Advanced Control System Design
Prof. Radhakant Padhi
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture No. # 07
Overview of Flight Dynamics – I

We will have, I mean we have already covered what is basics of flight dynamics as well as many things about classical control system as well as state place representation so far. So, this particular lecture next we will study a small overview about flight dynamics, so that we can connect the material what is going on here for aerospace guidance and control.

So, this particular lecture we will take you through this so called point mass equations as well as a major portion of what is called as (( )) equation. And then we will follow that up in the next class with further concept which will make us ready for understanding the relationship between what we are studying in the control theory to aerospace applications actually.

So, this is the first we will three point mass dynamics and it all depends on your application, how you want to visualize this aerospace dynamics actually. Suppose, you are interested in a long term trajectory then probably point mass equations are sufficient, because you do not know do not need to know the attitude of the vehicle at any point of time. You just need to know the location of the vehicle as well as its velocity and velocity direction in a gross sense.

So, really do not need to have all details about the vehicle attitude, vehicle roll rate and pitch rate things like that in for a long duration application. That is where this point mass equation becomes relevant, and it is mostly used for guidance applications actually. So, let us see what is the word is call as point mass dynamics, and we are essentially visualizing the entire vehicle as a point moving in space.

So, we will and then relevant equations we will see, I mean what all variables are there and things like that.

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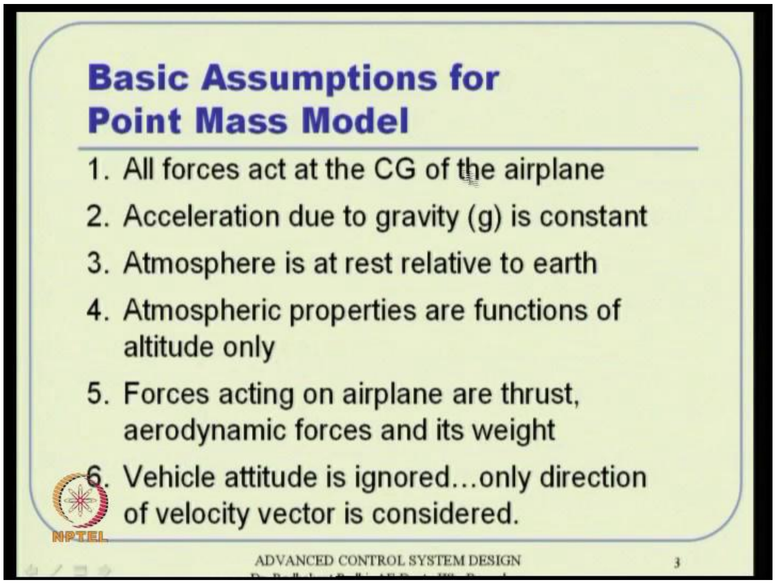

So, basic assumptions for point mass model, first of all we are all kind of collapsing the entire vehicle at the CG of the airplane. So that means, all forces act on the CG of the airplane the entire vehicle is just a point and acceleration due to gravity is constant, we are not bothered about variation of g unless you talk about a very long, very kind of interplanetary missions and thing like that when you really go far away from earth actually.

Otherwise, for most of the application we can fairly assume that gravity is constant, then atmosphere is rest relative to earth, we are not considered about this wind effects and all that, and atmosphere properties are also functions of altitude only. We will consider this specially we know that dynamic pressure is a quantity which is which depends on density of atmosphere and that varies with height.

So, atmosphere properties will consider only as a function of height nothing else actually, and the force acting on the airplane or thrust aerodynamic forces and its weight, we will see that in a in a Victorial diagram next slide actually. Vehicle altitude is ignored as I told, the which angle it is oriented and things like that is not of our concern here; however, the direction of velocity vector is a primary importance because that will govern the trajectory of the vehicle on the long run actually.

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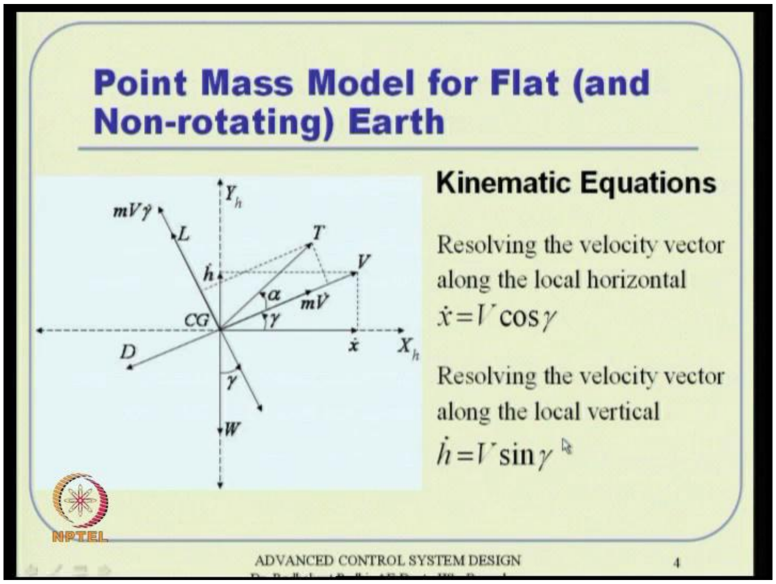

So this is picture really, this is where the vehicle is located the entire vehicle is just a point here, and let us say the vehicle is moving in that direction V, that is the total velocity of the vehicle and this x is actually like parallel to the local horizontal. So, whatever angle that this velocity vector makes with respect to the local horizontal plane that is called flight path angle or gamma, and most likely the thrust vector is is align with respect to the vehicle x axis, vehicle nose and things like that; however, that there may be kind of alignments small L, small angle with respect to the vehicle x axis, but it'll not consider that here.

What we will see is the thrust is align with respect to the vehicle x axis in that sense, this angle what you see is angle of attack alpha. So, Y is the is the altitude axis and the x is the down ranging actually; that means, wherever this flight goes we are considering only on that plane this is not a three dimensional picture it is just a two dimensional, in the pitch plane sort of thing actually. So, this this vehicle this velocity vector V is can rotate depending on this alpha actually.

Because is the thrust is the thing, it is not align with a velocity vector and hence it will have a component perpendicular to the velocity vector which will result in orientation, I mean which will take this velocity vector away; that means, gamma will alter because of alpha basically. So, with respect to that let us resolve the component in various planes now we have this we know that drag acts directly opposing to the velocity, so drag is straight, I mean straight away opposing to the velocity. And thrust I mean lift is perpendicular to the drug or perpendicular to the velocity vector so that is how it is.

And then weight we know it is perpendicular to the local horizontal, so weight will always act perpendicular gravitational weight, and thrust is anyway there. So, we are interested in this velocity level equation and isolation level equation actually. Now let us see velocity level equations are nothing but simply components of the velocity vector in X and Y direction actually, that is all. So, what you have here, this velocity vector this angle is gamma then this is actually x dot and that h dot and then this is x dot is; obviously, V Cos, V Cos gamma, and h dot is V sin gamma, this is simply the component of velocity vector in horizontal and vertical plane, I mean nothing there simple geometry tells us that one actually.

However, you want to have a moment level moment level equations, I mean sorry the force level equation

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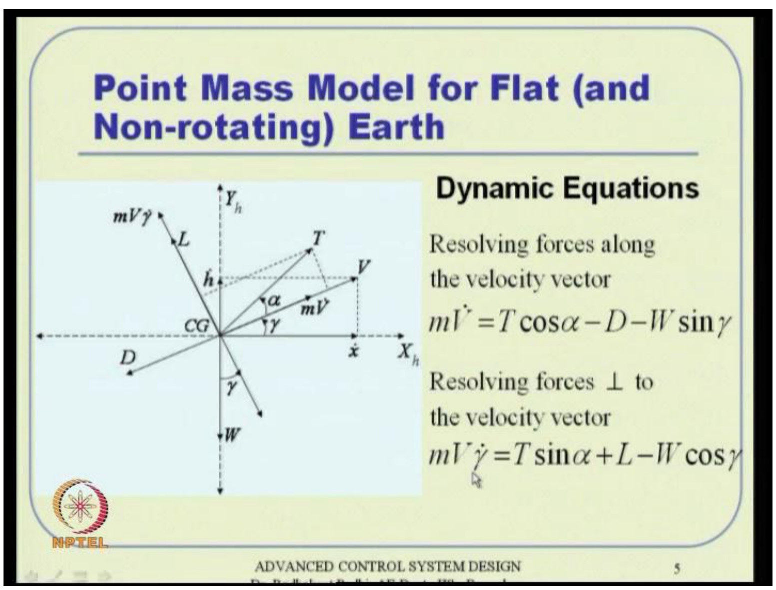

Next that will give that will be resolved again into this perpendicular directions and things like that. Before you do that let us say, I mean also remember that these velocity level equations do not see forces, and do not see vehicle parameters they are simply related by geometrical relationship, these are call kinematic equations.

So, you will have two kinematic equations coming here x dot is V Cos gamma and h dot is V sin gamma. The next is dynamic equations where your thrust and weight vehicle parameters everything will come into picture, let us see how to resolve that. So, this particular thing we will resolve; one, in the wall one along the velocity direction, velocity vector director and one perpendicular to that, all the forces that is acting on that.

So, where are the total forces acting on this particular direction? One component comes from thrust so that is given as T Cos alpha, one is opposing to that, that is minus d, So you have T Cos alpha minus D, and W was also a perpendicular, W has a component because there is a gamma angle that is acting on that actually, if you see W, I mean the W has a component which is W sign gamma which is negative to V actually.

So, if you have T Cos alpha minus D minus W sin gamma that is the net force acting along V direction, velocity direction, and the hence by Newton's second law this is m V dot. So, m V dot is nothing but T cosine alpha minus D minus W sin gamma whatever component, so that is what it comes there. And similarly, once you try to I mean kind of resolve this along the vertical direction, I mean this perpendicular to the velocity vector, then see that this is the there is a thrust component again, that is T sin alpha this time plus lift component minus W cosine gamma this component is also there.

So, that is what you see here and that is all equal to the net resulting force acting on the direction, remember that is like centrifugal, I mean centrifugal force and things like that you can assume that way. So, if you I mean this particular thing is nothing but m V gamma dot actually, m times V into gamma dot, remember this is actually a force quantity this isolation quantity V times gamma dot, and m V gamma dot is a force quantity and that is like centripetal force sort of thing actually.

So, if you imagine a circle somewhere here passing there, where velocity vector is tangent to that, if you imagine a circle passing through this point where velocity vector is tangent to that, then the net resulting centripetal force will be somewhere m V gamma dot actually. So, that is how you see that, that is how this gamma dot is will start changing actually, I mean this. So, if you divide the L the entire quantity by m you will get V dot, if you divide the entire quantity by m V will get gamma dot.

