

Optimal Control, Guidance and Estimation

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Module No. 13

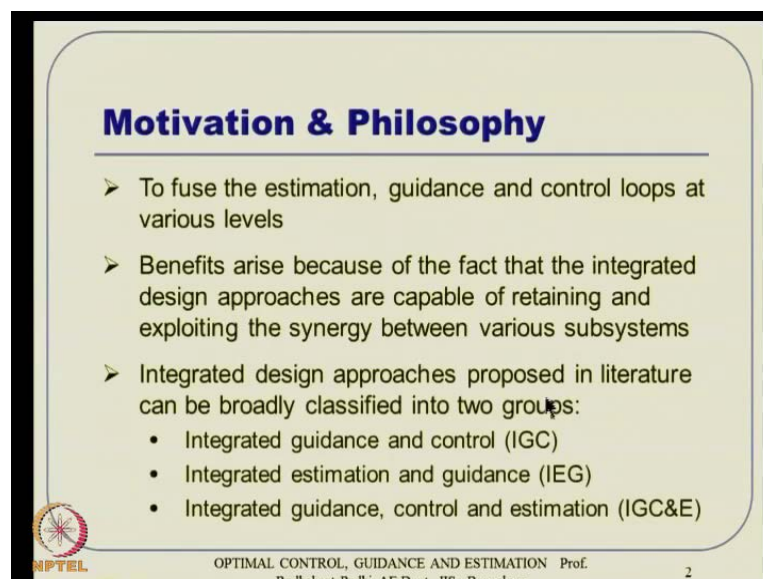
Lecture No. 31

Integrated Estimation, Guidance & Control-1

Hello everybody **we will** we have seen concepts of Optimal Control and also seen concepts of Estimation. Especially kalman filter and some external kalman filter and unscented d kalman filter think like that. And now we will migrate to a slightly advanced topic in aerospace engineering of course, which talks about Integrated estimation Guidance and Control. That means all these concepts that are available bits and pieces estimation itself guidance itself and control itself can we really talk some sort of in a integrated fashion.

And then, try to get some advantage out of it **actually**. So, the next couple of lectures I will talk about that at various level something like that; and primarily these are out of our room research **actually**, so some of our reference will be our own publications as well plus they started.

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Motivation & Philosophy

- To fuse the estimation, guidance and control loops at various levels
- Benefits arise because of the fact that the integrated design approaches are capable of retaining and exploiting the synergy between various subsystems
- Integrated design approaches proposed in literature can be broadly classified into two groups:
 - Integrated guidance and control (IGC)
 - Integrated estimation and guidance (IEG)
 - Integrated guidance, control and estimation (IGC&E)

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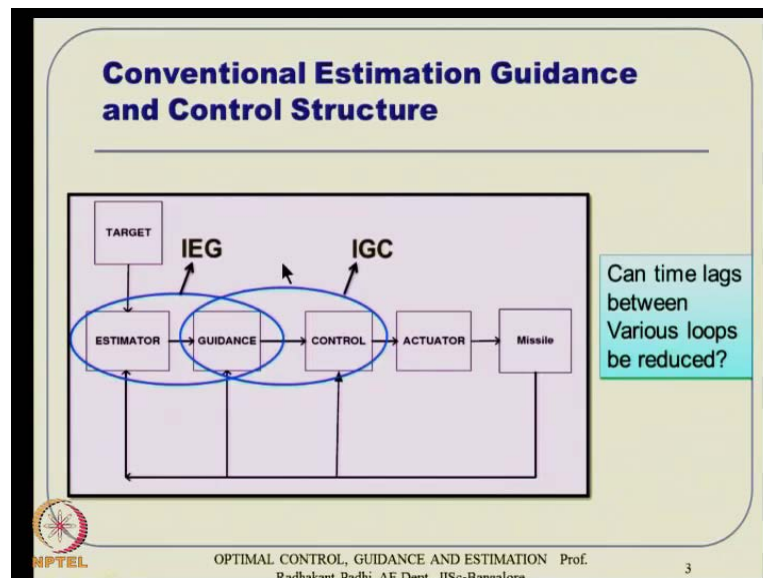
So, this motivation and philosophy like this, what your interested is to fuse the estimation guidance and control loops at various levels. And benefits arise, because of the fact that integrated design approaches are capable of retaining and exploiting the synergy between various subsystems.

So, ultimately its finally one system, so if you separate it out and then put estimation as a subsystem guidance subsystem like that. And ultimately everything has to act on the same system finally, and then if you **if you** do not exploit this synergy between them then, unnecessarily there are transient effect, there are large delay effect things like that **actually**. So, can we kind of separation of those by invoking some of these concepts **actually**.

So, the integrated design approaches proposed in literature can be broadly classified into two groups. And primarily it is integrated guidance and control or integrated estimation and guidance but, they also concepts of integrated guidance control and estimation together.

And popularly it is a IEGC some people call some people call as a IGC and E. So, Integrated Guidance Control and Estimation, **actually** could depends anyway, so the concept is more important.

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So, let us see what is going on here, so typically when you and these are largely in the context of missile guidance. Whereas, you can talk about other systems as well we have **actually** as to be experimented some of these concepts of IGC and think like that in u a v's and all that here but, here the primary focus will be on missiles **actually**. So, when we talk about missile guidance and control, we also need some information about target **actually**.

So, that target information is typically is captured either through radar or seeker, so we have a some sort of a target estimation loop. And this target estimation loop also talks about missile guidance, so position and think like that position awareness detective. So, that the relative error between the in this target and missile kind of gets estimated in all that **actually** and that is what it is fed to the guidance. Guidance give some lateral command and think like that which eventually goes to the control, control can have it is own loops, which are not putting in this log diagram.

It can have two three loops by itself, then it will spinally go to actuator; actuator will give some command of in deflection. If in deflection is something that couples with the missile aerodynamics and hence it orientation changes its force moments change and then it corrects it is path **actually**, so this is all what it happens. So, when somebody talks about estimation it typically ignores all the things inside this, guidance control like actuator all that, one talks somebody talks about guidance ignores everything else control similarly, think like that **actually**.

So, now the question is can the time log between various loops we reduce because the moment we talk about loops after loops after loop, what happens is inner loops should be sufficiently faster than the outer loop. In other words outer loops has to be sufficiently slow the meaner loop. So, purposefully this one has some actuator bind width and all that this control design what you are talking about has to be slower than that, guidance has to be even slower than that. And estimator has to be even further slower than that **actually**; in error closing sense basically that is in a.

So, the question here is can you really not do some guidance and control together because, if you see this the guidance is typically done based on point mass or kinematic level to some extent. Whereas, control design is typically done using this 6 top equations of motion **actually**. Now, the question is six top equation of motion is used in control

design in anyway, in any case then six top contains the position in velocity information also basically. So, why using a separate dynamics for guidance and then invoking some approximate dynamics and all that **actually**.

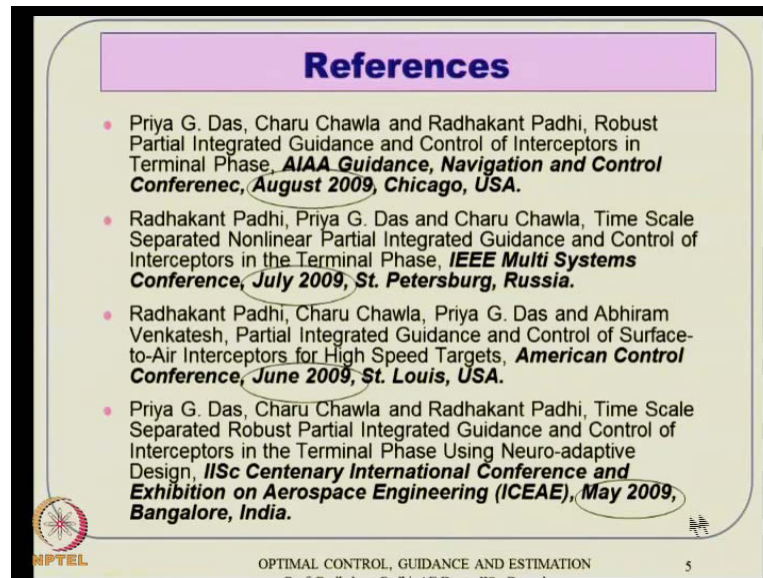
So, that is probably not needed that whole idea if you try I mean people have thought about and then try to combine these two loops and then concepts are something called integrated guidance and control starts appearing. Then some people think about well we can also do this way, we can combine estimated and guidance, because guidance essentially talks about target information all the time.

So, estimation in guidance why talking separately you can talk together so that that leads to this integrated estimation in guidance. And some people talk about you think together I mean this estimation guidance control at different level fusing, them together and that relates to this IEGC concept integrated estimation that is in control .

So, this IEG and IEGC will see that the subsequent lecture but, today we will talk about this fusion IGC integrated guidance control. And especially we **talk the** talk a take, I mean method that we have propose recently **called what we call** we are calling as partial integration guidance and control. So, alright so this is as a topic of discussion today primarily, because if you are interested in talking about I mean or learning about various things and IGC there are several literatures available.

Obviously it is not possible for me to discuss many of the concepts that is this is appeared, we can interested you can read yourself. But, one of the concepts that is eliminated from our own lab that is what we call is partial integrated guidance and control that is what I am going to talk in this lecture **actually**.

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References

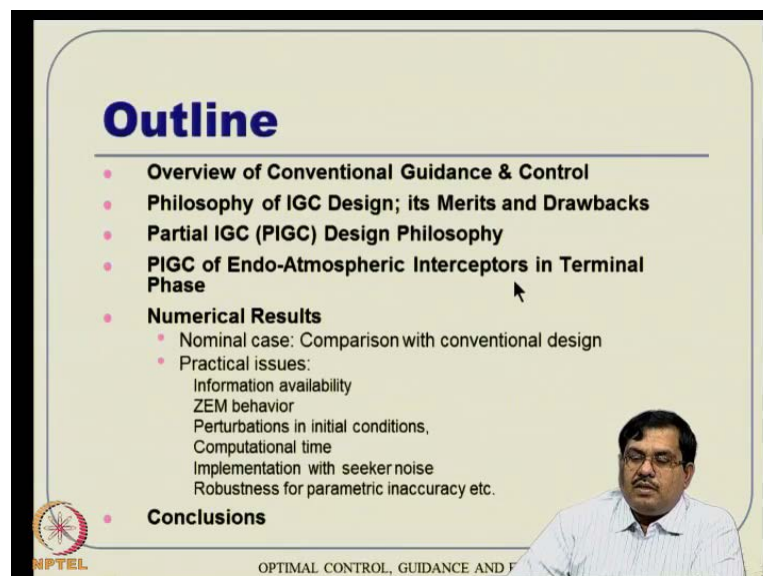
- Priya G. Das, Charu Chawla and Radhakant Padhi, Robust Partial Integrated Guidance and Control of Interceptors in Terminal Phase, **AIAA Guidance, Navigation and Control Conference, August 2009, Chicago, USA.**
- Radhakant Padhi, Priya G. Das and Charu Chawla, Time Scale Separated Nonlinear Partial Integrated Guidance and Control of Interceptors in the Terminal Phase, **IEEE Multi Systems Conference, July 2009, St. Petersburg, Russia.**
- Radhakant Padhi, Charu Chawla, Priya G. Das and Abhiram Venkatesh, Partial Integrated Guidance and Control of Surface-to-Air Interceptors for High Speed Targets, **American Control Conference, June 2009, St. Louis, USA.**
- Priya G. Das, Charu Chawla and Radhakant Padhi, Time Scale Separated Robust Partial Integrated Guidance and Control of Interceptors in the Terminal Phase Using Neuro-adaptive Design, **IISc Centenary International Conference and Exhibition on Aerospace Engineering (ICEAE), May 2009, Bangalore, India.**

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The primary references of these or several conference papers that appeared in 2009, I am interestingly appeared in sequentially may June July and August 2009. But, various level, so various techniques and various issues, etcetera and think like that and if you really have interested you.

So, just that you can **you can** read some of these literatures for more detail explanations and all that. We also in this process of writing some general paper and all but, so far it is not appeared anywhere **actually**.

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Outline

- Overview of Conventional Guidance & Control
- Philosophy of IGC Design; its Merits and Drawbacks
- Partial IGC (PIGC) Design Philosophy
- PIGC of Endo-Atmospheric Interceptors in Terminal Phase
- Numerical Results
 - Nominal case: Comparison with conventional design
 - Practical issues:
 - Information availability
 - ZEM behavior
 - Perturbations in initial conditions,
 - Computational time
 - Implementation with seeker noise
 - Robustness for parametric inaccuracy etc.
- Conclusions

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So, how to outline of the talk is something from the subsequent part of the this lecture is something like this, give a very brief overview conventional guidance and control. Followed by philosophy of IGC design it is merits and draw back at the philosophy level we will not go details inside **actually**. Then we will switch about to this partial IGC concepts.

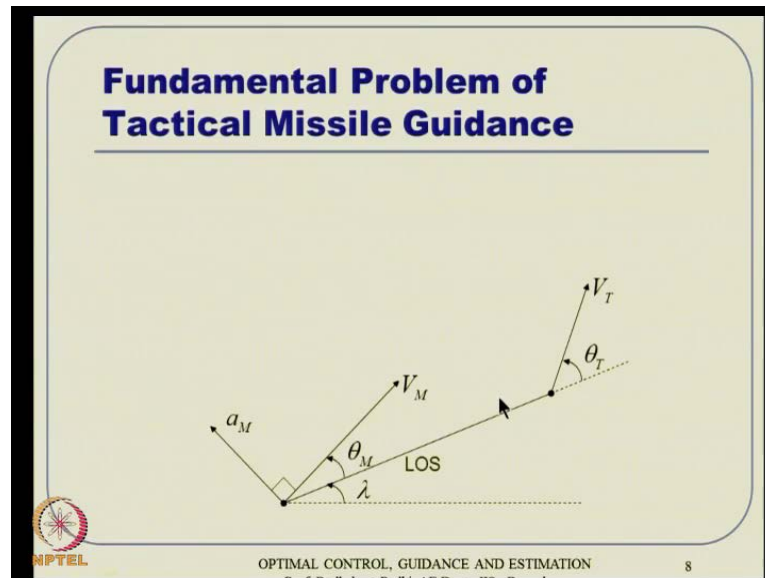
And then we will take a typical real life example of endo atmospheric intercept is engaging against **ballistic missiles** incoming ballistic missiles. And we are assuming that the ballistic missiles can take ballistic trajectory only, they cannot go through this typical this spiral motions and think like that. But, still I mean we assume that the target is some sort of a ballistic oriented vehicle with high speed. And then we have some sort of endo atmospheric interceptive were we want to intercept it before it, I mean before it comes ground **actually**.

