Optimal Control Guidance and Estimation

Prof. Radhakant Padhi

Department of Aerospace Engineering

Indian Institute of Science, Bangalore

Module No. # 07

Lecture No. # 17

Overview of Flight Dynamics - 111

Hello, everyone. Let us continue with our discussion on flight dynamics, which is relevant for flight control design, which we will use it subsequently in these courses especially. So, the last lecture we have derived point mass equations with various assumptions.

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Followed by a six degree of freedom equations, where we have derived the dynamic part of the thing, and then we will continue further on this particular lecture and try to finish it up. I will try to give a complete overview of this dynamics.

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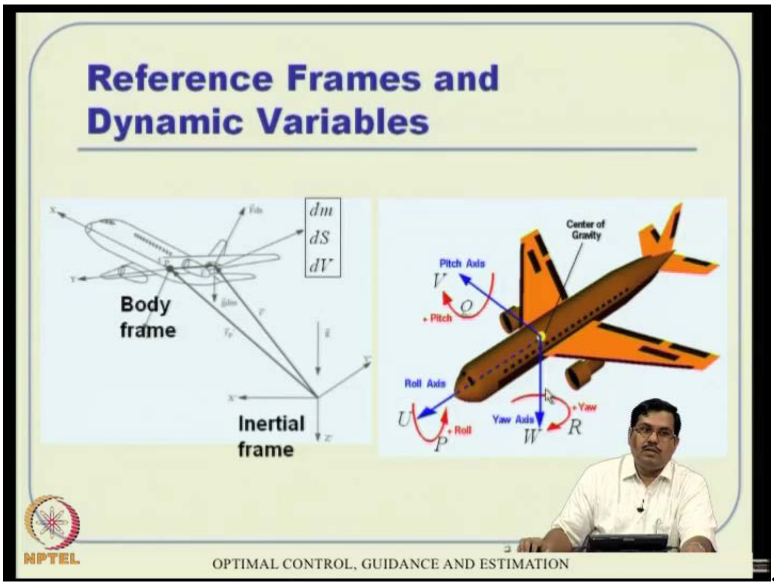

In this six degree freedom model, what we discussed in last class is that we essentially try to find a relationship in two coordinate systems. One is in inertial frame, which is I mean you can visualize that its fixed on the surface of somewhere and then there is a body frame, which is attached to the body. Then about the body frame, we discuss about several variables, which you call them as dynamic variables, which essentially consist of U V W and P Q R. U, V, W are the velocity components along body x, body y and body z and then around those axis, there are some rotation rates, which are called roll rate, pitch rate and y av rate. We have discussed that in little before as well.

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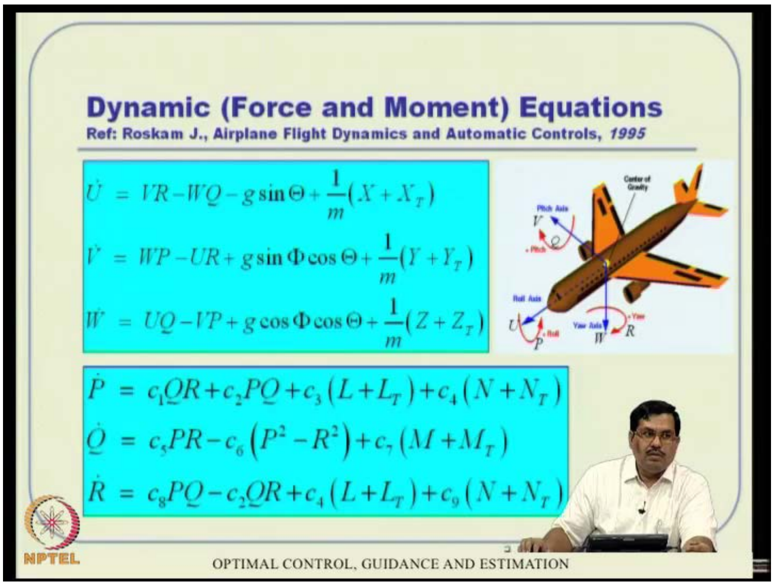

Then we entered and derived this set of equations, which essentially consist of translational resolutions resolutions/isolations/acceleration and rotational resolutions resolutions/isolations/accelerations terms. This consist of partly, this a coreolis component and partly from gravity and partly due to external courses. Similarly, we have these equations for rotational rates and all that. So, we have done that in detail in previous class. Now, if you see that there are some quantities like theta phi especially here, which also very big term, because that represents the attitude of the vehicle, so they are not fixed, they keep on varying with time.

So, we certainly want to have a rate of change of these variables as well. In addition to that we also require this aerodynamic components here, this X Y Z and l m n, which are also functions of height, because their functions of dynamic pressure and dynamic pressure consists of density of atmosphere which varies with height. So, we need to have rate of change of height - and especially if you want to have a trajectory of the work log like where it is going or what is doing, anything like that if you want visualize from some inertial frame, then we certainly need to have x y co-ordinates in the inertial frame as well.

So, inertial x y z, the negative z part of it will essentially give us the height part of it, I mean the height of the aircraft any point in time and X Y Z together will define this coordinate of the CG of the vehicle, which will essentially give of the flight path of the trajectory. So, we need to have this phi theta along with that we also need something called heading angle psi. So, we essentially want to derive phi dot, theta dot, psi dot as well as x dot, y dot, z dot in the inertial frame.

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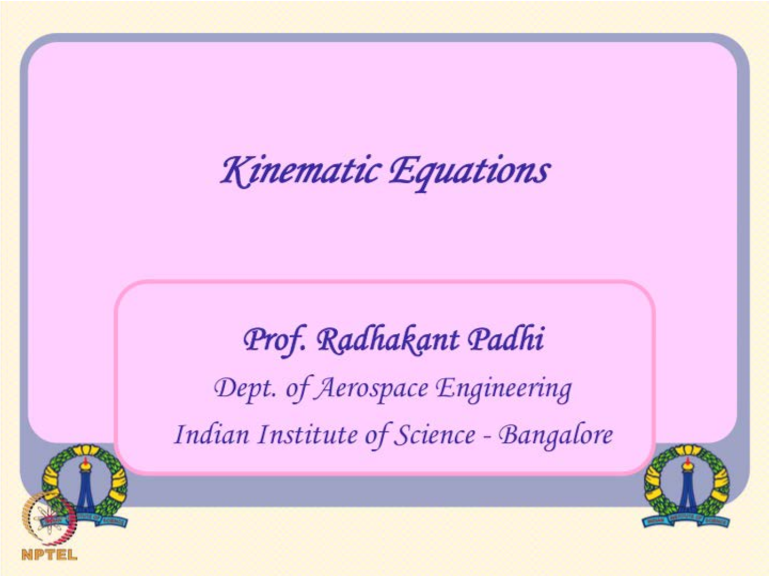

So, let us go and try to do that. Those are call kinematic equations essentially. They do not rely on external forces and moments, but they are - velocity level equations. What we saw here is an acceleration level of equation.

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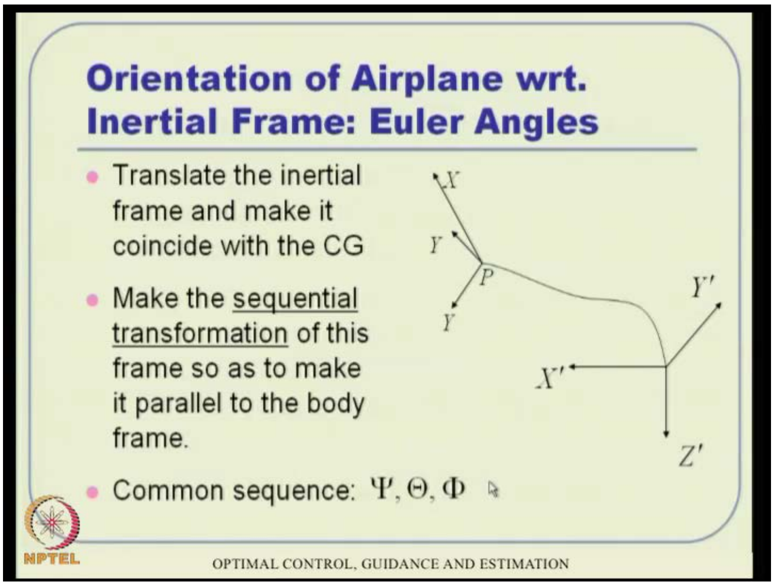

Now, let us continue with our required derivation. So, here we need the concept of something called Euler angles, and later we will see that this is not the only parameterization, we can also have various other parameterization like cosine and cotant things like that, but let us complete the derivation with Euler angles. So, what you really want here is there is an inertial frame here and there is a body frame out here. So, we want to know the orientation of this body frame X, Y, Z with respect to this inertial X Y Z- and that is our objective here. So, what we really want to do is let us we first want to take this and this co-ordinate system and just translate it over there. Then try to rotate some this to this coordinate system, one at a time. First with respect to Z, then with respect to y then with respect to x, things like that.

Ultimately, our aim is this translated coordinate systems should coincid

e with this body axis frame - then whatever angles we require those angles are nothing, but this angles are psi theta phi - (( ))

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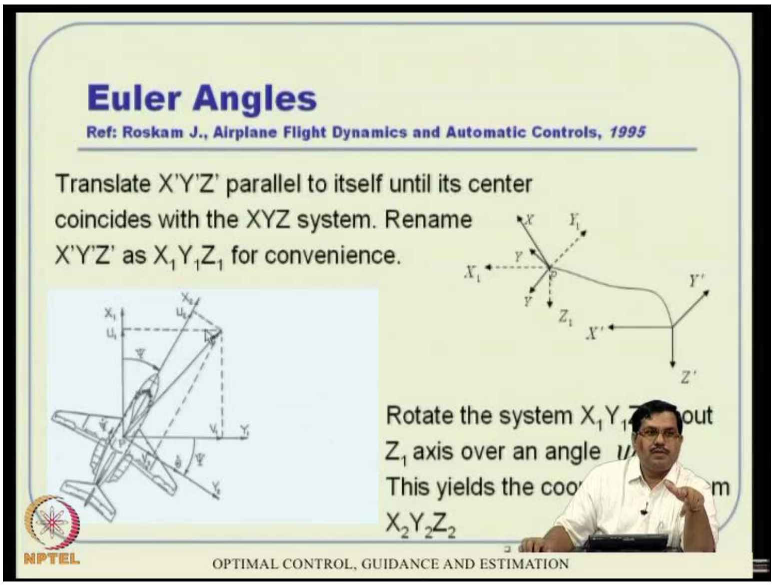

So, let us try to understand little better. So, first what we have interested is taking this coordinate system over there; simply translate there. We have to have some different notation there, just not to get you confused with that. So, this x does our x prime, y prime and z prime is rewritten in terms of X 1, Y 1, Z 1. This is simply just to translate it over there and nothing more.

Now, this X 1, Y 1 and Z 1, I can rotate it with respect to the body axis with respect to the body Z axis. So, that this entire x y frame whatever X Y frame was that previously it will try to coincide with this X Y. So, this X 1 Y 1 what you had by translating this plus this inertial co-ordinate over there, now will rotate it by angle psi, with respect to the body Z axis, Z axis is your probably middle finger in the left hand side. So, this Z axis you have to rotate it; Z axis you keep it fixed and rotate it by angle psi and that what the picture says. So, that is what you will do.

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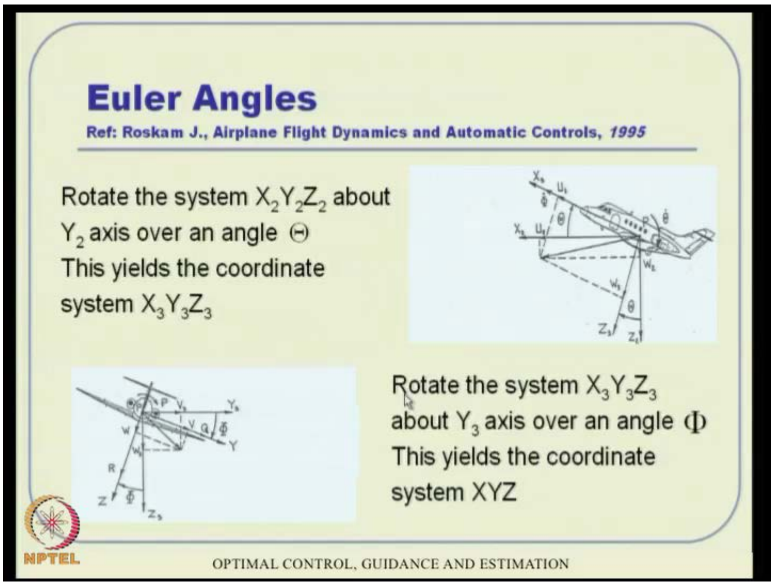

And then next what will do is this rotated frame whatever is there you rotate it by angle theta about the y axis. So, first with respect to the z axis, then with respect to y axis and ultimately with respect to X axis phi and that is what you will do here. So, the sequence of rotations is psi, theta and phi and these sequences of rotation yields the translated inertial coordinate system, which gets rotated about its axis, so that ultimately it coincides with the body frame. So, this is the concept. Now, let us try to see the relationship between them and anytime essentially what we are doing is we are doing a one angle rotation of a two dimensional axis frame at one point of time. About one axis we are rotating, so that the particular axis remains fixed and then the remaining coordinate system moves in one angle in 2D.