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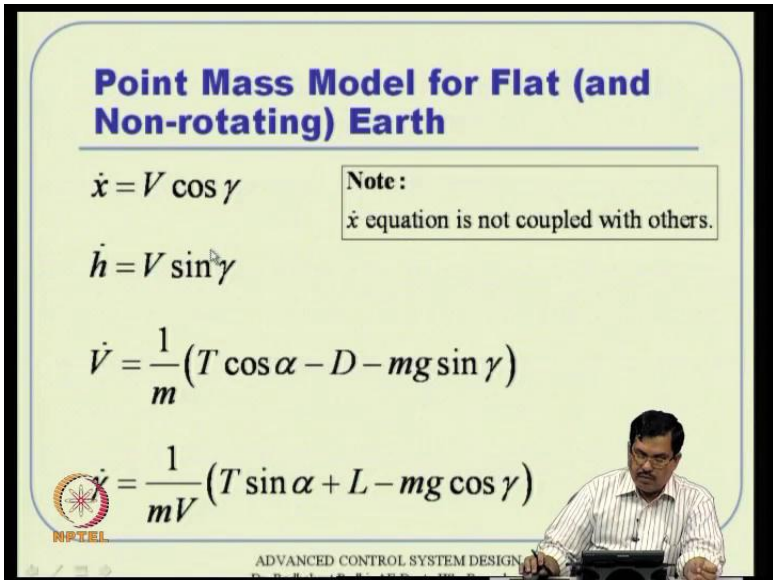

So, putting everything together what we get? x dot is V cosine gamma, h dot is V cosine gamma, V dot is that and gamma dot is that actually. And typically in this kind of a problem we interpret alpha as the control variable that is where we have the liberty of changing alpha basically. So, that is I mean the is suppose if you go back to that if you see a especially a rocket let us say, then we have thrust to vectoring facility; that mean, thrust can be tilted out I mean; that mean, this thrust reflection can happen it need not happen, need not be align with respect to the vehicle nose actually.

So, that is that that can that can act as a control variable really. Aerodynamic variable all sense also it is same, you can manipulate alpha by reflecting the control surfaces through aerodynamic forces actually. So, putting together this is the set of equations in state space is form that we talk as, long as we talk about flat earth model point mass equation. So, the earth is flat and it is not rotating, and under several assumption that we several other assumption that we talked here actually.

So, under those assumptions we will consider these are set of equations that to deal with where X h V gamma are state variables let us say, and then your alpha is control variable and D and L remember they are also functions of alpha, drag and lift they are certainly functions of alpha actually. And specially this mass quantity and thrust we interpret them as parameters actually, thrust can be constant in a in aircraft applications, thrust can be time varying in rocket applications actually, thrust and mass both can be time varying if it is a launch vehicle or missile application basically by the way.

So, thrust and mass are considered as time varying parameters, h X h V gamma are state variables, and g is also a parameter by the way and then alpha is a is a control variable here. So, that is how we these state equations are there in the in this point mass model, also note that this rest of the equation what you see here h V gamma are decoupled from x dot they are not really functions of x actually, h is I mean this V dot and gamma dot are functions of h because through dynamic pressure, this lift and drag will depend on h actually.

Whereas, this x is kind of decoupled, we still need to integrate and get a value for x if you really want to plot the trajectory, trajectory is dictated by X and h, the coordinate of this particular CG location is dictated by X and h, so we still need to use that equation for integrating; however, this for all practical purpose we can also solve a problem in h V gamma coordinate only basically.

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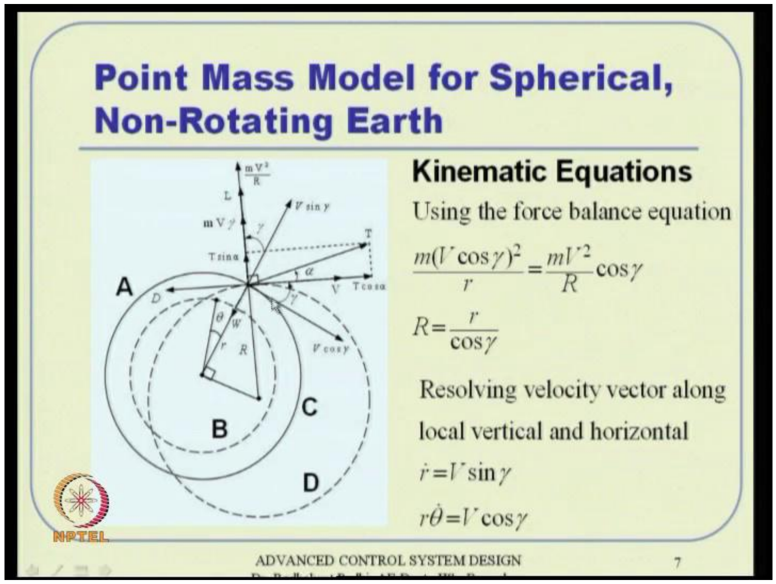

Now, next we go to lets slightly complicate the matter here, and then you tell about a spherical non rotating Earth, Earth is still not rotating, but we now will consider long duration of flights let us say for example, ballistic missiles they will really travel a very long distance before falling there somewhere actually. So, earth curvature we can never, we cannot neglect really.

So, this effects k h to be taken into account in that sense what happen, now let us say the for a moment the vehicle is somewhere here it is still a point, velocity is there somewhere remember this is no more tangential to the surface of Earth, surface of Earth is somewhere here, this dotted line what you see B is surface of Earth, and then this is a point where the vehicle is currently flying with a velocity direction that, which is certainly not parallel to the tangent which will resolve from there actually. So, it is certain has some applied path angle and all that actually.

So what you are interpreting? We are interpreting, let us say try to analyze the equation little bit better way, we will try to visualize this in a hypothetical circle parallel to Earth actually, so B is like surface of earth, A is the instantaneous hypothetical circle parallel to the surface of Earth, and these also like let us say, this another one this dotted line which will be require slight rate actually.

Anyway So, we coming back to this is a thrust direction again, which is again at an angle of alpha this is now local horizontal remember, this is this circle is parallel to this dotted circle so; obviously, this this tangent is actually a local horizontal tangent actually. So, then you have this angle which is gamma and that angle is alpha, and hence if you see that this is actually thrust direction which is, I mean sorry this is drag which is opposing to velocity and then lift has to be perpendicular to the velocity.

And we also know that negative lift direction is the centrifugal centripetal force sort of thing and hence, we can visualize another imaginary circle around that for which the center of the circle, if you join this point this will be like a along the lift vector actually. Now, we have several directions here so we have to resolve it properly, and also remember this is a reference line, this particular line is a reference line let say this is launch point or something about which you want to calculate, and you want to know how much range angle you have covered, theta is called as range angle actually.

The actual range is on the surface of earth if you multiply theta with radius of earth R e you will get the actual down range actually, on the curvature of the earth actually. Anyway so, now coming back to that first is let see again this kinematic equation sort of thing, we will also see this force balance equation first so if you see this particular direction, V Cos gamma is along this and V sin gamma will be along that.

So, if you see this m V, m V Cos gamma whole square by R, R is the instantaneous radius from center of earth, this is center of earth then that is the force that is acting along that actually, and that has to be balanced by m V square by R Cos gamma actually. So, m V square by R is here and into Cos gamma component is here actually, this particular component. So, you are resolving this force quantity, I mean this sorry this velocity level equations and all that, one is I mean this one is acting along that the the balancing is acting along that actually.

So, through that if you acquire these two you will get a relationship between this particular R, this is the radius of this hypothetical circle, and this is the R that is instantaneous radius from center of earth actually, that is the relationship they will have, this bigger and smaller they will satisfy this relationship actually because of this force balance. Now, we will go back and resolve this vector along local horizontal and vertical that is where to result in kinematic equations. So, r dot is again r dot is like h dot.

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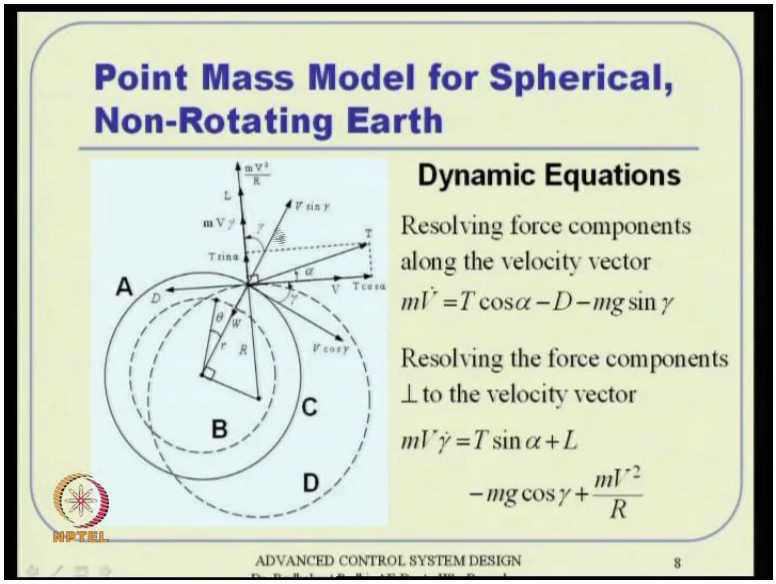

So, that is along that perpendicular sense that is nothing but V times sin gamma, **sorry** r dot is nothing but V times sin gamma actually, r dot is like h dot, so velocity is there which is not orthogonal to this this line and hence it will have a component actually, r dot is V sin gamma, and then r theta dot is nothing but the the angle that you cover actually, theta dot is remember this theta dot is angle of rate or change of this angle. So, R time is theta dot what you see is actually, what is covered in terms of this particular direction suppose, you take this way the velocity.

Velocity tangential to this, particular thing we will have r theta dot and that is nothing but V Cos V cosine gamma actually so, r dot is V sin gamma again that is similar and instead of x dot, I mean what you saw in the last time is x dot, that is like if it is flat earth it is x dot, now spherical earth is r times theta dot which is similar to x dot actually. So, r theta dot is V Cos gamma actually, again similar quantity here. Then the next one is slightly complicated and you see that what happens to the other one actually.

So, you are interested in m V dot and m V dot is almost similar to what you had last time again, this is T cosine alpha one component coming from thrust, minus drag and m g is acting along that direction W, W is nothing but m g, m g into sin gamma component will also I mean it along this direction actually, so m V dot is T cosine alpha minus D minus m g sin gamma. Now, we have to resolve one direction is that, the perpendicular direction to that is that one that is where this gamma dot equation will pop up.

Again this m V gamma dot is the net resulting force along this direction, and here we can see that one component comes from T sin alpha, from the thrust there is a lift vector acting along that, there is a gravity component which is there subtracting that m g cosine gamma, and there is a m V square by R which is actually, centrifugal force and here we have derived a relationship between big R and small r.

So, that is where you can use that substitute that, and then simplify this equation in terms of R remember, our equation is r dot and big R is not a decoupled I mean relationship from R actually big R and small r are related, so it cannot be an independent variable. So, we will substitute this big R in terms of small r, and then simplify the equation in the state place form again.

