Advanced Control System Design
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Lecture No. # 08
Overview of Flight Dynamics - II

Hello everyone, let us continue our discussion on flight dynamics which is relevant for flight control design, and which we will use it subsequently in this course especially. So, last lecture we have derived point mass equations in various assumptions, and followed by a 6 degree of freedom equations, where we have derived the dynamic part of the thing. And then we will continue further on this, this particular lecture and try to finish it off and try to give a complete overview of this dynamics actually.

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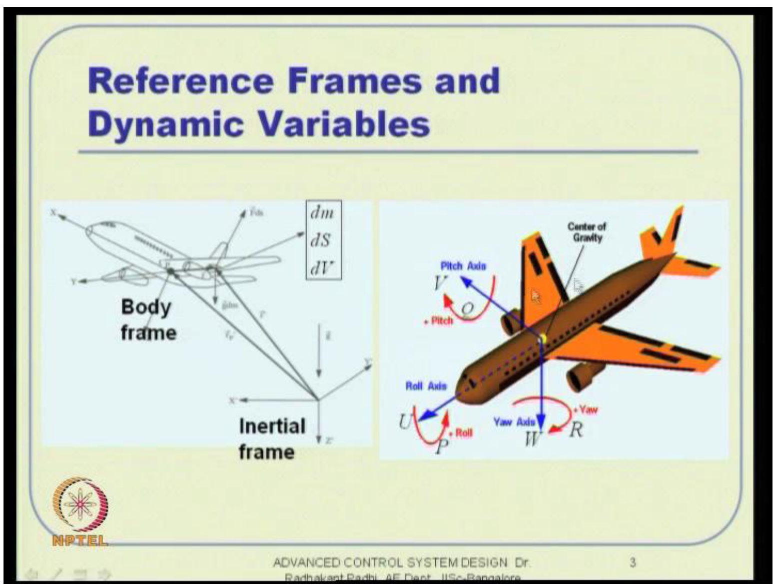

So, this 6 degree freedom model what we discussed last class is we essentially, try to find a relationship in 2 coordinate systems - one is inertial frame which is I mean you can visualize that as fixed on the surface of or somewhere, and then there is a body frame, which is attached to the body. And then about the body frame, we discussed about several variable which you call as dynamic variables, which essentially consist of U, V, W and P, Q, R; U V U V W are velocity components along body X body Y and body Z. And then around those axis, there are some rotation rates which are called roll rate, pitch rate, and yaw rate, we have discussed that in detail before as well actually.

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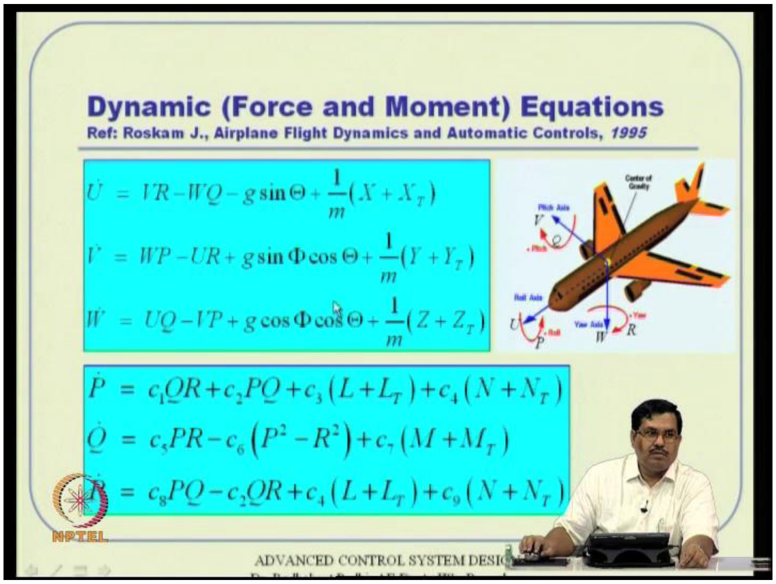

And then we went out and derived this set of equations, which essentially consists of a translational oscillations and rotational oscillations terms actually, and this consist of partly this carioles component and partly from gravity and partly due to external forces and similarly, you have this equations of for rotational rates and all that actually. So, we have done that in detail in previous class.

Now if you see that there are some quantities theta phi especially, here which also vary with time because that it represents the attitude of the vehicle? So, they are not fixed so they keep on varying with time. So, we certainly want to have a relationship, I mean rate of change of these variables as well. In addition the to that we also require this aerodynamic components here this X Y Z and the L M N which are also functions of height, because they are functions of dynamic pressure.

And dynamic pressure consists of density of atmosphere which varies with height. So we also have we need to have rate of change of height actually and especially if you want to have a trajectory of the aircraft where it is going what it is doing things like that if you want to visualize from some inertial frame, then we certainly need to have X Y coordinates in the inertial frame as well, so inertial X Y Z the Z the negative Z part of it will essentially give as the height part of it.

I mean the height of the aircraft at any point of time and X Y Z together will define this coordinate of the C G of the vehicle which will essentially give us flight further at trajectory. So, we need to have this phi theta along with that we also need something called heading angle size so, we are essentially we want to derive phi dot theta dot psi dot as well as X dot Y dot Z dot in the inertial frame actually.

So, let us go ahead and try to do that those are called kinematic equations essentially. They do not rely on the external forces and moments, but they are actually velocity level equations what we saw here is oscillation level equations.

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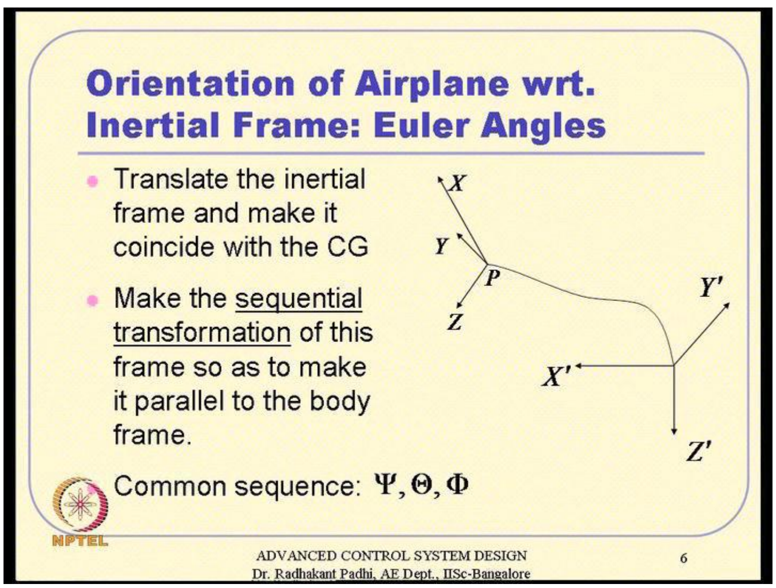

Now let us continue with our equal I mean derivation. So, here we need the concept of something called Euler angles and later we will see that this is not the only parameterization, we can also have various other parameterization like direction cosine and quaternion things like that.

Well let us complete the derivation with l o angles actually. So, what you really want here there is an inertial frame here, and there is a body frame out here so, we want to know the orientation of this body frame X Y Z with respect to this inertial X Y Z actually. That is our objective here. So, what you really want to do let us say first we want to take this and this coordinate system just translate it over there then try to rotate some this to this coordinate system one at a time.

First with respect to Z X then with respect to Y then with respect to X and things like that ultimately our aim is to this translated coordinate systems should co-inside with this body axis frame actually. Then whatever angles we required those angles are nothing but these angles psi theta phi actually.

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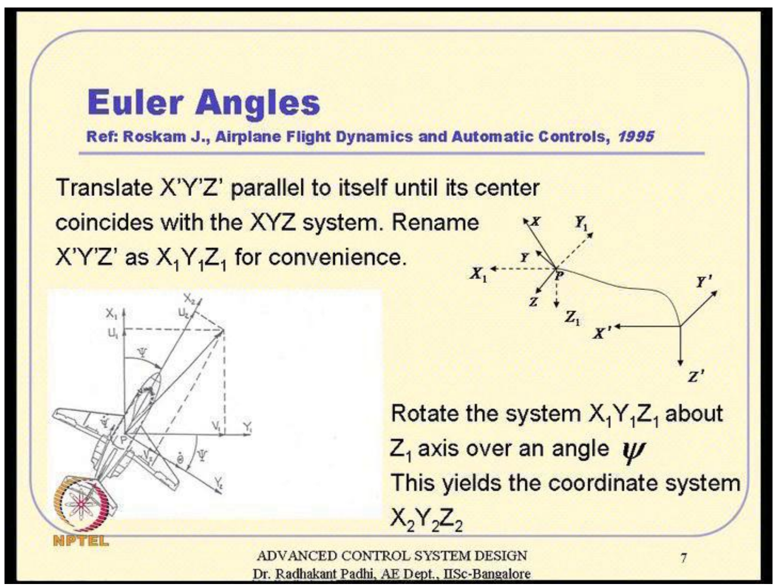

So, let us try to understand a little better, so what you first you first what you are interested is taking this coordinate system over there simply translate that and we have to have a some different notation there just not to get confused with that.

So, this X dash this X prime Y prime Z prime is rewritten in terms of X1 Y 1 Z 1 this is simply just translated over there nothing more actually. Now this X 1 Y 1 Z 1 I can rotate it with respect to the body X axis with respect to the body Z axis so. that this entire X Y frame whatever X Y frame was there previously, it will try to coincide with this X Y actually. So, this X 1 Y 1 what you what you had by translating this one this inertial coordinate over there.

Now we will rotate it by angle psi with respect to the body Z axis, Z axis is your probably middle finger in the left hand side actually. So, this Z axis you have to rotate it the Z axis you keep it fixed and rotate it by angle psi and that is what the picture says actually. So, that is that is what you will do.

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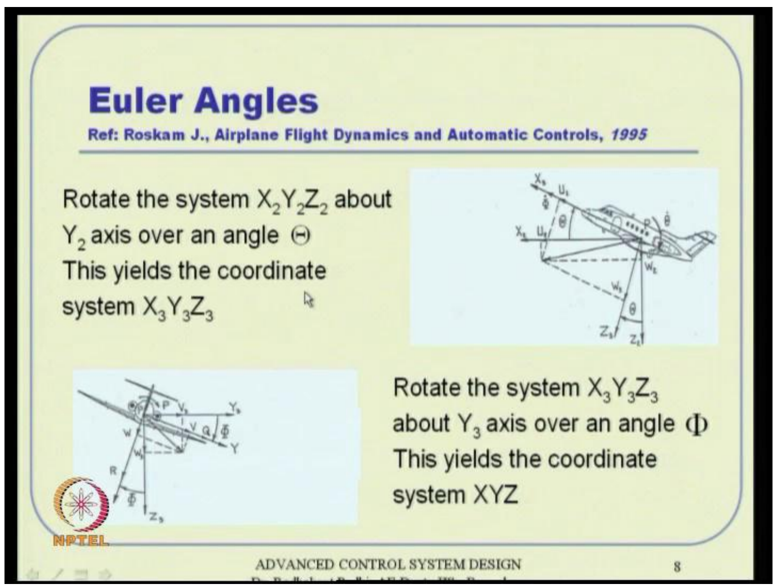

And the next what you will do is this rotated frame whatever is this you rotate it by angle theta about the Y axis. So, first with respect with Z axis then with respect to Y axis and ultimately with respect to X axis phi that is what you will do here.

So, the sequential rotation are, so as the psi theta and phi and this sequential rotation is the translated inertial coordinate system which gets rotated about its axis actually, so that ultimately it coordinates with the body frame. So, this is just this is just a concept actually now let us try to see the relationship between them and anytime you know the essentially what you are doing.

You are actually doing it one angle rotation of a 2 dimensional axis frame at 1 point of time there one about one axis you are rotating so, that particular axis is remains fixed and then the remaining coordinate system moves in one angle in 2 d actually.

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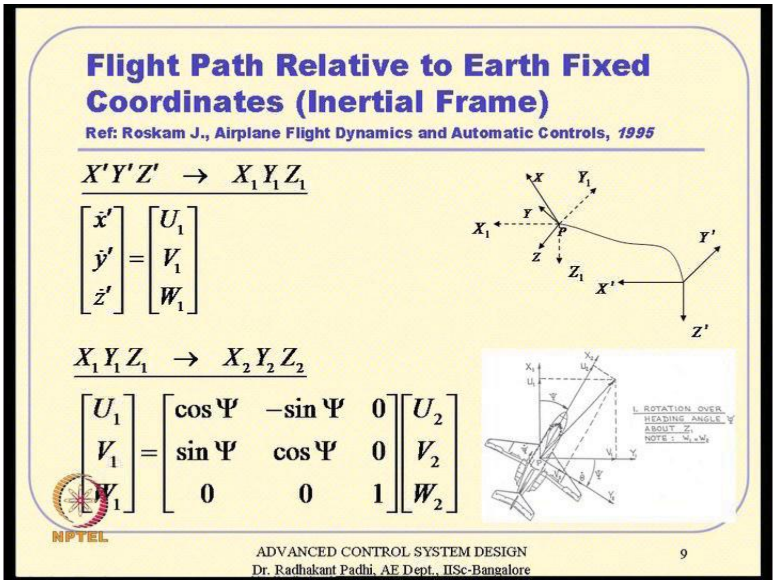

So, let us let us try to see what is going on here so, first we are doing that this coordinate frame we just translate there so, essentially this whatever doubts you have in the Inertial Frame you are nothing but U 1 V 1 and W 1.