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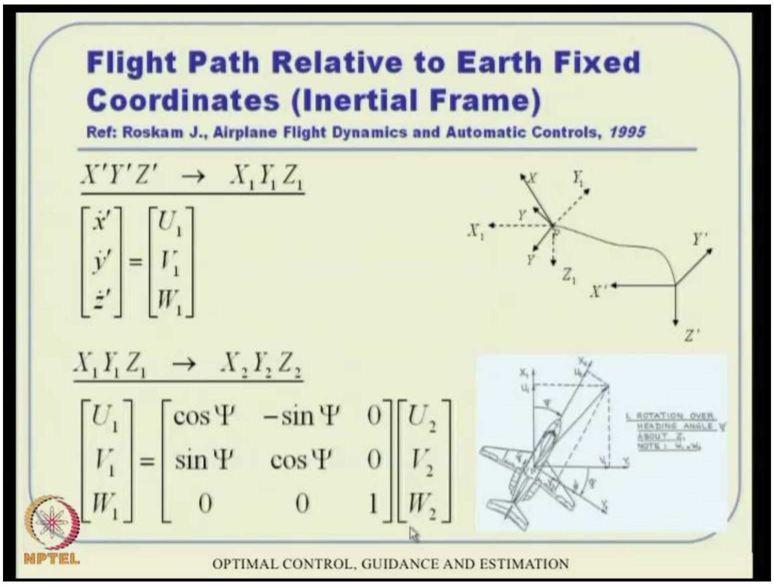

Let us try to see what is going on here. So, first we are doing that this co ordinate frame we just translate their, so essentially this whatever dots you have in the inertial frame is nothing but U 1, V 1,W 1. This is a simple translation. The velocity quantities do not change, but after that there is a rotation and is 2D rotation and what we want to visualize that is in 3D, actually. So, W 1 in this particular picture, if you see is just the angle psi. So, this angle psi happens to take this U 2 V 2 to U 1 V 1; that means whatever U 1 V 1 is there, suppose you already have U 2 V 2 then what is U 1 V 1? That relationship comes through these rotational 2 D rotational matrix we know probably.

So, if you do not know, then you can see some co ordinate geometry book something there which is very standard. So, they are some need properties out there, for example, the determinant of this matrix is always one, so this matrix is never singular or anything like that. This is an orthonormal matrix also. The various properties are associated in that, but the fact is suppose I have U 2 V 2 after the rotation, then U 1 V 1 is related by that particular 2D matrix here with respect to U 2 V 2, and W 1 is equal to W 2. So, this becomes zero, zero and that becomes zero, zero. So, you have one. So, this U 1 V 1W 1 is related to U 2 V 2 W 2 in that way.

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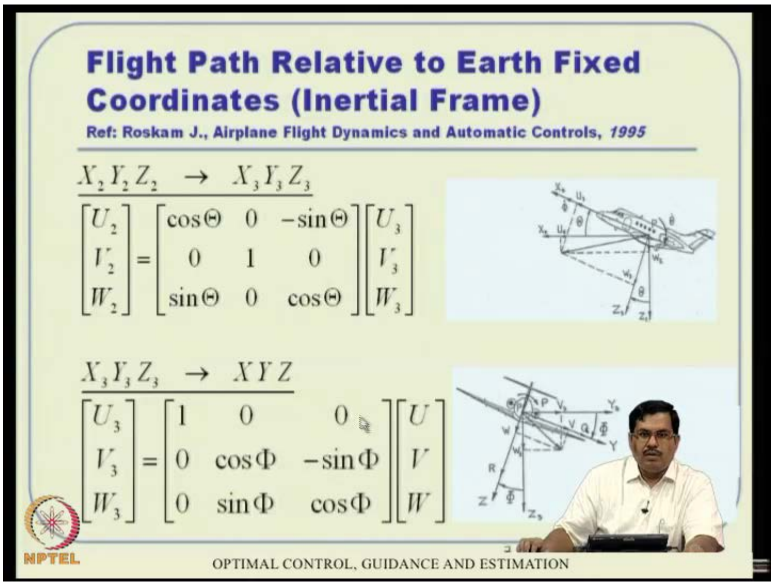

Then we have the next transformation by angle theta. So, this angle theta is with respect to U 3 and W 3. Remember, this is because Y axis remains fixed here. So, V 2 is equal to V 3, but the other variables U 2 V 2 W 2 is related to U 3 V 3 W 3 because of that - because our angle, this rotation angle theta. Similarly, this X 3 Y 3 Z 3 goes to X Y Z by angle phi ultimately. Now, this is the rotation about x axis - the transform x axis.

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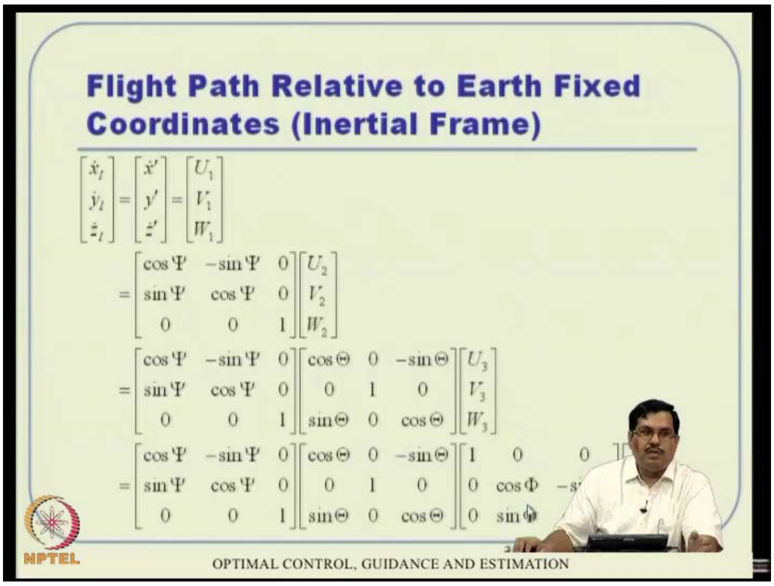

So, all this rotation sequences or rotations, once you try to combine them then what you combine? Let us say you want to have inertial rate of change of position; that means, velocities, which is nothing but that. That is the definition in our, because the definition that we are talking about the co ordinate frame is x prime y prime z prime. So, I mean normally I prefer to write it as x i dot, y i dot, z i dot at the end.

So, that is related or same as this and this is nothing but, same as this because of translation. That is how we started with. This is translational here, but this translation is now given as that one, just say that one in the first translation. Then this translation what do you see here is given by that. That is the second translation, I mean second rotational thing. This is translation; this is first rotation towards i. This is the second rotation and this is the third rotation.

So, ultimately if you see the rate of change of position inertial frame is related to phi theta psi all angles with respect U V W. So, if you know U V W components, then x i dot y i dot z i dot is given by that - this entire sequence of rotation and then you can simplify that. You can multiply all these three matrices and then find out one vector equation at a time.

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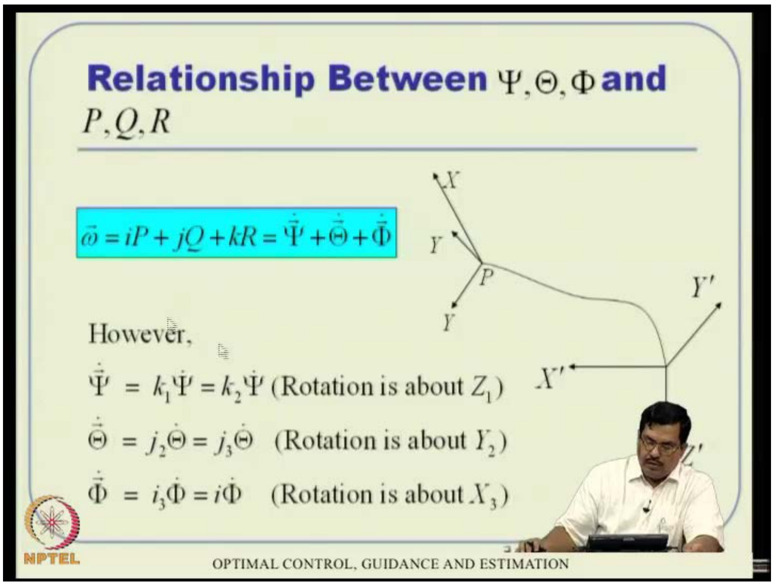

That is about this position; rate of change of position in a universal frame. So, if I use this relationship then I would be able to plot the vehicle trajectory in a universal frame. Now, obviously, this function of i theta size and then we know that this is also a function of i, means this U dot, V dot, W dot is also a function of theta and phi. So, obviously, we want to know this phi dot theta dot psi dot as well. So, how do they vary? Now, obviously, they are related to in this body rate of rotations P Q R. So, if psi theta size was this inertial coordinate frame rotation in a sequential way. P Q R is nothing, but the body rate of rotation about the vertices of the equal itself. So, in a vectorial sense you can say that i j k is the body from the unit vectors. So, this w bar is i times P plus j times Q plus k times R, which is also equal to, vectorialy speaking psi dot theta dot and phi dot together.

This is the same vector. They are two different coordinate systems. This is written in terms body axis and this is written in terms of inertial frame - a sequence of operations essentially. So, vectorially speaking, this two are equal. Now, we try to find out what all goes in there. Now, vectorially speaking psi dot is nothing, but k one times psi dot. Remember, whatever we did here, this X 1, Y 1, Z 1stands for i one j one k one, X 2 Y 2 Z 2 stands for i two j two k two like that.

So, in that sense if you can see that this psi dot is essentially k one time psi dot that is the first operation what you did. Essentially, k one and k two remains same, because that is the rotation about Z 1 axis; k component will remain same. Remember, i j k stands for X Y Z. So, k stands for Z component and it is a rotation above Z axis component, so k component will remain same. So, I can write it that way.

Similarly, theta dot is essentially j two and that is second operation we know that. So, because of the second operation, this I mean theta bar dot is nothing but, j two theta dot which also equal to j three theta dot. That is the rotation about Y 2 now and similarly, phi dot is like this. So, it is rotation about X 3. So, this is how these vectorial notations can be decomposed to. Now, we have to simplify this k two j three i three also those things to put it in single frame work and then try to end up with the relationship between them.

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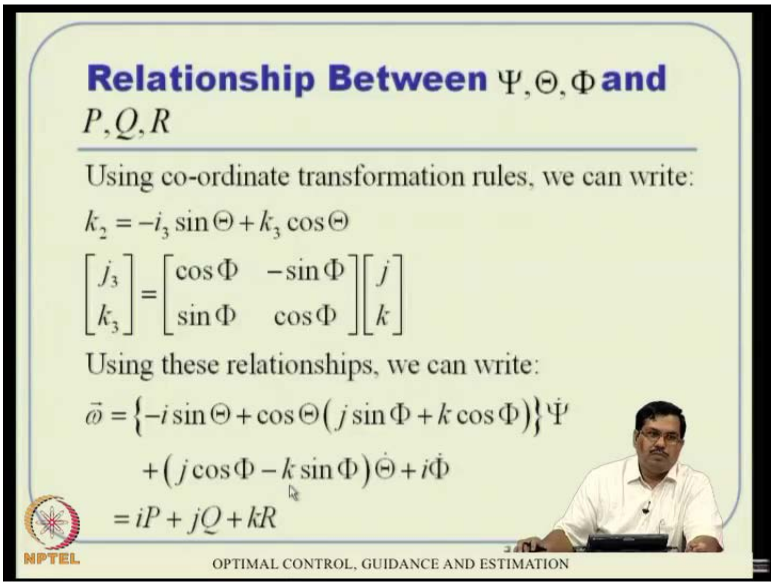

Then what happen is this k two, I can write it as something like this; i three and k three sort of thing. Essentially, you can write it in to something like i two and k two as a function of i three and k three. That is the way I have written it here below. So, this is what is written as j three and k three are related to j and k like this. Remember, j and k stands for body axis and j three k three stands for just before that. Last rotation is phi ok. So, this relationship is like that. Similarly, you can write it in terms of theta also. That means i two and k two with theta in between x and z, remember that.

So, i two and k two will be in the form of a similar relationship with pop up for i three and k three. You need only the component of that; I need only the k two component, so I just taken of that one. Now, j three and k three are given like that. So, using this relationship, this entire thing whatever you have, this vectorial dots I can represent this like this. This is like k two times i dot plus j three times theta dot plus i three phi dot if i want do that. Then I would kind of simplify all these because k two is like this; j three k three are like that. So, I put them together and then that is ultimately equal to i terms P plus j times Q plus k times R.

Now, that is where we started with. So, I have started with all these and then try to write all in terms of body coordinate frame; this phi dot theta dot psi dot and that is equal to this i times P plus j times Q plus k times R. So, now I can equate this component by component that these have three are orthogonal quantities; i j k are orthogonal to each other. So, I can equate this quantities component by component.

