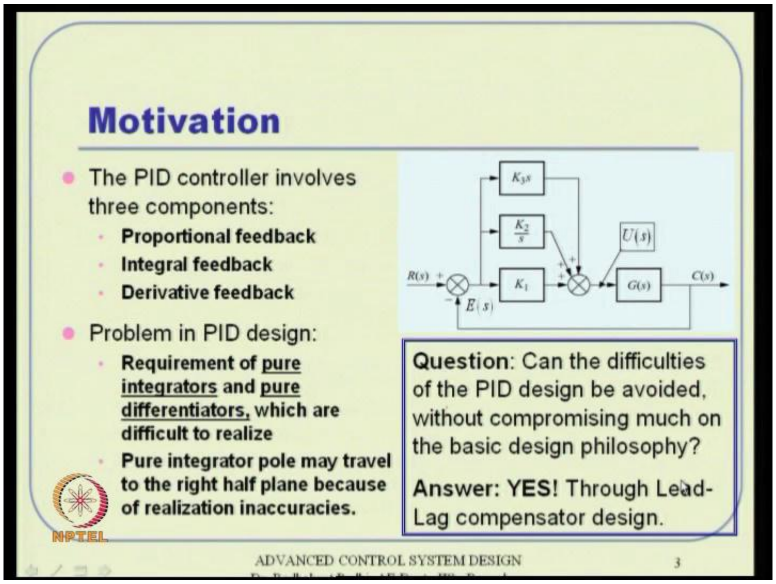
Advanced Control System Design
Prof. Radhakant Padhi
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Module No. # 02
Lecture No. # 05
Classical Control Overview - IV

First in the series is we have in first topic is lead lag compensator design, which is some what extension of PID control design. So, let us see how how it is operates actually.

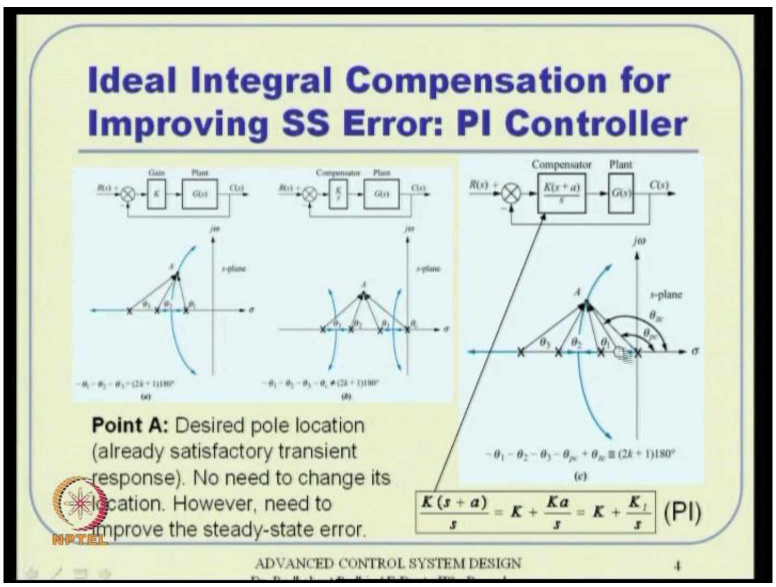
(Refer Slide Time: 00:33)



Last class, we saw that PID control design essentially has three loops; one is proportional feedback, and one is integral feedback and derivative feedback. This is how it operates; proportional feedback, integral feedback and derivative feedback. However, the problem in PID design is requirement of pure integrators and pure differentiators right; I mean, if you see these two loops, what you need to realize is, pure integrators and pure differentiators, which are normally difficult to realize. And in addition to that, there is also a small danger that this pure integrator pole, which is supposed to be there exactly at the origin that means, the pole is somewhat here in the origin, by realization difficulty like that inaccuracy or whatever. There is a danger that this pole may slightly be realized in the right hand side actually then the system becomes on (( )) and all that actually. So, it is not a good idea to do that kind of a thing.

So, the question arises is, can you avoid this difficulty, there all right just a second. Now the question is, can we can this difficulty of the PID control design be overcome, but we do not want to compromise to the too much on the basic design philosophy, we want to retain the basic design philosophy; where at the same time we want to avoid this difficulties of the PID control designs, can we do that? The answer turns out to be yes, and that is how we will do that through something called lead lag compensator design. Sometimes it is also called as lag lead compensator design, it does not matter the concept is same anyway.

(Refer Slide Time: 02:19)



So, let us understand the ideal integral compensation that is what typically we use for improving the steady state error performance actually. So, essentially this does not have a d loop, but it still has I loop that means it is a PI controller. So, the realization difficulty is having something like this, how do you do this, this is this is the original plant, the problem is like this, this is original plant, this is a root locus for the plant.

And then we have a point A, which is a desired pole location, because this particular point A gives us the required transient performance, transient performance there is no issue actually.

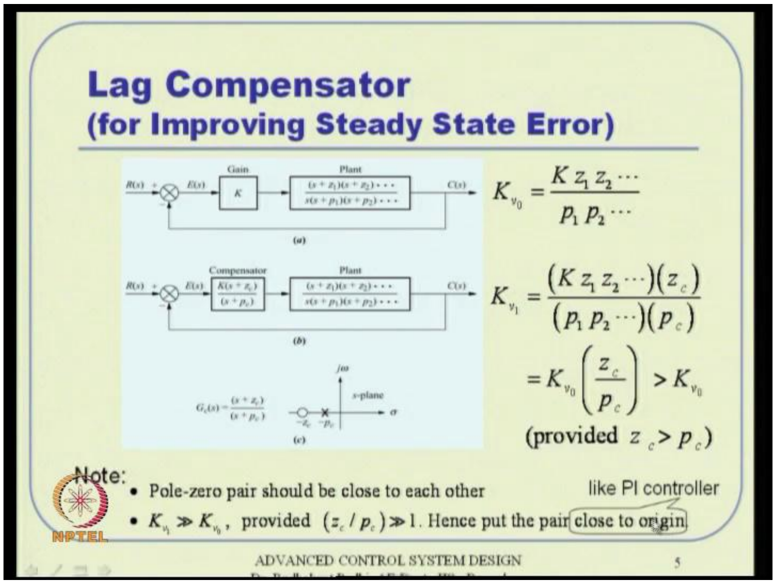
However, the steady state error is not good. So, we want to improve that through this integral compensation. But the problem here is once you have the integral compensation in in place then the root locus is no more like that. This root locus is somewhat different, so obviously, the point A which is your desired pole location is no more on the root locus; that means simple gain adjustment will not do the job actually.

So, now the question is how do you overcome that, obviously one idea is just put a 0 along with that pole actually very close to each other put a put a 0 close to the pole; once you do that, if you see the angle contribution essentially it again becomes or multiple of 180 degree. So, essentially the root locus passes through that, originally it was 180 degree or multiple of 180 degree, the moment you add one one pole then the extra angle contribution made it unequal.

However, you do not want the pole to go somewhere else, you just want it here itself, because that gives you require transient performance already. So, by adding of 0 close to each other then this two angles are roughly same; so, that means you will not you are not compromising on the transient performance, yet improving the steady state performance.

