**Optimal Control, Guidance and Estimation**

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**Module No. # 01**

**Lecture No. # 01**

**Introduction, Motivation and Overview**

Hello everybody. Welcome to this new course on optimal control guidance and estimation. Typically, it will be covering around aerospace engineering problems and applications thing like that, but largely the techniques, I mean tricks and techniques will be very generic. So, anybody from any other discipline can also use this concept at equal level. I am Radhakant Padhi working as a associate professor in Department of Aerospace Engineering, Indian Institute of Science, Bangalore.

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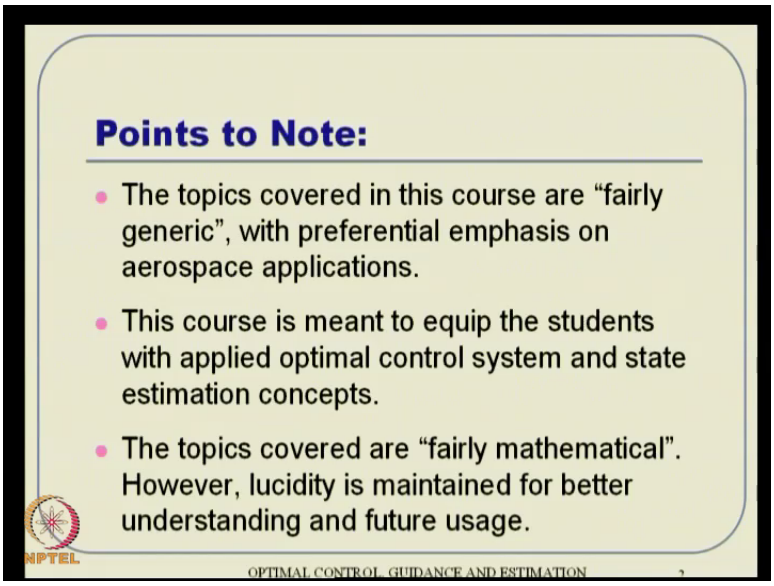


So, let us see some of this introduction, motivation and things like that, before we go to the details of the course. This is the main objective of this particular lecture actually. So, before we start begin the some points to note. The topics covered in this course are very generic with preferential emphasis on aerospace applications obviously, because largely if we see this optimum control applications have been given by aerospace engineering tragic optimum problems conventionally.

And we also want to kind of develop the course and this course is meant to equip the students with applied optimal control system and state estimation concepts. So, as part of it, we will also take this missile guidance and other equal missile guidance problems as well, which is nothing but an application of optimal control theory, and from that prospective, we will discuss actually.

And the topics covered as I told are fairly mathematical; that means somebody who loves mathematics can, can, enjoy it very well, but if you, if you are reasonably good in mathematics, I think you will be able to follow it well, but that is a necessary requirement of this particular course. And I also suggest all of you that, if you that as the course goes along, you take a set of paper, pencil and all use start kind of practicing everything yourself, then only the geometry comes here more actually.

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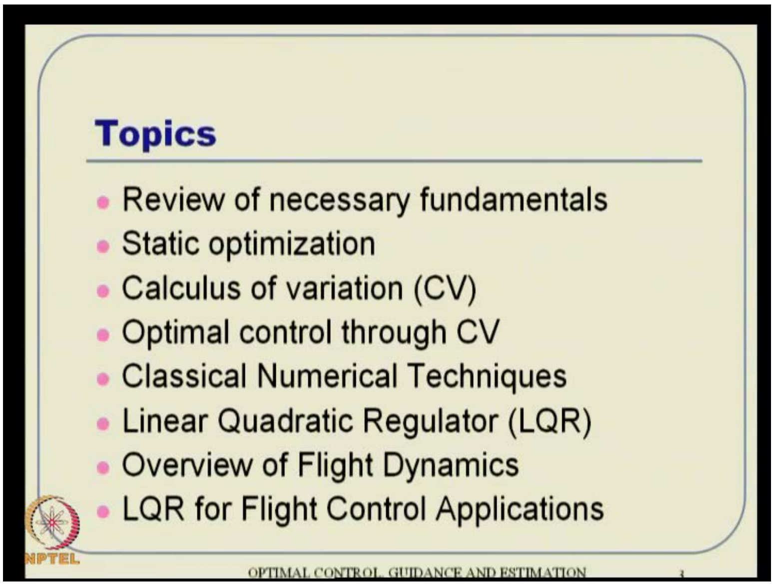


So, the topics to be covered in this particular course is a first thing is a some review of necessary fundamentals; that means we will talk about some numerical methods, matrix theoretical, matrix algebra and whatever necessary background material, that, that, will be quickly review. Then followed by static optimization concepts and that is a kind of a backbone for optimal control in away, because even if we see one, one, method of solution of optimal control is, is, first to convert it to a static optimization problem and then take the advantage of static optimization and rotation all that. So, that we need to know a little bit of a static optimization before we go ahead with the, with optimal control principle, which is nothing but dynamic optimization.

So then, we will go to calculus of variation and see some of the concepts of. So, what is this calculus of variation; obviously, it is a, it is a vast topic; we are not able to cover everything, but whatever is relevant to us, I think that much we will we able to overview before proceeding to calculus of variation application for optimal control and that is what we will talk next optimal control through calculus of variation, and then, that will be followed by some numerical techniques to solve this variation and calculus problem, which essentially leads to these something call two point one revalue problem. We will talk about that is the course develop actually.

But that is one of the reasons why it is computationally very kind of very challenging and computationally over well and rather, and then, we will proceed for other techniques, reason techniques and, and, classical as well as recent techniques to how to overcome some of those actually. Anyway, after covering numerical techniques, we will move to this something called LQR theory or linear quadratic regulator theory and that will be followed by some overview of a flight dynamics, because most of the concepts you want to use in aerospace application. So, it is better to know some of these flight dynamics on the way.

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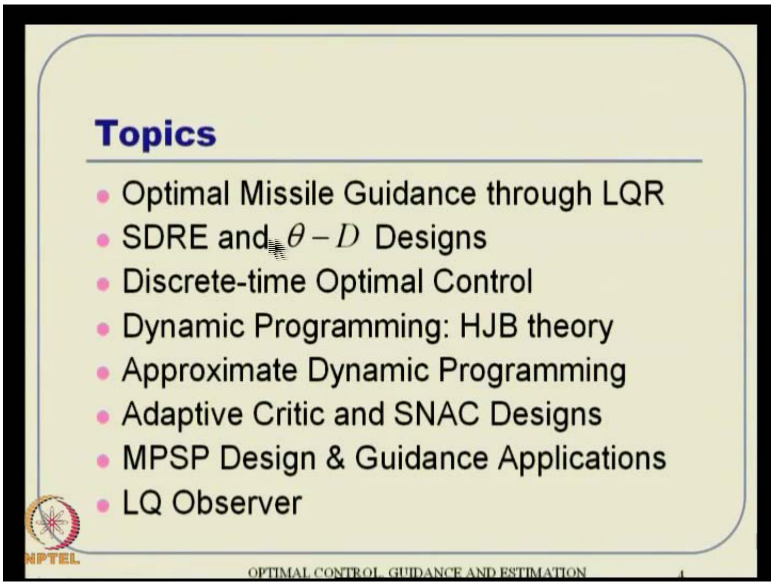

Then we will see how LQR is use for flight control applications. This is some of the application like stability turbulent let us and then many variable announcement and think like that. We will talk about that as an application of flow domain. Next their application LQR will be optimal missile guidance and that is what lot of people do using linear system, system, dynamics rather engagement dynamics between vehicle and target. That is what they do. The missile and target, if you take relative motion and all the using that as a constraint and then formulate some sort of a minimum problem, and then, solve it using l. So, those details will talk after that.

That will be followed by some of this. Then we will venture into this non-linear conserve. So, after LQR this which is largely kind of valid for linear systems, we will go and see how this, this, kind of simple concepts can we extend to non-linear problems as well and that will lead us to this something call state depending record equation approach, and then, there is a little bit further development on that idea which is called theta d design. So, those things will talk. Then we will quickly enter into this discrete time optimal control theory and then discrete time LQR and very quick over view sort of thing, and that, then we will follow to a very different approach. This is called dynamic programming and that will essentially lead to something called Hamilton Jacobi bellman theory very famous.

And then, we will see the, of that actually. So, essentially the H J B theory over comes this curves of complexity, but essentially it leads to course of dimensional so that those details will see that when we talk about dynamic programming, and because of this curves of dimensionality, people are thought about something called approximate dynamic programming in discrete settings. So, we will also discuss that, and then, use this approximate dynamic programming concepts for adapt something called adaptive creative design.

And that and that will be a little subset of that is something called single network adaptive critic design and that will be part of our discussion next actually. Then we will venture into a different class of problem, I mean different cross of formulation which is called model private to static programming design. And again, this particular design is huge potential for guidance applications, and we will see some of these examples actually there. Then after that, we will follow to this estimation part of the story and these are all complementary problems - estimation and optimal control sort of thing. So, we will see first what is called linear quadratic observer.

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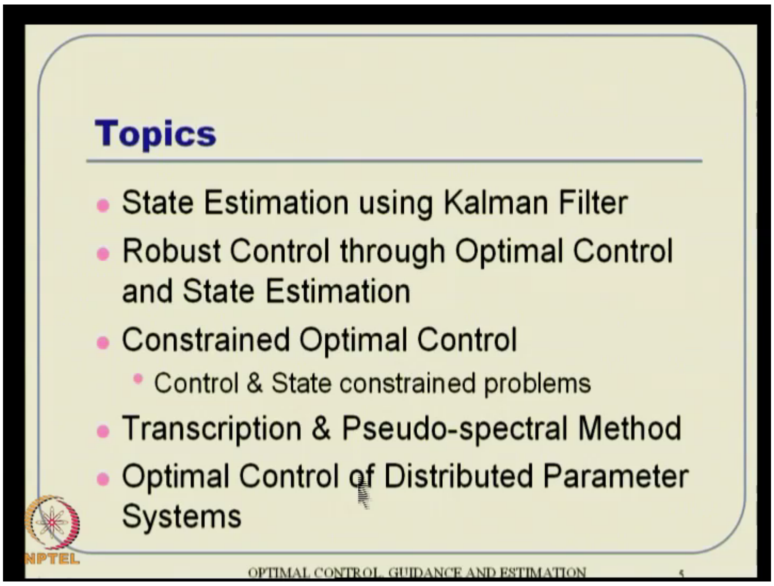


So, that is from there, we will proceed to state estimation using Kalman filter, and then, there is an idea of robust control through the concepts of optimal control and state estimation. Once we put them in together, it leads to something like robust control approach and all that we will see about that.