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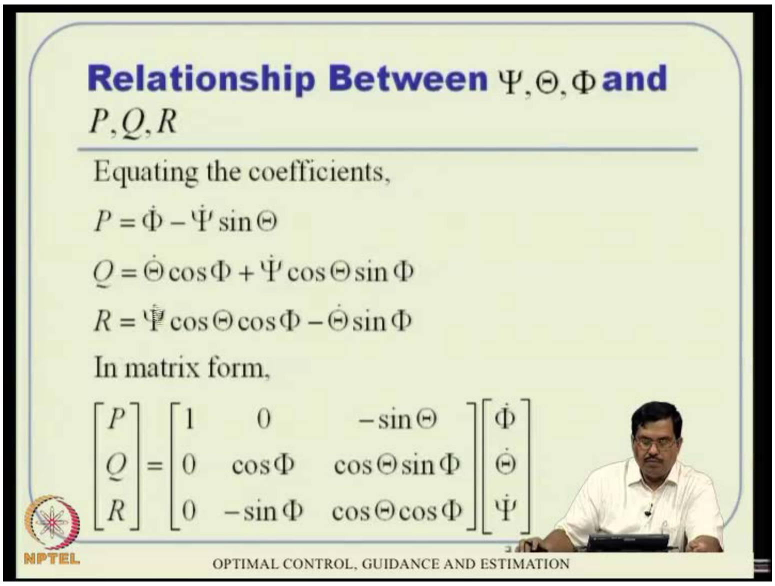

So, let us do that and then we will write this relationship. So, if you try to use this component by component and then this relationship will popup. So, vectorially speaking I can write it that way. But, normally what we do not want P Q R is a function of phi dot theta dot psi dot; we want the reverse one. We want phi dot theta dot psi dot as a function of P Q R. That is the dynamic equation and that is what you want integrate later. So, what you do? I can take an inverse transformation here.

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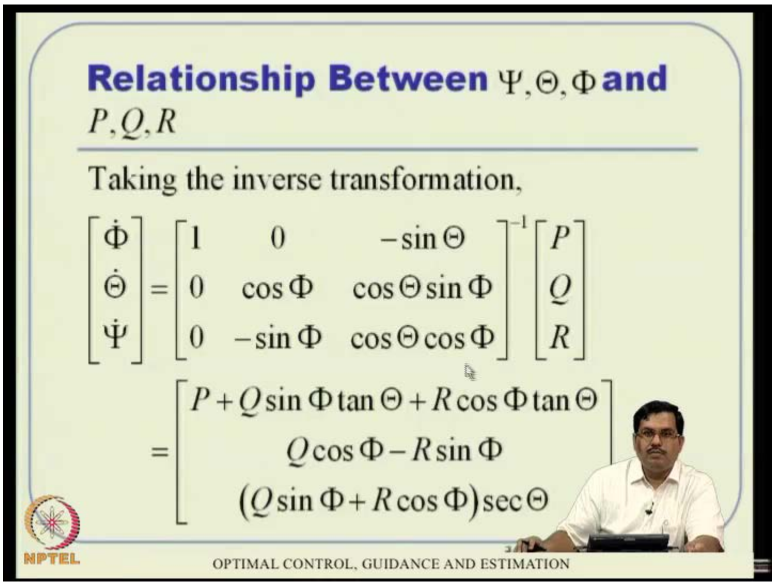

This inverse transformation, I can carry out and if you sit down with this inverse calculation there is one by determinant times at z metrics and then you simplify sort of things and then it will be popup like this. So, what you have is ultimately we have this we have this hmm x i dot y i dot z i dot, which will give us the trajectory part of it. Associated with that we have this phi dot theta dot psi dot also; everything needs to integrated together because they are all coupled equations. This is not the end of the story, we have this I mean you also see that this components in the last class we just derived from simple logic; simple integration. Now, can we do it in formal way? So, what part of this will go to v dot anything like that? So, if you let us try to quickly do that.

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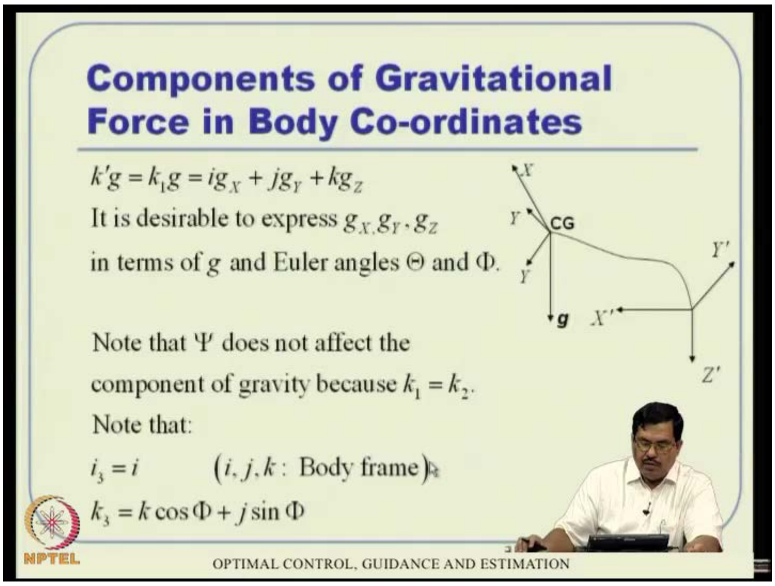

This is essentially the same way, even though gravity is perpendicular to the I mean parallel to the inertial Z axis all the time, no matter what ever is the vehicle altitude; gravity is always vertical with respect to surface of (( )), so that means, it is always parallel to the z axis; it is always point out to the centre - or earth centre.

So, that why this particular vector, now this need not be with respect to body axis - body X Y Z. However, because this is at orthogonal frame, which spans entire 3D space, so the g vector, any particular vector and then particularly this gravity vector I can decompose that into X Y Z components. So, k 1 times g, I am decomposing that in terms of i j k here. So, g x g y g z stands for the component of the gravity in body X body Y and body

Z coordinate frame. So, which part goes where? That is the question. Also note that psi does not affect the component of gravity because k one is equal to k two. Any time the vehicle take a rotation parallel to the surface of earth; that means, a some psi rotation - then the gravity is still pointed vertically; that means, the z vector, whatever it happens it does not effects its gravity. In other ways, the rotation of psi does not affect the gravity component at all.

Essentially, under the plot of the assumption that earth is flat. So, if I take some psi rotation somewhere, it does not affect anything about the body X Y Z. So, we will not worry so much about that. What you really need is phi and theta components. So, there are essentially functions of phi and theta, because the moment there is a theta component, I mean the pitch angle theta, essentially phi angle is also roll angle. So, pitching and rolling essentially, I mean they will make the componential gravity different in terms of X Y Z, but (( )) does not normally affect it under the flat as assumption. So, what you say here is i 3 again i. That is ultimately your body axis and that is the final rotation. Then this k 3 is equal to this k times cos phi plus j times sine phi.

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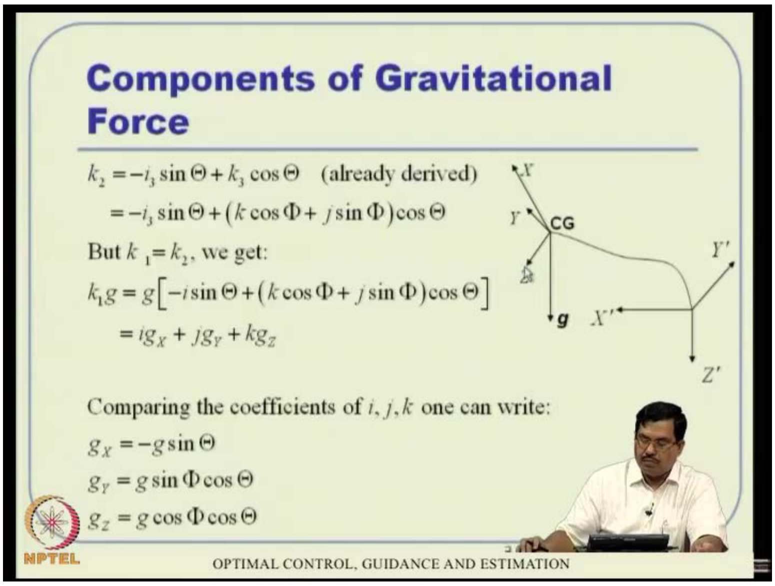

So, when k 2 is these one and so k 3 and k 2 are like this as a function of a theta obviously, and theta is related to i and k. So, phi related to k and j, and if you do a proper bookkeeping then k 2 turns out to be something like that; however, k 1 is equal to k 2, so that means, you can substitute this relationship and then you see that k 1 g is essentially like that. Whatever you see here, i j k, whatever components comes in i, j and k; these are body type, body X Y Z components. This is also body X Y Z component. So, if we equate the two we get this thing - and last class we just derived it intuitively, but this is much more formal way of deriving it using this Taylor angle rotation.

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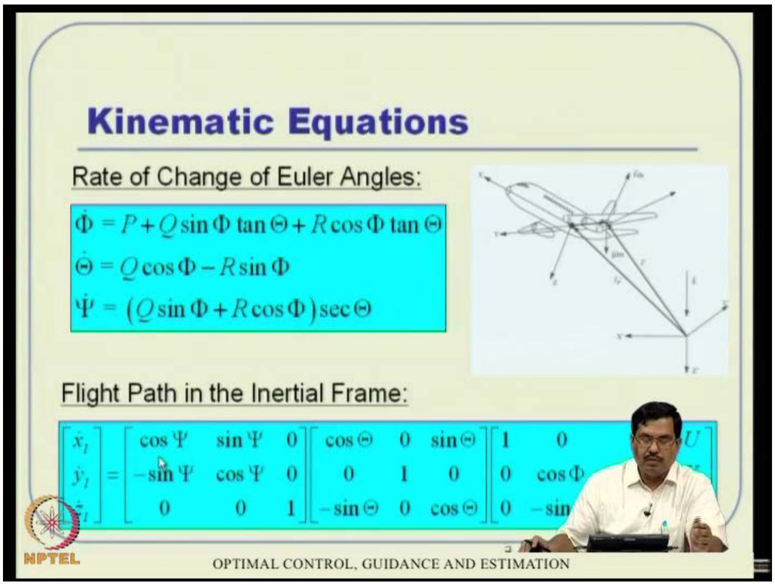

So, this is how the entire six-DOF equations is made of, but before going there, in last previous lecture, we have seen the dynamic level equations. Here, we see the kinematic level equations; that means velocity level equations. These are angular velocity term and these are translational velocity terms. So, these six equations coupled with the previous six equations what we see in the last class derives the, gives the complete set of six-DOF equations.

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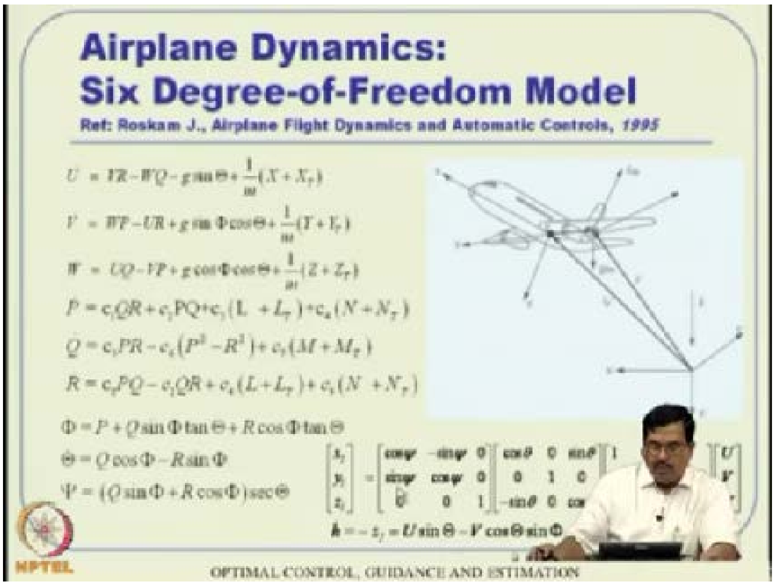

So, this is here. So, you see this consists U dot, V dot, W dot, P dot, Q dot, R dot, then phi dot, theta dot, psi dot and X i dot, Y i dot, Z i dot. So, these twelve equations will derive that and normally this height dot, h dot is negative of Z i dot. So, that is if you simply take out the last row and make a negative sign then it will provide whatever parts of is nothing but h dot actually. So, that is not an independent equation.

So, what is relevant here is only these twelve equations and fundamentally speaking of this twelve equation given, the first six equation are directly related through the body axis only and the last six equations are gives a relationship between inertial cell frame and body frame -especially this phi dot, theta dot, psi dot. Then this X i dot, Y i dot, Z i dot gives us the trajectory of the vehicle in the (( )) in form of the inertial coordinate frame. This is how it is all coupled together. Now, this set of equations what you see here they are corializ quantities. These are gravity and these are force quantities.