So, that is the type of problem they will follow with some numerical case numerical results past is nominal case and will also give some comparison with conventional design as well as a if IGC design **actually**. And will demonstrate that this partial IGC happens to be in some sense better than both **actually**.

Then we will advised some of these practical difficulty issues, information availability then zero of at misbehavior, perturbations with few initial condition. Computational time issue, implementation with seeker noise, robustness with respective parametric inaccuracy. All these things happens to be very practical issues un unless all these things are address sufficiently is not really very good design, so we will see some of this behaviors with respect to this.

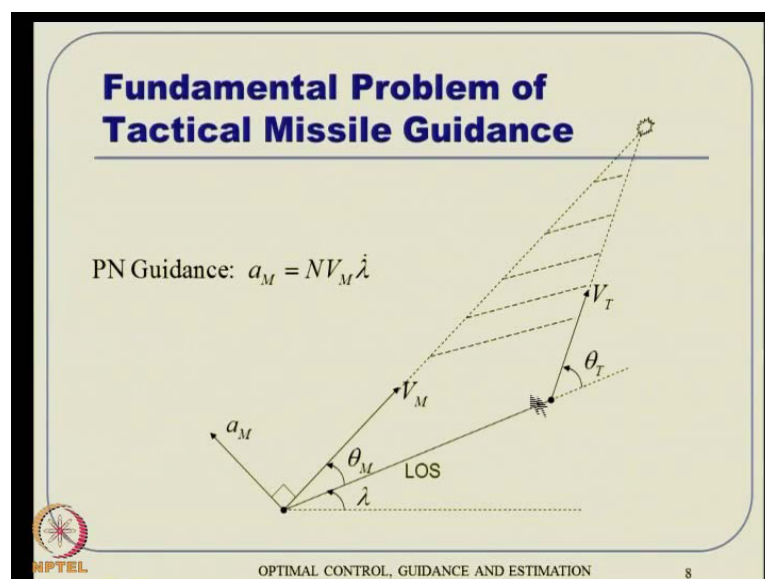
Then we will have some control being statements **actually**. Alright so very quickly what is motivation and philosophy of partial IGC.

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This is typical guidance scenario and some of these are very clear from the **from the** guidance lecture as well. So, we have a missile and we have a target going on its own way and missile goes in that probably; and then this missile if you continue to go in this direction it may not hit it may miss the target. So, what happens if there will be some sort of a direction correction and direction correction happens in such a way that the LOS does not rotate **actually** the typical 2 D picture.

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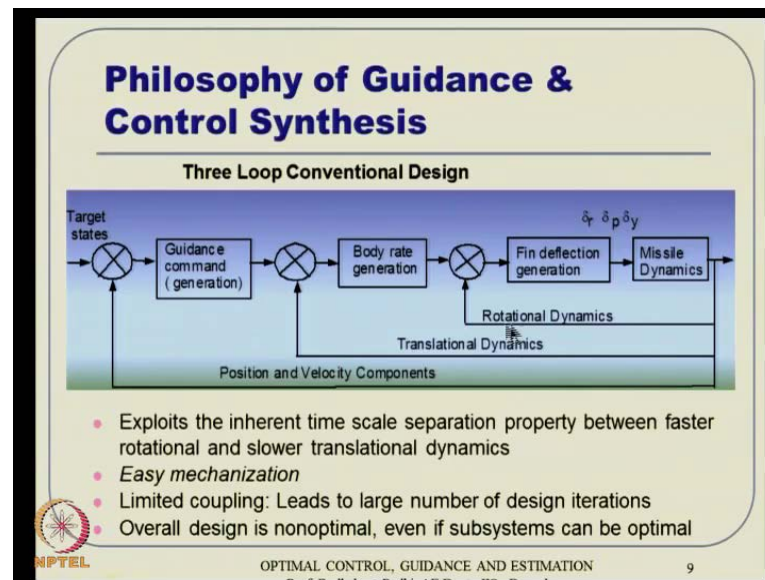
And so, ultimately the LOS lecture does not rotate and finally it leads to this collagen triangle sort of thing it has the there will be slight path correction sort of thing here **actually**. So, that finally this vector gets in a slight different directions sort of thing depending on what target does, and that is done typically through lift generation and lift by mass is a typically what is lateral resolutions.

So, M and a M on a when we apply this lateral resolution, this velocity vector gets turn and this missile that it what happens. So, how this a M is generated typically is gone through this **through this** very simple thing guidance laws, $N \cdot V \cdot M \cdot \lambda$ dot professional navigation and there are several variations of that.

There are something like true proportional navigation and there are several variations of that, there are something like true proportional navigation, $p \cdot r$ proportional navigation then, argument proportional navigation, modified proportional navigation think like that various things are available. But, the fundamental philosophy is like that it either a M is perpendicular to $V \cdot M$ or a M is perpendicular to V , I mean this LOS. Then if it is perpendicular to LOS, then it is something like $V \cdot c$ if is perpendicular to V I mean velocity vector it is $V \cdot N$. And there are again the discussion in the literature which one is better then over depending on the situation **actually** anyway.

So, this summery of this very conventional sense but, there are several issues there. But, once you generate the same and if you extend that to 3 D then one will be in the Z direction. And other one will be y direction side of them **actually** and remember in the acceleration in the velocity **in the velocity** vector direction typically we do not have the control it is **(())** equally. So, we have to leave with that and that have the engagement is assure typically based on the time to the information.

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Actually anyway, so this is receive what it is, so once you generate this lateral resolution this is the guidance command generation part of it. Then there is one more loop, which will take this guidance command and then convert it to equivalent body rate command **actually**. So, that is what body rate command generation will happen then you go to the inner loop and you tell I will take this rotational dynamics; in other words \dot{p} \dot{q} hydrostatic equation and all that.

Then I will use this equations to generate this **delta** δp δy by δp δy seen you are basically, then it will be fact to the activated dynamics. And then spinally it will be fact to the missile dynamics and then it the vehicle gets backwards correction basically. So, what are the things here first of it kind of exploits the inherent time scale separation property that is the very good part of it.

Because, one we talk about guidance we really do not worry, so much about the body attitude **actually**, the orientation of the word it does not matter ultimately what matters is taking the vehicle from point a to point b. So, in a point mass sense that is the guidance problem in other words very precisely the guidance use to translate the c c of the vehicle to a decide location wherever target is **actually** there.

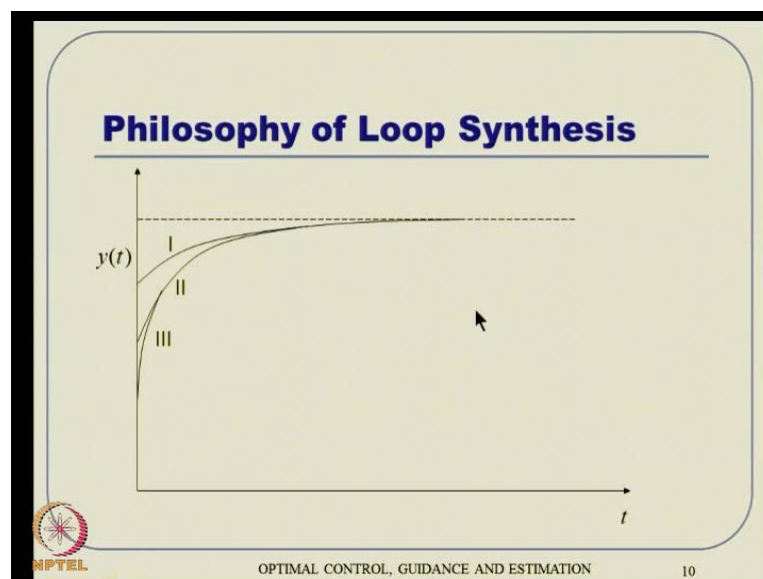
So, the other kind of thing, so we typically separate it out the body rate part and think like that what happens is body rates is typically fast **fast** dynamics. Whereas, velocity vector level well as u that u \dot{v} \dot{w} \dot{w} that component level or even point mass

dynamics level. So, equations are slow **actually**, in other words takes some time to evolve **actually**.

So, this time scale separation between faster and slower dynamics are typically retained here, which is very nice that is why the tuning difficulties are typically not there and all that **actually**. So, easy mechanization with easy tuning also relatively is a tuning ill that way and but, the drawback is overall design is non optimal, even if the subsystem becomes optimal. And also this limited coupling, because of this point mass here 6 top here and think like that, the because of this limited coupling it lead typically leads to large number of design nitration's as well.

So, this is the I mean the good part is exploits the time scale separation property, and hence it gives a **certain**, certain advantage including easy mechanization. And but, it has this the drawbacks **actually**.

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So, to address this drawbacks, I mean let us understand what is going on here, so we have this outer loop, the whatever you are talking about, the outer most loop here guidance changes in part of it, then there is a body rate generation. In other words this is let us say one can be assumed as guidance loop and this intermediate loop has to track this command, so that means II has to track this I, so is how t and III has to track II.

So, that III has to be, I mean III has to go and merge be II to basically. So, that is how ultimately III goes to this reference command outer line **actually** finally.

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Comments: Guidance Laws

- Many classical missile guidance laws are inspired from “observing nature” (e.g. Proportional Navigation (PN) guidance law is based on ensuring “collision triangle”)
- Control theoretic based guidance laws are usually based on “kinematics” and/or “linearized dynamics”. Hence, they are usually not very effective!
- Nonlinear optimal control theory is a “natural tool” to obtain effective missile guidance laws

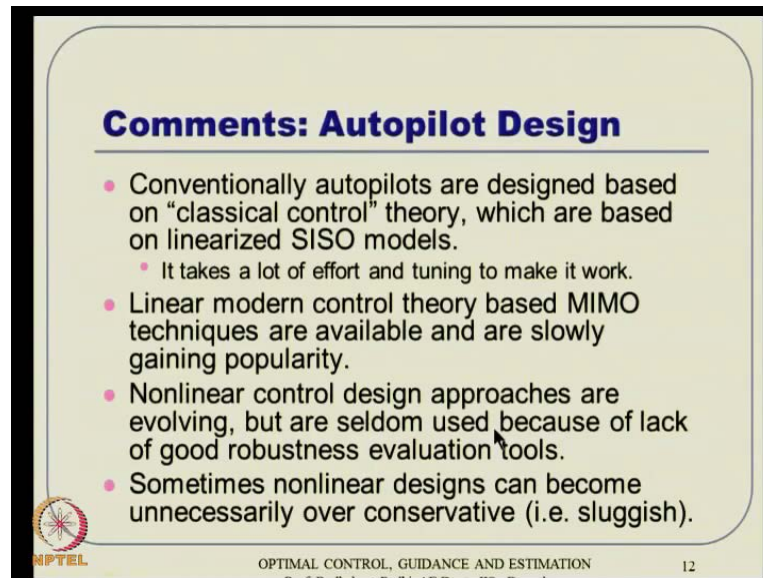
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So, that is how the I mean you can think of understanding basically but, what happens here have just one some observations before you move on, this many classical missile guidance laws are typically inspired from nature. And especially proportional navigation is inspired from, so called collision triangle conception and think like that typically geometrical variant.

Then, control theoretic basic guidance laws are also there but, that typical take either kinematics or maximum some sort of a linearized dynamics and think like that; so essentially they are not very effective also.

So, in my own view and so I mean the, which is typically all modern theory based modern control theory. And especially this non-linear optimal control theory based on guidance laws certificantly is a natural tool to obtain very effective missile guidance laws. The only constraint is probably this computational difficulty that work people for a long time can it be a work on **actually**.

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Comments: Autopilot Design

- Conventionally autopilots are designed based on “classical control” theory, which are based on linearized SISO models.
 - It takes a lot of effort and tuning to make it work.
- Linear modern control theory based MIMO techniques are available and are slowly gaining popularity.
- Nonlinear control design approaches are evolving, but are seldom used because of lack of good robustness evaluation tools.
- Sometimes nonlinear designs can become unnecessarily over conservative (i.e. sluggish).

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Whatever the autopilot design part, again autopilot design parts are also based on typical classical control theory like root locus but, a plot of $\log |G(j\omega)|$ vs ω control I mean lead lag combine set of think like that. And typically these are based on linearized SISO model Single Input Singular Output model

So, essentially it takes lots of effort into unique to make it work, so linear modern control theory base MIMO techniques are available and are slowly gaining popularity as well. However non-linear control design approaches are also evolving and they are typically not that much used, because of lack of good robustness evaluation tools then I have robustness. But, evaluation saying the tools are not there, hence the computation level is not there to go there **actually** like that, but, having saved that there are certain cases where people have **actually** went sort out; and then implemented non-linear control design also that the pilot in labial exploits **actually**.

And then sometimes this non-linear designs can be unnecessarily over conservative also for example, in non-linear H_∞ control if you want to use, sometimes it may be too sluggish also there is a reason, why it is not that popular. I mean said that, if you address some of the concerns then obviously the design candidates very, very powerful **actually**.

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Drawbacks of Three-loop Guidance and Control Synthesis

- Not much synergy between guidance and control
- Guidance either completely ignores the vehicle dynamics or only accounts for limited point mass dynamics
- Large overall loop delay (not good in time-critical applications, i.e. when t_{go} is small)
- Overall design approach is typically not optimal and hence it requires a good amount of effort (tuning) to make it work near-optimal.

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So, what are essentially the drawbacks of this conventional three loop guidance and control synthesis. First of all there is not too much of synergy between guidance and control. And in essentially the guidance either completely ignores the vehicle dynamics or only accounts for a limited point mass dynamics.

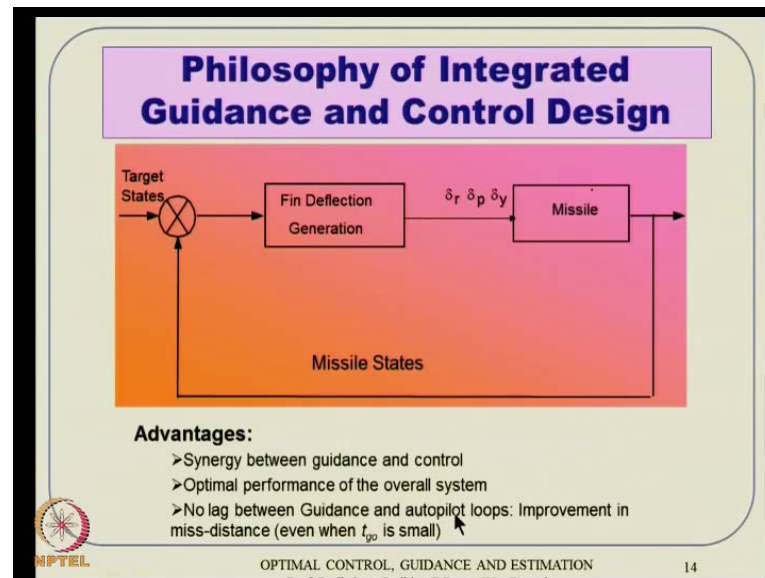
So, the dynamics happens at six top level that I mean assuming read it **read it** and think like that typically the six top is capable of capturing many things that goes on into the vehicle dynamics. But, the guidance typically completely ignores that or very limited sense in point mass essentially, it will take **it will talk** about that **actually**, then it essentially leads to large overall loop delay.