Then on the way, we cannot forget that there is a huge branch of optimal control that is of relevance which is called constrained optimal control; that means, whatever solution we want it, it need not be only constrained wise state to dynamics or system dynamics, but they can also contain some of these inequality constraints of control and state; that means your control for example can be bounded between certain values. So, those bounds can be incorporated here as well as the state bounce as well actually. So, these are the essentially some of these inequality constraints.

And that, within that formulation, how do solve this optimal control problems. So, it is possible to do and the some of this concept will see. Towards the end of the course, we will go to every again at a very different approach. This is called transcription method, and this transcription method is a very direct approach of solving optimal control problem. That is what I just talk that you convert this optimal control problems dynamic problem to a static optimization problem with large variables, large number of variables.

So, that is what is a, the problem get transcripted to a kind of a static optimization problem and that is why it is called transcription method. Some overview of that, followed by some little bit over view of what is called as Pseudo Spectral Transcription. So, that is one of the things that is getting lot of attention these days and also is a very fast approach in a way, so much fast that it can be probably we use online actually.

So, those kind of concepts can be discussed towards end of the course, and if time permits, I will, I will also give a climbs of the similar concepts, that is, optimal control systems is applicable to distributed parameter system, and distributed parameter systems are nothing but systems governed by partial differential equations lot of application.

There are lot of application again this flow control and then temperature control and flex flexible body control think like that is all fall under the distributed parameter system, whether system dynamics is govern by PDS or partial differential equation which is a not very common in OD domain basically. So, how do we solve or how do you take those problems into account that, that is the topic of discussing next actually. So, this is how the course of structured and then we very slight variation here and there, but largely we will do we will stick to this, this, outline actually.

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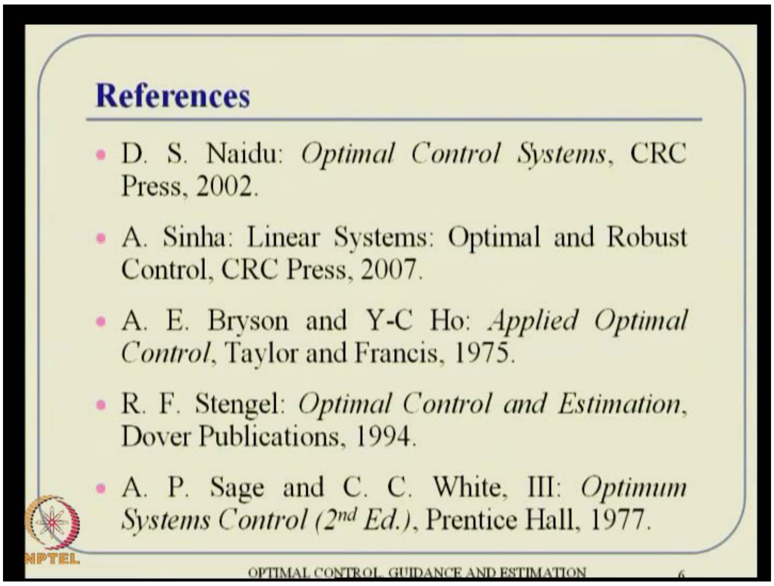


Some of references on the way you can, you can, think of probably buying some of these books. Some of them are still available; especially the first one is very much available and it is available in Indian edition as well. So, it is extremely cheap, and one of my favorite systems or favorite books for linear system optimal control is the second one is very clearly written and I like it. You can think of buying that, and especially the third one happens to be a very classic book. It, if we remember, if we can notice, it is published in 75, but it is still on print and it is per as the one book which is the most widely referred ever in the in the entire control system domain basically not necessary only in optimal control.

So, that is that kind of implied book. It is, it is never seen a second addition, but still the first addition contains enough material for everybody to kind of look at it and then still keep on learning from it actually. I strongly recommend the third one if somebody wants to get into these optimal control theory and applications as well. Then there are other book, which are again there is a Dover publications very cheap book from Stengel, talks about optimal control as well as estimation concepts.

And this, this, AB sage and C white book also extremely well written book. In fact, AB sage and book that was one of the very popular book actually and it sustains its legacy as well. In fact, this distributed parameter control and some of these sage equation concepts and all I will take it from C. White actually.

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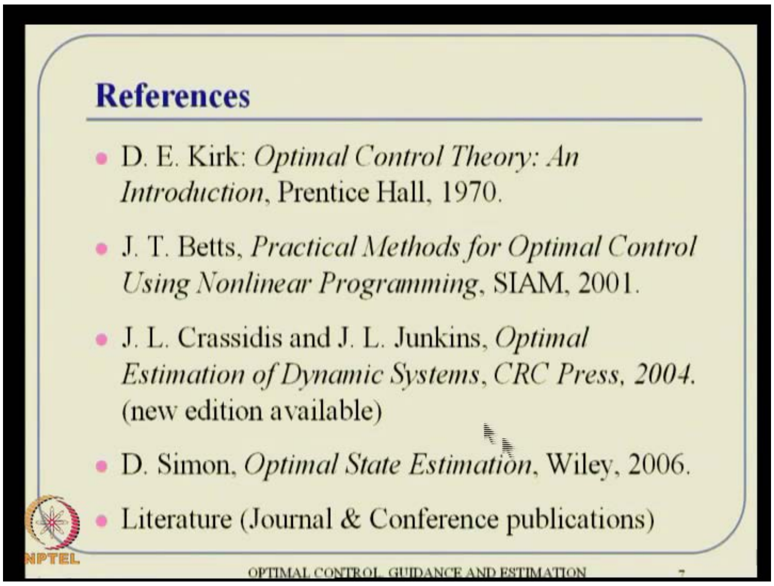


So, there are other books well which is this one is another very classic book, lot of numerical techniques are discussed very well here. So, some of those concepts can be read in detail there and there are other publications as well. For example, these JL Betts it talks about this transcription method very well; he is a very pioneer on that, and then, he published many things on transcription method as well and that is what the book talks about actually.

Then there are other estimation theory concepts and this is my most favorite book is Crassidis and Junkins very rigorously written, very much well return as well and those of you want to buy some estimation books surely you can by the this as well as d Simons book, and this is also extremely well written; however, rigorous part is much more in Crassidis, and I also think there is second edition coming up. It might already come up in late two thousand twelve or something that is my actually.

And obviously, there will be topics taken from literature as well; that mean some journal and conference publications are also included as part of this course actually. So, take a close note of all this things and ensure everything will be quite enjoyable experience actually. So, let us go to some, some, degree of introduction and motivation first before we go into this math, math, details and try to explore more.

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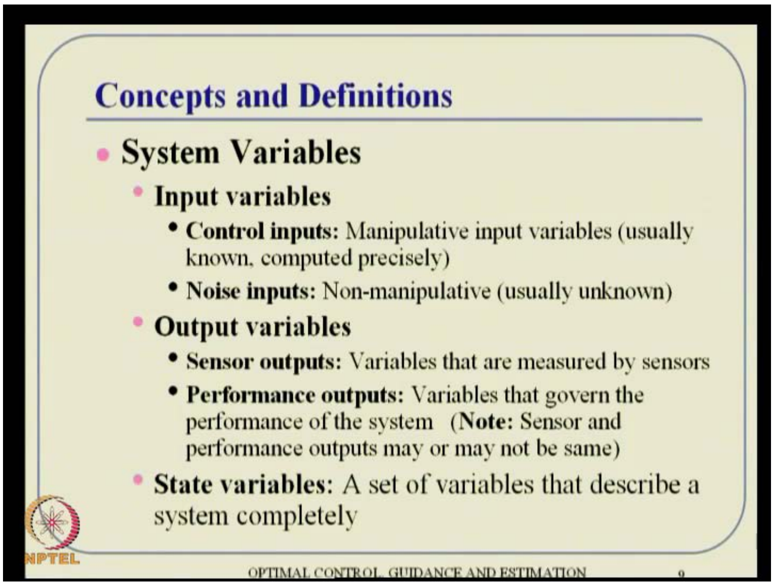


So, come to the introduction and motivation first very basic concepts. So, first thing is what is called a system variable. Then we talk about two type of variables - one is I mean largely it can be classified something like input variable, output variable and kind of state variables. So, what is input? Input can again be like some control input which is a nothing, but a manipulative set of variables. That you can or we can rather vary as we wish, and then, this can be computed precisely as well, but there are other class of inputs and remember these are actually inputs to the system which is called noise input. In other words, they, they, do come in input the system and they do disturb the system. In other words, for example, if you go on a car, then the road condition and all will affect the car drive. That is not a control system, but that is a noise input to the system. Similarly, when aircraft flies, then there is a something called atmospheric dust phenomenon and all. So, that will disturb the vehicle motion as well. So, those are noise inputs. Usually they are non manipulative not in our control, but we have to live with that; we have to handle that and that is how this necessity of robust control comes and all.

Then there are two types of output variables as well, and one is a one set of output variable that you can measure by sensors. That is called sensor output, but there need not be something that you want control. The control can be something same or different and the variable that you really want to control the system or you want to have a performance measure performing whether the system is performing well are not. Those things will be describe something called performance output actually.

So, sensor and performance output are may or may not be same. Then there is something called state variables and the state variables are nothing but a set of variables that describe a system completely; that means you will see some examples, I mean you might have already seen in lot of text books early in system theory. State variables are, there are certain restrictions. They cannot be, you cannot take more state variables are necessary. You cannot take less state variables than necessary either. There are again problems and advantages of that and like that, those of you are interested can see a classic text book on these concepts actually.

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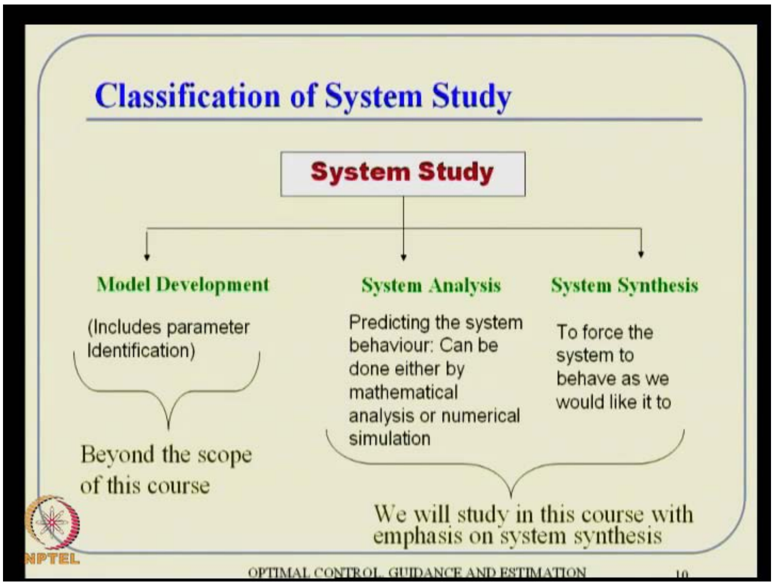


And when we talk about system study in general and there are three category of system study - one is model development which you can also include parameter identification in general and typically will not talk about that. In other words, we will assume that the model is typically available to us as far as control system synthesis problem is concerned, and other words as a control system designer will not worry too much on modern development issues and all that actually. So, we will assume that it is, it is, known to us in a away.