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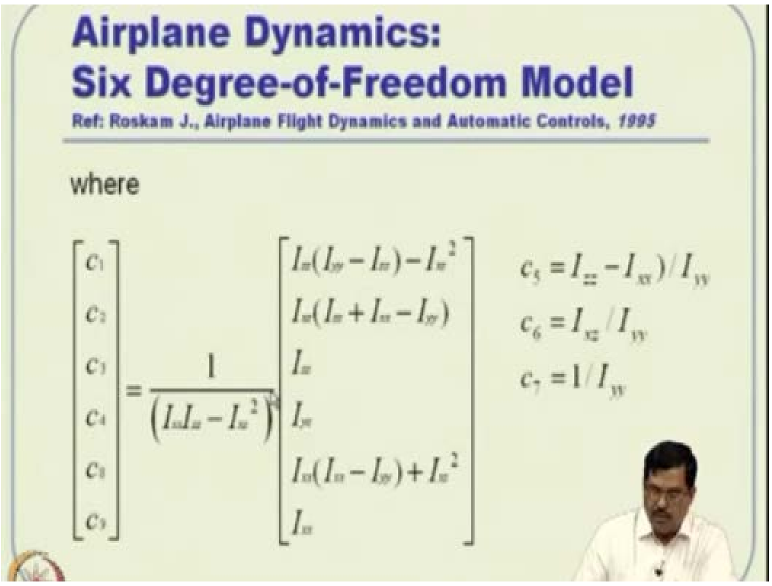

And then I mean if you go and see that this c 1, c 2, all these things we saw that last class also, whatever c 1, c 2, c 3, I mean all this c with this suffix. What you see here in P dot, Q dot, R dot equations are essentially functions of moment of inertia which is given like that. We have derived that in the last class. Rest of the equations are kinematic equations, so which are given here. And then whatever you see these components are, this external forces and moments are the only one which is different from vehicle to vehicle.

All rest of the things is same, no matter whether a bird flies; of course, bird has to be rigid body, which is not really true. But, whether any flying objects, which is a rigid body, which is governed by all set of equations. The only difference comes is because of this external forces and moments and that is where lot of study goes into the aerodynamic study. So, this is like aero dynamic component X, Y, Z, L, M, N and x t, y t, z t, and Lt, M t N t are nothing but, thrust components. So, thrust component are not very difficulty usually. It does not vary unless we have a thrust deflection mechanism, which happens in missiles and launch vehicle especially, with (( )) we have thrust deflection mechanism also, for better manual ability and all that, but normally this X Y, this commercial aircraft sort of thing, these are fairly known. It all depends on engine power and how much you do and what orientation you are fixed and that is where this phi t and psi t are coming actually.

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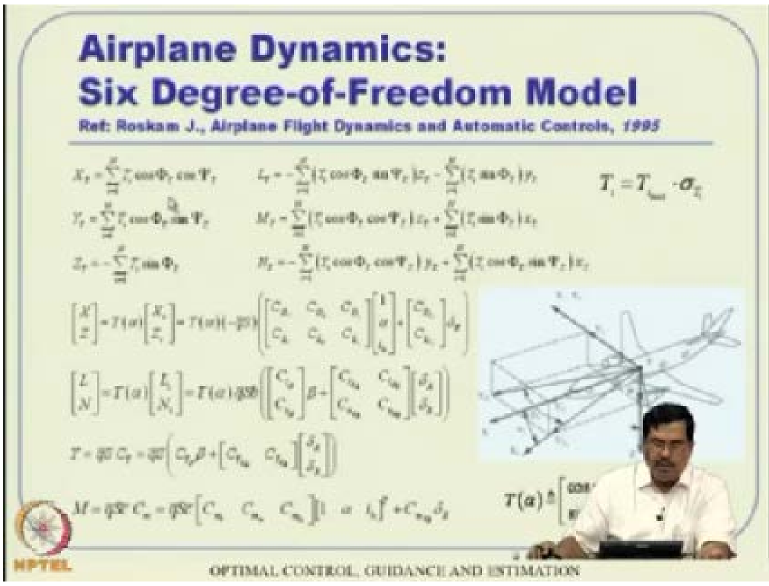

Phi t, psi t are not Euler angles, but they are the angle engine orientation angles as fixed to the, I mean as seen in the inertial, as seen in the body frame. With respect to the body frames, these engines are typically oriented by angle phi and psi or certain beneficial properties essentially. I will not talk too much on that. One thing that you can visualize is that if it contributes something on the vertical direction, which is inertial vertical direction and that essentially helps aiding to the lift to sustain the weight.

So, you do not really have to a very big wing to sustain your lift, if thrust as a small component let say some sign of two degree or three degree, but thrust is a large quantity. So, that quantity into multiplied by sign pi degree also plays a big quantity, which will which will act to the lift. Like that there are various ways. They talk about that jet should not hit the tail wing because the engine jet is there. So, it should never hit the tail wing because that is very unpredictable, I mean the circulation and all that. So, want to have something like this way. So, the engine exhausts goes somewhat like that it does not hit the vertical thing. There is various ways in these various regions why these are all done that way and then this aerodynamics forces and moments are typically generated two ways. One is that there three levels of doing that or three or four levels you can say; aero dynamic forces and moments.

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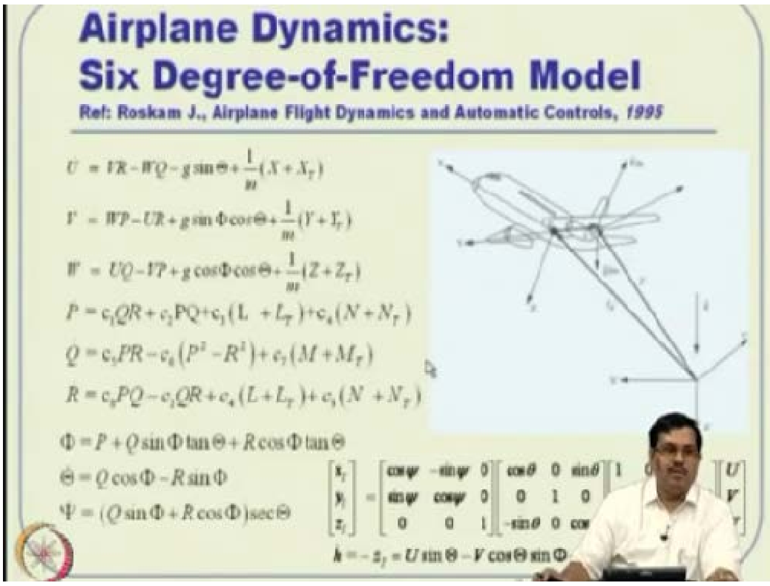

The first thing people do is just have a first thought guess, which comes from first principles and that is some something to start with. If we simply give the aircraft configuration then there are thumb rules available and basic relationships available from which I can get basic idea of what is there. Then it goes to the next level of what is called CFD. This computational fluid dynamics where they talk about generating in a grid point sense or using finite element, finite volume sort of things, so this will give you little more or better accurate sense. Remember, these are not; the flow pattern over this aircraft body is not very much in tune with whatever assumptions we do in the first craft model. The first craft model is all keen configurations, there is nothing there, I mean nothing objection are there in. The surface is fairly good and all sort of things.

So, going to one level of higher, the CFD thing will account for all this protrusions and then all sort of wing configuration, whether you have the fin out there, all sort of things it will take away taking by account and try to give a better accuracy on that. Then it goes to the next level and you get people talk about the wind tunnel study where most of time it is the model of this prototype. I mean, it is not a very big air craft that you cannot put in a wind tunnel, so you do a moderately smaller version of that. Then try to see instead of the vehicle flowing in the medium, now medium is flown over the vehicle; that means, the air is flown over the vehicle, essentially the relative velocity matters there.

So, we can get fairly better accuracy ideas, I mean better about this forces and moment, but, this internal study can only talk about generating what is called as static forces and moments; the dynamic level forces in moment it cannot generate. So, whichever the forces and moments that are components of these rates, the rotational rates, for examples P Q R; those components cannot be predicted in a wind tunnel. So, for that you need the flight test. So, this goes through several rounds of refinement and then by mean while if you design, the configuration itself goes through refinement again. You do one more round of wherever you want to start, either from CFD or wind tunnel or flight test.

So, that is where these models will keep on getting updated. When anything that is generated from flight test, it is essentially falls in the bracket of what is called parameter identification, because that is where you want to identify on how this process and moments act under the vehicle. So, roughly speaking, this aerodynamic interaction of this vehicle is not very well understood, from physics point of view. That is why we need to have several rounds of experimental study and then combine with deterministic model to get fairly accurate model.

That means this model what we are talking about you can visualize that. So, it is some sort of a hybrid model. One is the deterministic part and they will be the large component of that, fairly equally good component of that, which is aerodynamic dependent, which are somewhat theoretically understood and there are components which are not understood, but then there function feeds from the experiment, so it is largely data driven in that sense. So, all these things are part of the six-DOF model.

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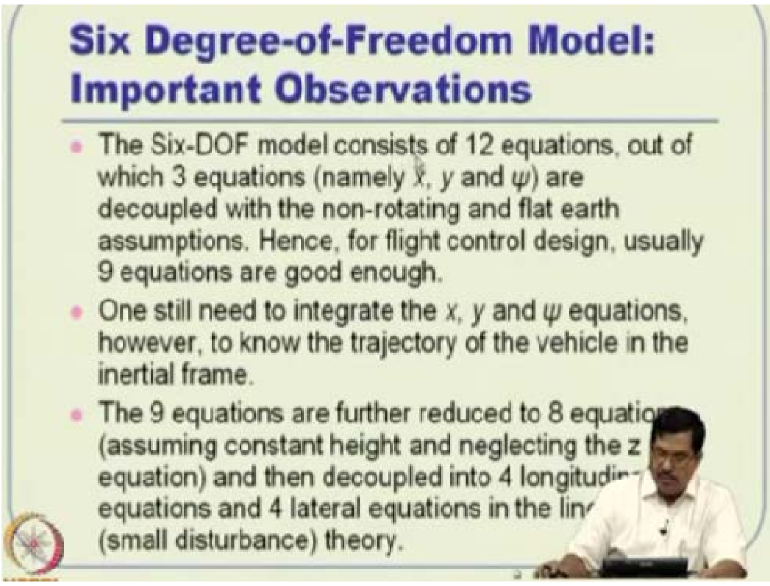

Now, before you go ahead, let us have some further comments. First thing is Six-DOF model, consists of twelve equations, out of which three equations are decoupled with, I mean, with assumption that the earth is non-rotating and flat. Like what I told on the last class, if you see all those equations, essentially the psi - and then x y do not come in any of the rest of equations. So, psi and x, y you can probably decouple that and tell even if there is a twelve equations under the non rotating part assumption, I can just work with nine equations as for as local control design is concerned. Control design is typically localized in the short duration stability problem something like that, they are not long duration affects and all that. So, that is fairly ok.

But one should also remember that one still need to integrate this x y and psi equations to get the trajectory of the vehicle in the inertial frame. Suppose you really want that and trajectory information is sincerely evaluated in the guidance design. So, if you really want to talk about a proper guidance design that means take up from point one and go to point two and things like that you really need this trajectory information. So, there is still need to integrate that and there are better models available under this spherical earth assumption and then the rotating earth assumption, and things like that.

So, forgetting this factor, whatever nine equations are there, this equation can be further reduced to eight equations because you can tell that locally the height does not change; that means, that height dot, h dot, I still just consider a reference height and then h dot equation, I do not have to integrate. I know about my operational height, which height I am talking about and then I do not want to integrate this h dot equation. Height remains constant around that, then the nine equations become eight and out of the eight equations we normally decouple that into four longitudinal equations and four lateral equations. These two we can be visualize them as different. The weak coupling that exists between the two under this linearization approximation, we can neglect that. So, these four equations are group together and other four equations grouped together, I will talk about that, of course.

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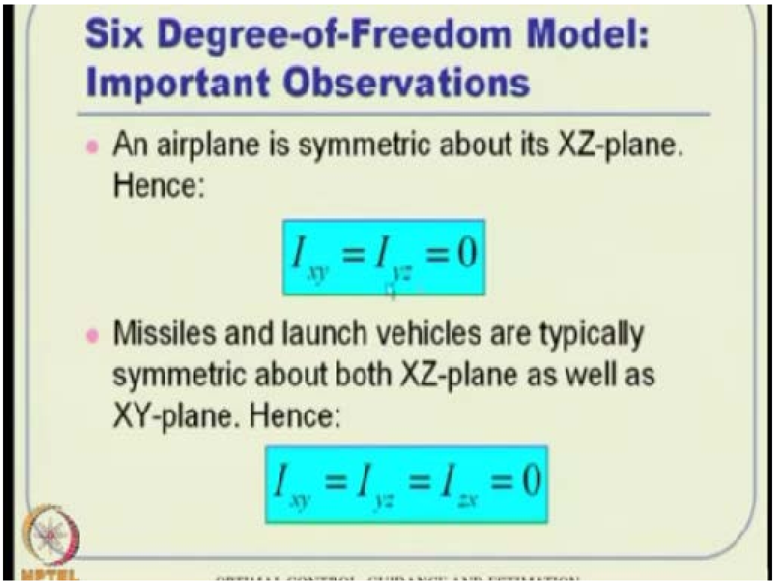

And then look, this can be viewed in this way. Then also remember that airplane is symmetric and that is one of the assumptions. So, that is why this cross moment of inner cells are zero and missiles and launch vehicles are typically symmetric about both x z plane x y plane and hence all the three cross moments of inertia are zero.