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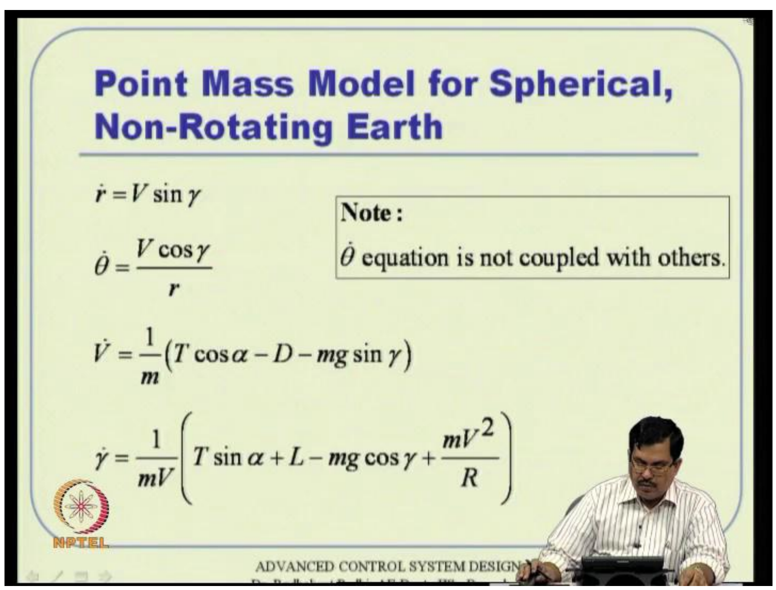

So, what you are getting, r dot is V sin gamma and r theta dot is V Cos gamma hence, theta dot is V Cos gamma by R, and then V dot is what you saw here that is V dot equation divided by mass and gamma dot is divided, all this divided by m V actually

So, if you simplify that you will get everything that reaches. So, this is what you will get remember, this R is also a function of this R and gamma that is what we saw here actually, you have to substitute that big R actually. So, these are the set of equations that you have to deal with, when you talk about spherical, but not rotating earth; that means, the earth is not rotating, and earth rotation does play a role depending on if they I mean depending on the system equation actually. Suppose you want to start with latitude, longitude, you go to different lat-long and things like that, then as long as they are not in the same latitude plane then things will be different actually. So, for that I will not derive the equations and all, but there relationships are available.

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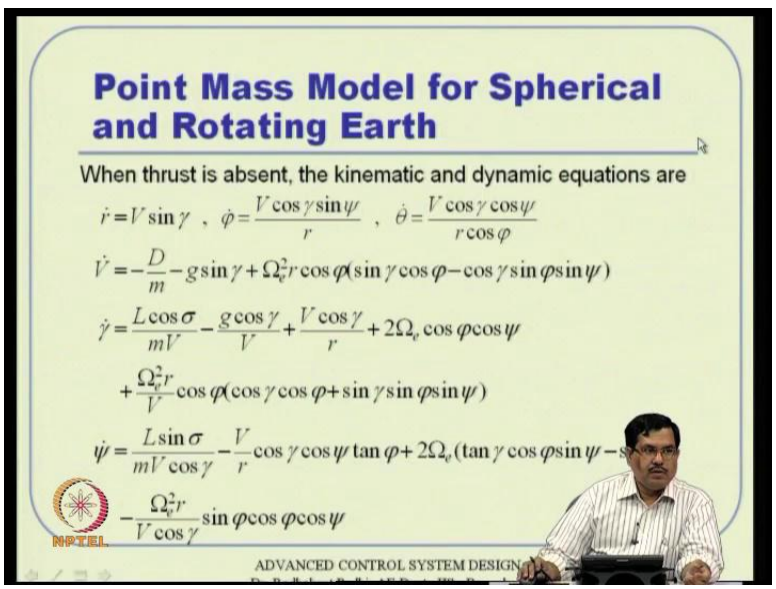

So, then things will be getting slightly more and more complicated as you talk about more and more effects coming into picture, and this will be more and more close to reality of course. So, for spherical and rotating earth if you talk about then you have r dot, now you cannot talk about only theta because that that was a along what is called as. I forgot to mention this is this dynamics what you are deriving is all valid in the great circle actually, great circle means suppose, you are going from one point to other point.

And suppose, we well let us say your launch point is somewhere here, you want to go somewhere here on the surface of earth wherever you want to go. So, that is launch point, target point and center of earth, there are three different points actually. So, these three points when you connect; obviously, that will go through into that will intersect the earth center and it will result in the greatest circle actually, anything parallel to that will be small radius circle.

So, because of that it is called a great circle and whatever equations we saw here is all valid in the great circle plane actually. So, we are not consider the 3 D equations, I mean this cross I mean the cross range motions and things like that this is all down range motions still, but here we will not worry about that, you are talking about the entire 3 D movement sort of thing, we consider rotating earth also into picture and hence all the equation, all the things you have to take into account actually. So, phi and theta are lot-long positions here.

And r dot is still V sin gamma, but you will have to have latitude dynamics, longitude dynamics, V dot gamma dot then there is a psi dot equation also and things like that, all these things you have to talk together all these six equations actually. And remember this omega e, big omega e is earth rotational rate actually, what about two 2 pi rotation over 24 hours, whatever that fellow value comes in radiant per second that you have to take here actually. And so, this if you see this is more and more relevant to this I mean this reality and all that.

But, depending on the application we will talk about that, I mean we will have to take select a particular dynamics for example, if you are just firing a small gun or a small artillery you really do not need to talk about all this complicated thing here because the range is small, the time of flight is small things like that, but if you talk about a ballistic missile application or a satellite launch launch vehicle things like that, then you have to very careful about accounting for all that so, depending on the application we will have to to select whatever appropriate thing is suitable for us actually.

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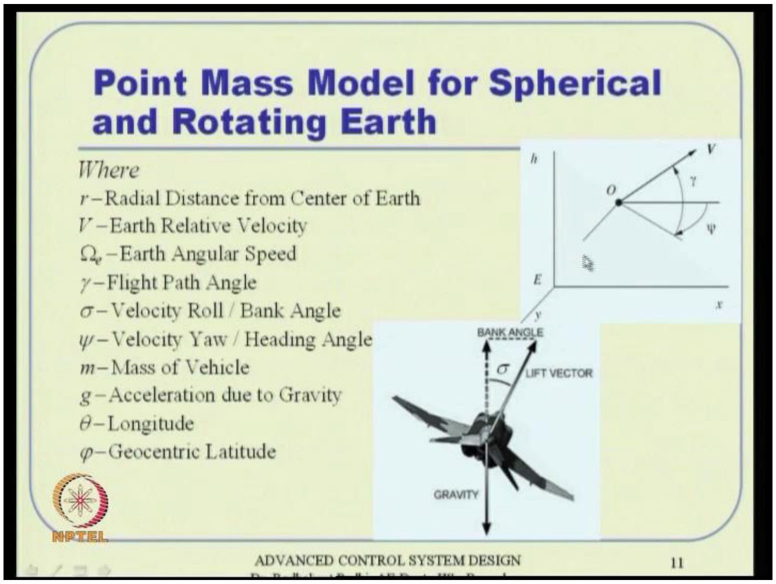

So, these are the various notations which go along with all these equations actually, that is also I will study out this specially, there is a bank angle also where the lift vector turning, the vehicle can bank and that will result in that, and this if you have a this velocity vector can now be resolve into both gamma and psi, and this is if you consider a plane parallel to this local horizontal, and there is a velocity vector direction that, this is a projection of the on that plane, so that whatever gamma this velocity vector makes, this is flight path angle and with respect to a reference line this is actually heading angle psi.

So, this is how we have to visualize the problem and theta is longitude and phi is latitude all sort of things are available here actually. So, all these variables intersections everything is given here. Anyway, so these are still point mass equations remember that, and here the you can think that we have taken sufficiently complex problem, but things can be even more complex sometimes, that people tell what about oblate earth effect; that means, you have this earth is really not spherical and north pole and south pole they are really flat actually. So, if you have oblateness effect, coming into picture then there are correction terms available actually, depending on and where, if you really want to go towards north pole then things will be slightly different there.

And then correction terms will be available associated with this actually, we will not go too much into that actually, for all our many of many applications, staring from artillery to kind of ammunition to aircraft to missiles to launch vehicles everything, it will fall under this set of equations actually, either you consider flat non rotating earth or you consider spherical, but not rotating earth or you consider spherical and rotating earth, it should be good enough for many practical applications actually.

Only on very high accurate missions and things like that, especially ballistic missile applications we talk about this oblateness corrections and all that, otherwise it is not typically not needed actually. Now, these are still what we discussed is all point mass equations, then more details detailed dynamics will come from what is called as six degree of freedom motion actually, 6 DOF model. So, how do you how do you describe that part of it, and this particular class whatever time remains I will not be able to completely derive that, but I will derive a significant portion of 6 DOF equation actually and we will follow on that in the next class also.

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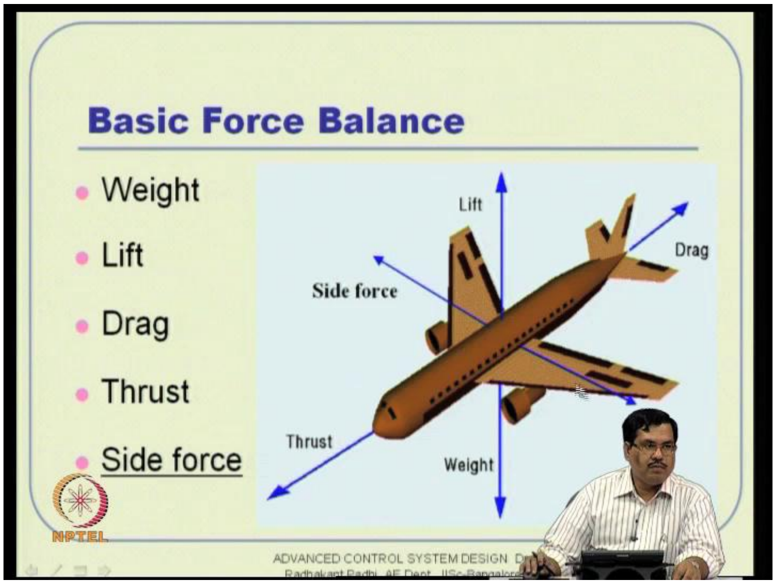

And if you remember one of the previous classes, specially lecture seven this is some of these concepts in a very rudimentary preliminary way we discussed actually. So, if you see any flying object, we talked there are very basic forces acting on that one is weight, one is drug and then there is a thrust force to counteract the drug, and then there is a lift which will balance the weight. Along with that there is a side force, but typically this side forces are balanced out in a in a good flight actually, even in turning when you call coordinated turn this side forces will be balanced out actually. When all the forces will be there, but side forces will be kind of very close to 0 basically, we do not want side forces actually.

