taken something a plus minus 10 percent and think like that and evaluated it some sort of cunal points actually, you take all this plus, plus, plus perturbation all the is kind of minus, minus, minus perturbation some plus some minus like that actually.

So, then **then** you repeat repeatedly simulated again and then take this standard mean and standard derivation. This 100 test cases are done for each other cases like this and then **then** do some sort of mean standard derivation sort of thing and it turns out to the

numbers are the are even if you take a worst cases probably that turns out to be. So, 3 I mean 3 into 0.6 1.8 plus 1.2, so 1.3, so 3.1 actually.

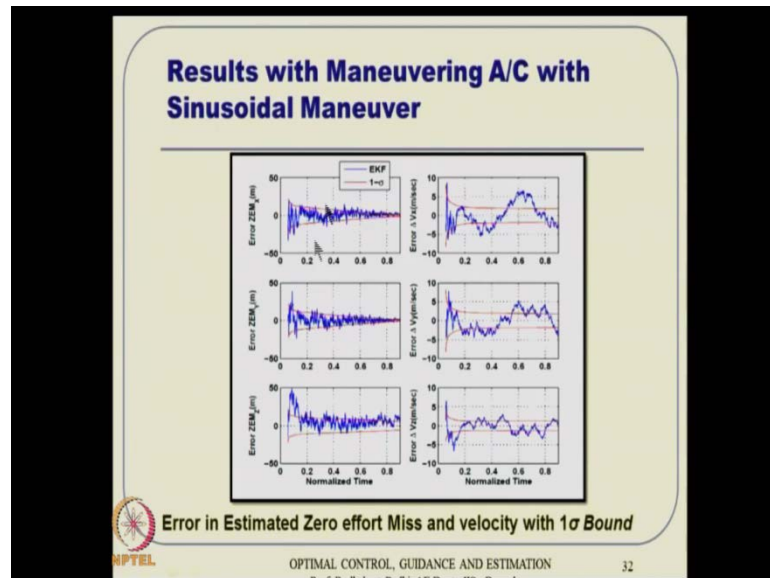
So, about 3 meters misty turns is a worst case actually, essentially the message is it works very well and it works even it time to go is very small that is point actually.

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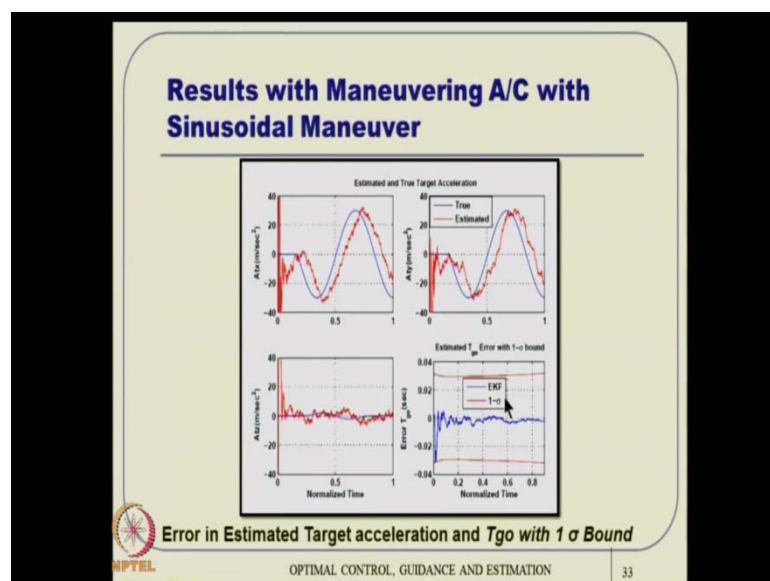
Then the last case is the next case is sinusoidal maneuver, if you does something like this what happens target comes like that what your able to do ready what the target doing to the estimation some process. And hence we are having a good enhancement, now here we can see this **this** plane it is good but, that plane is not very good initially here I mean target is here initially 0. So, some degree of error basically but, **but** also remember this is actually flat error; that means, this depression is not very good and not very high. So, that is why the accuracy is not very well actually.

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Again similar, we have here you can see all the and you go coming down what you need really remember is ultimate ZEM by T go square. So, along ZEM s estimation is very good. And T go estimation is also very good and think like will be following place actually. Obviously velocity level because the target is continuously doing some **some** sinusoidal maneuver sort of thing going this sort of that. This is not exactly boundary v^2 in 1 sigma values what if you really put 2 sigma 3 sigma bounds and it will come back to it actually.

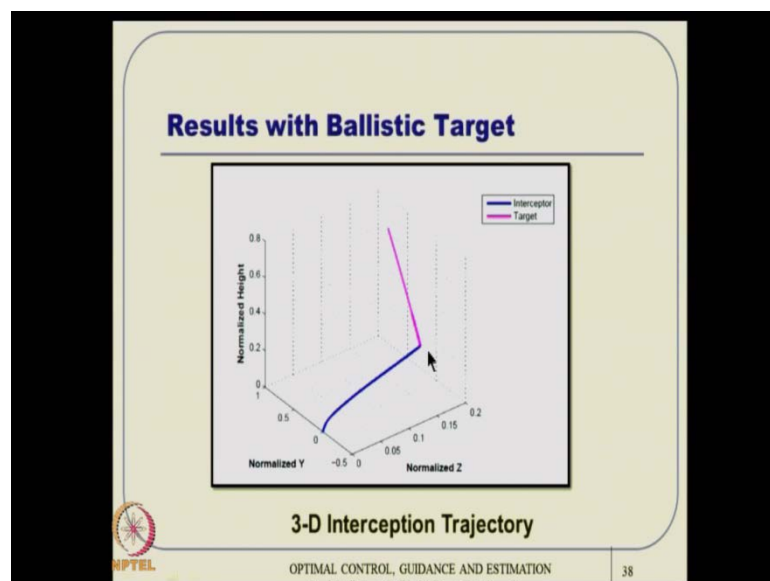
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So, these are $I_t \times I_t$ that target acceleration components $x \ y \ z$ and the T go remember T go $1 \ \sigma$ bounds good I mean. So, high, but the estimation value is, so good the estimated T go information what you getting what the actually T go is **is** very close to 0. So, here we have this ZEM components are estimated very good and T go is also estimated extremely good that is reason why everything works very nicely actually. You can similarly sort of maneuver (Refer Slide Time: 39:09) and do not want to keep on.

