Optimal Control, Guidance and Estimation

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Lecture. # 16

Overview of Flight Dynamics - II

We will I mean, we have already covered what is the basics of flight dynamics as well as many things about classical control system as well as the state space representation so far. So, this particular lecture in next will study a small overview about flight dynamics, so that you can connect the material what is going on here for aero space guidance and control. So, this particular lecture will take you through this so called four point mass equations as well as major percent of what is called a six-DOF equation, and then will follow that of in the next class with further concept, which will make us ready for understanding the relationship between what we are studying in the control theory to aero space applications actually.

So, this is the first, we will see point mass dynamics and it all depends on your application, how you want to visualize this aero space dynamics actually. Suppose you are interested in a long term trajectory, then probably point mass equation are sufficient because you do not know, do not need to know the attitude of the vehicle at any point a time. You just need to know the location of the vehicle as well as it is velocity and velocity direction in a gross sense. So, really do not need to have all details about the vehicles attitude, vehicle load rate and in width rate thing like that for a long duration applications that is where this point mass equations becomes relevant, and it is mostly used for guidance applications actually. So, let us see what is called as point mass dynamics, and where it is essentially visualizing the entire vehicle is point moving in space. So, we will and then relevant equations will see what I mean what all variables are there and think like that.

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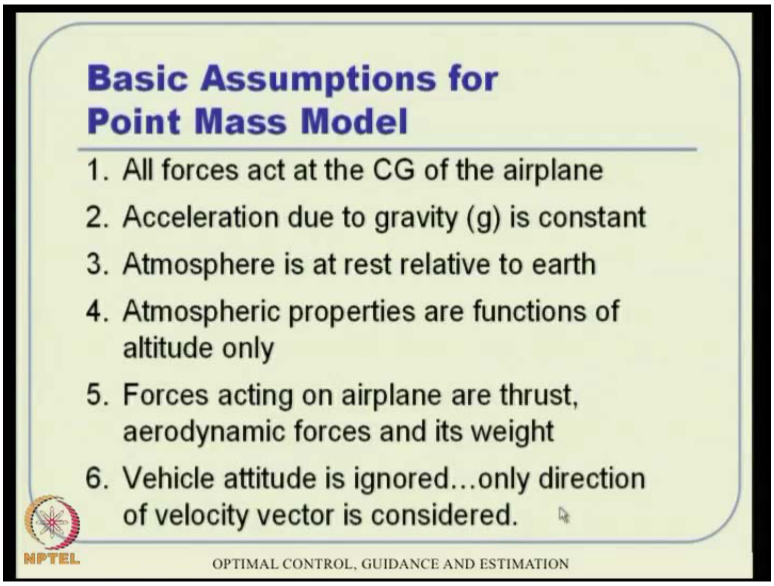

So, basic assumptions for point mass model. First of all, we are all kind of collapse in the entire vehicle at the CG of the airplane; that means, all forces act on the CG of the airplane, then the entire vehicle is just a point; and acceleration due to gravity is constant. We are not bothered about variation of g, unless you talk about very long very kind of inter-planetries missions something like that, when you really go far away from earth actually.

However, it is for most of the application, we can fairly assume that gravity is constant, and then atmosphere is rest relative to earth. We are not considered about this wind effects and all that; and atmosphere properties are also functions of altitude only. We will consider this especially we know the dynamic pressure is a quantity, which is which depends on density of an atmosphere and that varies with height. So, atmosphere properties will consider only as a function of height nothing else actually. And the force acting on the airplane or thrust, aerodynamic forces and its weight, will see that in a vectorial diagram next slide actually.

Vehicle attitude is ignored as I told, which angle it is oriented and things like that, is not of all concerned here. However, the direction of velocity vector is a primary important because that will go when the trajectory of the vehicle and the long actually.

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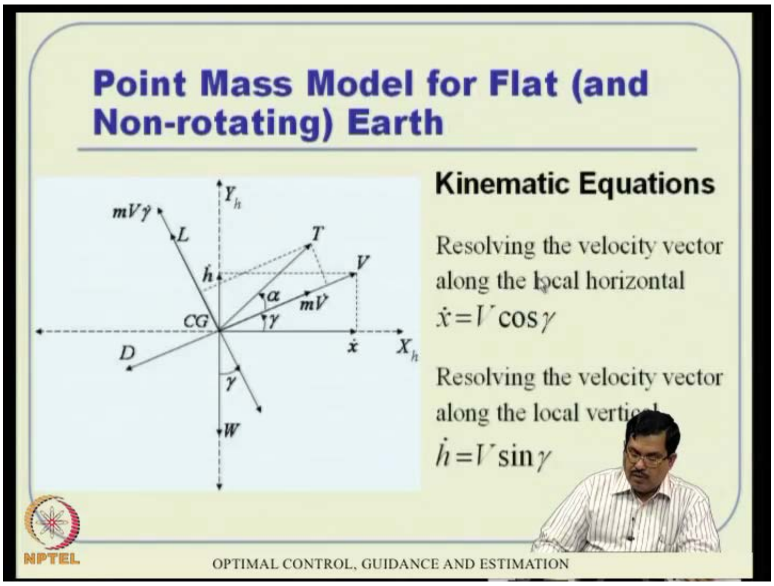

So, this is pictorially, this is where the vehicle is located. The entire vehicles is just a point here; and let us say, the vehicle is moving in that direction V that is the total velocity of the vehicle, and this x is actually like parallel to the local horizontal; so, whatever angle that these velocity vectors max with respect to the local horizontal plane that is called flight path angle or gamma. And most likely, the thrust vector is align with respect to the vehicle X axis, vehicle knows and thing like that. However there may be the kind of alignments, small angle with respect to the vehicle X axis, will not consider that here.

What we will see is the thrust is aligned with respect to the vehicle x axis and that sense this angle what you see, is an angle of attack alpha. So, y is the altitude were Y axis and x is the down (()) actually; that means, wherever this flight goes, we are considering only on that plane .This is not a three-dimensional picture. It is just two dimensional in the pitch plane sort of thing actually. So, this vehicle, this velocity vector V can rotate depending on this alpha actually because thrust is thrust is the thing, it is not aligned with the velocity vector and hence it will have a component perpendicular to the velocity vector which will result in orientation, I mean which will take this velocity vector away; that means, gamma will alter because of alpha basically.

So, with respect to that, let us result the component not in various planes. Now, we have this; we know that drag it is directly opposing to the velocity. So, drag is straight full, I mean straight away opposing to the velocity and thrust I mean, lift is perpendicular to the drag of or perpendicular to the velocity vector. So, that is how it is. Alright and then weight we know, it is perpendicular to the local horizontal. So, weight will always at perpendicular gravity a small weight and thrust is anywhere there. So, we are interested in this velocity level equation and resolution level equations actually yes.

Now, let us see, velocity level equations are nothing but simply components of the velocity vector in x and y direction actually, that is all. So, what you have here, this velocity is, if this angle is gamma, then this actually x dot and that is h dot and then this x dot is; obviously, V cos gamma and h dot is V sin gamma. This is simply the component of velocity vector in a horizontal and vertical plane; nothing the simple geometry tells of that one actually. However you want have a moment level moment level equation, I mean sorry, the force level equation, next that will be resolved again into this perpendicular direction something like that.

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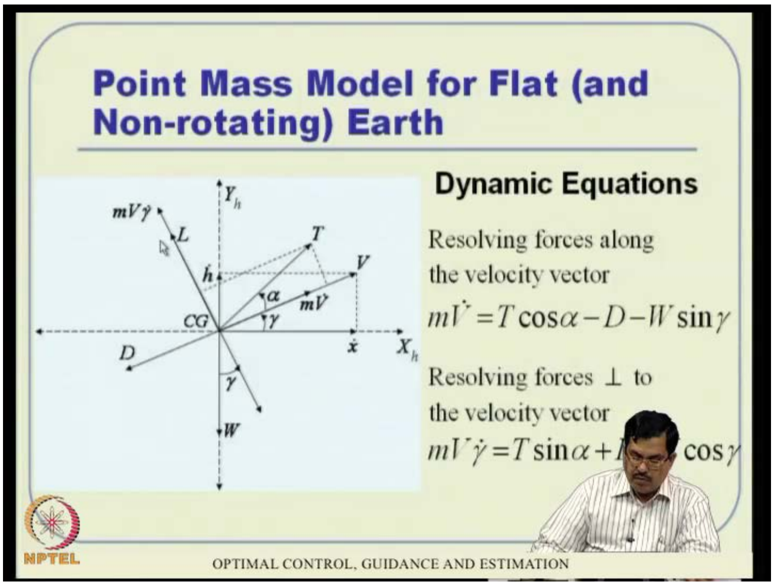

Before you do that, let us I mean also remember that is the velocity level equations do not see forces and do not see vehicle parameters. They are simply related by geometric relationship. These are called kinematic equations. So, you will have two kinematic equations coming here; x dot is V cos gamma and h dot is V sine gamma. The next is dynamic equations where your thrust and weight vehicle parameters everything will come into picture. Let us see how to resolve that.

So, this particular thing will resolve one in the one along the velocity direction velocity vector direction and one perpendicular to that all the forces that is acting on the three. So, what are the total forces acting on this particular direction, one component comes from thrust, that is given as t cos alpha. One is opposing to that; that is minus D. So, you have T cos alpha minus D and W is also a perpendicular W is a component because is a gamma angle that is acting on that actually.

If you see W, I mean the w was the component which is w sin gamma which is negative to v actually. So, if you have T cos alpha minus D minus W sin gamma, that is the net force acting along V direction velocity direction and I hence by Newton second law this is m V dot. So, m V dot is nothing but T cosine alpha minus D minus W sin gamma whatever component; that is what it comes to…

And similarly, once you try to prove I mean, the kind of resolve this along the vertical direction I mean, this perpendicular to the velocity vector, then see that this is the this thrust component again that is t sin alpha this time plus lift component minus w cosine gamma. This component will also there. So, that is what you see here and that is all equal to the net resulting force acting on the direction. Remember that is like I mean centrifugal force anything like that you can assume that way.

So, if you I mean, this particular thing is nothing but m V gamma dot actually. m times V into gamma dot remember this is actually force quantity this acceleration quantity v time gamma dot and m V gamma dot is the force quantity and that is like a centripetal force sort of thing actually. So, if you of you imagine a circle somewhere here passing there, but velocity vector is tangent to that, if you imagine a circle passing through this point where velocity vector is tangent to that, then the net resulting the centripetal force will be somewhere m V gamma dot actually. That is how you see that. That is how this gamma dot will start changing actually.

I mean this. So, if you divide that the entire quantity by m, you will get V dot if you divide the entire quantity by m V will get gamma and dot. So, putting everything together, so, what we get x dot is v cosine gamma, h dot is V sine gamma, V dot is that and gamma is that actually and typically this kind of problem.