So, what you what you look at it, you made a compensator not like that, but somewhat like that; and what is it (Refer Slide Time: 04:22), this particular compensator can be visualized something like this. If you if you expand, it turns out to be a pure integral controller I mean pure PI controller, but you want to avoid that.

(Refer Slide Time: 04:41)



So, what you do, the whole idea is can we shift this pole 0 pair whatever pole 0 pair it it appear to be, can we shift it slightly left slightly left; see, this is almost like on the origin, but it is not really on the origin actually, it will only do the job actually that is the question.

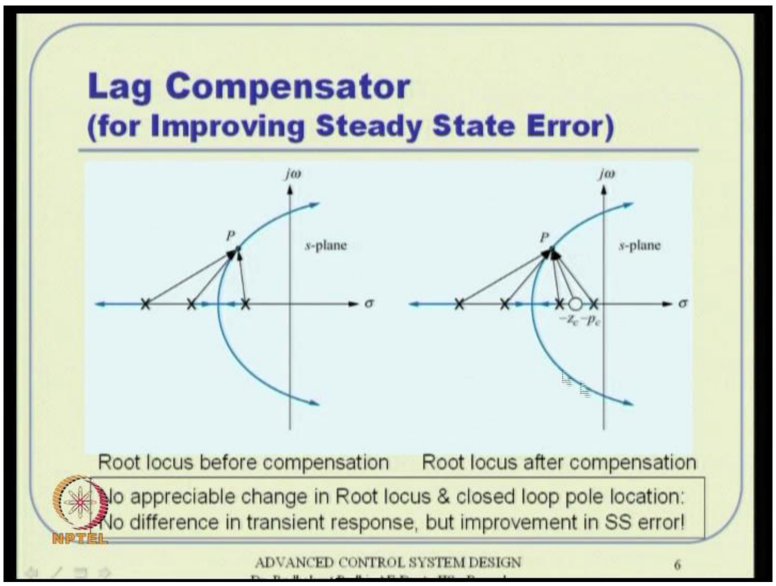
Now, it turns out that if you if you I mean analyze this error constant for example, let us say you talk velocity error constant then, originally it was something like that without any I mean with only with the proportional loop, not I loop. However, with the compensator remember this no more a PI controller, it is a lag compensator; the lag term probably comes, because the 0 is lagging the pole in that sense actually, the 0 is never (( )) probably there will get (( )) difference may be.

So, because of this compensator now this K v 1, the new error constant turns out to be that one, because the extra 0 will come here, extra pole will come here. And as long as that z c is greater than p c that means this error constant will be greater than the previous error constant. And also remember that, compared to that, if you have a type one system, then the steady state error is inversely proportional to that actually.

So, I mean the if if this one, this error constant becomes larger than the previous one obviously, the steady sate error becomes lesser. So, in that sense we are still able to do the job without compromising on the the PI control philosophy image. Now, the question is how do you do that how do how do you make it sufficiently large compared to that actually?

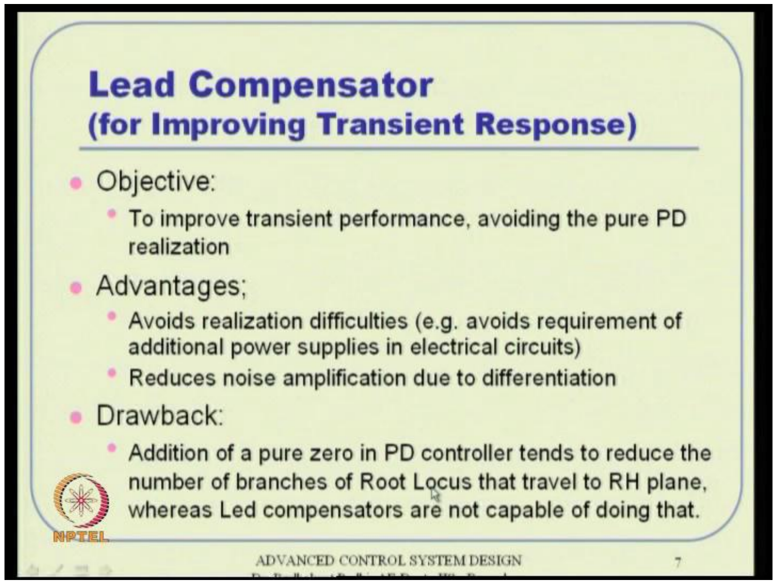
So, first thing is you were to put this pole 0 pair close to each other, because that will not add to any angle contribution much actually. Second thing is to make this substantially larger you make this ratio substantially greater than 1. So, how can how can that happen? This this ratio substantially greater than 1 you can I mean you can do that, by putting the pole pole 0 location very close to origin actually; because if this is let us say 0.01, this is still 0.21 let us say, then you have 0.1 divided by 0.01 which is like 10 times actually. So, you can still improve this this ratio by 10 times by putting close to origin actually. So, that is the first philosophy of lag compensator essentially.

(Refer Slide Time: 07:01)



So, by putting it very close to origin it is like a PI controller actually. So, that is what that is what it is. So, this is the the overview of originally this pole was here and then by by a using a lag compensator, we have been able to roughly retain the same root locus same root locus, yet improving the steady state error actually, that is the whole idea there.

(Refer Slide Time: 07:20)



Now, coming to the lead compensator, the objective is to improve the transient performance instead of the steady state error, what we can I mean ideally we can do that using of your PD realization, but again we want to avoid the pure PD realization, because we want to avoid the differentiation loop.

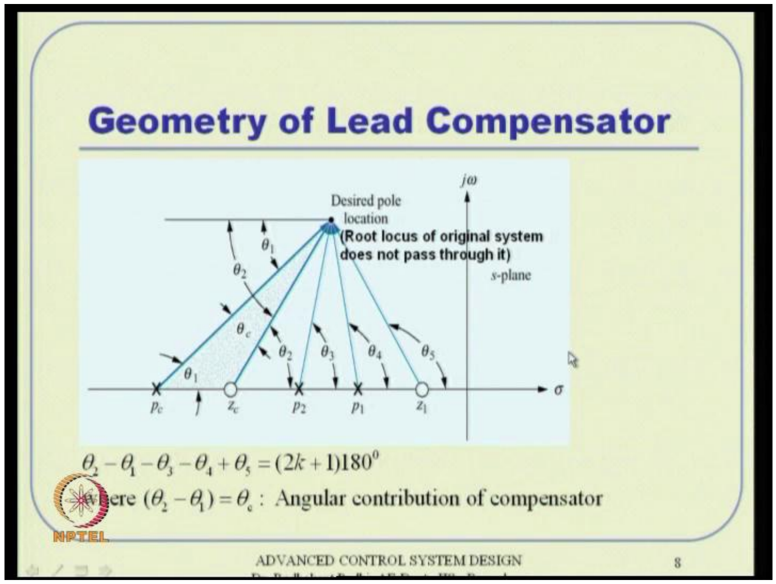
The advantages, primarily it avoids realization difficulties that is it avoids requirement of additional power supplies in electrical circuits for example; that means, if your differentiators can be realized through some through some devices, which requires additional power supplies that can be avoided you can realize this through passive devices.