The commanded value or achieved value are very good that are also tells us that the auto pilates syntheses is very good. Again the mean standard even sense it is very, very small and we have this 0.32 this one is a 0.07 in this case 7 centimeter other. So, and then the level I mean confident is a very known is very good actually. Again, similar exercise and then similar results also, what we get 1.3 and 0.6 here.

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What about ballistic target, where the target comes in the striate line, but there is **some** some desolation some dragon all that about it cannot be maneuver. So, that situation this is a coming that way, this usually going in their again 2-D plane again similar sort of behavior T go estimation also good here. And that ZEM is also good here (Refer Slide Time: 40:07) and then all other things are very similar. So, and also see the **the** body rates commanded and achieved also nice, only 2 hours the end the some problem that is that is respected actually.

Later lateral acceleration is expected to change drastic towards actually (Refer Slide Time: 40:25) it because essentially the guidance operation PN philosophy. So, the one of the draw backs of in guidance 2 hours the end the **the** will lateral similarity all that actually. So, that will be here as well again mean and tunnel deviation turns out to be very small is a 0.4 times 3 sigma even if you take 0.75, 0.75 plus 0.4 something like 1.2 around that figure. So, essentially it is also good again perturbation studies and then results also good is nothing to be along and worst case of area the turns out to be around 2, 3 meters in this estimation actually.

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Conclusions

- A new Integrated Estimation and Guidance algorithm is introduced for Interceptors in Terminal Phase.
- The design implicitly uses the *time-to-go* as well as guidance parameter like *zero-effort-miss* in the estimation process to reduce the consequences of estimation errors and delay on guidance performance.
- This new IEG formulation has been applied for various targets, which demonstrated a substantial improvement in the result
- This scheme also possesses the potential to satisfy the hit-to-kill requirement.

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So, they will the small conclusion about this particular work is something like a it is a new method, new technique of integrated estimation and guidance. And the design essentially is not very difficult the only difficulty is how do put the output equation, once you understand how do put output equation in the form **of state**. I mean in the form that is required that y equal to h half x then everything will follow into place actually, all other things or other striate forward sort of thing.

Various target cases, which demonstrated and then it also tells that the miss distance satisfy this hit to kill requirement actually. This is **that part** this part of the work, now the second part, we talk about integrated estimation guidance and control, so the all the three things are taken together.

(Refer Slide Time: 41:56)

Reference

P. N. Dwivedi, S. N. Tiwari, A. Bhattacharya and Radhakant Padhi, ***A ZEM Dynamics Based Integrated Estimation Guidance and Control of Interceptors for High Speed Targets***, AIAA Guidance, Navigation and Control Conference, **2011**, Portland, USA.

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And this is subsequent follow of work this one in presented into 2011 and also just to command this both the refers we trying to kind of put it into general paper. And try to see whether you can publish in some sort of general paper in the little more police result and **some conse** small consents little more recursively actually. Let us see **that is the** that is I cannot say this two to reference is already available you can ready to actually.

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Features of Proposed IEGC

- Integration of Estimation, Guidance and '**Outer loop of Control**'
- It estimates the following 'guidance parameters' and utilizes them to compute the **necessary body rate commands** directly
 - Zero effort miss (ZEM)
 - Relative velocity (ΔV)
 - Target acceleration
 - Time-to-go (T_{go})
- Inner loop of the control is still synthesized in a separate loop: **Time scale separation between G & C loops is preserved explicitly** (similar to the concept of '**Partial IGC**')
- **Benefits:**
 - Overall reduction of loop delay
 - Optimality of the overall system

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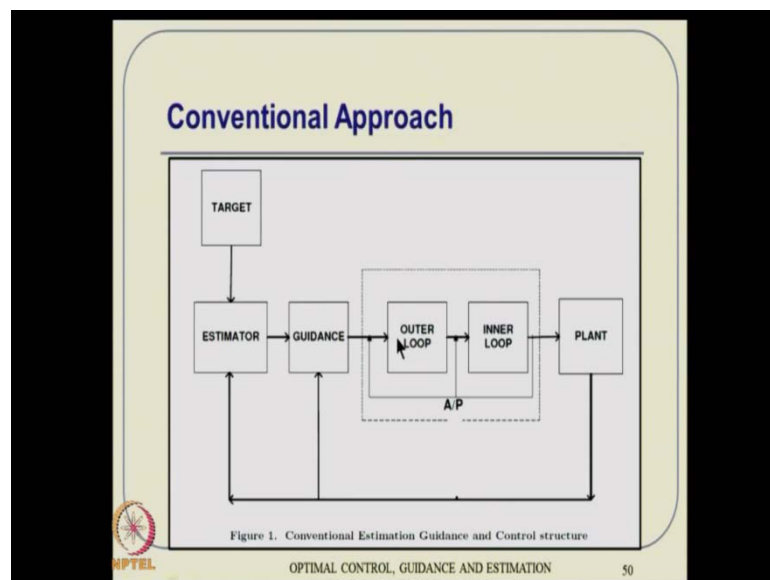
So, the features of proposed integrated estimation guidance and control is before and estimation guidance and outer loop of control **is a** is kind of used the together. And in

some sense you can also interfere the something like partially integrated estimation guidance and control. Remember the last lecture I talk about partial IEGC. So, this is something like partial IEGC basically.