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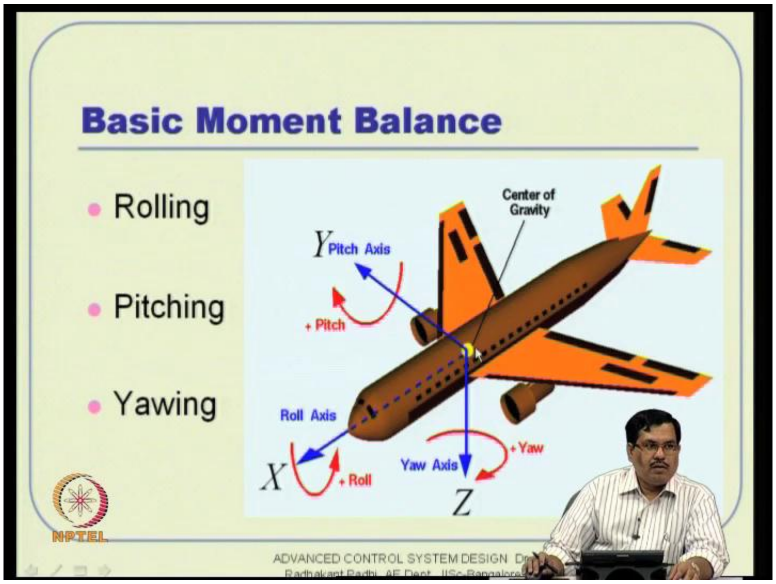

Anyway these are basic forces and then there are basic moments about this axis now we remember, we talked about body axis frame and we visualize at the CG n axis frame now because three direction motions you have to talk about and there is X axis, there is Y axis and Z axis which will form some sort of a left handed coordinate system actually, left handed coordinate system where your where your fore finger will point towards the nose cone, and then the your thumb will point towards the Y axis, and the middle finger will point to the downward direction Z actually.

These axis frames are very critical in understanding the six DOF equation, because you have to resolve this motions of the vehicle in all possible directions actually, all these three directions now once you have once you define the coordinate frame you can graph your finger in the right hand coordinate sort of thing, and then this your if you point your thumb to the fore side then the directions of this your other fingers will give you the positive direction of the rolling action pitching action things like that actually.

So, if you have X Y Z defined that way then the positive roll will be like that positive pitch will be like that, and positive yaw will be like that. So, these are the notes notions we have to remember before understanding anything more actually. So, we basically talk about these forces, ignore side force for a second, then you have weight, lift, drag and thrust and then we have this roll pitch yaw motion actually. So, in this axis frame is still available to you, so remember this is X axis, this is Y axis, this is Z axis here, and this is what you see here X Y Z.

So, along this X Y Z we have forces, and around this X Y Z we have moments actually. So, force level equations will come from, I mean the pull define this translational dynamics and moment level equations will define this rotational dynamics actually. So, that is where you have to see that, and there also we discussed about this control action, then how these control actions can be accounted for and you saw that this Aileron motion, Ailerons are typically employed for rolling about X axis. So, that is the job of the Aileron. And that the then we discus about pitching action which is through this Elevator action, it can this is a rotational motion about the Y axis that is rotational motion about the X axis.

And then you talked about a rotational motion about the Z axis also. So, all this three things are possible as far as rotational motions are there and translational it can go in X direction, it can have a component along Y direction, it can have a component along Z direction. So, all these translational three and rotational three we will have to discuss, and remember Newton's laws are all second order equations; that means, you have six second order equations which will essentially resolved in twelve first order equations later. So, that is what we will have to derive and see the intergraded relationships and all that actually.

And these are all coupled in a way because this, I mean depending on like what you are talking about you can visualize in a approximately decoupled system, but in truly speaking they are all coupled with each other actually. So, you have to account for all these coupling affects and things like that.

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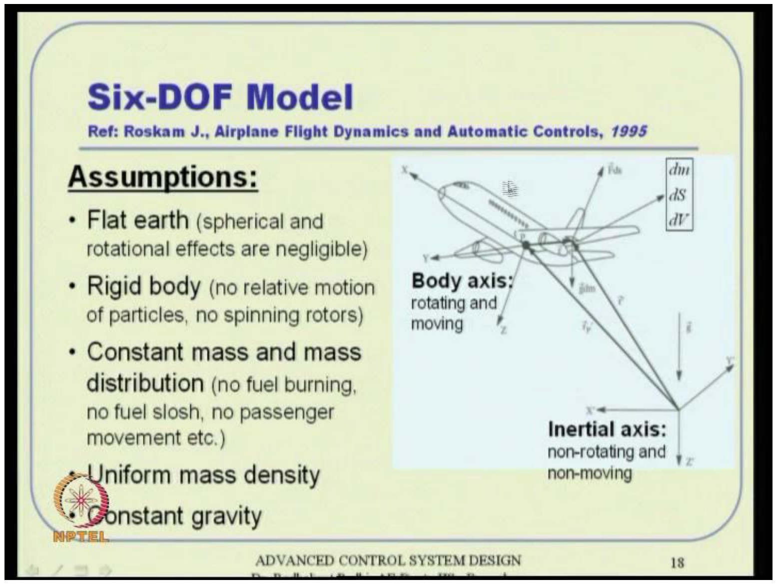

So, let us see what we discuss here. So, assumptions involve again, first we will start with flat earth assumption, so spherical and rotational effects are negligible here, and models are available which will account for that also. So, this is not the restriction that you have to really deal with, but to understand the the equations this is good enough actually.

Then the we have also assumed the vehicle is rigid body; that means, there is no relative motion of particles and no spinning rotors also. So, we are not talking about like helicopter motion things like that actually, that will have a slightly different mechanics coming into picture and that is why helicopter dynamics are also lot more complex than aircraft dynamics really. We are considering here aircraft dynamics and probably launch vehicle and missile dynamics here.

And then we also assume that the there is a constant mass as well as mass distribution; that means, we do not talk about, theoretically speaking we do not talk about fuel burning; that means, we do not talk about CG movements and all that, CG remains fix, the mass remains fix, the no fuel slosh happens, no passenger movement in inside the aircraft, lot of assumption, but remember passenger weight is hardly a fraction of the entire weight so that is that is justifiable motion actually, I mean justifiable assumption really.

We also talk about uniform mass density; that means, if you see anywhere the density of the aircraft remains same actually. So, ultimately it is not a very strong assumption either because we are all talking about rigid body motion where this effect of density is integrated over the entire volume. We will see that, that is where this moment of inertia or other things will come actually. So, it is not a very strong assumption either actually.

Then again we will assume constant gravity; gravity force remains constant throughout the airplane body actually, which is very much justifiable. Aircraft is just a, such a small object compare to Earth dimension basically, just a theoretically justifiable reason actually. So, we just put it, I mean we do not talk about, see theoretically speaking gravity can vary along with height, but we do not talk about that much height variation here so that we have to account for that that gravity variation with respect to height we will not talk about that actually. So, under all these assumptions, let see how we derive all this this complicated kind of equations of motion.

And here we are also, have to visualize one more axis frame which is called as inertial axis actually, it is a non rotating and non moving axis frame, typically it is let us say launch of launch, I means launcher point for the launch vehicles or let say like a airport for a for a aircraft application or it can still be an imaginary point somewhere in the, on the Earth, and sometime people tell, its it is fixed at the center of Earth and thing like that. So, we will not discuss too much on that, all that you are telling is there is inertial axis, and then there is a body axis which moves along with the body.

Inertial axis does not move, but body axis moves actually, and if the aircraft is oriented then the body axis is also oriented along with that because body axis is tightly fixed with the body basically. So, if the aircraft turns the body axis also turns basically along with that. Now if this axis this body axis moves along with the aircraft, then on that axis frame we cannot define the position of the aircraft, on that in that axis frame the position of the aircraft is always 0 0 0, so we cannot get a information of position of the airplane basically. So, what you have to do, and as well as attitude of the airplane, attitude of the airplane because there is if the airplane is rotated the axis frame is also rotated. So, the the attitude information is also lost actually.

In the body axis, the attitude angle is always 0, and the position of the vehicle is always 0. So, for that reason we need another axis frame which is decoupled from that behavior, and hence you can visualize the position of the vehicle as well as attitude actually, that is where this inertial axis comes into picture

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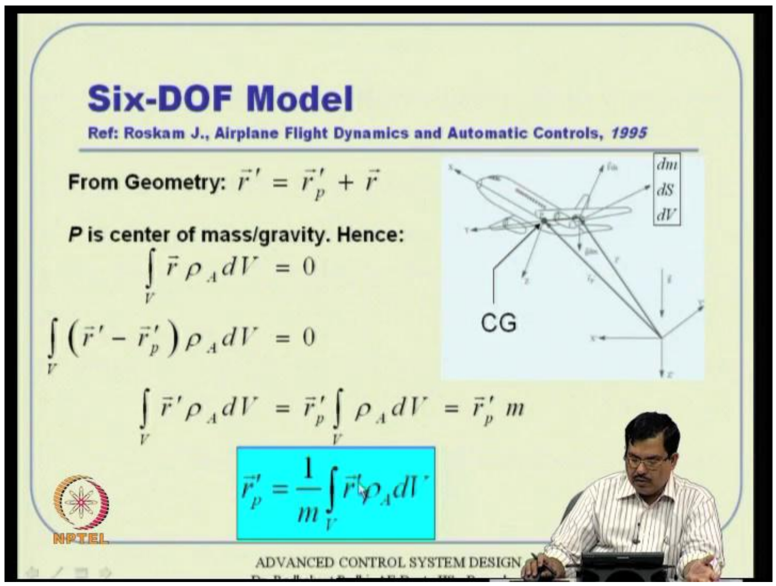

Now let us say as from simple, now we will consider some geometrical aspects and all that, and if you see this we will consider this vector as r p, and we will consider this as some particle on the aircraft somewhere in the aircraft so that vector is r, I mean r prime rather r, this is r p prime and this is R actually.

So, then if you consider this from simple geometry this r, r prime rather is nothing but r p prime plus r actually, and because P is the center of gravity or center of mass in a uniform gravitational field, center of mass and center of gravity are same anyway. So, we talk about P is a center of mass and hence if you consider this is nothing but dm, rho is density of the aircraft, d V is the control volume that you are talking about. So, if you just take R times this dm and integrate over the entire volume; obviously, that is 0, that is the definition of center of mass.

Around center of mass if you just take every individual particle, and then do this volume integral then it should come to 0 actually. So, if you substitute this R, this is r is nothing but this relationship r prime minus r p prime, this r is nothing but this minus that. So, you substitute that and hence what you get is this integral of this R prime rho, rho A dV this particular thing equal to that that is there, but you also know that R prime what is there actually, this particular R depends on where you are situated actually.

But this particular r p does not depend on this particle, I mean particle location, r p is that are the CG that is all. So, I can take out this r p outside the integral, and consider this because of this is nothing but the entire vehicle mass actually. So, hence my r p prime that is essentially the vector which will give me the position of the aircraft, with respect to this inertial frame that is nothing but this actually. So, this relationship you will be use it later actually, keep that in mind. Now we will derive this dynamic equations and this kinematic equation we will derive next class actually.

