Optimal Control, Guidance and Estimation

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Module No. # 08
Lecture No. # 18
Linear Optimal Missile Guidance using LQR

Hello everybody; we will have a different lecture today of application of optimal control theory. So far, you have seen linear quadratic regulator design lot of details about that and then we have also seen this flight dynamic entire detail actually we will go back and try to kind of relate these two concepts and then can ask the questions whether you can do this missile guidance using L Q R ok.

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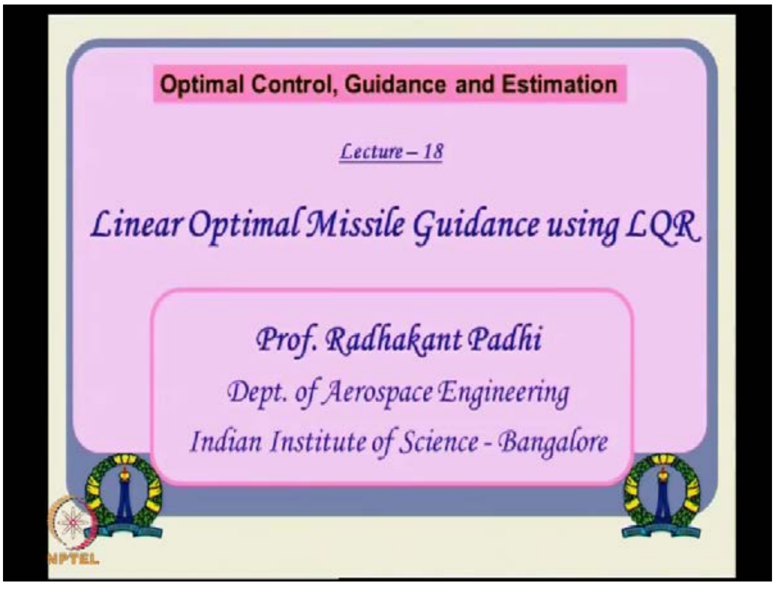

That the topic of lecture and let us the way quick glimpse of what all we can do through L Q R theory and very nicely it will fall down to the same classical control I mean, cases and all that we will derive it and so, and one of that we have already seen in one of the previous lectures actually. But, here we will have a different case derivations and something like that and alternatively you will be able to see little more generalization of that concept. In addition, to what you all classical theory's actually when classical missile guidance theory gives us alright.

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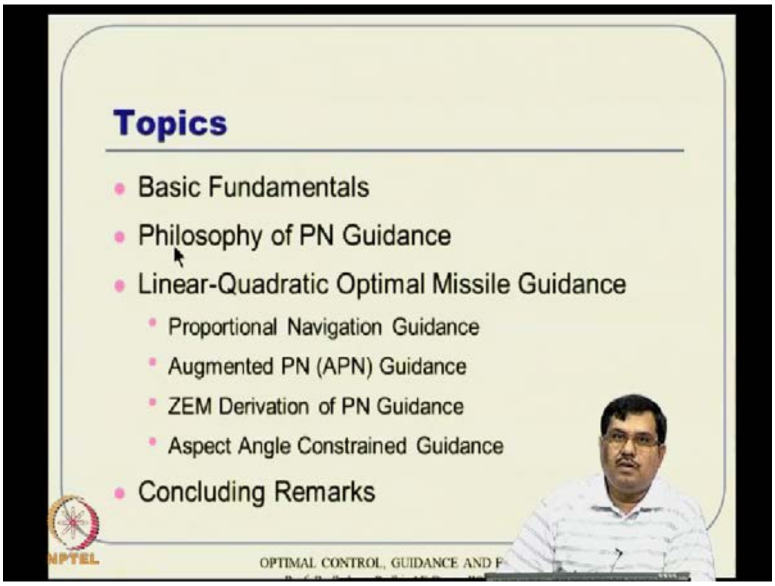

The topics of this lecture is something like this a little bit basic fundamentals of what are missiles what are the classifications what are component things like that then one quick glimpse of what is this philosophy of this. So, called proportional navigation ok then, we will try to revisit the L Q R theory and then formulate the problem in the frame work of linear quadratic optimal control and come up with optimal missile guidance actually.

Then in that framework we will study various classes actually mostly in proportion 2-D proportional navigation guidance augmented proportional navigation guidance then the same derivation through this 0 effort missed concept of the P N guidance and also aspect angle constrained guidance actually and have some concluding remarks at the end of the lecture. Very quickly some basic fundamentals of missile and then components like that.

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So, classification of missiles you can have various classifications and the very basic things that comes to mind is either you have surface to surface or surface or surface to air to air to surface.

And somewhere here talks about surface to surface that can either be strategic g can be simply tactical. So, strategic missiles have typically large compact the normally carry I mean some sort of nuclear was ead and things like that they are normally not in use very rarely used and we. So, far know that only one time of was used in second world war that is all actually what, but tactical missiles which are limited impact, but target impact and that is where the uses become much more because we do not want have collateral damage and we do not want to have unnecessarily locked damage and things actually. So, that is where tactical missiles come.

Then surface to air is largely defensive in nature see if we if you know for safe that some enemies attacking around that before it does some harm we have to find out and nullify that is the surface to air sort of missiles. Then air to air is all I mean, it is also a kind of a defensive missiles I mean it is something like air come back missiles. Then, air to surface is lastly attacking sort of things actually. So, all sort of things classification somebody can do various things in a different sense.

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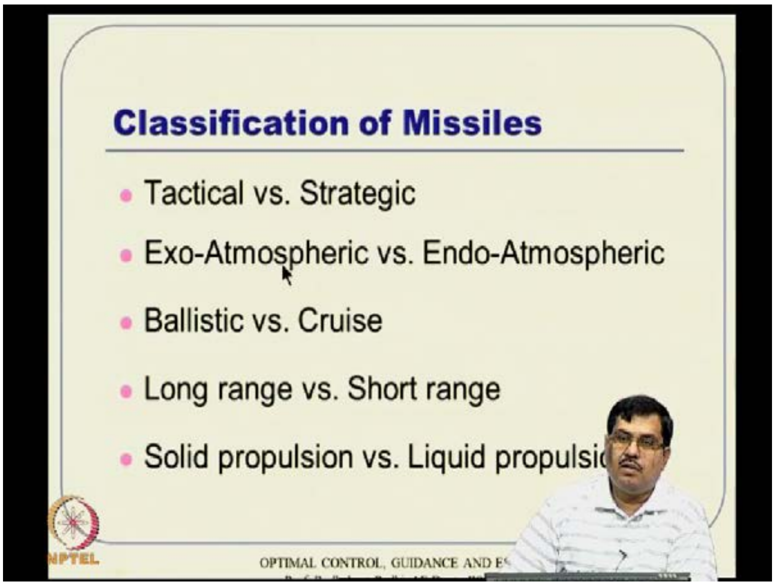

They can also be classified in different ways; somebody can talk about tactical versus strategic. I have already talked a little bit about that; then somebody can also talk about exo atmospheric versus ends atmospheric; that means, finally, the interception happens beyond atmosphere; that means, aerodynamic forces in means are not valued there ok. Alright, we can have endo atmospheric engagement where still talking about engagement with in atmosphere actually ok.

So; obviously, also dynamic force moment reply a large role there then we can classify from ballistic versus cruise. So, ballistic missile is largely follows are called ballistic projectery in other words the most one is largely influenced by only gravity or purposely by drag only they cannot be any left like thing like that alright. Then on the other hand there is a cruise missile is almost like aircraft very close to earth it travels and then travels per short range long range depending on the mission and things like that largely if travels because of the lift like aircraft.

There are various pros and cons; obviously, I mean ball missile it can corer large distances with less energy where as cruise missile is asceptive I mean, if it if is like good amount of stiltedness actually; that means, because of the earth curvature unless and until the vehicle is very close to the target we cannot detect it actually. So, that advantage of missile I the air and then somebody can also talk about from long range versus short range depending on hero far you want to go like that and then there are criterion like solid propulsion and versus liquid propulsion and things like that and that the major departure by the way the various issues we with respect to solid propulsion and liquid propulsion thing one of that we will talk little bit when we go ead in the lecture series little later actually.

So, and this is not the end of the story because, various classification also exits other then these things and it does not matter. So, much actually the fundamental physic remains the same and the fundamental ideas remains the same in a way one depending on the nature of the problem one depending on missiles require requirement things can be different actually.

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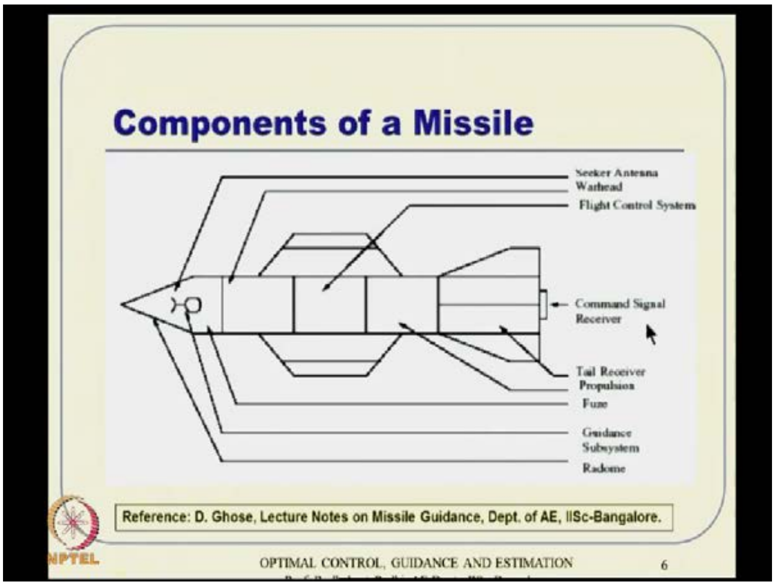

So, typically components of a missile we can you classify something like command signal receiver at the end which can receives the commands from ground station order something is like that and then we have various components at of a vehicle very tightly packed by the way the inside thing the space is very limited and then it is very tightly packed typically.

Here, you can see this add on thing this which is this actually protects the vehicle from low dynamic loading around that and this part is a very sensitive part this is where all your seeker and then I mean this sensitivity is towards if actually and next to that is ear head and then the flight control system a can take care of some of here and somewhat here and then 3 pins here things like that aerodynamically activated surfaces. Then you have this propulsion system somewhere here and then this configuration is not unique by the depending on the designing part of it actually. But, lastly this components are the part of the vehicle if has to have a propulsion it has to have a guidance subsystem if has to have seeker system it has to have various other things varied of course, I mean if it has to have various other system that is necessary for a missile lecture actually; alright, most of operation.

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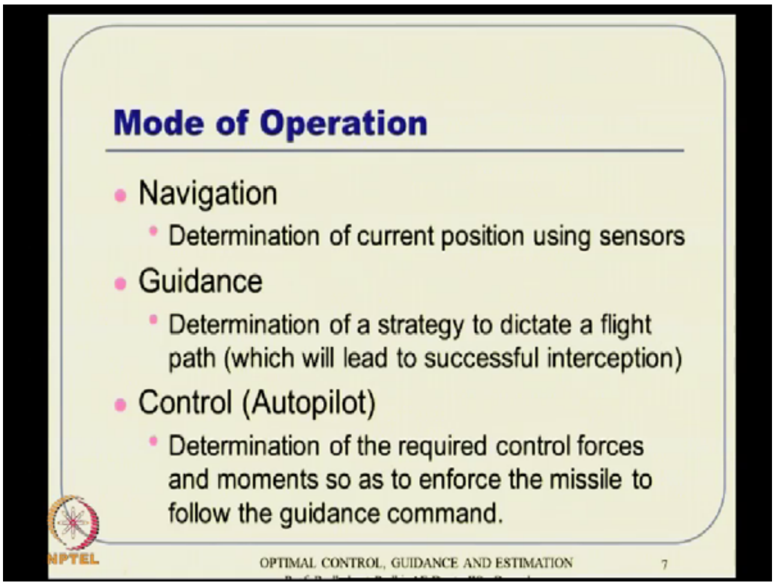

Typically, when we talk about we are not bothered too much about propulsion structure since, like that we are all bothered about navigation guidance and control that is the primary motivation for as to see something actually. For the navigation is nothing but the determination of current position. So, it all that it does is using some sensors and that is a big task of course, using some sensors it finally, finds out where the current location is large typically in the dimensional good at pun actually.

So, that this so called higherness sensors in (()) GPS system and we have this diffuser of this you got to know may things are there which actually tells us the current position of the vehicle and in addition to that the current velocity vector coordinates the vehicle as well if does not talk only about the position ok. The position and velocity it will gives us actually and both in an I mean, in a transverse the motion as well as in the rotational motion; that means, altitude and altitude rate also if will tell actually

Then after you know all that from navigation system the next text is guidance. That is nothing but determination of a strategy to dictate a flight path actually. Or in other words, we got objective to meet from point A to point B. So, this guidance system sill come up this strategy to dictate a flight path actually. So, the aim; obviously, ambition of the flight path is to leave the vehicle for a successful interception ya. So, the navigation followed by the guidance and that is what lastly we talk in this lecture and after that is done and the next task is controlled; where lastly it is known as auto pilot in the I mean, in the missile community and it is that I mean the objective here is determinate of the required control forces and moments. So, as to enforce the missile to follow the guidance command.