Then there are system analysis concepts; that means once you have a system model or a mathematical set of equations for representing the model behavior, then essentially you want to know whether system is behaving properly or not so that, that will lead to something called system analysis. The typical example is whether the system is stable or not stable, whether the system is on controllable or not controllable. So, those kind of issues are some part of system analysis problem, and if the system analysis output is good, that means system is behaving well; we do not want to do anything, but if something that needs to be done, then we will go to the system synthesis part; that means, we design a set of control input in such a way that the system performance is satisfactory actually.

In other words, we force the system to behave as we would like to see actually. That is all, and typically will kind of give emphasis on the this type of issues and especially on the system synthesis part of it, and analysis will be there in a weak sense, but largely we will talk about system synthesis; otherwise how do we actually design a control variable using these concept of optimization or dynamic optimization. That is what the main objective of this course actually.

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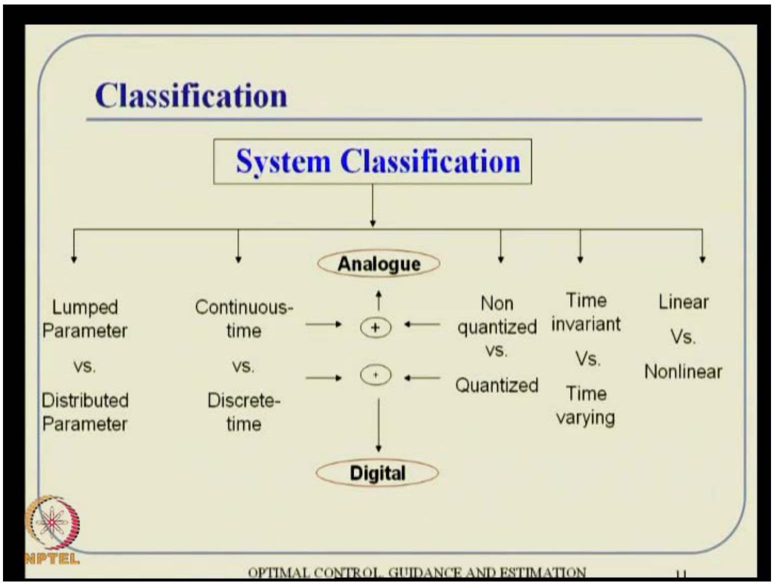


Then, we need, there are various classification again as a, when you talk about system, we can classify in variety of ways - one is lumped parameter systems, where there is no relative motion between two molecules of the system, and in other words, everything if the center of gravity moves, then every other particle moves in the same way, same direction, same velocity like that. So, that is a lumped parameter system, and then, there is a distributed parameter system where we have relative, I mean the relative motion between two molecules are also there as a along with the whole system getting (( )). So, those are the system that can be described only using partial differential equations, whereas this follows lumped parameter system can be used in, I mean can be described in using this ordinary differential equation and all that actually.

Then, there are different classes of systems. When you talk about continues times systems, time variable is treated as continuous variable versus there is a discrete time system, where we talk about only discrete weak points, and then, see the way read those weak points (( )). Then there are ideas of quantized system, non quantized system think like that, and there are something called quantized means the variables are given at discrete number. The quantity of the variable can change only in discrete numbers actually. Non-quantized is a something that varies continuously. In other words, all rational numbers are also accounted for numbers actually, I mean, then there is a concept called time invariant system and there is something called time varying system basically; so that means, if the parameter of the systems can change in time, then this leads to time varying system. For typical example is when, when, an aircraft moves in a fuel and rocket moves in one fuel, the system is see's the different mass as time goes actually; so that means mass keeps on changing, and if that does not happened, then we talk about something like time invariant system. You can think of, roughly you can think of it is a kind of applying under in electric propels. If it is electrical propels, there is no expendable mass goes there; it is only the charge and all that way.

But then, every other thing remains constant. There is no person moving there. So, they do not move around; the mass remains same; moment of inertia remains same, and I mean this kind of systems can be thought about something like time invariant system actually. Then there are something like linear system or and verses non-linear systems, and largely linear systems turns out to be linearised systems of non-linear system because nature typically gives a non-linear systems.

Anyway, coming back if you somebody analyzes the system using discrete time theory as well as quantization theory, then it essentially leads to something called digital control, whereas continuous time and non quantized variable combination leads to analogue control, and it is so happens most of the time that we confine in courses to the first line; that means we discuss lumped parameter system largely in continuous system domain using non quantized variables assuming time variable systems and linearizing the system, whereas the reality is all in the bottom actually mostly in the bottom rather.

So, the reality most of the systems theoretically speaking at least can we describe in a distributed parameter system. Even though largely lumped systems are also from the engineering point of prospective point of view, and then, there are discrete time theory typically now a day's not given that much of importance and quantized theory as well because computers has becomes very fast continuous time and non quantized and all. Even if you quantize it very, I mean if we quantized it very fast rate and then it becomes almost equivalent to the kind of continuous time and a non quantized and all that. So, that is not that much of a big deal. However, time varying parameters as well as non-linear systems are typically reality; I mean we just came to visit away, but unfortunately most of the courses in classes we are talking about linear systems.

And this particular course, we will talk both linear system as well as a non-linear system as whole entire thing to see in totally lot of thing. So, if it happens be a linear system, well and good; it has its own advantage and all that, but we do not want to confine everything in and around linear system actually.

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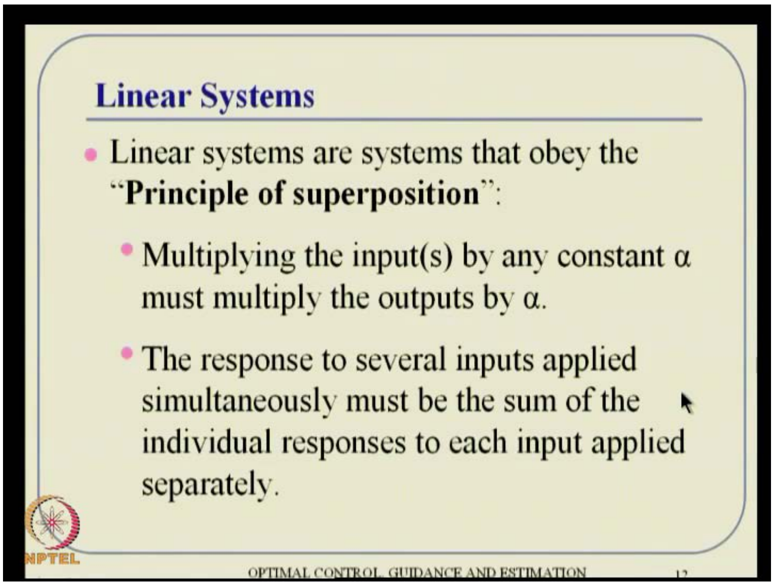


So, the, let us move on then very quickly what is a linear system. A linear system is a something some system that obeys principle of superposition. In other words, multiplying the input by any constant alpha must multiply the output by the same constant alpha as well as this, this, is the law of multiplication and this is law of addition as well. So, this is response to several inputs applied simultaneously must be sum of the individual inputs to each input applied separately. So, all these things I think is a very standard thing in, in, text book, in linear system theory especially.

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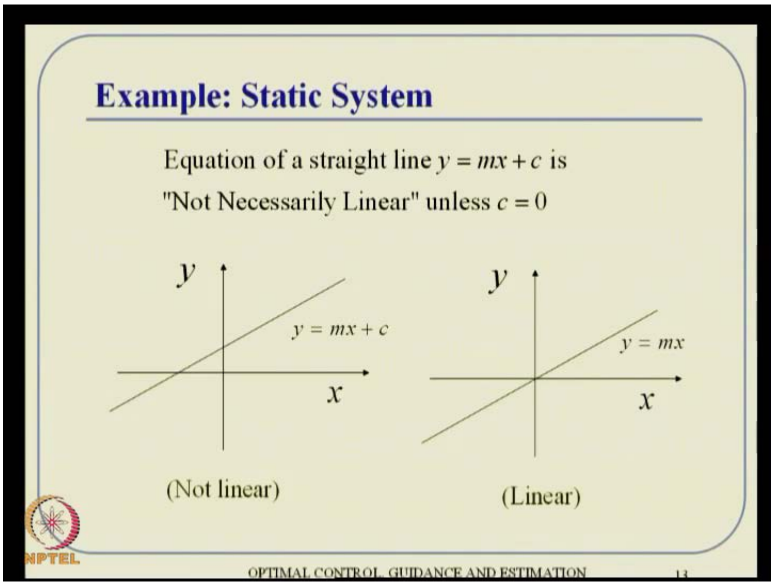


So, you can, I mean this course is not towards, so I will not find too much of time on those lines actually, but still I will just give a quick example. If it, if you consider a static system, that means there is no differential equation anything, but the system is given by this kind of equation. Typically the very first impression that you get is a linear equation, correct, is a linear equation but it is not a linear system, because it does not necessarily linear unless c is 0. Actually it is not linear. You can very quickly see y 1 if it like that and y 2 is m x 2 plus c, y 1 is m x plus c, then y 1 plus m in 2 x 1 plus x 2 plus 2 c.

So, that 2 c is not equal to c and hence it will not satisfy this, this, law of addition and all that actually. So, it is essentially it will satisfy everything if provided c is 0 basically. So, that means if the line passes through the origin, then it can consider that as a linear expression. Otherwise it is a equation of a straight line; it is certainly not a kind of a linear equation actually. So, those kind of ideas typically we forget it actually, just keep it in mind.