And then more important it actually reduces the noise amplification due to differentiation, because it is not a pure differentiation. We know that, pure differentiation is a noise amplification property. So, that one gets reduced actually.

The drawback turns out to be that pure zero in the PD controller tends to reduce the number of branches of root locus that travel to the right half plane. So, having a pure zero I mean in the PD controller changes the root locus behavior, the the root locus does not go to the right half plane actually. So, that gives you additional flexibility of improving your gains and all that, because root locus does not go to right half plane anyway.

However, this one I mean using this lead compensator you are not capable of doing that that is only drawback actually.

(Refer Slide Time: 08:49)

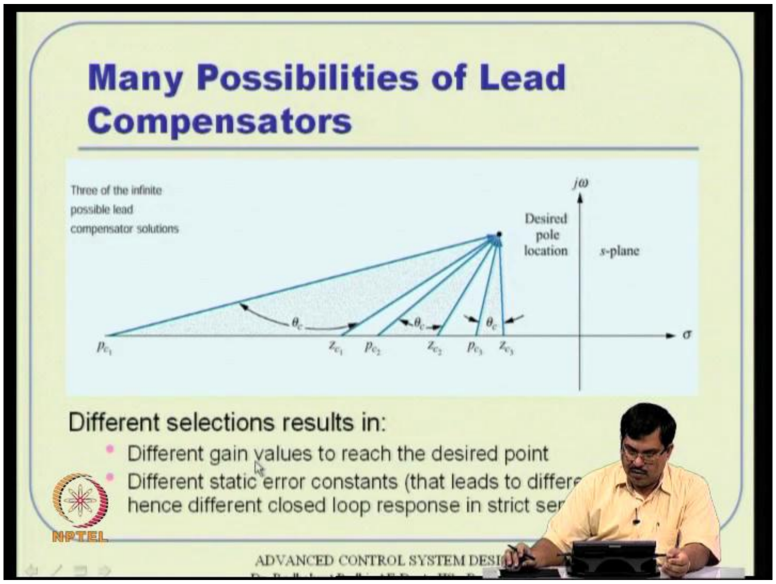


Now, let us see, the understand the philosophy. So, what you are doing here is the desired pole location was somewhere else, we want to make it here actually somewhere in the desired root loop I mean desired pole location actually. And remember, because this is not part of the original system that means the original root locus does not pass through it. So, hence we cannot just I mean tune the Gains. So, that there I mean select just tune we are not able to tune the gains, so that we will get the desired performance actually.

So, how do you do that then, primarily there is a problem, because this angle what you really want; remember root locus is primarily coming from angle condition, (( )) the angle contribution 0s minus pole should be (( )) 180 degree. So, if I add a pole 0 pair again here something like this, then what happens actually? There is additional angle contribution through this difference theta 2 minus theta 1 that is the additional angle difference which will that this angle; if you see this this side, this theta 2 minus theta 1 is that angle actually.

So, this additional angle will make sure that, the root locus passes through that particular point that you are interested in. And hence you know this what is a gain corresponding to that and you will be able to select their gain actually. So, that is the whole idea. Now, what it what it tells us, as long as this angle is same this angle is maintained, the pole 0 locations can be anywhere actually. That means I can put this pair anywhere on the l axis wherever I want, as long as they contribute this same this this angle difference out there.

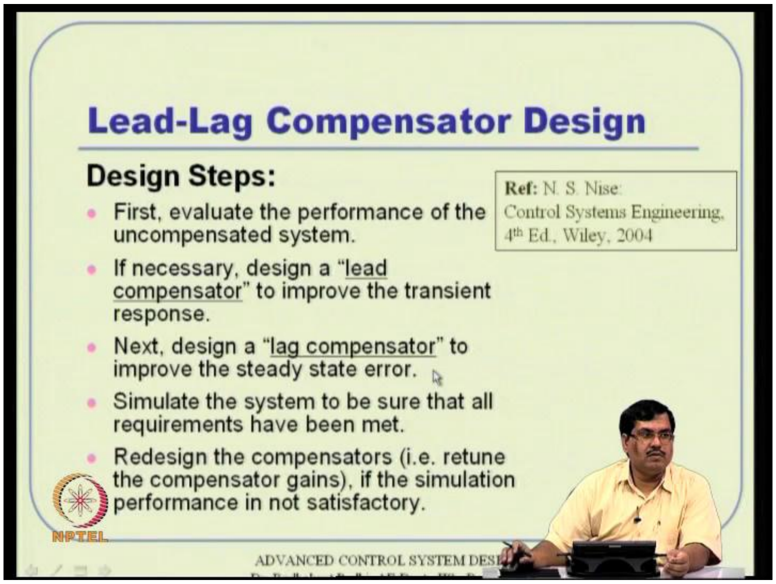
(Refer Slide Time: 10:25)



So, that essentially gives us a flexibility of putting in a number of locations and different selection results in obviously, different other performance behavior. And essentially different gain values to reach the desired point because, if you select different locations you you really need to select different gains actually. So, that like I mean corresponding to this pole 0 location, there will be different I mean there will be particular gain corresponding to this phase there will be another gain thing like that actually.

And also remember that as far as static error constants that mean the steady state error behavior is concerned that depends on this pole location actually pole 0 pair location. So, that is in that sense, there is a requirement of tuning this locations also basically. So, that is the philosophy of lead compensation.

(Refer Slide Time: 11:10)



Now, what is lead-lag compensator design, essentially it is a combination of both the two I mean, both lead and lag compensator. So, what you do? First, evaluate the performance of the uncompensated system that is that is true for any control design by the way; just see how the system performs and if there is a necessity then only do something otherwise, if the system is already good then do not try to unnecessarily design a controller actually.

Now if necessary, then design a lead compensator first and try to improve the transient response actually. Next, you design a lag compensator to improve the steady state error over the transient performance. Then you do like you have to above you have to do the simulations to make sure that all requirements have been met. Remember many of the times we are force to do some sort of a second order approximation of the actual plant which is not true.

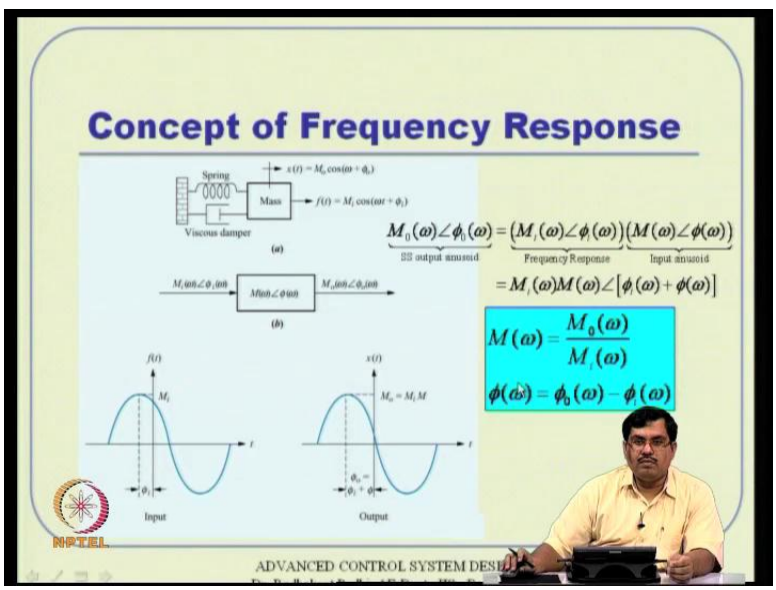
So, hence there is a necessity to simulate the system actually. The dominant pole and other things will come as approximate second order plant and all that that is not true. So, (( )) you tune of all these with respect to that approximation second order plant, go back to the main plant and try to simulate and make sure that all things have been met actually; if not redesign the compensator that means retune the compensator gains and probably pole 0 locations as well actually.