And essentially, this is not very good in time critical applications, that is in the that is my strong way **actually**. That means when time to go t_{go} is small, then you do not have a luxury of having this long settling time **actually**, so that will demonstrate in the next slide probably.

So, some times the non-linear designs can become unnecessarily over conservative that I already talked but, anyway. So, the point is the it may result in a large overall loop delay **actually**, that is when t_{go} is small it is not acceptable. So, overall design approach is typically non optimal and hence it requires a good amount of effort, that means really requires a good amount of tuning effort to make it work near optimal. I mean you can

talk optimal but, if really want to work it near optimal, then essentially need lot of training effort **actually**.

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So, for the for this fact people have thought about putting them everything together I IGC sense and the inter local what about one loop the every information is content target information you have missile information we have. So, simply just do some math and then to generate these spin deflections or likely basically. So, essentially it leads to this line of thought that it results in some at big advantage essentially, synergy between guidance and control optimal performance of the overall system.

Because nothing called point mass equation anymore everything this is of level and then essentially no lag between guidance and autopilot loops and essentially. And hence we assume I mean we expect some improvement in the miss distance **(())** but, what happens here that I mean.

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**Advantages and Drawbacks
(Single Loop IGC Design)**

- ❖ **Advantages:**
 - Synergy between guidance and control
 - Optimal performance of the overall system
- ❖ **Drawbacks:**
 - Works, but only with small perturbations about the collision triangle
 - Design tuning is very difficult because of conflict in "problem objective" and "control effectiveness"
 - Control attempts to alter the translational dynamics directly, rather than through rotational dynamics
 - This causes the rotational dynamics to overcorrect and hence the vehicle becomes unstable

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So, is advantage wise it is synergy between guidance and control and optimal performance here. And what is the drawback? Drawback is in our own observation it works but, it works only with small perturbation about collinear triangle otherwise the tuning becomes quite difficult **actually**. So, design training is difficult, because of the conflicting problem objective and control effectiveness, problem objective is to translate the c c from point a to point b which happens to be the target position.

Whereas, the control effectiveness happens in the rotational dynamics, because what happens typically is a, if you **if you** deflect of a control surface did not then there is a small delta force generation. But, force generation is not important, what is important is movement generation, because of these entire this long momentum and then long momentum results in lot of movement.

And hence the everybody gets rotated **actually**, which is typically ignored in the **in the** point mass level but, your intention is ignored. So, the any amount of controls, I mean any amount of the control surface deflection has to account (Refer Slide Time: 19:06) this body rates explicitly **actually**, if it does not then it is, it leads to lot of training difficulties.

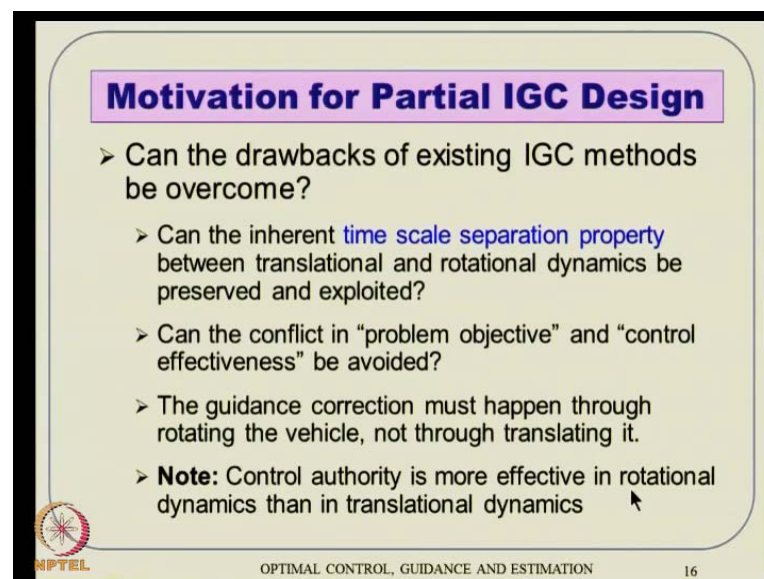
Because, what happens is your this δp δq δr are δr δp δq by they also appear in the velocity level equation \dot{u} \dot{v} \dot{w} \dot{t} or \dot{v} $\dot{\alpha}$ \dot{w} $\dot{\beta}$. So, if your objective is to translate the vehicle from point A to point B, then implicitly in the design

process this spins and deflections get generated to the velocity level equations, which are not good **actually**, were to go through the revetment level equation, then the training will be covered as **actually**.

So, this is what we have written here control attempts to alter the translational and dynamics directly rather than through the rotational dynamics, that is the whole observation there, so this causes the rotational dynamics to overcorrect.

And hence the vehicle becomes, unstable if you are not careful about this the spins and deflection sort of thing. Because, what you are doing is typically it generates the deflection from a relatively lesser powerful component, and then the deflection becomes more. And if it is becomes more then the rotational level it is affects the effectiveness is very high, so it will rotate the vehicle further and then it will go to unstable instability and all that **actually**. So, these are the draw backs.

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Motivation for Partial IGC Design

- Can the drawbacks of existing IGC methods be overcome?
 - Can the inherent **time scale separation property** between translational and rotational dynamics be preserved and exploited?
 - Can the conflict in “problem objective” and “control effectiveness” be avoided?
 - The guidance correction must happen through rotating the vehicle, not through translating it.
 - **Note:** Control authority is more effective in rotational dynamics than in translational dynamics

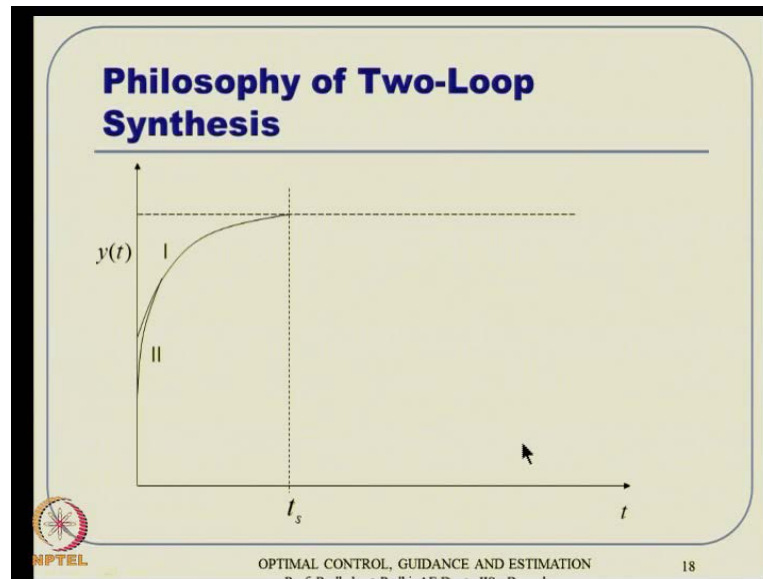
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So, the motivation for partial IGC is like this, in other words we want to retain the benefits of the IGC but, we want to work on the drawback of it **actually**. So, the questions are like this, can the inherent time scale separation property between translation and rotational dynamics be preserved and exploited. We want to keep this property and exploit that as well but, the problem that the conflict between the problem objective and control effectiveness has to avoided. In other words the guidance

correction must happen through the rotation of the vehicle and not through the translation **actually**.

So, the point here is control authority is more effective in rotational dynamics, then the transform translational dynamics and hence that is that is the requirement **actually**.

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So, let us understand this concept again, so we have this guidance followed by body guide generation followed by in deflection. So, guidance takes a long time and let us say settling time of the guidance loop is typically here, so problem objective is done submerge here. In other words if you time availability is more than t_s , then probably this miss systems is very small that is not a problem, so much **actually**.

But, what happens is we do not want to look at those problem, we want to look at those problem were t y is very small especially, when the incoming ballistic missiles comes with a very high speed. Even though separation distance is let us a 30 kilometer one skier looks I mean opens upon and sees the target.

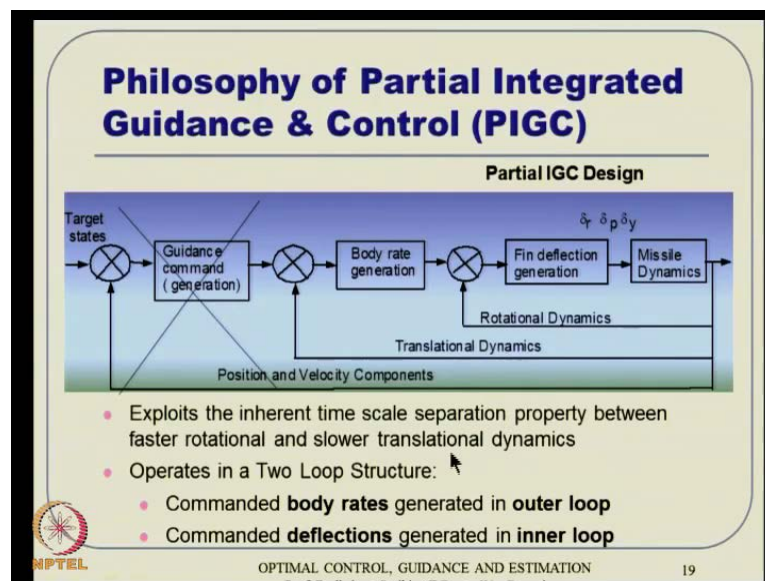
The 30 kilometers distance is typically covered in and 3, 4, 5 seconds **actually**, so that becomes a small t go problems really. So, then in those cases if what happens here is we want to do something, so that this outer loop is avoided. So, remember this inner loop inner most loop is capable of following the second loop, were that the by design the

second loop has become quite slower, because it has to track the I loop that is the whole idea there.

Now, let us get out of this loop I, were guidance loop then directly talk about body rate generation, can you do that. If you do that probably this body rate generation can try to catch up this commanded value very fast. And hence these three will also goes up with that is what is reflected here. So, instead of having this of a picture, we will have something like this kind of the picture.

So, essentially what is happened is initially you our settling time was somewhere here but, by doing this we have **we have** been able to kind of reduce this settling time that much **actually**. So, that means even if this even if let us say the some small t go problem the real t go happens to somewhere here, then also we will be able to do it because our settling time has reduced to here. And our t go more than settling time now, so will be able to do these all **actually**.

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So, the idea is like this, we have this guidance but, you body rate spin deflection and all. What you are telling here is like come out of the typical guidance loop and whatever is left out is what your calling as partial IGC design **actually**. So, it exploits the inherent times scale separation property and then it operates essentially into loop not in really one loop but, we want to operate in two loops **actually**. And so, the commanded body rates

are generated in the outer loop directly and then, they are using those commanding body rates, we generate the spin deflections in the inner loop that is the whole idea here.

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Advantages of Partial IGC (PIGC)

- Partial IGC: A New philosophy
 - Combines the advantages of IGC and conventional designs
- Very minimal tuning requirement
- Very less computational time (due to usage of MPSP and Dynamic inversion)
- Successfully verified for a large number of initial conditions of both interceptor and target
- Comparison between conventional three-loop design and a SDRE based one-loop IGC design: Partial IGC is better than both!

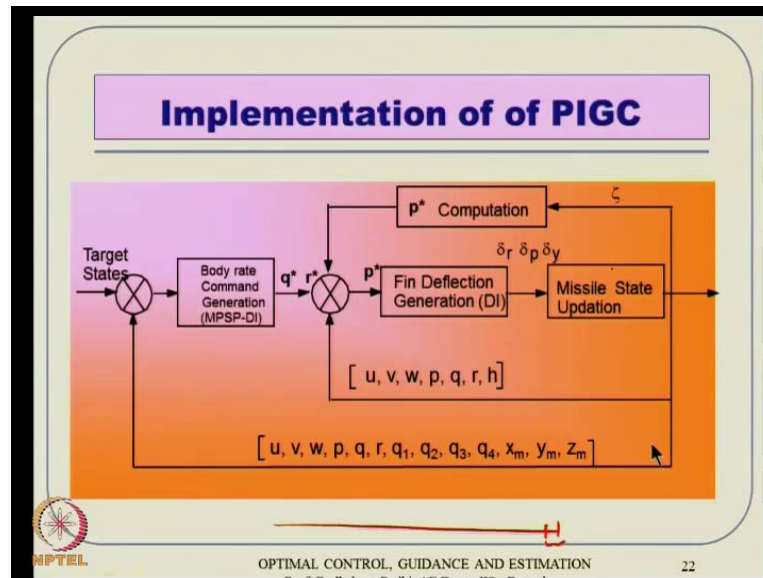
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So, advantage is again just to summarize were is a new philosophy all together and tries to a kind of combine the advantage of IGC and conventional design both very minimum training requirement. And that the big fact **actually** in my view if you **if you** happen to see the easy community and think like that happen to talk to them, most of the time the most of the designs have this tuning difficult **actually**. But, in PIGC really do not have this training difficult **actually**.

So, very less computational time because that is something we cannot avoid and it has to have otherwise all these nice concepts are useless. So, what you are doing here is outer look at proposing this these MPSP or MPSC that, what you are discuss before these two techniques. And then inner loop we have time to use this dynamic conversion concept, so this non-linear control design approach. So, successfully verified for a large number of initial conditions for both interceptor and target as well and comparison between conventional three loop and a SDRE based one loop design is also there **actually**.

So, sincerely the conclusion turns out the partial IGC better than both but, instead of simply telling a in these words will also go through the some of the results. And then see why it is happening again these are the concept that is **that is** available in some of these references you can **you can** find more details on that **actually**.

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So, implementation of this partial IGC is typically done this way remember body rate generation either MPSP or MPSC whatever we are talking about, then the this one will have this q^* and r^* available **actually**. So, this q^* r^* generation mechanism will give us this the desired body rates, w^* and r^* basically and then what happens to tell the little more story **actually** here, say whatever t go we get from in this way law and think like that initial ways. Essentially, the whole idea here is to nullify the errors in the two channel to two exceeds and then stick to that zero error **actually**.