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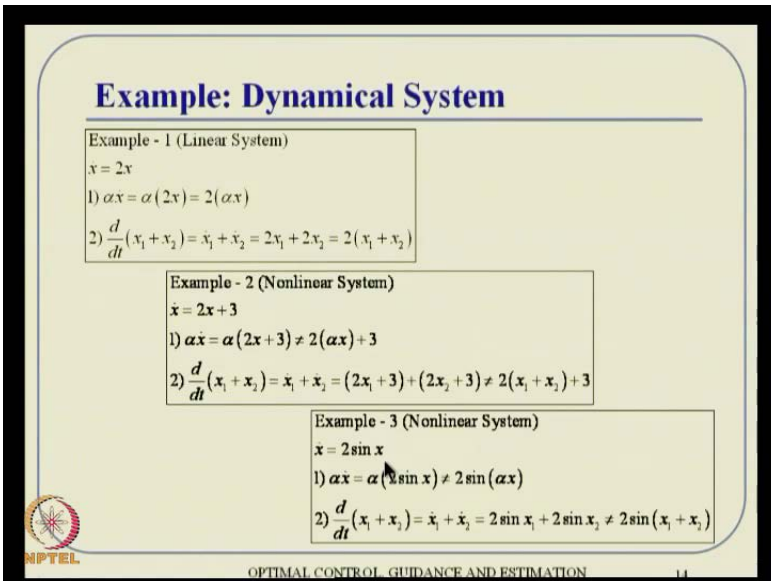


Similarly, in dynamical systems as well, if have a differential equation and all, if it is in this form, then it is actually linear form. Then very quickly you can verify this law of multiplication and addition actually. So, if you take like alpha into x dot and you can multiply with alpha the right hand side and then put 2 into alpha x, so that equal to that, and second equation is like that actually. If you take x 1 and x 2, and then, try to valid all that; it will turn out to be like that actually. Then, if you have this kind of example 2 which is actually non-linear system, the moment you have a bias there it will not satisfy, and hence, it is not a linear system. And if you talk about this kind of a system as well, again it will not satisfy this principle purpose for this reason 2 sin x and think like. So, again it is not a linear system actually.

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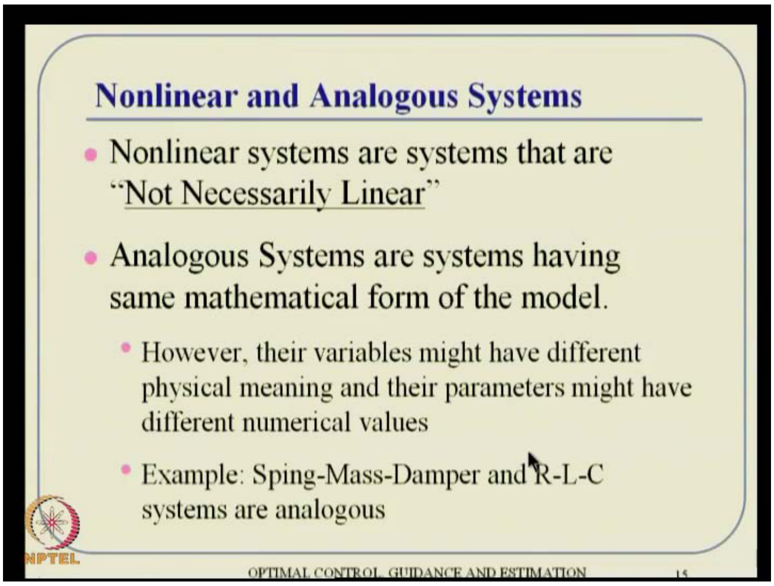


So, by the way, a, in non-linear system by definition is something that is not necessarily linear; that means, all linear systems can be thought about as part of non-linear family as well actually, but not necessarily vise versa. So, in other words, if you study a concept which is applicable for non liner systems, it is actually equally valid for linear systems as well.

So, do not forgot that part of it actually, and then, there are some concepts called analogous system; that means the, if you take two systems for which the mathematical representations are similar, but their physical meanings may be different. So, mathematically these systems are kind of similar behaviors. They will have similarity, but physical meaning sense they will having different way things actually. For example, in spring mass damper system, in mechanical systems, it is very equivalent into RLC systems in electrical systems. So, these two systems, as far as system theory is concerned are considered as analogous system. So, the math theory and all that it will be very similar. It is the physical interpretation that needs to be different actually.

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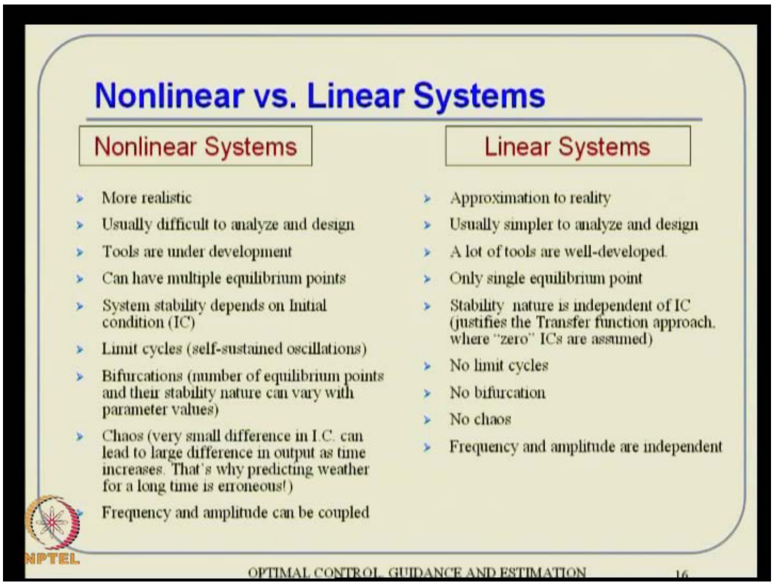


So, now coming back to the some differences between linear systems, non-linear systems, some similarity all that thing you can summarize something like this, and this is very easy to see that non-linear systems are more realistic and liner systems as I told usually do not exist in nature. All that we do is make simplicity. You take expression and then forgot the rest of the series and keep it only the linear term. So, that is certainly an approximation to reality. Then non-linear systems they are typically more difficult to analyze and design, but linear systems are usually simpler in those aspects.

So, the linear system there are lot of tools already well developed, and here, tools are getting double of their partly done partly not done. So, there is lot of scope in doing exercise as well actually, but the main difference that you see in non-linear system. The difficulty starts from here. It tells the system, system, can have multiple equilibrium points. Very easy to give examples; for sake of time, I will not give that, but suppose you take, let us say x dot equal to like x q minus x something like that, then it is, if you take x dot 0, then it is x equal to 0 and plus or minus 1. It will also satisfy. So, in a equilibrium sense, you will have minus 1 0 and plus 1 and all these things are actually.

But if you take a linear system, that is not a choice actually. You will always land up with single equilibriums, I mean point and that equilibrium point happens to be a origin actually, and now, the more difficult is the system stability depends on initial condition as well for non-linear system. So, it is not that you have only multiple equilibrium points, but the system stability behavior itself will, will, change depending on what equilibrium point you are talking about. Around that, the system can be stable. Around some of the equilibrium point, it can be unstable as well actually.

That kind of behavior is typically not seen here. The stability nature is. In fact, getting independent of initial condition here, because here, only one equilibrium point and whether the system is stable or unstable, does not necessarily depend on that equilibrium point; it does not actually. In fact, if your system is stable, it is globally synthetically stable, I mean global exponential stable rather. In other words, no matter wherever the initial condition is there; the system is stable means for all those points initial conditions, that is, trajectory double of zero actually. If it is unstable for, from every point other than the equilibrium point the system trajectory will get to, go to, instability, go to, go to, infinite actually. So, those kinds of behaviors are not seen in linear I mean non-linear systems. They, they, are critical. It depends on the equilibrium point as well actually.

Then there are concepts called limits cycles which is all kind of a self sustained oscillation that consists bifurcation. In other words, this number of equilibrium points and their stability nature do not stay constant. They can vary with, with, parameter values as well, and especially it is time varying system the parameter values are changing. Then you can essentially lead to this bifurcation issues and all that actually. Now, there is concept called chaos, and then, chaos are very small difference in initial condition can leads to large difference in prediction.

In other words, if you have a very small two initial conditions that are very close to each other, but they are not identically same. Then if you propagate the system dynamics for some time, then the behavior will be very different; the dispersion will become wider and wider actually, and it will point very quickly will see that the difference going up towards the system simulation is no more available actually and very common example of chaotic system is weather system.

So, that is the short duration weather prediction becomes easy; the long duration becomes difficult, because no matter how much good instrumentation and computation you can have. The system being chaotic; you cannot propagate the system dynamics or the system model for a long time, and then, tell your trajectory will be reliable and all that. So, that is the problem of chaotic system actually. Now also there are other issues like frequency amplitude can get coupled and then you have associated problems with that.

So, you cannot take certain sort of signals and analyze only in frequency and amplitude like that and these are other problems that frequency domain analysis does not typically hold good here. So, in other words, if you see a frequency domain analyses, especially using this Laplace Transform and like that is typically vary for linear systems only and do not forgot that part. So, when you talk to non-linear system, Laplace Transforms are or even Fourier Transforms something's like that, that are typically not valid this. So, you lose this frequency domain luxury here, frequency domain interpretation, and then directly here to deal with domain only. So, that is that is the non-linear systems are actually.

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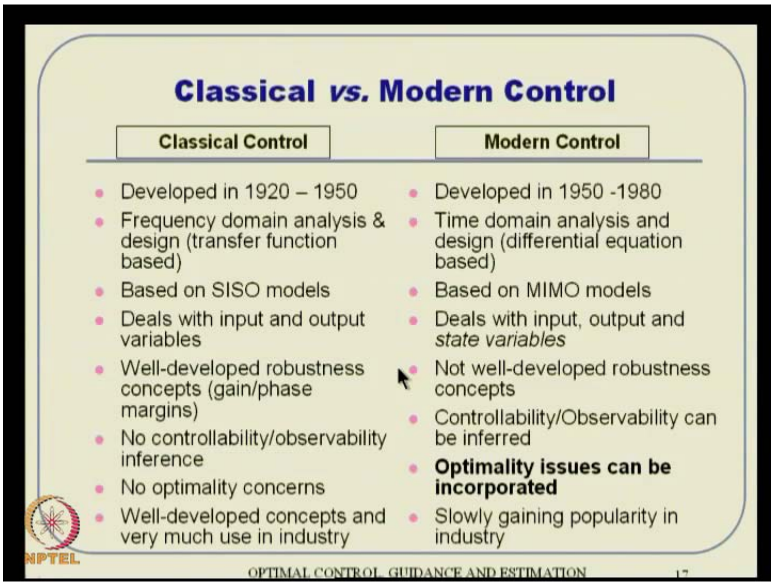


Then a quick over view what is classical control verse modern control. There are all little bit old concept largely classical control is developed largely between first world war and second world war. Modern control, what I called as linear modern control (( )), that is developed between these things, after the second world war lastly, and then, typically these are frequency domain analysis, whereas, these are time domain analysis and classical control is based on single input, single output models, and whereas modern control based on multiple input, multiple output, and remember, optimal control theory is all based on this, this, modern control concepts actually.