So, if there is a simulation performance is not satisfactory do that, if it is satisfactory you are done actually. So, that is the that is the procedure for lead lag compensator design.

So, that essentially completes this this particular thing. And next big topic in classical control is something called frequency response analysis. It is an it is a parallel development and probably, the earlier development as well. Earlier people when they started looking at feedback control systems, this is the one that came to the the mind first actually.

And then root locus came much early much later I mean if I if I if I am correct then, this frequency response analysis somewhat in between first and Second World War; and the root locus and modern control and all the things happened after Second World War. So, it is a very important concept, there are many things that you can be you can be done in a very intuitive manner following this frequency response analysis.

(Refer Slide Time: 13:26)



So, let us see the concept. So, here we will be primarily interested in sinusoidal input, no more constant and ramp input and thing like that actually, no more step input ramp input like that. Let us see, what happens if you give something like a sinusoidal input, this is the input that I am giving and as here I am considering a spring mass damper system which is also equivalent to R L C circuit actually in electrical system. Most of the mechanical systems including aerospace engineering are typically spring mass input systems anyway.

So, what I am looking it, I am looking it a sinusoidal signal and it turns out, if the system is a linear system then, the output will also be sinusoid. And it interestingly turns out this this sinusoid output sinusoid will also contain the same frequency as the input, frequency part remain same. So, what is different then, the different is difference is in the magnitude and phase actually, this is a magnitude difference of the output and there is a phased phase lag of the output as well actually, there is a phase difference actually.

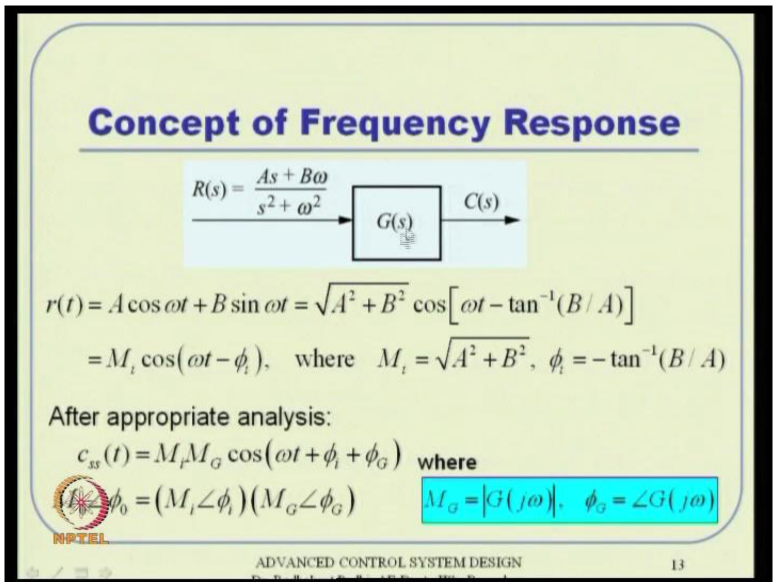
So, if you really consider the and however, this magnitude and phase difference is is a function of this frequency. Suppose, this frequency is different then this, magnitude and phase will be will also be different actually. So, I can I can interpret this input signal as some sort of in a phasor form what is called. So, it is a magnitude and phase angle if I if I give that then I will get another signal with a different magnitude and phase; obviously, all these are functions of that particular frequency that this this signal contains then, I can interpret that this particular system that I am talking about, also contains some sort of a phasor form I mean characteristics actually. So, that is the frequency response of the system actually.

So, what I am looking at, if I am looking at the output signal the steady state remember we are talking about steady state output not transient output here; giving a signal and waiting until it reach I mean I I wait until it reaches to the steady state condition, then I am analyzing that particular signal actually.

So, steady sate output of the sinusoid whatever looking it here is something like the frequency response of the system multiplied by the input sinusoid actually; and it will also turn out that, this and this magnitude if you do the proper analysis in time domain also, details are there in (( )) book as well; if you do that, it turns out that this, this magnitude, magnitude will multiply and the phase angles will will sum up actually.

So, what is looking at if I am looking at this particular magnitude of the system, then this magnitude response is something like magnitude of the output divided by the magnitude of the input; and the phase response is the phase difference between the two actually. So, together this this M of omega and phi of omega magnitude and phase, they contain the system properly together and these two pair is known as frequency response actually of the system. And I I mean its very clear that, if I vary omega then this two will also vary actually.

(Refer Slide Time: 16:33)



So, what is little more detailed if I see that in Laplace domain I am giving you some sort of a combined sinusoid, a cosine and sin omega t together I will I can interpret that as some something like this, this is the magnitude and this is the phase angle actually (Refer Slide Time: 16:47). So, this is in the form of M i cos of omega t minus phi i, where M i M i and phi i are defined like that.

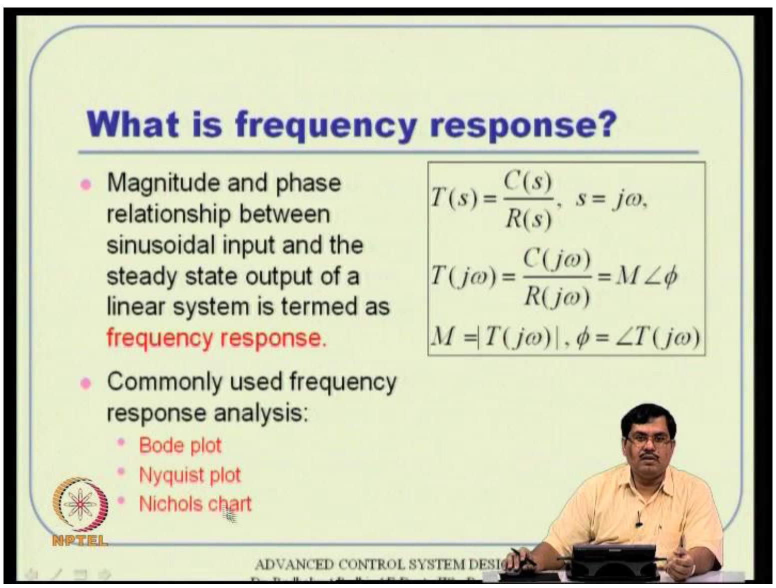
And after appropriate analysis what we are looking at is the time domain I mean time domain response for this C of S that means C of t; and that we are essentially interested in steady state response actually. And if your system is stabilizing remember that this this steady state response will will approach to the reference signal actually.