This gives the status of the vehicle I mean the navigation system guidance gives the way to go flight path to take and all that and the control autopilot essentially generates all the necessary control forces and moments to ensure that the guidance path is followed actually if means that whatever the guidance is going is command it is actually getting trapped by that of the pilot system.

So, all the three are uni generally and if one system tends somewhere the entire will go by it actually. So, everything happens should have lot of correlation and everything is important actually for a successful mission for this particular lecture we will largely concentrate on on guidance actually.

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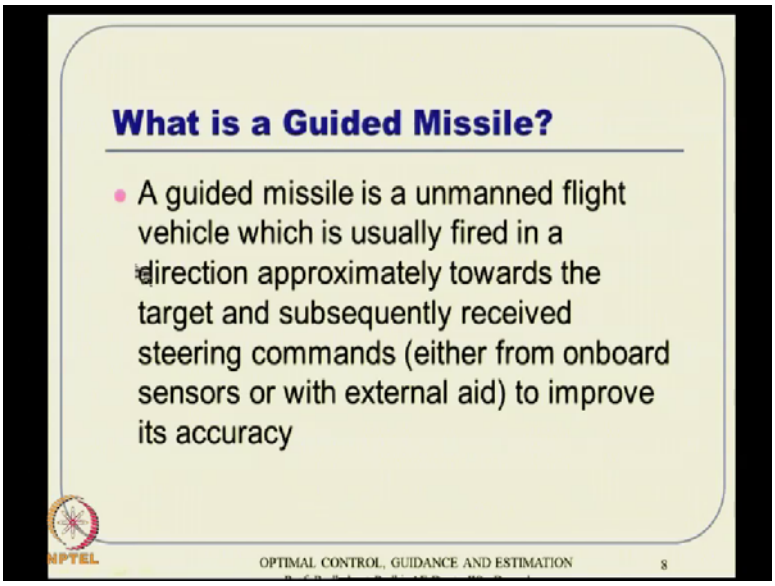

So, what is guided missile formal definition somebody can write if this way a guided missile is a unmanned flight vehicle which is usually fired in a direction of direction of approximately towards the target. Of course, we will not fire in the reverse direction which is fired usually toward the target direction and subsequently received the steering commands to compare its accuracy and the steering commands can come form on broad sensors or with external aid also basically that so; that is guided missile concepts is actually.

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Basically, and again you can various guidance laws has been proposed in the literature over several years and these few accomplished things with tactical missile guidance law and all that when you can have versions classical or empirical calls you can various mathematically formulation and that is what we are going to do this lecture particularly then you have this optimal control class of guidance laws and we also have predictive guidance which we will slightly do later in the course and then the differential games ideas also which information there is no scope to about it here.

Are there also nice areas where the capability of the target is equivalent of the missiles capability thing like that. So, the target gave here is equally capable basically is that those situations the differential game guidance becomes much more useful actually ok.

The kind of extension of optimal control theory is you can that we will not talk about too much on that lastly we will concentrates on optimal control guidance things and then subsequent lectures we will talk little bit on predictive guidance which is very effective actually.

Anyway conway to that the classical and empirical thing can also be classified in some sort of 2 ways one is conceptual sort of ideas and implementable sort of ideas actually ok. So, something can be conceptual means its starts with very neat ideas and all that and its all mathematical very rigorous and its very appealing rather or they may be some sort of practical implementations difficulties and all that actually. So, those things will be slightly modified and then the some then this sort of the branch results actually ok.

So, you have this P N class of guidance if you see P N falls in here proportional navigation is true proportional navigation and pure proportional navigation and if you also land up with this theory you will also land up with this t P N P P N and augmented P N also basically. So, we will see that in this particular class have these things which is already been proportional from gementrical conseclution can also be mathematically rigorously shown that thay are nothing but optimal guidance law actually alright this is the summary of all the guided laws.

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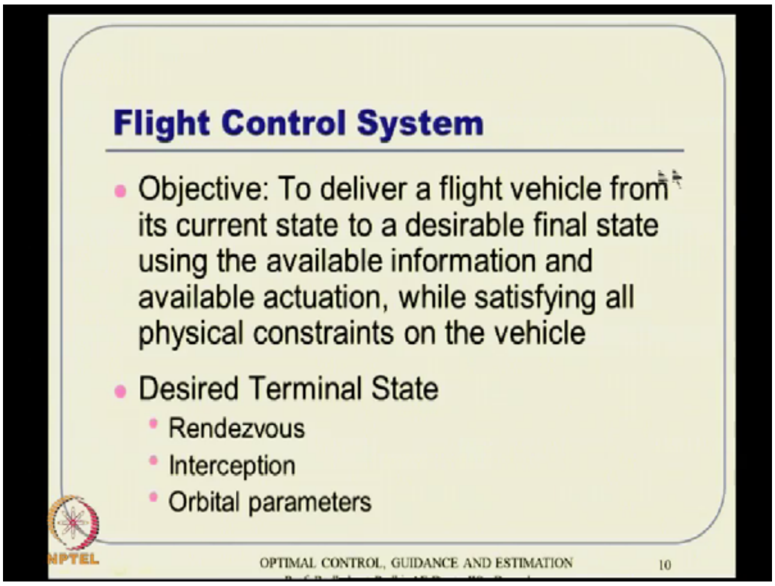

And, in general our visions what is broader picture of a flight control system the objective is to deliver a flight vehicle from its current state to a desirable final state using the available information and available actuations while satisfying all physical constraints on the vehicle. So, cannot talk about ignoring these things physical constraints such as a like a angle at a constraints related angle at angle of constraints or even various body rate constraints security constraints all sort of thing will happen actually. So, those constraints is to be honored and with in the those constraints it is to deliver the flight vehicle from its state to its desirable final state; obviously, it is to be the target interception paint and all that actually alright. So, that is the objective.

Then, what are the desired terminal state then can be several application in different ways one can be can be rendezvous problems. So, that one can simply some and join together and continue to go together; that means, position since there may be close to each other or velocity sense there is no relative velocity between them actually.

So; that means, they continue to coexist sort of thing the very good applications for example, aerial refilling and the rendezvous fighting control and things like that actually. Then this is the interception where you are we are all interested in this lecture to nullify that target with appropriate interception; that means, velocity vector need not relative velocity should be as high as possible while the position vector should be 0 actually this is interception and also like some desired terminal state can be orbital parameters when somebody talks about just launching a satellite. So, what is the final of final desired condition that is nothing but any orbited parameters on the orbit from where it will continue to circulate, actually. That is why depending on the various flight applications there are few application we can talk about various flight control systems actually.

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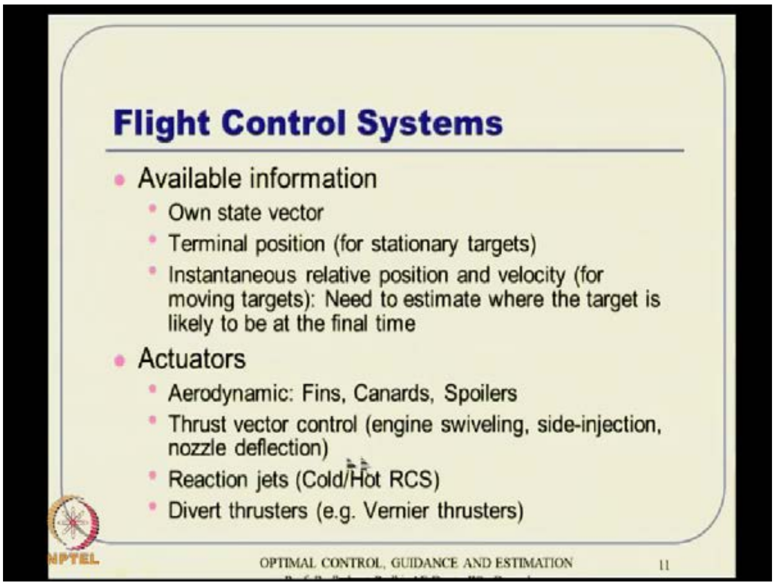

Now, if talks about available information what are the available information first of all won state vector if will log through the own rich sensor information with this information is typically very good because the sensors are very smart these days with any they helped if in the capability is much higher and then the noise component is almost negligible actually.

So, own state vector is very much reliable and that what is available from the any system or into entirely their navigation system from the vehicle actually. Then the other reliable information is terminal position for the stationary targets or the instantaneous relative position and velocity for the moving targets either using a radar or seeker or the things like that ok.

So, in this case these is a need to estimate where the target is likely to be at the final time and that is where you want to go not to the current position of the target, but where you want if will go at the final time that is where you want to go actually. So, that is that is what missile guide concept is then there are actuators various sort of various types of actuators aero aerodynamically control actuators such as fins, canards spoilers and things like that which are typically useful within end atmospheric invasion actually where you can generate a lifts moments and then manipulate that and there are very good because normally these aerodynamic forces moments are what you call as available free actually well not to be very free because you have to deflect fine and things like that, but almost in the sense the lift vector is anywhere there you just have to linger a little bit and then you get a huge impact on your vehicle actually.

So, that a very little effort, but the output is very good actually. So, where ever aerodynamic controls are available they recommend as must to go for that actually then there are thrust vector control like enquire swiveling side injection I mean the mechanism nozzle deflection mechanism think like that by which the thrust vector is purposefully deflected away from the missiles excel l missiles excesses I mean, excess basically then we get lateral component of that which will give you some sort of moment to term actually.

Then, there are reaction jets can also talk about cold RCS or hot RCS like that and again depends on the cold RCS just pressure gas and just let if go through expand through nozzle. Whereas, hot r c s means you kill a small locate engine sort of thing where actually burn and get your thrust and then deflect it through different jets and all that.

Anyways, then there are divert thrust which is Venier thrusters these are typically located at the cc of the very final correction at the end actually if rae they are not the correction does not happens through moments here if directly happens through the I mean through the translation like motion of the vehicle Vernier thrusters critically pull the place close to c c. So, directly it will translate wherever you want; obviously, the capability will be lesser. So, people do not do it just like that, but the very end suppose you want to correct the small ever then it is worth doing if actually.

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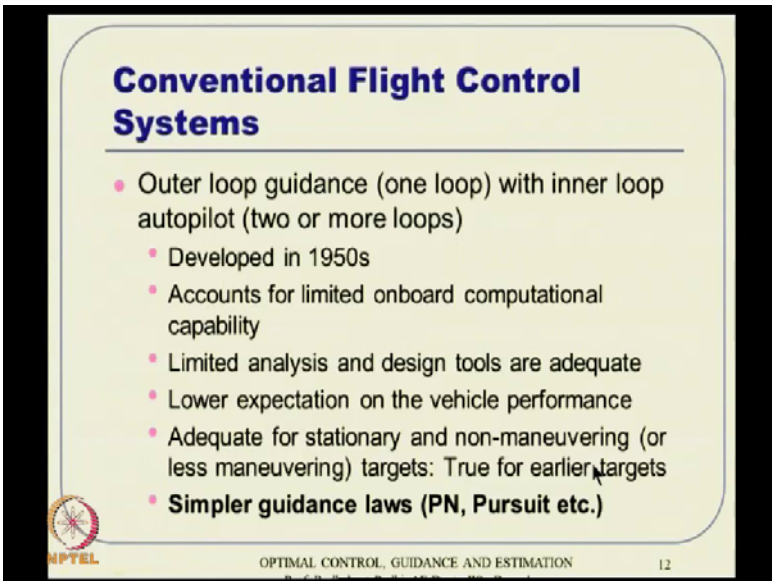

Alright, see then conventional flight control system is typically designed this way where this outer loop guidance. Obviously, one loop with inner loop auto pilot; that means, whatever guidance command gives let lets listen to commands and things like that inner loop divided if can be synthesized either into loops or more then 2 loops actually ok.

So; that means, 2 loops means certifically the guidance gives us body and gives us lateral isolation commands sh the guidance loop will give later less less commands and then the inner loop autopilot will convert that first body rates and negatives your rates and then that is one loop and then with in your guides using control surface reflection that one more loop. So, minimum 2 loops are necessary in that and more in that we put some times integral feedback for a bet better performance and then you have detected the loops inherently sitting like that, think like that.

So, you have at least one loop the guidance and then inner loops are there at least 2 loops in the autopilot of this concept almost developed in 1950's where many missile guidance ideas got developed and then if also account for limited on board computational capability because each of the loop is small subsystem sort of things. So, you do not have to computationally board on your system then limited analysis and design tools are adequate because the dimensions of the system is not lost complexity is not lost things like that and then the lower expectation on the vehicle performance was also effective for such as synthesis loop actually.