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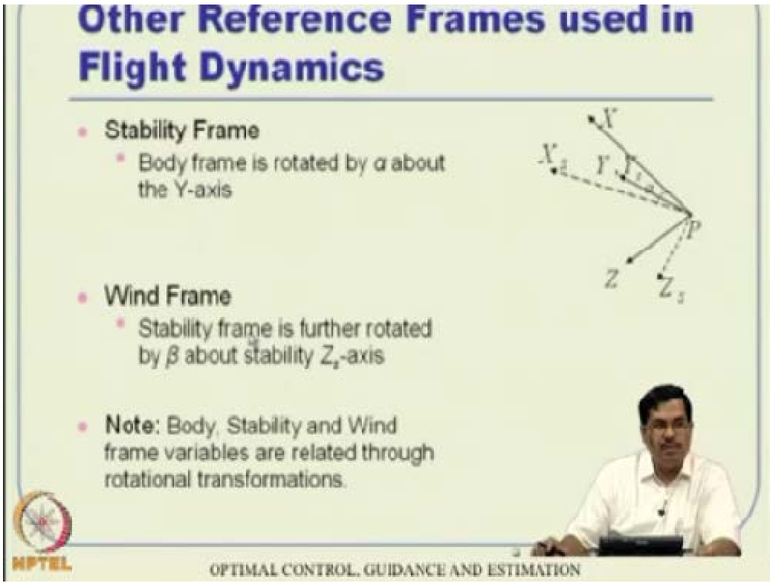

Then also remember that this is not the end of the story. We, in flight dynamics, people talk about various other frames; one of that is stability frame, the other one is wind frame. So, there are neat properties out of that, especially this wind frame, the aerodynamics lifts and drag that typically act on the wind frame. In these experiments sometimes the wind terminal data will be given in terms of stability frame. So, what is this? Stability frame is nothing but, like a body frame, just the rotate the body frame by angle alpha about the Y-axis; Y-axis remains the same and then you rotate whatever X Z axis by angle alpha and that will give you the stability frame. Wind frame is that stability frame you rotate further by angle beta which is something called side slip angle.

So, it will perfectly align, the X vector of the wind frame will be perfectly aligned to the extreme velocity. That is how we got the wind frame. The details, I will not talk about, but again they are all rotations only. Again this relationship will be reflected by this rotational matrix through this rotation matrix that cos alpha in the diagonal and minus sin alpha and plus sin alpha in the (( )) elements. So, the details and all you can find out in flight dynamics books and one of the books is probably Stevens and Louis with aircraft control and all that - aircraft simulation and control ability.

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So, let us move further and then we talk about what I just pointed out and we can discuss something called Small-Disturbance Flight Dynamics; that means local effects. You do not talk about the non-linear effect, everything coupled and thing like that. So, can we do some sort of a decouple motion, longitudinal and lateral and thing like that within small disturbance assumption. Many of this material I will take it from this particular book, which is also I think is very well written for these concepts.

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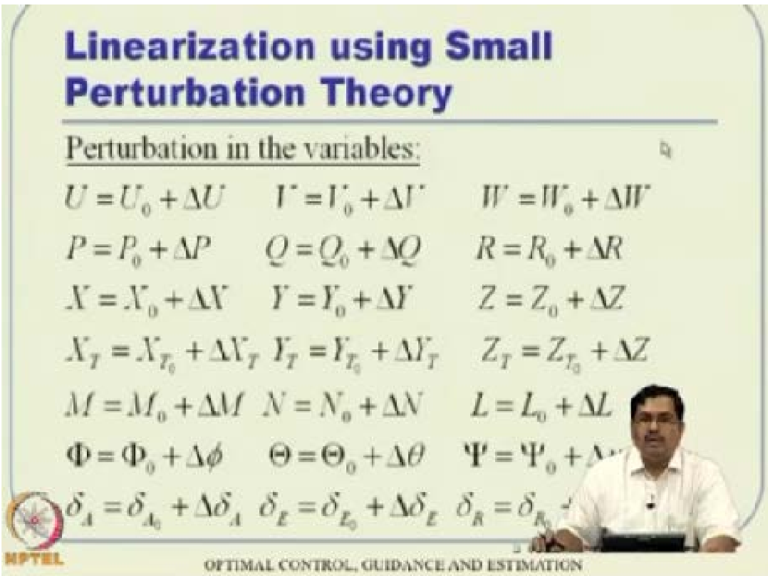

So, what are you doing roughly is you are doing all the variables whatever you had in the system dynamics, you interfere that as perturbations around some nominal values. For example, U zero, V zero and W zero, these are all nominal values around which there are perturbations. Like delta V delta W, all sorts of thing and the assumption is that the perturbations are small. So, what do you do? In the linearization, which I think we did not discussed yet, but in one of the further classes we will discuss about that formally. So, in the Taylor series expansion you neglect the higher order terms of these quantities. You can have the 6 degrees of freedom equation that is a non-linear coupled equation. So, you want to liberalize this around some nominal operating condition.

So, these variables which subscribe zero are operating conditions and these perturbation quantities, you assume them as small and try to put that in that Taylor series expansion and then neglect the higher order terms. We can do it in a different way also. If you see that book especially it is done in a slightly different way.

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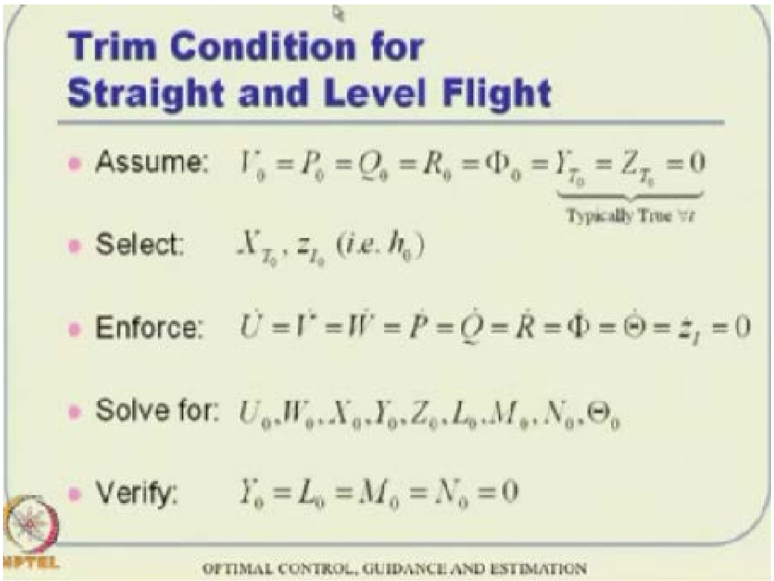

What it essentially does then? So, first we have to find out this operating point. Like you need to find out what are all these U zero P zero Q zero all sort of things are. So, for that there are various steady state conditions available which is called trim condition and trim does not necessarily means straight and level flight, but that is one of the very typically trim condition. If it goes straight and level that is like a steady state conditions, which essentially is called as trim condition. There are various other trim conditions also available. Trim condition means nothing changes, your control surface and all that are deflected by a constant quantity. Then suppose you want to take a vertical loop, it can also be considered as a trim; horizontal circles can also be considered as a trim. But, most popular thing is straight and level flight, if I just go straight and level and then do some stability analysis around that.

What we have seen here is straight and level flight, all those V zero and P Q are zeros, like P zero Q zero R zero, these are rotational rates. V zero is the velocity component along body Y. So, these quantities are all zero and these are like force quantities in the Y and Z direction coming out of thrust. Typically these are true for all the time. So, body Y and Z component, thrust does not contribute any component largely, especially in commercial flights. So, under these assumptions you select these quantities; that means, you tell what is my thrust level, thrust force, there is a engine I want to operate in five percent of thrust, ten percent of thrust, fifteen percent of thrust or whatever it is; the maximum thrust, I mean here. So, you fix that thrust quantity and then fix what height you want to fly as this vehicle flies. So, once you do this; that means, all these variables are either zero or known.

Then you want to enforce, remember these nine equations are there for us. So, all these nine equations should, I mean the rate of these variables should be zero there. There is no rate of change for these entire U dot V dot W dot and P dot Q dot R dot phi dot theta dot and Z i dot. This is the height part of it. So, all these quantities if I force it into zero this will essentially give me various free variables. Out of those free variables I know these and I have to find these. Remember, this is just one way of finding the trim conditions, these are other ways also. So, what I mean is essentially you are (( )) here that this happens to be zero; all this rate of sense happens to be zero and then you solve for that.

In that process, you can also verify that what you really get that also has to be zero. So, you just verify that there is no force from aerodynamics; Y has to be zero, no moment from aerodynamics about the body axis. That is just the verification; I mean if that does not satisfy then there is something wrong here. So, you solve these and essentially what I mean is all these variables with subscript zero are now available.

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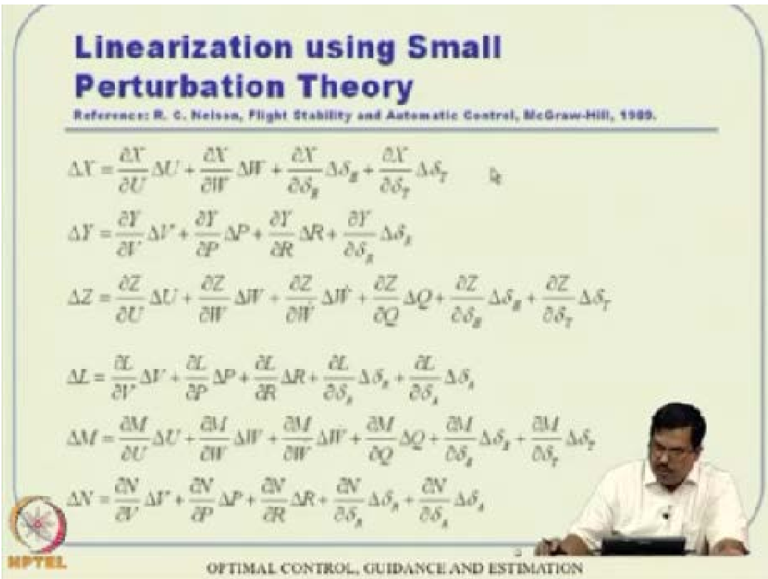

Now, you can take, these aerodynamics forces and moments around that is perturbations around that I can visualize in Taylor series expansion and I want to keep the major components only and this series does not stop. Suppose, somebody wants to write this other functionalities then they can still write it and as for as perturbation and delta X is concerned, then remember this delta X is the aerodynamics force in ver d X direction. Then this aerodynamics force can be a function of U, function of delta W, function of delta g, function of delta thrust, all sorts of things. The series can be expanded also and these are the major components. By the way, this way of writing is something called component build up. So, component by component you are building up, whatever components come here ultimately. Similarly, delta Y you expand and delta Z you expand and delta L, delta M, delta N also you expand.

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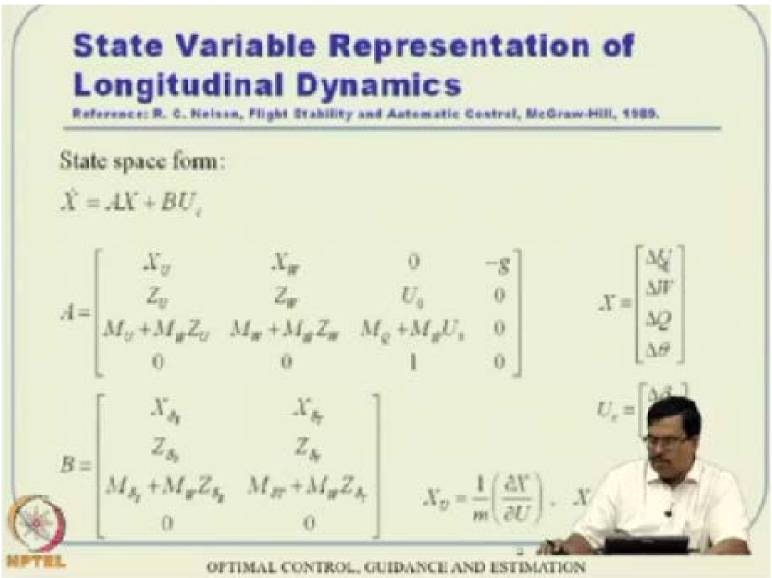

Then you can write this. Then the inter dependencies you see, out of that, still if I club these variables together, the delta U delta W delta Q and delta theta, they became independent of rest of the equation. They become only functions of themselves and these two quantities delta g and delta T. Essentially, I will be able to write that these X dot equal to A plus B U e, in states space equation form. So, linearized state space equation form for longitudinal dynamics happens to be like that. It is a four dimensional system where state and control are defined like that. For notational simplicity, these are also defined that way. X u means this del X by del U with one over m normalized and that is well acceleration quantity. X is a force quantity, but X u is an acceleration quantity. Now, one by m, so it is the force divided by m becomes something like acceleration quantity.

Anyways, so with these notations entered whatever you see here in the matrix delta naught will not to be like that. So, in flight dynamics suppose somebody gives us this A and B matrix numbers, these numbers essentially mean these quantities. That means, there are coming from there.

These are the nice consequences. Now, I can further divide this four by four into two, two by two systems and then I will take. One time I will talk about this delta U delta theta together. Other times, I will consider delta W and delta Q together in a way. These are all large and which is a function of what and thing like that.