So, very quickly you I mean we will **we will** have this in other words, if **you I** your t go is somewhere like this your estimate can we somewhere here but, the real thing you can very close to that. So, during this period whatever period you left out you can think of implementing some DI ways to guidance thing there and that is why it is written MPSP DI. And all that up to here it is MPSP any small time that is left over that means error is gone to 0 in two components, it makes to remain at 0 for the remaining time. So, the remaining part is typically drawn through here details **actually**, you can see that in that paper **actually** anyway, so that is that part of it **actually**.

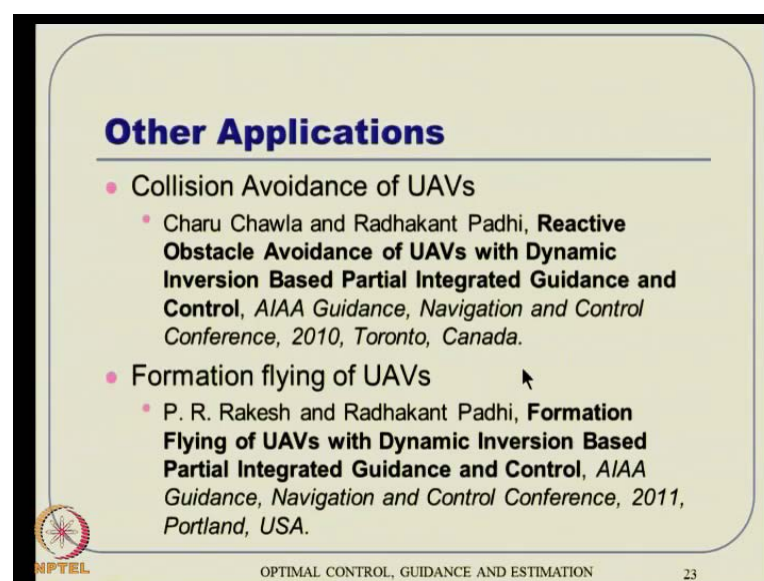
So, but, also remember we need to stabilize the roll rate, because without roll rate stabilization things can go very bad, because seekers can not see the target spin deflection resolution. In other words, what your generating the δp δy ultimately any missiles have **actually** four deflections four things $\delta 1, 2, 3, 4$.

So, there is some spin deflection I mean logic where you generate this delta 1 to 4 using these things. And then recombine wake and think like that, so that details are not seeing here but, this can be done in a good way provided the body rate is stabilized. That means no further body rate is there, the body rate is very small in that sense that can be done.

But, typically roll rate is also a sensitive channel, because your I_x says the moment of inertia I_x is very small and the moment that gets generated is large **actually**. So, your \dot{p} becomes non zero and it double words very quickly, so that means your body will start rotating violently and think like that **actually**. In those situation even though guidance is good and all it now think and we done **actually**, so we really made a stabilized roll rate to do anything like guidance and all that **actually**.


So, that part is assured by assuming this some **some** zeta variable, where \dot{zeta} is assumed to be p and then this variable is paid us, so that \dot{p} can be computed and then **sorry** p^* can be computed. And in p^* q^* r^* are now available at this loop **actually**. So, this p^* q^* r^* can be can be utilized and then directly you can get the spin deflection and then operate it **actually** that way; anyway more the entire the guidance I mean the body rate the command generation can be though about like a outer loop guidance sense. So, everywhere the full information is required that all the variable that goes into the six top equation has to be fed back **actually**. And this q_1 , q_2 , q_3 , q_4 are typically quaternion component **actually**.

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Other Applications

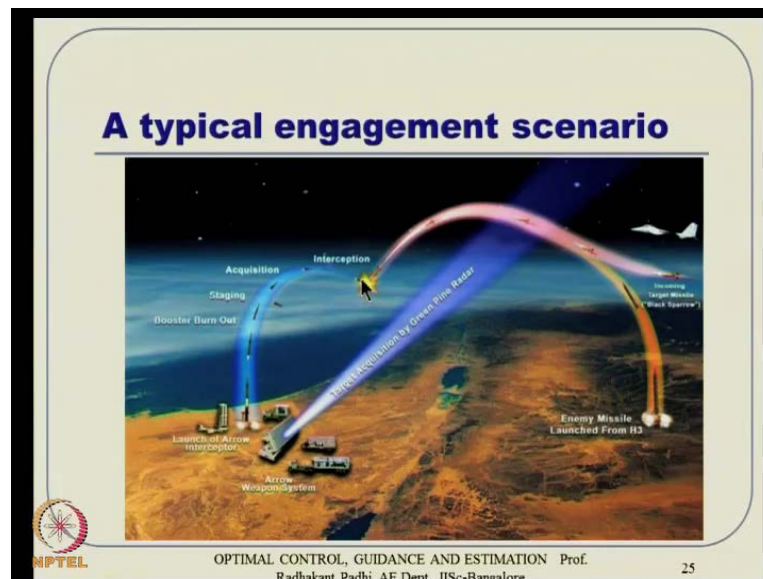
- Collision Avoidance of UAVs
 - Charu Chawla and Radhakant Padhi, **Reactive Obstacle Avoidance of UAVs with Dynamic Inversion Based Partial Integrated Guidance and Control**, *AIAA Guidance, Navigation and Control Conference, 2010, Toronto, Canada.*
- Formation flying of UAVs
 - P. R. Rakesh and Radhakant Padhi, **Formation Flying of UAVs with Dynamic Inversion Based Partial Integrated Guidance and Control**, *AIAA Guidance, Navigation and Control Conference, 2011, Portland, USA.*

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Other applications as well we have done collision avoidance of UAV's and then you can also implemented. So, perforation trying of UAVs similar concept different techniques and think like that, you can see some of these literature again if I interested in more about the concept of partial IGC. The way it is implemented for this missile problem is different from these things but, still the concept remains the very similar **actually**.

And typically, these are dynamic inversion waste here also, here also nothing called MPSP MPSC and all that if you that way. I mean coming back to this particular problem this partial IGC design for endo atmospheric interceptor for ballistic targets especially.

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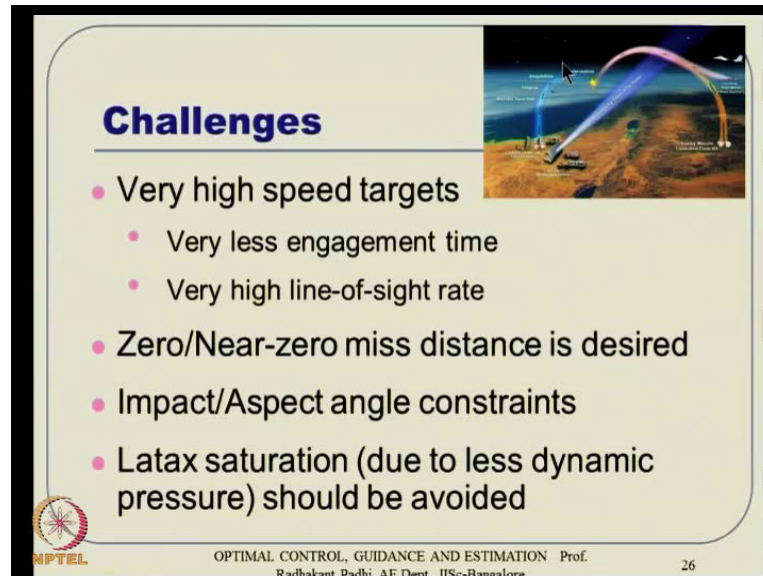


So, let us see to more little more on to that, so there is threat missile which is the learn somewhere and all that, this picture is a taken from internet I do not know the website I forgotten but, there typically taken from internet. So, this is threat missile, which is which is launch towards the target somewhere but, this is a defining region basically. So, this is where typically the depending missile is to be has to be large and before it intercepts, I mean before this the set missile comes down.

It has to be intercepted wherever ground **actually** and this such a logic operates for this class of vehicle, then it can also engage with the conventional targets like aircrafts and think like that way. So, this is whole idea of typical engagement scenario information's are given based on radar and then this vehicle itself, will have a seeker towards the end

and think like that. What we are talking is the very last segment of from here to here for the interceptor and probably some here to here for the target **actually** kind of thing.

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Challenges

- Very high speed targets
 - Very less engagement time
 - Very high line-of-sight rate
- Zero/Near-zero miss distance is desired
- Impact/Aspect angle constraints
- Latax saturation (due to less dynamic pressure) should be avoided

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So, the challenges here is first of all very high speed targets, that means very less engagement time availability is very soft and also remember very high line of sight. I mean that is the fundamental thing for any guidance no matter what technique you use **actually**. If your line of sight rate changes, I mean you have to really in a reactive yourself very fast also basically. So, if your line of sight rate is high, because the vehicle is let us say going in some sort of I mean cross plane sense, if you are engaging remember, this picture if you see if it is going **it is going** in some plane and we are engaging in some other plane **actually**.

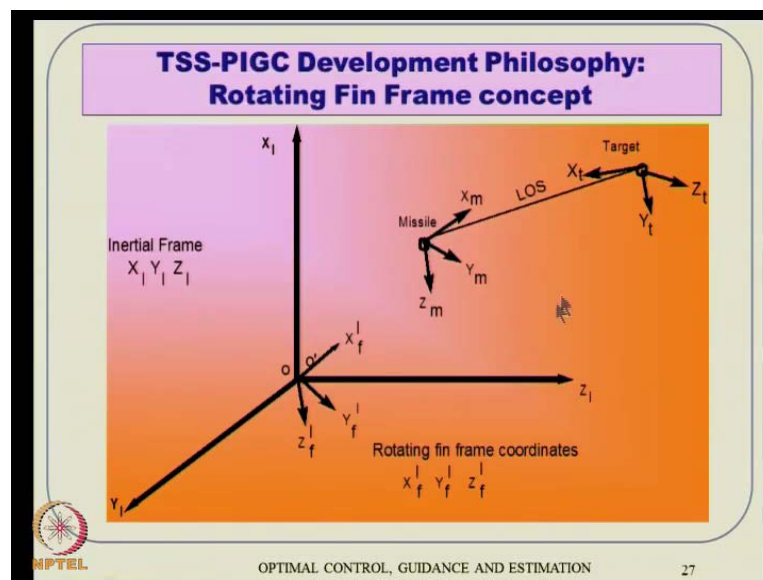
So, that kind of hesitation happens, then anything that comes under very high speed here it reflects in large LOS rates **actually** and that generates lot of difficulties. Ultimately we want zero miss distance are very close to zero miss distance, because this incoming targets are also having very good sealing for their own weapon system and think like that. So, unless there is a direct heat or very close to there and the very good in fact or that if it does nothing happens there **actually**; so we have to go there if the zero miss distance finally.

So, that is why this plan loss an all not very effective one line to this is singular towards and it towards an **actually**. So, that we can say very series and then this a complaint of

impact angle an aspect angle think like that, so that also may has to be accounted for that not only the engagement is happen the zero to miss distances but, it to happen particular angles.

So, that can happen an that due drawback is a this lateral resolution it was dynamic is less claim up an down. Once, a dynamic pressure level drops down double, this engagement is to happen in a, I mean this difficulty comes out turning the vehicle **actually**, like that are else **(())**.

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Alright, this is a target as is own access an system and all this is LOS a side, vehicle are intercepted missile lesson we call, goes to an target **actually**. So, it has two system this entire promolisium are any guidance and problem or inverse an access problem **actually**, the vehicle has it is own body access system, it has a inertial frame.

And then what you are telling is the entire equation of motion given this pin frame, which is difficulty rotated by 45 degrees about is one body of it is **actually**. And that in frame, you are imagining one more pin frame sort of thing, located **at the in the** in the inertial frame center. That is the, what mathematical formulation problem **actually**. This is what we seen inertially located fin frame an **actually** that referred here, here to here the target **actually**. The orientation keeps changing but, the position does change for this frame but, this one, both the inertial and orientation keeps changing **actually**.

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Missile System Dynamics

(No approximation needed in nonlinear 6-DOF model)

$$\begin{aligned} \dot{u} &= vr - wq - \frac{QSC_D}{m} - Q_{11}g \\ \dot{v} &= wp - ur + \frac{QSC_{NBV}}{m} + \frac{QSC_{NB\delta_Y}}{m} - \left(\frac{Q_{21} + Q_{31}}{\sqrt{2}}\right)g \\ \dot{w} &= uq - vp + \frac{QSC_{NAV}}{m} - \frac{QSC_{NB\delta_P}}{m} - \left(\frac{Q_{31} + Q_{21}}{\sqrt{2}}\right)g \\ \dot{p} &= \frac{-QScC_{RM}}{I_{XX}} + \frac{QSc^2C_{lp}p}{2V_m I_{XX}} - \frac{QScC_{l\delta_R}}{I_{XX}} \\ \dot{q} &= \frac{QScC_{mq\delta_A}}{I_{YY}} - \frac{QSC_{NB}(X_{cp\delta} - X_{cg})\delta_P}{I_{YY}} - \left(\frac{I_{XX} - I_{ZZ}}{I_{YY}}\right)pr + \frac{QSc^2C_{mq}q}{2V_m I_{YY}} \\ \dot{r} &= \frac{QScC_{mr\delta_B}}{I_{ZZ}} - \frac{QSC_{NB}(X_{cp\delta} - X_{cg})\delta_Y}{I_{ZZ}} - \left(\frac{I_{YY} - I_{XX}}{I_{ZZ}}\right)pr + \frac{QSc^2C_{mr}r}{2V_m I_{ZZ}} \end{aligned}$$

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The equation of motion are given like that in involve in that it is complete full 6-DOF model. So, here is n dot, w dot, v dot, p dot, q dot, r dot equations, so this are quarter line components, there are gravity terms of this zee term are seen here. This are disturbance terms, which a difficulty to sum extent it is here known, something your known qualification known for that an then if we go back to dynamic learning (()) component and this quadrennial component.

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$$\begin{aligned} \dot{q}_1 &= \frac{q_4 p - q_3 q + q_2 r}{2} & \dot{q}_2 &= \frac{q_3 p + q_4 q - q_1 r}{2} \\ \dot{q}_3 &= \frac{-q_2 p + q_1 q + q_4 r}{2} & \dot{q}_4 &= \frac{-q_1 p + q_2 q + q_3 r}{2} \end{aligned}$$

Rotating Fin Frame Equations:
 Ref: P. K. Menon and E. J. Ohlmeyer, *Integrated Design of Agile Missile Guidance and Autopilot Systems*,
 Control Engineering Practice, Vol. 9, 2001, pp. 1095-1106.

$$\begin{bmatrix} \dot{x}_{rm} \\ \dot{y}_{rm} \\ \dot{z}_{rm} \end{bmatrix} = T_{LV2F} \begin{bmatrix} \dot{x}_T^J \\ \dot{y}_T^J \\ \dot{z}_T^J \end{bmatrix} - \begin{bmatrix} u \\ v \\ w \end{bmatrix} - \begin{bmatrix} q z_{rm} - r y_{rm} \\ r x_{rm} - p z_{rm} \\ p y_{rm} - q x_{rm} \end{bmatrix}$$

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And then we are not talking about position of vehicle itself but, relative position between the vehicle and the target. And this relative position is seen in this rotating position in fin frame **actually** (Refer Slide Time: 33:48), when you have derivatives taken in a rotating frame position **(O)** cross turn as to be accounted an doing all that but, will end of this something like this equation an **actually**.