This is a simply translation, so the velocity quantities do not change actually, but after that there is a rotation and it is a 2 d rotation what you want to visualize that in 3 d actually. So, W 1 this particular picture if you see that is the angle psi actually. So, this angle psi happens to take this U 2 V 2 to U 1 V 1 that means whatever U 1 V 1 is there suppose, you already have U 2 V 2 then what is U 1 V 1 actually.

That relationship comes through this rotational 2 d rotational matrix we know probably. So, and if you do not know you can see some coordinate geometry book or something there which is very standard. So, there are some neat properties out there this for example, the determinant of this matrix is always one, so this matrix is never singular length and I think like that actually so, this is actually an orthonormal matrix also then there are various recognized properties also associated with that.

But the fact is this suppose I have U 2 V 2 after that rotation, then U 1 V 1 is U related by that that particular U V matrix here with respect to U 2 V 2 and number 1 is equal to W 2 so this becomes 0 0 and that becomes 0 0. So, you have 1 actually. So, this U 1 V 1 W 1 is related to U 2 W 2 that way

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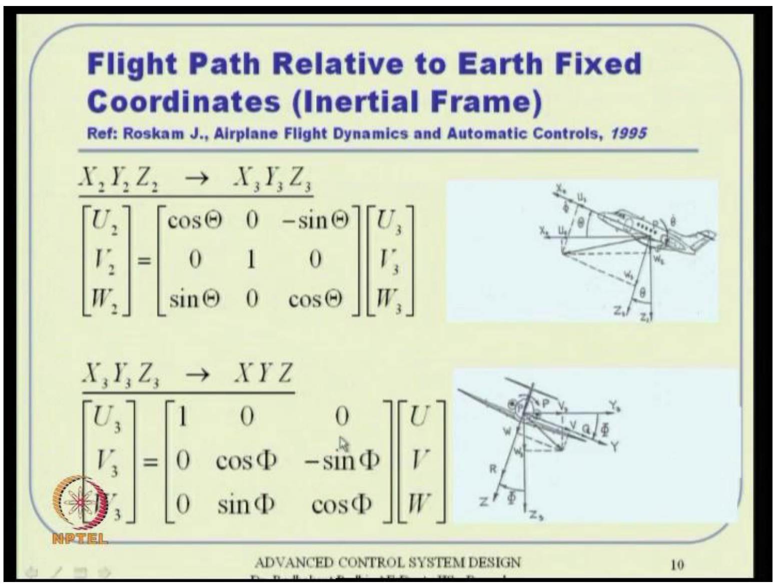

Then we have this next transformation by angle theta. So, this angle theta is with respect to U 3 and W 3 remember that, because Y axis remains fixed here, so V 2 is equal to V 3 but the other variables U 2 V 2 W 2 is related to U 3 V 3 W 3, because of that actually because of angle this rotation angle theta and similarly, this X 3 Y 3 Z 3 goes to X Y Z by angle phi ultimately. Now this is a rotation about X axis actually the transform X axis.

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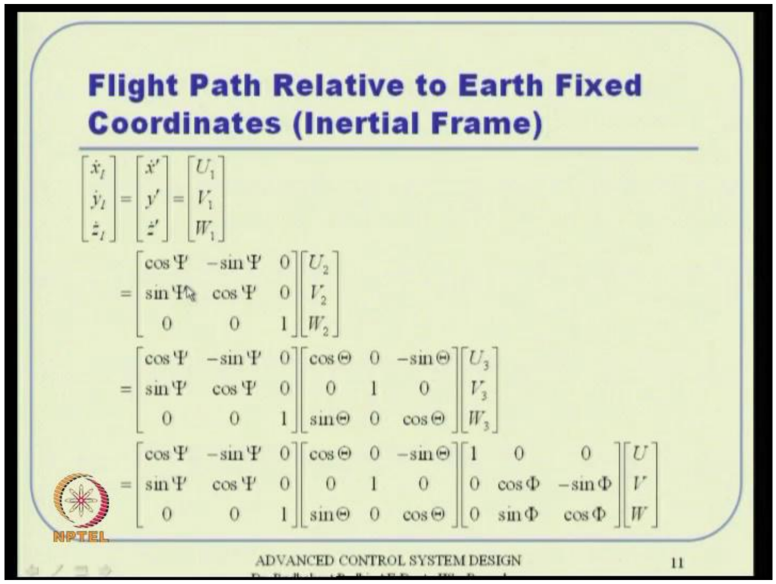

So, all these rotations sequence of rotations once you try to combine them what you combine. Let us say you want to have inertial position rate of change of position that means velocities which is nothing but that, that is the definition you know because the definition that you are talking about the coordinate frame is x frame y frame z frames. So, I mean normally I prefer to write it as x I dot y I dot z I dot that is the end actually. That is related that is same as this and this is nothing but same as this, because of translation right that is how we started with this is just translation here.

But this translation is now given as the back one that is just a self one in the first translation, then this translation what you see here is given by that that is a second translation. I mean second rotation thing this is translation this is first rotation V of psi this is the second rotation and this is third rotation. So, ultimately, if you see the rate of change of position in your inertial frame is related to phi theta psi all angles actually with respect to U V W.

So, if you know U V W components then x I dot y I dot z I dot is given by that actually. This entire sequence of rotation and then you can simplify that; you can multiply all this 3 matrices and then find out one vector equation at a time actually. So, that is just that is about this position rate of change of position universal form. So, for is this relationship then I will be able to plot the vehicle trajectory actually and the inertial frame.

Now obviously, this is function of I theta sin and then we also know that this is also function of I mean this V dot U dot W dot is also function of theta and phi. So, obviously we want to know this phi dot theta dot psi dot as well actually

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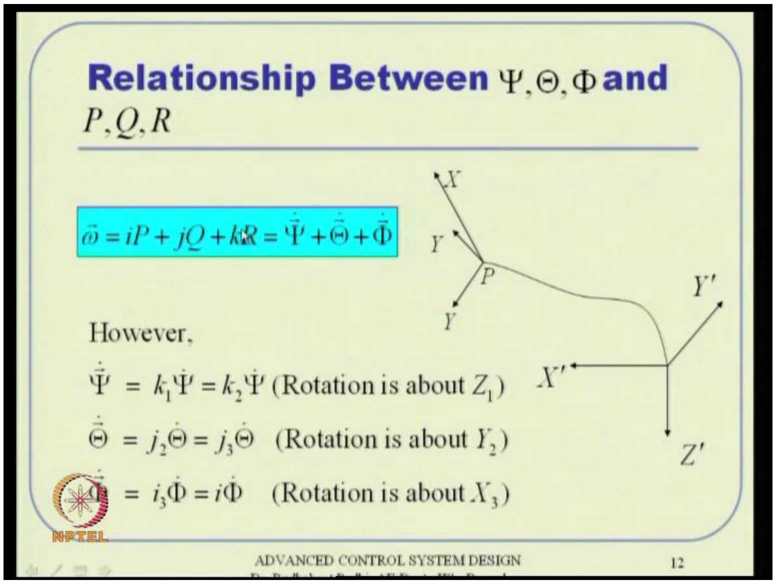

So, how do they vary actually? Now obviously they are related to in this body related rotations P Q R. So, if phi theta psi is what this inertial coordinate frame rotation in a sequential way.

And P Q R is nothing but the body rate of rotation about the body axis of the vehicle itself so in vectorial sense, you can tell i j k is the body frame unit vectors. So, this W bar is i times T plus j time Q plus k times R which is also equal to vectorially speaking psi dot theta dot and phi dot together now this is the same vector they are 2 different this one coordinate system actually this is written in terms of body axis and this is written in terms of inertial frame actually sequential operations essentially.

So, vectorially speaking these 2 are equal. Now you have to try to find out what I will what I am placing there vectorially speaking psi dot is nothing, but k 1 times psi dot. So, remember whatever we did here this X 1 Y 1 Z 1 stands for k 1 in i 1 j 1 k 1 X 2 Y 2 Z 2 stands for i 2 j

2 k 2 like that actually. So, in that sense, if you can see that the psi dot essentially k 1 time psi dot, because that is the first operation what you do actually.

And essentially k 1 and k 2 remains same because that is rotation about Z 1 axis k component will remain same i j k stands for X Y Z. So, k stands for Z component and this rotation about Z axis so k component remains same actually so I can agree that way similarly, theta dot is essentially j 2 because that is a second operation you know that so the because it is a second operation this theta bar I mean the theta dot is, nothing but j 2 theta dot which is also equal to j 3 theta dot.

Ok that is rotation about Y 2 now and similarly, phi dot is like this actually. So, it is rotation about X 3 alright. So, this is how this vectorial notations can decompose to now we have to simplify this k 2 i j 3 i 3 all sort of things you have to put it in single framework and then try to kind of see the relationship between them actually.

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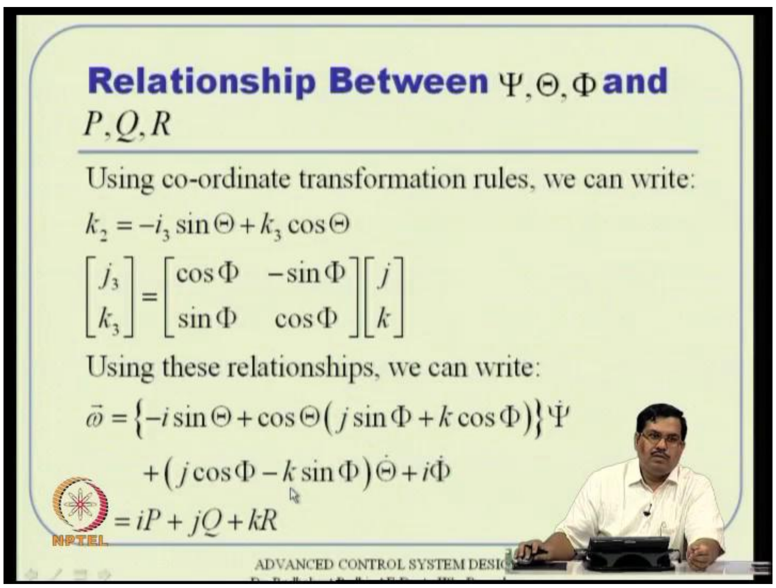

So, if you let us do that then what happens this k 2 you I can write it in something like this i 3 and k 3 sort of thing.

Essentially, you can write it into something like i 2 and k 2 as a function of i 3 and k 3 the way I have written it here the below so, this is what is written as j 3 and k 3 are related to j and k like this, remember j k stands for body axis and j 3; k 3 stands for just stands before that last rotation is phi. So this relationship is like that similarly, you can write it in terms of theta also.

And that means i 2 and k 2 with theta theta is between X and j remember that, so i 2 and k 2 will be from a similar relation will crop up for i 3 and k 3. So I need only the component of that I need only the k 2 component actually. So I just have taken out that one now j 3 and k 3 are given like that so, using this relationship this entire thing whatever w you have, so this vectorial dots I can represent this like this actually this is like k 2 times I dot plus j 3 times theta dot plus plus i 3 times phi dot, if I would not do that.

And then I kind of simplify all these, because k 2 is like this j 3 k 3 are like that. So, I put them together and then that is ultimately equal to I time speed plus j time Q plus k times R. Now that is where this we started with actually. So, as I started with all these and then try to write all in terms of body coordinate frame this phi dot theta dot psi dot and that is equal to this I term speed plus j term Q plus data k R.

So, now I can equate this component by component that these three are orthogonal quality i j k R orthogonal to each other. So, I can so I can equate these quantities component by component actually.