So, all these analysis details are there in the (( )) book actually you can see the details and then, what I am looking it the steady state time domain response is given by something like that very clearly comes to that analysis actually; you can take Laplace transform and then invert the Laplace transform do partial fraction also so things actually that is there, (( )) are there.

So, interestingly it turns out this, this M G and what are what you are looking at this M G and phi G this two angles I mean this two conditions, this two variables are clearly given by these actually (Refer Slide Time: 17:49). So, M G is given as magnitude of this G j omega and phi G is given as angle of G j omega.

So, because of that, the the frequency response characteristic of this particular system G s is nothing but G of j omega. If I simply simply represent that in a in a in a complex plane probably I mean in a phasor form, then I can simply contain this magnitude and phase together and talk G g of j omega is nothing but the frequency response of this particular series, it contains both magnitude ring and as well as phase thing actually.

(Refer Slide Time: 18:25)

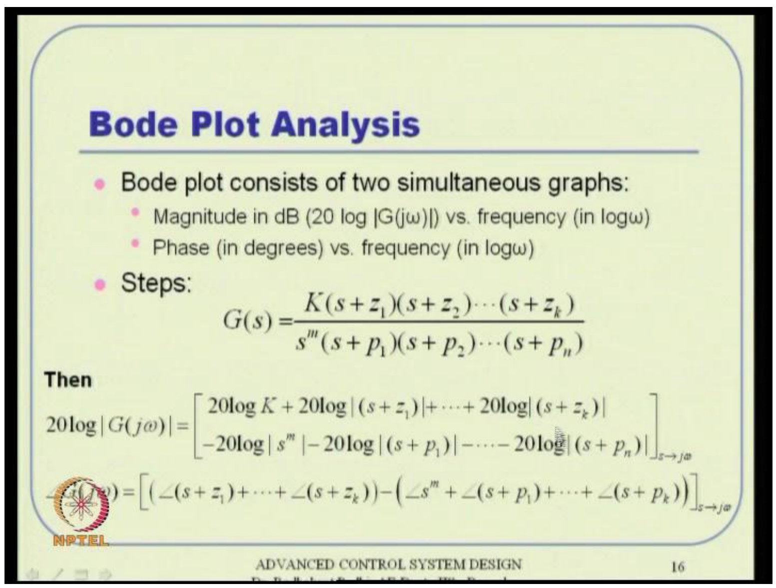


So, what is a frequency response formula? Now, you know that this closed loop transfer function is C S by R S. So, if I substitute S equal to j omega then, T of j omega contains a magnitude and phase; and the magnitude and phase together are called frequency response. It is a I mean we have been telling that over two three slides now, this is one of the same thing actually.

Now, to analyze this frequency response formula, there are many nice tools available actually. And very intuitive thing that comes to mind is bode plot followed by Nyquist plot then Nichols chart constant eminent circle thing like that actually. We will not talk too much details, but essentially it probably starts with Nyquist plot and then it list to Bode plot.

However, for our better understanding will study Bode plot first and then go to Nyquist plot. Nichols chart are probably we will skip actually, that is also a easy to follow it and see the book also.

(Refer Slide Time: 19:21)

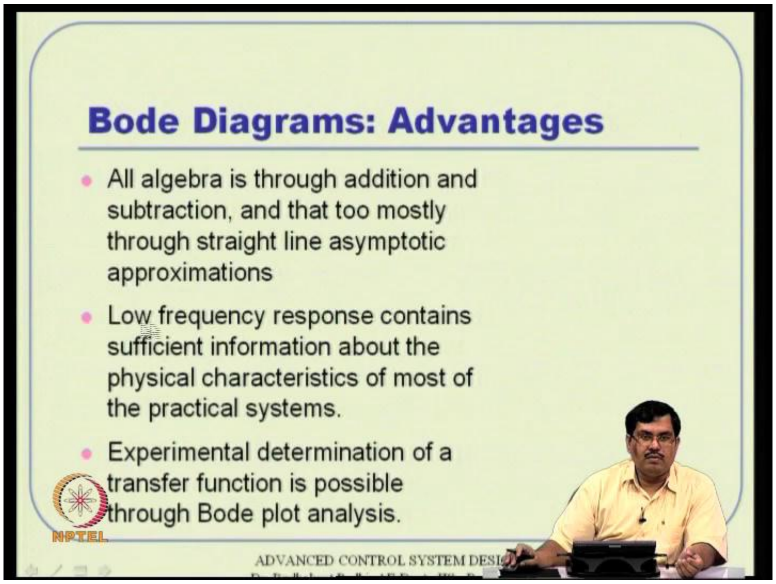


So, let us see first what is Bode plot analysis? So, you have a transfer function something of this form, and then what we are interested in, we are interested in studying the G of j omega its its magnitude and its phase angle actually, that is all we are interested in studying. So, what I meant, what I am looking for is magnitude of G j omega, but we are interested in studying this this particular magnitude of G j omega for a wide range of frequency actually, for a very small frequency to a very large amount very large number. So, better way of studying that is with respect to logarithmic scale.

So, what I am looking it, is something called decibel unit. So, I am looking at 20 log of magnitude G j omega that is that is the magnitude in d B that is in decibel actually. What is that frequency, and frequency also is a very wide spectrum very wide range. So, I will also interpret that in terms of log omega. So, what I am doing actually, as far as magnitude is concerned I am expanding this this 20 log of magnitude G j omega which will turn out to be like this (Refer Slide Time: 20:26), this by using simple logarithmic expressions. And then the angle part will turn out to be angles contributed from the 0's minus angles contributed from the poles actually, all evaluated with S equal to j omega all these things will evaluated are S equal to j omega.

So, what is the beauty? Beauty is both leads to addition in subtraction only; there is no multiplication after that. So, this addition subtraction in the magnitude by taking logarithm we have been converted it to a to a multiplication I mean converted from multiplication and division to to addition and subtraction. And by nature of this phase is anyway given in terms of addition and subtraction. So, the algebra becomes very simple and it it is possible to plot by hands actual the frequency response characteristics.

(Refer Slide Time: 21:13)



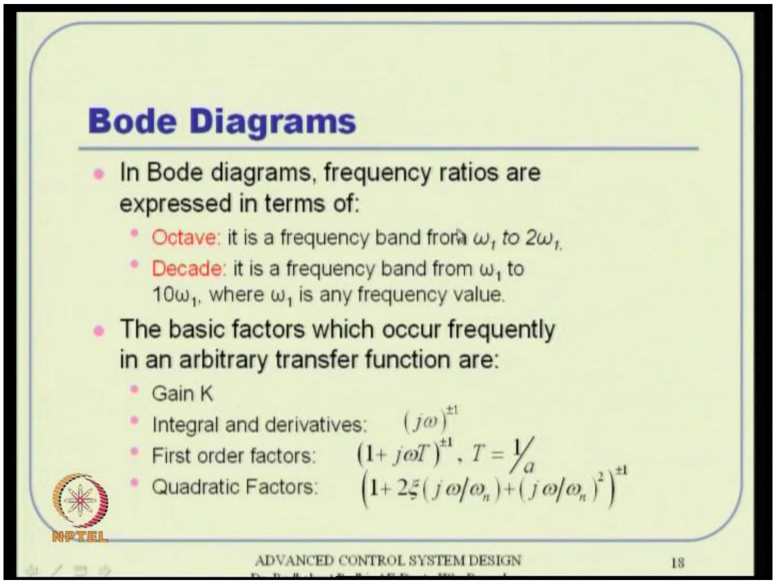
That is why I told that, the Bode diagram has several advantages very intuitive things of analyze very intuitive tools to analyze. As I told, algebra is through addition and subtraction, and that too mostly through simple straight line asymptotic approximation. We are not interested in exact curvature of the response and think like that, will be able to do this very quick analysis you think what is what are called asymptotic approximations of the response actually.