So, that is what we are talking here and then something like what you are interested is the necessary body rate commands are evaluated directly. Then know lateral acceleration command we want took kind of estimate or kind of know the desired, body rate commands directly. So, the states are very similar to what you done before ZEM relative velocity target acceleration time to go. And the filtering part the problem or the estimation part of the problem remains very same is what you done before, only the **the** instead of going through a lateral acceleration generation.

And then realizing inner loop sort of thing, we want see whether we can do a little alternate way and then kind of from ZEM itself can we put it some sort of a directly body rate commands actually. Alright, so this actually borrows this idea some separation between G and C loops. So, similarly this also is retain here, because dynamic loop is not fused inner loop is still separate actually.

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Alright, so, this is what we talking here, this outer loop this inner loop and what we are talking here is this part of the design is together **alright**. So, this looks like this estimator guidance and outer loop all together and pass through inner loop and then inherit to the plant actually.

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Mathematical Formulation of IEGC

State Equation In State formulation zero-effort-miss, relative velocity, target acceleration and time-to-go have been taken as a state variable, all in 'inertial frame'.

$$\dot{X} = f(X) + G(X)U$$

$$X = [Z, \dot{Z}, \ddot{Z}, \Delta \dot{V}_x, \Delta \dot{V}_y, \Delta \dot{V}_z, a_{tx}, a_{ty}, a_{tz}, T_{go}]^T$$

Measurement Equation The measurement variables are the direct output of seeker in 'seeker polar frame' (range, range rate, gimbal angles and gimbal angles rates)

where,

$$Y = h(X)$$

$$Y = [r_m, \dot{r}_m, \phi_{ym}, \phi_{zm}, \omega_{gym}, \omega_{gzm}]^T$$

Objective: To compute the body rates directly from these estimated variables

$$Z = g(X)$$

$$Z = [q, r]^T$$

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Alright very similar I do not want to repeated all the things, these are all state equation these are frame state. So, and estimation part of the formulation remains exactly identical. So, I do not have to this is the philosophy it barross the idea of little simplification and notification instead of ZEM retain as Z. So, that represent Z represent ZEM and all that but, other things are very similar to what you done before these are this is ZEM in y the rate and relative velocity dot and T go dot actually.

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States Equations

$$\dot{Z} = \Delta X + \Delta V T_{go}$$

$$\ddot{Z} = \Delta X + \Delta V T_{go} + \Delta V \dot{T}_{go}$$

$$\begin{bmatrix} \dot{Z}_x \\ \dot{Z}_y \\ \dot{Z}_z \\ \Delta \dot{V}_x \\ \Delta \dot{V}_y \\ \Delta \dot{V}_z \\ \dot{a}_{tx} \\ \dot{a}_{ty} \\ \dot{a}_{tz} \\ \dot{T}_{go} \end{bmatrix} = \begin{bmatrix} \Delta V_x + (a_{tx} - a_{mx}) T_{go} + \Delta V_x \dot{T}_{go} \\ \Delta V_y + (a_{ty} - a_{my}) T_{go} + \Delta V_y \dot{T}_{go} \\ \Delta V_z + (a_{tz} - a_{mz}) T_{go} + \Delta V_z \dot{T}_{go} \\ a_{tx} - a_{mx} \\ a_{ty} - a_{my} \\ a_{tz} - a_{mz} \\ -\dot{a}_{tx} \\ -\dot{a}_{ty} \\ -\dot{a}_{tz} \\ -1 + (\Delta R \Delta \dot{R}) / \Delta R^2 \end{bmatrix}$$

Where
 Z : ZERO effort miss
 ΔV : Relative velocity
 a_t : Target acceleration
 a_m : Interceptor acceleration
 T_{go} : Time to go
 ΔR : Relative range

$$T_{go} = \frac{-\Delta R}{\Delta \dot{R}}$$

$$\dot{T}_{go} = -1 + (\Delta R \Delta \dot{R}) / \Delta R^2$$

where ,

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$\Delta \dot{R} = (\Delta x + \Delta V_x + \Delta y + \Delta V_y + \Delta z + \Delta V_z) / \Delta R$$

$$\Delta \ddot{R} = (\Delta R + [\Delta x + \Delta V_x + \Delta y + \Delta V_y + \Delta z + \Delta V_z] - [\Delta x + \Delta V_x + \Delta y + \Delta V_y + \Delta z + \Delta V_z] + \Delta \dot{R}) / \Delta R^2$$

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And this is all happens I mean explain just take few minutes back the this state equation can be realize this way. Now output equation I mean this the small **difu** instead of ZEM we have just re return in terms of ZEM actually that is the only deference here between this two slides actually. **Alright**, so after that the output equation exactly remains identical is before, so this **this** is also I do not have to work kind of explain too much, this is range and range rate. And then this LOS angles and rates the especially the rates will be used in **in** this formulation here (Refer Slide Time: 45:29).

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**Measurement Equation
(in Seeker Gimbal Frame)**

$$\begin{bmatrix} r_m \\ \phi_{ym} \\ \phi_{zm} \end{bmatrix} = \begin{bmatrix} \frac{\Delta x \Delta V_x + \Delta y \Delta V_y + \Delta z \Delta V_z}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}} \\ \tan^{-1} \left(\frac{n}{-l} \right) \\ \tan^{-1} \left(\frac{m}{\sqrt{l^2 + n^2}} \right) \end{bmatrix} + \begin{bmatrix} \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}$$

Where
 r_m = range
 ϕ_{ym} = gimbal angle along y
 ϕ_{zm} = gimbal angle along z
 $\dot{\omega}_{ym}$ = gimbal angle rate along y
 $\dot{\omega}_{zm}$ = gimbal angle rate along z

Unit LOS vector in body frame (l, m, n) is

$$\begin{bmatrix} l \\ m \\ n \end{bmatrix} = C_b^t C_t^b \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \phi_y \cos \phi_z \\ \sin \phi_y \\ -\cos \phi_y \sin \phi_z \end{bmatrix}$$

C : Transformation matrix

$$\begin{bmatrix} \dot{\omega}_{ym} \\ \dot{\omega}_{zm} \end{bmatrix} = C_b^t C_t^b C_t^b \begin{bmatrix} -\dot{\lambda}_y \sin \lambda_y \\ \dot{\lambda}_y \\ \dot{\lambda}_y \cos \lambda_y \end{bmatrix} + \begin{bmatrix} \eta_5 \\ \eta_6 \end{bmatrix}

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So, the finally, you have this range and then the range rate and the gimbals angles and then the rates actually.