So, this equations what you have six equation here, four here and three here thirteen equation total but, we will also able quadrant constraint equations, which is like square a q 1 plus q 2 plus q 3 plus q 4 square is equal to 1. That also we constant to learn to me so this that is where complete 6-DOF equation is given for this vehicle more on this rotating fin frame concept one can see will be seen **actually**.

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Target Model

$$\dot{V}_T = \frac{-D}{m_T} - g \sin(\gamma_T)$$

$$\dot{\psi}_T = 0 \quad \blacktriangleleft$$

$$\gamma_T = \frac{-g \cos(\gamma_T)}{V_T}$$

$$\dot{x}_T^I = V_T \sin(\gamma_T)$$

$$\dot{y}_T^I = V_T \sin(\psi_T) \cos(\gamma_T)$$

$$\dot{z}_T^I = V_T \cos(\gamma_T) \cos(\psi_T)$$

Assumptions:

- Endo-atmospheric engagement
- Ballistic entry: No lift, only drag
- Gravity-turn is accounted for
- No intentional and/or Spiraling Maneuvers

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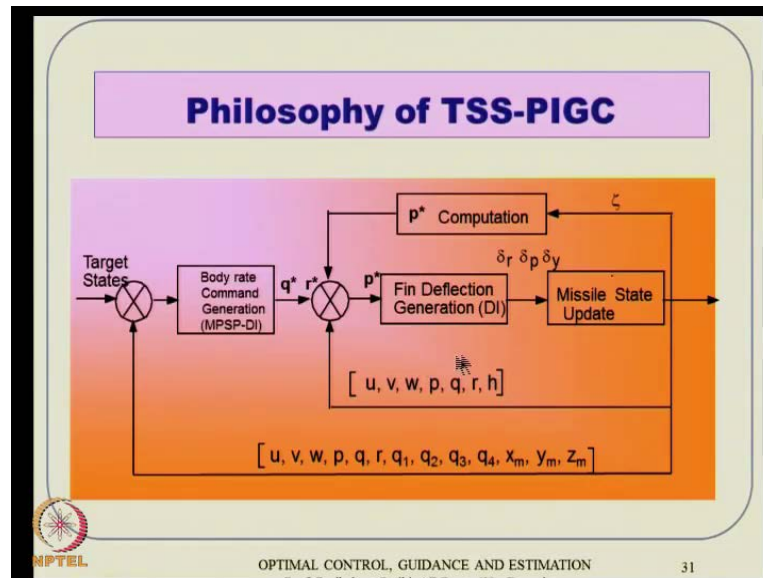
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The target model is difficulty **(O)** model, because no sensors are available as if now it can be 6-DOF level detail clarity of target in is, it is obviously impossible to do is current technology concern. But, you certainly get the informer an at a point mass level and typically becomes sufficient **actually**. So, this **(O)** level equation three dynamic component an three quadratic component remember everything accounts in 3-D **actually**.

An also there sum essential about target in a means first, of all we seen endo atmospheric engagement will seen ballistic entry that means no lift only drag is there. Then we have gravity turn were accounted here and no internal an I mean no intentional and or spiraling maneuvers **actually** I mean no internal and no intentional **actually**. So,

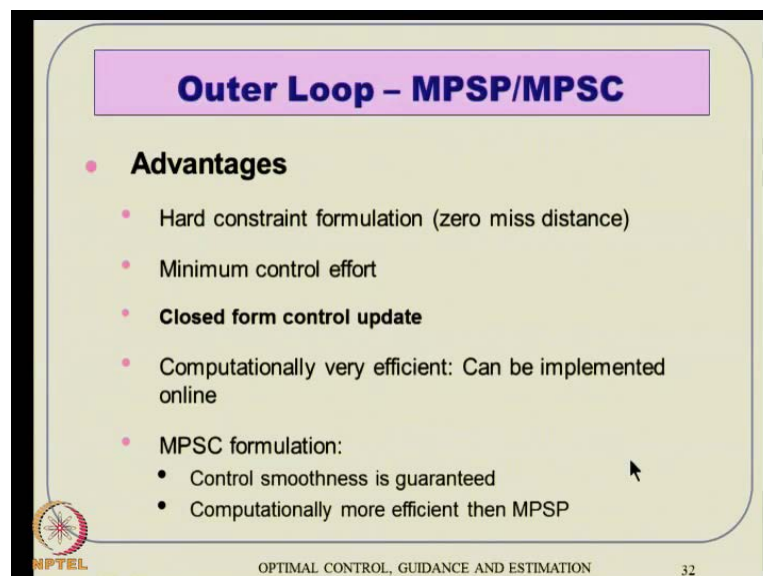
sometimes this spiraling maneuvers or intentional also because the physics of the problem that happens **actually**. But, that a reality I agree with that, this particular experiment that proposing a new consultant together. So, we thought we will propose with respect to non targets but, it has dragged **actually** which is a major component. So, that is how which is define **actually**.

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Any way so the philosophy of this time spin separated something like this, we have already talk about that.

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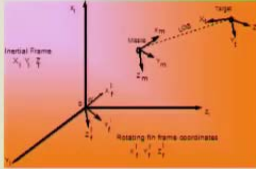
And then advantages already we talked about many thing was outer loop were this MPSP or MPSC we both are familiars. And both the things we discussing detail in one of the previous lectures **actually** about the details. So, advantages of using this technical outer loop it can serve it can it **actually** as a hard constraint formulation that means one of the thing we are looking for zero miss distance.

We can aim for now, we have the technique which can do that, here minimum control effort kind of formulation here, it also closed form control updates, computational time was less. And essentially it can be implemented online that what we strongly think **actually**.

So, comfort to this MPSP as MPSC can this particular think MPSC as little more advantage but, controls weakness is granted by formulation. And computationally slightly more efficient then MPSP.

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MPSC in Outer-loop Design



$$X = [u, v, w, p, q_1, q_2, q_3, q_4, x_{rm}, y_{rm}, z_{rm}]^T$$


$$U = [q_c, r_c]^T$$

δ_{Roll} , δ_{Pitch} , δ_{Yaw} and p are considered as time varying parameters

Output Vector: $Y_N = [y_{rm} \quad z_{rm} \quad \dot{y}_{rm} \quad \dot{z}_{rm}]$

Goal: $Y_N \rightarrow Y_N^*$, where $Y_N^* = [0 \quad 0 \quad 0 \quad 0]^T$

A reasonably good guess history is obtained from PN based three-loop design



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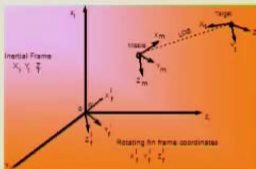
So, that is then outer loop design, we taken all this things u, v, w, q, p. Remember q an r control variable here so p happens to be a time variable parameters sort of thing. These are the stares that are considered in outer loop an the other control that is assumed in the outer loop **actually**. So, the output vector tells out like this remember (Refer Slide Time: 37:19) this is frame work was this is particular domain. You need this stud dynamics, you need a outer vector, which has to go to some desired output values and all that at t goes to t a.

This output are typically, this is what it is, we have to y r m and z r m relative dynamic in the fin. Inertially located in the fin frame in y an z direction they have to goes to zero and it is a velocity in y and z directions also goes to zero that is the formulation **actually**.

So, this techniques is also requires some sort of a guess history, so lot of implementer P N based three loop design than took a guess history from there. So, (Refer Slide Time: 37:55) this design will talk about that before we have linear parameterization and then this constant equation detail method already discuss, why do not want to talk about it here.

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MPSC in TSS-IGC Design



$$X = [u, v, w, p, q_1, q_2, q_3, q_4, x_{rm}, y_{rm}, z_{rm}]^T$$

$$U = [q_c, r_c]^T$$


$\delta_{Roll} \quad \delta_{Pitch} \quad \delta_{Yaw} \quad \text{and } p$

are considered as time varying parameters

Output Vector: $Y_N = [y_{rm} \quad z_{rm} \quad \dot{y}_{rm} \quad \dot{z}_{rm}]$

GOAL: $Y_N \rightarrow Y_N^*$, where $Y_N^* = [0 \quad 0 \quad 0 \quad 0]$

A reasonably good guess history is obtained from existing three-loop design with MPN law for guidance

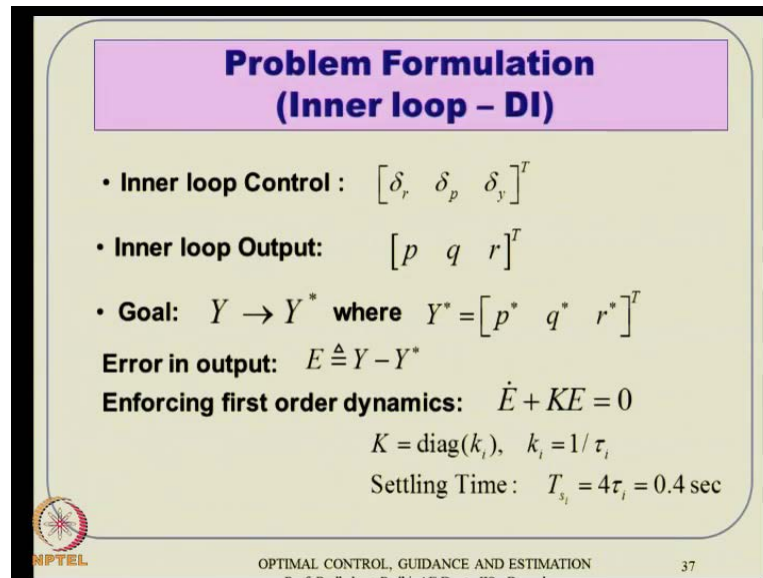


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But, essentially it is operating based on the same technique **actually**. So, these are the designed vectors 0 0 0 this is the actual vehicle parameter this decide vehicle parameter when t goes to t f **actually**. And we implemented everything with respect to the three loop design based on the these modified P N loop followed by dynamic inversion control synthesis and all that. That gives us a kind of guess history to start with and all also gives us platform to compare our results with respect to those results **actually**.

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**Problem Formulation
(Inner loop - DI)**


- Inner loop Control : $[\delta_r \quad \delta_p \quad \delta_y]^T$
- Inner loop Output: $[p \quad q \quad r]^T$
- Goal: $Y \rightarrow Y^*$ where $Y^* = [p^* \quad q^* \quad r^*]^T$

Error in output: $E \triangleq Y - Y^*$

Enforcing first order dynamics: $\dot{E} + KE = 0$

$K = \text{diag}(k_i), \quad k_i = 1/\tau_i$

Settling Time: $T_{s_i} = 4\tau_i = 0.4 \text{ sec}$

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Then coming to inner loop control, we have this fin deflection and inner loop output something like this and the formulation y should go to y^* in the dynamic inversion sense. We define this error and enforce aero dynamic, K is diagonal matrix with diagonal entries been this one of the tau sort of thing **actually**. And settling time, how small we have assumed settling time have assumed as 0.4 seconds.

So, less than half a second we want the settling time to happen in the inner loop **actually**. So, outer loop can close in something like 3, 4 times of that so wherever one and half two seconds time, we expect that thing will be the error will go, so any t time to go which is more than two seconds probably will be able to handle **actually**; so that is a whole idea here.

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Inner Loop Design Based on Dynamic Inversion

Control Equation:

$$U = [G_y(X)]^{-1} [f_y(X) + K(Y - Y^*)]$$

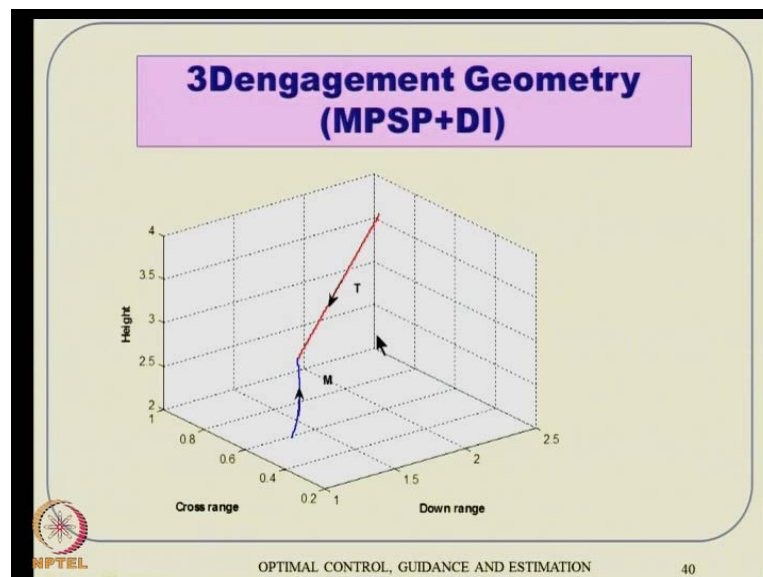
$$f_y(X) = \begin{bmatrix} \frac{-QSc_{RL} + QSc^2 C_{\theta P}}{I_{xx}} \\ \frac{QSc C_{\omega \theta \theta}}{I_{yy}} + \left(\frac{I_{xz} - I_{zx}}{I_{yy}} \right) p r + \frac{QSc^2 C_{\omega \theta q}}{2 I_{yy}^2} \\ \frac{QSc C_{\omega \theta p}}{I_{zz}} + \left(\frac{I_{xy} - I_{yx}}{I_{zz}} \right) p r + \frac{QSc^2 C_{\theta r}}{2 I_{zz}^2} \end{bmatrix}$$

$$G_y(X) = \begin{bmatrix} \frac{QSc C_{\theta}}{I_{xx}} & 0 & 0 \\ 0 & \frac{QSc_{\omega \theta} (X_{\omega \theta} - X_{\theta})}{I_{yy}} & 0 \\ 0 & 0 & \frac{QSc_{\omega \theta} (X_{\omega \theta} - X_{\theta})}{I_{zz}} \end{bmatrix}$$

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The control update equation is in close form, so that you do not know dynamic inversion I encourage to see one of my lectures in the previous. I mean the another parallel cost advance control **in this** in this NPTEL program and then you can see, why this dynamics are coming like this **actually**. So, details also you can see in the papers **actually**.