And, toward the end of this course we will also see that this concepts can be relaxed and people not talk about integrated design approaches integrated guidance control thing like that where this loops can be minimized as much as possibly actually ok then the next point is adequate if turns out to be typically adequate for stationary and non-maneuvering targets or rather little bit maneuvering; that means, less maneuvering targets also it will resolve because if if guidance law and control design gets several updates on the way basically that why if happens.

And ultimately the if actually results in simpler guidance laws typically this P N for this P N guidance for simpler guidance like that it will result in that these are very simpler guidance like that it will result in that these are very simple to implement on basically.

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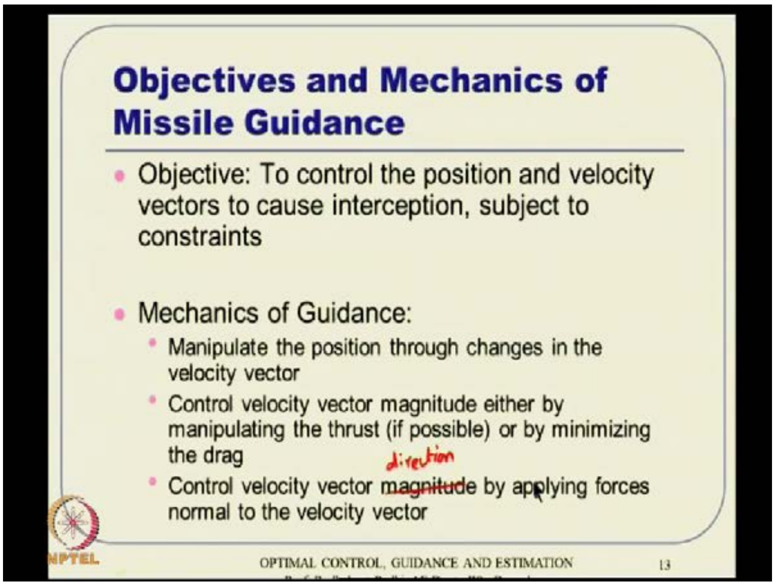

Whereas, the objectives and mechanics of the missile guidance objective is to control the position and velocity vectors to cause interception subjects to constraints that state forward objective. And mechanics of guidance is to manipulate the position through changes in the velocity vector that is how you can influence that. So, you to manipulate the position through changes in the velocity vector, but how to do the changes in the velocity vector that is through the control.

The controlled velocity vector magnitude either by manipulating the thrust if possible or by minimizing the drag also his and the way of doing that is only for reqenorousment and things like that actually I can also change the control velocity vector magnitude by applying forces normal to the velocity vector magnitude by applying forces normal to the velocity vector that is the well vel control the velocity vector direction like that sorry this is not magnitude, but this is direction ok.

You can always control the velocity vector direction and that is what is done typically lastly applying forces normal to the velocity vector and that is where the course correction happens actually.

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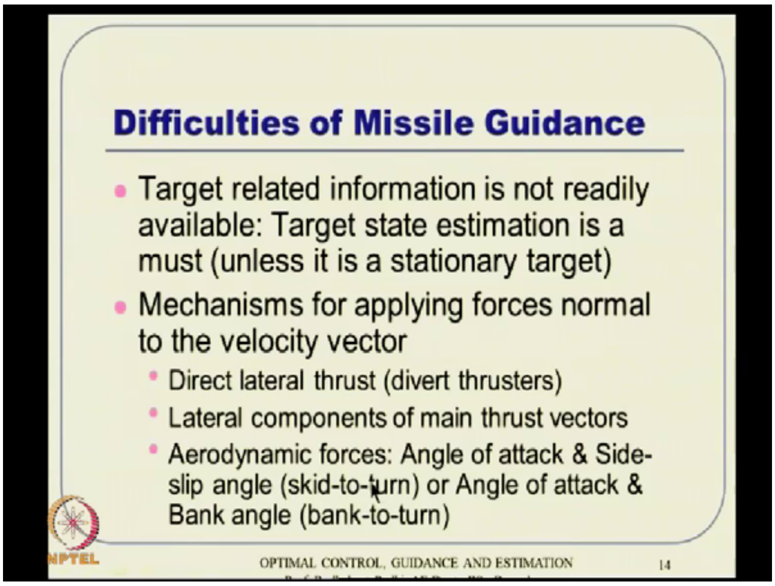

Alright, what are the difficulties of missile guidance first of all target related information is not readily available target related information is not going to give target information free actually. So, obviously, we have to estimate the target information that the must actually.

In spite if it turns out that the final missed distance is the strong function of target estimation behavior property actually that is very important factor we should not miss out actually we they we are not talking in this class. Anyways then, what are the mechanisms for applying forces normal to the velocity vector we can have a direct lateral thrust something like divert thrusters.

Or you can have a something hub like lateral component of the main thrust vectors noise that s thrust deflection and all that if will deflect the thrust there will be a lateral component which will generate moment and all that. And also there is aerodynamic forces that can be generated by this angle of attack and side slip angle if we can generate both together especially we can do that when 2 planes are symmetry like 4 fins and all that in a missile setting which is possible.

So; that means, you can consider the side slip angle something like angle of attack and then it will be like both angle of attack side slip angle can be generated in utilizing the control surface deflection and that concept is called control surface deflections and that concept is called skid to from and suppose that mechanism is not there. Then, the vehicle has only 2 big wings and all that is also possible and especially for long duration practical things typically if you do that to minimize the drag actually.

So, when you have something like that you have also longer time for doing your corrections and because of that this concepts called bank to turn where you name the control influence is given to the vehicle through angle of attack and bank angle commands actually.

So, either you directly generate for fin deflection and generate 2 moments here and directly go there called skid skid to turn or you have the bank in the appropriate duration and then apply the control. So, as so, that you can go towards that and that is called bank to turn actually. And lastly I mean, it is typically done skid to turn in missile things and other applications such as aircraft or air control thing like that this is there typically bank to turn actually ok.

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There are several undesirable consequences also everything does not happen. So, nicely that we like to say and then first of all lateral force components are not happening at the c c then obviously, there are and undesirable moments actually and largely that they also result in non-minimum phase behavior especially when you have tail control vehicle and then the vehicles are also designed aero dynamically unstable they are not design stable vehicle because they are design they are design unstable because to have better l o d ratio lift over drag ratio lift over drag ratio and better maneuverability because lift will be high and all that actually hence the motion arising from the manure force may destabilize the vehicle also that mean if do not do a very careful exercise the vehicle inherently unstable.

So, if you excited little more and then it will go to rotational un stability very quickly actually and also remember that large at lateral acceleration command is a sad reality; that means, a targets are becoming more and more powerful they have more and more very incapability and think like that. So, we do not have the luxury any one not to have large lateral commend resolution is I mean this modern type missiles are typically they have to have large lateral acceleration especially this defensive tactical missiles actually.

So, for example, some air to air missiles also talk about 20 30s z of lateral resolution that is quite large actually. That is the sad reality and where there is the rapid command response is a bad necessity also. So, that is where you have to be extremely careful that occurs something some effects will not come which will be destabilize the vehicle actually. These are typically difficulties of the missile guidance theory.

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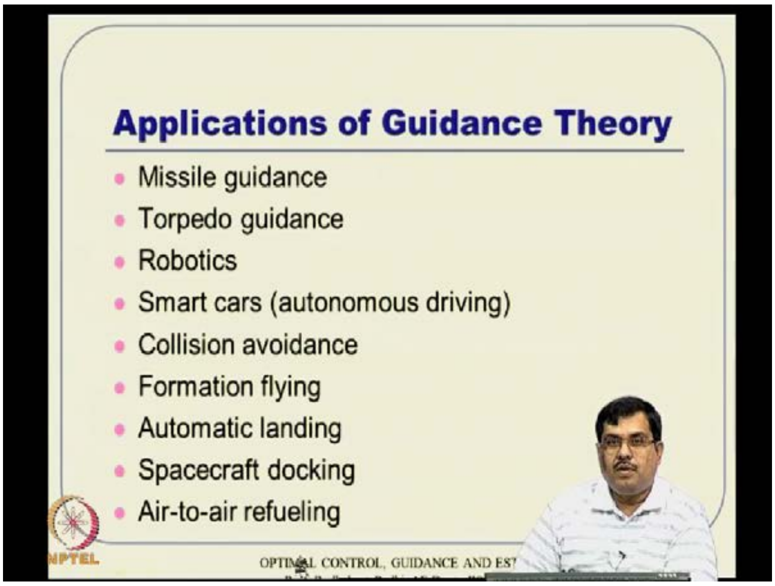

Then applications of guidance theory is at I mean, before you go ead it is good to see this because even though without lesser then generally to have talking about missile guidance here for the same concerns it can be used in several variety of applications including for torpedo guidance robotics smart cars even collision avoidance (Refer time: 27:00). That means, missile guidance is all about collision, but you can also have problems exactly opposite where you can talk about collision avoidance has safety. Basically, safety considerations and also used the similar consideration formation flying automatic landing spacecraft docking air to air refueling and, but not actually there are several is I mean domain are applications rears with this guidance theory is very much useful actually.

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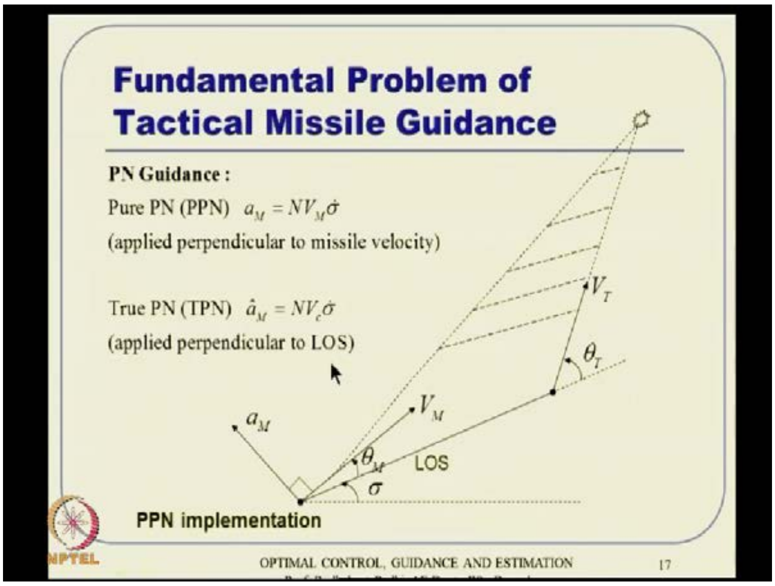

Alright. so, let us provide with little bit fundamental problem of tactical missile guidance and see some of the concepts very quickly to something the engagement is typically like this you have a target which is running away from in this direction of the with the velocity v t and there is a changing vehicle which is called as missile which is having it is velocity vector v m and ultimately it has to and hit the target.

And, normally it does not happen automatically the whole idea here in the very beginning sense is something like this if you continue to go in this direction if may not to go because this may run away. So, you have to you have to take a little bit course correction and the direction changes essentially. So, that ultimately both the vehicle as well as the target will go towards the same intercept point actually finally, they are intercept each other.

So, this course correction is typically done by applying a lateral acceleration command which can either be perpendicular to the LOS vector or perpendicular to the missile zone velocity vector. Obviously, if it is proportional to the LOS vector things as to be things are mathematically good vector because, ultimately this guidance relation the philosophy that the LOS vector should not rotate, but initially it can rotate after some time this LOS vector instantaneous position one for the after in position one for the target position 2 position 2. Here, if you can keep on changing that then this LOS vector should remain parallel to each other there should not rotate actually.

So, that is the concept that we was used in skid guidance and all that before and that is where the concept the terminology is barrowed like navigation when of this is a guidance concept nothing do with navigation really basically.

Anyway. So, this are this are the 2 variations if you are am is if your aim is to make sure that the L O S vector dose not rotate then the; obviously, the lateral acceleration command and that needs to be applied these should perpendicular to L O S actually that is where it is mathematically more elegant conceptually that is the reverse the thing, but again practical difficulty of implementation do exists because, if you have something some am something am perpendicular to L O S vector then obviously, it will have a component on along v m and component along v m means component along thrust axis direction typically and that is normally not possible to manipulate. May be drag, but drag is not a control variable; you cannot manipulate that actually. So, the idea here is why do not we apply force which is perpendicular to the velocity vector then which is easy to implement because it I mean, do not have to worry about the other aspect that I cannot really implement the other component actually because by design and defect is something which is perpendicular to the velocity vector which I can generate through my lift generation mechanism actually.

So, these are the 2 concepts and depending on that you can have 2 variations of that and if it is something like perpendicular to the velocity vector it is called pure proportional navigation which is the conceptive like these you have this sigma dot as a sigma is L O S angle which is with respect to arbitrary reference line. So, what to what is more important the sigma dot. So, their guidance law talks about a m equal to navigation constant n which is a number times v m velocity of the missile time sigma dot and in in other hand 2 proportional navigation tells that I will not apply perpendicular to the velocity.