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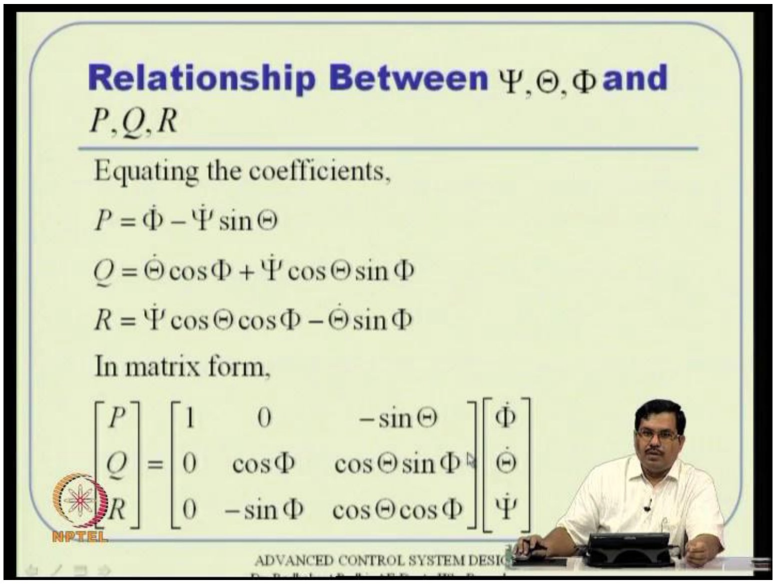

So, let us do that and then we will have this relationship, so if you try to use this component by component and then this relationship will pop up actually. So, vectorially speaking I can write it that way, but normally what we do not P Q R is a function of I dot theta dot psi dot we want the reverse one we want phi dot theta dot psi dot as a function of P Q R, that that is the dynamics equation that is what we want to integrate later. So, what you do I can take inverse transformation here etcetera?

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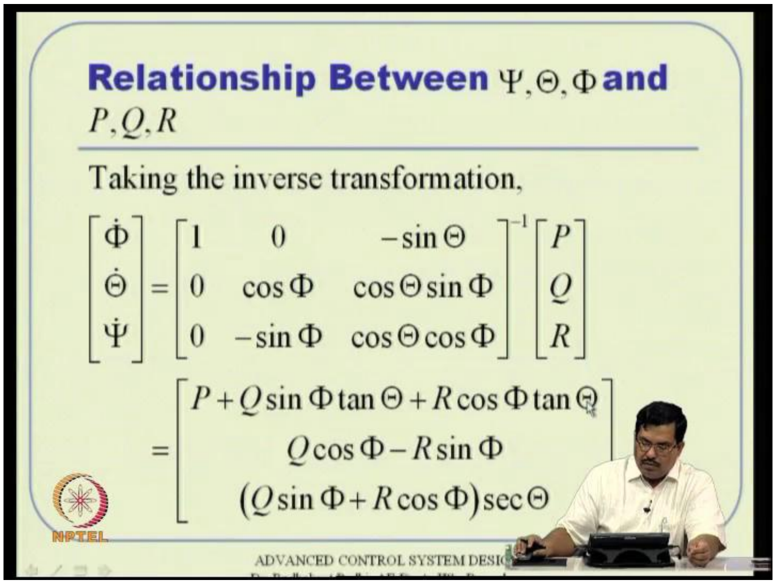

So, this inverse transformation I can carry out and if you sit down with this inverse calculation there is one by determinant times object matrix and then you simplify all sort of things then it will keep up like this actually. So, what you have ultimately we have this we have this A X I dot Y I dot Z I dot which will give us the trajectory part of it and associated with that we have this phi dot theta dot psi dot also.

Everything need to be integrated together because we are all proper equations actually. So, and then this will not the end of the story we have this, I mean you also see that these components we last class we just derived from simple logic simple intuition. Now can we do it in a formal way actually? So, what part of this will go to U dot what part will go to W dot and things like that?

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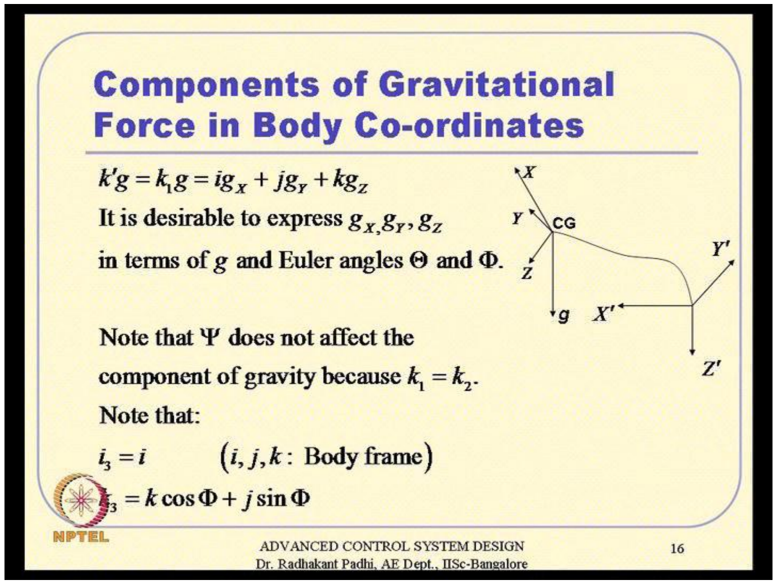

So, if you let us try to quickly do that and this essentially, the same way remember gravity is perpendicular to the I mean parallel to the initial Z axis all the time, no matter whatever is the vehicle attitude gravity is always vertical with respect to surface of attitude only. So that means it is always parallel to the Z axis it is always pointed to the center actually earth center.

So, that way this particular vector now this may not be oriented to with respect to body axis actually body X Y Z. However, look at this is orthogonal frame which points is entire 3 d space. So, the g vector any particular vector and particularly this gravity vector I can decompose that into X Y Z components and that is what I am doing actually. So, k 1 time g I am decomposing that in terms of i j k actually here so, g X g Y g Z stands for the component of the gravity in body X body Y and body Z component body Z coordinate frame actually.

So, what part goes away that is that is the question actually and also note that the psi does not affect the component of gravity because k 1 is equal to k 2 so, anytime the vehicle takes a rotation parallel to the surface of earth that means some psi rotation actually, then the gravity is still pointed vertical actually that means the Z vector whatever, it happens it does not affect it.

The gravity in other words the rotation of psi does not affect the gravity components at all actually. Essentially under the part of the assumption I mean, if the earth is flat, so if it takes some psi rotation somewhere, then it does not affect anything about the body X Y Z actually. So, we will not worry so much about that what we really need is the pi and theta components actually. So, they are essential functions of phi and theta. Because in the moment there is a theta component then I mean, the pitch angle theta essentially sincerely we have and phi angle is also roll angle actually.

So, pitching and rolling essentially I mean they will let the components of gravity different in terms of X Y Z, but yawing does not normally affect it actually under the part of assumption. So, what you see here your i 3 is again I ultimately your body axis that is the final rotation and then k 3 is equal to this k times Cos by Cos a times sin phi.

So, and k 2 is the other one so, k 3 and k 2 is like this is a function of theta obviously right and theta is related to I and k, so phi is related to k and 0 like that

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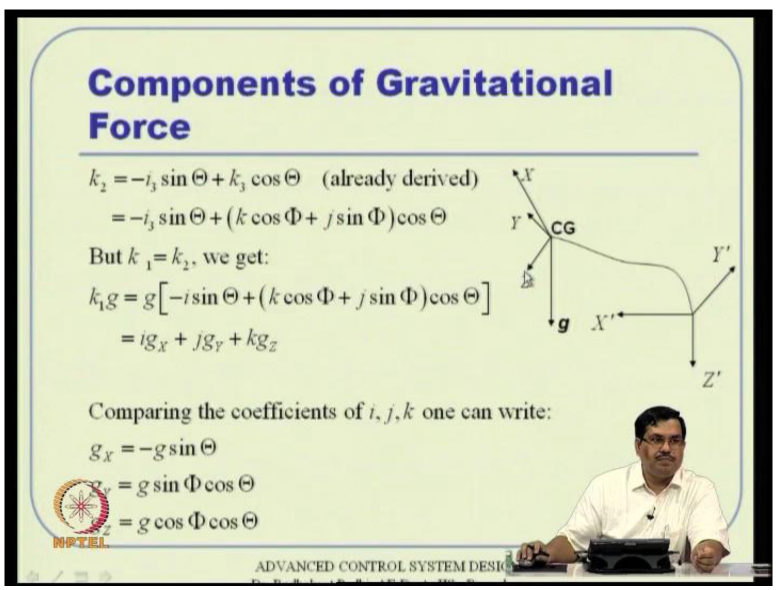

So, if you do it with proper book keeping, then k 2 turns out to be something like that. However k 1 is equal to k 2, so that means you can substitute this relationship, and then we see that k 1 g is essentially like that actually.

Whatever you see here i j k, whatever components consists i and j and k; these are these are body X Y Z components, and this is this is also body X Y Z component. So, if you equate the 2, you get these things actually. And last class we just derived it intuitively but this is much more formal way of deriving it actually using this Euler angle rotations actually.

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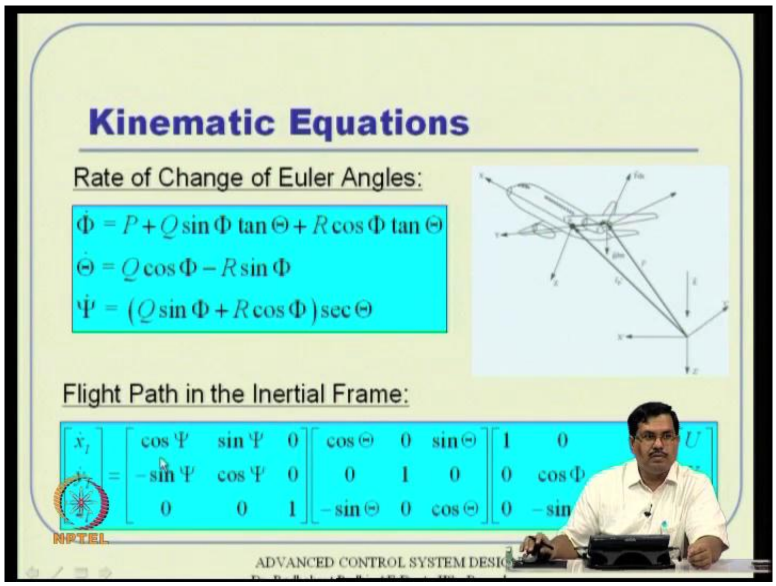

So, this is how the entire six of equation is made up, but before going there, so last previous lecture, we have seen the dynamic level equations, here we see that kinematic level equation. That means velocity level equations, these are angular velocity term and these are the translational velocity terms. So, this 6 equations couple with the previous 6 equation what we say that last class derived the complete set of 6 equations.

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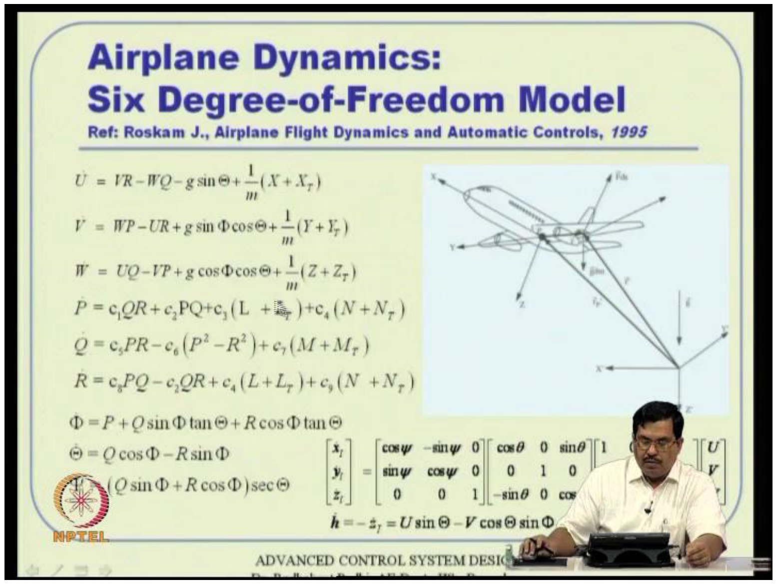

So, which is here, so you see this consist of U dot V dot W dot P dot Q dot R dot then finite theta dot psi dot and then X I dot Y I dot Z I dot. So, this 12 equations we will derive that and normally this height dot this s dot is actually negative of Z I dot that is, if you simply take out the last row and make a negative sign, then it will provide a and nothing but X axis.

So, that is not an independent equation so, what is what is relevant here, is only these 12 equations actually and fundamentally speaking this a 12 equation gives this first 6 equations or directly related to the body axis only and the last 6 equation or gives a relationship between inertial frame and body frame actually. Especially, this phi and theta root side and then this X Y dot X I dot Y I dot Z I dot gives us the trajectory of the vehicle in the s in the inertial coordinating frame.

So, this is how it is all coupled together sort of thing. Now this set of equation what you see here these are coriolis quantities these are gravity and these are force quantities actually.