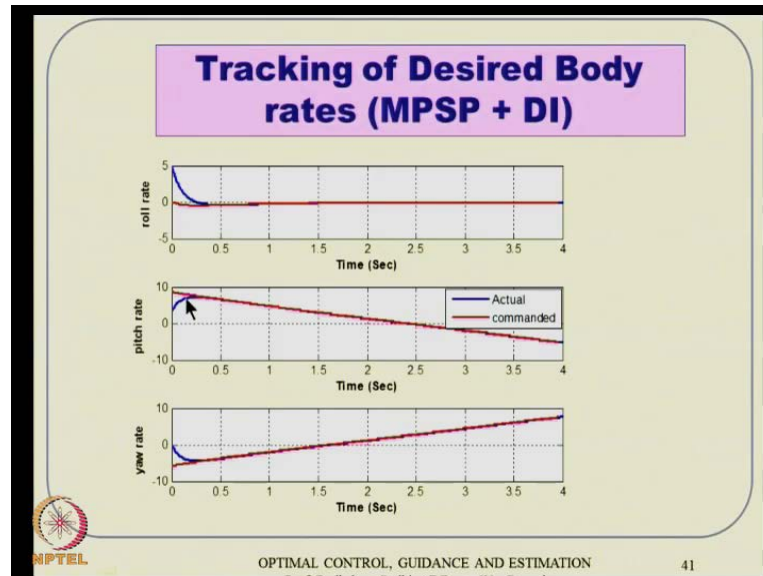
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Some stimulation results this is, what it is we have this the target coming in see velocity target typically come in a straight line sense. Because they know many where their but, the speed at various locations are different, because drag is there **actually**. So, typically

this is target is coming missile is going and engaging here so can picture really the zero mass to sense **actually**.

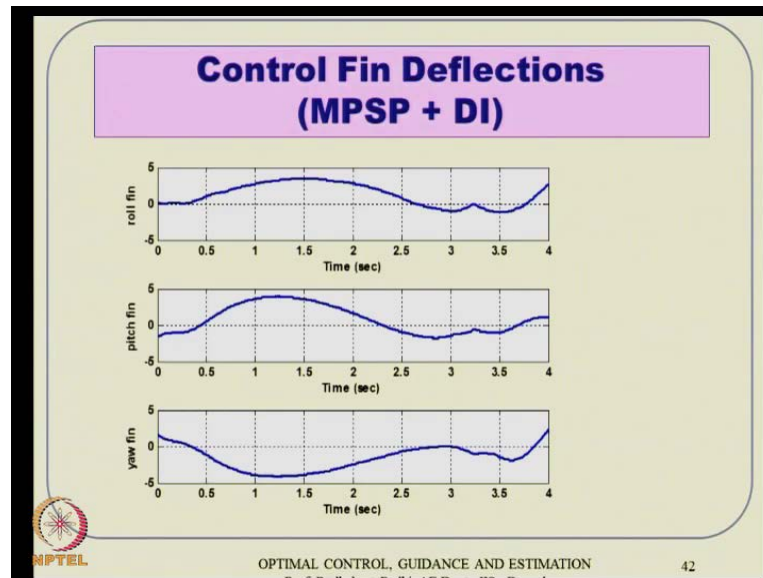
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But, we will see the values also basically, that way anyway this is the roll weight and all you can see very, very soon, they are civilizing whatever initial conditions they were some I mean. So, remember the roll rate has to go zero that is what the role, role rate stabilization has to happen it is happening and where as the pitch and your rates are typically generated from the guidance loop that did not go zero; because you really need to turn the vehicle to go towards the target **actually**.

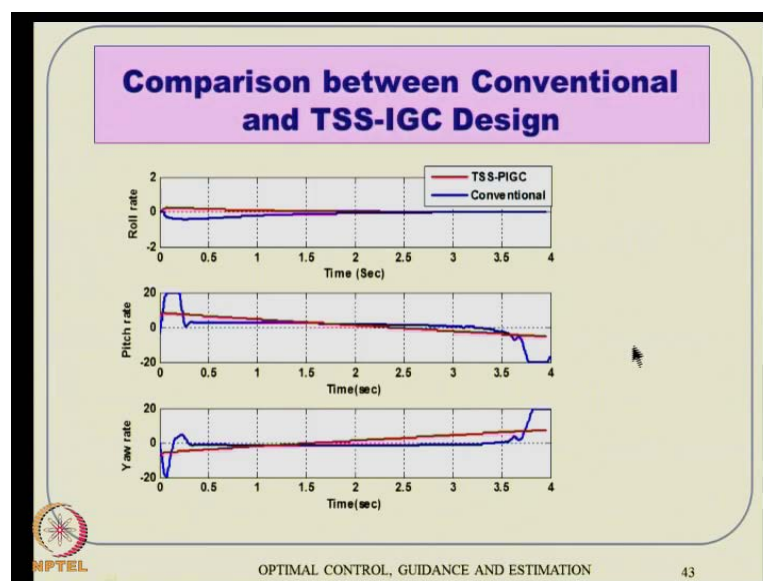
So, this is what the spinal solution is, this is the desired pitch rate and this idea rate. Interestingly it happen to be a kind of straight line **actually** very close to, I mean you can see this these are straight line **actually**, even if it is implemented in MPSP for a frame work where you are not really enforcing on a straight line equation basically. But, anyway so we can see that very quickly the inner loop tracks the designed dynamics. And hence everything happens in a nice way remember settling time is 0.4 seconds that is what we have assumed here, so 0.4 seconds how much here **actually**.

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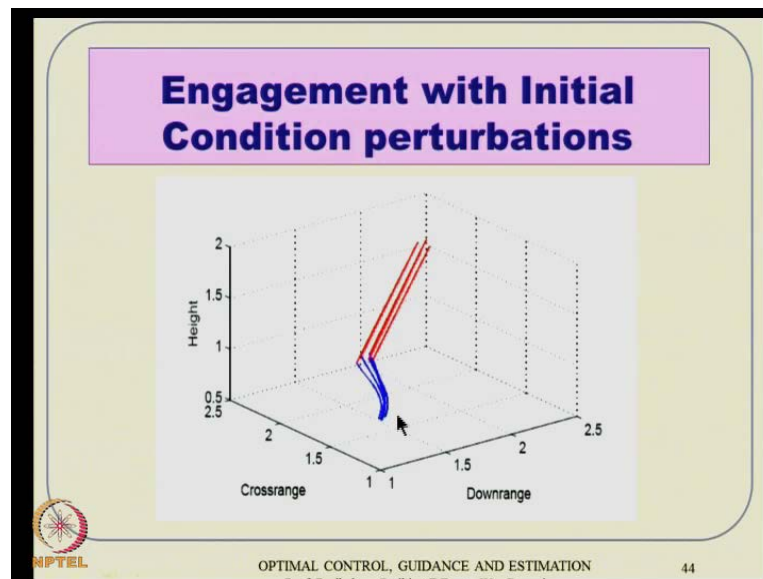
So, the then spin deflections sense also you can see their very smooth they do not have this this similarities use and think like that. And also the bounce have their blow typically plus minus 5 degrees which is I very less and typically the bounce will something like 20, 30 degrees reflection, where to where whereas, your utilizing only about plus minus 5 degrees reflection. Otherwise the scaling becomes in from t_0 to t_f in such way that the demand does not becomes very high towards the end to it is typically happens in a conventional design sense.

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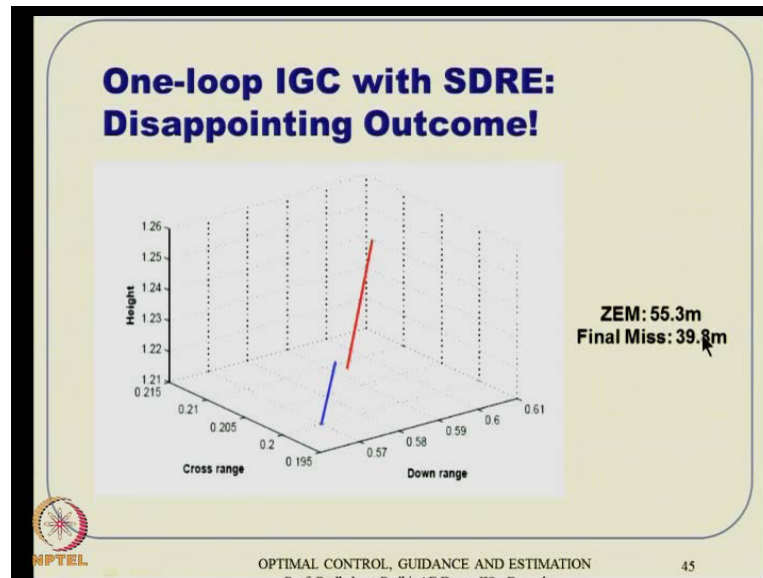
You can see this, this is also comparison between this conventional and typical I mean the partial IGC that we are proposing. So, this is this I mean roll rate is not too much of a difference but, if you see pitch your rate that is what the guidance look close as **close as** in and see some saturation here and, and more important towards end **actually**. That is what will deflect reflect in a system **actually**, because here we are purposefully bounded it toward to the 20 degree per second limit. And that is why it is it is stabilized their **actually**, not allowing it, if allow it will go lot higher than that **actually**

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So that is avoided completely in this new formulation and engagement with different initial condition, we can even try it out for several, several cases; once you tune it just forget it you just keep on changing the initial conditions and your program **actually**. So, that is what is done here, you can see various initial conditions of the missiles various initial conditions for that target also, both in position as well as velocity sense, no matter what it is able to engage the target **actually**.

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And this was a little frustration if you have **actually** implemented one IGC concept based on a theory and tried several things for a couple of months altogether. But, no matter whatever we did the spinal disappointing news is something like this **if the** if the initial error without doing anything happens to be around 55 meter, we are able to correct only up to 39, 40 meters **actually**.

There is not too much of gain that way but, again this S D R E technique is a little bit funny technique it depends on S G C and think like that. So, we do not claim too much into that what you claim is to the best of ability and then tuning this is what results that we got. And typically if the design methodology is good then it should not based on designer's experience and think like that.

So, that is what the good method will demand **actually**, I mean will result in that basically. So, without claiming I mean without claiming too much what you claim is our best effort if the miss distance is initial 0, 0 effort means without doing anything is happens to be the outer of 50 60 meters. Then some correction may happen but, this is nowhere close to what we really want **actually**, so this in our experience this one loop IGC was a complete failure **actually** in some sense.

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Comparison with conventional Three-loop and One-loop IGC designs

SIMULATION RESULTS FOR DIFFERENT INITIAL CONDITIONS

Cases	Conventional Three-Loop Design		TSS-PIGC
	Zero Effort Miss (m)	Miss Distance (m)	Miss Distance (m)
1	652.97	0.3141	0.2241
2	596.62	2.0196	0.6835
3	452.29	1.7659	0.6732
4	525.02	0.8401	0.6310
5	478.139	2.8238	0.6691

SIMULATION RESULTS FOR DIFFERENT INITIAL CONDITIONS

Cases	Conventional One-Loop IGC Design		TSS-PIGC
	Zero Effort Miss (m)	Miss Distance (m)	Miss Distance (m)
1	55.27	39.767	0.1311
2	53.17	32.147	0.1556
3	27.15	11.88	0.0919
4	50.49	24.85	0.1458
5	44.82	14.56	0.0945

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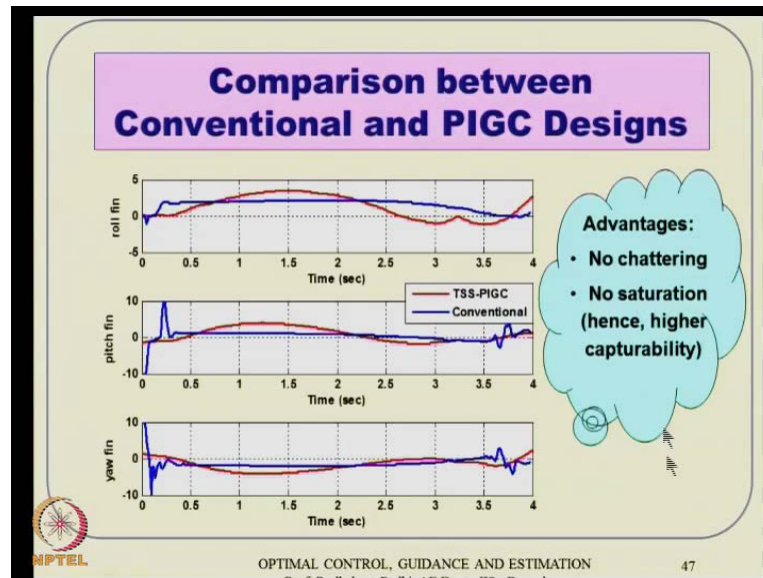
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And see the numbers here, see this conventional designs if you have this zero effort means, without any corrections it refers to be outer of 500, 600 kilometers like that, 600 meters like that half a kilometer. Then no matter whether you **whether you** implement this, **this** conventional IGC, I mean conventional guidance and control three loop design then also the miss distance is very low.

If you use our thing time scale separated partial IGC this is also very low but, if you compare them very closely, these are these are lower than these values **actually**. Whereas, compare the one loop IGC these are 50 meters, 40 meters like that then only it works as **if**, if you happens if initial conditions are shows that initial zero forty minutes is of the order of this high half a kilometer like that does not work. It does not, I mean correct their vector **actually** anyway.

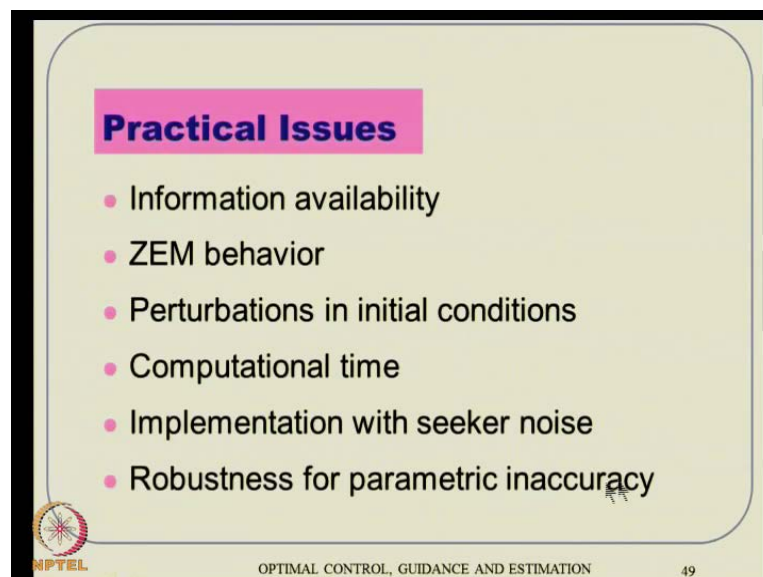
So, coming back to this, **this is the** if this happens at least then there is some correction, I mean 55 become 39, 53 become 32, 27 becomes 11 like that but, now we are close to what it is this is a less than 1 meter level accuracy, what you get it here **actually**.