And the second, second point is this low frequency response contains a sufficient information about the physical characteristics of most of the practical systems; the high frequency vibration for example, you are not that much interested, we will consider that as a noise actually very high frequency vibration. What you are interested, a vibration response is something, if you just give a some sort of a dip to the system or some sort of excitation to the system then, how it really vibrates in a low frequency mode, that contains the system characteristics that we are interested in primarily actually.

So, low frequency response is what your control system is able to do and that is what our primary intension is actually. However, this this Bode plots are for all frequency range. So, Bode plots will be able to plot it for a wide variety of frequency actually. Then as a by product what it turns out, this experimental determination in experimental determination of transfer function is also possible through Bode plot analysis, using this corner frequency and all that.

Suppose, you do not know the transfer function in the beginning, but if you go back to this (Refer Slide Time: 22:44), and this this values you want to find out actually, this location of 0's and location of poles; and it will essentially turn out that, these are also locations of this corner frequencies in the Bode plot we will see that in a while actually. So, if you roughly see the Bode plot and then find out the corner frequencies and there's roughly the nature of the plot actually, then you will be able to predict these values in a good way. So, that is what system identification is about through through Bode plot, you will be able to do that actually. That is an additional byproduct, but you are not going to discuss too much on that actually you are.

(Refer Slide Time: 23:18)

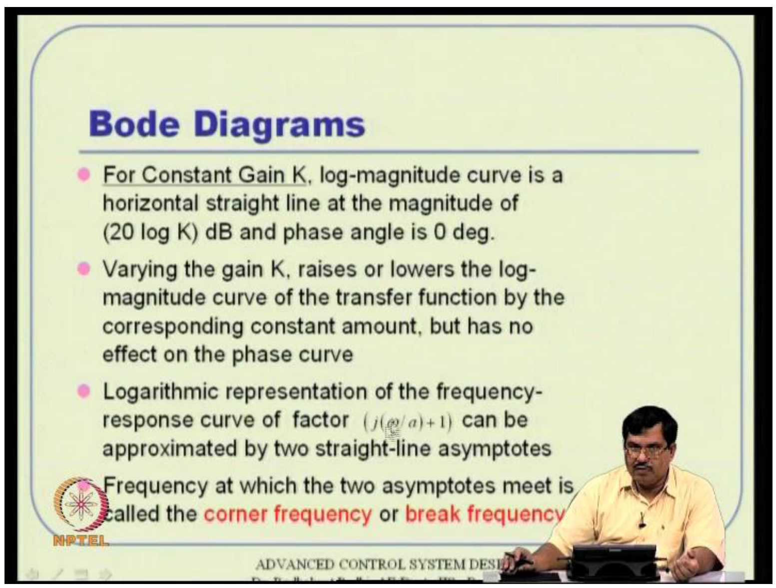


So, what is Bode diagram I mean it is essentially Bode diagrams is frequency response of magnitude and phase angle together. And then we are interested in plotting that in a logarithmic scale and then in the it turns out that, there are two notions for that; one is octave and the second is decade actually. So, the frequency band from like some omega particular some omega 1 to 2 omega 1 essentially if you double the frequency whatever happens that is in logarithmic scale that is called octave. And if you would multiply frequency to 10 times whatever happens in the logarithmic scale, it turns out that is known as decade actually. And something that is that frequently appears in our transfer functions if you see this transfer function (Refer Slide Time: 24:03), is it actually all written in first order form, but they would may appear to be partly second ordered form also. So that means, instead of this S plus p 2 this particular term I can interpret that as some sort a second order polynomial also basically like that way.

So, it make sense to study some of this standard thing that appear, that is this suppose you have a integral terms either you can have a pure gain, pure gain has no S, S to the power 0. Or you have a integral or derivative term which is S to the power either plus 1 or minus 1 depending on whether it is a 0 or it is a pole actually. Or you have some term like 1 plus S either in the numerator or in the denominator. Or you have this 1 plus 2 zeta omega and plus omega and square sort of thing actually. All these are written in normalized form by the way, that is a standard practice that is that is done in Bode plot analysis.

Once you normalize the the entire quantity, suppose I take normalized that divided by G 1 I mean z 1 here z 2 here p 1 here p 2 here then all that I am doing is multiplying this gain by some other constant. So, it turns out to be an overall gain factor actually. So, that helps me in in drawing the Bode plot in a good way actually. So, that is the reason for getting normalization and all that.

(Refer Slide Time: 25:15)

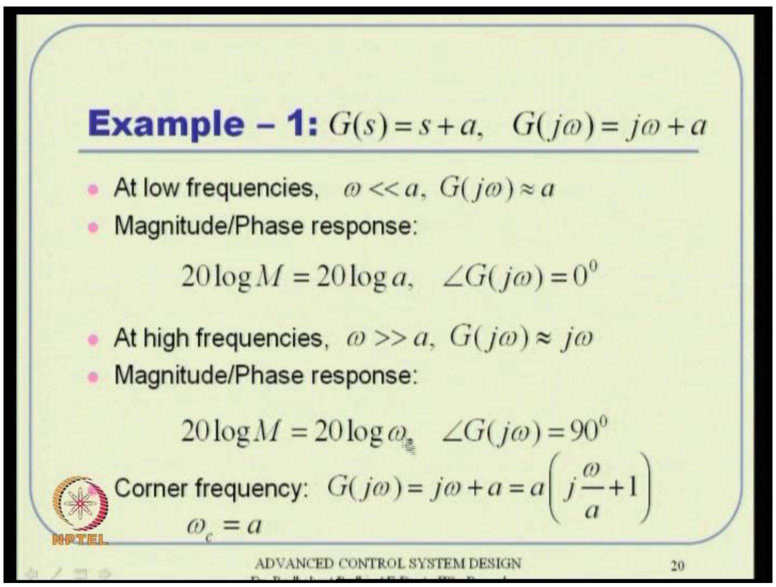


So, details Bode diagram for constant gain. And then some varying the gain K what happens and all that so, all sort of things are possibly. So, what is what is happening for constant gain K? If the gain is constant, then the log-magnitude curve is also a constant line right, if you take if you take a logarithmic of a constant value number, it also turns out to be constant.

So, it the constant line turns out to be, 20 log K in d B that is it actually; and because it is a it is a scale of number sitting in the numerator that too, so the phase angle is 0 actually. So, whatever number is that I will interpret that the phase angle is 0; and then for what I am doing is, for varying the gain K if I really have gain variation what happens here, if I gain initially was K 1 and later is K 2, then with respect to K 1 then K 1 and K 2 the Bode plot will essentially shift up and down; there is if I multiply by some quantity let us say the original gain, then multiplication turns out to be addition in a in a logarithmic scale.