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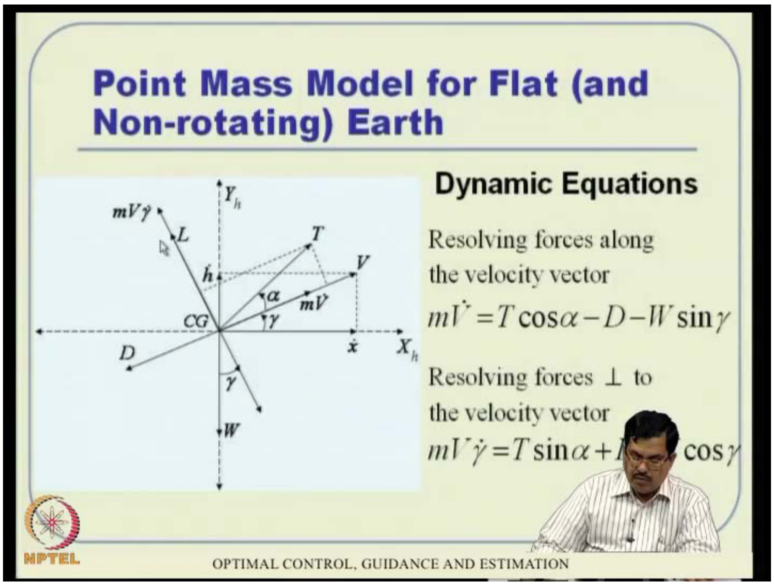

So, we interoperate alpha as the control variable. That is where we have the liberty of changing alpha basically. So, that is I mean, I suppose if you go back to that, if you see as especially locate, it will say then we have a thrust vectoring facility; that means, thrust. Thrust can be tilted out I mean, that means, this thrust deflection can happen, did not happen did not be align with respect to the vehicle knows actually. So, that is that can that can acts as a control aerodynamic variable all sense also it is same. You can manipulate alpha by deflecting the control surfaces to an aerodynamic forces actually.

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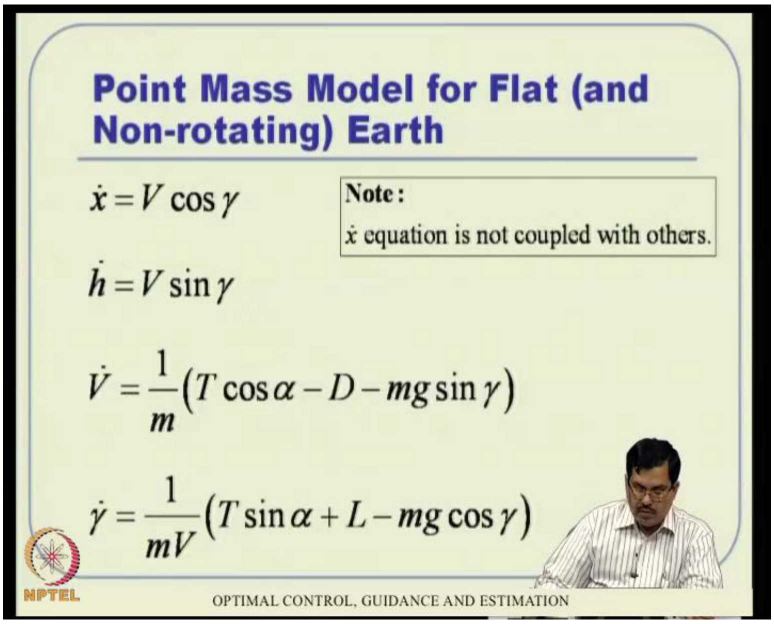

So, putting together, this is the set of equations in state phase is as form that we talk as long as we talk about flat earth model of point mass equations. So, the earth is flat, and it is not rotating and also several assumptions that we several other assumption that we talked here actually. So, under those assumptions, we will consider, these are set of equations that to deal with where x, h, v, gamma are state variables let say, and then your alpha is control variable and D and L, remember if there also functions of alpha, the drag and lift; they are certainly functions of alpha actually. And especially this mass quantity and thrust, we trip it them as a parameters actually. The thrust can be constant in a in a aircraft applications; thrust can be time varying in rocket application actually; thrust and mass both can be time varying as by if it is launch vehicle or missile application basically (( )).

So, thrust and mass are considered as time varying parameters; x, h, v, gamma are state variables and g is also parameters by the way, and then alpha is a is a control variable here. So, that is how this state equations are there in the in this point mass model. Also note that this rest of the equation what you see here h v gamma are decoupled from x dot. They are not really functions of x actually h. I mean, this v dot and gamma dot are functions of h because through dynamic pressure. This lift and drag will depend on h actually whereas, this x is kind of decouple, we still it to integrate and get a value for x, if you really want to plot the trajectory. The trajectory is dictated by x and h; the co-ordinate of these particular CG location is dictated by x and h. So, we still it to use that equation for integrating; however, this for all practical purpose, we can also solve a problem in h, v, gamma as a co-ordinate only basically.

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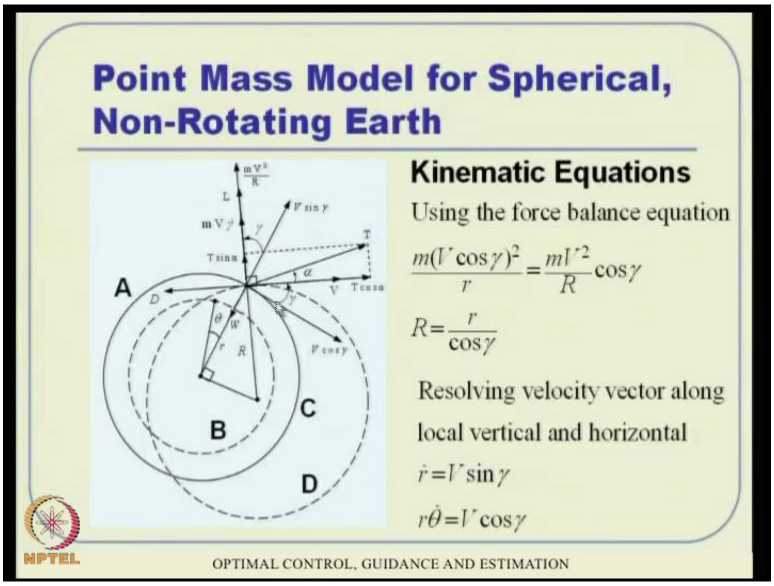

Now, next we go to let us slightly complicate the matter here, and then tell how about a spherical, non rotating earth is still not rotating, but we now consider long duration of flight, let us a let us for example, velocity missiles they will really travels a very long distance before falling the somewhere actually. So, earth curvature you can never you cannot neglect really. So, this affects has to be taken in to account in that sense what happens?

Now, let us say, for a moment the vehicle is somewhere here and still a point, velocity is there somewhere remember, this is no more to tangential to the surface of earth; surface of earth is somewhere here, this dotted line what you see B is surface of earth. And then, this is a point if you are the vehicle is currently flying with a velocity direction that which is certainly not parallel to the tangent, which will result from there actually. So, it is it certainly has some flight path angle and all that actually. So, what you are interpreting? You are interpreting let say it try to analyze the equations little bit better way. We will try to visualize these in a hypothetical circle parallel to earth actually. So, B is like a surface of earth, A is the instantaneous hypothetical circle parallel to the surface of the earth and D is also like a let us have these another one, this dotted line which will be which will require slightly that actually.

So, we will come back to this as a thrust direction again which is again at an angle of alpha, this is now local horizontal, remember this circle is parallel to this dotted circle.

So obviously, this tangent is actually a local horizontal tangent. So, then you have this angle which is gamma and that angle is alpha and hence if you see that this is actually thrust direction which is, I mean this is drag which is opposing to velocity, and then lift has to be perpendicular to the velocity, and you also know that negative lift direction is the centripetal force sort of thing, and hence we can visualize any of the imaginary circle around that for which the centre of the circle if you join this point this will be like a l m the lift factor.

Now, we have several directions here. So, we have to resolve it properly and also remember this is a reference line, this particular line is a reference line, let say this is a launch point or something about which we want to calculate, we want to know how much range angle you have covered theta is a called as range angle. The actual range is on the surface of earth, if you multiply theta with a radius of earth r e, you will get the actual down range actually on the curvature of the earth.

So now, coming to that, the first is let see again the kinematic equations sort of thing. We will also see this force balanced equations first. So, if you see this particular direction, V cos gamma is along this and v sine gamma will be along that. So, if you see this m V cos gamma whole square by R is the instantaneous radius from centre of earth; this is centre of earth, then that is the first that is acting along that actually and that has to be balanced by m V square by R cos gamma actually.

So, m v square by r is here and m 2 cos gamma component is here actually this particular component. So, you are resolving this, this force quantity I mean, this velocity level equations and all that one is I mean, this one is as acting along that the balancing is acting that actually.

So, through that, if you equate this two, you will get a relationship between this particular r. This is the radius of this hypothetical circle and this is the r that is the instantaneous radius from centre of earth actually that is the relationship they will have. This big r and small r will satisfy these relations actually, because of this force balance. Now, we will go back and resolving this vector along local horizontal vertical, that is that is what will result in kinematic equations.

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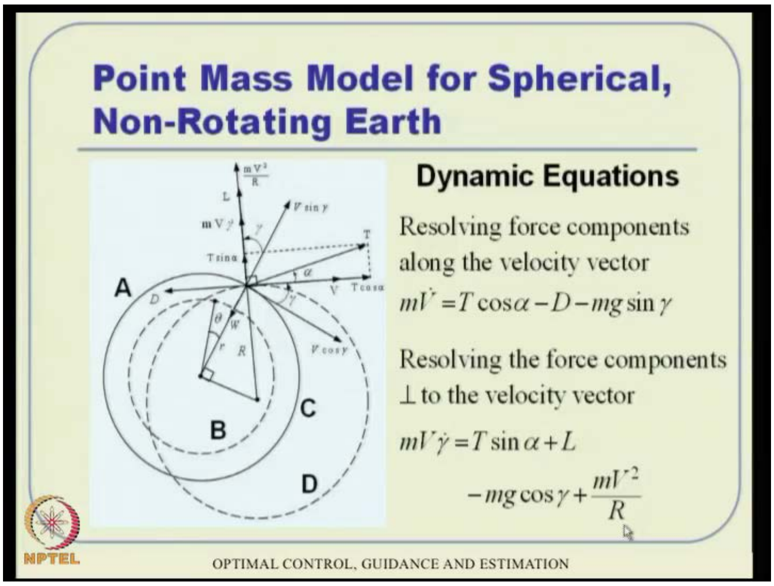

So, r dot is again r dot is like h dot. So, that is along that perpendicular sense, that is nothing but v times sine gamma sorry r dot is nothing but V times sine gamma actually R dot is like s dot. So, velocity is there which is not orthogonal to this line and hence it will have a component actually R dot is V sine gamma and then R theta dot is nothing but that the angle that you cover actually, theta dot is remember, this theta dot is angle of rate of change of this angle. So, R times theta dot what you see is actually, what is covered in terms of this particular directions, suppose you take this way the velocity; velocity tangential to this particular thing will have R theta dot, and that is nothing but v cos v cosine gamma actually.