So, we will we will largely this course will contain, I mean this kind of over around this modern control concepts actually. So, there are other things as well you can read some of these things. The, the, very good reason why this classical control is still popular is because of this issue. There is something called gain and phase margin concept which is very well defined in classical control domain, where people get comfortable with the robustness properties of the system and the design actually.

So, that is typically not available here even though in a phase margin sense you can talk about gain margin in gain margin in sense there is some misuse in that. So, it is not extremely well connected that way, but things are under development as well, but where this things comes into big ways something called controllability and observability issue. In other words, if the system is controllable, then only it makes sense to kind of go ahead and find various control designs schemes that are and then evaluate them. The system is not controllable, then it implies that no control system is going to work, no design is going to work.

So, it is better that we do not waste our time actually. Similarly, if it is, system is not observable, then no observe is going to work. So, there is no point in naming for an observer and or an estimator think like that and then struggling and wasting our time and all actually. So, those are the concepts that are available in the modern control domain only, and the big point is we not only see this classical control, we can is largely stability base theory and it is actually whether the system is stable or unstable.

Whereas in the modern control part of it, I mean we can also talk about something called optimality. In other words, in what sense the control is optimal, whether it is minimum time, minimum distance, minimum work actually. We can talk about the maximum profit. All sort of things we can discuss in a very neat sense actually from the optimum, from the modern control prospective and that is where optimal control comes into picture actually.

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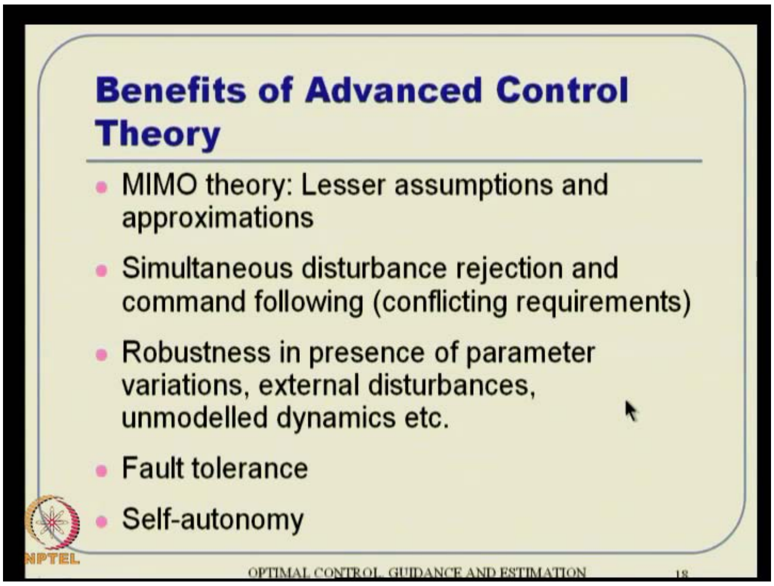


So, some of the benefits of the advanced control theory first of all, it say mimo theory, that is, multiple inputs, multiple outputs so that essentially leads to lesser assumptions and approximations. So, we do not have to ignore this cross coupling effects and all that. You account for that and then design your control actually. Then there are concepts of simultaneous disturbance rejection and command following; that means, there are typically conflicting requirements. We have, we have to just reject the disturbance and assure that command is been followed.

So, that is typically a kind of conflicting requirement, but can be done because you are not ignoring the cross coupling effect and we are taking, taking, power of modern. So, it is possible to take into account of all that. Then you can also talk about robustness issues in presence of parameter variations, external disturbances, unmodelled dynamics or neglected dynamics or inaccuracy parameter variables think like that. Then even we have things we can talk about something called fault tolerant control; that means, if your system is under operation, your aircraft is flying, and then, you have developed some fault in actually. In other words, this actuator stuck and all that. So, how do you how do you still make sure that your aircraft lands or still make sure that your system behaves in a little tolerable way at least. So, it is very big implication if you talk about, let us say nuclear power plant. Then if something goes bad in a, it should not really lead to a catastrophic disaster actually. So, those kind of control system can be thought about, can be analyzed, can be developed all using this advanced control theory.

Then there are concepts of self autonomy and some of these things we will see that. When we talk about guidance, the guidance essentially leads to the self autonomy sort of concept where the algorithms and all are designed in such a way that there is intermittent human intervention is not necessary. Once you fire it, you fire it and then the logic takes over; then the sensors are integrated to the actuators; then the sensor information will be processed to the computer and it will pass to the directly actually. So then, the system is autonomous, I mean you really do not have to control anything; you do not need to bother about that as well actually.

So, those are the concepts for autonomy and there are very challenging resource problems as well. For example, if you have some unmanned aerial vehicle, you have a (( )), then you have camera there. Then how can you make use of the camera photograph so that you can sense the obstacle in a and. So, those kind of things you do not have to manual it on right and left and then go and all that. So, it will take care of that itself. Similar concepts are there in robotics as well and link that. So and typically when we talk about autonomy, this vehicle guidance comes into picture, and here, we are going to see how to design guidance schemes using optimal control theory. So, essentially optimal guidance scheme itself.

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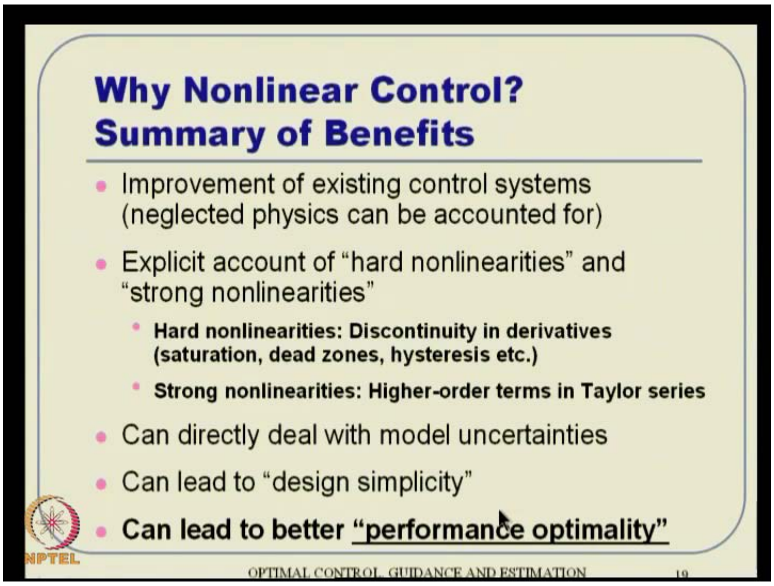


So, why non-linear control in general, there are certain benefits. First thing is we can think about improving existing control systems and some neglected physics can be accounted for, and then, there are something like some explicit accounts of hard nonlinearity verses strong nonlinearity. What is hard nonlinearity? This talks about discontinuity in derivatives. For example, saturation dead zone hysteresis all that, those are nothing, but hard nonlinearity, that can be accounted for.

Then there are strong nonlinearity essentially higher order terms in Taylor series. So, that kind of both the thing can be handled here actually. Then it can directly deal with model uncertainties as well and sometimes it can lead to design simplicity as well and that is a little bit surprise, but it turns out to be true. In other words, if you go through linear control system, the theory may be easy, but it remember it is locally valid; that means, you have to really go for designing a set of non-linear systems everywhere, that is, that is of interest to you and then you talk about stitching them through something called gain scheduling.

So, that gains scheduling processed turns out to be quiet challenging. It is a quiet cumbersome; it is it is also problem dependent; it is discipline dependent; it lot of exposure comes and lot of prior experience will be helpful, and if the system goes through a design iteration and the system dynamic changes, then you have to repeat the entire exercise. So, those kinds of ideas are not necessarily true when you talk about non-linear control in general basically.

And it can also lead to better performance optimality. So, if you a branch of non-linear control which can be designed through optimal control and all that, then obviously you can talk about performance optimality actually. So, that is, that is the another biggest advantages come into picture.

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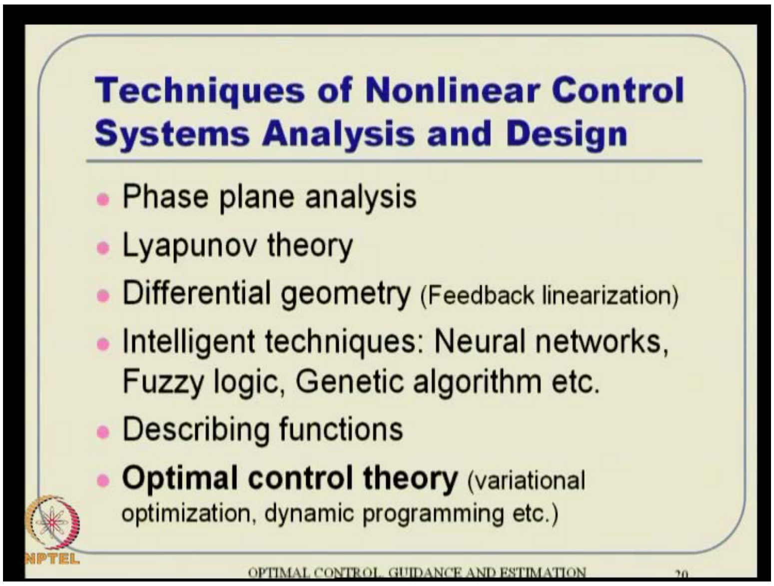


So, in general, techniques of non-linear control can be thought about many things. First thing very simple in a, if you, if you were state phase contains two or at the maximum three states, then you can think of something called phase plane analysis. In other words you plot different states x 1 verses x 2 or x 1 x 2 x 3 and eliminate the time part and that will have certain meaning actually. That kind of things is called phase plane analysis. Then there is a branch called Lyapuno theory base things.