So, essentially I will add a add it some constant value through out or I will subtract a constant value throughout actually. So, that is like it will it will take the Bode plot up or below as per as the magnitude plot is concerned, phi angle is 0 in anyway actually. So, similar analysis you can do and specially, if you have a 1 plus S term 1 plus omega by a in a in a normalized manner, then essentially it can be approximated by two straight line sort of things.

(Refer Slide Time: 26:53)

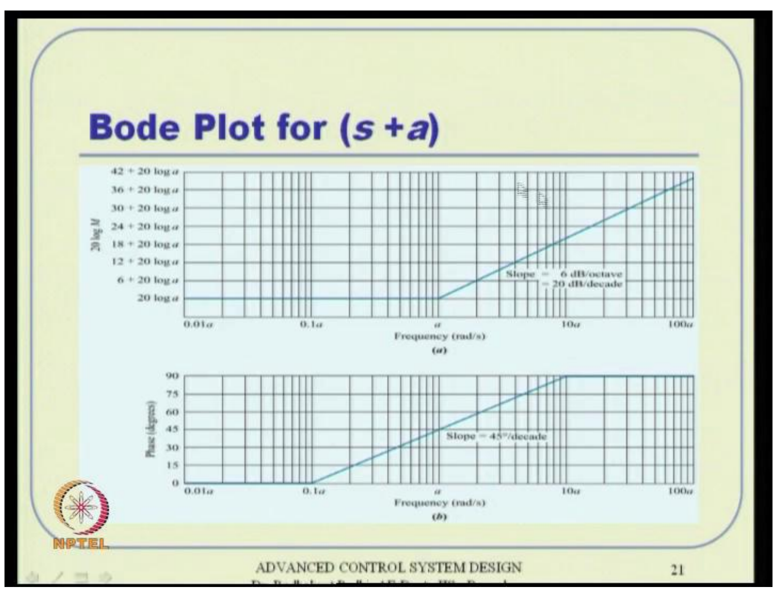


So, let us study that part of thing actually. So, either you I mean G of S is S plus a. So, what you have is G of j omega is, j omega plus a actually. So, at low frequency what happens, if omega is very less than a I mean compared to a omega is very less, then I can approximate this quantity is nothing but simple a right, j omega is is almost 0 actually that is that is gone. So, I am left out with G of j omega is a. So, magnitude and phase of that particular time, when omega is very less low frequency turns out to be this number and the M phase angle is 0 actually. At at high frequency, what happen? The reverse happens, this I mean the approximation sense I can forget the a component, because omega is very high compare to a.

So, G j G of j omega is nothing but simple j omega actually and if you see that phase part of it, phase part of is 90 and the magnitude part turns out to be like that (Refer Slide Time: 27:51); which is actually a straight line this particular thing if you see 20 log omega is a straight line with respect to log omega in the x axis. We put that if if you put this in y axis and then, log log omega in the x axis then it is nothing but somewhat y equal to like 20 times x basically, that is the straight line.

So, what happens and then when does this change actually, this first it is a constant then, this is a straight line approximation, when do when would when the change would happens? The change happens to something called corner frequency. And that is where, if you normalize this and interpret it that way, when this I mean when do you consider that this is high or this is low depending on the whether this is less than 1 or greater than 1 actually; if it is less than 1 I will dump it as something like omega is very less than a, if it is greater than R greater than a I mean greater than 1, then I will consider that a second case actually. So, the corner frequency happens to be just magnitude a.

(Refer Slide Time: 28:48)

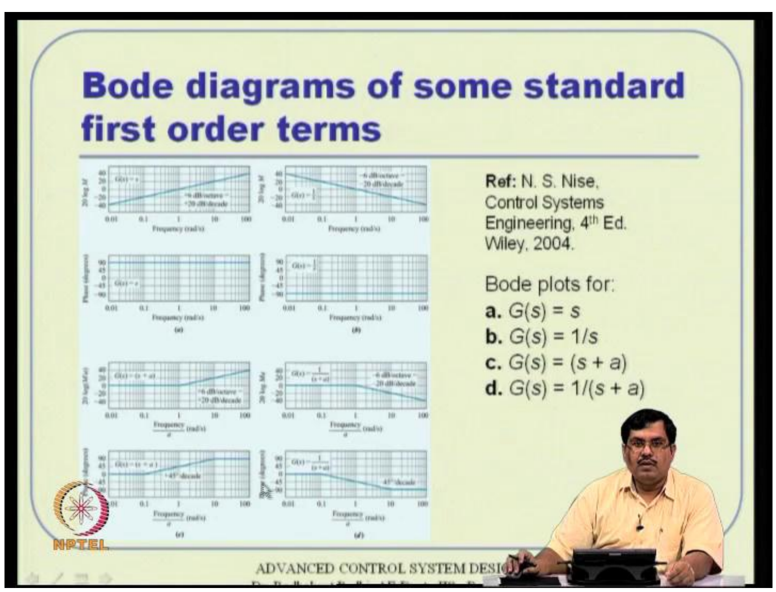


So, this is what it is actually. So, the the lower side of the frequency that is just a constant line and that a, this is a straight line starting from that actually. So, these are like asymptotic approximation actually. So, in this logarithmic scale that we are looking it, log omega in the x axis and 20 log M in the y axis, it this turns out to be like that actually this this particular S plus a. And phase angle remember that, phase angle is something like starts with 0 goes to 90 and this starting and going to 90 should happen around that corner frequency, corner frequency is a. Remember, the Bode plot needs to be plotted on the same scale, next to each other just to one below the other actually on the same x axis scale.

So, if you see this a around that, this frequency change I mean this phase angle change happens to be 90 degree. So, I will consider that if I if I go 10 times below this particular a, this is all kind of thumb rules actually if I go 10 times below that, then I consider from there onwards everything is negligible that means that is a straight line. And if I come 10 times higher than that, from there onwards everything in a S is valid in a high frequency range sort of thing.

Remember, this high frequency range is 90 degree; low frequency range is 0 degree. So, anything that is 10 times lesser I will consider that a 0 degree, anything that is 10 times higher I will consider that is 90 degree; in between there is a straight line approximation actually, this it varies through that actually. So, that is the Bode plot for magnitude and that is the Bode plot for phase actually.