Dynamic equations are are little more complex kinematic equations are not that complex to visualize actually.

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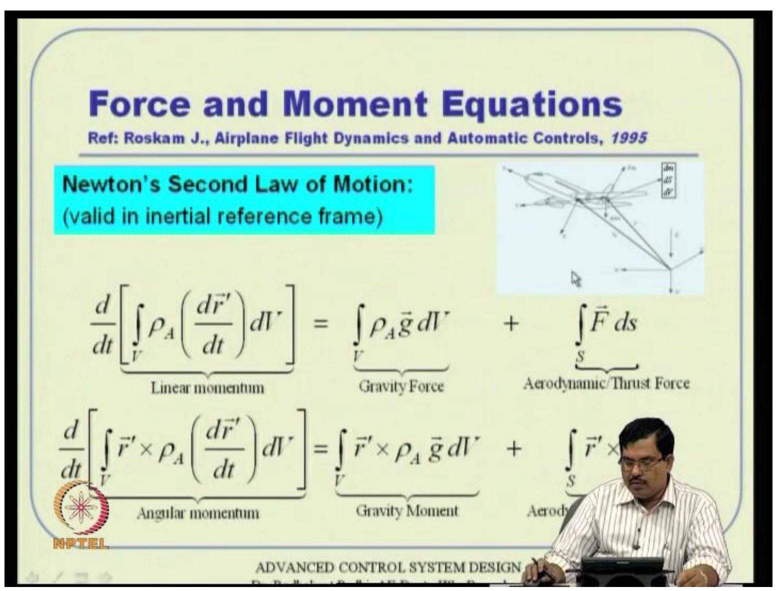

So, dynamic equations are the equations which will directly see Newton's second law of motion actually, now if you consider this airplane again and then want to apply this Newton's second law and things like that, there are two level, two ways we have to, I mean you can we have to apply that one is the linear momentum and another is the angular momentum actually.

So, rate of change of linear momentum, this is the linear momentum of the particles integrated over entire volume. So, the total linear momentum and this is rate of change of total linear momentum, nothing but total applied force, total applied force is, one is gravity force, another is aerodynamic force, remember aerodynamic force depends on surface area basically, if you integrate it over the entire surface area then you will get something. So, this is surface area integral and this is a volume integral, and all that actually.

So, rate of change of linear momentum is total applied force, partly through gravity and partly through aerodynamic and thrust forces actually, and similarly rate of change of angular momentum almost same expression, but there is a cross product that you have to take with respect to r prime actually, and that is r prime. So, you have to talk about that and that the rate of change of that that is the angular momentum is nothing but again r prime cross that, whatever force you have and then r prime cross that whatever force you have.

So, this is nothing but rate of change of angular momentum is equal to the moment generated due to gravity force, and moment generated due to thrust and aerodynamic forces actually

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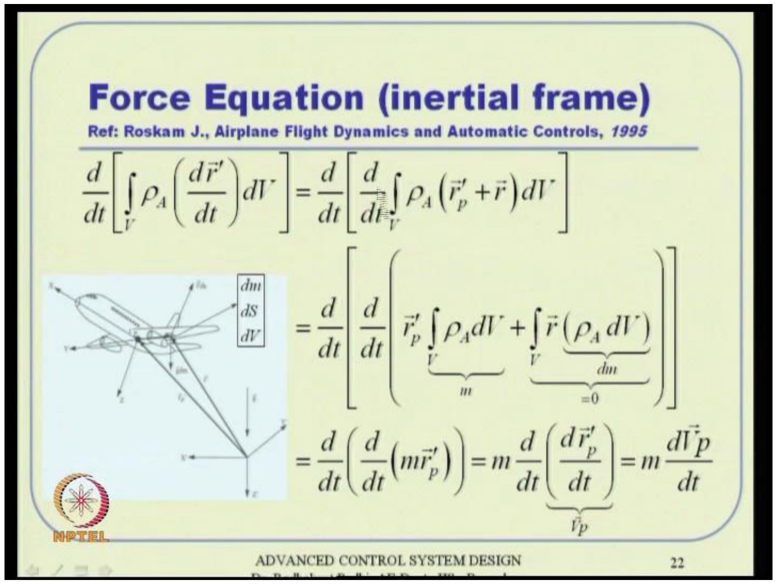

Now, let us analyze these equations slightly more and then tell what is going on here actually. So, this is what you want to analyze here, and remember R prime, R prime is nothing but r p plus r actually. So, this is what will we will substitute here rate of change of this force that you are discussing here actually.

Then it is, this is like this, and remember this integral I mean this derivative I can take out outside the integral now because finite volume integral anyway, and then I can I try to kind of visualize, I mean simply this actually. So, this d by dt out now, and then I talk actually rho a times T V now r p this is nothing to do with this mass where it is located actually, because r p is directly this vector. So, I can take out outside the integral.

And then I live with this r, r is this see this volume integral once I want to take this volume integral, this r I cannot take out of that because this r is inside this volume, it is a function of this where it is located actually. So, I cannot take out that, but r p prime I can take out because that is that is independent of where this fellow is situated actually. So, with that and then again what is this actually, because that is by definition this is nothing but 0 because that is say just one second. So, this is that nothing but center of mass. So, again that is by definition center of mass this is 0.

So, we are left out with this d by dt, d again d by dt of this fellow is mass total mass of the vehicle and something like this actually. So, we are left, ultimately you are left out with m into dV p by dt. So, V this velocity vector whatever is there with respect to this center of gravity, the total velocity vector will pop up like that actually, m into the total rate of change of this entire velocity vector, that is all we are talking here this entire expression actually

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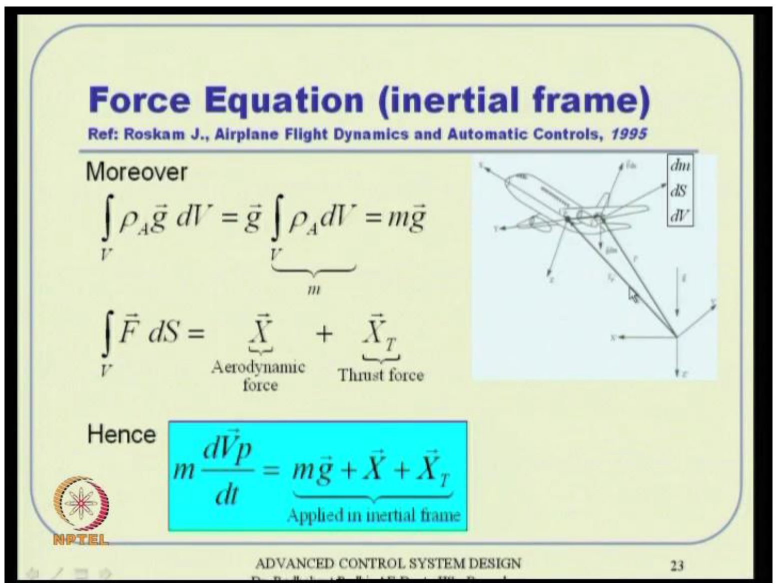

Now what happens here, this particular thing, this is all about left hand side, what about right hand side. Right hand side terms are nothing but this is easy because gravity is constant uniform gravitational field that is what you talked about.

And this integral is nothing but the entire mass of the vehicle. So, it is having m time's g basically, and this thrust is actually a surface integral, it is not a volume integral, let me correct that, this is surface integral this is nothing but aerodynamic force and thrust force actually taken together. So, what you have here, this is like, this is the left hand side that we saw, left hand side expression is equal to the right hand side expression, now this gravitational force is nothing but m g. So, that is remember these are all in Victorial notation actually. So, all the the all the components level and all we have not discussed yet if the total Victorial thing that we are talking here actually.

So, m g plus this aerodynamic force is that X vector, and thrust force thrust force is nothing but X T vector. So, this is how the relationship turns out in the Victorial notation actually

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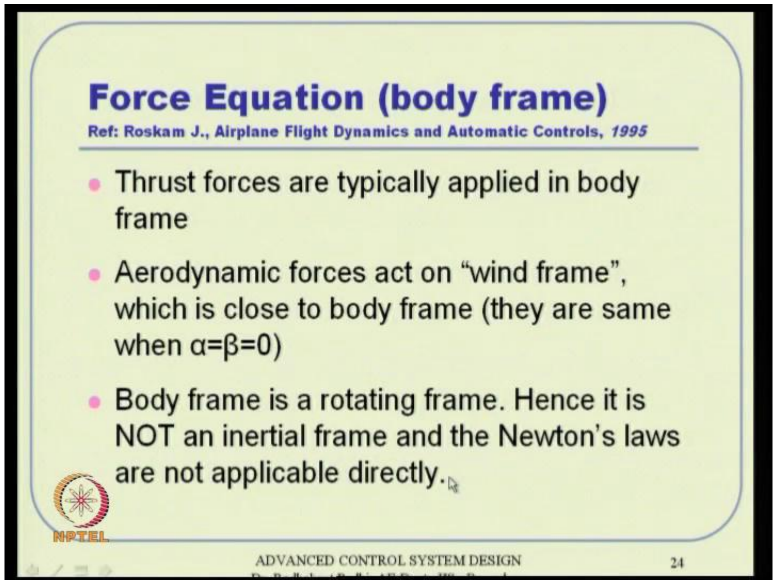

Now the problem here is, thrust forces are typically applied in the body frame they are not applied in the inertial frame right, what you discussed here all this thing Newton's laws are remember they are valid in the inertial frame, they are not valid in the body frame because body frame is really not in inertial frame.

So, what happens the thrust force is, but the thrust and aerodynamics forces typically happen in the body frame anyway, all forces movements happens they are acting on the vehicle in the body frame. So, aerodynamic forces typically act on what is called as wind frame, which is actually close to body frame and they are exactly same when alpha and beta are 0, angle of attack and sideslip angle when they are 0 then these things happen to be 0, more on that we can see from applied dynamics book actually.

So, body frame is a rotating frame and hence it is not an inertial frame. So, that the locking effect that is coming here is, we cannot rely on this thing as it is actually. So, you have to take the help of something else before you visualize the comments of the forces actually. So, the because Newton's laws are not applicable directly in the body frame, what we saw here is all happening in the inertial frame actually, everything happen in the inertial frame. I want to have a body frame, I mean the body frame levels of equations of motion, because that is where the real action takes place as far as forces and movements are concerned actually.