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IGC&E Formulation: Outer loop

➤ Enforced Error Dynamics: $\ddot{Z} + 2\zeta\omega_n\dot{Z} + \omega_n^2 Z = 0$

➤ However,

$$\ddot{Z} = \Delta\dot{V} + \Delta\dot{V}T_{go}^i + \Delta\dot{V}T_{go} + \Delta\dot{V}T_{go}^i + \Delta\dot{V}T_{go}^i$$

$$\ddot{Z} = (\dot{a}_t - \dot{a}_m)T_{go} + \Delta\dot{V}(1 + 2T_{go}^i)$$

➤ Hence, from the enforced error dynamics,

$$\dot{a}_m = \dot{a}_t + \frac{(2\zeta\omega_n\dot{Z} + \Delta\dot{V}(1 + 2T_{go}^i) + \omega_n^2 Z)}{T_{go}}$$

➤ Note: This has three components

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Then here is a new concept after getting the Z E M we do not want to kill generate P N sort of guidance ZEM by T go square and all that actually. What we well the one more small command before I go actually no **no** it is **alright** we need this state I mean this estimation part of the formulation remains identical. Then what you need is instead of going through this lateral acceleration generation and all that, we want enforced this sort of error dynamics. And this part is derived kind of motivated from **from** dynamic inversion few back in philosophy actually.

So, we want to put finally, remember what is the guidance problem guidance problem essentially talks about Z E M going to 0 as T goes to or T goes to 0 basically. So, this equation if you **if you** put it that I mean if you put it that way, when T go goes to 0 T go goes I mean T goes to T f. And if you assume T f is infinity I mean the infinity subjected to the selection of guidance and all that if you guidance also very high, even per second can infinity really. So, if you assume that here T f is **is** high, so that my gains are very high. So, that what about T f is there that can be interrupted is infinity.

Then this equation can **can** be put it I mean you can **can** we put. So, that when T goes to infinity; that means, T goes to 0, T goes to infinity means T goes to T f; that means, T go goes to 0. So, when that T go goes to 0 then Z and Z dot also will go to 0 all that and just Z Z will go to 0 that is are objective what in this **this** one Z will go to 0 and Z dot also go to 0 basically. So, that is and all that degree of good thing any **any any** inaccuracy of T

goes to estimation suppose small error there and because $Z \dot{}$ also 0 it will not blow suddenly.

So, Z will be remain to be 0 for time it actually so; that means, small amount of and certainly t estimation will also already here. **Alright**, so coming back to this **this** is how it is this **this** equation is what we want enforced where ω ζ these are parameters essentially this is one gain and this is two gain I mean another gain. Position gain rate gain sort of thing that way, then $Z \ddot{}$ can be I mean you have got this **this** $Z \dot{}$ equation from here you put one more derivative, $Z \ddot{}$ and estimated and all that actually **alright**.

So, $Z \ddot{}$ is $\delta v \dot{}$ plus all these can be taken, but here it come of cross the T go $\ddot{}$ also and that is also what here assuming to be 0 that need not to be 0, but here assuming to be 0. And that assumes that $T T$ go is really does not change that basically it can change the changes also captured into $T \dot{}$ but, t go $\ddot{}$ level we do not capture it is assume to be 0. Then the within simplification and assuming and knowing that $\delta v \dot{}$ is nothing but, these we can actually put it $Z \ddot{}$ is something coming from here. **Sorry** this one **this one** is nothing but, that and coming like this and then these two terms combined rate to that way actually.

Now, remember it is not a t minus a m it is a $t \dot{}$ minus a $m \dot{}$ actually. So, from this equation, we can that a $m \dot{}$ is something like this and then a $m \dot{}$ is a is something lateral acceleration rates sort of thing.

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IGC&E Formulation: Outer loop

>Next

$$\begin{bmatrix} \dot{\alpha}_c \\ \dot{\beta}_c \end{bmatrix} = \begin{bmatrix} a_{mzb} \frac{\partial a}{\partial z} \\ a_{mgb} \frac{\partial \beta}{\partial y} \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} a_{mzb} \\ a_{mgb} \\ a_{mzb} \end{bmatrix} = C_i^b \begin{bmatrix} a_{mz} \\ a_{my} \\ a_{mz} \end{bmatrix} \quad \begin{bmatrix} a_{mzb} \\ a_{mgb} \end{bmatrix} = \begin{bmatrix} QS(C_{y\alpha} a + C_{y\beta} \beta) \\ QS(C_{y\alpha} \beta + C_{y\beta} a) \\ m \end{bmatrix}$$

>However

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} q - \frac{1}{\cos \beta} [(p \cos \alpha + r \sin \alpha) \sin \beta + \frac{1}{mV_M} (-0.5\rho V^2 SC_L - mg \cos \gamma \cos \mu)] \\ p \sin \alpha - r \cos \alpha + \frac{1}{mV_M} (0.5\rho V^2 SC_Y + mg \cos \gamma \sin \mu) \end{bmatrix}$$

>Hence

$$\begin{aligned} q_c &= \dot{\alpha}_c + \frac{1}{\cos \beta} [(p \cos \alpha + r \sin \alpha) \sin \beta + \frac{1}{mV_M} (-0.5\rho V^2 SC_L - mg \cos \gamma \cos \mu)] \\ r_c &= \frac{1}{-\cos \alpha} [\dot{\beta}_c - p \sin \alpha - \frac{1}{mV_M} (0.5\rho V^2 SC_Y + mg \cos \gamma \sin \mu)] \end{aligned}$$

> Note: Roll rate command (p_c) is chosen to be zero.