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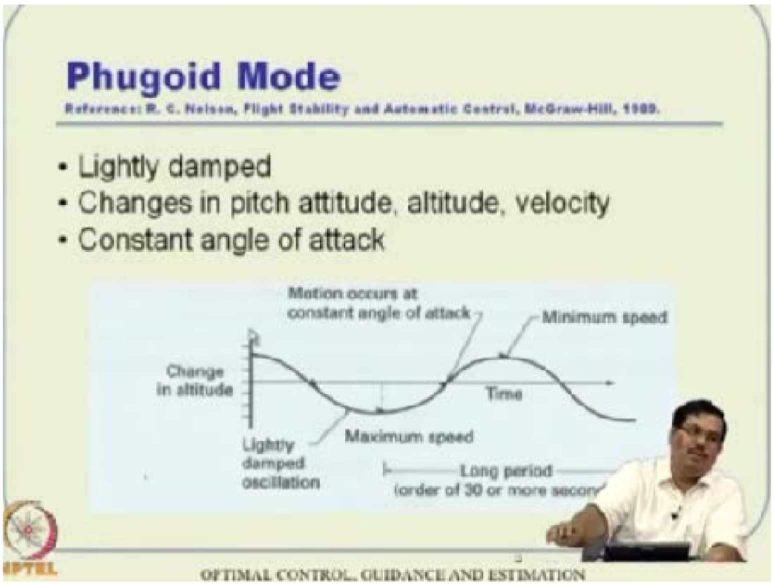

They are two types of nice perturbation happens around the steady state level fly. So, one thing is which is called as Phugoid mode. Essentially, the aircraft drops it height to a significant thing and then goes off. The vehicle altitude does not change; altitude remains same. The entire vehicle will keep on going and then coming down and then going up. It is like a roller coaster ride. So, this happens with something like a constant angle of attack.

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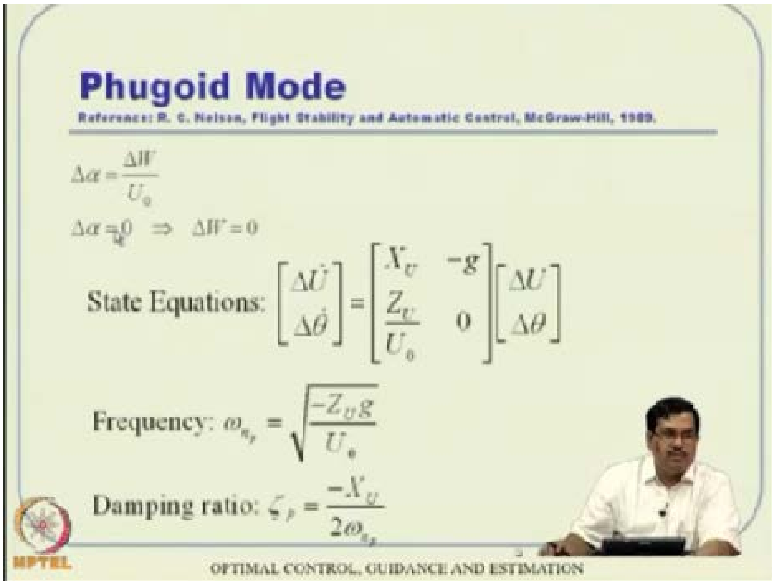

So, under these assumptions delta alpha becomes zero because alpha remains constant.

And then we got delta alpha and delta alpha is defined that way. So, delta W also becomes zero. So, you are left out with this nice small matrix and I mean homogeneous linear system, which you can analyze it in a very clear way. This is second order system in a way. Delta U and delta theta are just analyzed together. So, we can find out the Eigen values and then we can still look and write it as a second order of equation and tell that s square plus two theta omega minus plus omega n square equal to zero. That kind of equation we can write and tell that this omega n turns out to be like that and theta turns out to be like that. So, from the figuring note there is a natural frequency and there is a natural dumping and which are given like that and the motion is something like that.

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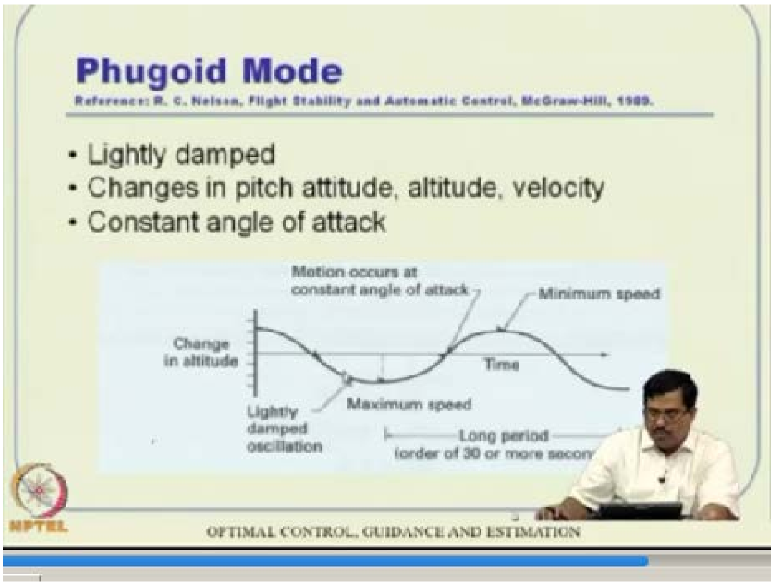

It will go to a maximum speed at the down then it will again quick up and then it is a minimum speed at the top. Then it will come down and it will go up. It is a nice roller coaster ride. Obviously, nobody in the passenger aircraft will like that. So, you want to control that also, physically.

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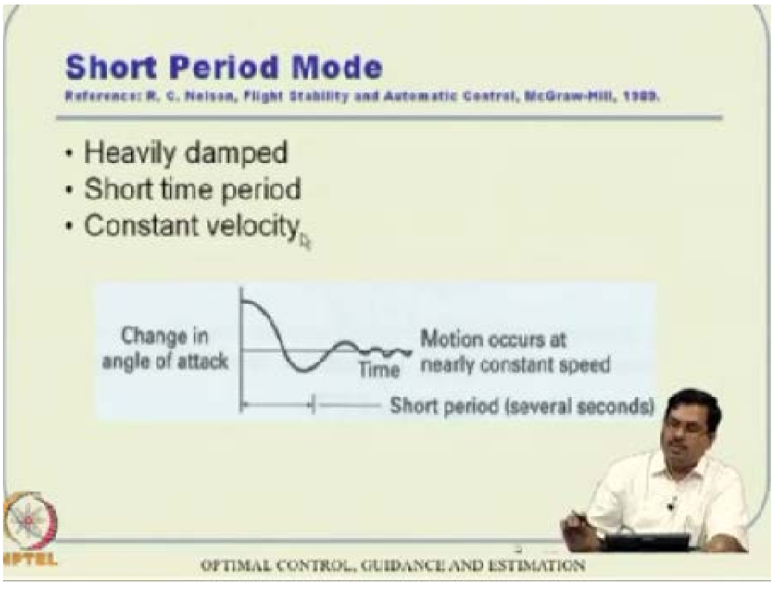

Now similarly, there is another motion which is called short period; that means, this vehicle does not go through large amount of height variation; however, there is a vibration around that. This happens to be something like this. So, this oscillation about this vehicle (( )) sort of thing, but, this is little bit more unpleasant experience for the fellow passengers. So, what happens there it is change of angle of attack and this oscillation fortunately happens to be heavily damped. So, that means, this oscillation typically die out first and some of these you might have experienced while commercial flights, sometimes this wind gust and all that naturally died down also without too much of control like.

Some control can be exited also for that. Suppose, the damping is not satisfactory then obviously, we need a control for that also and both these are controlled by mean elevator - a elevator held on an (( )). So, these are longitudinal motions and these are functions of elevator, delta e quantity, remember that here. So, this delta e quantity what we see here is largely utilized for surprising these two phenomena.

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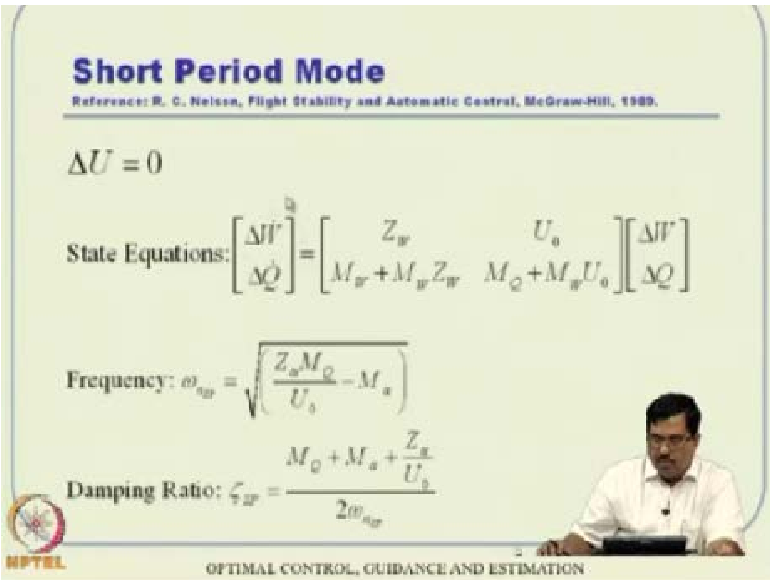

Again, this short period modes again you can visualize that dynamics in terms of delta W and delta Q. Again, a two by two matrix, again you can do this naturally, I mean frequency and damping ratio. It happens to be like that and remember these are all functions of aero dynamic parameters, which are also functions of vehicle configuration.

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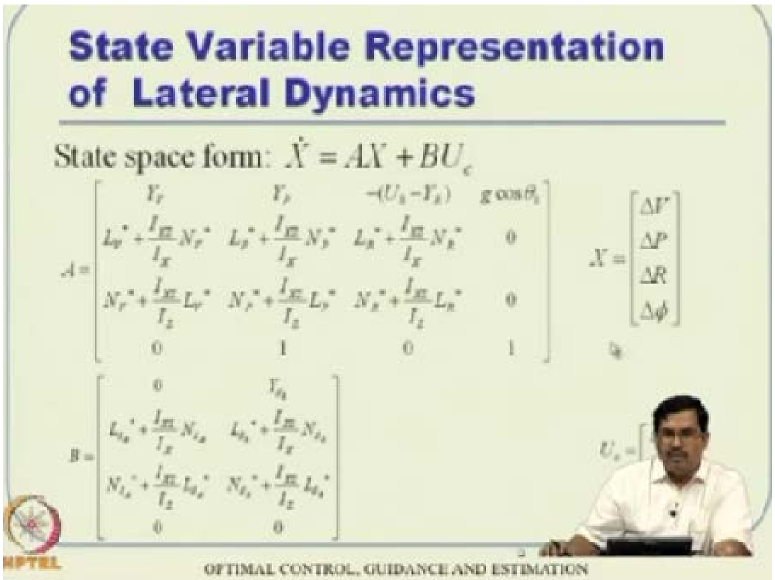

Now, similarly, in the lateral dynamics, you can decompose that into again four more variables, which are not same as the other variables. Here we talk about V, P, R and phi. Phi is the role angle over there and then the A and B matrices are given like that. All these parameters values are that.

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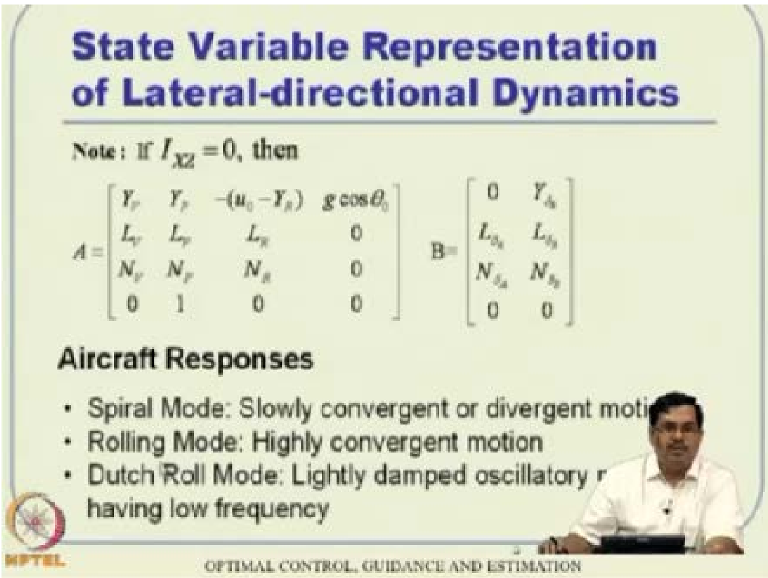

Similarly, because if it is I xz is also zero, which happens to be missiles and on all that, then these whatever you see here, will further reduce these, for the simple looking form and this will term out be like that. So, aircraft response as far that is concerned, we can see that in this lateral dynamics there are two modes again; one is Spiral mode and another is Rolling mode and there is a Dutch roll motion also, which is a couple between these two.