So, R dot is V sin gamma again that is similar, and instead of I mean what you saw in the last time, it is X dot, that is like if a flat earth it is X dot. Now, spherical earth is r times theta dot, which is similar to x dot actually. So, R theta dot is V cos gamma actually again similar quantity here. Then the next one is slightly complicated and will see that what happens to the other one actually. So, you have interested in m v dot and m v dot is almost similar to what we had the last time again. This is the t cosine alpha by 1 component coming from thrust minus dot and m g is acting along that direction w. w is nothing but m g m g into sin gamma component will also I mean, along this direction actually.

So, m V dot is t cosine alpha minus d minus m g sin gamma. Now we have to resolve one direction is that the perpendicular direction to that is that one; that is where this gamma dot equation will pop up. Again this m V gamma dot is that net resulting force along this direction and here you can see that one component comes from T sin alpha from the thrust, there is a lift vector acting along that, there is a gravity component which is which is there subtracting that and the cosine gamma and this m V square by R which is actually centrifugal force and here we have derived relationship between big R and small r. That is where you can use that substitute that, and then simplify this equation in terms of R. Remember our equation is r dot and big R is a is not a decoupled I mean relationship from r actually. So, big R and small r are related. So, it cannot be an independent variable. So, we will substituted this big R in terms of small r and then simplify the equation in the state case from again.

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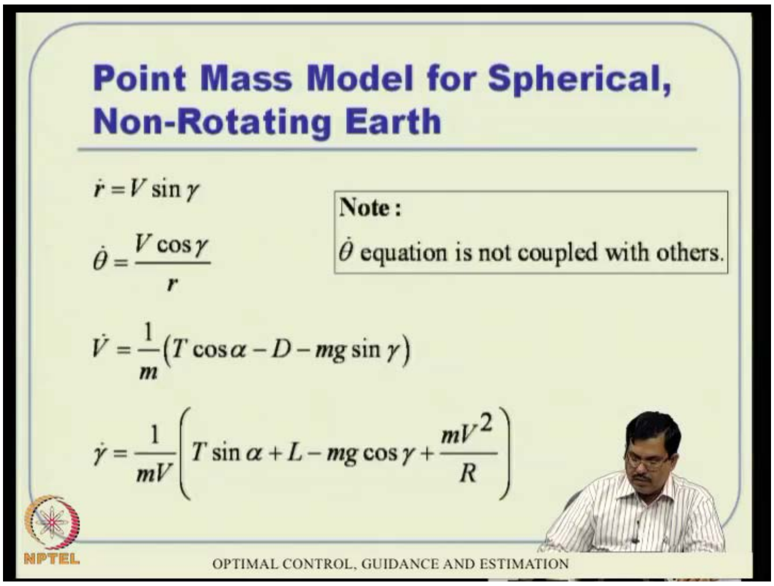

So, what you are getting? r dot is V sine gamma and r theta dot is V cos gamma. Hence theta dot is V cos gamma by r and then v dot is what you say here that is V dot equation divided by mass, and gamma dot is divided and all these divided by m V actually.

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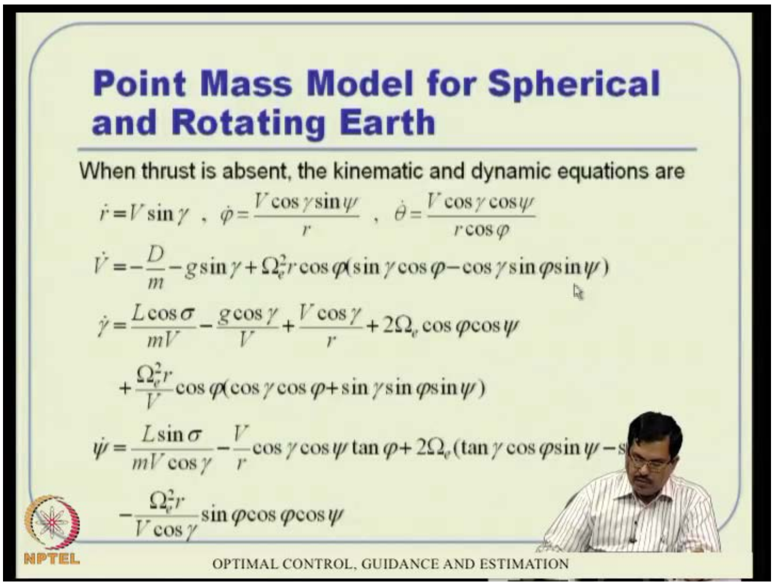

So, if you simplify that you will get everything there each other. So, this is what will get remember, this r is also a function of this r and gamma that is what we saw here actually, you substitute that we got actually. So, these are the set of equations that we have to deal with as when we talk about spherical but not rotating earth; that means, the earth is not rotating and earth rotation does play a role depending on the system equation actually. Suppose you want to start with latitude, longitude go to different lot at long thing like that, then as long as they are not in the same latitude plane, then things will be different actually.

So, for that, I will not derive the equation an all, but there is relationships are available. So, then things will be getting slightly more and more complicated as you talk about more and more effects coming into picture and this will be more and more close to reality of course. So, for spherical and rotating earth, if you talk about, then you have r dot now you cannot talk about only theta because that that was a along the what is called is, I forget to mention, this is this dynamic what you are deriving is all valid in the great circle actually.

Great circle means suppose you are going from one point to another point and suppose well let us say we launch point is somewhere here, you are you are you want to go where here on the surface of earth where you want to go. That is the launch point target point and center of earth they are three different points actually. So, these three points when you connect obviously, thus that will go through into that will interest at the earth center and it will result in the greatest circle actually. Anything parallel to that will be small radius circle. So, because of that it is called a great circle and whatever equations we show here is all valid in the great circle plane actually.

So, we are not consider the 3 d equations I mean this cross problem, I mean cross length motions something like that this is all down in the motion still. But here will not worry about that we are talking about the entire 3D movement sort of the thing. We consider rotating earth all sin to picture and hence all the equations, all the things you have to taken into a count actually. So, phi and theta are lot long positions here and r dot you still V sin gamma, but you will have to have latitude dynamics, longitude dynamics, V dot, gamma dot, then there is a psi dot equation also and thing like that; all these things you have to talk together all there six equations actually.

And remember, this omega e, big omega e is earth rotational rate actually. What about 2 pi rotation over twenty-four hours, whatever that value comes in radian per second, that you to take here actually. So, this if you see this is more and more relevant to this, I mean this prodigality and all that, but depending on the application will talk about that. For I mean, we will have to take select a particular dynamics for example, if you are just firing a small gun or a small utility, you do not need to talk about all this complicated thing here because the range is small, the time of plate small thing like that, but if you talk about a velocity missile application or satellite launch, will launch vehicle thing like that, then you have to very careful about accounting for all the you know. So, depending on the application, we will be able to select whatever appropriate thing is suitable for us actually.

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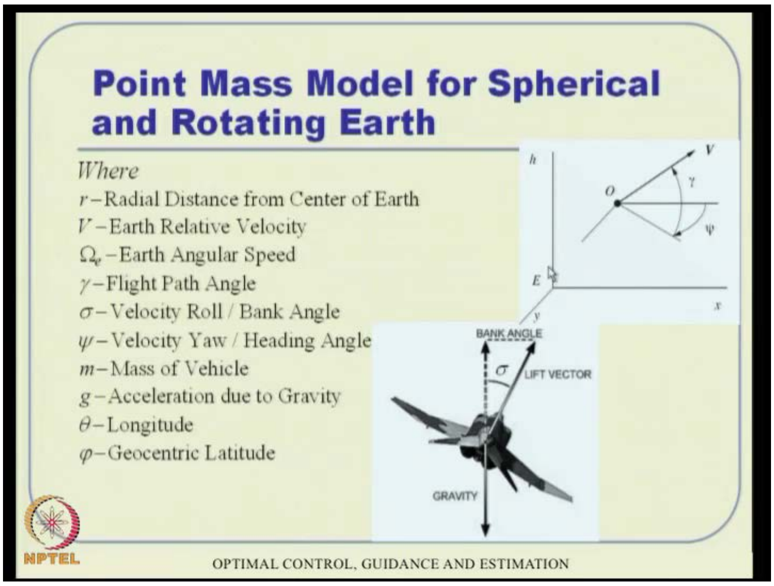

So, these are the various notations which go along with all these equations actually. That is also I listed it out; this especially this bank angle also the lift vector turning, the vehicle can bank and that will result in that; and this if you have a this velocity vector can now be resolved into both gamma and psi. And this is the if you consider a plane parallel to this local origin term, and there is a velocity vector direction that that is this is the projection of the on that plane. So, that whatever gamma this velocity vector max, this is flight path angle and with respect to a reference line this is actually heading angle psi.

So, this is this is how you have to visualize the problem, and theta is longitude and phi is latitude also the sort of thing are all available here actually. So, all these variable interact everything even here anyway. So, there are all still point mass equations are remember that and here the you are you can think that we have taken sufficiently complex problem, but things can be even more complex sometimes that people tell what about oblate earth effect; that means, you have this earth is really not spherical on north pole and south pole they are really plot actually.

So, if you have oblateness of effect coming into picture, then there are correction terms available actually depending on… If a really want to go towards north pole, then things will be slightly different there and then correction terms will be available as a set to do with this actually, will not go much into that actually. For all our many applications starting from utilary to kind of (( )) to aircraft to missiles to launch vehicles; everything it will fall under this set of equations actually. Either you consider flat, non rotating earth or you consider spherical, but not rotating earth or you consider spherical and rotating earth. This should be good enough for many, many practical applications actually.

Only on very high accurate missions something like that, especially velocity missile applications, we talk about this oblateness corrections and all that, otherwise is not typically not needed actually.

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Now, they are still, what you discussed is all point mass equations, then the more details to detailed dynamic will come from what is called as six degree of freedom motion actually six degree of freedom model. So, how do you describe that part of view; and this particular class, what about time remains along able to completely derive that, but I will derive a significant portion of six-DOF of equations actually and will follow on that in the next class.

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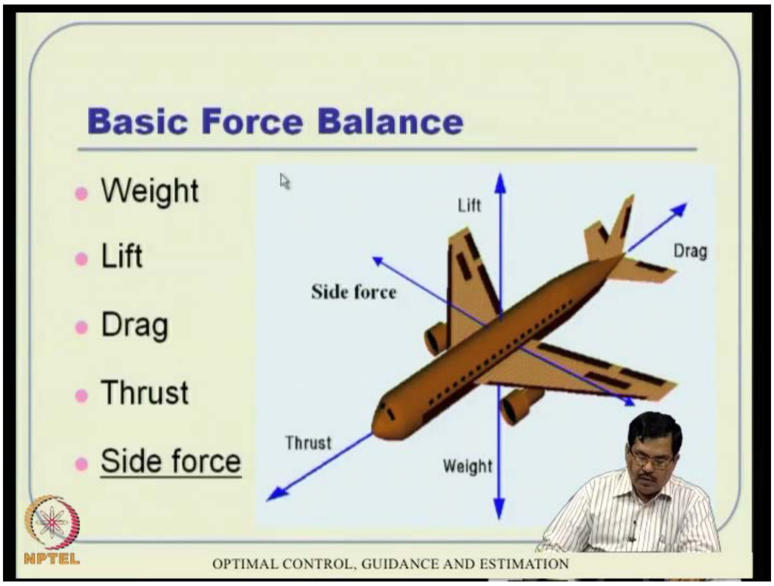

Also and if you remember one of the previous classes, especially lecture seven, this is some of this concepts when a very rudimentary preliminary way we discussed actually. So, if you see any flying object we talked, there are very basic forces sitting on that; one is weight, one is drag, and then there is a thrust force to contract the drag and then there is a lift which will which will wall of the way. Along with that, there is a side force. What typically the side forces are balanced out in a in a gook flight actually. Even in a turning, when you come coordinated turn, this side forces fully balanced out actually. So, then when all the forces will be there, one side forces will be kind of very close to 0 basically, we do not want side forces actually. Anyway these are basic forces, and then there are basic moments about this axis.