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And this is three components again and these three components what we see here or can be, so remember this is what is used in the **in the** inner loop of the auto pilate syntheses a t m dot. I mean this is this is what we purposefully is generated assuming z is a function of purely alpha a y is function purely beta. Now it is actually directly available from this expression sort of thing. So, you can use it and then alpha c beta c all the generated that way and then you have this generated body rates that q c and r c actually.

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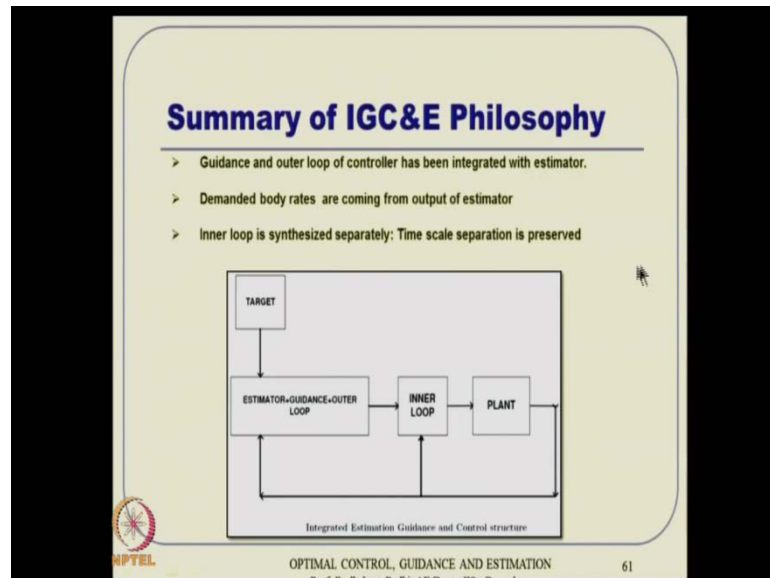
Inner Loop: DI Based Nonlinear Autopilot

For tracking of demanded body rate feedback linearized Dynamic Inversion Autopilot has been used as:

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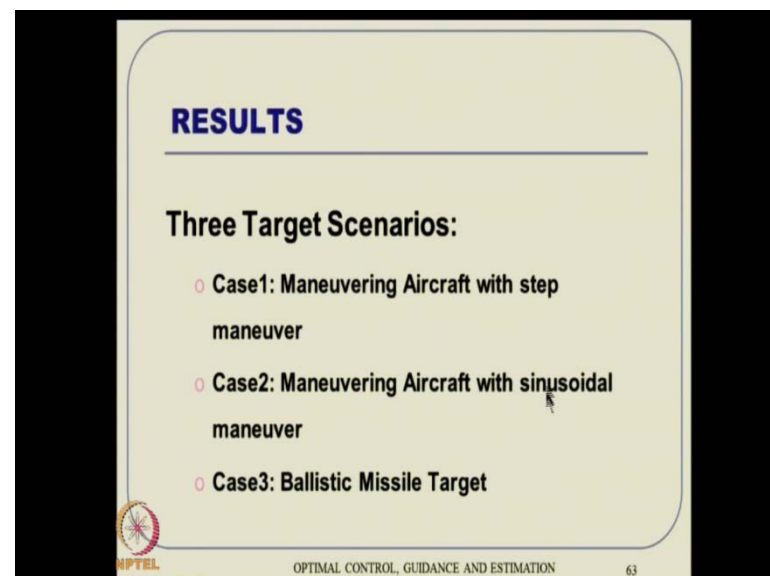
And again similar things are there a little bit lightly different picture are takes a little bit more details of about auto pilate in sort of thing it is a populated here. So, more and detail more detail can be found in the paper let me not, I mean explain too much here.

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Alright summary of this this I G C and E, I mean Integrated Guidance and Control and Estimation what we are talking or I G C also some people called different ways. What about is summary of philosophy is like this out, guidance and outer loop of controller is a integrated as well as estimation is also basically, that is the bottom line actually.

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So, results are something like this, again results are a generated using six- DOF platform we have second order exo dynamics here. And for sensing the body rates and acceleration a second order gyro and accelerometer model also used. And then realistic seeker model has been used for generating seeker measurements as well. So, all this things are part of the six-DOF simulation actually; six-DOF simulation platform again the same cases aircraft maneuvering the step maneuver and then sinusoidal maneuver and then finally, ballistic target.

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P₀, Q & R matrix

Table 1. Seeker (one σ) measurement noise

$r(m)$	$\dot{r}(m/s)$	$\phi_y(deg)$	$\phi_z(deg)$	$\omega_{gy}(deg/sec)$	$\omega_{gz}(deg/sec)$
15	5	0.3	0.3	0.03	0.03

Table 2. Filter tuning parameter(P_0)

Z_X	Z_Y	Z_Z	ΔV_x	ΔV_y	ΔV_z	a_{tx}	a_{ty}	a_{tz}	T_{gp}
10^3	10^3	10^3	200	200	200	10^5	10^5	10^5	0.0008

R matrix :
Diagonal matrix with square of 1 σ measurement noise

Table 3. Filter tuning parameter(Q)