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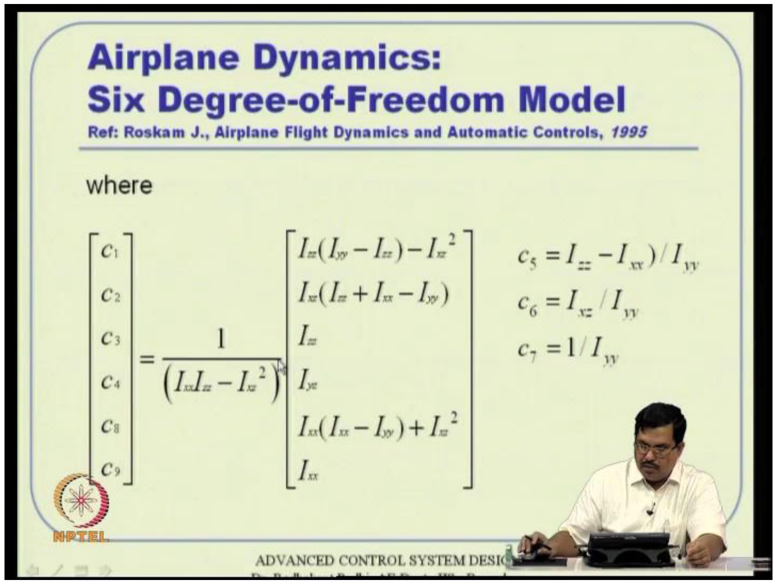

And then I mean if you go and see that this c 1 and c 2 all these things we saw that last class also, whatever c 1 c 2 c 3 I mean all this c with a suffix what you see here in P dot Q dot R equations are essentially functions of moment of inertia which is which is given like that actually.

We have derived that in the last class rest of the equations are kinematic equations. So, which are given here actually and then this whatever you see this components are actually. This external forces and moments are the only ones which are different from vehicle to vehicle with all rest of the things are same no matter whether a bird flies of course, bird has to be a rigid body which is not really true, but whether any flying object which is a rigid body which is dictated which is governed by all these atomic equations actually.

The only difference comes is because of this external forces and moments and that is why lot of studies goes into in the aerodynamic study actually. So, this will like aerodynamic component X Y Z L N M and X 2 Y 2 Z 2 N m T N T N T are nothing but thrust components so, thrust components are not very difficult to visualize it does not vary unless you have a thrust deflection mechanism which happens in missile and equations specially with some clouds we have thrust reflection mechanism also for vector maneuverability and all that actually.

But normally this X Y this commercial network sort of thing actually these are fairly known actually it all depends on engine power how much you do and what orientation you are fixed actually that that you actually.

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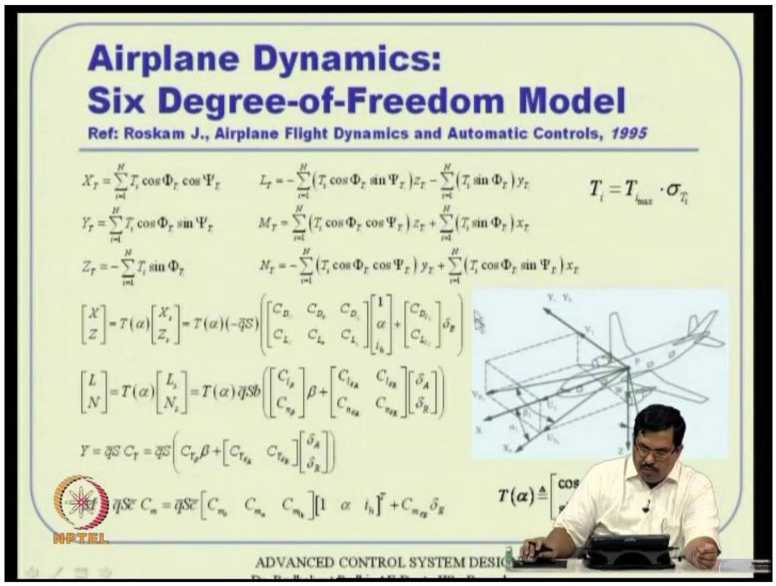

That that is where if this phi T and psi T R coming actually phi T and psi T are not Euler angles, but they are the angles engine orientation angles as fixed to the I mean as seen in the inertia in the body frame.

With respect to the body frame this engines are typically oriented by angle phi m psi for certain beneficial property is essential, I will not talk too much on that because one thing you can visualize is, if it contributes something on the vertical direction inertial vertical direction, then essentially it helps aiding to the lift to sustain the weight actually. So, you do not have really have to have a very big wing to sustain your lift.

If thrust has a small component let us say let us say some sign of 2 degree or 3 degree but thrust is a large quantity so, that quantity multiplied by sin phi degree also plays a big quantity which will aid to the lift actually anyway. So, like that there are various ways then they are talking about jet should not hit the tail wing so because the engine jet is there, so it should not hit the tail wing actually because it is a very unpredictable decision the circulation and all that.

So, you want to have something like this way so the engine just goes somewhere like that it does not hit the vertical thing actually. So,there are various ways with various reasons why this these are all done that way and then this aerodynamic forces and moments are typically generated to ways one is the 3 levels of doing that and 3 or 4 levels you can say aerodynamic forces and moments (Refer Slide Time: 19:40) first thing people do is just a first code guess actually, which comes from first principles that is some something to start with.

If you simply give the help of configuration there are some rules available basic relationship available from which I can get a basic idea of what the what is there, then it goes to the next level of what is called c f d actually. So, is computational speed dynamic, so where they talk about generating it in a grid point sense or for using finite element finite volume all sort of thing. So, it will give you little more better accurate sense.

Remember these are not the flow pattern over this aircraft body is not very much in tune with whatever assumptions we do in the first code model first code model is all in configuration there is nothing there I mean nothing objections are there, I mean the surface is fairly good and all sort of things so going to one level of higher the c f d thing will account for all this protrusions actually, and then all sort of wing configuration whether you have a fin out there all sort of things will be taken into account and try to give you better accuration there.

Then it goes to the next level when you people talk about internal study, so there is actually you I mean most of the time it is actually a model of this prototype I mean it is not a very big aircraft you cannot put in a internal. So, you do a small version of that and then try to a see instead of the vehicle flowing in the medium. Now medium is flown over the vehicle that means the air is flown over the vehicle essentially the relative velocity matters there. So, you can get fairly better accuracy ideas I mean better idea about this forces and moments.

But this internal study can only talk about generating what is called a static forces and moments actually, dynamic level forces and moments it cannot generate. Now so that means whichever the forces and moments components of these rates rotational rates for example, P Q R those components cannot be predicted in internal actually, so for that you need the flight test. So, it goes through several rounds of refinement and then by meanwhile if you design configuration itself goes through refinement again you do one more round of wherever you want you start either from c f d or internal operator test actually.

So, that is where this models will keep on getting updated then anything that is generated from flight test that is essentially falls in the bracket of what is called parameter identification that is where you want to identify the how this forces and moments actually act on the vehicle. So, it is a roughly speaking this aerodynamic interaction of this vehicle is not very well understood actually from physics point of view.

That is why we need to have several rounds of experimental study and then combine with a deterministic model to get a fairly accurate model that means this model what you are talking about you can visualize that. So, it is some sort of a hybrid model 1 is deterministic part there is a large component of that there is a fairly equally good component of that, which is aerodynamic independent which are somewhat theoretically understood and there are components, which are not understood, but then they are function fields from the experiments actually. So, that is a largely dotted even in the sense. So all these things are part of 6 of model actually.

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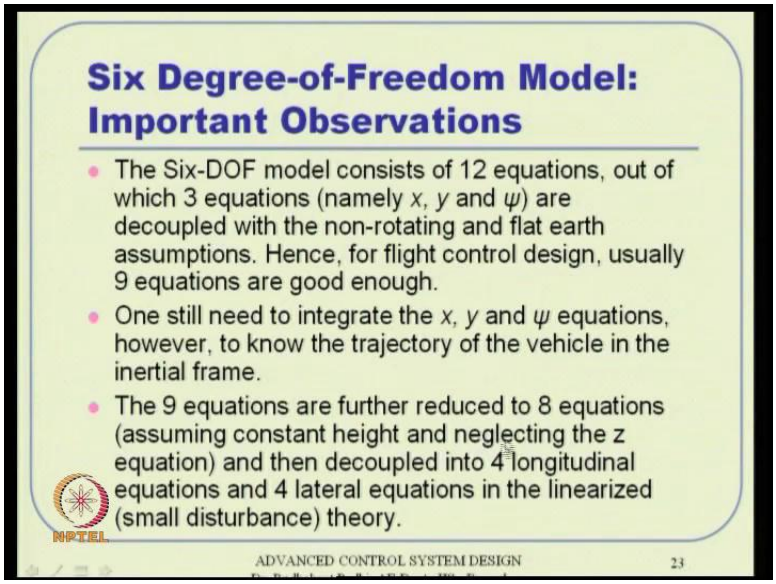

Now before you go ahead there are further comments. First thing is six of models consists of 12 equations out of which 3 equations are decoupled with the assumption that the earth is non rotating and flat like thing, like what I told also last class if you see all those equations essentially the psi actually, and then X Y do not come in any of the rest of the equations actually.

So, psi and X Y you can probably decouple that and tell even if there is actually 12 equations under the non rotating plot of assumption I can just work with 9 equation as far as local control design is concerned actually, so and control design is typically local it is a short duration stability problems and things like that, they are not large long duration a section of that so that is that is fairly ok. But also remember that 1 still needs to integrate this X Y and psi equation to get the trajectory of the vehicle in the inertial frame.

Suppose you really want that and trajectory information is essentially embedded in the guidance design actually. So if you really want to talk about proper guidance region that means you start take up from point 1 and go to point 2 and things like that you really need this trajectory information actually. So, there is still you need to integrate that and there are better models available under this spherical earth assumption and then the rotating earth assumption and things like that actually, so that is also there.

So, forgetting this factor whatever 9 equations are there this equation can be further reduce reduced to eight equations, because locally the height does not changes actually that means that height dot s dot. I do not I still can just consider a reference height and then Z dot equation I do not have to integrate actually. I know very what is my operational height which side I am talking about then I do not want to integrate this Z dot equation height remains constant along that actually.

So, then you write 9 equations you can write and out of that 8 equation, we normally decouple that into 4 longitudinal equations and 4 lateral equations these 2 we can visualize them as different actually, the we coupling that adjust between the 2 under this under that relation of approximation or we can neglect that actually whatever. So, this 4 equations are grouped together, and other 4 equations are grouped together. I will talk about that of course, and then you tell this can be that way actually.

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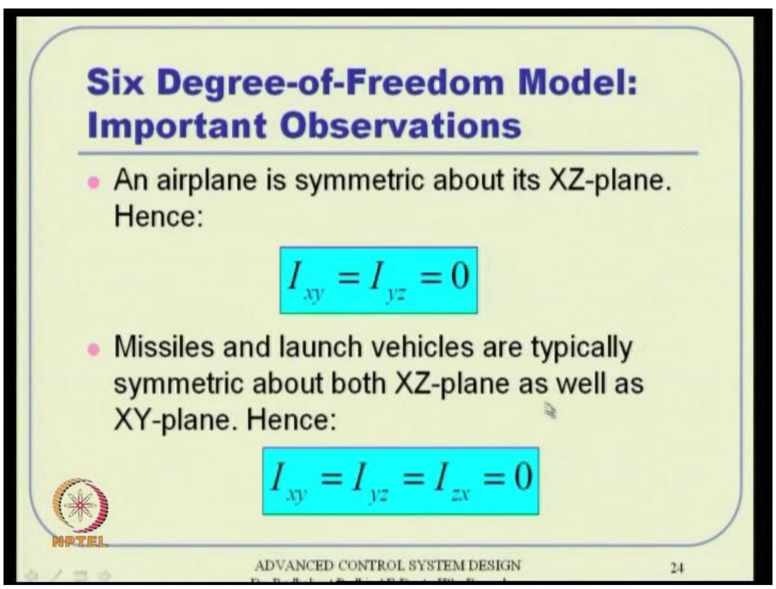

Then also remember, that airplane is symmetric its one of the assumptions. So, that is why this cross moment are 0 and missiles and launch vehicles are typical symmetric about both the both X Z plane and X Y plane and hence all the three cross moment of inertias are here.

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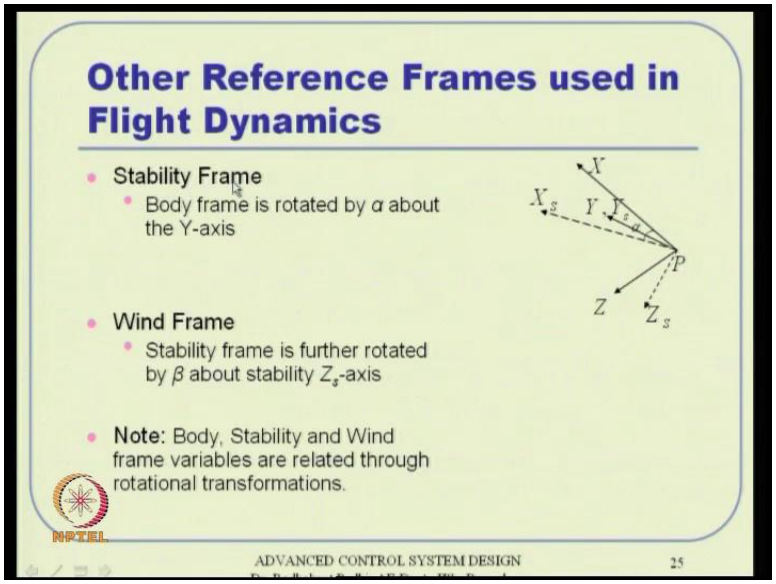

Then also remember, that this is not the end of the story we in flight dynamics people talk about various other frames one of them is wind, I mean stability frame the other one is wind frame. So, there are neat properties out of that especially, the let us say this wind frame the aerodynamic side the lifts and drag that typically acts on the wind frame the aerodynamic side lifts and drag that typically act on the wind frame some about this experiment sometimes the internal data will be given in terms of stability frame.