So, you define a process definite function, and if the derivative of that time derivative of that turns out to be a negative derivative function and the system is stable and those concepts fall under this, this, Lyapunov theory and think like that, and I mean this something called differential geometry base thing or otherwise known as feedback linearization, and then intelligent techniques as well using neural network fuzzy logic, genetic algorithm thing like that, biological inspired computing and much moral actually. Then there are concepts of describing functional analysis and lastly but hugely, I mean were hugely concerned about this which is called optimal control theory comes from this variational optimization or dynamic programming approach, etcetera like that actually. That is where our, our, focus will be for this entire course actually.

I mean just a comment before I move on, if somebody is interested in Lyapunov theory and differential geometry based control theory using this feedback linearization as well as lyaponav theory based this analysis and all think like that, you can probably refer to my first course, which is already available under this NPTEL program actually.

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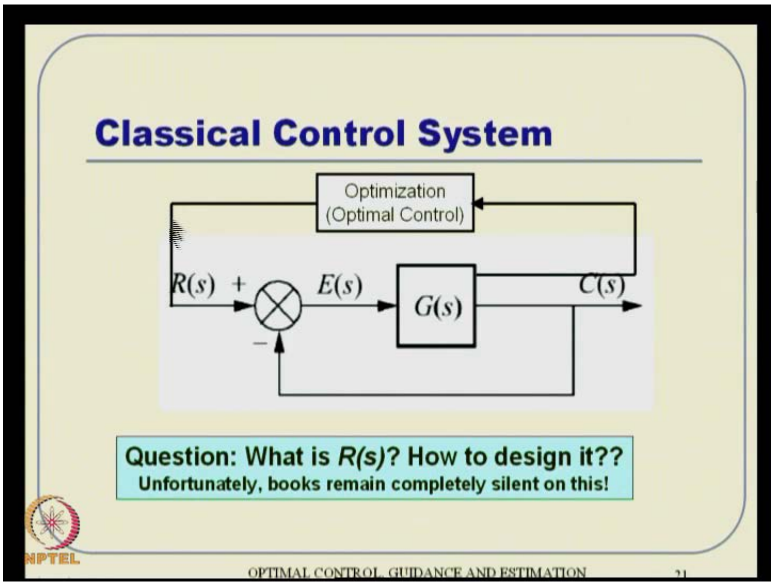


So, what is this, this, optimal control for a, in a nut shell sort of thing, and to understand that, if you see any classical control system book, probably you will come across this kind of a diagram, where this g of s is supposed to be a system plant dynamics and we have something like a output and this output needs to be a kind of tracking problem sense; output has to go R of s; that means C of s should approach R of s with time. So, that means we have an error signal there and then this error signal is manipulated in a, in a, control gain and that is back to that.

So, in a control and plan together sense, this g of s comes into picture. Here, this unity feedback or nonunity feedback and all sort of things there. So, this is a very standard thing actually. So, essentially, if we look at it, this is nothing but a stabilizing control design problem sort of thing. You design a control gain in such a way that the close loop planned dynamics whatever you see here is stable; in other words, it drives to 0 actually.

All that things are fine and no, no, questions on that, but if, if, I ask a little question that what is r of s. Most of the books will turn out that this r of s is nothing but this step input, ramp input and sort of thing actually. Typically those step input ramp input are certainly not reference signal for a plan to operate. So, for any realistic system, if you just confine yourself to those kind of signals, then obviously the, you are not going to go anywhere actually, but still we want to test our control system for that the reason being any arbitrary signal can be actually decompose to this, I mean kind of constant signal, then ramp input and also using Taylor series actually.

So, that is the backbone why it is done. If you any arbitrary signal, if your system behaves well with respect to let us say step input, ramp input and parabolic; that means you actually testing for first three terms in the Taylor series. That is why it is done actually but moving ahead. In other words, what is the R of s that will lead me for the system optimization, overall goal to be achieved; that means, that is where this R of s will play a major role, because if I have to guide a missile guide a missile, then I have to guide it towards the target.

Once the guidance part is taken care, that means r of s is designed well, then it will be tracked actually, but what is the problem that how can I do that actually? So, this R of s is coming, come up with a value of r of s actually. So, this is what I want to talk a little bit here. So, if you that is the thing. Question is what is R of s and how to design it. Unfortunately, many other books remain silent on this, but think a little, and if you just, just, think about doing this actually. Take the similar output sort of thing and then you design an r of s is, r of s in some sort of outer look actually.

And that is typically done in using some optimization criteria; that means if you are running a power plant, then your overall load should be at a particular level for your system to operate nicely. If you are talking about a emission guidance problem, then your should be at zero then. So, all sort of things nothing but optimal control problem. So, this is the loop that you are talking about in designing here in optimal control actually.

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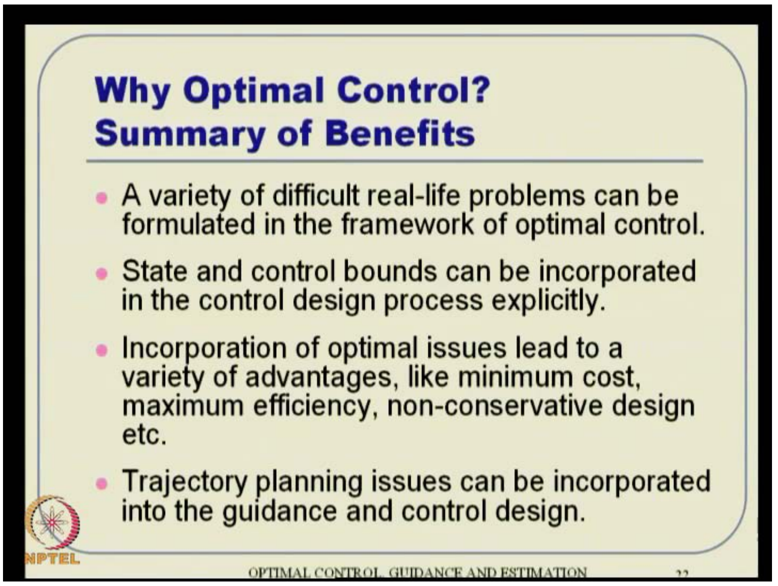


And what are the summary of benefits for, for, using optimal control, and first of all, a variety of difficult real life problems can be formulated in the framework of optimal control. Do not think that optimal control is just another approach of solving actually. The many difficult problems which cannot be solved; otherwise using any of these Lyapunov theory based techniques or feedback based techniques and all can, can, be actually handled very well using optimal control; that means it actually gives us a very powerful technique or powerful platform to handle those problems actually.

Then state and control bounds can be incorporated in the control design process explicitly. Typically it is not true in stability based design. In other words, if you design a system, I mean stabilizing controller and evaluate it using this bounce and all that. If it does not, then go back and retune it and that tuning process can depend on your experience and all that. Actually here, you are not talking about that state and control bounce. If it is known Apriory, then those can be explicitly incorporated while designing the control itself. So, that is that is a big departure philosophically.

Then obviously the, you can also incorporate optimality issues for a variety of advantages. For example, minimum cost, maximum efficiency, and then, it will lead to this non conservative design and all that actually. That is big advantage that we can actually talk that the controller is optimal in this particular sense. So, that we will be able to discuss as well.

Then essentially it tells, I mean if your optimal control can be solve very fast, then essentially it leads to some trajectory planning issues can be, I mean this essentially optimal control is nothing but a trajectory planning issue basically. So, how the trajectory evolves in a period of time; that means the only problem there is computational problem. Now, the computational problem is taken out. In other words, there are techniques which can use it for solving it very fast. Then essentially it amounts to the trajectory planning issues can also be incorporated into guidance and control design actually. That is a nothing this kind of big advantages are not possible and the stability base design

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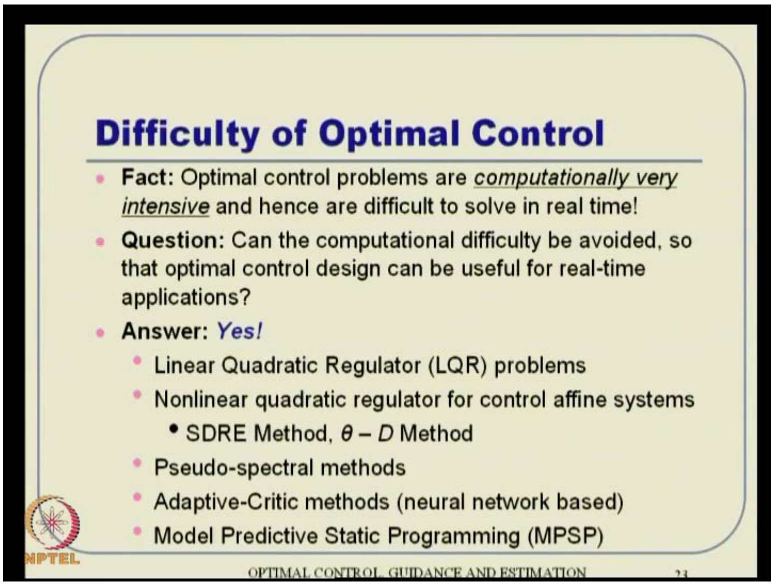


So, what is the difficulty, I mean these are all nice (( )). So, these are the benefits part of it, but what are the difficulties? The difficulties very (( )). It is a kind of open fact that is known to everybody that optimal control problems are computationally very intensive and do not think that your modern computers can solve it. These are nowhere, nowhere, close. Even the super computer of today is nowhere close to solve a real good project optimization problem in real time actually.

So, these are actually computational difficult problems, and hence, they are difficult to solve in real time as well. Now, the question is when the computational difficulty be avoided at least to some class of systems, class of problems think like that so that the, it can actually useful for real time applications and answer turns out to be yes and we can do that for a variety of things, and the first thing is LQR problem.

Traditionally it is all done in sixties and seventies think like that so, but the system has to be linear first of all and linear time invariant most of the time. So then only you can use think of using this LQR problem. There are also you need to solve something like a Riccati equation which may or may not be solvable online if your numbers of states are more actually, but for a reasonably decent practical application, you can think about using LQR technique and Riccati equation can be solved online and that is what is SDRE method actually essentially.

So, this, this, SDRE method is a little more than that; obviously, we will talk about that when we go there, but essentially it amounts to solving Riccati equation online basically. That is what it is. Theta d tries to avoid it. It, it, kind of gives you something one Riccati equation off line solution and once of lyapuno equation online solution because it is own computational advantage itself. Then there are other techniques Pseudo-spectral method, Adaptive-critic, this MPSP method and think like that. So, these are the things that we are going to talk in this course as well. So, there are, I mean all these things I have put putting here is actually we will talk in detail as we go along actually.