I will apply perpendicular to the L O S rather and my lateral acceleration command and happens to be n v c times sigma dot and it is not v m v m for that v c is the closing velocity along L O S actually times sigma dot again. So, this is the fundamental concept of two dimensional guidance in general.

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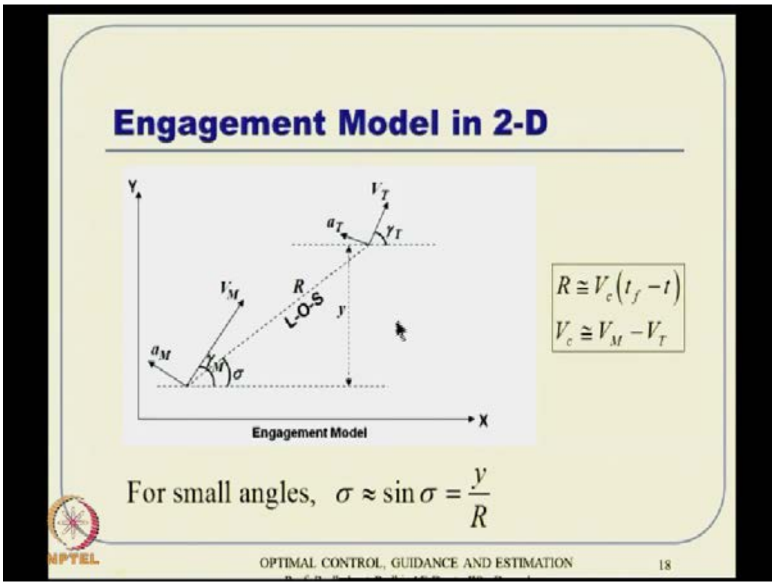

Now, if you talk about incase model engagement models and 2-D and all that you can have the same. So, you can put in actually in x is frame another probably is in a y direction you can see that I can write I mean the reference line happens to be parallel to the x axis other one is perpendicular to that and then you have this sigma which is approximately sin sigma provided sigma is not high that can approximated is by something like sigma is equivalent to y over r and then r is nothing but vc times t f minus t per in time and final time and v c is the closing velocity along final site. So, are is canto v c times star f minus t; obviously, and then v c is nothing but v m minus v t or v t minus v m dependently on the what people write actually. So, if you again depend on what the definition you have, if you typically if you v t minus v m there there will be a sign change in the radiance law later actually. So, v c is the velocity along velocity component long los actually. So, that is some typical geometry and then concepts used initial guidance.

Now, the whole idea have is I can I can nullify this difference between y that is it goes to 0 where is this direction I mean, it should also go to 0 then only it it can happen then one then only it can intercept. So, the y direction error will be killed through the formulation pest of it whereas, x direction error will be killed through appropriate kind of estimation of time to go rather basically. Alright, let us move on to optimal missile guidance how do you formulate in all that.

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Be a little bit content that many classical missile guidance laws are in spite from observing nature something like P N guidance and all that. Then, there are control theoretic based guidance law that are usually based on kinematic and are all linearized dynamics in that is what we are more interested in this lecture to come up with optimal control theoretic guidance laws which are based on kinematics of engagement actually, but they are usually not very effective even though they can do the job of with a loss of void and all that and not be effective in general.

The later subsequent lectures we will see how it make more and more effective actually. So, in general optimal control theory happens to be a natural to obtain the effective missile guidance laws became we are we are equipped with a good formulation of boundary condition at the end initially something happen, but it can imposed boundary condition that we need and that is that is the platform it gives us formulating a problem and getting a solution history of the control variable in other words the lateral acceleration variable actually.

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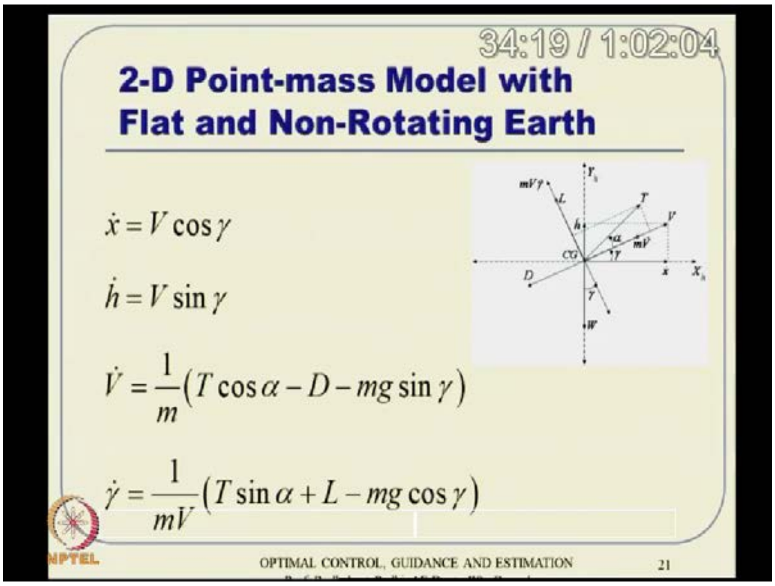

Typically, people talked to about 2-D point mass equations we have talked about that in flight dynamics lecture also that if you really simply flat earth and non rotating earth and it simply 2-D point mass model you can fix a x v system like that and come up with these 2 kinematic equations and these 2 dynamic equations it does not in a this part of the equation does not even talk about target dynamics at what you really interested is having some sort of a formulation where the relative dynamics is accounted for actually between target and vehicle. So, that was see that very quickly and then this is what you have talk is 2-D optimal guidance laws using l a v and here.

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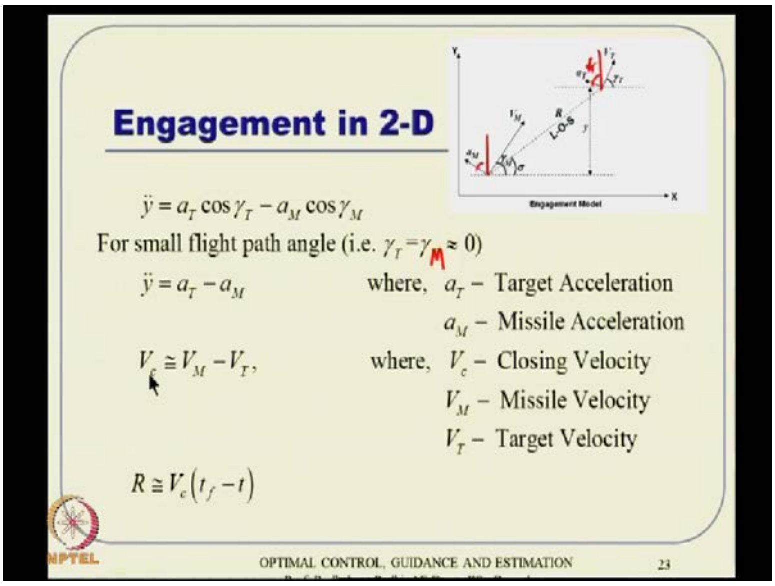

The engagements scenario again it will go back revise it and tell what is my y double dot if I see that y double d dot happens something like it it times in mean gauss gamma t ok. Well, at if we put a little bit if you put vert in vert vertical x is then again this angle is going to be gamma t basically. So, this is because at cross gamma t in the in vertical direction minus am cross gamma n for the same reason actually if you put it here alright. So, we are take about y double t g double t like that again you assume gamma t and and gamma m is very close to 0 then t it is can that gauss gamma is one and you have y double t at minus am this is the dynamics in y direction that will account for these nothing do any particular equation in other words while confirmation should be general formulation while I can generate this at accordingly I mean, a m accordingly basically any way. So, this is small discrepancy also you can see hare that whereas, I double dot is at minus am the concept of that we are going to using in the lecture is v m minus v t not v t minus v m actually.

Anyway, so, you change it if you it really it like to can sign change will happen at the end actually. Anyway, so, this is what it is now let us quickly derive this pure p m value using L Q R. So, the system dynamics is remember which is y double dot is a t minus a m and something like a ma let us not (( )) a small correction here will in a formulation purpose you will also make it compatible rather will tell this is y double dot is a I mean a m minus a t concept actually. So, will plus n minus at. So, like that actually that makes as make. So, a life easier for derivation sort of things. So, this will go ead in the this y double dot with this restriction I mean this constraint and first case that we will talk is at is 0 the lateral acceleration command for the target flat you are assuming is 0; that means, target continuous to move in same direction basically.

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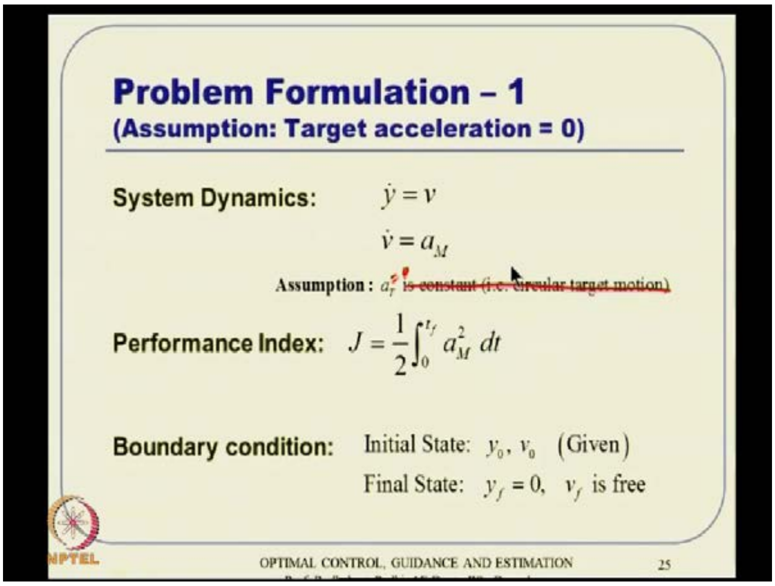

So, when we to target acceleration is 0 then y double dot which is v dot v dot actually y dot is v. So, you double dot is v dot. So, v dot is a m. So, this constraint actually. So, that what we are assuming is I mean, at assumption is actually wrong here. What we are assuming here is a t equal to 0 that is what you are assuming. So, this next case we will see that t is constraint in all that right now at is 0. So, this is why v dot is a m actually.

Now, performance index; obviously, I will like to a minimum lateral acceleration I want to generate a minimize t object to this 2 system dynamic equation very simple actually initial state is known to me y dot v dot and final state is also known to m e what I want to do is y of is goes to 0 where as v f is free actually ok.

So, if this is very standard will can can start and you can put some simultaneous and they like flat coessentially you can see there is also a L Q R problem this is a linear system dynamics is the quadratic cross function. So, you can directly got if using L Q R theory also and that is what we have done in one of the pervious lectures using this state transition solution and thing like that. Now, here we will directly go through this hamiltanious sort of the necessary conditions and all that.

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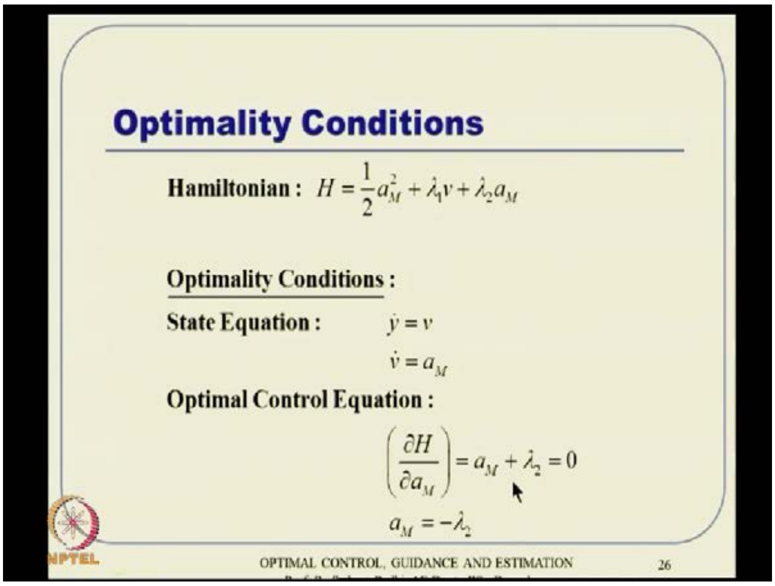

So, we put Hamiltonian like this the optimatic conditions tell as that there three things state equation optimal control and core state. So, this is the state equation and then optimal control tells us that del s by del a m has to be equal to 0; that means, lambda 2 has to be minus a m sorry a m has to minus lambda 2 actually. So, once we get lambda 2 we are done, but how do get the lambda 2.