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That is the more important part of it these are again comparison between conventional p IGC designs in terms of spin deflection sense. And see, the momentary deflections here are quite large, where is the red line is smooth and then, small everywhere **actually**. Essentially no chartering and no saturation and hence you can think it has higher capability.

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And all that **actually** now coming to some real quick practical issues, as I told in the beginning there are **there are** issues some like something like information availability

resuming, so much information feedback are they available or not. Then the ZEM behaviors we referred finally what happens their, then perturbations initial conditions, computation of time issue, implementation with seeker noise and robustness parameter in accuracy.

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Practical Issues in Partial IGC (PIGC)

- 1) Is all information required for PIGC implementation available?
- 2) Is PIGC successful for perturbations in initial conditions (including process noise)?
- 3) Can the computation be done in real-time?
- 4) How does the ZEM plot (holding control at current value) behave?
- 5) Is it robust wrt. parametric uncertainties?
- 6) How to deal with seeker noise?

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So, this questions are like that you can it is a kind of put it in a question primer sort of thing.

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Information Availability

Information used	Available means
$u \quad v \quad w$	INS
$q_1 \quad q_2 \quad q_3 \quad q_4$	INS
$x_m \quad y_m \quad z_m$	INS
$x_t \quad y_t \quad z_t$	Seeker through $\Delta x \quad \Delta y \quad \Delta z$
$\gamma_t \quad \psi_t \quad v_t$	Seeker through $v_{xt} \quad v_{zt} \quad v_{yt}$

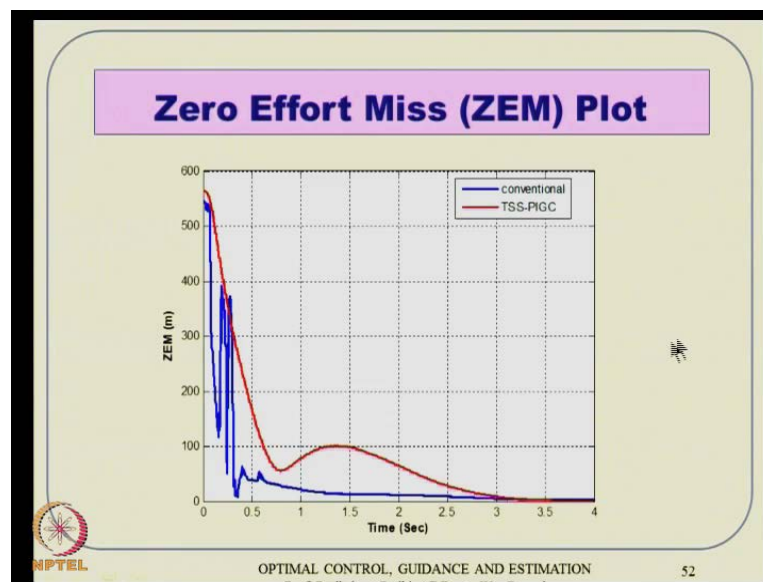
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So, the information availability issue sense what is what you are thinking is this you all these things are equal for implementing **actually**. Now the good thing is this u v w and quaternion main components as well as position these are all available from the INS system and INR system is typically very good, I mean you do not really need a filter in the loop also for they blindly rely on this information **actually**.

Now, also need these information target position and target velocity vector direction and all that, that is typically available through some sort of this seeker information. But, also remember this anytime what we want this information, you would also utilize some sort of a filter design and typically that is on through canal filtering. So, that part we not talking here, so remember we are fusing guidance and control, we are not touching the estimation loop.

So, but, typically this information's also used **in the** in whatever best possible way you are using for the conventional design, probably same thing can be thought about extract this information and then properly that **actually**.

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What about ZEM behavior you can see the conventional loop, it is very lot of settling here initially at lease. I mean this is the real reason what why you required lot of battery power consumption and think like that **actually**. Your delta dot requirements will reflect all sort of things but, anyways we will not go too more detailed into that.


What happens here is if you deflect the spin deflection and do not do anything for the rest of the time. Then you have some miss distance that one you collect keep collecting and then float **actually**. If it happens to be smooth then the design is much better **actually**, and you can see compared to the blue line, how better is the red line **actually**.

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Neuro-Adaptive Design for Robustness Enhancement

- Problem:
 - The DI design (used in inner loop) is sensitive to modeling (parameter) inaccuracies
- Solution:
 - Enhance Robustness by augmenting the DI with "**Neuro-Adaptive Design**" in the inner loop (as inner loop is more sensitive to this issue)

i.e. $[P \ Q \ R]^T \rightarrow [P_d \ Q_d \ R_d]^T$


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Now, what would to do with the robustness part of it, the some issues like sensitivity with respect to the modeling or parameter in accuracy thinks like that. So, for that part we propose that you can incorporate this mirror adaptive design, in the inner loop only basically.

So, that means this rotational dynamics, where you generate the spin deflection if that that is done properly, then everything else will be taken care and that is typically done through this mirror adaptive design.

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Neuro-Adaptive Design for Robustness Enhancement

Desired output dynamics:

$$\dot{Y}_d = f_{Y_d}(X_d) + G_{Y_d}(X_d)U_d$$

Actual output dynamics:

$$\dot{Y} = f_{Y_d}(X) + G_{Y_d}(X)U + d_Y(X)$$

Objective: $Y \rightarrow Y_d$ as soon as possible

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And what is the philosophy there we have this \dot{p} , \dot{q} , \dot{r} or this \dot{y}_d sort of thing is desired dynamics **actually**. And this dynamics can be thought about without any perturbation without any **any** accuracy issues and all that but, the actual output dynamic is different it has this **this** disturbance term in other words this gets generated from this parameter inaccuracy and think like that.

And it happens to be a kind of critical issue for any aero dynamically controlled vehicle, because aero dynamic is some part of it we will spind out a tabulated data. And the confidence level will be fairly good but, you expect some 10, 20 percent error all the time **actually**. So, that is the part which will result in this \dot{y}_d and also they feel and I feel some neglected dynamics some think like that, it also appears in the form like this. So, the concept here is we know this too, so the idea is can we do something so that \dot{y}_d will go \dot{y}_d very quickly **actually**.

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Neuro-Adaptive Design for Robustness Enhancement

- Dynamics of auxiliary output:
$$\dot{Y}_a = f_{Y_a}(X) + G_{Y_a}(X)U + \underbrace{\hat{d}_r(X)}_{\text{NN Approx.}} + K_a(Y - Y_a)$$
- Strategy:

The diagram illustrates the control strategy. It shows three nodes: 'Actual State' (Y), 'Approximate State' (Y_a), and 'Desired State' (Y_d). Solid arrows point from Y to Y_a and from Y_a to Y_d. A dashed arrow points from Y to Y_d. The text 'Approximate State' is above Y_a, 'Actual State' is below Y, and 'Desired State' is below Y_d.

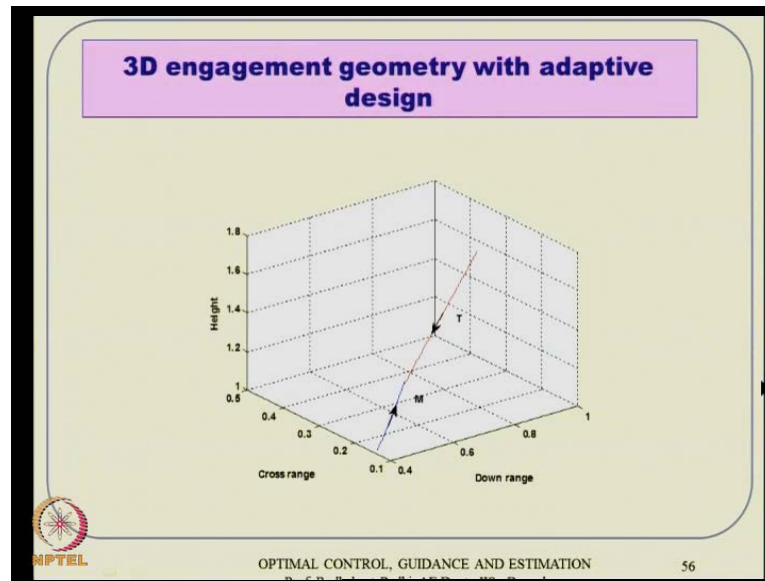
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So, this not cannot be done directly because this part is not known, so the whole idea is can we know it first. And that knowing part has typically done through some neural network approximation, so whatever you do not know here, that one is we attempt to do that using a neural network **actually**. And once you do that and there is some sort of additional term for making it a kind of a completely observed dynamic sort of thing **actually**. So, then this term is helps us in **in** having this smaller bounds and think like that also basically and those of you are interested you can this 2009 GNC paper a double like GNC paper.

And you can see on that **actually**, the whole idea once you put it here this term as an approximation and neural network and this additional term is their, this is not same as anything like this. But, our objective lies here but, typically it is done in two ways, that is Y should go Y a and Y a should go to Y d that is the design. Whereas, the objective wise Y should go to Y d basically what we do is, we force Y to go Y a and then Y a go to Y d and Y a to Y d is done directly in that part.

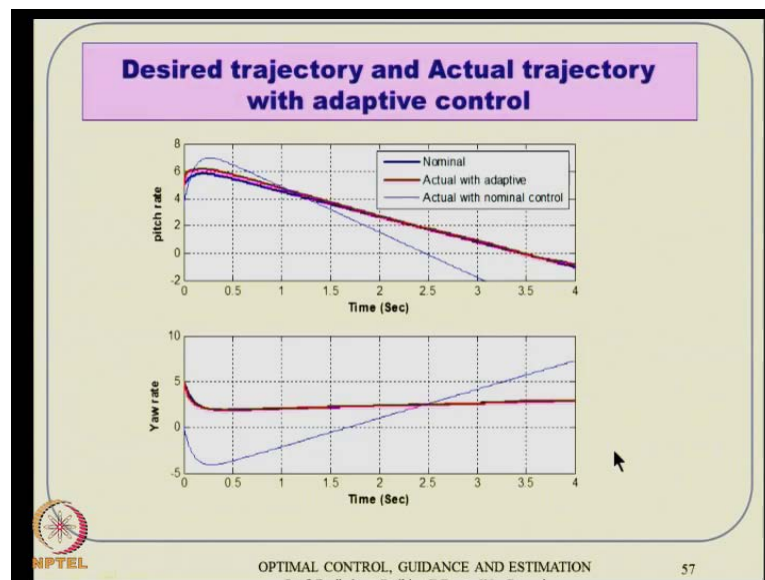
But, Y to Y a is typically done, so this design think like that **actually**, details I will not say it will different branch of mathematics all together using think like that way.

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But, I did not said that this is the engagement scenario of with adaptive design, with parameter in accuracy of vehicle sitting there, as part of the modeling **actually**.

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Now, you can see the difference here and this is the three plots here, one is axial with nominal control, this is the one more to nominal is a first is nominal control apply to the nominal plan. So, this is thick blue line what you see here whether if you do not do anything do not do any adaptation and think like that. And then you will operate simply based on feedback, that your feedback formula will be same, what you are state will

come from actual state and think like that **actually** but, even then the **the** see the deviation **actually** the requirement is somewhere their but, the actual happens to be far away from their and that is here also here it also.


And because of that there will be heavy motive penalty for the miss distance. However if you put the adaptive control break into action, then what happens is this red line, what you see here is **actually** very close to thick blue line, that means nothing as appendix **actually** subject to the this d is approximate very quickly **actually**.

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Results: Miss distance study

Sl.No	Case	Nominal(m)	Adaptive(m)
1	$[X_{cp_{max}}, C_{N_{max}}, C_{N_{\delta_{max}}}]$	10.09	1.049
2	$[X_{cp_{max}}, C_{N_{max}}, C_{N_{\delta_{min}}}]$	10.09	1.029
3	$[X_{cp_{max}}, C_{N_{min}}, C_{N_{\delta_{max}}}]$	40.34	0.651
4	$[X_{cp_{max}}, C_{N_{min}}, C_{N_{\delta_{min}}}]$	40.34	0.649
5	$[X_{cp_{min}}, C_{N_{max}}, C_{N_{\delta_{max}}}]$	37.36	2.800
6	$[X_{cp_{min}}, C_{N_{min}}, C_{N_{\delta_{max}}}]$	13.24	1.430
7	$[X_{cp_{min}}, C_{N_{max}}, C_{N_{\delta_{min}}}]$	37.36	2.809
8	$[X_{cp_{min}}, C_{N_{min}}, C_{N_{\delta_{min}}}]$	13.24	1.424

Outcome: Substantial enhancement of robustness!



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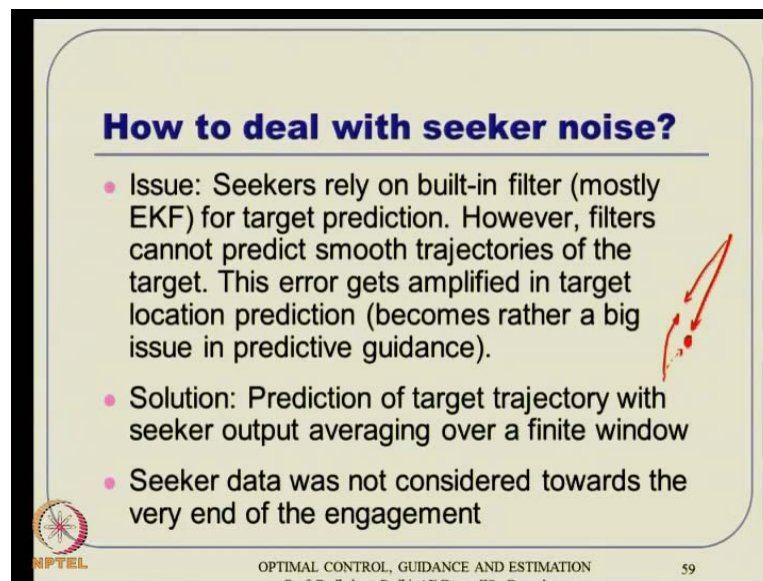
Now, these are perturbations that we have are the parameter that are effect in vehicle dynamics widely. So, we have taken some nominal and some minimum maximum value some sense think like that and then you can see that, if the nominal design is degraded earlier remember it also sub meter level accuracy, within one meter and all that. Now, if you parameter equation like that, then the same thing will result in higher mist stance 30 meter, 40 meter like that **actually**.