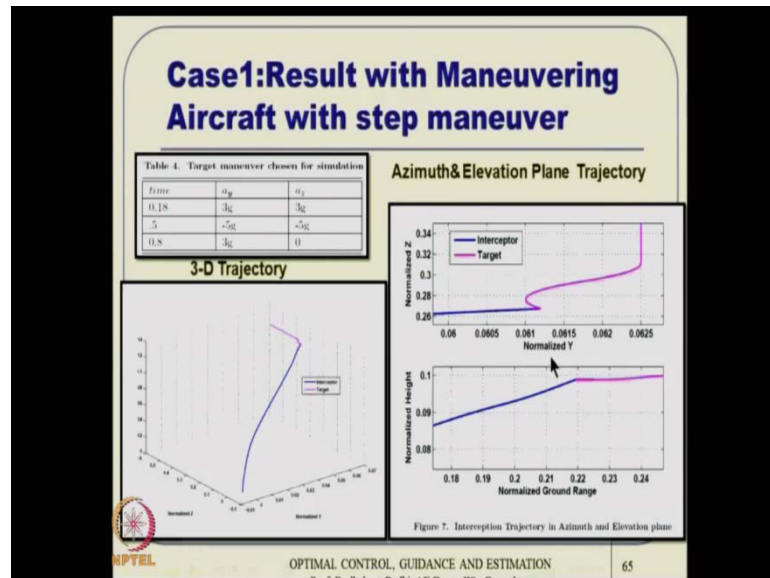
Z_X	Z_Y	Z_Z	ΔV_x	ΔV_y	ΔV_z	a_{tx}	a_{ty}	a_{tz}	T_{gp}
50	50	50	0.5	0.5	0.5	1.0	1.0	1.0	0.00001

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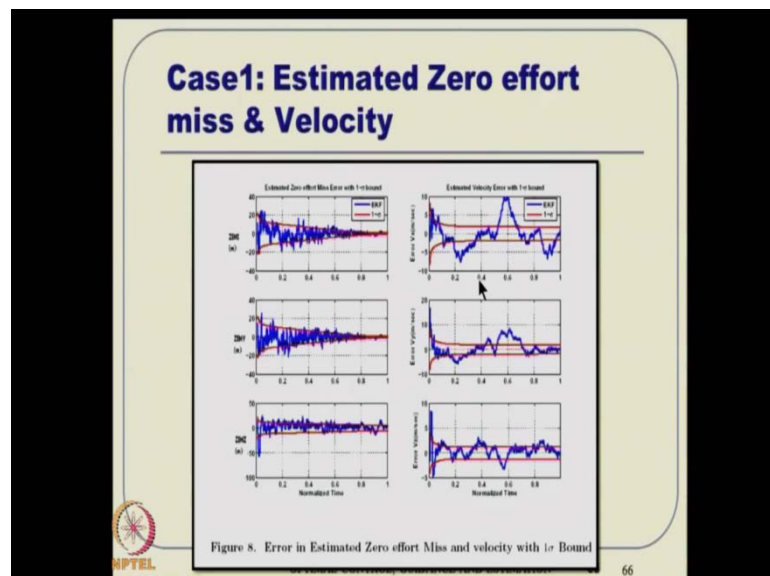
So, how does the thing P this P not Q R can be slightly different then what we **what we** have as much of possible mostly similar values.

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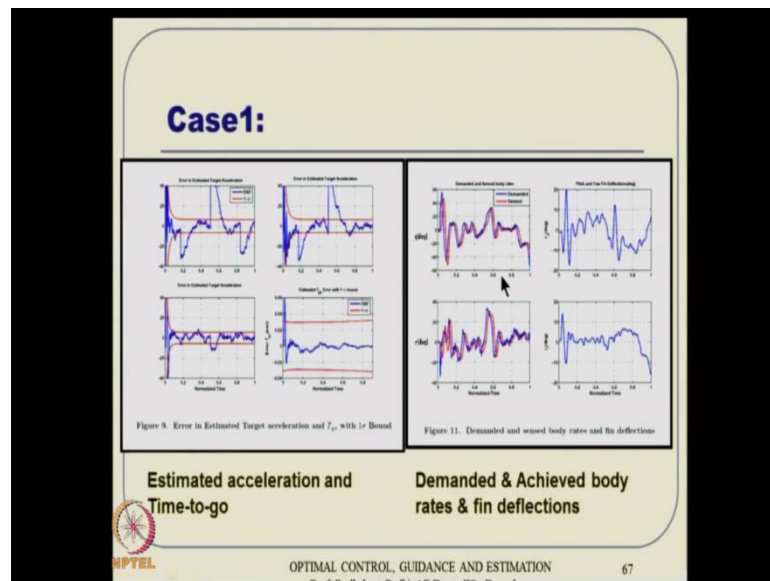
And then these are the results actually, we have this similar case of target comes with one acceleration some sort of 2 hours and then acceleration and all that actually. And here it is you can see two **two** times it changes actually one time it changes here initially, then after some time it changes some value. And then two words very handy it also another value actually, so that all that thing are happening but, still you have able to capture that actually. So, anyway this is output is **alright**.

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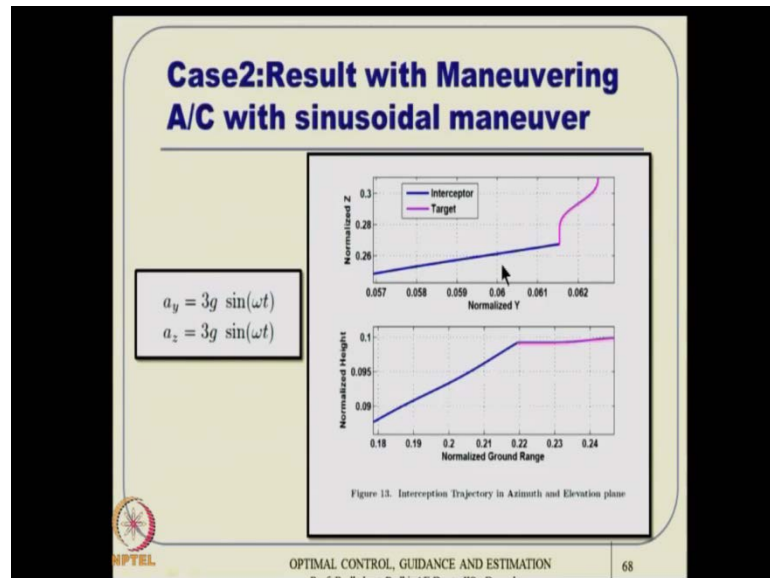
So, this is output is the results are like this here Z E M x y z components are estimated again very well and this **this** comes momentary out, because a target acceleration here as well as something like here. So, we have this **this** things given anyway this values are see normalize time again. So, the actual time can be out of 3, 4 second what we see here is normalize values that actually. Anyway that does not matter what it matter is there are double surprises towards the end but, the still able to get it here very good.