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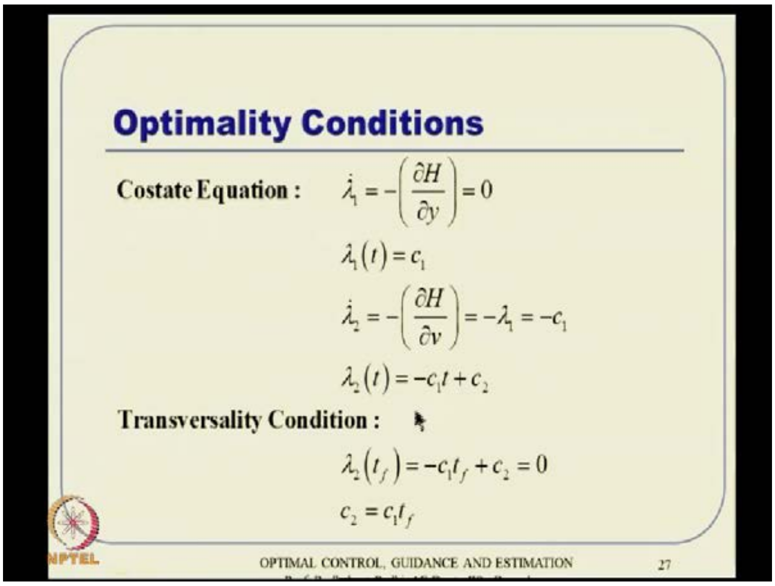

So, for that you have to see the costate equation also if fortunately turns into lambda one dot is 0 because Hamiltonian does not contain y.

Whereas first at is y and Hamiltonian is not a function of y. So, lambda dot is partial derivative Hamiltonian with respect to y which is 0. So, lambda one is essentially constant similarily lambda 2 dot is del s by del v which happens to be minus lambda one anal lambda one is c one. So, you can put c one there the lambda 2 happens tho be this one minus c one t plus c 2 actually.

Then here we can involve this tranversality conditions to evaluate this c one and c 2 sort of things. So, one of the transversality condition is lambda 2 at t f has to be equal to 0 because v of t f is free basically. So, now, lambda 2 of t f is if you put t equal to t f this is what it is. So, you get this constraint equation and hence c 2 is nothing but c one into t f. So, we have a constant, but they are not able to not able to found the values for c one and c 2 then only we have done find c one c 2 then lambda 2 is known and then (( )).

So, how do you get that one equation is available what is the second equation that also you have to impose that y of t f has to be 0 actually.

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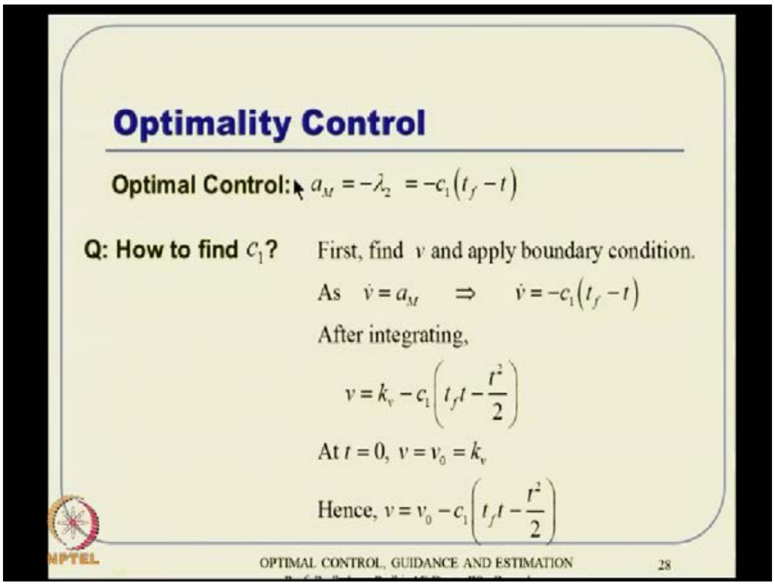

For that you need a solution of these conditions. So, then all come back to the system dynamics equation for where we want to integrate the v dot equation and then y dot equation thing like that actually.

So, we have this little minimization is minus lambda 2 is something like this and c one times t f into c 2 is t f actually. So, now, put it c 2 equal to t f it will turn to be c one into t f minus t. So, that is what it is and then, you can apply first find v and apply the boundary conditions. So, v dot is. So, v dot is a m. So, v is nothing but this one and a m is something like this. So, a m is substitute that we get v dot you got integrate one time and then we will get something like k v that is constant of integration minus integral of that which is c one times t f t and because of t is t square by 2 actually now we apply the boundary conditions that t equal to t not v equal to v not so; that means, k v is nothing but v not. So, k v is once you put that is so; that means, v is nothing but k v k v is v not here. So, v not minus this expression actually.

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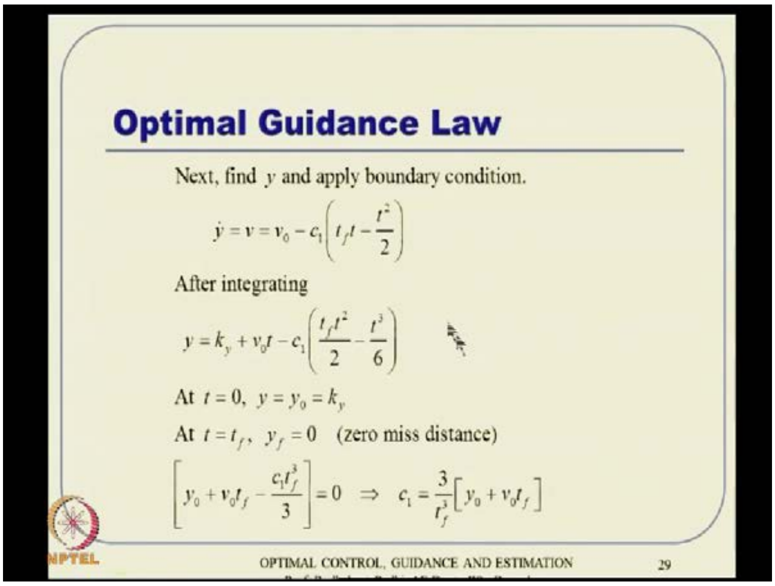

Now, about y dot expression and then tell y dot is v. So, now, v I know v is something like this. So, put it here and then we integrate one more time and tell this is my y actually and this integral constant of integration is evaluated again at the t equal to 0 and t equal to 0 we have y equal y not. So, that will give us that k y is nothing but y not actually and hence at t equal to I mean we got the value for k y now. So, entire thing will turn out to be, but that t equal to t f and y f is also 0 that is the miss distance actually.

So, the one 0 miss distance in y direction. So, you put everything here that is equal to 0 and this is give us the finally, the constant that we are looking for c one actually. So, c 1 into t f minus t that we knows that (( )). So, now, we got c 1 here.

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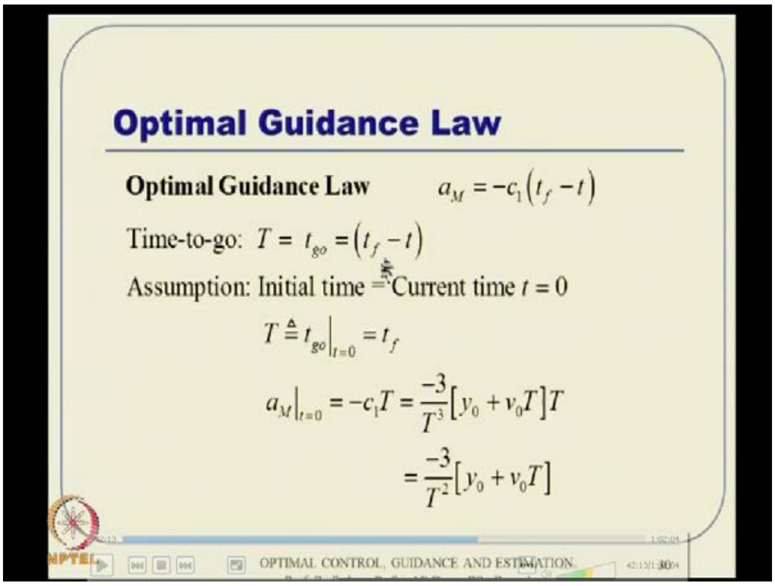

Then, the optimal guidance law you can this is because like that and tell I will define something like time to go is nothing but t f minus t is equal to the least at the guidance literature and also is that initial time is nothing but the current time that is so; that means, t equal to 0 I can assume.

So, essentially time to go sometime it is written a t sometime it is t go essentially with the assumption that t equal to 0 is nothing but t f actually. So, a m assuming that current time is the initial time is nothing but something like this expression actually minus there over time t go square times y not plus v not into time t go ok.

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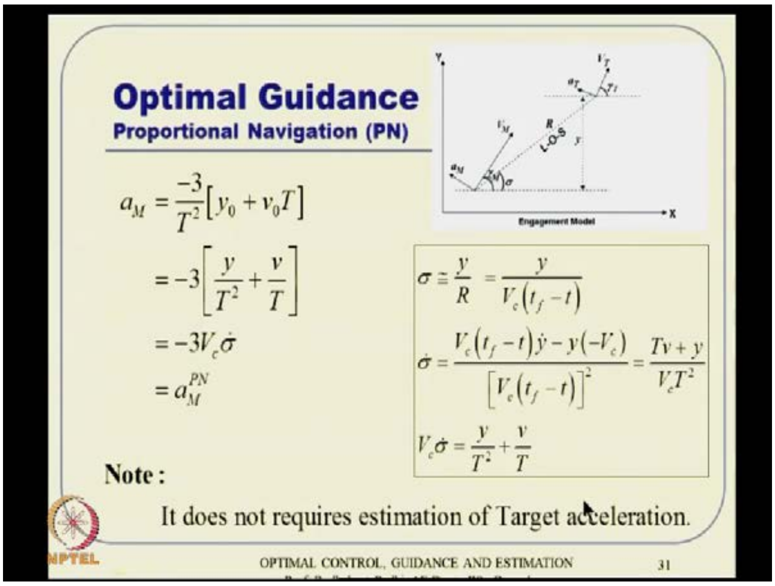

Now, the interesting this. So, happens that if you look at the engagement geometry and then and henceyou have this approximation sine sigma equal to sigma sigma is y over r that what is we mention in one of those like before. Now, if y one r r is nothing but we see closing velocity along r times t f minus t. So, if you take sigma dot sigma dot turns out to be like this denominator times dot of the numerator like the standard formula now it turn out to be like this. So, if you t one e you cancelled out and all. So, and v c takes at this side it turns out turns out v c times sigma dot is nothing but this expression and this expression is something that we are getting here basically.

So, that is why we are getting this kind of thing a m equal to minus these v c sigma dot and also remember thus is nothing but a m and this small sine sine incompatibility if you see that this minus or plus again depends on the definition of v c and here the definition of v c we have use I mean something like v m minus v t, but if you take the t minus v m then; obviously, if is going to be plus actually. So, this is what it is. So, a m what you will end up with and; that means, optimal control theoretic derivation is also the a m P N actually. So, this expression is the expression for this t P N sort of things so.

The beautiful thing about this guidance law is does not require a estimation of target acceleration; that means, a target acceleration is actually a very difficult thing to estimate. Actually, the target velocity I mean, position you can to velocity we has to some extent i, but we have seen the estimating target acceleration is different all come all together actually if (()) is corrected with nice and the information may not be reliable actually.

So, that the idea here is that does not require that. So, whatever error is all is not able to it will respond t that actually; however, if you really know a large component of the target acceleration around this there is a deviation and all this small kind of errors and all it is actually some extent to implement that and then c.

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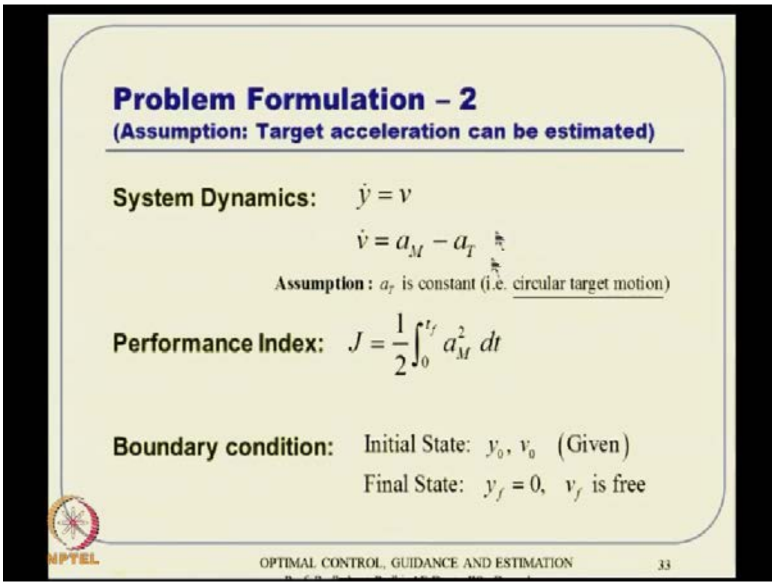

Actually, that leads to the concept of augmented P N and where we are not we are telling that a t it is not really 0, but a t is somewhat constant it is a bias term about this some there are errors and all which I mean I may I may ignore get sincerely I will not ignore the bias component actually.