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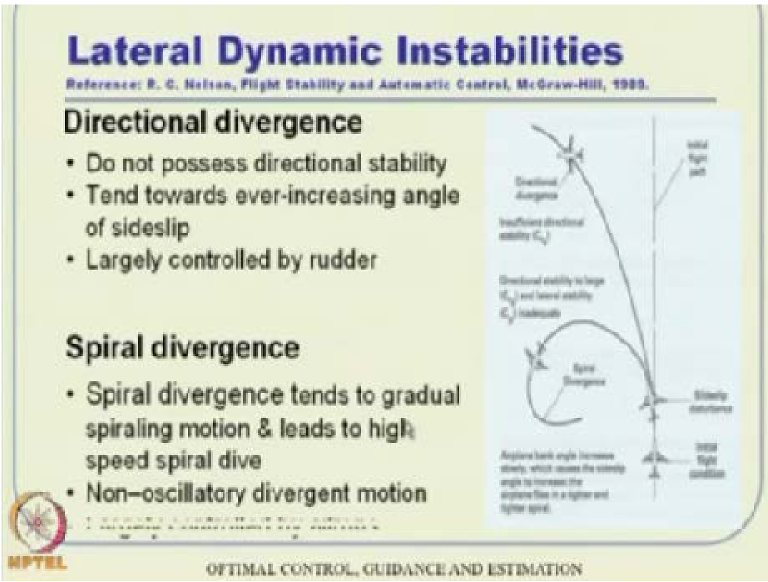

So, let us see what that is. So, this is directional divergence first and what it does is they does not possess too much of directional stability. So, in other words there will be an increase of this sideslip angle beta. The U V W components, remember that, so the V component starts building up. If the V component starts building up then the vehicle will largely deviate from his intentional path. In a long duration if it is suppose to go in that direction, it will not go there, it will try to diverge. And then at some point of time beta will couple with phi and then it roll dynamics will start all sort of things.

So, that is called directional divergence. The other one is spiral divergence, which is - largely responsible from this gradual spiraling motion, that means, the rolling stabilization suppose the vehicle starts rolling and all that. Then it will also lead to dive and that we know that. The moment the vehicle rolls, it dives also. So, that will not be stable then it will excite spiral mode that mean this radius of curvature becomes very fast and if also loses sight and it crashes ultimately. So, this two are and directional divergence is not that bad. In other words, if it is not controlled in times then it will lead to be further bad things. This is even further bad in the beginning itself because it rolled, this spiraling action gets amplified very quickly.

This particular aspect is largely controlled by rudder and this particular effect is largely controlled by airlons. Remember, it is a motion due to phi; roll angle, and this motion is due to beta. So, beta is slide slip angle. So, beta is controlled largely through rudder and roll is largely controlled through airlons.

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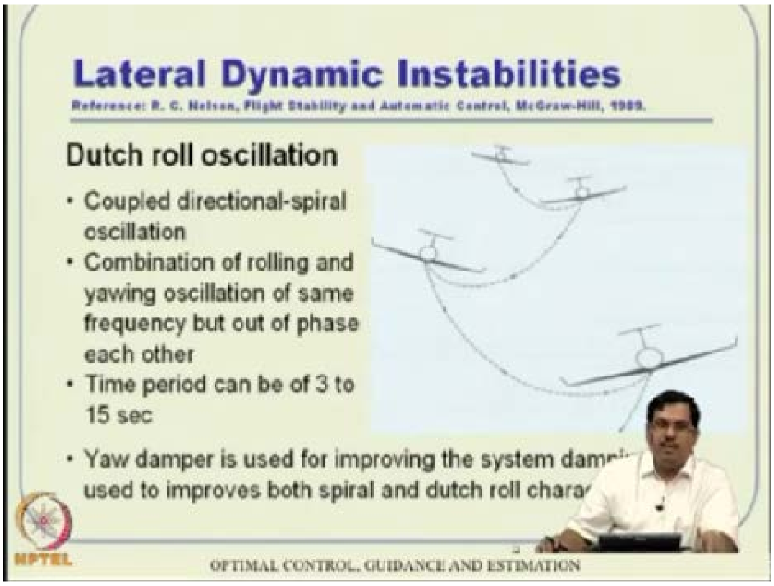

From there, there is a nice coupling with the two. This is what is called Dutch roll oscillation. So, here it is that the directional spiral oscillation will be coupled together. It will roll and hence its beta will low, it will go somewhere and then it will starts in opposite directional sort of thing and then it will go like this and then it will go like that. These keep on and remember this amplitude will keeps on building also. This typical name comes from this ice skating.

When we do ice skating then one side you go and then you come back to other side. So, that is typically done by people like that in ice skating and also that is how its terminology has been given. So, time period can be of three to fifteen second and then typically this yaw damper is used. Yaw damper is largely by the way you design the vertical fin. The part of the vertical tail which is ahead of the rudder, that vertical fin, if you define it properly or design it properly then it got proper yaw damper.

But, in addition to that you can always control it using both airlon and rudder together. This motion you cannot de-couple into two, two by two system. You have to talk at least about one four by two systems. And then where both these airlon and this one rudder will come and hence you design them together. That is the power of this state space design approach. You talk about the coupling effect also in a design process.

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Now, before I stop this class, I will also take you through some various attitude representation, which has this entire six-DOF and all, you derived in terms of Euler angles, but there is the strong drawback for that and we will see that. There are also alternate to attitude representation, people have thought about that seriously. And these problems are further amplified if you have this satellite control problem, for example, or missile dynamics which have large angle of rotations coming to the picture. So, satellites can turn; it will completely turn 360 degrees and keep on doing that in a space and hence the missiles also. Their high angle of attack means it can take big turns and all that. So, large angles, when we talk about the Euler angle are typically not good. So, we want to have some other thing.

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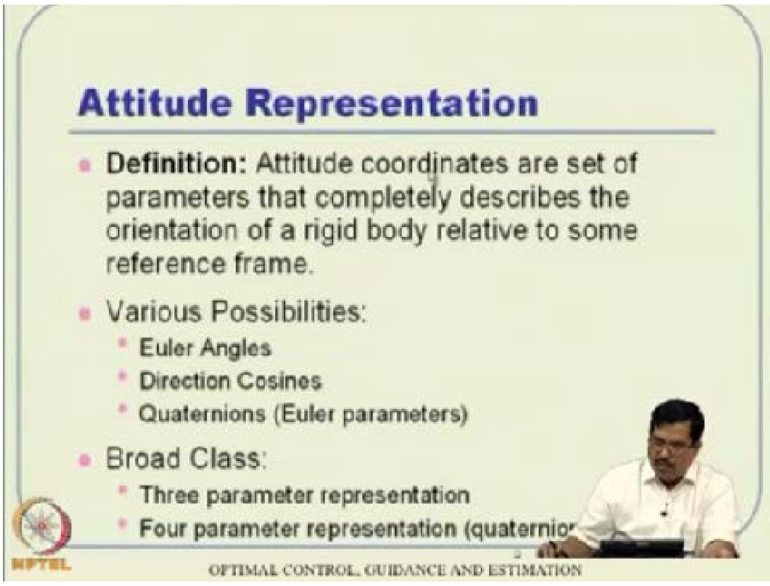

So, let us see that. So, attitude representation by definition is essentially the attitude co-ordinates or set of parameters that completely describe the orientation of a rigid body relative to some reference frame. One set, we have already seen, the Euler angles. The other things that are available to us are direction cosines (( )). In fact directional cosines are the first step before the Euler angles and then it does not stop here, the people talk about well Rodrigues parameter, then modified radix parameter and the family of that. They are given and take. So, in broad class, all these parameters were to see in two broad classes. One is like a three parameter representation and another is a four parameter representation, essentially, quaternion in the four parameter representation. So, three parameter representations are nice, because the attitude is a three parameter representation.

Whatever you see, any attitude is three angles somehow, but there is a serious problem. We will see that and if you want to quickly look at that I can also go there and come like probably. So, this Euler angle representation (Refer Slide Time: 48:33) if you see here there is tan theta term involved. Tan theta is sine theta by cos theta and here is a sec theta which is also one by cos theta. So, that means, the theta goes to ninety then the cos theta becomes zero, and if something is divided by zero which many these rates will go to infinity. That is something called (( )). No matter, whatever degree of accuracy, we use like delta T takes it to various small quantities; even then the integration cannot be drawn. These dots go to infinity. The angles are okay and the angle values are fine, but the rate of change goes to infinity, which is not nice.

So, in this set up this rate of change is function of the sequence of rotation. How we do that? You did that sequence of rotation from psi theta phi. If you do a different side of equation then the variable with the tan theta will appear happen to be tan phi or tan psi. Somewhere, we will be getting locked. So, no way you can get out of this mechanism. So, that is where it happens and this is actually bad because it happens in ninety degrees. 90 degree is a small angle. Now, the whole idea for the MRP, modified Rodrigues parameter, they are already taken into something called that 360 degree or multiples of 360 degree, and then only you can have singularity.

So, one full round you can take and then still can evolve singularity and then still talk about three parameters representation, which is nice. The three to three is like there is a redundancy variable does not come into picture. So, that is why. So, let us talk about that now.

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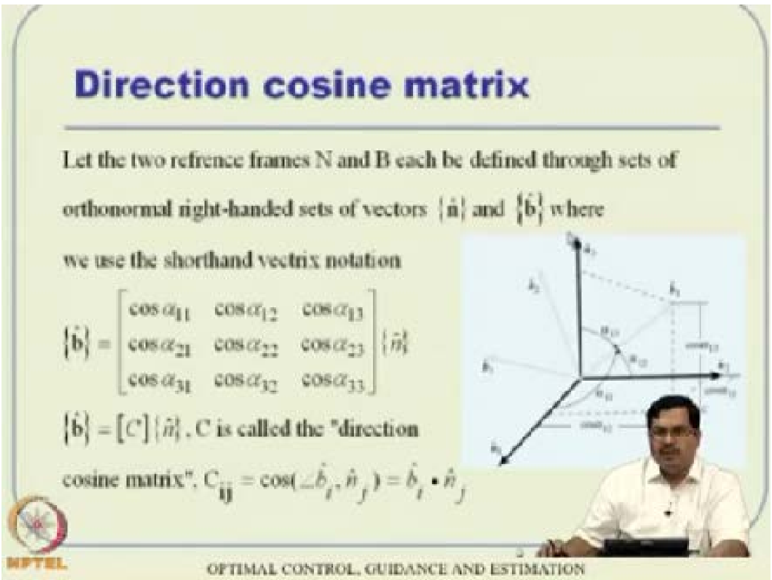

First is the direction cosine, so direction cosine is if you have a reference frame and if you have a vector, any vector I can talk about direct angles. This vector whatever I have, I can take direct angles with respect to this axis. If I take cosines of those angles and put them in a matrix form then this is the matrix that I am talking is direction cosine matrix. Also the components of this vector, whatever this vector this something like a b 1 component, b 2 component, b 3 component; each of the component what I mean is they will help direct angles with respect to this other frame. See there are two frames, one is n frame and one is b frame. You can visualize n frame as an inertial frame and the b frame is the body frame. So, each of the components of the body frame makes an angle with all the three axis of the inertial frame directly. So, it means, that you have nines of equations.

So, all those things, if we have put to them together then the b frame is related to the n frame through a matrix representation, which is directly like the cosine terms and all that. We know that and if you take dot production you can derive it and that is not a problem. The two vectors if you take dot product that will the cosine angle will come in to picture and that is why this cosine terms come. So, this b which results from this algebra is something related to n, through this c matrix and c matrix is essentially called direction cosine matrix. That is the definitions actually. Now, remember we really required three, but you have nine parameters. That means there are six redundancies here. That is the problem here.

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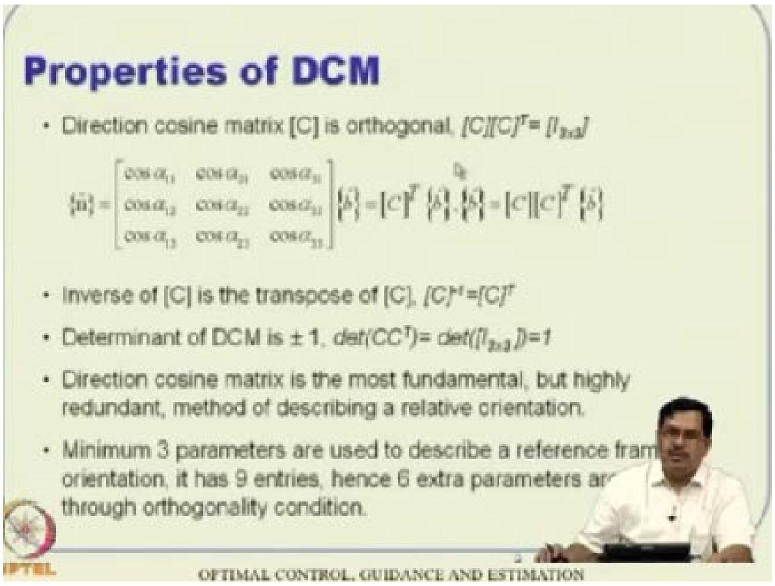

Now, before we talk about that let us observe some nice properties. Like, that c is essentially an orthogonal matrix and orthogonal is a nice property that C times to C transpose is identity; that means, inverse of this matrix is transpose and inverse always exist. So, there are nice properties of that. Inverse is transpose and determinant of this is always plus or minus one, depending on what angle it takes and direction cosine matrix is the most fundamental, most natural way of visualizing things. But, it is also highly redundant and because of this we required three, but it has given as nine parameters, nine entries.