But, if you put the head of control break and to extent and then again it come back within 1 to 2 meter **actually** maximum 2.3 meters like that. So, you can see the number different and you can claim that very easily, you can see that there is sustentations announcement of probation **actually**. The miss distance directly, if it not that the vehicle is getting fragmented it is roll at very high it is gone unstable and think like that.

Even though it may operate in little bit stable manner, it is going somewhere else not towards the target thus whole difficult **actually**. And we cannot effort 10 meter, 30 meter, 40 meter miss distance this problem does not allow that **actually**; you remember you have zero miss distance or something which is very close to zero **actually**. And also remember somebody calls zero miss distance not physically zero, it is between somehow relative radius of the target and the radius of missile.


If you take them together anything that any number comes below, then that can be considered zero miss distance, because then it result in typical **actually**. So, anything that typically less than 1 meter, 2 meter probably depending again on target on missile radiance and all that. So, you can think about it zero miss distance less than 1 meter, within 1 to 2 meters or 1 to 3 meters around that figure, you can think about something like here zero miss distance, that is what is happening here after, you put the adaptive control great **actually**.


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How to deal with seeker noise?

- Issue: Seekers rely on built-in filter (mostly EKF) for target prediction. However, filters cannot predict smooth trajectories of the target. This error gets amplified in target location prediction (becomes rather a big issue in predictive guidance).
- Solution: Prediction of target trajectory with seeker output averaging over a finite window
- Seeker data was not considered towards the very end of the engagement



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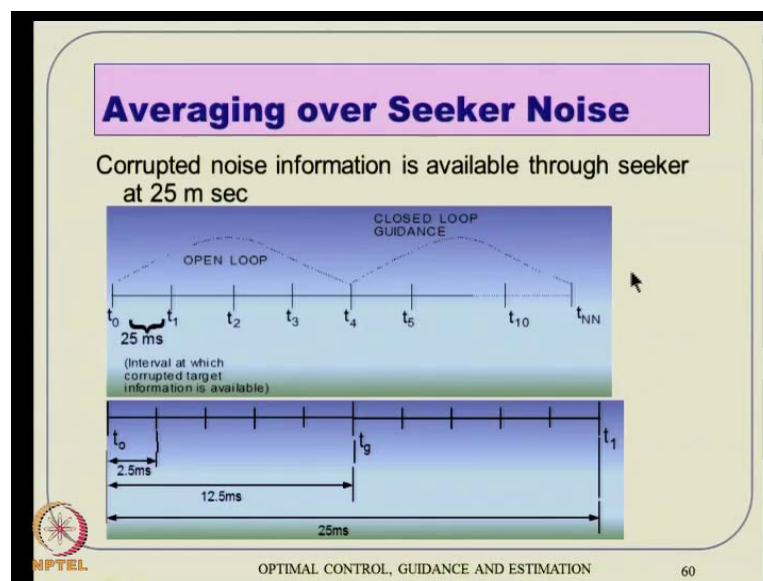
Now, there is another issues that works out is how to deal the seeker noise it ultimately no matter, what common filter EKF. Whatever you put their will be some sort of residual error **actually**, it is not never filter of any filter does not filter of noise totally, there will be some degree of in accuracy of the output of the filter. And remember these are the predictive logic PSP, MPSP all that **actually**. So, any amount of velocity directions error sitting there, will result in lot of three four questions **actually**, because if you are aiming

if you let me picture of that. This is the target here, we have estimation tells that something here but, if you small error in the angle there, then it may come out with something there.

So, your vehicle will typically goes towards that, where as the real target is somewhere here. So, that is that is that is the difficulty if the real target is somewhere here, your guidance should actually taking towards that, not towards the other one actually. But, as for information content is their it is telling that it is going towards that direction, obviously it gets miss guided actually anyway. So, how do handle this, so then the idea here is we have, what we have telling is prediction of the target trajectory is done seeker output averaging over a finite window actually, I will demonstrate that in picture actually, will take little bit finite window.

And then do kind of some sort of averaging of whatever value you are getting over a period of time and then silt that period and then keep operating as in that actually. And also remember that towards the very end seeker data becomes very high in I g and think like that because of several other issues like other things like there. We do not talk about that towards very end less than 1 second and to go and think like that we really do not take up for that. We simple propose that we can that we can go prediction mode prediction mode actually, each is still in divisibility in a way, how far we can tolerate how less you can tolerate, that becomes still a kind of topic of reasons actually.

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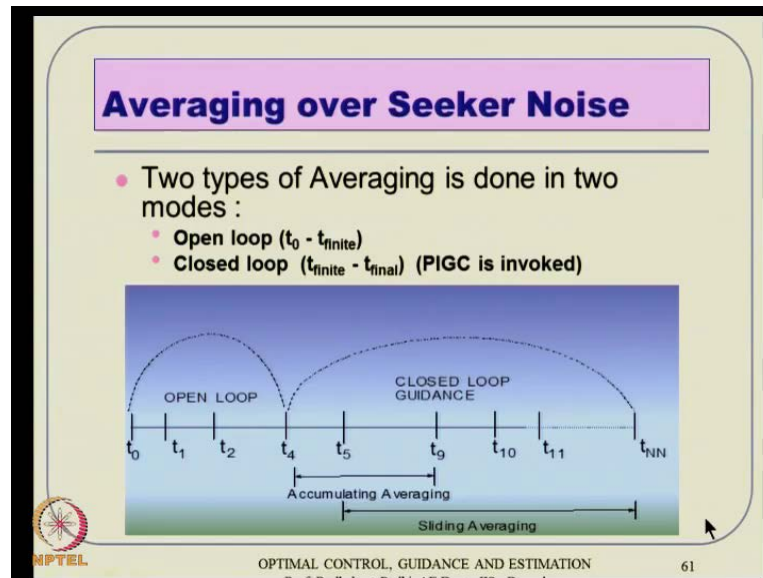
Anyway coming to coming way to that this is what is done. So finally you remember 25 mille seconds, what is assume for the seeker of data, so what you are telling is first five data will simple observe, will not react the guidance direction will not happen, will simple observe what the are target is doing which direction is going. And one this guidance look closer happens t g we already have some at least some spinite window data simple **actually** some four five data depending on what level the mechanism want to do **actually**. So, here will have take some t o, given some information will projective to from t 4, t 1 as given some information will projective to t 4, t 2 as given something will project again to t 4 like t 3 to t 4.

And t four still going on to some data, so everything what you are getting t 4 you can just try to kind of average it out and this particular case, everything will head off and then divide by five. Then that will be much better information compare to what information we got only at t 4, this noise averaging is not happening always **actually**. And then from t 4 to some spin infinite window will keep accumulating, so that means up to here it is less than t 0 to t 4 then here when you go will have this five data, one more data averaging and then one more data will accumulate, accumulate.

So, then after while you cannot keep on accumulating thus there are too many data's **actually** and also make sense kind of remember the most recent data, rather than getting what we have to do that apply very far away. So, we do not want do that the recent data suppose much more accurate think like that, so what you do that here is this accumulation loop, there is the guidance closer loop, this is what just missile loop will do no guidance in action. Then further the there will be some sort of accumulation averaging and all missile way start reacting to that **actually**, and after while there is spin finite window and that spin finite window will flied **actually** in other words.

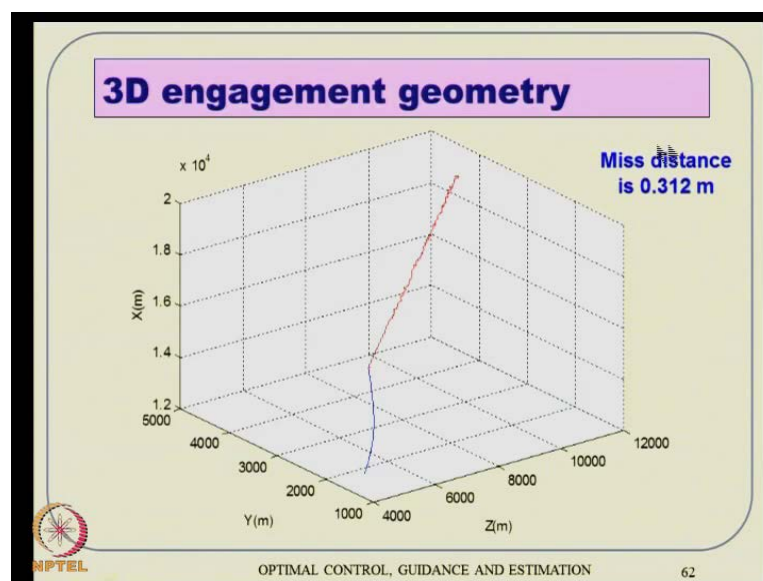
One more new data will come this the previous that the last and the very first data will be more **actually**. The recent data will kept and the very last data will be ignore and then the window will be silting **actually**, then do also averaging, then you take it, that means when you talk about t 10. Let say then what is happening here is t 0 is projected to t 10 in production mode, t 1 is projected to t 10, t 2 is projected to t 10 like that **actually**. And everything happens to t 10 that s the average out everything out of 1 divided for 10 alright. So, that becomes the target information and based on that information we get machine **actually**.

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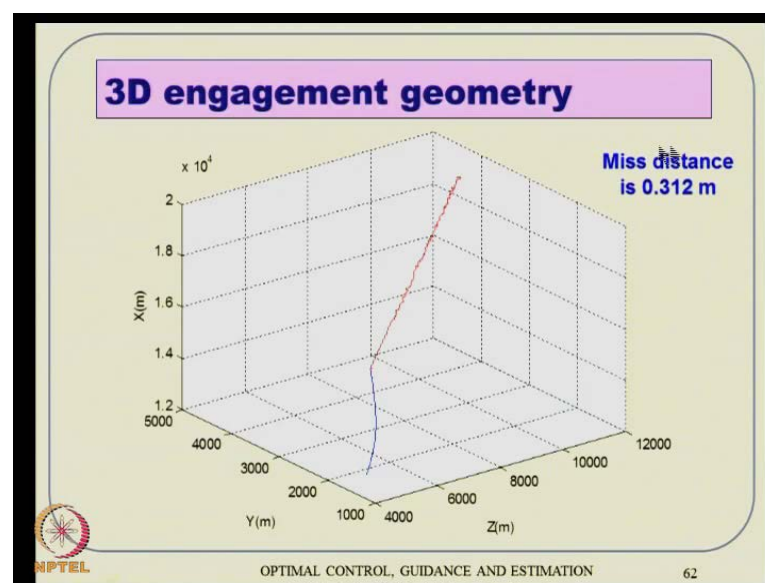
So, this is all explain in the wordings and all two types of averaging first is open loop then is close loop and all the picture will tell you. There will be open loop, there will be cumulate averaging in t_0 to t_9 , then there will be silting averaging in operation and then this operation see us little bit before the real t basically. Because, towards the end what you observers is if the data has not been corrected by that time, then there very very less time to do anything **actually**. So, it is does not do very good job **actually** after that, so we will leave of little bit time before t_f , more easily simple operate on the prediction mode.

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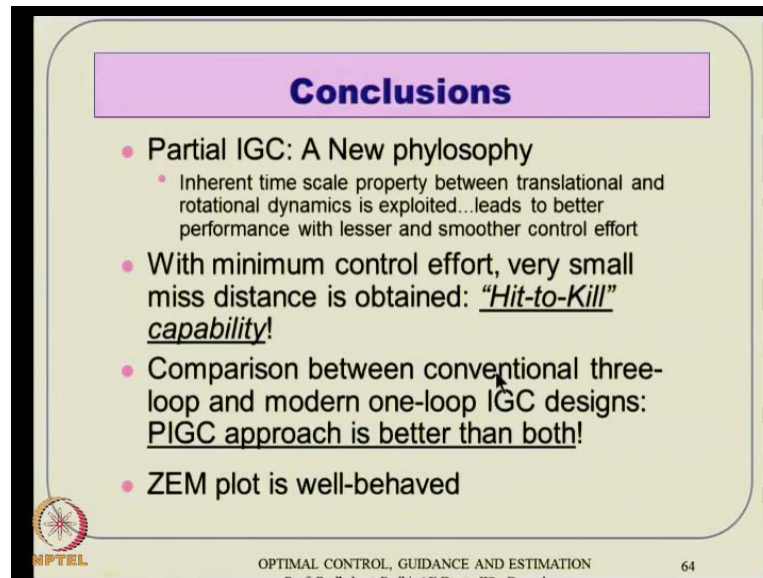
Alright this is what you see after doing this, remember this is not a straight line any more but, this is little bit setting here and this is the information aids sin by the seeker information **actually** anymore. The real target an go still in straight line what our seeker will keep on telling something is like this **actually**, that is does not matter still operates based on the information that we have towards the end no more from here to here, there is no further information **actually**. When we just operate its prediction more and this kind of logic the miss distance happens to be 0.3 meters very small and then, if you see this averaging and all that.

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(Refer Slide Time: 59:41) So, that different value of the miss distance these are the various things, it take various random cases and all. And then average it out and see what happens in the main some sort of model lessons **actually**. So you can see that the maximum miss distance that we observe 10 above 4 meters is very small.

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Conclusions

- **Partial IGC: A New philosophy**
 - Inherent time scale property between translational and rotational dynamics is exploited...leads to better performance with lesser and smoother control effort
- With minimum control effort, very small miss distance is obtained: "Hit-to-Kill" capability!
- Comparison between conventional three-loop and modern one-loop IGC designs: PIGC approach is better than both!
- ZEM plot is well-behaved

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In conclusion sense it is **actually** personalize some sort of a new philosophy, essentially what happens here inherit times to suppression prospective, we can translation outer dynamics is exploited. And hence it lead to better performance with lesser and smoother control loop **actually**, with minimum control effort, very small miss distance obtain that means it result in some sort of hit to kill capability of a the vehicle.

And in away it a our claim it is happens to better than both **better than both** conventional design and as well as one loop IGC design **actually**, also observe that seem plot is very well behaved.

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Conclusions

- Information required for implementation are available
- Successfully verified for perturbations in initial conditions of both missile and target
- MPSP/MPSC (with “iteration unfolding”): Very less computational time.
- PIGC with (DI + N-A) in the inner loop: Very good robustness for parameter uncertainty, without compromising on performance

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And the information required from implementation are available and successfully verified for different perturbations without any separate cast to cast I sense. And then somebody thinks that this M P S C is cannot be use us, then it still operation hydration and all that, still we can have some sort spin infinite, hydration operation and that also experimented with just one iteration at a time and the result also very good **actually**. So, essentially leads to very less computational time as well.

So, PIGC with D I, N A in the inner loop this neurotic in the inner loop, it also gives us lot of robustness for parameter uncertainty and all that **actually** without compromising on the performance too much. Again the references are like this any of your interested you can see the some of the references and get more details out of it **actually** that is all for this lecture thank you.