So, it means a t the constant component on that event actually. So, if that is the case then this is something like this assumes that at he constant; that means, it user and wherever something (()) some lateral acceleration is constant it results is some sort of a circular motion behavior. So, that is what we called it is a circular motion target and assumption and all that actually. So, anyway, so, the difference here is some sort of a m it is become a minus a t everything else is same actually.

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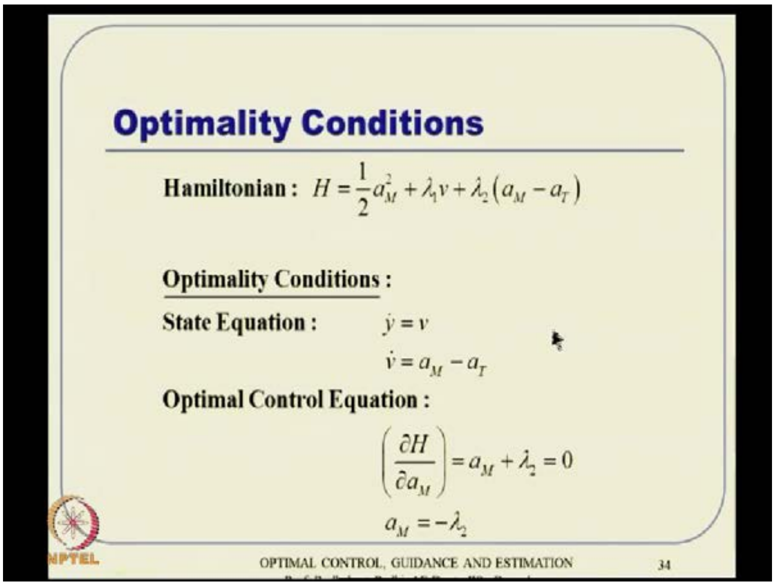

So, v dot is become a m minus a t, so, accordingly there will be changes everywhere and the term Hamiltonian is nothing but that is a m minus a t. So, now, and the nevertheless the optimal control is still minus lambda 2 that is not affected.

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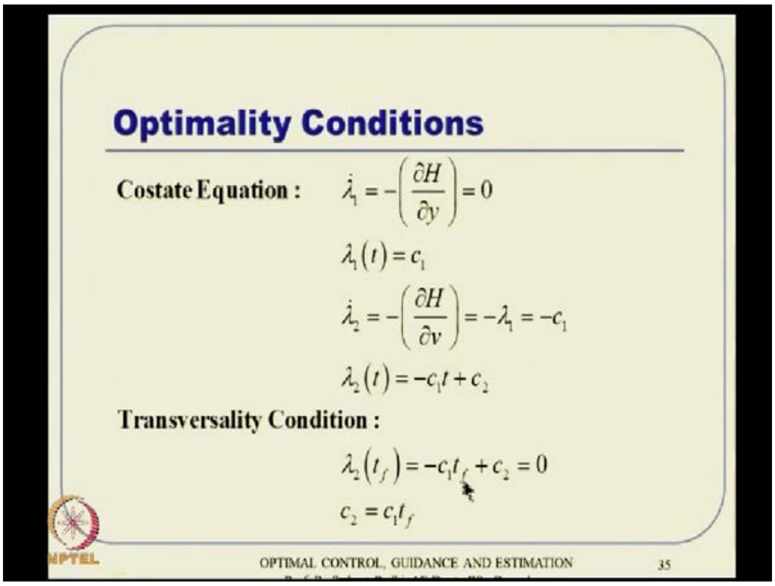

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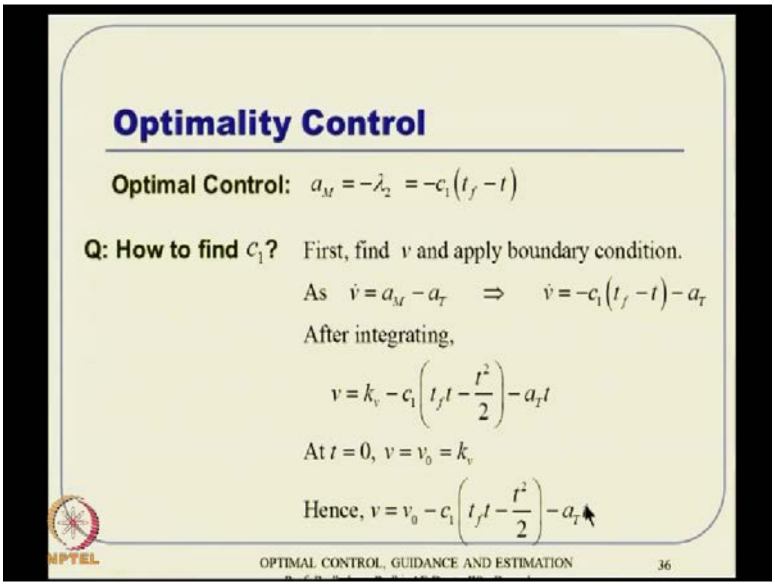

Now, the same philosophy we start with lambda 1 dot lambda 2 dot solve it and lambda 2 will turn out to be like that again tranversality equation you put same constant will get and then your aim is nothing but that what the value of c 1 will be different here compared to what we have before that the boundary condition is different actually, alright (()).

So, follow the various similar steps and when it really comes to the final boundary condition it also reveals same the only difference is this v dot actually v dot is this minus a t component it coming off and that complicates the matter will be. So, now, we can talk this call this v solution also contains this minus a t term and hence every solution will also contain this minus a t term a t times t.

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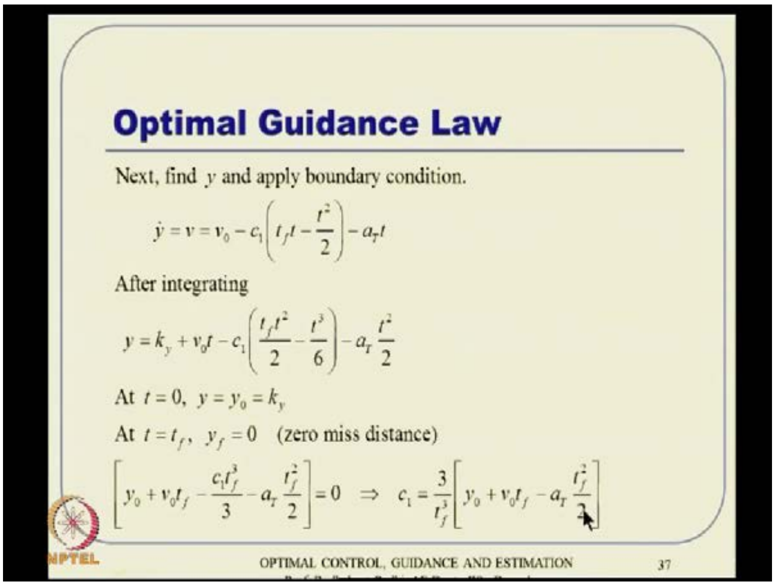

So, but the procedure essentially remains same. So, finally find the y again apply this similar boundary conditions and you will end up with this kind of expression where c 1 turns out to be an expression containing fairly similar to what we had one component and there is a additional component actually, ok.

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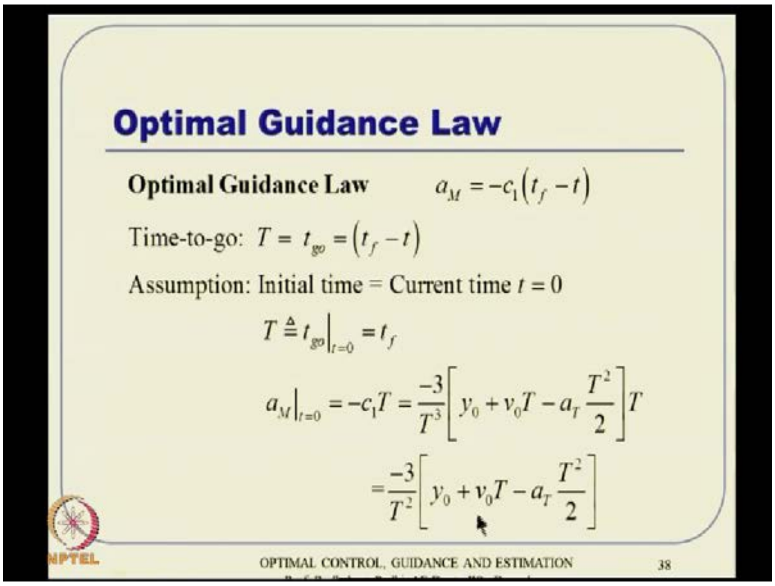

So, then optimal guidance law accordingly will turn out to be like, so, you go back and put everything there and a m at t 0 for t 0 means current time actually and sort to be in this expression form.

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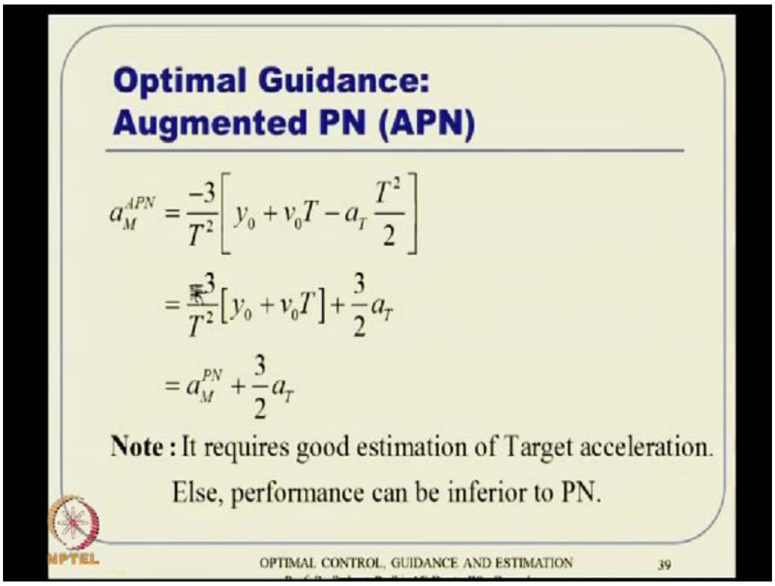

So, if you really simplify try to simplify a little t it contain the first of component which is which happened something before you can for clarity you can put it something like a t P N if you want to here. So, that is so, augmented proposal navigation command guidance guidance command is nothing but the same command that comes from (()) at least there is a component which comes from the target acceleration three by 2 what is actually. So, this is good as long as the target estimation itself is good at least the bias tam is good actually, but if that is not good that the performance can lie infector to P N also you know that actually. So, when we when we talk about P N I mean in this entire optimal control talk about a P N anyway ok.

So, if it is I mean the target information is good then this is suppose to perform better, but if the information is bad then the perform is worst actually. Very quickly you can also see that z e m sort of ideas that is what is used very repeatedly and very vast amount is the missile guidance literature.

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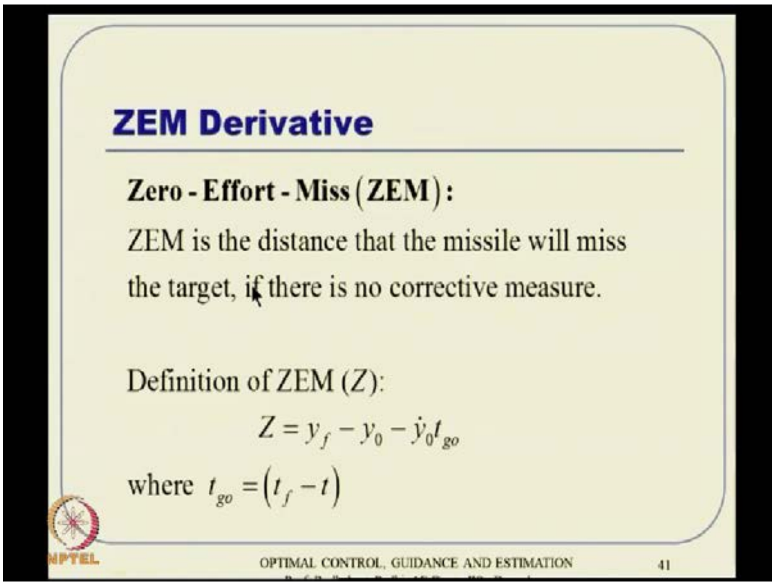

The 0 effort miss this concept is something like the distance that the missile will miss the target if there is no corrective measure ok. So, that is that is a very neat concept and using that there are several ideas that work of in even including current day literature. Actually, if you do not do any further correction the direction correction or the velocity (()). Let it go continuously wherever you want to go and then finally, we will end up with some miss distance that is called 0 effort miss there are no effort and whatever means z and f refer actually then, obviously, the missile guidance can be interpreted this way the final 0 effort miss at t equal t f or t equal to 0 that has to be 0 actually the correction should happens is such a way that the final 0 should be 0. Anyway, to mathematically speaking this idiom can be interpreted something like this y f minus this term; this term this is the predicted y f sort of thing y not plus y not dot times t go this is a predicted t f, but y and (()) z is nothing but y f minus this quantity.