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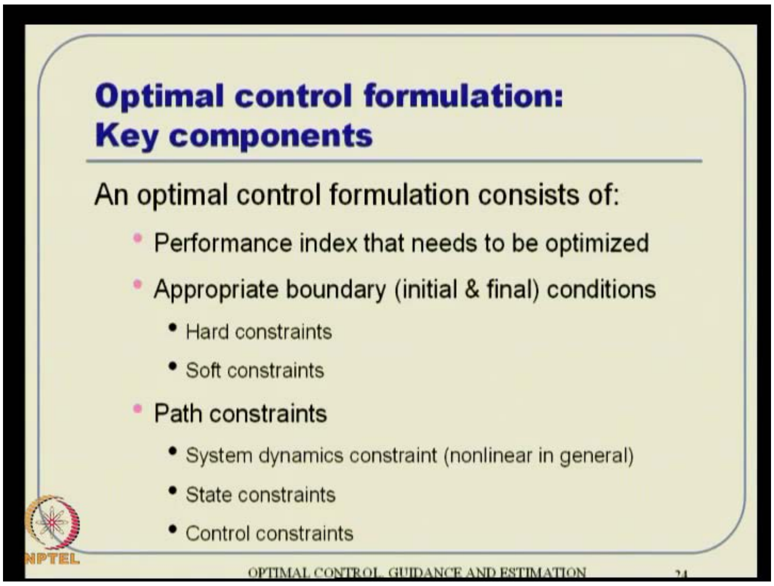


So, what are the key components of an optimal control formulation? Mathematically speaking first of all it needs a performance index that needs to be optimize and then, there is to be some sort of appropriate boundary condition, and essentially one somebody talks about boundary condition, it amounts to initial and final condition. So, we have a set of initial conditions and we have a set of final condition, and again, both can be either soft constraints or hard constraint; that means, if you talk about my initial condition value has to be equal to some value or finally, I has to be equal to 0.

So, those kind of demands are hard, and hence, they are called hard constraints, but if we talk about this constraints can be minimum, I mean I do not really bother about it is 0, but it is within some let us say 5, 10 meters and all that, but I do not have very not, I am not very particular that it has to be exactly 0 or exactly 1 meter think like that. Then it amounts to something called soft constraints actually and both are possible in this framework.

Then there are path constraints, and as I told first thing, first is the system dynamic constraint. So, every system as their own dynamic equation and that as the, that is accounted for explicitly is set of path constraints actually and there they can be non-linear in general actually. So, then there are state constraints and control constraints, and as I told sometime before, they can be equality constraints actually. So, all this things can be in cooperated part of the problem formulations itself.

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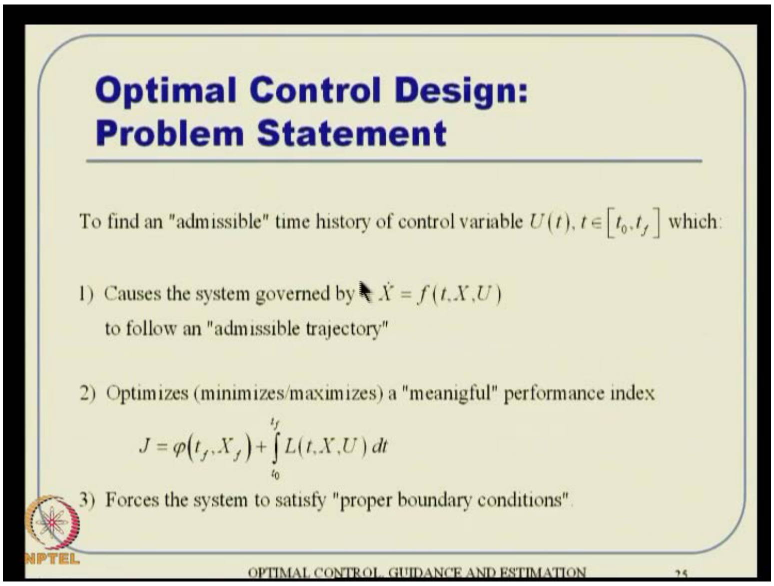


So, mathematically speaking it amounts to like finding an admissible time history of control variable with in this initial time to final time, which causes the system govern by this set of non-linear equation. Remember, this is a non-linear difference set of differential equations so with number of states and time varying as well actually. This is this actually non autonomous sort of system. So, as time explicitly appears in system available that, that is also possible.

Most of the system will not have that in our modeling and all, but even if it is there, it is possible to account for that. So, essentially it amounts to find the control variable accounting for this system dynamics, and then, not only that, it will find a, it needs to find a solution for an admissible trajectory. Then the state variable x of t whatever it turns out, it has to be admissible as well as, and on the way, it has to minimize or maximize a meaningful performance index, and a performance index in general can be given something like this, but it is not the only form; it can be you can design your own curve, I mean own cross function as well also called as cross function either. You can talk about performance index cross function either way.

And this is also as to satisfy certain appropriate boundary condition. So, all these things as to be part of the formulation itself, and if something is missing, then obviously not a optimal solution actually, and personally speaking I mean in a, in a, personal sense, I think satisfaction of these constraints have the biggest advantage of this, this, optimal control theory, and as a byproduct, we also optimize certain cost function and all that. So, that is, that is a my opinion actually.

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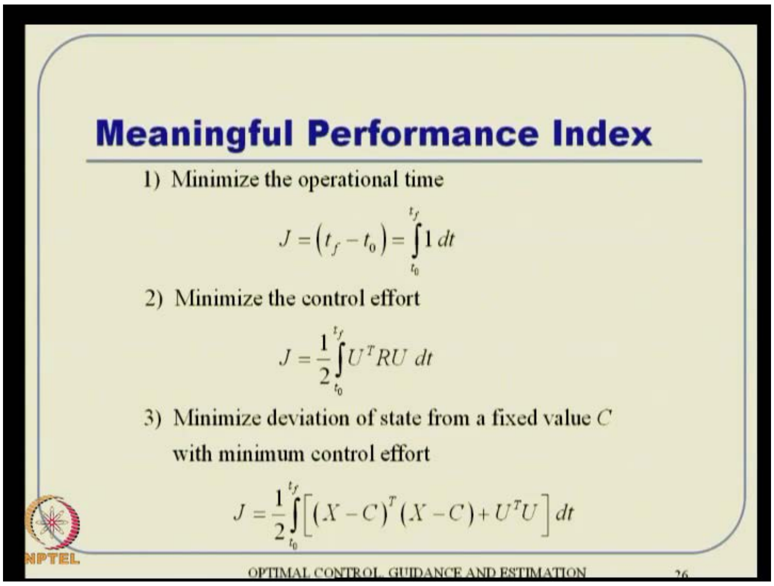


So, there are certain some examples of meaningful index in this, in this, frame work. You can think about some minimum time problem. Then it is nothing but minimizing t f minus t 0. So, I can, in, in, this platform, I can think about putting this that one I can take l equal to 1 and 5 equal to 0 and I can, I can, talk about a minimum time problem and minimum control effort problem can we thought about something like U transpose R U and U transpose R U turns out to be something like R 1 times U 1 square plus R 2 times U2 square think like that. As eventually, if you minimize that, it will lead to everything being 0 or close to 0. That is where you get minimum control effort.

Also if you think about like let us say minimum deviation of a fixed value, I mean states should being having minimum deviation fix value and also we want a minimum control effort. You can think about this kind of problems something like helicopter hovering problem for example. Then you can think about this is a, this x is a position x y z and then this should be a specific location c and that is what you want to minimize and this a quadratic term everything, I mean this is something like x minus c is an error quantity sort of thing.

So, you can take of that term, and then along with that minimum control effort can also be there, and also you can give relative weights here which I have not given, but if you, if you think about that, you can give some, some, relative weight some something like U here and something like R here, and then, there are some requirements that give us Q positive and R actually. So, these kinds of things are also possible where you can talk about relative weight as been assigned to different terms actually.

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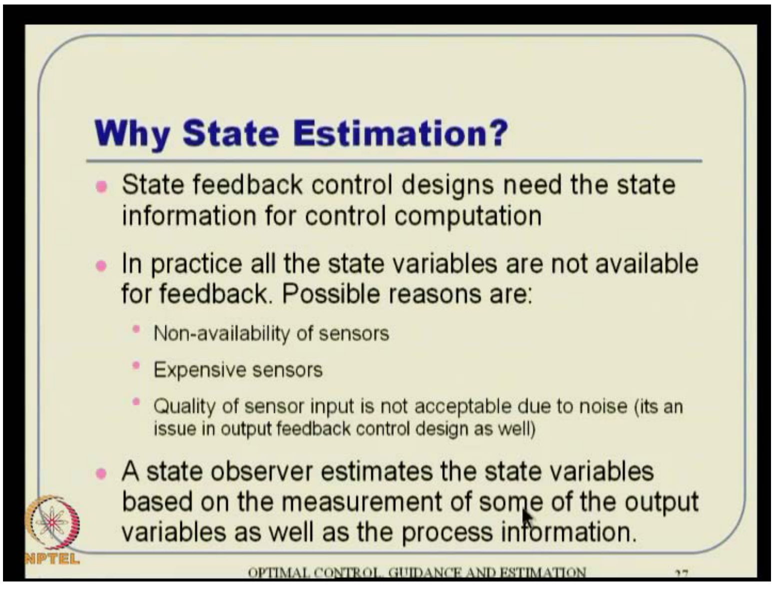

So, those things are, are, possible here. Then this, why state estimation so also remember there is a part of the course talks about state estimation, and there are such a needs actually. First of all the state feedback control design need some sort of a state information for, for, control computation; you, you, need state variable for that and your sensors may not give that.

So, you need some sort of estimation taking their, and why it is not, I mean not their start. In other words, all state variables are not available. There are, that, that can be various reasons. Some possible reasons can be some something like non availability of sensor or may be the sensor is available, but it is a, but it is a expensive, and then, it is also not a possible, and then, sometime the quality of sensors may not be acceptable due to noise, that also is a concerned. So, any of these issues can tell you I will not able to use all sort of sensors to get my all state variables.

So then, we go to this observer theory and the state observer essentially estimates the state variable based on the measurement of some of the output variables as well as the process information. That is critical actually. Not just dependent on the output variable only which is actually sensor output. So, we have given to us from sensor values, but it will also account for something like system dynamics. So, that is where the process information comes into picture.