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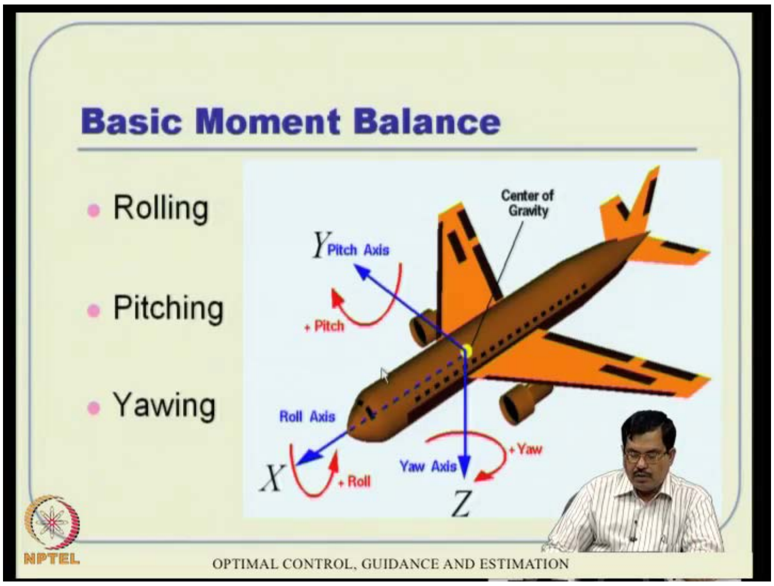

Now we remember, we talked about the body axis frame and we visualized, at the cc and X axis frame. Now because three direction motions, you will talk about and there is X axis and Y axis and the Z axis, which will form some sort of a left hand co-ordinate system actually. A left hand co-ordinate system, where you are where your fore finger will point towards the nose core, and then they thumb will point towards the Y axis and the middle finger will point to the downward directions Z actually. The axis frames are very critical in understanding the six-DOF equations, because you have to resolve these motions of the vehicle in all possible directions actually, all these three directions.

Now, once you define the co-ordinate frame, you can grab your finger in the right hand co-ordinate term sort of thing and then this your if you point your thumb to the fore side, then the directions of these your other fingers will give you the positive direction of the rolling action, pitching action, thing like that actually. So, if you have X, Y, Z define that way, then the positive roll will be like that, positive pitch will be like that and positive yaw will be like that. So, these are the notions, we have to remember before understanding anything more actually.

So, we basically talk about this forces; ignore side force for a second, then you have weight, lift, drag and thrust and then we have this roll, pitch, yaw motion actually. So, in this, this X axis frame is still available to you. So, remember this is x axis, this is Y axis, this is Z axis here and this is what you see here X, Y, Z. So, along this X, Y, Z we have forces and around this X, Y, Z we have moments actually. So, force level equations will come from I mean, the pull defined this translational dynamics and moment level equations will defined this rotational dynamic actually. Well, so that is that is where you to see that.

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And also we discussed about this control action. Then how this control actions can be accounted for and you saw that the aileron motion. Aileron motion are typically employed for rolling about x axis. So, that is that is a good job of the aileron and then we discussed about pitching action which is through this elevator action. You can… well this is a rotational motion about the Y axis, that is the rotational motion about the X axis and then we talked about your rotational motion about the Z axis also.

So, all these three things are possible as well as rotational are there and transnationally, it can go in a X direction, it can have a component along y direction, it can have a component along y direction, at can have a component along Z direction. So, all these transnational three and rotational three will have to discuss and remember Newton's laws are all second order equations; that means, we have six second order equation which will essentially result in twelve first order equations later.

So, that is what we will have to derive and see the integrated relationships and all that actually and these are all coupled in a way because I mean the depending on a like what you are talking about, you can visualize an approximately decoupled system what in truly speaking, they are all coupled with each other actually. So, we have taken on for all this coupling effects something like that. So, let us see what we discuss here.

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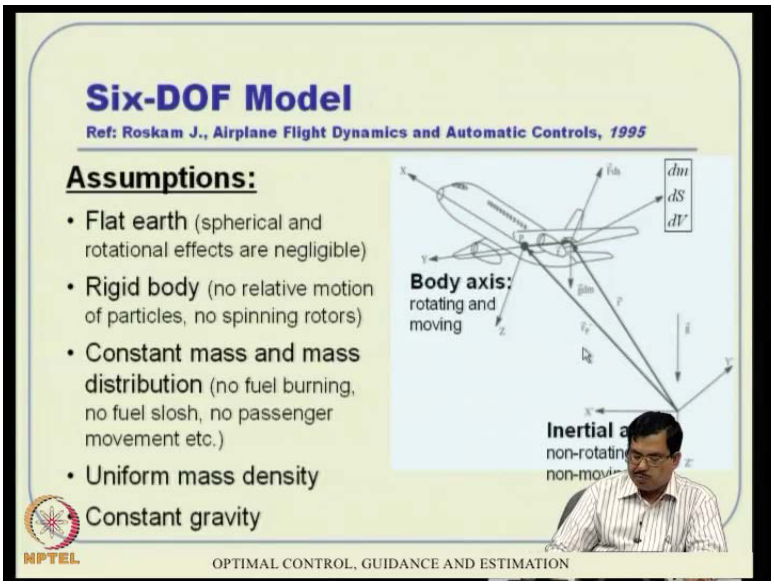

The assumptions involve again: first, we will start with the flat earth assumptions. So, spherical and rotational effects are negligible here and models are available, which will account for that also. So, this is not the restriction that we have to really deal with, but to understand the equations this is good enough actually. Then also assume the vehicle is rigid body; that means, there is no relative motion of particles and no spinning rotors also. So, we are not talking about like helicopter motion thing like that actually; that will have a slightly different mechanism coming into picture. That is why helicopter dynamics are also lot more complex than aircraft dynamics really also.

We are considering here aircraft dynamics and probably launch vehicle and missile dynamics here and then we have also assumed that there is a constant mass as well as mass distribution; that means, we do not talk about, theoretically speaking we do not talk about fuel burning; that means, we do not talk about c c movements and all that. If c c remains fixed, the mass remains fixed, there is no fuel loss happens, no passenger movement in the inside the aircraft; lot of assumptions we have (( )). But remember, passenger weight is hardly may fraction of the entire weight. So, that is justifiable motion I mean, justifiable assumptions really.

We will also talk about uniform mass density; that means, if you see anywhere the density if the aircraft remains same actually. So, ultimately it is not a very strong assumption either because we are all talking about rigid body motion where this effect of density is integrated over the entire volume. We will see that; that is where this moment of energy all those things will come actually. So, this is not a very strong assumption either actually. Then again, we will assume constant gravity; gravity force remains constant throughout the aero plane body actually which is very much justifiable aircraft is just a small object compare to earth dimensions basically. It just the theoretically justifiable reasons actually.

So, we just put it I mean we do not talk about see theoretically speaking gravity can vary along with height, but we do not talk about that much height variation here so that we take on for that that gravity variation with respect to height here. So, we will not talk about that actually. So, under all these assumptions, let us see how you derive all these complicated kind of equations of motion and here we are also we are we are also have to visualize one more axis frame and this is called as inertial axis actually.

It is a non-rotating and non moving axis frame. Typically it is let us say launch of launch I mean, launch off point for the launch vehicles or let us say like airport for aircraft application or it can still be imaginary point somewhere on the earth and sometimes people say, it is it in fixed at the center of earth and thing like that. So, we will not discuss too much in that. All that you are telling is there is an inertial axis and then there is a body axis which moves along with the body. The inertial axis does not move but body axis moves actually and if the aircraft is oriented, then the body axis is also oriented along with that. So, the body axis is tightly fixed with the body basically. So, if the aircraft turns, the body axis also turns basically along with that.

Now, if this x is body axis moves along with the aircraft, then on that axis frame, we cannot define the position of the aircraft. So, on that in that axis frame, the position of the aircraft is always 000. So, we cannot get information of position of the airplane actually. So, what we have to do and as well as attitude of the airplane, attitude of the airplane because if the air plane is rotated, the axis frame is also rotated. So, the attitude information is also lost actually. In the body axis, the attitude angle is always 0 and the position of the vehicle is always 0. So, for that reason, we need another axis frame which is decoupled from those behaviors and hence we can visualize the position of the vehicle as well as attitude actually. That is where this inertial axis comes into picture.

Now, let us say as from simple domain. Now we will consider some geometrical aspects and that and if you see this, we will consider this vector is r p and will consider this is a some particle on the aircraft somewhere in the aircraft; that vector is r, I mean r prime rather r, this is r p prime and this is r actually. So, then if you consider this from simple geometry, this r prime rather is nothing but r p prime plus r actually and because p is the center of gravity or center of mass in a uniform gravitational field center of mass and center of gravity are same anyway.

So, we talk about P is a center of mass and hence if you consider this is nothing but d m rho is density of the aircraft, d v is the control volume that we are talking about. So, if you just take r times this d m and integrate over the entire volume; obviously, that is 0. That is the definition of center of mass. Around center of mass, if you just take every individual particle and then do this volume integral, then it should come to 0 actually. So, if you substitute this r, this r is nothing but this relationship; r prime minus r p prime. This r is nothing but this minus that so you substitute that and hence what you get is this integral of this r prime rho d v, this particular thing is equal to that that is there.

But you also know that r prime… what is there actually? This particular r depends on where you are situated actually. But this particular r p, it does not depend on this particle mass I mean particle location; the r p is that at the c g that is all. So, I can take out this r p outside the integral and consider this because of this, this is nothing but the entire vehicle mass actually. So, hence my r p prime, that is essentially the vector which will give me the position of the aircraft with respect to this inertial frame that is nothing but this actually. So, this relationship will use it later actually. So, keep that in mind.

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Now, we will derive this dynamic equation, and this kinematic equation will derive next class actually. Dynamic equations are little more complex, kinematic equations are not that complex to visualize actually. So, dynamic equations are these are the equations which will directly see Newton's second law of motion actually.