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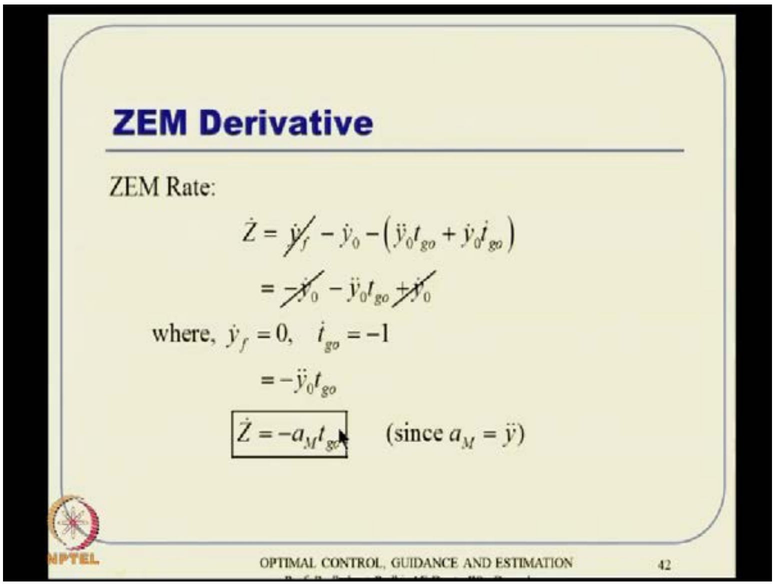

So, what is the z e m rate and that is the, that is what we want to impose as a system dynamic equations. So, z z dot is this one where we assume that y f is a fixed quantity. So, y f dot is 0 and then if you tell this t I mean t go dot is nothing but minus 1 and t go by definition is this. So, t go dot is nothing but minus 1 with t f constant. So, that minus 1 if I put this becomes plus y not. So, these 2 terms also cancelled out it lands up with like something like this actually. So, y not double dot y not double dot times t go you also know that our aim is nothing but y dot. So, this z dot is minus a m times t go actually. So, this becomes a constraint equation to empower in the system dynamics as a system dynamics in the formulation.

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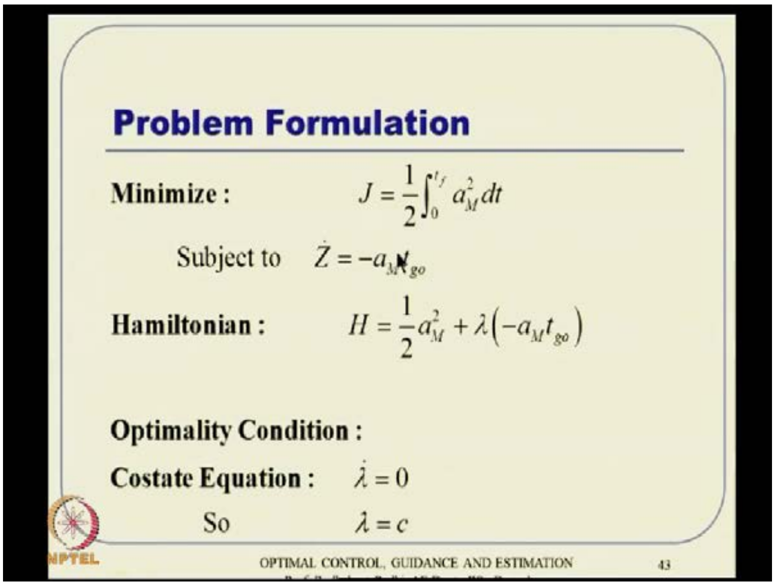

Again, this concept remains same. So, minimize the let the same letter resolution, but subject to this single z z dot remember this is a single equation now is not is not a vector it is a scalar equation constraint actually. So, other way becomes most is here also in a way. So, we have this Hamiltonian optimality conditions again costate equation terms out that lambda is just a constant and then optimal control it terms out that a m is nothing but t go times lambda if you put it here and then thus is equal to 0.

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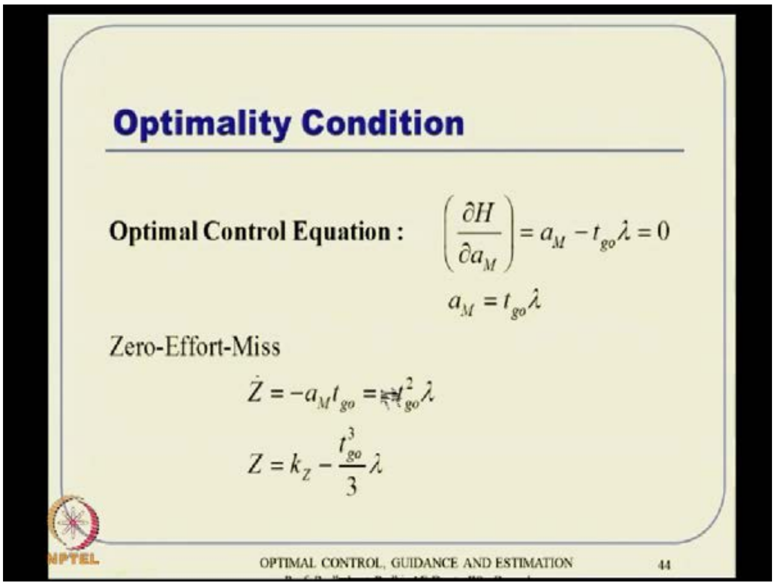

So, a m equal go times lambda now the 0 effort miss dot sense this is this is the equation z dot equal to that. So, a m equal to that. So, you put it by here. So, z if you integrate it it terms out to be something like this and then you put the boundary condition that when t go equal to t go initial t go then z is z naught and when t go equal to 0 finally, the z equal to 0 we know about this z dot equation actually double of some backward of some sense actually anyway.

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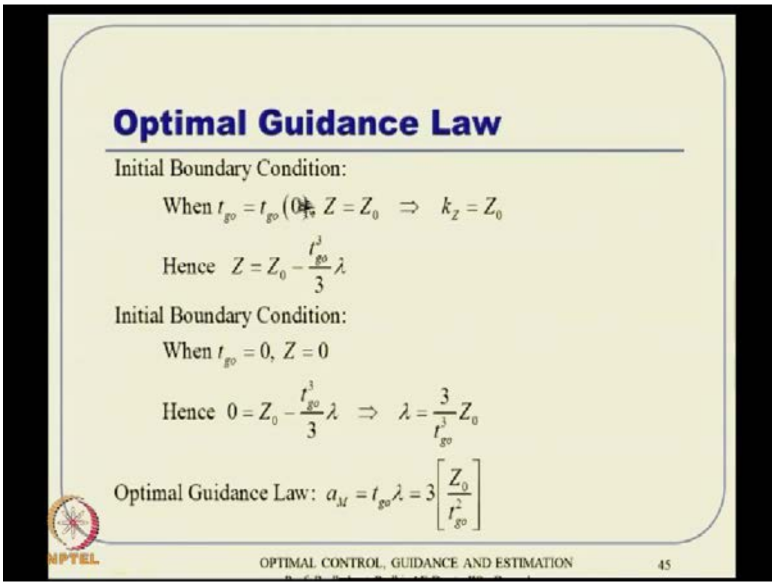

So, this is if you interpret this if you put these boundary conditions; then it turns out that k z is nothing but z naught. So, the z is nothing but that and then if you put additional boundary condition like this then this is nothing but 0 equal to z naught times this one. So, lambda you can get a solution like this very easily once you get lambda the am is nothing but t go times lambda. So, a m is nothing but three times z naught by t go square. So, what you have interpreted that a m in the previous lecture say something like this it has been represented something like this actually. So, either you can have a m equal to this expression minus 3 by y t square plus v by t or you can have minus 3 v c times sigma dot or the same expression. You can write it a three times; something like z m z m by t go square actually alright. So, 3 equal to a m is nothing but z naught by t go square the initial z m by t go square. So, if you somehow compute this z naught. So, z naught z by definition is like that. So, z naught is if we put all these initial values of t go and thing like that. So, this if you compute z naught that is a come with every instant you compute this z a m, but these are current values of z am and then put it there and we will get it what we want actually not that alright let us go to that . So, this is where we are actually alright now finally, when we will also see will a different application and tell can you really do something better because we are using optimal control theories are going finding out this distance 0 is not the only condition. Actually, when we to do little better than that then this idea of terminal aspect angle of constraint gradient comes. So, that means, you not only suppose to go to the target and yet it, but also you have to hit in a particular angle of the velocity vector. So, that concept leads to this terminal angle constraint gradient sort of thing.

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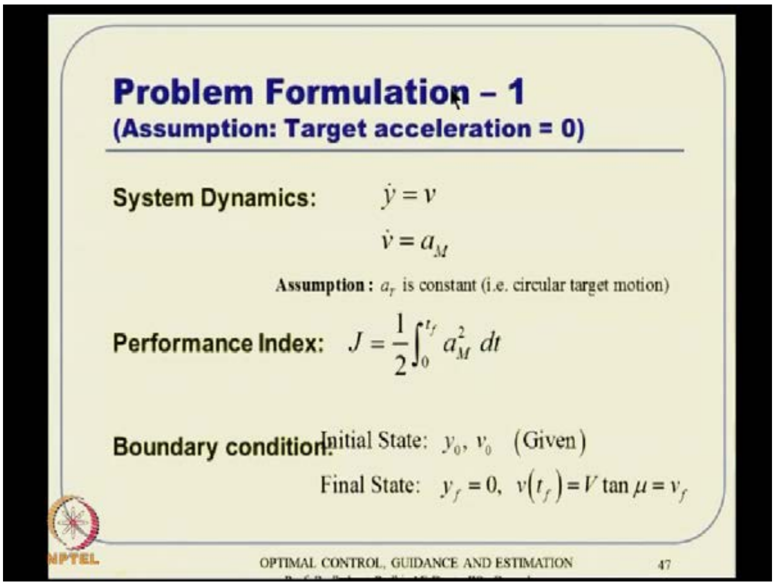

So, this is again a different we will go back to the formulation one what we heard with a target acceleration 0, but if there is a boundary condition that is also some actually v f v f of t f is not free anymore, but v of t f is just I mean, this is the desired angle and using this v times tan mu if you put it that is our view that is we want actually v of t f as should be v f now if you see this entire formulation I mean just a close note of it you we know this optimal control theory frame work and you know the problem objective. So, once you formulate this objective when you put it in the proper mathematical framework rest of the things are simply algebra actually you do not have to I mean brake over ead thinking so many things about geometry.

Concepts how the trajectory (()) should happen all sort of things it will automatically take care of all those concerns to a mathematical formulation and that is the beauty of optimal control and that is why we call it is naturally grade of towards this part finding problems sort actually and estimation guide an. So, be naturally falls out to form optimal control theory. So, everything remains same only the little bit change in the boundary condition of course, you when you account for this boundary condition v of t f f is that one then; obviously, the solution naturally going to change we will land up to the different formula actually.

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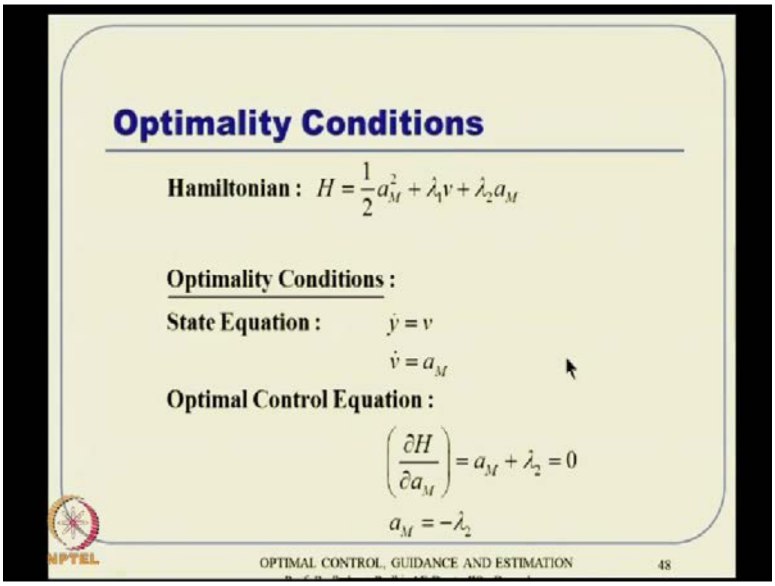

Alright, so, we start similar way again Hamiltonian is that system state equation is like that control is equation is like that. So, you have an equal to minus lambda 2.