So, what is this the stability frame is nothing but like a body frame just rotate the body frame by angle about the Y axis, Y axis remains same and then you rotate whatever X Y X Z axis, by angle alpha that will give you the stability frame and wind frame is that stability frame you rotate further Y angle beta, what is sort of which is in its something called sideslip angle.

So, it will perfectly align the X vector of the wind frame will be perfectly align to the to the extreme velocity actually. So, that is how you get the wind frame actually, so details I will not talk about, but again these are all rotations only. So, rotation things are again these relationship will be reflected by this rotational matrix through this rotation matrix that Cos alpha Cos alpha and the diagonal and then minus sin alpha and Cos sin alpha in the orthogonal elements actually.

So, these are details and all you can find out in flight dynamics books and 1 of the books is probably Stevens and Lewis with a aircraft control and all that aircraft simulation and control. All right, so let us move further and then we talk about what I discovered I just pointed out can we discuss something called small disturbance flight dynamics that means, local effects you do not talk about the non-linear effect or everything coupled or thing like that so, can we do some sort of a decoupled motion longitudinal lateral and thing like that with a small disturbance something actually. And many of this material I will take it from this particular book which is also, I think very well written for these concepts.

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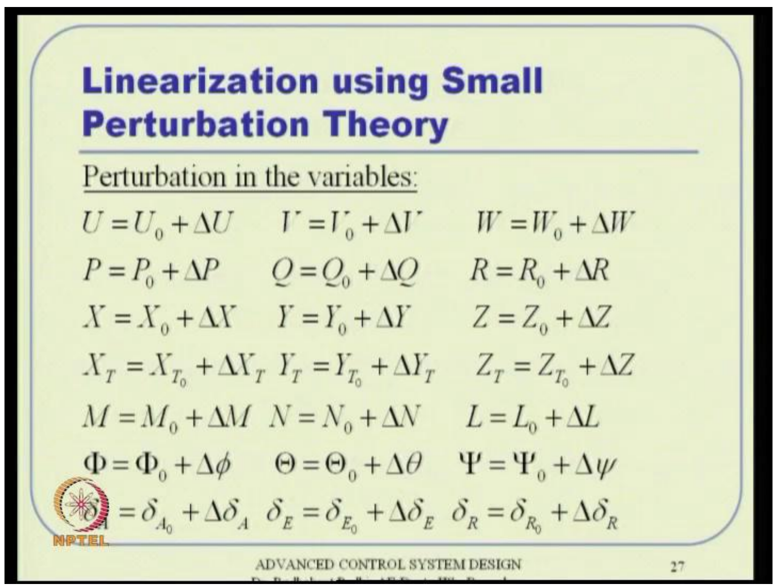

So, what I am doing roughly what you are doing is all the variables whatever you had in the 6 of dynamics, you interpret that as perturbations around some nominal values that for example, U 0, V 0, W 0 all sort of thing these are all nominal values around which there are perturbations actually delta V delta W I mean all sort of things.

And the assumption is that these perturbations are small actually. So, what you do in the lineal equation I think we will not discussed it, but one4 of the further classes we will discuss about that formally. So, in the Taylor series expansion you neglect the higher order terms of these quantities actually you can you have the 6 of six degree of freedom equation that is the non-linear coupled equation.

So, you want to linearize this around some nominal operating condition actually. So, this variables, which exclude 0 are operating conditions and then this perturbation quantities, you assume them as small and try and put that in the Taylor series expansion and then neglect the another terms actually. Then you can do you can do it in a different way also and you can see the book especially is done slightly in a different way as I suppose.

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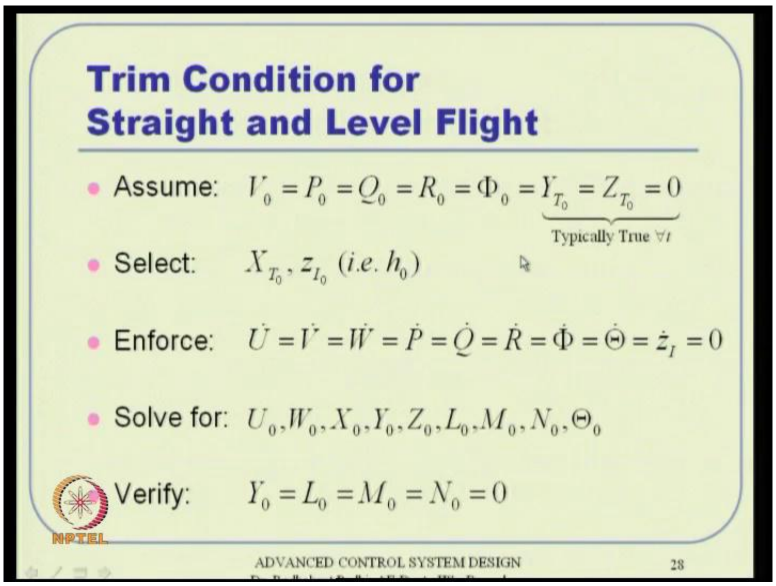

So, what essentially does actually then, so first we have to find the one first we have to find out this operating point actually, like you need to find out what all these this U 0 V 0 P 0 P 0 all sort of things. So, for that there are various steady state conditions available which is called trim condition actually and trim does not necessarily mean straight and level flight, but that is one of the very typical trim condition.

If, it goes straight and level that is actually like a steady state condition which is essentially called as a trim condition actually, but there are various other trim conditions also available trim conditions means, nothing changes your control surface and all that are reflected by a constant quantity, then suppose you want to take a vertical loop you can also you also reconsider with the trim with horizontal circles and all that.

You can also reconsider the trim and things like that, but most popular thing is straight and level flight, if I just go straight and level and do some stability analysis around that actually. All right. So, what we assume here is straight and level flight all this V 0 and P Q R 0s like P 0 Q 0 R 0 these are rotational rates V 0 is the velocity component along body Y

So, these quantities are all 0 and these are like force in I mean force quantities in the Y and Z direction coming out of thrust actually and typically these are true for all time so, body Y and Z component thrust does not contribute any component largely especially in commercial flights actually, So, then so under these assumptions you select these quantities that means that means you tell what is my thrust level thrust force, there is a engine, I want operate in 5 percent thrust ten percent thrust fifteen percent thrust whatever it is maximum thrust what I mean .

So, you fix that thrust quantity and fix what how if we want to fly, I said this is the vehicle flies so, once you do this that means all these variables are either 0 or known, then you want to enforce remember these 9 equations are therefore, so all these 9 equations should I mean the rate of these variables should be 0, if there is no I mean, this rate of change actually for all this U dot V dot W dot all sort of U dot Q dot R dot phi dot theta dot and then Z I dot this is the height part of it actually.

So, all these quantities, if I enforce into 0 this will essentially give me various 3variables and out of these 3 variables, I know this and I have to find out this actually and remember this is just 1 way of finding the trim condition there are also other ways also basically. So, what do you mean I mean the essentially you are somehow making sure that this happens to be 0 all these rate of change happens to be 0, and then you solve for that.

And in that process you can also verify that what you really get that also has to be 0 actually. All right so, you just verify let the there is no force from aerodynamics Y has to be 0 no moments also from aerodynamics about the body axis actually, that is sounds very if that does not satisfy then there is something wrong here actually. So, you solve this so, essentially what you mean is all this variables will substitute 0 are normal variables actually.

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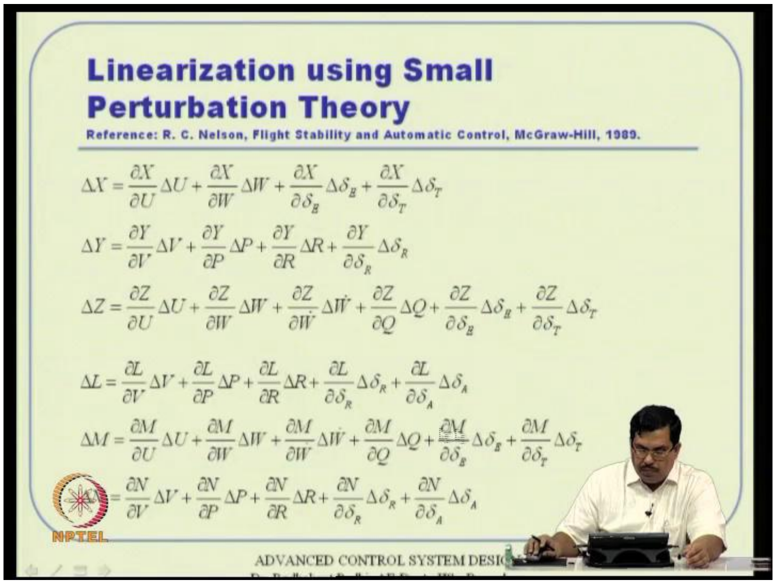

Now you can look this aerodynamic forces and moments is perturbation around that, I can visualize in Taylor series expansion and I want to keep the measure components only and this series does not stop actually, suppose somebody wants to write this is the other function of T actually, then I can still write it rest part is perturbation and delta X is concerned then remember this delta X is the aerodynamic force in body X direction actually.

And then this aerodynamic force can be function of U function of delta W function of delta E function of delta cross all sort of things and if, the series can be expanded also about these are the measured components actually. By the way this way of writing is something called component build up actually. So, component by component you are building up whatever components come here ultimately actually.

Ok similarly, delta Y X 1 and delta Z X 1 and delta L delta M delta N also expand actually, then you can you can write this then the interdependence is if you see out of that

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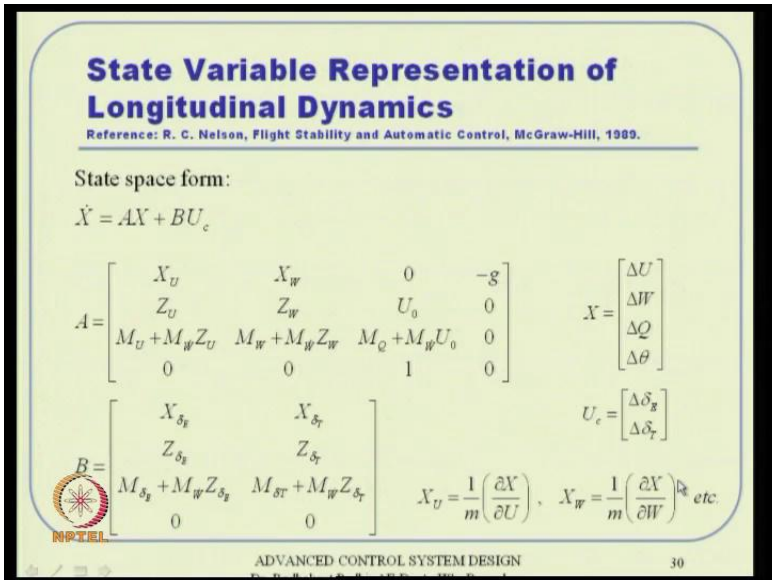

See if I club this variables together delta U delta W delta Q and delta theta, they become independent of rest of the equation they become only functions of themselves and this 2 quantity delta E and delta d actually. So, essentially I will be able to write this X equal to A X plus B U what we know I mean state space equation form basically. So, linearized state space equation for longitudinal dynamics essentially like that happens to be like that it is a 4 dimensional system where state in control are different like that and for notational simplicity these are also defined that way.

X U means this L X by del U with 1 over M normal a that is a kind of well X U X N quantity well X is a force quantity, but X U which is quantity actually 1 by m. So, it is a the force divided by m becomes something like some quantity basically. Anyway, so with these notations intact these all these, whatever you see here in the matrix will turn out to be like that so, in flight dynamics suppose somebody gives us A and B matrix numbers these numbers essentially mean this quantity actually.

So, that means they are coming from there and these are right, so that is just the just something and it is the nice consequences actually. Now I can further divide this 4 by 4 into 2 2 by 2 systems, then I will take 1 time I will talk about this delta U delta theta together, out of that delta U and delta theta together, other time I will consider delta W and delta Q together in a way they are all large whichever function of what and things like that I mean that is that is what it comes out.

And there are 2 types of nice perturbations happens actually in the in the around the steady state level flight.