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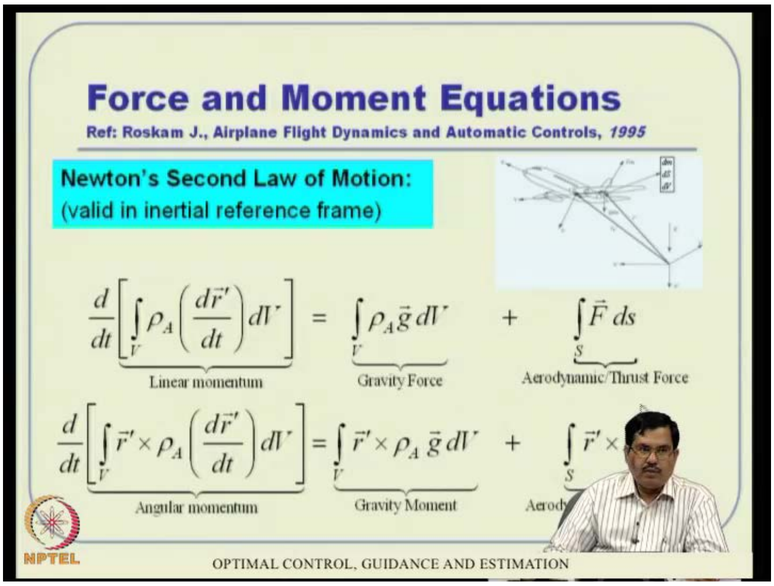

Now, if you consider this air plane again, and then want to apply this Newton's second law and thing like that, there are two level to two ways we have to interfere I mean, we can we have to we apply that one is the linear momentum, another one is the angular momentum actually. So, rate of change of linear momentum, this is the linear momentum of the particles integrated over entire volume.

So, the total liner momentum and this is a rate of change of total linear momentum is nothing but total applied force. Total applied force is … one is the gravity force, another is aerodynamic force. Remember aerodynamic force depends on surface area basically. If you integrate it over the entire surface area, then you will get something. So, this is surface area integral and this is a volume integral m times and all that actually.

So, rate of change of linear momentum is total applied force partly through gravity and partly through aerodynamic and thrust forces actually and similarly rate of change of angular momentum is a almost same expression, but there is a cross product that we have to take with respect to r prime actually. Anymore that is that is that is a alpha d. So, we have to talk about that and that is a rate of change of that, that is the angular momentum is nothing but again r prime cross that whatever force you have and then r prime cross that whatever force you have. So, this is nothing but rate of change of angular momentum is equal to the moment generated due to gravity force and moment is generated due to thrust and aerodynamic forces actually.

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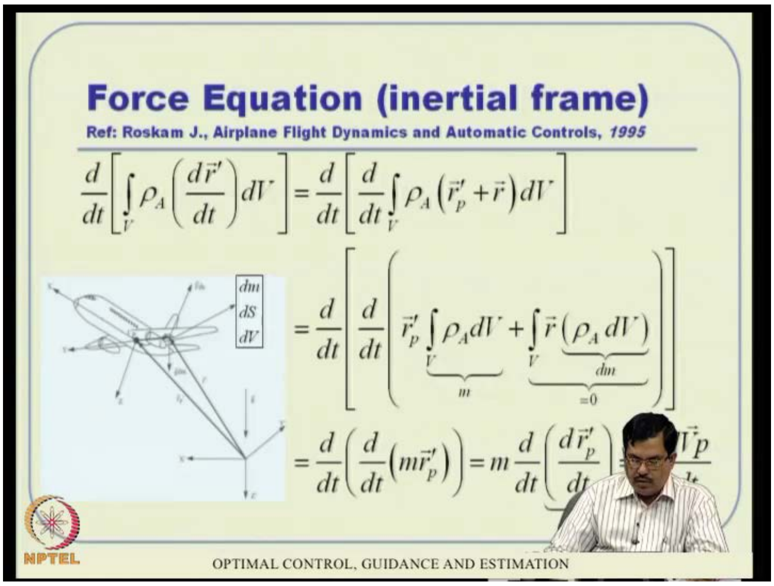

Now, let us analyze this equation slightly more, and then tell what is what is going on here actually. So, this is what you want to analyze here and remember r prime, r prime is nothing but r p plus r actually alright. So, this is what we will substitute here, rate of change of this force that we are discussing here actually. Then this is like this, then remember this integral I mean, this derivative, I can take out outside the integral now, because of final value integral anyway and then I can, I will try to kind of visualize, I mean simplify this.

So, this d by d t act now, and then I talk actually rho a times d V. Now r p, this is nothing to do with this mass distribution where it is located, because r p is directly this vector. So, I can take out outside the integral, and then I will leave this r. r is this volume integral, once it wants to decrease volume integral, this r I cannot take out of that because this r is inside this volume. It is a function of this where it is located.

So, I can take out that, but r p prime I can take out because that that is independent of where this fellow is as situated. So, with that and then again; so with that in what is this actually? That is by definition is nothing but 0 because that is, just one second alright. So, this is nothing but centre of mass. So, again that is by definition of centre of mass is 0. So, we are left out with this d by d t, again d by d t of this follow as mass total mass of the vehicle and something like this. So, we are left ultimately, you are left out with m into d v p by d t. So, this velocity vector whatever is there with respect to this centre of gravity, the total velocity vector will popup like that actually m into the total rate of change of this entire velocity vector; this all we are talking here, this entire expression actually.

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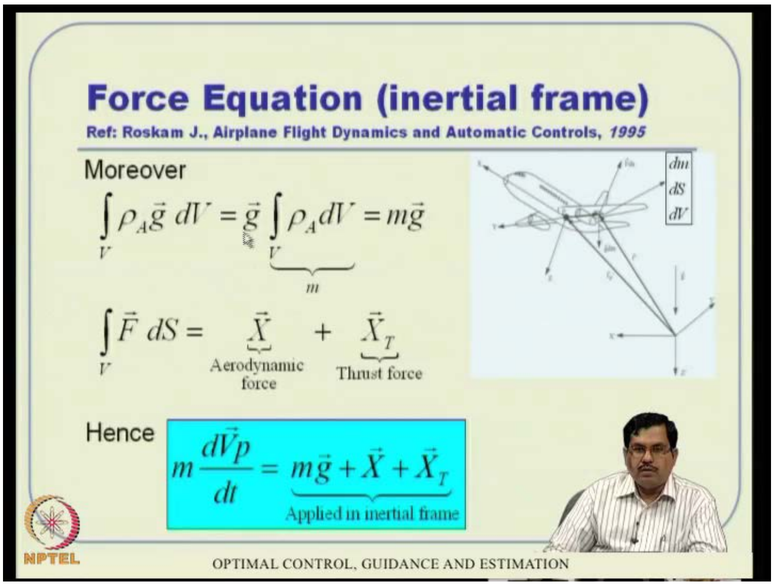

Now, what happens here? This particular thing, this all about left hand side what about right hand side? Right hand side terms are nothing but this is easy because gravity constant uniform gravitational field. That is what we talked about and this integral is nothing but the entire mass of the vehicle. So, it… I mean m times g actually and this thrust is this actually surface integral. It is not a volume integral. Let me correct that. This surface integral, this is nothing but aero dynamic force and thrust force actually taken together.

So, what you have here. This is like, this is a left hand that we saw, left hand side expression is equal to the right hand side expression. Now thisgravitational force is nothing but m g. So, that it remember they are all in vectorial notation actually. So, all the components level and all we have not discussed yet. It is the total vectorial thing that we are talking here actually. So, m g plus this aero dynamic force is that x vector and thrust force thrust force is nothing but x t vector. So, this is this how the relationship turns out in the vectorial notational actually.

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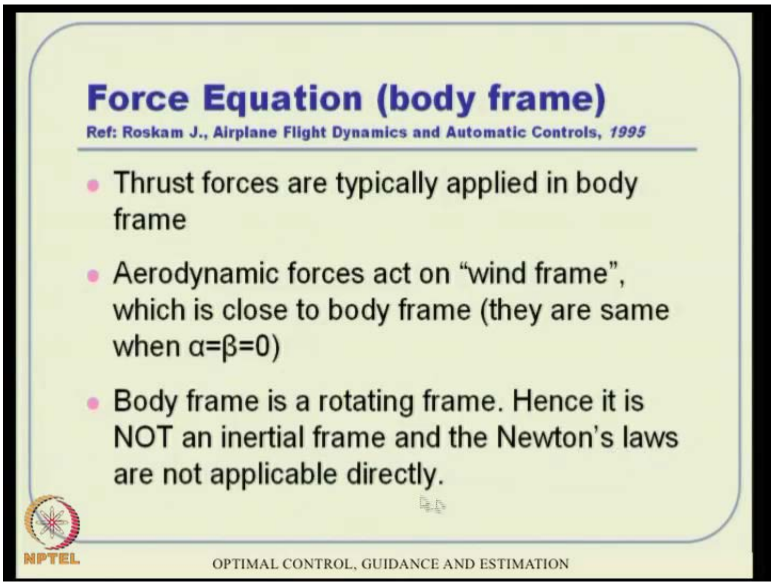

Now, the problem here is thrust forces are typically applied in the body frame. They are not applied in the inertial frame. What you discussed here, all these things Newton's laws or even more valid inertial frame, there not valid in the body frame, because body frame is really not in an inertial frame. So, what happens, the thrust force is typical what the thrust and an aerodynamic forces typically happening body frame any way. All forces moments happen there acting on the vehicle on the body frame.

So, aero dynamic forces typically act on what is called as wind frame, which is actually close to body frame and they are exactly same when alpha and beta are 0, angle of an attack and sizably being l; when they are 0, then these things happens to be 0 more on that you can see from flight dynamic book actually. So, body frame is a rotating frame, and hence it is not an inertial frame, so that the that locking effect that is coming here is we cannot rely on this thing as it is actually. So would yet to be taking the help of something else before you visualize the components of the forces.

So, they are because Newton's laws are not applicable directly in the body frame. What we saw here is all happening in the inertial frame actually. Everything happens in the inertial frame. I want to have a body frame I mean, the body frame levels of equations or motions, that is where the real x n takes place as for as forces and moments are concerned actually. So, if you are really want to visualize in body frame, then I have to talk about a standard results in vector theory.

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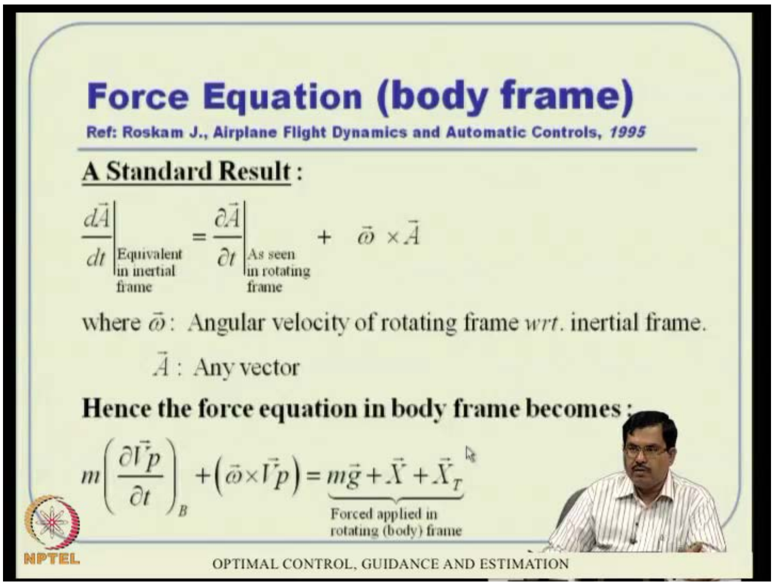

If you take any vector A, then this equivalent what will interpret in inertial frame here is nothing but as in the rotating frame that you take about whatever rotating from a fake plus this omega cross A. The rotational rate will come into picture. So, whatever angular velocity of the rotating frame with respect to the inner cell frame that is omega and a can be any vector. So, this relationship we have to account for, if you really want to apply that in the body force actually, in the body frame.