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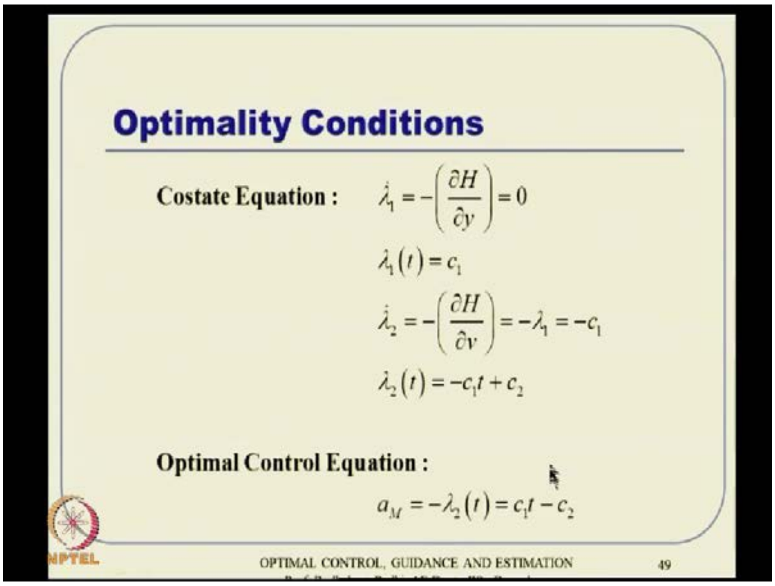

Then, costate equation we have this lambda 1 dot is same as 0. So, lambda 1 is c 1 lambda 2 dot is again minus c 1. So, you have this same expression for lambda 2 t also the optimal control equation again similar minus lambda 2 which is c 1 t minus c 2 the question is how the c 1 and c 2 will be different here.

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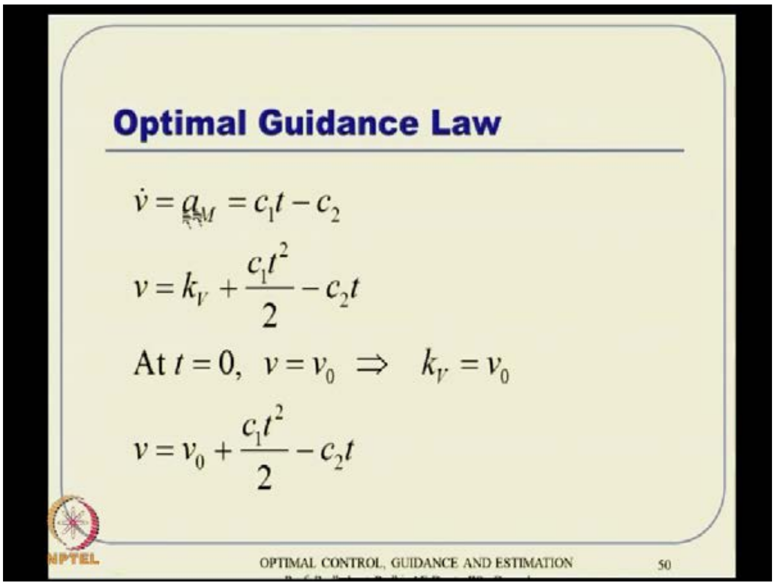

We have this v dot again will go against all the system dynamic equations now v dot is a m which is like this and hence v. If you integrate it, you will get something like this with an integration constraint, but t equal to 0 we know v is v not. So, we have this k v equal to v not. So, ultimately we will put k v equal to v o then v is nothing but v not plus c 1 t square minus c 1 t square by 2 minis c 2 t. So, we have this to still now c 1 c 2 actually.

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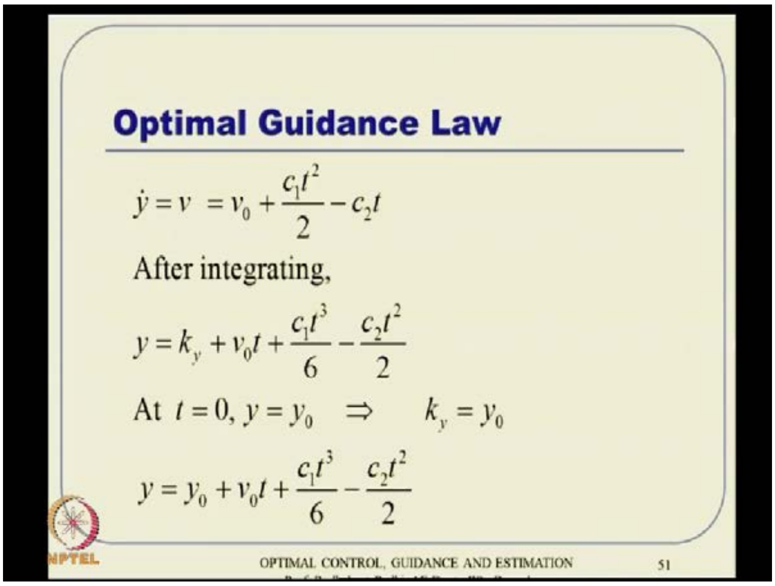

So, we have this optimal guidance now; which is I mean, this for deriving that we need this expression for y also because, what we utilize the boundary condition for y. So, we now put y dot is v which is nothing but this and then after integrating let this additional integration constraint again from initial condition that t equal to 0 y equal to y not we get k y equal to y not ok. So, you will get from this expression; you will get this expression for y. So, we have this expression for v and this expression for y.

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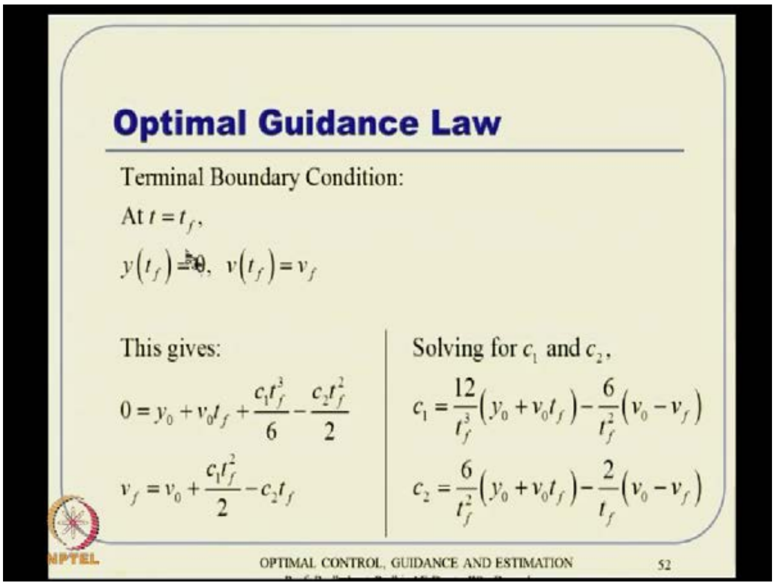

This is the, I mean, you know, ready for applying the terminal boundary condition which is given in terms of v f and y f actually. So, at t equal to t f here y of t f is 0 and v of t f is v f that is what is given as constant equation for the problem; that means, when you go when you substitute this in this expression and that expression we will land up with these 2 equations now there are 2 equations and to one one c 1 c 2. So, we can always solve for this and after solving we will get something like this. Actually, I suggest that you solve it yourself and solve it either in a vector matrix formulation or simply using this method of elimination with only 2 variables anyway. So, we write c 2 as a function of c one and then put it back here and hence alpha c one and then come back hence alpha I mean get the expression for c 2 that is what lead to something like this. So, you got everything that you wanted.

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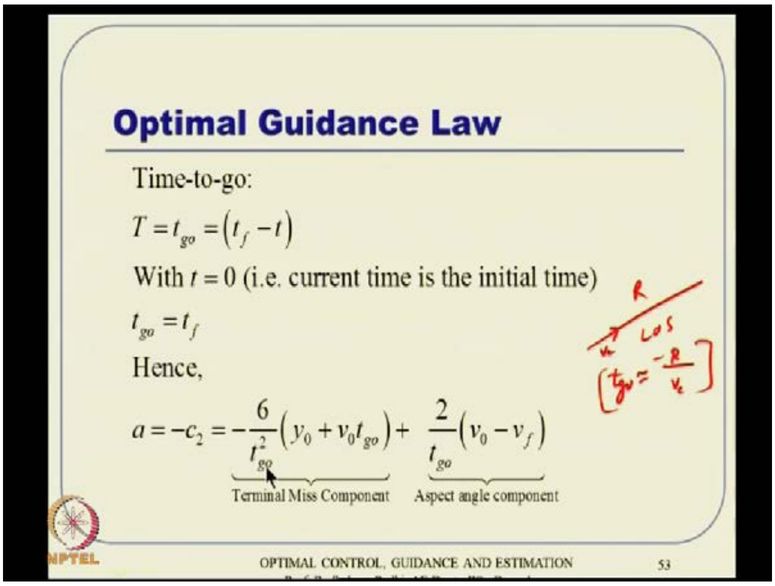

So, that means, this time t go information again we put it something like this by the way this time t go when somebody talks about it is also like there are this critical parameters and optimal missile guidance and normally, you will take this is in LOS and something like r is may a lowest distance of t go and it is approximated typically by r by this v c v c is closing velocity v c is typically negative. So, formally we will put minus y r by v c. So, this is the expression that is heavily used as the approximate time to go where there are there are many different ideas for estimating this time to go in a better sense you can the argument is it is again getting updated. So, obviously, it will I mean it is not that weight and it happens to be like that, but anytime the use of, you rely on this update sort of ideas then the fluctuation are will be large actually and it will keep on getting different values at different final time and hence here the entire magnitude will become different. So, here control oscillations will become more actually when that you heard that it is better that you estimate a good t go which does not fluctuate mass actually for that you have to estimate circular path this path I mean depending on the trajectory surface that we are talking essentially that talks about the entire path the line integral of the path divided by average velocity on the path then you will get a very good.

Estimation of t go I will not (()) that we are encouraged to read anything. So, on that actually, but this is something the people conveniently use actually anyway. So, you refine back which is t go and go back to the expression that our letter resolution is nothing but c one t minus c 2. So, we have got c one and c 2 now. So, we put it back here c one and c 2 and simplify little bit; that means, with t equal to 0 the t go will be turned out to be t f and inclined that you simplify little bit will turned out to be just t equal to 0. So, this c one t will go and only c 2 minus of that. So, it will turn out to be like this actually. This is you can very clearly notice that there is a terminal miss component and there is a aspect angle component this component came because of the formulation actually. So, you can see now the power of optimal control and also then these 2-D things it can be extended to 3-Ds. So, simplify mathematical formulations and all that actually; so that is possible also. So, let us what I mean various things you can talk about while formulating the guidance problem in the frame work of optimal control theory.

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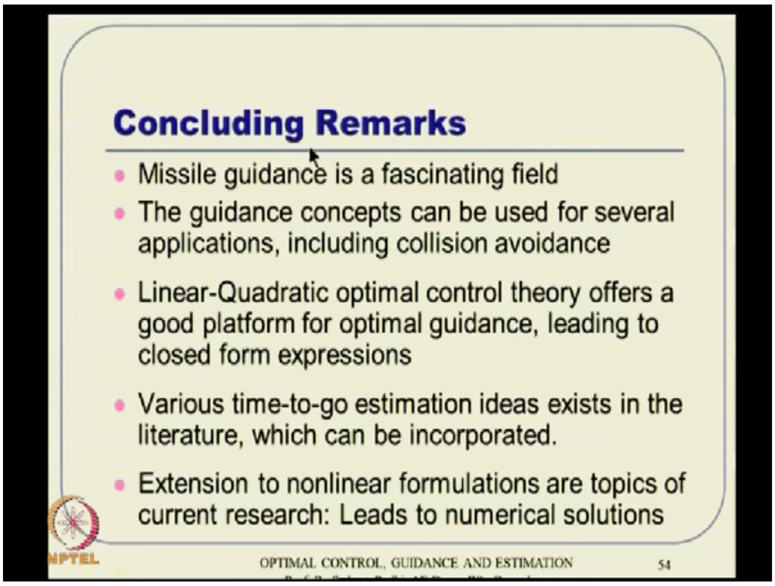

Some, concluding remarks in general missile guidance terms out to be a very fascinating field in my view and then the guidance concepts can be used for several applications which is even more exciting than it is not necessarily only missile guidance. So, the various applications in one of the slides including the exactly the opposite problem which is collision avoidance that is also we can formulate and use it in a in a good way then linear quadratic optimal control theory. Of course, a very good platform for optimal guidance already into very good closed form expressions actually and; obviously, t go appears to be a concern, but various time to go estimation ideas exists in the literature which can be incorporated for better loss. Actually, the extensions to non-linear formulations are topics of current research essentially this beauty of closed form solutions will be compromised for numerical solutions, but as long as you get numerical solutions in real time thing should be alright actually and those will be much more powerful compared to this linear approximation guidance laws and there we will talk about the real vehicle dynamics as a constraint equation. So, then the solution nature will be much more relevant actually. We will see all that as we go a long in this lecture series actually.

So, with these applications and a sort of one lecture I taught, it gives you a good motivation and the ideas of why you want to read more in the application sense. We can talk various challenging problems in general actually alright I will stop here, thank you.