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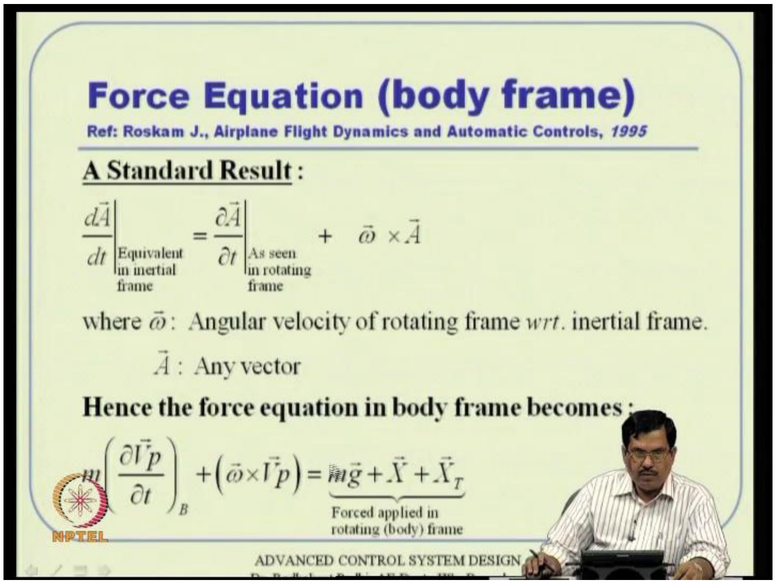

So, if I really want to visualize in body frame then I have to talk about a standard results in vector theory, if you take any vector A then this equivalent what will interpret in inertial frame is nothing but as in the rotating frame that you take about whatever rotating frame effect, plus this omega cross A the rotate rotational weight will come into picture. So, whatever angular velocity of the rotating frame with respect to the inertial frame that is omega, and A can be any vector. So, this relationship we have to account for if you really want to apply that in the body force actually, in the body frame.

So, the same equation that we have here, suppose I want to apply that in body frame from the left side of the equation, I have to modify that and tell now that is valid, if this the right hand side happens to be in the body frame then the left hand side (( )) should be this way not the other way. Otherwise if the right hand side happens to be in the inertial frame I can simply live with that. So, this is where the critical relationship comes actually, and this is where it will generate this, what is called as carioles components actually, these are this omega cross V p which is actually, that actually makes the aerospace dynamics complicated if you ask me, that is the only reason why aerospace equations look complicated actually. So, this cross sort of thing we have to account for that all the time actually.

Anyway, but we have a relationship now so we do not have to be confine with respect to only to inertial frame. So, let us visualize this expression all happens, as it happens in the body frame actually.

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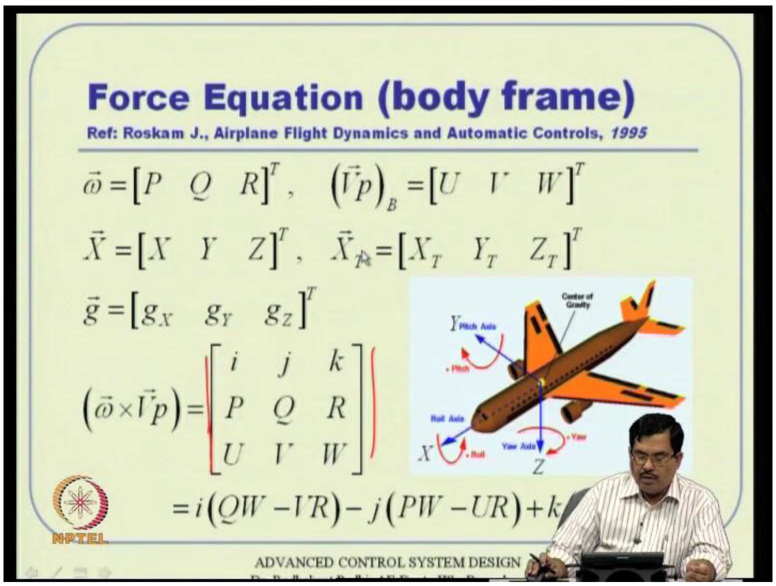

So, before we move, I mean before we move along we will to decompose this now we will decompose this vectors notations into components now. So, this entire rotation rate is consist of P Q R, P is about X axis, Q is about Y axis, R is about Z axis like that, in the body frame similarly, V P the total velocity vector will have U V W component, the total position will have X Y Z component like that actually.

So, we decompose that this X remember are aerodynamic forces, this is not position coordinate they are forces actually. Similarly thrust forces will be decomposed into X T Y T Z T actually, gravity also will have a component X g X g Y g Z, because this X Y Z is not parallel to the inertial frame, these are oriented with respect to inertial frame actually. Now this omega cross V p, this particular term we have to evaluate that way, this matrix form determinant form like that actually, we know that the cross product evaluation basically.

You have to evaluate this sort of determinant notation basically, and this is if you really want that then probably you have to put one more bar here, determinant there and then it is like if you evaluate that. So, I times this Q times W minus V times R minus Z times p times W minus U times R plus k times this P V minus U Q, so all that is there actually. So, this term I can write that in I j k component in the body frame like that actually. So, once I do that, this particular thing other things are very straight forward anyway, this is this is in body frame, so this is m U dot m V dot m m W dot, this is very straight forward.

M g X m g Y m g Z will be there, and this will have X Y Z component anyway basically.

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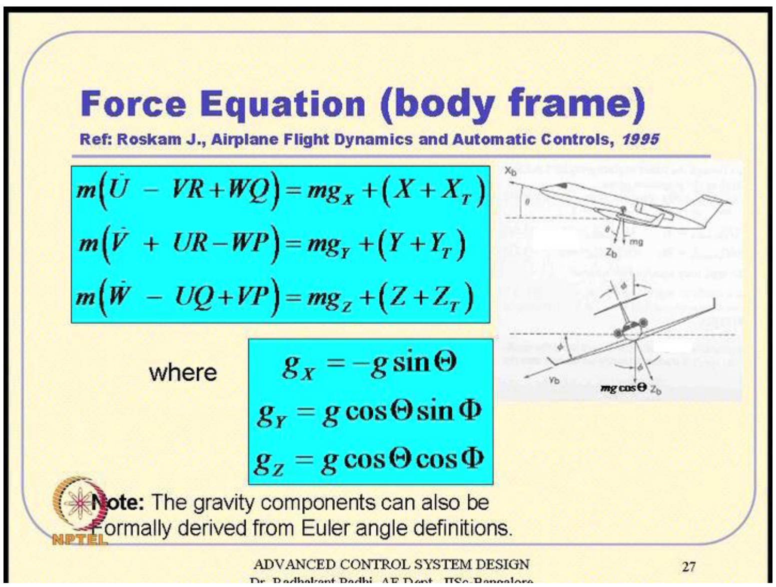

So, now, taking into account all that, I will be able to write this equation that way, Remember, this is the first m U dot, m U dot comes from here, then this I component of that which is nothing but this component actually, so that will come here. And then that is equal to the gravity component and all that m g X will come here, and then this is X direction forces actually, so that will remain there like that. So, similarly it'll happen to all U V W components actually.

Now, what about g X g Y g Z? Eventually, it can be derived in a more formal way this these components and all that. Now, this attitude angles will come into picture, it is a function of for this pitch angle theta and roll angle phi basically, and very quickly you can visualize that this is the gravity force that is acting alone. So, if the aircraft is pitched by angle theta, then this particular component g X, remember g X is opposing to this X direction basically. So, this particular component will be nothing but minus g sin theta basically, this angle is theta. So, we will have a sin theta component along that direction basically, little component there.

So, that is where it will have, and then if it is not only pitched, but after pitching it is also rolled actually, this is a pitch angel there is a roll angel also, and then this roll angle will generate these two components actually. So, one is g sin theta which is opposing to X vector, then g Cos theta will have two more components; g Cos theta into sin phi, and g Cos theta into Cos phi. So, one will act along the Y direction, one will act along the Z direction, it can be derived more formally also, we will try to see when the next class probably. But pictorially you can see that one component what is coming as m g Cos theta here, that will have two more component depending on this phi angle actually, so m g Cos theta resolve into this Cos phi sin phi component will act along that actually. So, once you put that together and then try to sort out what is U dot V dot W dot, that is where you'll we will get U V W dot actually.

We will see…. You got it right? If put this here, and then solve for U dot equal to something, V dot equal to something, W dot equal to that. So, you will get three state equations from there.

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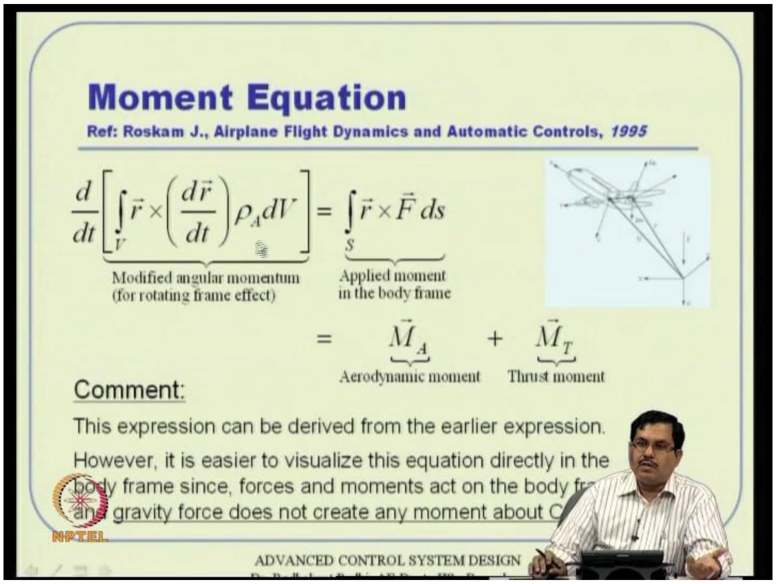

Now, these are all about force level equation, what about moment level equations? Again, moment level equation we have seen this is the left hand side of the equations, and then this can be derived from inertial frame to body frame through certain algebra and all that, you can see that details in this particular book, but I will try to simply the matter by directly applying these forces, these moments in the body axis itself.

So, and remember that the resulting moment of the gravitational field about the center of mass is 0, that is the uniform gravitational field anyway. And right, I mean the gravity force is acting everywhere in the same quantity actually, all the particles of the vehicle. So, it will not result in any specific moment because of gravity, the entire vehicle is pulled down basically, there is no rotation effect, there is no differential force between frame molecules and all that actually. So, I will ignore that term and then proceed further actually.

So, what you have? This is nothing but modified angular momentum term for rotating frame effect. We have noted that already that this also has to be taken into account, that is where it makes complicated actually. make life complicated Because, that vector a can be any vector, and this applied momentum is also vector, so you have to talk about a rotating effect with their also actually. So, what is that?