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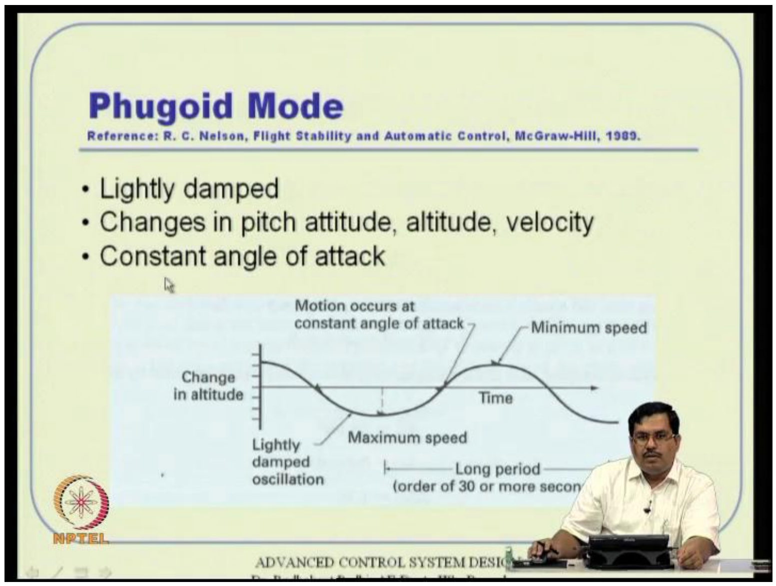

So, one thing is which is which is called as Phugoid Mode, sincerely the aircraft drops its height to a significant thing and then goes up actually the vehicle attitude does not change the attitude remains same the entire vehicle will keep on going and then coming down an then rise like a roller coaster ride actually. So, this happens with something like constant angle of attack actually.

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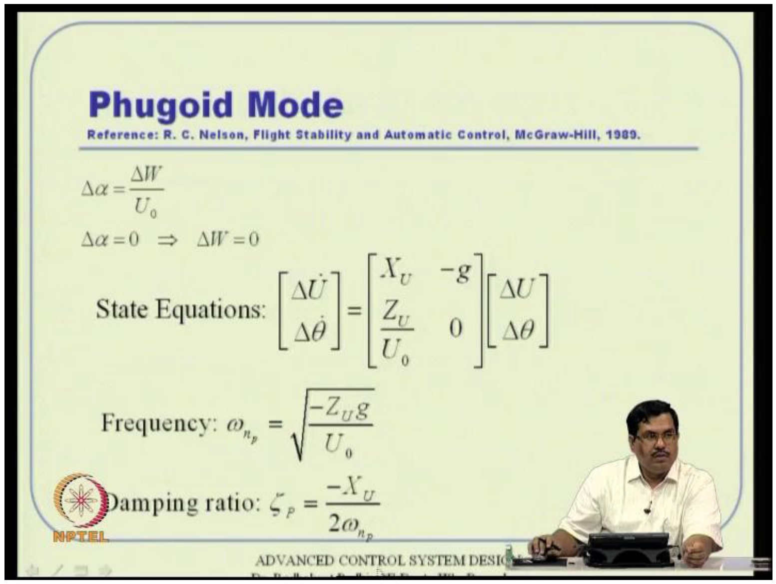

So, under these assumptions delta alpha become 0 with alpha remains constant and with delta alpha delta alpha is defined that way. So, delta W also becomes 0, so you are left out with this nice small matrix and I mean homogeneous linear system which you can analyze it very clearly, this is a second order system in a way delta U and delta theta you have to analyze together basically.

So, you can you can find out the Eigen values, and then you can tell and write it in second order equation and then tell this square plus 2 zeta omega and s plus and omega square equal to 0 that kind of 3 equation, you can write and tell this omega N turns out to be like that and theta turns out to be like that. So, in the phugoid mode there is a natural frequency and there is a natural damping which are actually given like that.

It will go to a maximum speed at the down then it will again pick up and then at the minimum speed at the top, then it will come down and then it will it is a nice roller coaster ride actually. Obviously, nobody in the passenger aircraft would like that so, you want to control that also basically.

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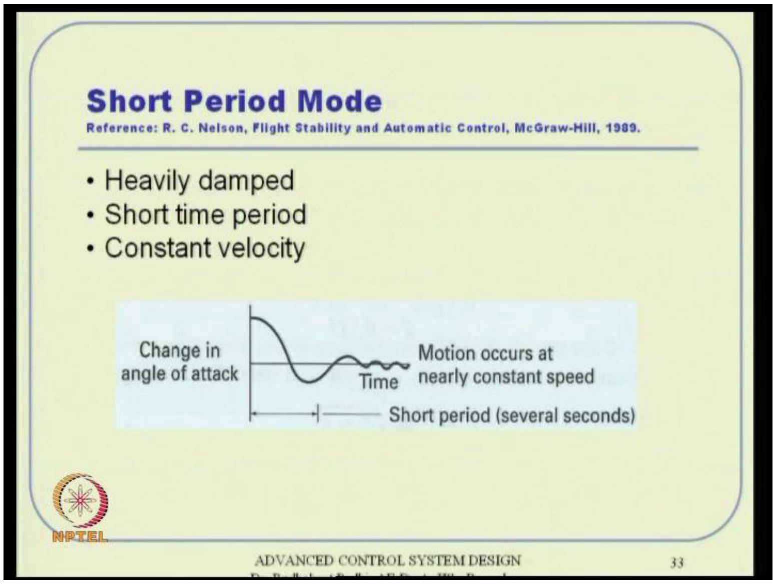

Now similarly, there is another motion which is called short period that means it has if the vehicle does not go through a large amount of height variation, over there is a vibration around that, so this happens to be this happens to be something like this actually.

So, this oscillation this is that oscillation about this vehicle Y axis sort of thing or this is actually little more unpleasant experience actually for the passengers, so what happens there is the change of angle of attack and this oscillation fortunately happens to be heavily damp actually. So, that means these oscillations typically die out first and some of this you might have experienced while commercial flights sometimes, this wind burst and all that they are naturally die down actually, also without too much of control action control can be exerted also before that.

Or suppose that the damping is not satisfactory then obviously, we need a control for that also and both of these are actually controlled by elevator I mean actually. You have elevator ailron and (( )). So, these are longitudinal motions and these are functions of elevator actually delta E remember that here. So, this delta E quantity what you see here is actually largely utilized for suppressing these 2 phenomena’s actually.

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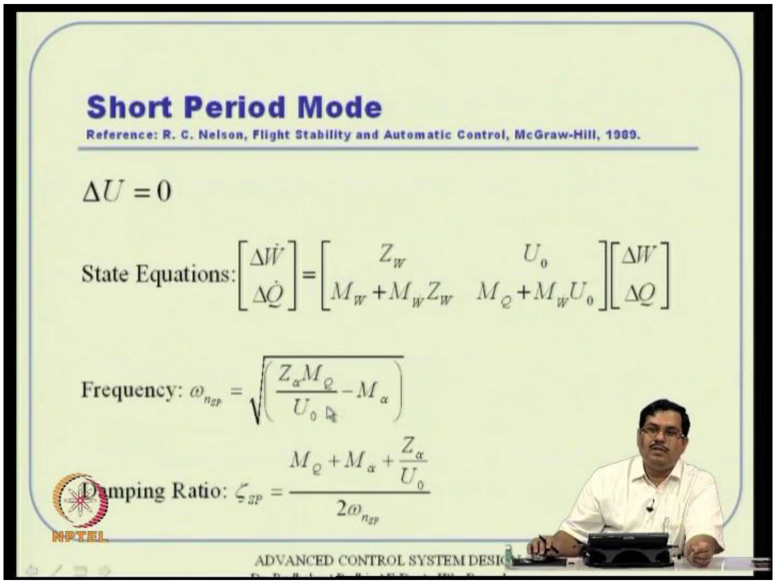

So, this is again short period mode, so again you can you can visualize the dynamics in terms of delta W and delta Q again a 2 by 2 minus and again, you can do this natural I mean frequency and dampingness actually, it happens to be like that and remember these are all functions of par dynamic parameter which are also functions of vehicle configuration so, like that actually

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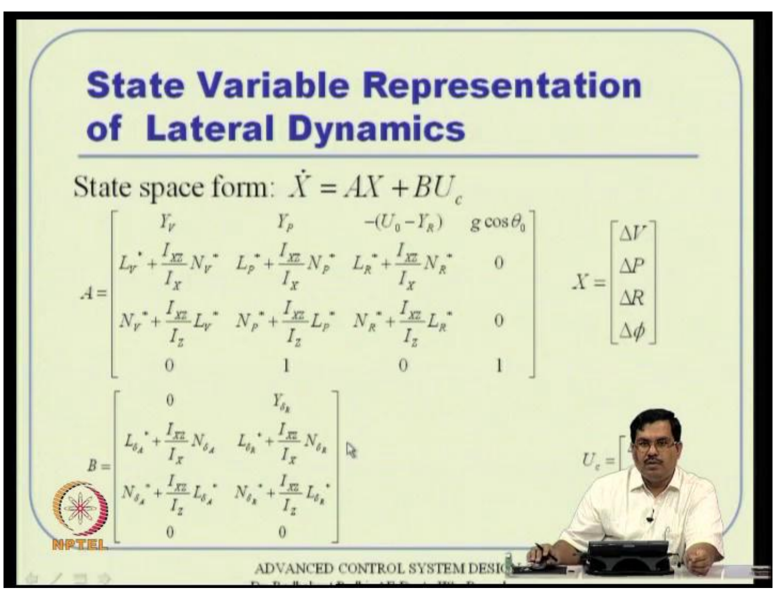

Now similarly, in the lateral dynamics you can decompose that into again four more variables which are not same as the other variables. Here, we talk about V P R and phi, phi is the role angle you know that and then the end limit responses are given like that all these parameter values and all.

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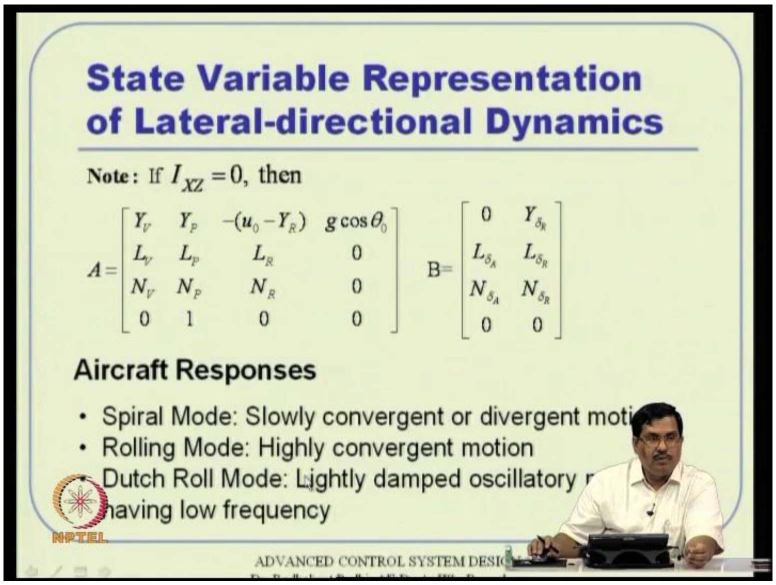

And similarly, because if it is I X Z is also 0 which happens to be in missiles and all that then this whatever you see here, it will further reduce to this rather simple looking form and this will turn out to be like that actually.

So, aircraft response as per what is concerned you can see that in this lateral dynamics there are 2 modes again one is Spiral mode and the other is Rolling mode and there is a Dutch roll motion also which is coupled between these 2.

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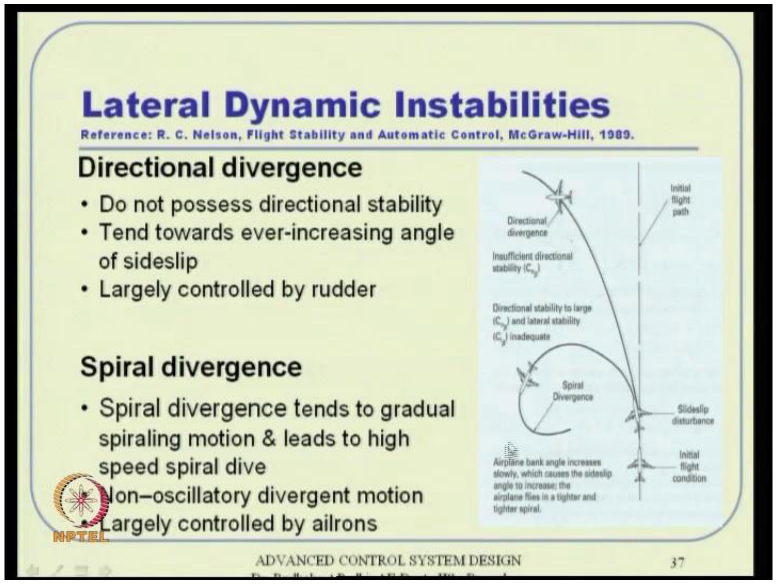

So let us see what is that, so this is spiral I mean directional divergence first so what it does is it does not possess too much a directional stability so in other words there will be a increase of side slip angle beta, so the U V W component you know that.