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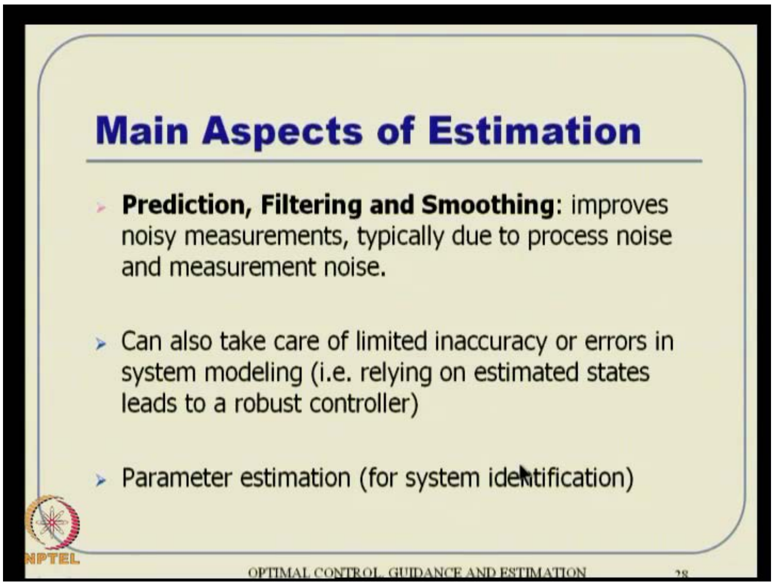

So, accounting for all that, it will try to predict which is which the prediction becomes much more kind of robust actually. So, what is the main aspect of estimation? There are, there are different things - one is prediction; you have to predict in the future. Then filtering current value whatever it is suppose to be we will find it out. That is called filtering, and then smoothing; that means, if the process is over and you go back and see, what, what, should have happened and think like that. That process is called smoothing actually. So, the, so, the three aspects of estimation is prediction, filtering as well as smoothing itself.

It can also take care of limited inaccuracy of system modeling in other words because you are typically relying on estimated states. So, this sensor information is going to help you even if your model is inaccurate actually. So, that kind of thing you see a little bit robustness actually, and this estimation can also be used for parameter estimation techniques or it essentially useful for system identification as well.

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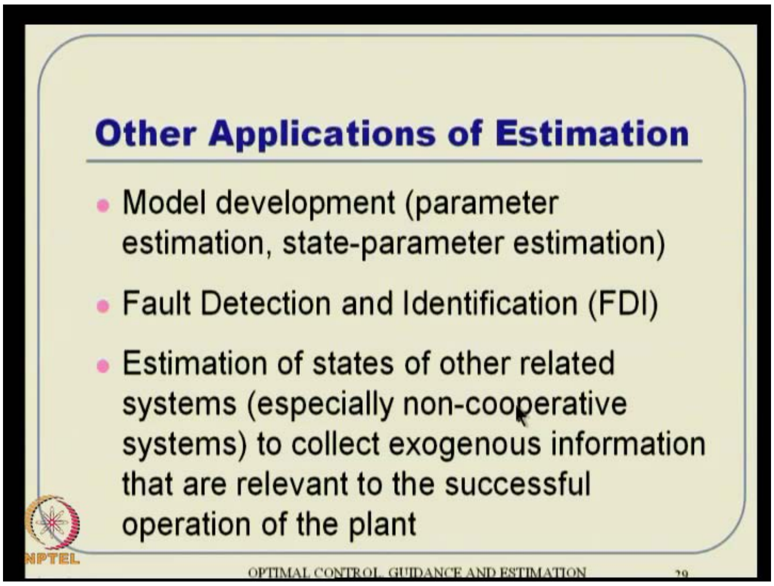


So, you can think of some other applications of estimation, which is first thing is you can think about something like model development which I told you can estimate the parameters and that is part of the model development, and you can also talk about state parameter estimation combine actually. So, that that can be (( )).

And people are using that actually in fact. So, estimation take means especially filter and all that are extremely useful for both state estimations, parameter estimations, parameter state estimation combine, and if there is other application, use number of other applications as well. Once you go along, we will see that. It can also talk about something like fault detection and identification. There that is part of this reconfigurable control if we think about control theory point of view, but essentially if you simply want to have information where the system is going fault, I mean where is the fault lie, where does the fault lies and when it happens and think like that.

So, that can be falling under this, this, FDI, that is, fault detection and identification. So, one branch of reconfigurable control leads to this, I mean realize on this FDS concept; that means I will detect where the fault lies and I will identify their properly and then there is a revised plant model for that, and then using that model, I will resynthesis the control actually. So, that will go that way actually. So, that is possible to do this fault detect and identification using estimation conception.

Then, there are estimation of states and other related systems especially non cooperative systems and exogenous inputs and things like that can also be there. You can very it is related to aerospace problems especially for let us say missile guidance actually. So, you need some target information, but target is not going to declare its position or velocity or intension too many were and think like that. So, this is something that you have to estimate as part of your sensor measurement and all that, and that is an integral part of requirement.

So, target state, without target state estimation, initial guidance probably as a very little significance actually. So, that is where the, you need these estimation concepts to make your system over all system work I mean as per your requirement actually.

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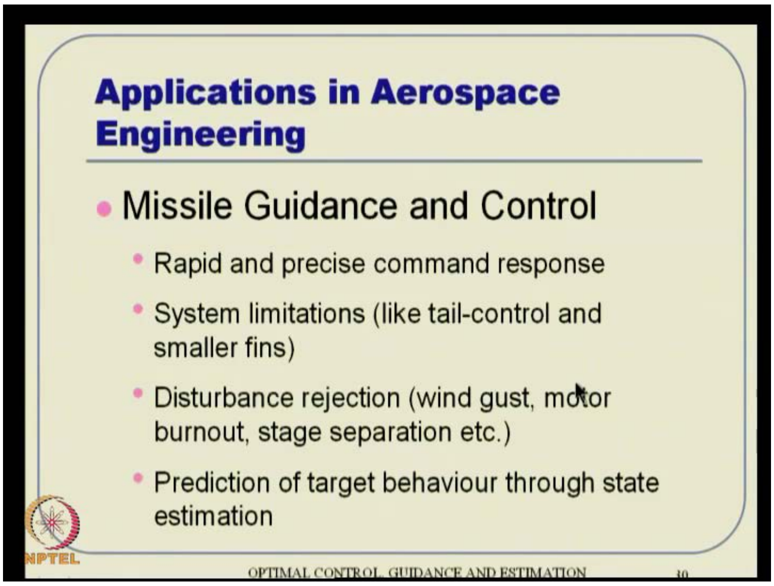


So, there are applications as I told. First application in guidance and control of missiles, and essentially I am talking about applications of optimal control and estimation some both actually. So, first thing in missile guidance and control, we required a several reasons well. The first thing we require is rapid and precise command response and it should also take care of system limitations. For example, the missiles do not have large fins for small lot of signatures and all that, and they can, they can be tail controller; that means, it leads to the something called non minimum phase systems and all. So, those are the systems limitations that you have which needs to be accounted for and it can optimal control can account for each other.

And then there concepts called disturbance rejection for example, wind gust motor burnout state separation things like that will lead to this system dynamics getting disturb as it operate. It has to tolerate all this actually. Then the prediction of target behaviors through state estimation. I have already explain about that actually.

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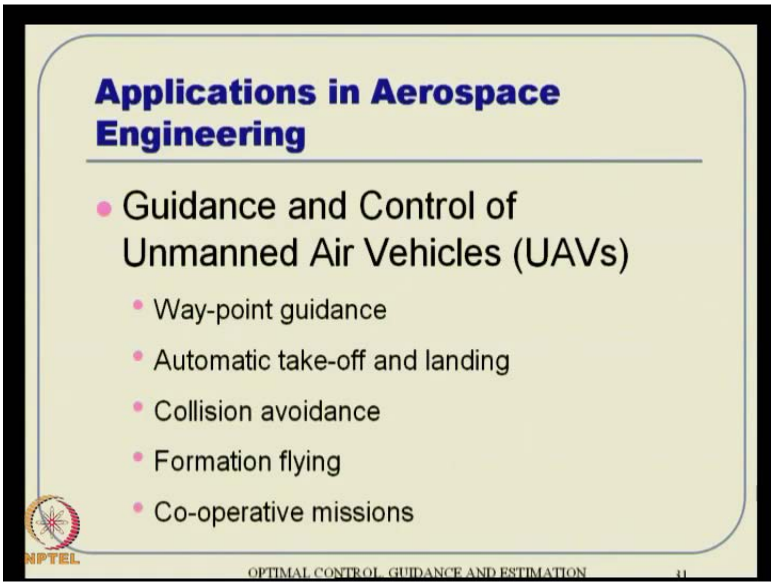


Then moving on in aerospace domain itself, you can take some of these concepts and then talk about lot of these UAV application these days, and first of all, UAV autonomous level you can think about way point guidance where you can define this way point 1, 2, 3, 4. The vehicle has to move around and come back and like that. Those kind of autonomy can be brought in, and then, there are automatic take off and landing algorithms where that has to be autonomously done.

Then there are collision avoidance problems, so where it flies and tries to avoid collision either for known obstacle or unknowns obstacles on the way. So, those kinds of problems can be handled using optimal control theory and you have this formation flying issues so that one. So, numbers of equals have to fly information or numbers of robots as to go in coordination and think like that. So, those can be handle.

And then cooperative missions where large number of different small systems said to operate cooperatively those kind of with and without leader formulation and thing like that are available. So, those are those can be handle as well.

Then there are aircraft flight control issues as well. They can think about something like a fly wire system. There, there, is a something like with a open loop system is unstable where there is part of the feedback which is automatically done irrespective to input pilot and all that, where that, where that will make the system stable. That is something called stability augmentation. Then you have this maneuver enhancement things are also possible using feedback theory, that kind of thing, and especially this control and all stability augmentation, you can think of using LKR theory and think like that. That is actually done in that way also LQG theory essentially which is LQR theory plus like that.

Now, there are consist for load alleviation and structural mode control and then flutter margin augmentation things like that. So, enormous number of problems, enormous challenging problems rather in aerospace engineering and in other engineering as well actually. I am not kind of confining only to aerospace, but you can think about many other problems, like vibration control problem flow control and all that. It is equally well it for mechanical civil engineering problems. You can also you think a power system optimization an electrical engineering problem. There are MEMS problem, there are nano system problem. Think like that, which, which, are very amenable for, for, optimal control application including probably by medical engineering as well; people have actually use that for solving cancer treatment and things like that.

So, this is all I want to talk in this particular lecture. So, there are enough motivations for us to, to, go ahead and look all details of optimal control theory in general. Before moving further, we will talk some or this review of some or the other concepts, so that we will be feeling comfortable itself. This is the part of this lecture. I thank you very much. Thank a lot. Bye.