So, the same equation that we have here, suppose I want to apply that in body frame and the left side of the equation, I have to modify that and tell now that is valid. If this fluid, the right hand side happens to be in the body frame, then the left hand side happens should be this way, not the other way. Otherwise, if the right hand side happens to be in the inertial frame, I think simply live it that.

So, this is this is where the critical relationship comes actually, and this were it will generate what called as coriolis components actually. These are this, so omega across v p, which is actually that actually makes the aerospace dynamic complicated, if you ask me that is the only reason why aerospace equation look complicated actually that. This cross producting we have to account for that actually. Anyway what you have a relationship now? So, we do not have to be confined with respect to unnecessarily initial frame. So, let us visualize this expression all happens as it happens in the body frame actually.

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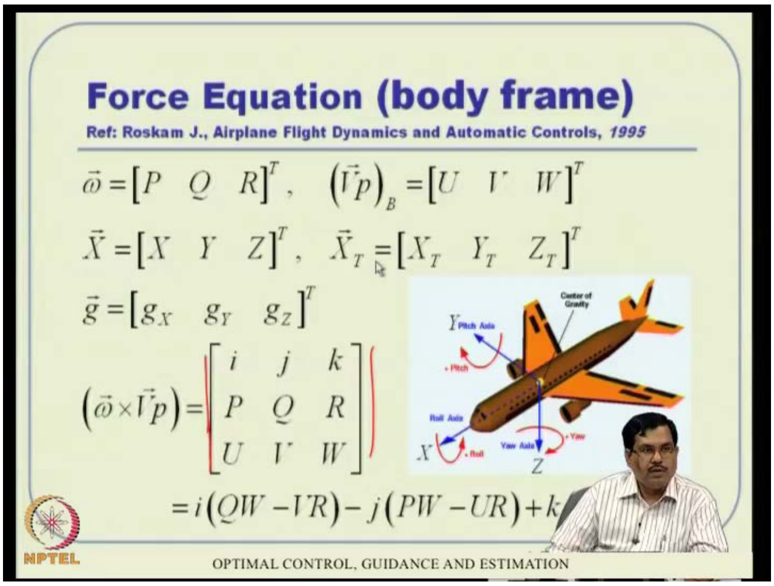

So, before you, I mean before we move along, we will decompose this. Now, we will decompose this vectors notation into components now. So, this entire rotational rate consists of P, Q, R; P is about X axis, Q is about Y axis, R is about Z axis like that in the body frame. Similarly, V P the total velocity vector will have U, V, W component; the total position will have X, Y, Z component like that actually. So, you decompose that. This X, remember are aerodynamics forces. This is not position coordinate, they are called forces. Similarly, thrust forces will be decomposed into X T Y T Z T actually. Gravity also will have a component x x g x g y g z, because this X, Y, Z is not parallel to the inertial frame. These are related to with inertial frame.

Now, this omega cross V p this particular term we have to evaluate that way, this matrix form, determinant form like that. We know that the cross product evaluation basically. You have to evaluate this sort of a determinant notation basically and this is the… If you really want that, then probably you to put one more bar here, determinant there, and then it is like if you evaluate, that is i times this Q times W minus V times R minus z times P time W minus U times R plus k times this P V minus U Q; so all that is here actually.

So, this this term I can write that in i, j, k component in the body frame like that actually. So, once I do that, this particular thing, other things are very straight forward. Any way this is body frame, this is m u dot, m v dot, m w dot. This is very straight forward m g x m g y m g z will be there and this will have x, y, z component anyway basically. So, now if you are taking into a account all that, I will be able to write this equation that way. Remember, this is the first m u dot, m u dot comes from here, then this i component of that which is nothing this component. So, that will come here, and then that is equal to the gravity component, and all that m g x will come here, and then this is a this x direction sources. So, that will remain that like that. So, similarly it will happen to all u, v, w components actually.

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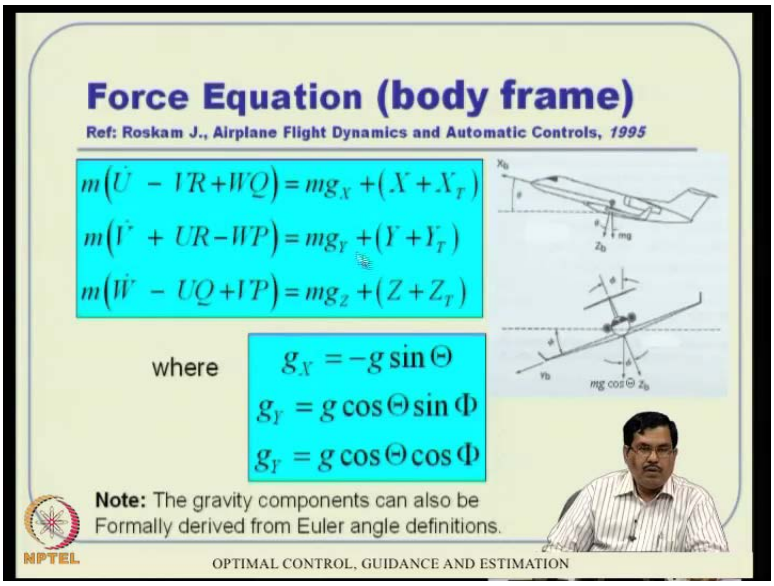

Now, what about g x, g y, g z? Eventually, it can be derived in a more formal way take this component and all that. Now, this attitude angles will come into picture. It is a function of this pitch angle theta and roll angle phi actually, and very quickly you can visualize that, this is the gravity force that is acting along. So, if the air craft is pitched by pitch by angle theta, then this particular component g x, remember g x is opposing to this x direction basically.

So, this particular component will be nothing but minus g sin theta basically. This angle is theta. So, will have a sin theta component along that direction basically, little component there; so that is where it will have. And then if it is it is not only pitched, but after pitching, it is also round actually. This is a pitch angle; there is a roll angle also and then this roll angle will generate these two components actually.

So, one is this sin theta which is and all opposing to x x vector, then g cos theta will have two more components: g cos theta into sin phi and g cos theta into cos phi. So, one will be along the y direction, one will be along the z direction. It can be derived more formally also we will try to weather in the next class probably. But pictorially, you can see that one component what is coming as m g cos theta here, that will have two more components depending on this phi angle; m g cos theta resolves into this cos phi sin phi component look like to along that actually. So, once you put that together, and then try to sort of what is U dot, V dot, W dot, that is where you will we will get U, V, W dot actually.

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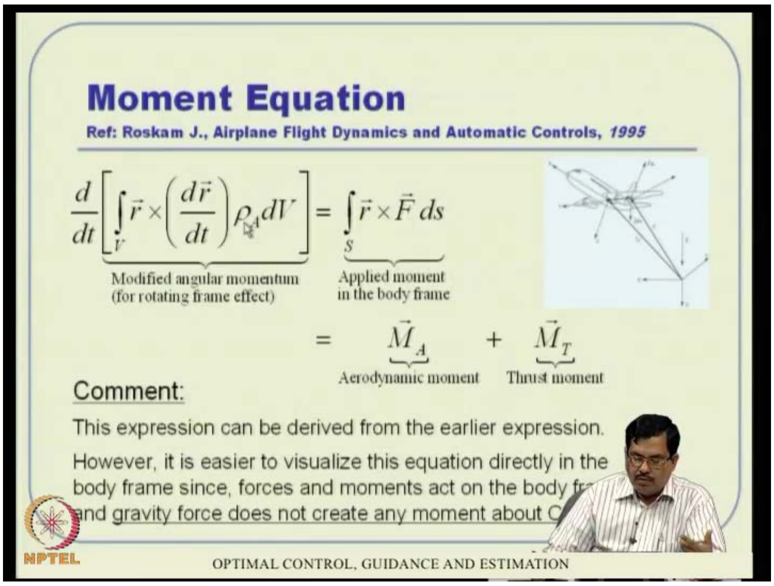

We will see you got it, right; you put this here. And then solve for U dot equal to something, V dot equal something, W dot equal to that. So, you will get three state equations from there. Now, these are all about force level equation. What about moment level equations? Again moment level equation we have this is the left hand side of the equations and then this can be derived from inertial frame to body frame through certain algebra and all that you can see that details in this particular book.

But I will try to simplify the matter by directly applying these forces, these moments in the body axis itself. So, and remember that the resulting moment of the gravitational field about the central mass is 0. That is the uniform gravitational field anyway and right, I mean the gravity force is acting everywhere in the same quantity actually all the particle of the vehicle. So, it will not result in any specific moment because of the gravity. The entire vehicle is pulled out basically. There is no rotation effect, there is no differential force in the between the same molecule and all that. So, I will ignore that term and then proceed further.

So, what you have this is nothing but modified angular momentum term for rotating from an effect. We have noted it down already, that this also has to be taken out. That is where it makes it complicated actually. It makes like complicated because that that vector a can be any vector and this applied a momentum, momentum is also a vector. So, you too talk about rotating efforts, which there are also.

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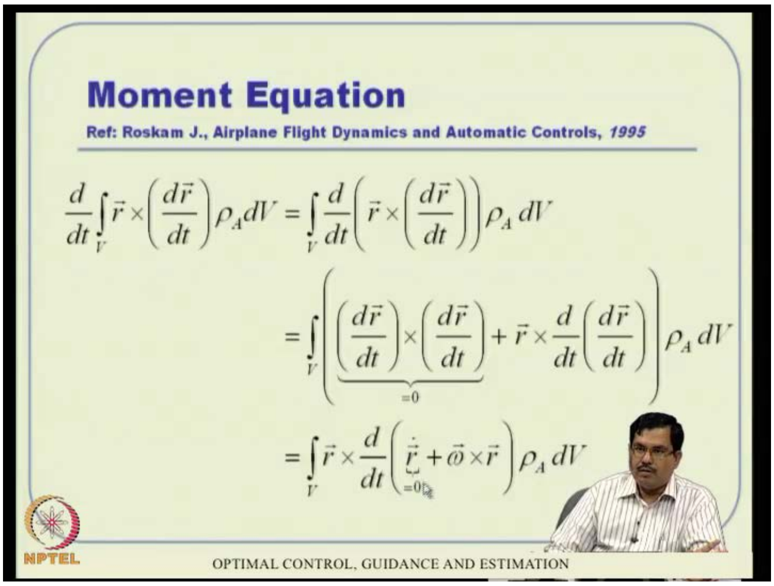

So, what is that? So, this particular term what you seen in the left hand, this d by dt of that, this nothing but that and this particular thing this d y by d t of a cross b. So, that is d a by d t cross b plus a cross by d t of v basically, the interpret that a vector that is a d vector. So, d by d t a cross b a nothing but d a by d t cross d, b vector is that plus a vector which is r cross d by d t of b vector and if you simplify, this is any vector with cross product with respect to that particular vector is 0. We know that. Cross product of the same vector is that 0 and then that is what is left out is again d by d t of that and again that is to be resolved; this vector notation components component actually.