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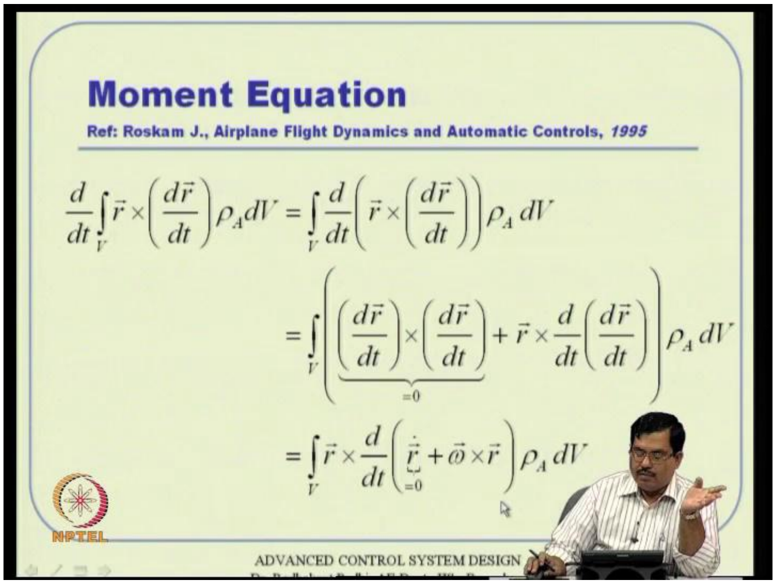

So, this particular term what you see in the left hand side, this d by dt of that this is nothing but that and this particular thing, this is the d by dt of a cross b. So, that is d by d a by dt cross b plus a cross d by dt of V basically. We interpret that as a vector that is d that is b vector. So, d by dt of a cross b is nothing but d a by dt cross b, b vector is that plus a vector which is r cross d by dt of b vector. So, and if you simplify this is any vector with cross product with respect to that particular vector is 0, we know that. Cross product of the same vector that is 0.

And then that is what is left out is again d by dt of that, again that has to be resolve into this this vector notation component actually. So, this particular thing is nothing but first is r dot, then plus omegas cross R also. This particular quantity what you see here, we cannot directly apply that actually, you have to apply through this angular relationship actually, this cross product relationship. So, that will result in r dot, and remember r dot is 0 because there is no movement of the particle with respect to the body frame axis. So, I mean a particle zone to move they do not vibrate, I mean there is a rigid body airplane right. So, because of rigid body dynamics r dot is 0, and then we are left out with only these quantities.

By the way, this book derives this equation in a slightly more complicated way, I try to simplify it as much as possible actually. It keeps this here and then derives, then further down the line it takes it 0 actually, it will result in double dot and things like that, we do not need to do that, right away you put it 0, that is what it is actually. Then we talk about this particular quantity what you have here. This results in this particular thing, r cross d by dt of omega cross r that is what you are left out with, again you have to apply this moment, this rotational effects and all that.

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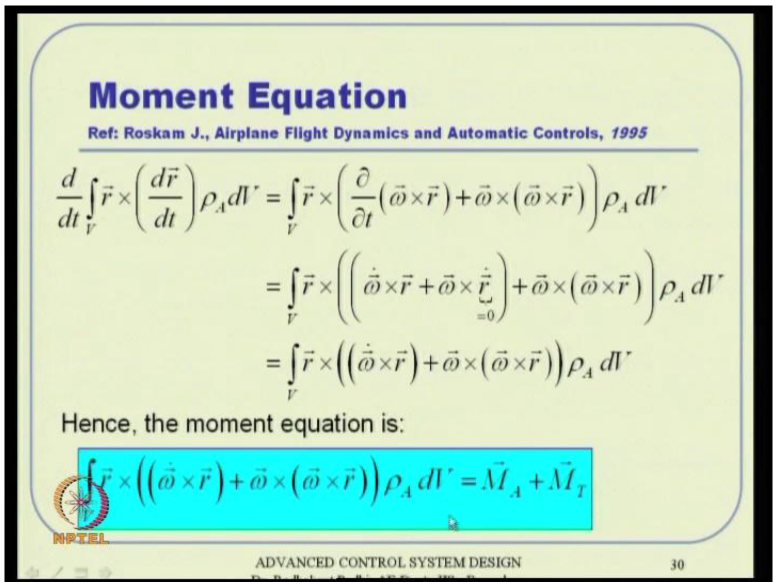

So, r cross d by dt omega cross r. So, this is r cross del by dt omega cross r plus omega cross omega cross r, again and again you have to keep on applying that to simply that actually. Wherever you see d by dt, you have to keep on applying that way actually, that is where thing become more and more complicated basically.

But, none the less it is, I mean you can make it you can track it basically, what is happening. So, if you simply this equation, now if you put it there and then omega dot and, then R and then r dot is 0 again, and you will be leaving out with that actually. So, the moment equation if you see, this left hand side is something like this complicated expression what you see here is nothing but the applied moment actually, through aerodynamic as well as thrust. So, this is rate of change of angular momentum through that, and then you have to simplify that; obviously, component level thing.

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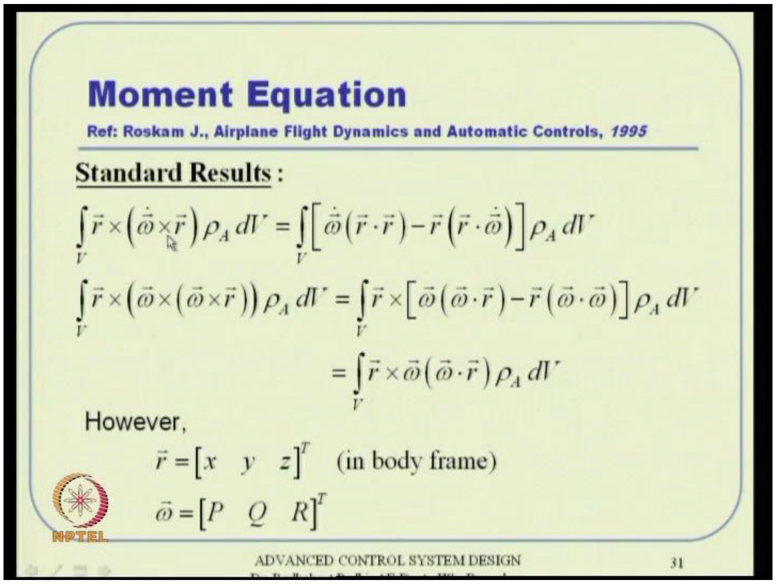

There we have to talk about some standard results in vector theory again. So, we have a triplet basically, a cross b cross c, that relationship you bring in. And then you try to put in all this expression again a cross b cross c, and then c also production and all that actually. So, you apply all this, it is not difficult to kind of see this long end algebra, and then you r is nothing but x y z components in the body frame, the body frame remember that it is not the position of the vehicle, it is x y z with respect to the center of gravity of that particular d n that we are talking about, that particle actually.

So, x y z is the distance from center of gravity of that control mass or something, control volume what you are talking. Now, omega has P Q R components, and then you are able to kind of decompose that because you know this, what is beauty of that? Entire thing now resolves, I mean whatever this cross product will have dot products now, dot products are nothing but scalar component, and then we will have one one vector associated with that. So, that is easy to simplify after that actually.

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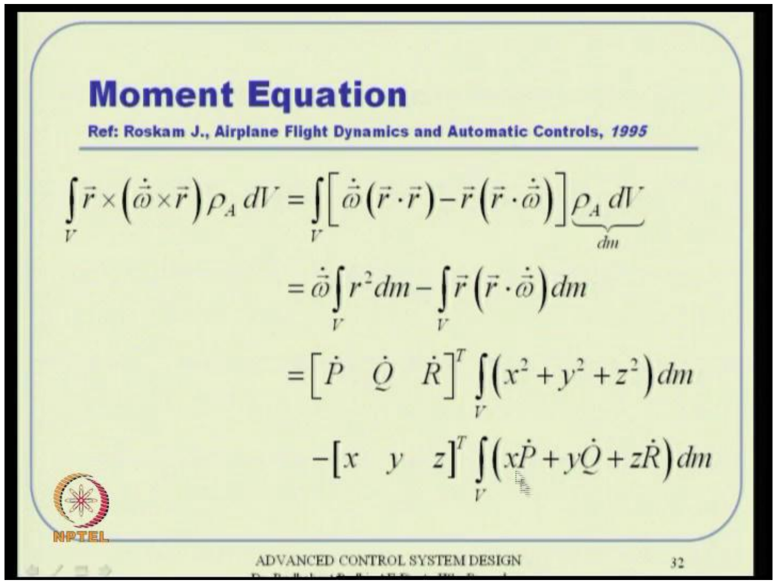

So, you put that all these things together and then try to solve, try to simplify this results in dm. So, it will go through this for example, this one r square, r square is nothing but x square plus y square plus z square. And similarly, r dot omega dot actually, that will result in that; these are dot products now it is easy to see that.

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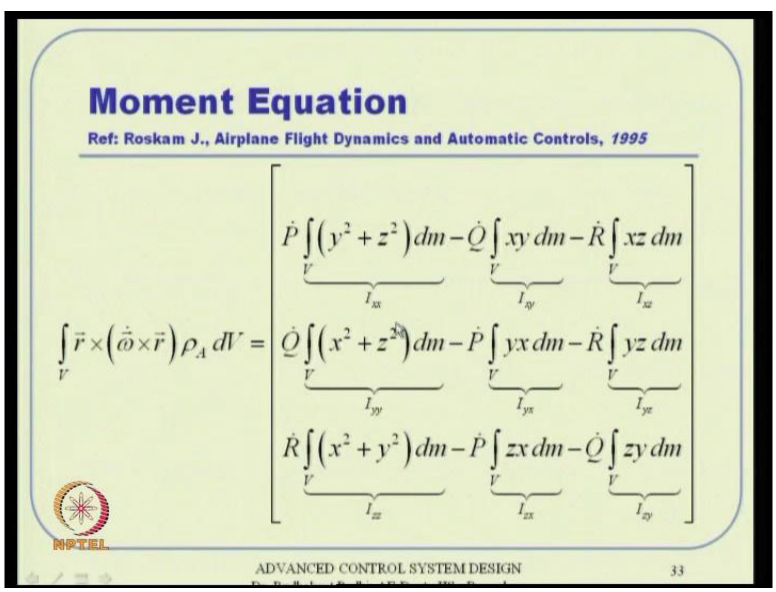

And then once you simplify this thing, it will result in some expressions like that, and this is need to see because all these things are nothing but moment of inertias, these terms what you see here is nothing but I xx, this term what you see here is I yy, this is I zz principle moment of inertias.

And these terms are nothing but cross moment of inertias actually. So, as long as the rigid vehicle dynamics is concerned, I really do not need to know the mass distribution and things like that, I just need to know the lumped quantity called moment of inertia. So, that is typically supplied to us actually, for the control design us actually, supplied to us from structural engineers essentially.