So, six extra parameters are redundant through this orthogonality condition. Basically, orthogonality condition tells us that each of this vector is orthogonal to that and because of this vector, matrix is orthogonal, the vectors also orthonormal. So, if I take dot products of any two rows, they are zeros. If I take normal two rows that will happen to be one. These are orthonormal, each of the vector are normal to each other. So, that those are the redundancies that I am talking about. So, if you have these combination, the three combinations of dot products and then three normalization quantities. So, these six redundancies we have to talk about.

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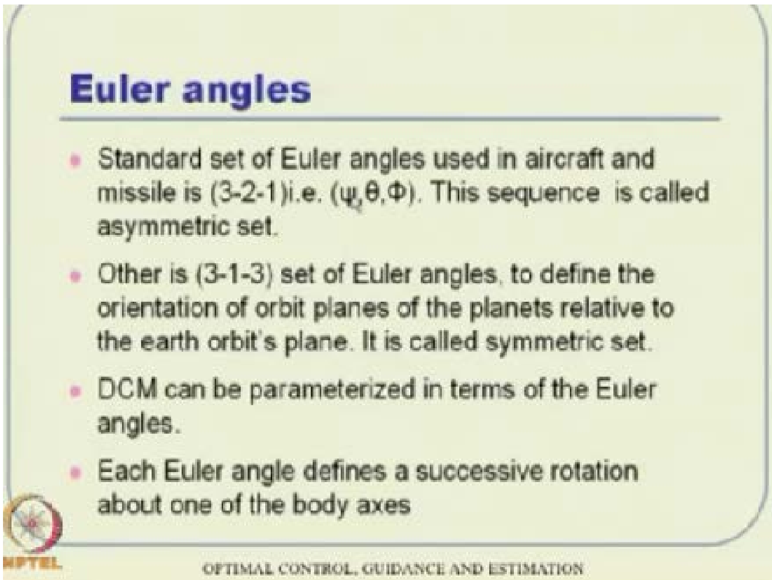

Now, Euler angles, we have discussed everything on that probably, but still to visualize that, the standard set of Euler angles is what we have done. it is a psi theta phi rotation. But, this is also called asymmetric set. What people use in satellite dynamics and all is what is called as three one three set of rotations. This we will not talk too much on that any way, but that is also called somewhat called symmetric set. You do not have to rotate it using this three two one, you can also go for three one three, three means three stands psi, two stands for theta and one stands for phi. So, what you are talking here is psi phi and again psi. We can also do that and that typically standard for satellite dynamic and all.

And then there is a direction cosine matrix, which can be parameterized in terms of Euler angles. We will see that relationship also and obviously, Euler angle's defines successive rotations and we discussed all about that.

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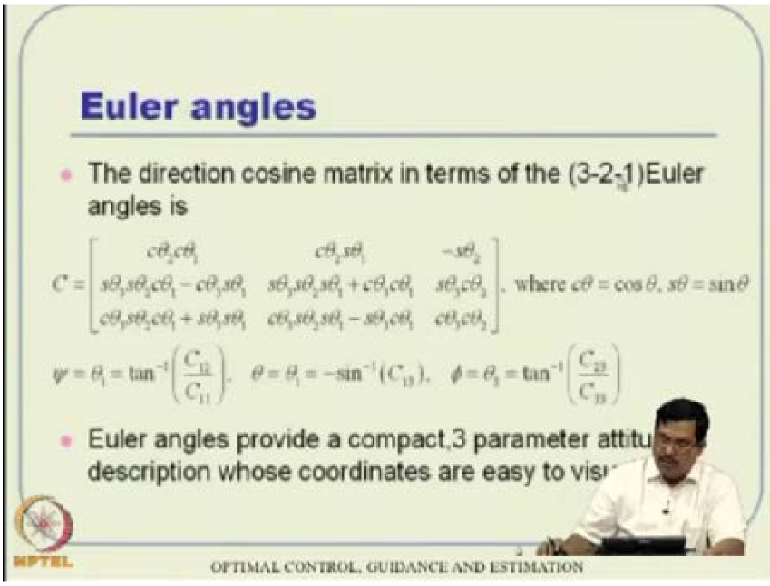

So, this is the representation. like if you have a Euler angles, the theta one, theta two, theta three, I am not writing in terms of theta phi, psi; you can also write in terms of one two three notations, theta one, theta two, theta three. Then the direction cosine matrix turns out to be it like that where c theta stands for cos theta and s theta stands for sine theta, especially. Then out once you do that then this psi theta, phi can also be given in terms of inverse transformation that means, somebody gives me the direction cosine matrix, then I can also recover these Euler angle quantities. So, the vacant four transformations available, once you know one representation you can find out the representation of the one.

So, then finally, let us talk little bit on quaternion. Quaternion is essentially to get it of the singularity problem. Unfortunately, what happens is anything that you use either in direction cosine, Euler angle, modified Rodrigues parameter, whatever you use, all these three parameter representation do suffer from singularity. It is a matter of where the singularity occurs, but it has a singularity. Now, can you talk about singularity free transformation and that is why this quaternion algebra is useful.

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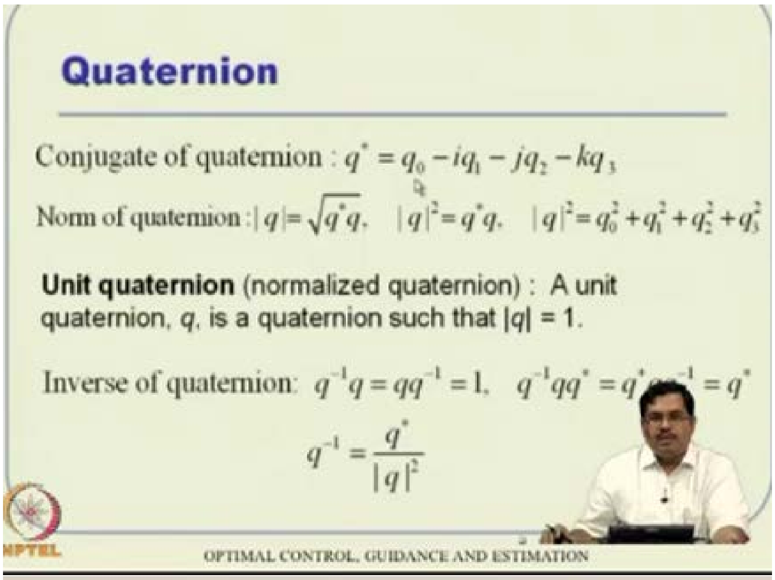

So, what it turns out is we will come back to this probably, but the fundamental thing is something here. So, for every attitude change that I am talking about, I can always visualize some x is somewhere around which I can take one rotation and finish it off. This is for all the sequence of rotation that I am talking, from inertial to body, whatever, from one axis to another axis frame, there is one vector which is called the principal rotation vector around which I can take one rotation, so that these two axis will merge together. That is the concept and that is the theorem.

Now, obliviously we have to design that axis frame and that is given by three quantities like q one, q two and q three. And then there will be something like one rotation quantities also have to discuss about it and that is the quantity phi. So, quaternion will consists of four parameters, q zero and then q one, q two, q three. So, q one, q two, q three, will typically give some sort of something like the direction of this rotation vector and then it is the phi angle, which you can be take out form this q zero component. So, phi by two is cosine inverse of q zero. That way the phi angle is the rotation about that vector, which will make it coincident.

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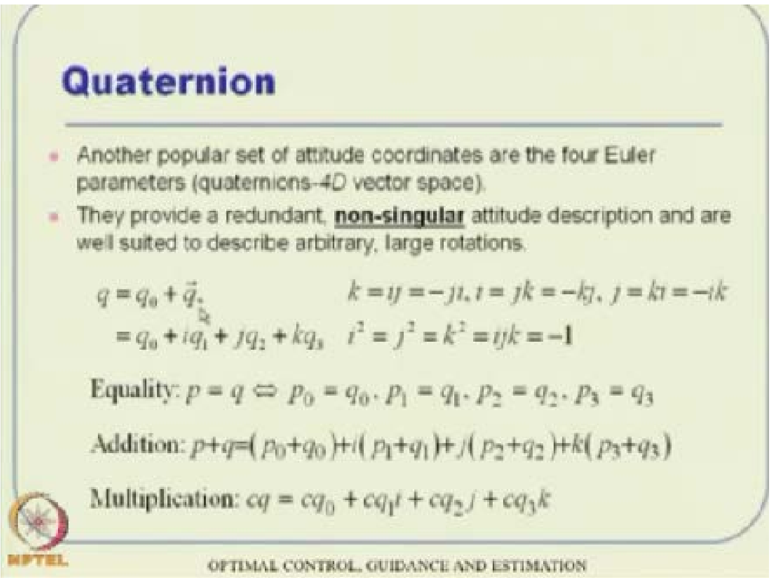

Now, going back to that, this is a quaternion form. This q zero plus q bar, q bar consist of q one, q two, q three, and then this i j k you can see that the square of that and i times j times k all equals to minus one. There is some neat algebra associated with that as I said to be. So, you can some of this and equality sense also like there is various further things. I think some of you were interested in this particular book which I update it from the book named as analytical mechanics of space system. There is nice software for doing all these.

Anyway coming back to this, so addition, we can define it that way. Suppose, you have two quaternion, let us say p and q, so p will have p zero plus p one p two p three; q will have q zero plus q one q two q three and then with respect to that you can define addition multiplication and like that.

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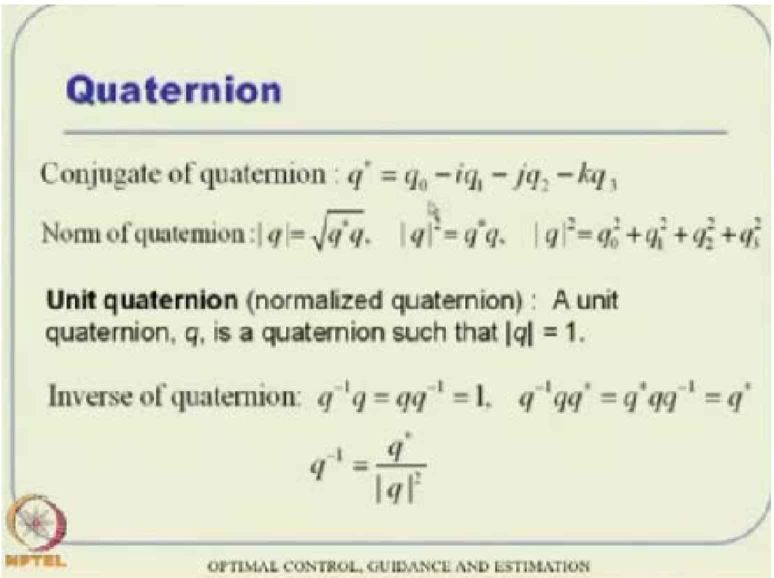

Then there is a conjugate quaternion, which is a negative of all these quantities and the norm of the quaternion is defined like that. So, obviously, this q square whatever you have seen here is one. q zero square plus q one square plus q two square plus q three square happens to be one and that is called unit quaternion. That is actually a constraint and that is a constraint that you have to operate with. So, all these parameters, this has to satisfied that and that happens to by problem in your integration, for which we need to normalized it force fully and for which there is no unique way of doing that. That is the difficulty there.

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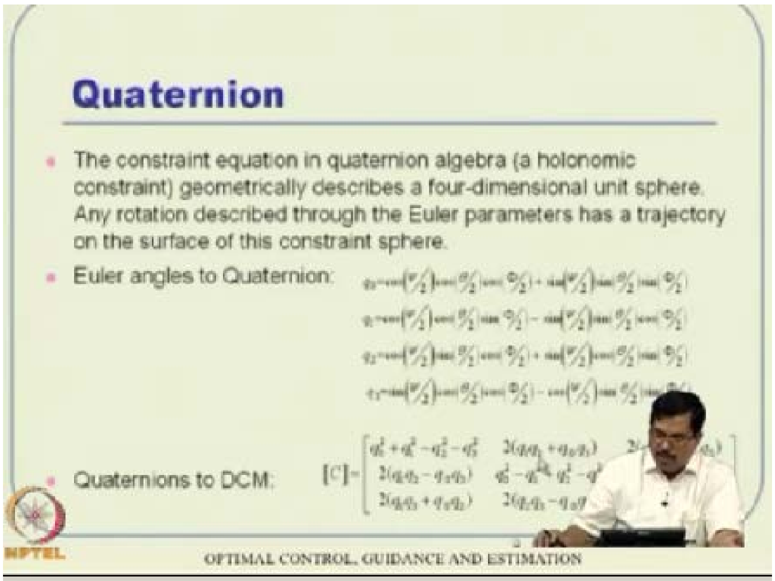

So, quaternion also satisfies this quaternion to Euler angle. Suppose, you know Euler angle then you can find out quaternion. Suppose, you know quaternion you can find out direction cosine matrix, things like that. there are there are transformations available which we can extract one information from the other. So, I request you to see some of these chapters for more details. This is a control theory class, you cannot talk too much detail on that, but with also the flight mechanics ideas and all that, we will be able to utilize all control theory ideas for flight control guidance and that is my motivation. With that, I will probably stop this class. Thanks for the attention.