So, the V component starts building up actually, and if the V component starts building up then the vehicle will largely deviate from its intentional path in a long duration it is supposed to be on that direction it will not go there, it will try to deviate that and then at some point of time beta will couple with phi and then roll dynamics will start all for all sort of thing actually that is that is directional divergence, the other one is spiral divergence which is actually largely responsible from and this gradually spiraling motion actually.

That means if the rolling stability suppose the vehicle starts rolling and all that actually, then it will also lead to E I that we know that the domain the vehicle rolls it also. So, if that is not stable then it will excite a spiral mode that means this radius occur which are becomes very fast and it also loses sight and it crosses actually ultimately. So, these 2 are actually directional divergence is not that bit in other words, if it is not controlled in time then it will lead to further bad things or this is even more further bad than the beginning itself, because it rolled this spiraling action get simplified very quickly actually.

So, then it is a both this particular aspect is largely controlled by rudder and this particular effect is largely controlled by ailrons actually, remember it is a motion due to phi roll angle this a motion due to beta. So, beta so beta is a sideslip angle so beta is controlled largely through a rudder and roll is largely controlled through ailrons.

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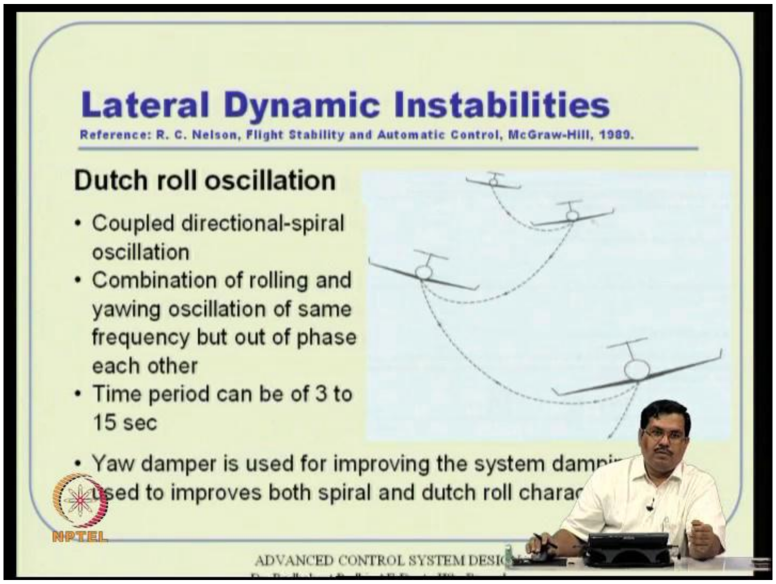

And then there is a nice coupling with the 2, this what is called a Dutch roll oscillation. So, here it is it will the directional spiral oscillation should be coupled together actually. It will roll and hence it will beta will be low it will go somewhere then it will stabilize.

It will start in opposite direction sort of thing and then it will go like this and then it will go like that and this keep on remember this amplitude keeps on building also and with this typical name comes from this ice skating actually. When you do ice skating, this is 1 side you go something and then you come back to other side and things like that actually. So, that is typically Dutch people like that ice skating and all so that is how it the terminology is been given actually.

So, time period can be of 3 to 15 second and then typically this yam damper is used yaw damper is largely by the way you design the vertical thing the part of the vertical type which is ahead of the rudder that vertical plane, if you define it properly design it properly then that will design the proper damping actually. So, N addition to that you can always control it using both ailron and rudder together this motion you cannot decouple into 2 2 by 2 system.

You have to talk about at least 1 4 by 4 system and then we've this both ailron and this one rudder income and hence you design them together, that is the power of this states phase design approach. You talk about the coupling effect also in the design process actually. Now before you stop this class I will also take you through some various attitude representation, which assesses this entire 6 degree we derived in terms of Euler angles but there is a strong drawback for that.

And we will see that and then we will there are also alternate attitude representation people who thought about seriously actually, and these problems are further amplified if you have this satellite control problem for example, or missile dynamics which have large angle of rotations coming to picture. So, satellites can turmoil it can completely turn 360 degrees and keep on doing that phase.

And hence, if the missiles also they are high angle of attack maneuvers you can take big turns and all that, so large angles when you talk about the Euler angles are typically not good actually, so we want to have some other thing.

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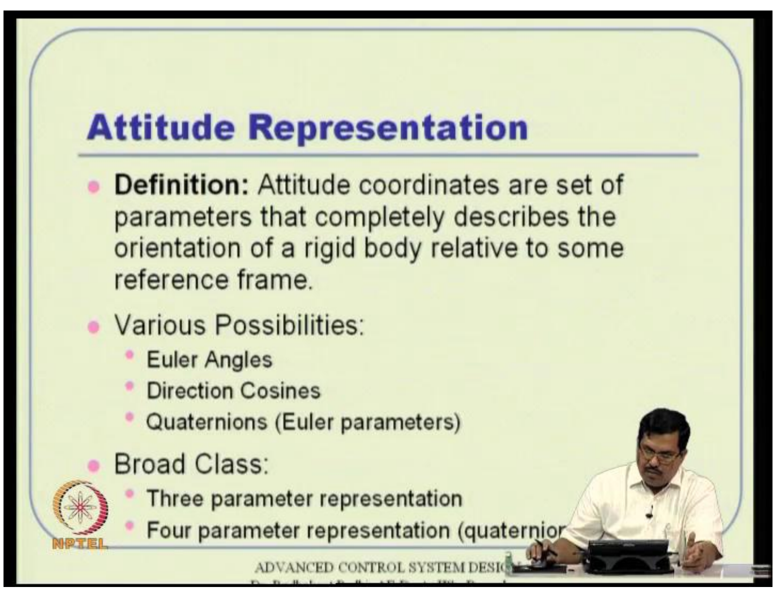

So, let us see that, so Attitude Representation by definition is essentially the attitude coordinates are set of parameters that completely describe the orientation of a rigid body related to some reference frame.

And one side we have already seen Euler angles, the other thing that are available to us is direction cosines and quaternion's in fact direction cosines are of the first step before even Euler angles, and then it does not stop here people talk about well Rodriguez parameter then modified Rodriguez parameter and they are in the family of that actually, there are given it actually.

So, broad class all these parameters what you see there are 2 broad classes one is like a three parameter representation and another is a four parameter representation. Essentially, quaternion's fall in the 4 parameter representation. So, 3 parameter representations are nice because the attitude is actually there parameter representation whatever you see any attitude these three angles come out,

But there are there is a serious problem, we will see that and then a response if you want to quickly look at that I can also go there and come back probably

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So, this Euler angle will representation, if you see it here, there is tenth theta term involved right what you see here and ten theta is sin theta by cos theta and here, is a sec theta which is also 1 by cos theta. So, that means if theta goes to 90, then the cos theta becomes 0, and there is and there is something divided by 0, which means these rates will go to infinity that is something called (( )).

Actually, that means no matter whatever degree of accuracy use delta T take it to various small quantity, then these integration cannot be done this dots suppose to be infinity the angles are the angle values are fine, but the rate of change goes to infinity actually which is not nice. So, this set of this and this rate of change is actually a function of the sequence of rotation. Now you do that you do that sequence of rotation from psi theta phi, if you do a different set of equation then, if the variable with some 10 theta will appear not 10 phi or 10 psi somewhere you will be getting locked actually.

So, there is a no way you can get out of this mechanism actually. So, that is where it happens and this is this is actually there because it happens in 90 degree, 90 degree is a small angle. Now the whole idea of this m R P and all that modified Rodriguez parameter they have already taken into something called 360 degree or multiples of 360 degree, then only you will have similarity.

So, one full round you can take and then still can have similarity and all that, and then you can still talk about three parameter representation which is nice the 3 to 3 is actually like there is a I mean, this redundancy variable does not come into picture actually so that is that is what.

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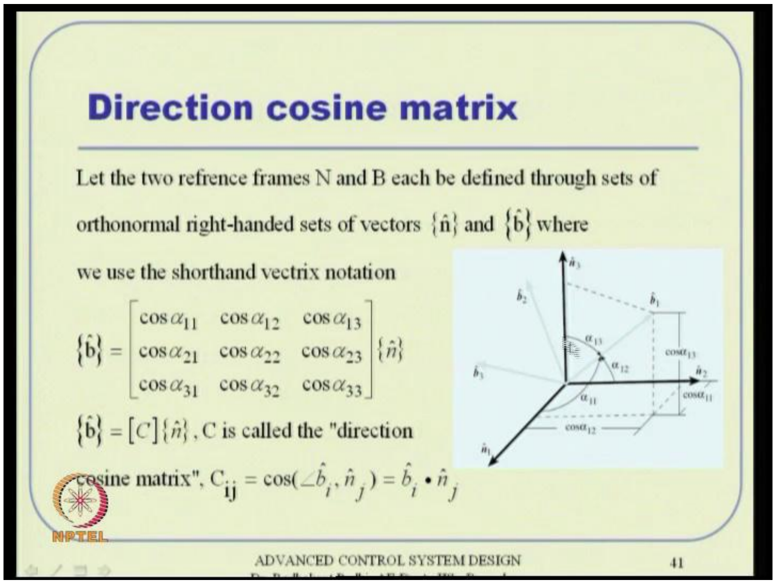

So, let us talk about that now, and then so first is direction cosine so, direction cosine is if you have if you have a reference frame and if you have a vector any vector I can talk about direct angles actually. This vector whatever I have I can take direct angles with respect to this axis and if, I take cosines of those angles and put them in a matrix form, then this is the matrix that I am talking as direction cosine matrix.

So, that means the components of this vector whatever this vector this like a b 1 component b 2 component, b 3 component each of the component what I mean, they will have direct angles with respect to this other frame see there are 2 frames, one is N frame and one is B frame you can visualize the N frame as inertial frame and the B frame is body frame.

So, each of the components of the body makes an angle with all the 3 axis of the inertial frame directly that means you have 9 such equation actually. So, all these things if I put them together then the B frame is related to the N frame through a matrix representation which is directly like cosine I mean the cosine terms and all that we know if you take the products and you can derive it actually that is not the problem.

The 2 vectors if you take dot product that the cosine angle will in the picture and that is why the cosine terms come actually. So, this B what it results from this algebra is something related through alienated to and through this C matrix and C matrix is essentially called direction cosine matrix, that is the definition actually. Now remember you really require 3 what you have line parameters actually that means there are 6 redundant is here that is the problem here actually.

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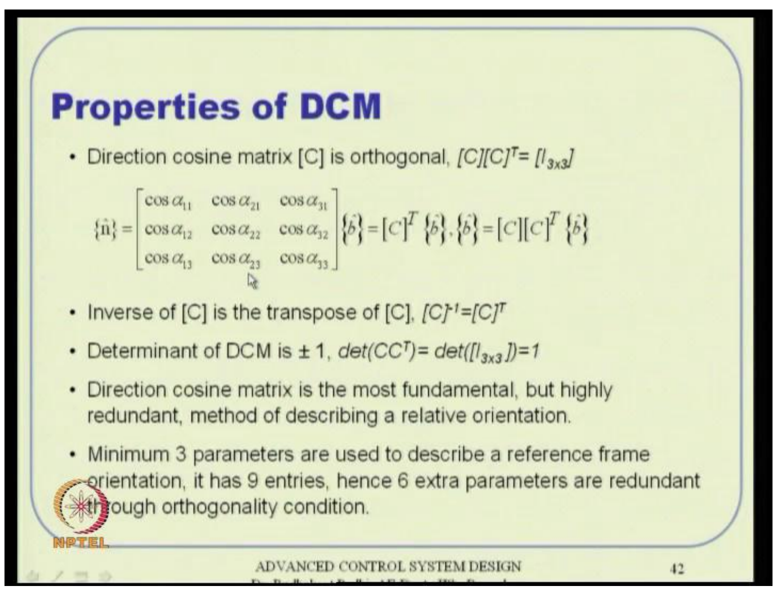

Now before do that before even talk about that, let us say observe some nice property, that C is essentially an orthogonal matrix and orthogonal matrix is a nice property the C time C transpose is actually identity, that means inverse of this matrix is transpose and inverse is all inverse always exists in this. So, there are nice properties of that actually. Inverse is transpose and determinant of this is always plus or minus 1, depending on what angles it takes and there are 2 direction cosine is probably the most fundamental most naturally of visualizing things.

But it also highly redundant, because of this we require through what it has given of 9 parameters actually. So, 9 entries so 6 extra parameters are redundants through this orthogonality condition basically orthogonality condition tells us that each of the vector, that this vector is orthogonal to that and this vector is orthogonal to that and because this vector is matrix is orthogonal the vectors are also orthonormal. So, if I take dot products of any 2 rows actually the a R 0s and if I take normal of any row that will happen to be 1 these are orthonormal each of the vectors are orthonormal to each other actually.