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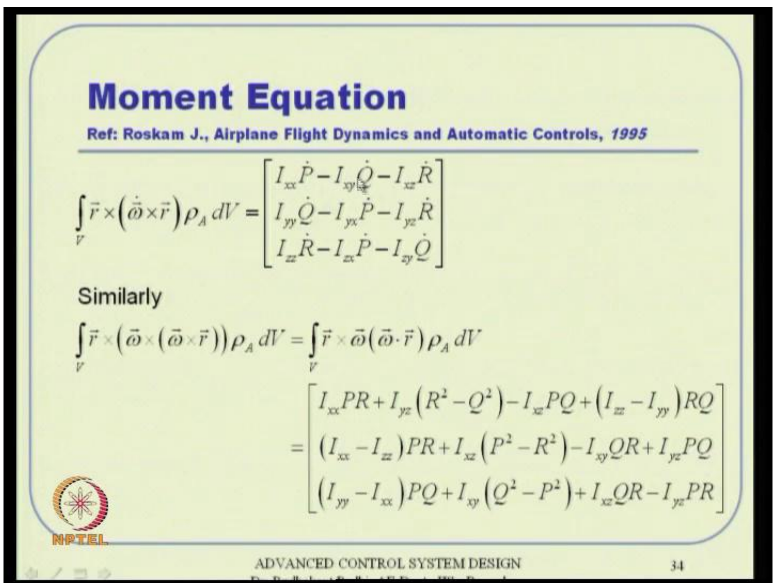

So, this entire quantity in terms of moment of inertias, I can write it in a very neat way. So, entire thing which look so complex here, this level is all reduces to in terms of moment of inertias, this is rather simple looking expression basically.

Similarly, this other component that we left out can also be derived in the similar manner, and you have equivalent expression like that actually. So, you have this now you have that.

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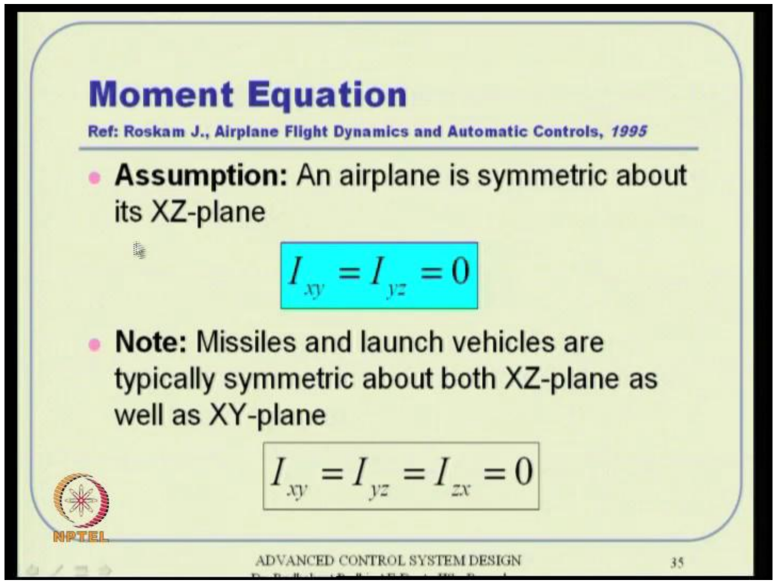

Now, put them together, and then bring in the assumption that airplane is symmetric about X Z plane, in the X Z plane if you tear the aircraft, it is all symmetric with with left and right side. So, in that situation I x I x y and I y z will be 0, this close moment of inertia. And in addition to that missiles and launch vehicles will have symmetricity about X Z and X Y plane both actually; they are symmetric about both the planes.

So, all this cross moment of inertias will be 0 in those situations. Otherwise, this is anyway true for aircrafts also basically. So, that will bring in further simplicity in this equation basically.

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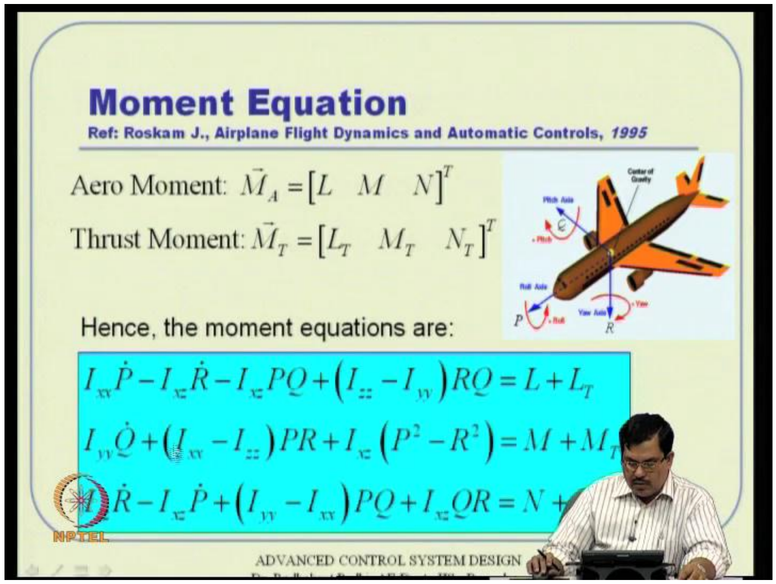

So, you will be left out with only these equations later actually, what you see here. Because applied aero dynamics components are L m n and thrust are like that, so that is you are left out with that. So, we can solve this P dot Q dot r dot from here, because remember this is a linear equation anyway, it is very clear here.

P dot R dot are coupled through I X Z, the moment it is not therefore, missiles and all this is also even not there. So, directly you get p dot Q dot and R dot here, because I X Z is not really 0 for airplanes, you will have some coupling effects for P and R actually, P dot and R dot, and nevertheless these three equations are linear. So, you can solve it, and then get it for P dot Q dot R dot.

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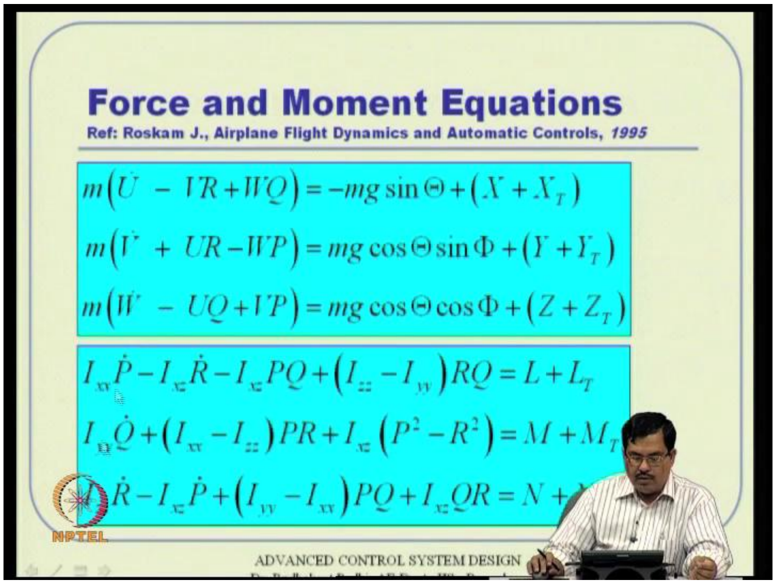

So, what you are getting here ultimately, is these force level equations and this moment level equation actually.

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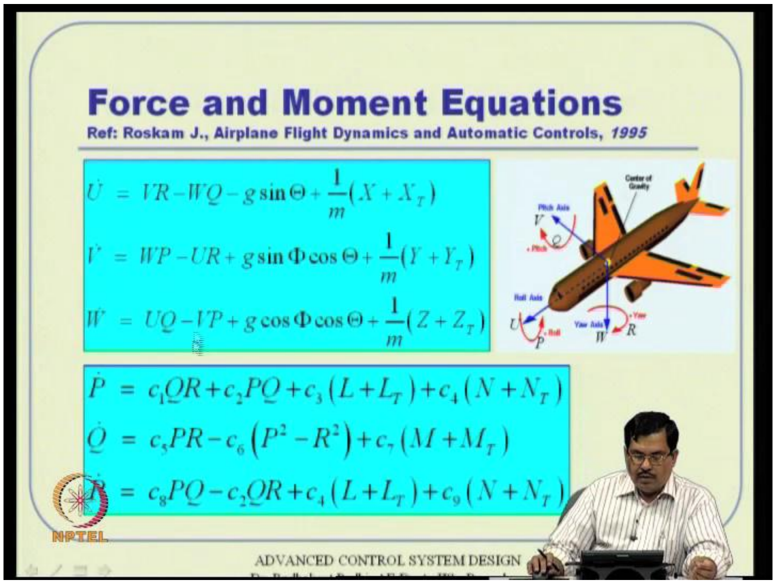

And if you solve this for U dot V dot W dot here, p dot Q dot R dot here, that is why you get it actually, U dot V dot W dot and p dot Q dot R dot.

So, these are what is force level equation, this is moment level equation, these are coriolis components comes from rotational effects and all that. This is gravity term, and this is the aerodynamics and thrust moments, external forces. And similarly, here the gravity term does not come remember that, moment level gravity does not play a role, and this is the coupling equation that you talk about.

Here the control surface actions will be significant here by the way, and this will have a very minimal effect, aerodynamic control surface action basically. So, details will be, I mean once you understand this details you will be seeing these things actually. So, now this c 1 c 2 what you see, again this c 1 c 2 c 3 all these are functions of moment of inertia which is given like that, it can be easily solved. I mean, from these equations what you have, you will solve for P dot Q dot R dot, as a byproduct we will be able to solve this actually and you will get c 1 to c 9 all the things are like that actually. So, it is easy to compute these constants actually.

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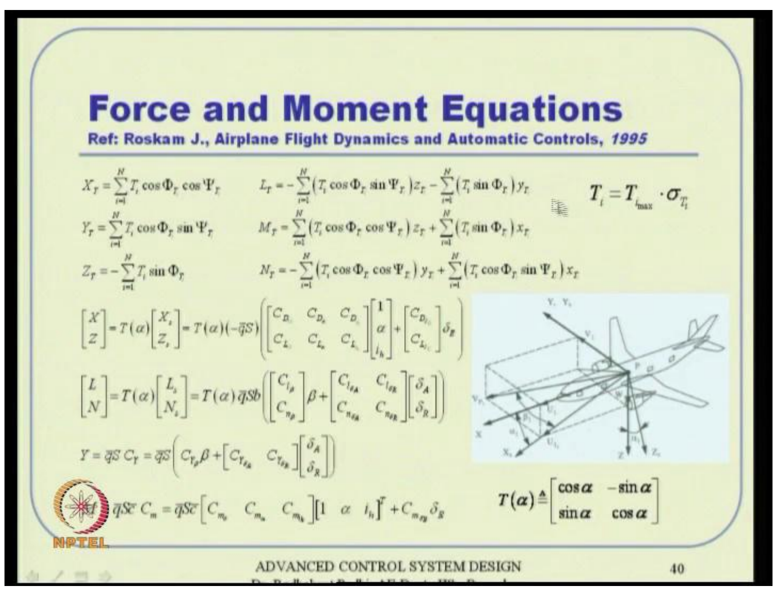

And then force and moment equations makes life more complicated, because of this aerodynamic forces and thrust forces and all, will be given in the several component level equations, considering various angles and things like that. So, this particular thing this remember X T Y T Z T is force level equation, I mean and then L T M T M T L T M T N T are they like moments and all that. So, all this expressions, you have to put it together in those equations to get these force and moment level equations actually.

That is where this dynamics become complicated; however, once you understand this, I mean this is we still can talk about all different directions of the velocity, and when we can in a component level and as well as moment actually. So, this is all dynamic equations, so kinematic equations and all I will talk in the next class actually. Thanks a lot.