So, this particular thing nothing but first is r dot then plus omega cross also, in this particular quantity what you see here, in the directly apply actually. You have to apply through this angular relationship actually. This cross product relationship, so that will result in r dot and remember r dot is 0, because there is no movement of the particle with respect to the body frame axis.

So, I mean particle do not move, they do not vibrate, I mean this rigid body (()) right. So, because of the rigid body dynamics, r dot is 0 and then we are left out with only these quantities. By the way, this book derives this equation in a slightly more complicated way to try to simplify it at one as possible. It gives this year, then derive, then further down line it takes it 0. Actually it will result in double dot something like that .We do not need to do that right away it. Put it 0, that is that is for it actually.

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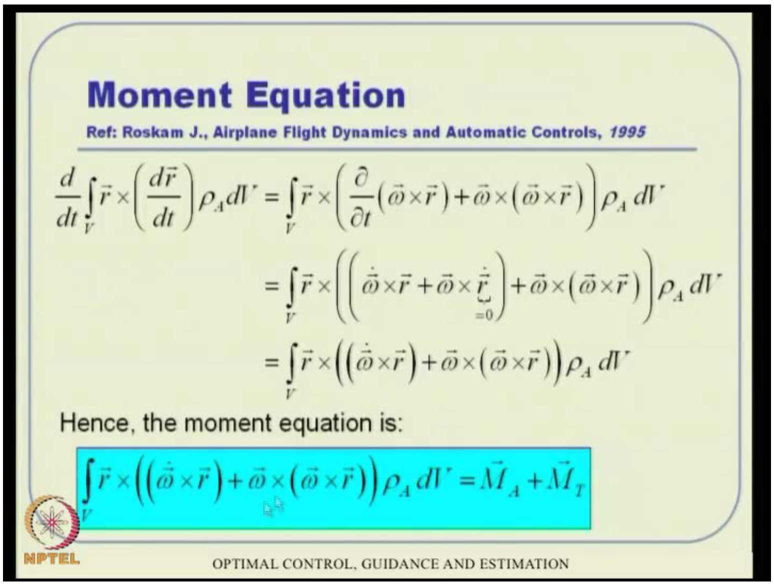

Then, we talk about this particular quantity, what you have here, this results in this particular thing, r cross d by d t of omega cross r. That is what we are left out here and again you have to apply this momentum of this rotational, these rotational effects and all that; so r cross d by d t omega cross r. So, this is r cross del by d t omega cross r plus omega cross omega cross r. Again that and again you have to keep on applying that into simplify that. Wherever you see d by d t, keep on applying that. That way, that is where things become more and more complicated basically, but non velocities occur, I mean you can make it, you can track it basically, what is happening.

So, if you simplify these equations, now if you put it there and then omega dot and r and then r dot is 0 again and you will left you will be leaving that. So, the moment equation if you see, this left hand side is something like this complicated expression what you will see here and nothing what the applied moment actually through aerodynamic as well as thrust. So, this rate of change of angular momentum is through that and then they have to simplify that; obviously, when a component level thing. There we have to talk about some standard results in vector theory again.

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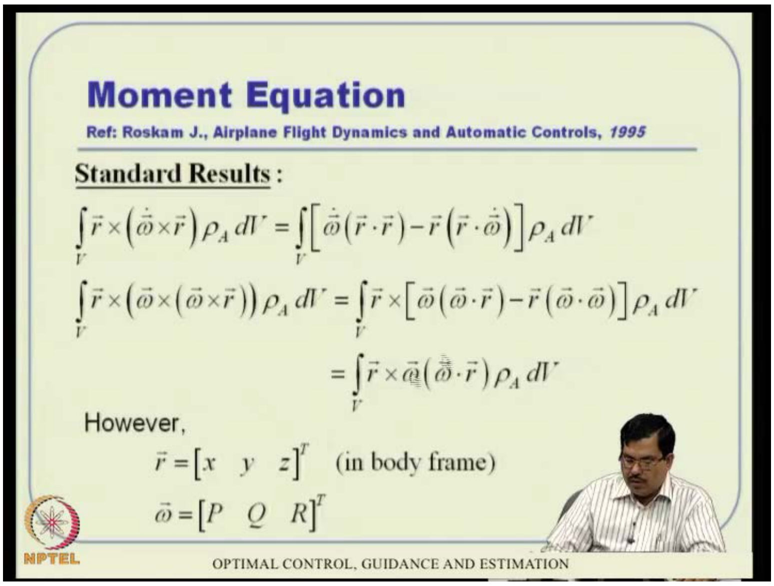

So, we have a (()) basically, a cross b cross c that that relationship bring in, and then you try to put in all these expression; again a cross b cross c, and then c is also that to production all that actually. So, apply all these is not difficult to kind of see this long and algebra and then you r is nothing but x, y, z components in the body frame. The body frame remember that is not the position of the vehicle; its x, y, z with respect to the centre of gravity of that particular d f that we are talking about that particle. So, x, y, z is the distance from centre of gravity of that control masses something control volume what we are talking.

Now, omega has P, Q, R components and then you are able to kind of decompose because we know this. What is beauty of that, the entire thing now results in mean what about this cross product. We will have dot product now. Dot product is nothing but scalar component and then will have one one vectors associated with that, so that easy to simplify after that actually. So, you put that all these things together and then try to solve try to simplify this results in d m.

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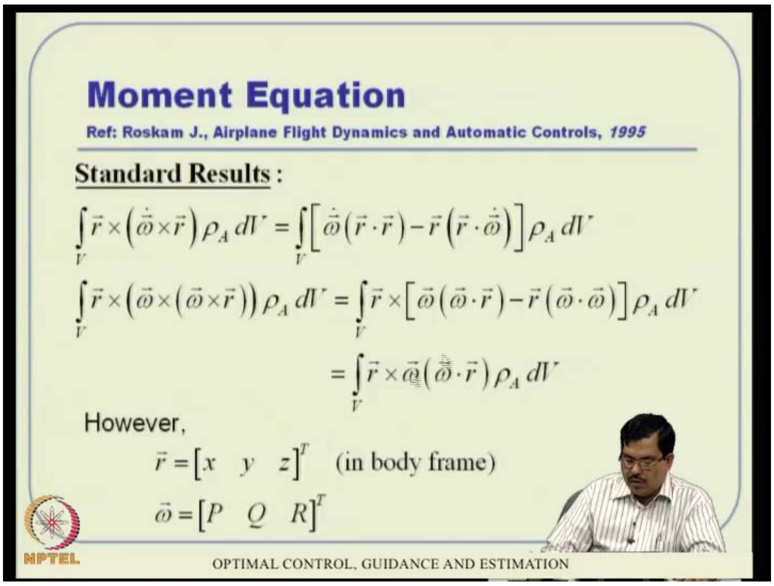

So, it will go through this for example, this one, r square is nothing but x square plus y square plus z square; and similarly r dot omega dot, that will result in that. We will result in that, these are all dot product.

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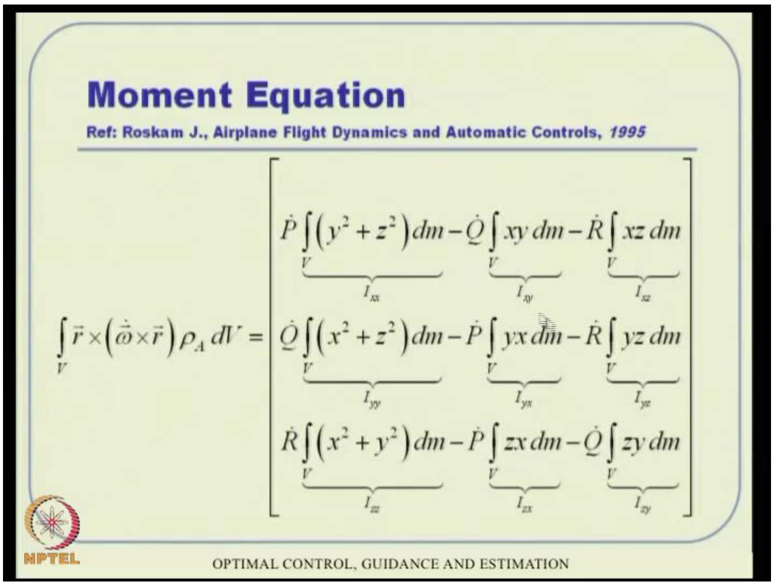

Now, it is easy to see that, and then hence once you simplify this thing, it will result in some expressions like that and this is needed to see, because all these things are nothing but moment of inertias. These terms what you see here is nothing but I xx, this terms are what you see here is I yy, this is I zz; principle moment of inertias and this terms are nothing but cross moment of inertias.

So, as long as there the rigid body rigid vehicle dynamics is concerned, I really do not need to know the mass distribution and thing like that. I just need to know the large quantity called moment of inertia. So, that is that is typically supplied to us actually is for the control designers actually, so supplied to us from structural engineers.

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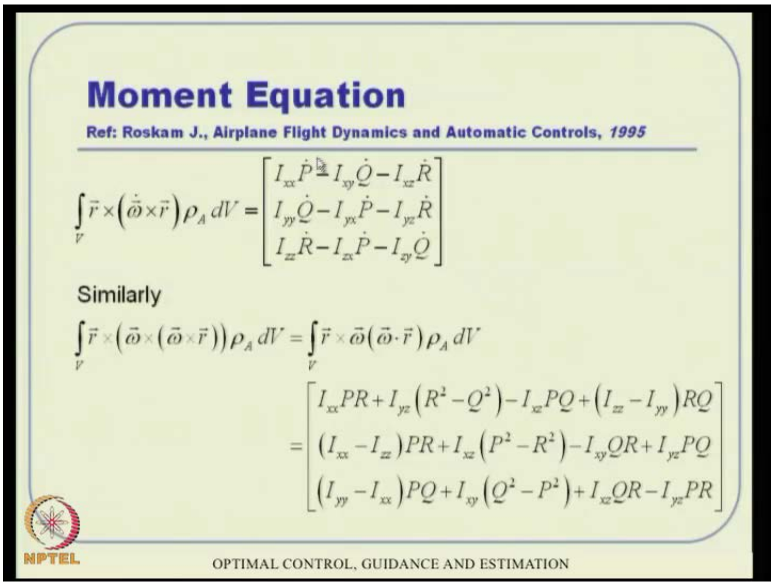

So, this funder quantity in terms of moment of inertias, I can write it in a very neat way, so entire thing which looks so complex here, this level is all reduces to in terms of moment of inertias; this is rather simple looking expression basically. Similarly, this other component that we left out, you can also be derived in a similar manner, and you have equivalent expression like that. So, you have this from you that.