So, that those are the redundant, that I am talking so, you have this combination since 3 combinations of dot products and then 3 normal relation quantities actually so, this 6 redundancies we have to talk about actually

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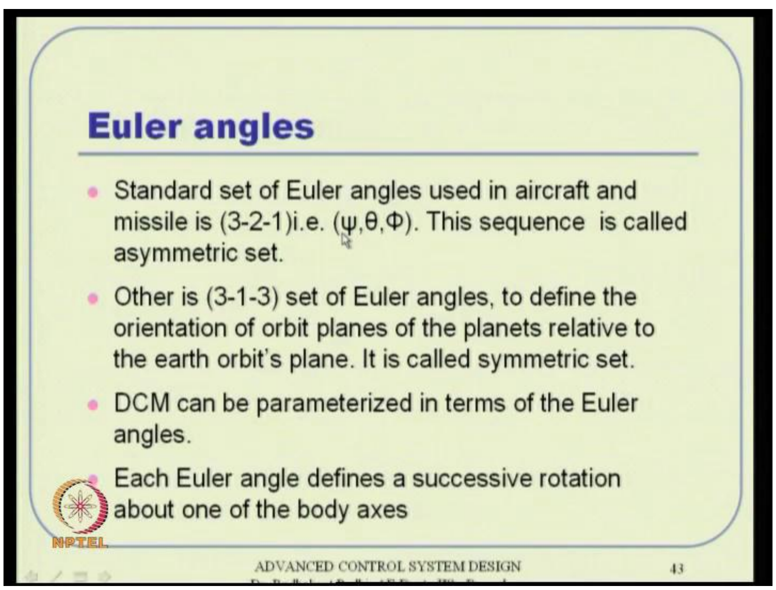

Now Euler angles we will discuss everything on that probably still to visualize that, the standard set of Euler angles is what you have done it is a psi theta phi rotation actually or this is also called asymmetric set what people use in satellite dynamics and all is what is called as 3 1 3 set of rotations actually. So this we will not talk too much on that anyway, but that is also called something called symmetric set actually.

You do not have to rotate it using this 3 2 1 actually you can also go for 3 1 3, 3 means 3 stands for psi 2 stands for theta and 1 stands for pi. So, what you are too talking here is psi phi and again psi you can also do that actually, because typically standard force and satellite dynamics and all. And then there is a direction cosine matrix can be parameterized in terms of Euler angles we will see that in relationship also.

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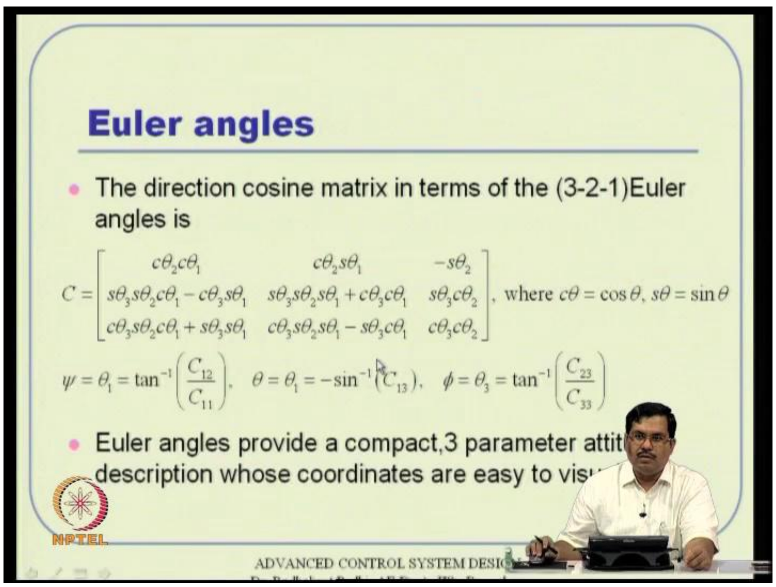

And obviously, Euler angles define successful rotations and all we discussed about that so this is the representation actually. Like if you have Euler angles theta 1; theta 2; theta 3 I am not writing in terms of theta phi psi you can also write in terms of 1 2 3 rotation actually theta 1; theta 2;theta 3.

Then the direction cosine matrix terms out to like that where c theta transfer Cos theta and s theta transfer sin theta especially, then out once you did that do that then this psi theta phi can also be given in terms of inverse transformation, that somebody gives me the direction cosine matrix then I can also recover this error angle quantities. So, they are vacant for transformations available actually.

If once you know, 1 representation you can find out the representation of the 1 actually

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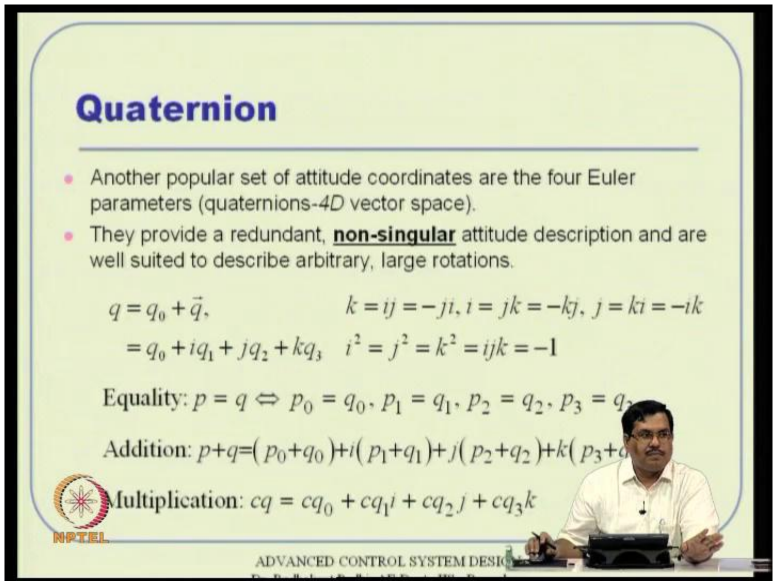

So, then finally, we will let us talk a little bit on quaternion, quaternion's essentially to get rid of this singularity problem. Now unfortunately what happens is anything that you use either you direction cosine Euler angle modular Rodriguez parameter,whatever you use all these 3 parameter representation do suffer from singularity.

It is a matter of where the singularity occurs, but it has a singularity now can you talk about singularity free transformation and that is where this quaternion algebra is useful actually

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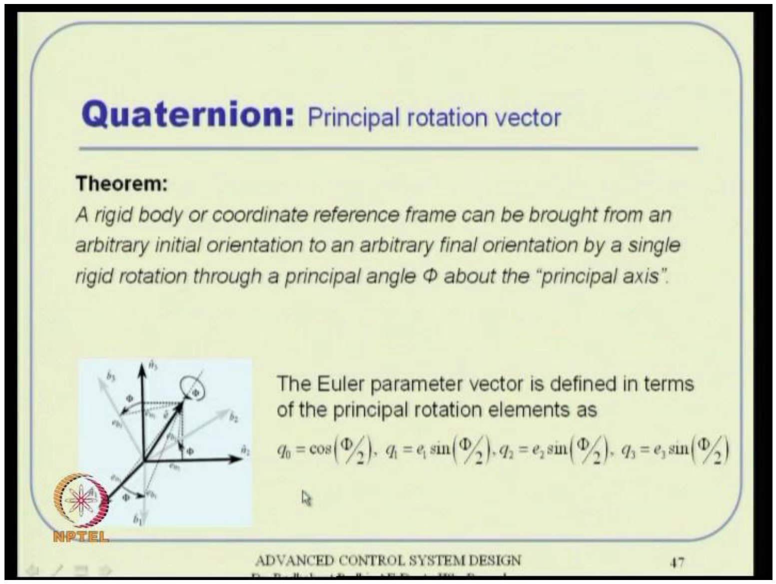

So, what you what it turns out we will come back to this probably, but the that is fundamental thing is something here. So, for every attitude change I am talking about I can always visualize some axis somewhere around which I can take 1 rotation and principal.

This is for all sequence of rotation that I am talking from inertial to body, whatever 1 axis frame to other axis frame there is one vector what is called the principal rotation vector, around which I can take one rotation, so that this 2 axis will merge together actually that is the concept that is the theorem actually. Now obviously we have to we have to design the axis frame and that is given by 3 quantities q 1 q 2 q 3.

And there will be something like, one rotation quantities also you have to discuss about so, that that is the quantity for each actually. So, quaternion will consist of 4 parameters q 0 and then q 1; q 2; q 3 and q 1; q 2 and q 3 will typically, give some sort of a something like direction of this rotation vector, and then this phi angle which you can take out from this q 0 component actually, phi by 2 is cosine inverse of q 0 is basically, that way phi angle is a rotation about that vector which will make it coincident actually all that thing.

Now going back to that this is a quaternion form this q 0 plus q bar q, bar consist of q 1 q 2 q 3, and then there is a I mean this i j k you can see that they are a square of that and I times k times j all equal to minus 1, there are some neat algebra as you said would be that actually. So, you can see some of these and equality sense also there are like various sorts of thing.

I think some of you who are interested in this you can see this particular book which I had taken it from actually analytical mechanics of space systems is a nice chapter for doing all this actually anyway so coming back to this addition you can define it that way suppose you have 2 quaternion's let us say p and q so p will have p 0;plus p1;2 to p 3 q you will have q 0 ;plus p 1; plus q 3 and then with respect to that you can define addition multiplication like that actually.

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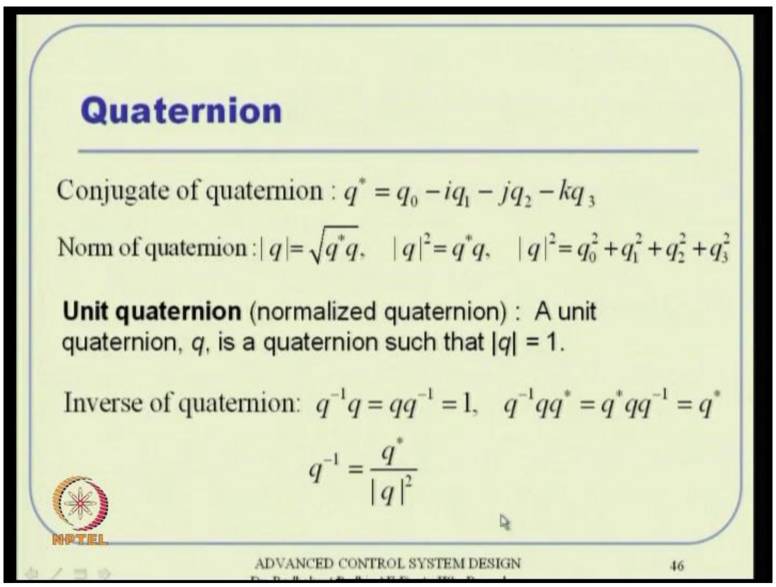

Then there is conjugate quaternion which is negative of all these quantities and the norm of the quaternion is defined like that. So, obviously this q square this what whatever you see here, is actually 1 q q 0 q square q 0 square plus q 1 square plus q 2 square plus q 3 square happens to be one actually that is called unit quaternion. So, that is actually restraint that is a constraint that you have to operate with actually. All this parameter this says to satisfy that and that happens to be a problem in numerical integration for which, you need to normalize it forcefully. And with that there is no unique way of doing that that is that is the difficulty actually

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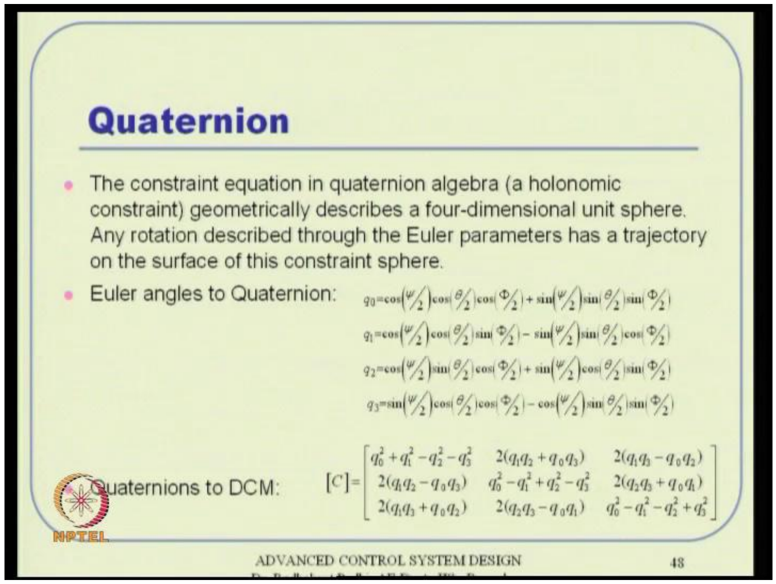

So, quaternion are also satisfy this quaternion 2 Euler angle suppose no Euler angle then you can find out quaternion suppose you know quaternion’s you can find out the direction equation matrix things like that, there are there are transformations available which you can extract one information from the other actually.

So, I request you to see some of this surface for more details this is a control theory class we cannot talk too much detail on that, but with all sort of wide various mechanics ideas and all that we will be able to utilize our control theory ideas a far flight control and radiance actually. That is my motivation so, with that I will probably stop this class thanks for the attention.