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We can see how this body rates are followed; that means, the sort of pilot design is also good. This is small amount of delay here, that delay does not matter too much a in **in** a estimation in finally, estimation actually.

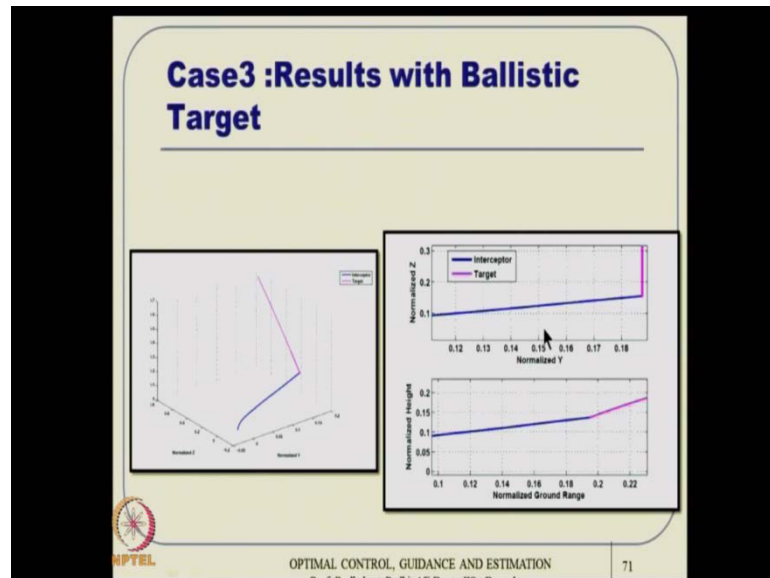
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Then finally, this n sinusoidal maneuver, I mean the next this sinusoidal maneuver a similar maneuver sort of thing, again the results good ZEM estimation is extremely good here. Almost 0 in this component, this component at least only z direction some value **which is** which results in some estimation actually.

Alright, so but remember this x y z is not body x y z, these are all inertial x y z then you converted back to body components and then that happens automatically formulation and all that actually. So, this is what it is (Refer Slide Time: 53:32) and then results are something like this you can see that the finally.

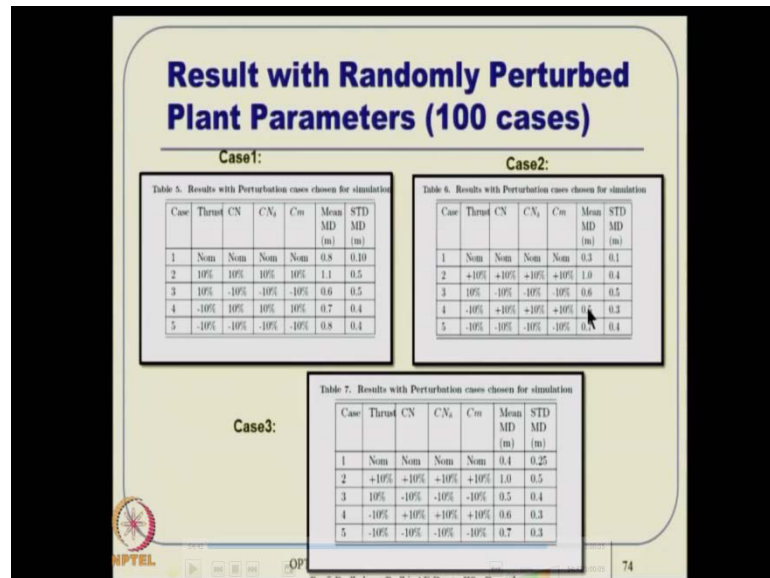
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The experiment with respect ballistic target, ballistic target comes like that that one before it cannot maneuver, but this a big amount of deceleration. So, as per as target I mean the problem of inertial guidance is conceded it accuracy maneuver target when target deceleration happens acceleration also. And 3-D sense it is actually nothing but, maneuver it has it sees deceleration in the acceleration anyway basically.

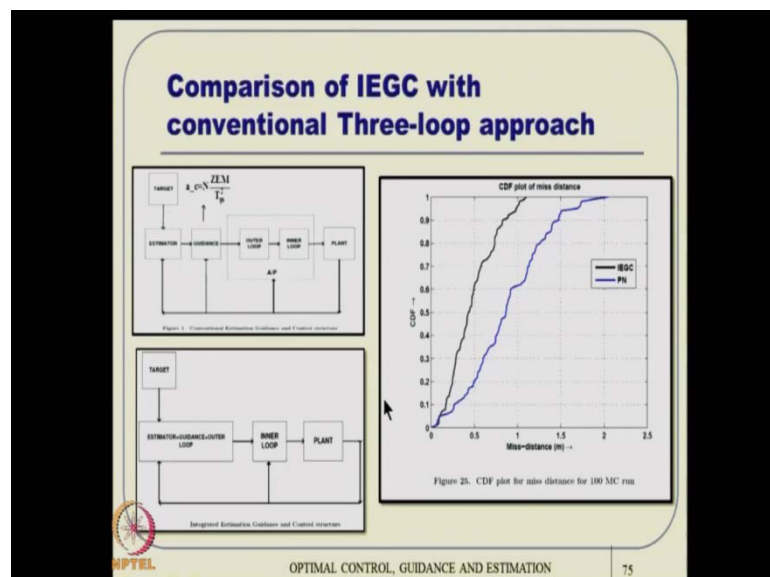
So, even then it is able to capture that in a good sense, this also you can see this is no difference usually you cannot say any difference between both the, I mean both the two plot actually.