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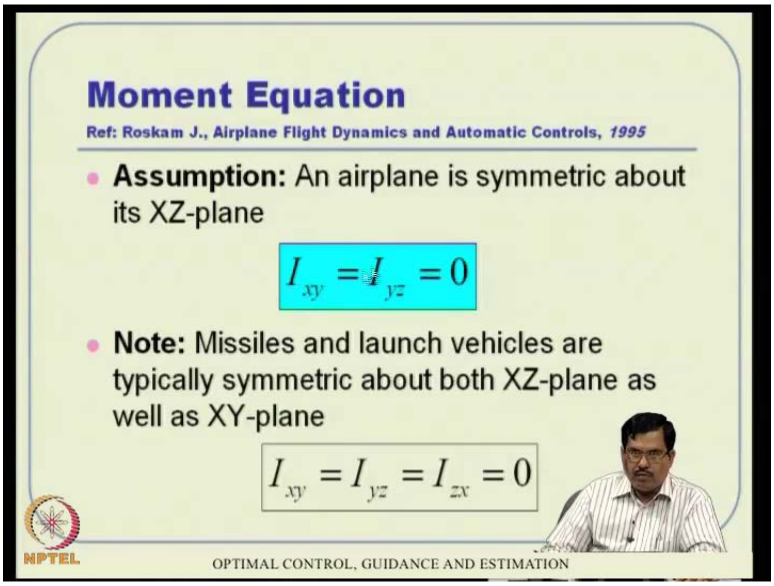

Now, put them together, and then begin the assumption that airplane is symmetric about x z plane. In the X Z plane, if you tear the aircraft it is all symmetric with left and right side. So, in that situation I xx, I xy and I yz will be 0. This is close moment of inertias and in addition to that, missiles and launch vehicles are will have to symmetric about XZ and XY plane both actually, there symmetric about both the planes. So, all cross amount of inertias will be 0 in those situations; otherwise this is anyway true for aircraft also basically. So, that will bring in further simplicity in this equation basically.

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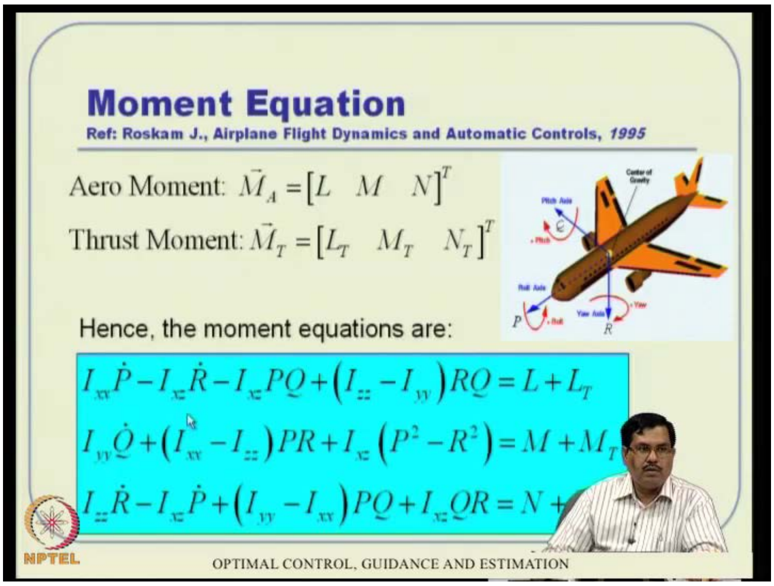

So, you will be left out with only these equations later actually, what you see here because applied aero dynamics components are l, m, n and thrust are like that. So, that is left out with that. So, you can solve this P dot, Q dot, R dot from here, because remember this is linear caution anyway is very clear here. The P dot, R dot are coupled through I xz, the moment it is not therefore, missiles this is also even not here. So, directly you get P dot, Q dot and R dot here, because I xz is not clearly 0 for airplanes, you have some coupling effects for p r actually p dot and r dot for nevertheless these three equations are linear. So, you can solve it and then get it for P dot, Q dot and R dot.

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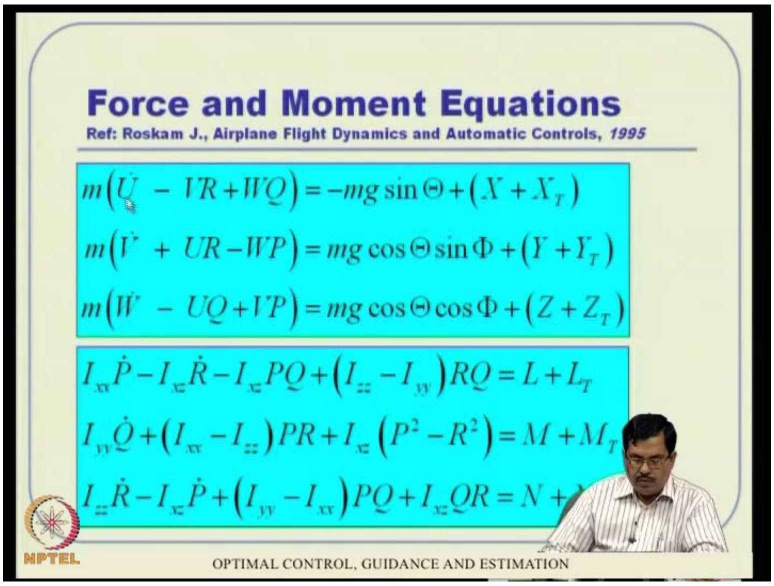

So, what we are getting here ultimately is this force level equations and this moment level equation and if you solve this for U dot, V dot, W dot here P dot, Q dot, R dot here.

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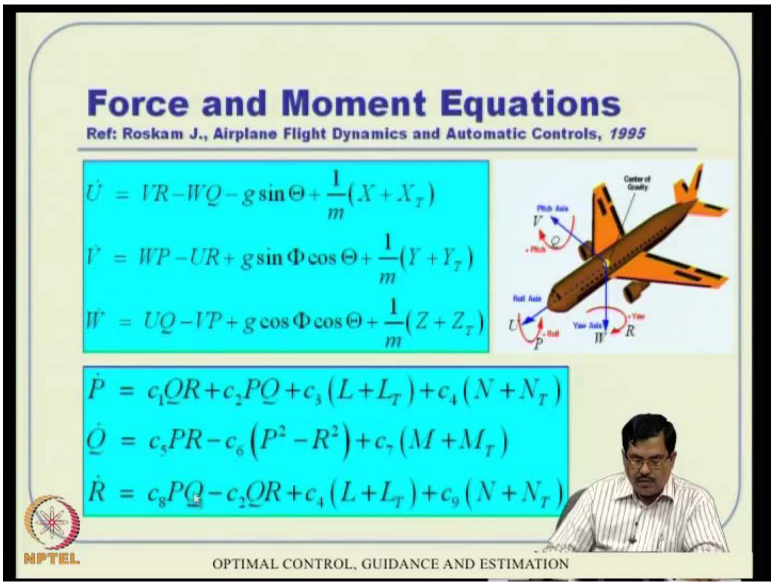

That is what you get it actually U dot, V dot, W dot and P dot, Q dot, R dot. So, these are what is force level equation, this moment of caution, these are corolis component comes from rotational effects and all that. This is gravity term, this is the aero dynamics and in thrust moments external moment forces and similarly here the gravity term does not come here that. Moment level gravity does not play role and this is the coupling equation that we take talk about. Here the control surface extents will be significant here by the way at this will be going very minimum effect aero dynamic control surface extents basically. So, details will be, I mean once you understand this details, you will be (( )) this actually.

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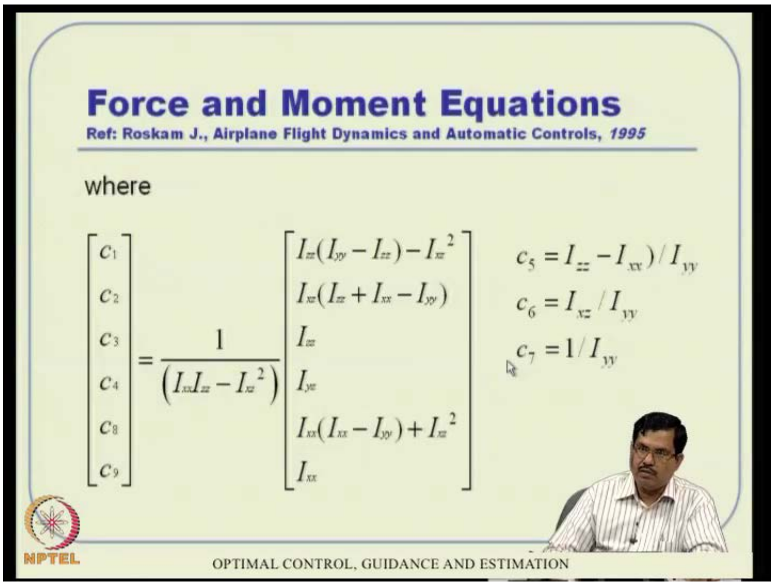

So, this now this C 1, C 2 what you see again the C 1, C 2, C 3; all these are the functions moment of inertia which is given like that. It can easily solve I mean from this equations what you have, you will solve for P dot, Q dot, R dot as a byproduct, we will be able to solve this actually and will get C 1 to C 9; all the things are like that. So, it is easy to compute this constant.

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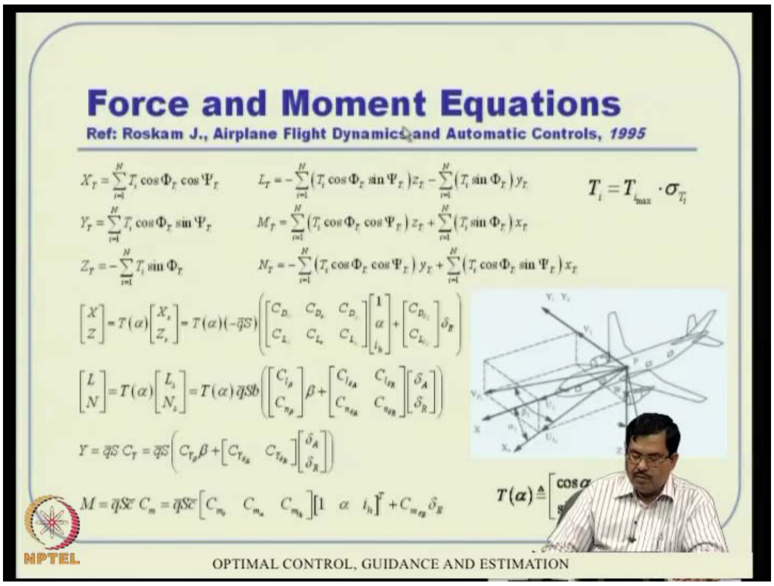

And then, force and movement equation makes like more complicated because of these aero dynamic forces and thrust forces and all, we will be given and the several component level equations, consider various angles and things like that. So, this particular thing, this remember X T, Y T, Z T is force level equation, I mean and [l t m t/l t m n t] L T, M T, N T are the like moments and all that. So, all this expression you to put it together in those equations, to get these force and moment level equation; that is where this dynamic become complicated. However, once you understand this, I mean this is we still can talk about all different directions of the velocity, I mean and on a component level and as well as moments. So, this is all dynamic equation, so kinematic equations and all, I will talk in the next class. Thanks a lot.