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So, these are the results and then finally, going to the metrical simulation sort of thing again, you can see that mean and standard deviation values are 1.1.5 in then 1.4 like that actually here. We can take 1.1.5 is the worst case maneuver, 1.5 is 1 sigma 3 sigma is 1.5 1.5 plus 1.0 is to 1.6 sort of thing. So, that happens to be the worst case this system actually within the scenario consider in the all this simulation actually.

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And this is a histogram sort of **sort** picture that typically give from larges simulation sinusoidal and all if you get 100 metrical runs. This is what will get in conventional

design; that means, you are guaranty to capture everything within 2 meters miss distance say, but here the miss distance will reduce to 1.1 sort of thing actually.

So, this picture tells us comparatively how **how** much benefit will get by incorporating this integrated design of process actually. So, some reasons this is what you see here, this is the conventional approach very outer loop, inner loop sort of thing guidance is computed that way. And this not fused I mean if you fuse it then this is IEGC inertial happen here is not fuse actually. So, this is conventional way of doing thing, this is integrated estimation guidance and anyway doing of things actually.

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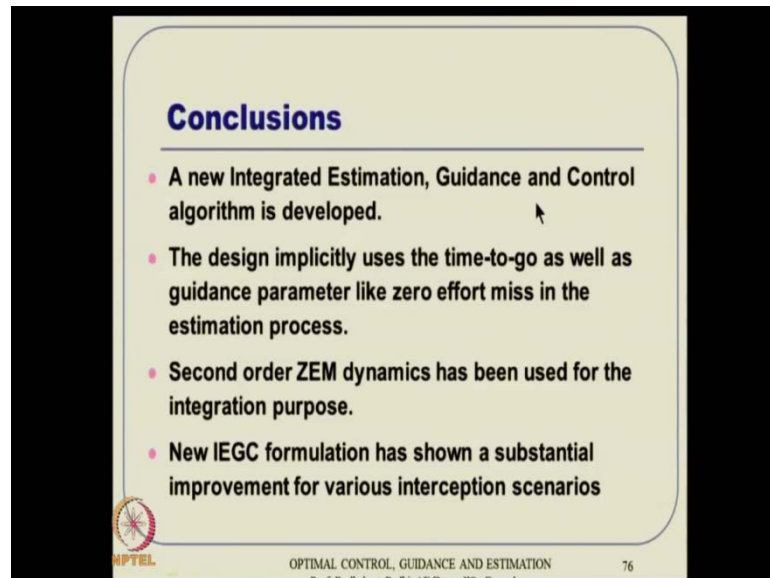
Conclusions

- Integrated designs are more natural to the flight vehicles
- Integrated designs bring more synergy between various subsystems
 - Necessity of having a compatible point mass equation in parallel is completely avoided
- Integrated designs lead to better performance in general.
- Integrated designs can be proposed following various philosophies: IGC, IEG, IEGC etc.

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So, conclusions essentially we saw also knew idea a new integrated estimation guidance and control algorithm. And that is what we T go, but also another literatures various literatures a few from **from** varies people, you can see that different ideas are looking of for this design approach and essentially the second order Z E M dynamics is key here. So, if you go back the formulation part of it what was the difference, the main difference is this one (Refer Slide Time: 56:21). Once you have this and then **then** compute that way that rest of the thing you compute directly using this components actually; this components I mean then take body components and all that actually. **Alright**, so that is the three differences actually.

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Conclusions

- A new Integrated Estimation, Guidance and Control algorithm is developed.
- The design implicitly uses the time-to-go as well as guidance parameter like zero effort miss in the estimation process.
- Second order ZEM dynamics has been used for the integration purpose.
- New IEGC formulation has shown a substantial improvement for various interception scenarios

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Alright so, then this new IEGC formulation has shown like substantial improvement for various interception scenarios. And also remember one body can deal from 2 meters can kind of 1.19 meters, well that is **that is** the real story. I mean the going close to that is suppose somebody wants let us say 20 meters mystery that is a very dual problem in a case. That is standard it is taken **(())** the it can be done, but running closer and closer you want really hit to kill sort of behavior.

And you want to view within this kind of 1 meter level accuracy 2 meter level accuracy like that the problem gets tougher and tougher see here, what you say in the blue line the conventional way, but conventional way also contains non-linear auto filter that actually remember that.

If you **if you** put the classical auto filter it still happens to be 5 6 meter and we need also talks about innovations way inner rather way of filtering, I mean is if you do not have that again go for classical filtering sort of thing mystery can be the outer matters. So, all these benefit if you put good filter and good training and put non-linear auto filter their and then that is leads to 2 meter, now every into IEGC concepts and all then you can ask 1 meter.

So, think like that it is not forget actually as you go closer and closer the balcony is much **much much** more difficult actually. So, this is **this is** what we getting advantaged by invoking this integrated and all of course, actually. And especially these are important

when T goes is very small, target surprise many verse of the end, which are also true for integration in away because towards the very end the when the inertial is a in atmosphere we are talking about end of atmosphere intersection. Then the target may not be doing intentional maneuver, but it can just happen, because of the physics that the target can go through maneuver, which is also surprise that actually.

So, this consideration can motivates to do more and more and then extract every piece of thing that you can extract from problem really actually. Modern control theory, modern optimal control, non-linear control everything comes into picture estimation theory everything comes into picture to realize this **this** dream of going hitting the target actually many were.

So, in a broad sense the broad conclusions something like this integrated designs are typically more natural to the flight vehicles, because flight vehicle does not understand what is point mass all the **the** physics happens the integrate I mean in the detail level only. So, the integrated designs are typically more natural six-DOF dynamics is directly used. And that typically derive the lot of synergy various subsystems and the necessary of having a compatible point mass equation is avoided integrated designs lead to better performance in general that we seen that also.

And integrated designs can be proposed following various philosophies, it is integrated guidance and control, integrated estimation guidance as well as integrated estimation integrated estimation guidance and control all together sort of thing. so, this is **this is** what it is more and that you can see again in this two papers and various other paper that other paper available papers actually. So, that comment I will stop here thank you.