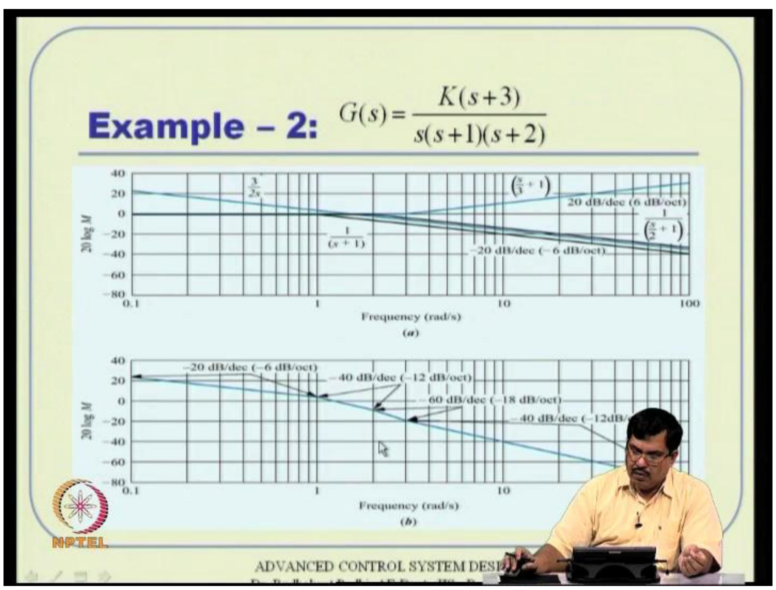
(Refer Slide Time: 30:21)



Now, similar analysis we can do for various things either for (( )) what we did previous slide is S plus a, we can also do that for S, we can do that for our 1 over S, we can do that for 1 over S plus a also. And there are very very easy to do rather, because these are the simply straight line approximations anyway. So, if you consider S this is just starts from beginning to end, 1 over is the reverse way.

Now, we studied this 1 S plus a then if it is 1 over S, it just happens to be in the in the other direction, we we had this kind of characteristics remember that, this this type of characteristics; if a 1 plus a sorry 1 by S plus a then it will turn out to be thus the reverse way, then the slope will be negative. So, that will start with the corner frequency a, and this all these are normalized plots remember that, frequency divided by a sort of thing actually. So, it will all start with that corner frequency and instead of going up it will go down and here also instead of going from 0 to 90, it will go 0 to minus 90 actually. So, these terms are rather easy to do actually.

(Refer Slide Time: 31:26)



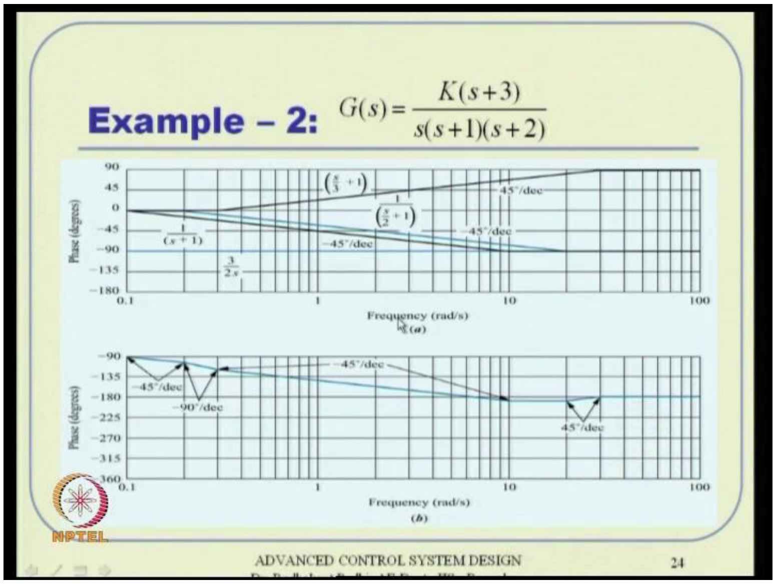
Now, what about a general transfer function like this? So, then what is happen what happen this nice property that we studied here comes to picture, this these are all summations and I mean additions and subtractions. So, what we will interpret is individually will plot the magnitude and phase angles, then we will simply take the summation I mean addition or subtraction sense actually.

So, that is what is happening here for a generic generic way I will just do that for constant gain, then S plus 3, then S, then S plus 1 and S plus 2 individually. And then I will just sum it up actually, that is what is happening in the first sub plot individually. For example, if you see this the 3 by 2 S that is a gain this one, remember this is a normalized, so you have to divide it everywhere divide by 3 and then divide it by 1, divide it by 2 like that actually. So, 3 come out from here and 2 comes out from here, so that is 3 by 2 K will appear here actually.

And what is that, that as a K if I do just a little bit. So, this is actually something like K 3 S plus S by 3 plus 1 divided by S into S plus 1 into I will take 2 common 2 into S by 2 plus 1. This 2 and 3 that is so that is what, this this this K into 3 and this 2 will lead to this this 3 by 2 that that is what here actually. So, all right all right. So, this is how what is happening. So, 3 by 2 S is just just like 1 by S sort of things, so it will start with and then go down in a straight line manner. Then, if it is coming to like S plus 3 for example, this is the term, so the corner frequency happens to be 3 and 3 is this is remember this a logarithmic scale that mean this is 1, this is 2, this is 3. By the time you come here you are 10 actually, so this is that way.

So, at at value 3 this starts to going starts going up actually right. And then similar thing it if you see the pole, pole should go down and then S plus 2 is somewhere here, corner frequency is 2 actually. So, it at 2 it will start going down actually like that you plots everywhere. And then once you done with all the terms you simply add it up actually. So, you just start adding up then wherever there is a first corner frequency there will be some change there, then I will go to the next corner frequency with a different slope; because slopes will get added actually here plus minus ends actually, then it will go to the next corner frequency and like that actually. So, that is that is the phase angle sorry that is the magnitude part.

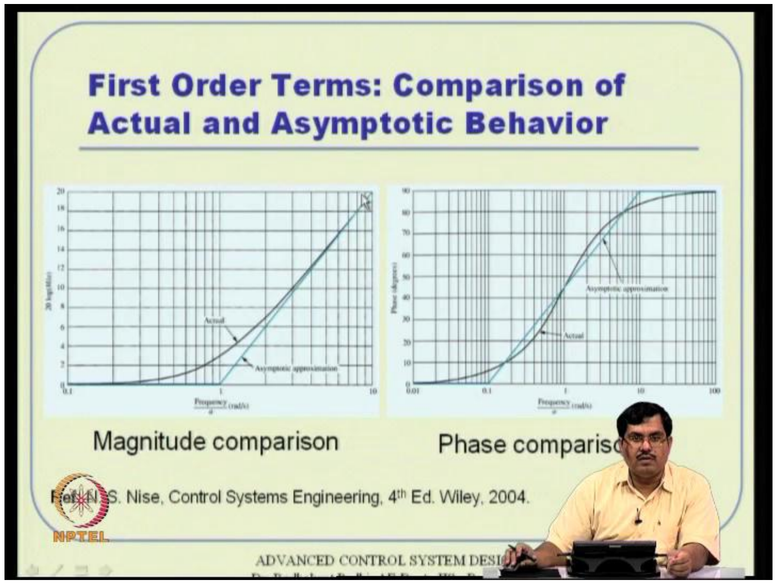
(Refer Slide Time: 34:14)



And similar thing we can also do for the phase angle part actually. Phase angle part will draw individually first, and then sum it up actually. So, it is just possible even to take your logarithmic graph sheet and then, do our take a scale and pencil and then, try to do this actually; I mean it is just possible to do that and that is that was a very beautiful part of it, when computers were not available actually.

Now a days of course, you can just log it this particular transfer function and generate the whatever you want root locus, (( )) plot, Bode plot everything in computer probably using standard software's like mat lab, all these things are easy. However, if you still want to do it by hand, it is possible to do draw this diagram using a graph sheet actually.

(Refer Slide Time: 34:57)

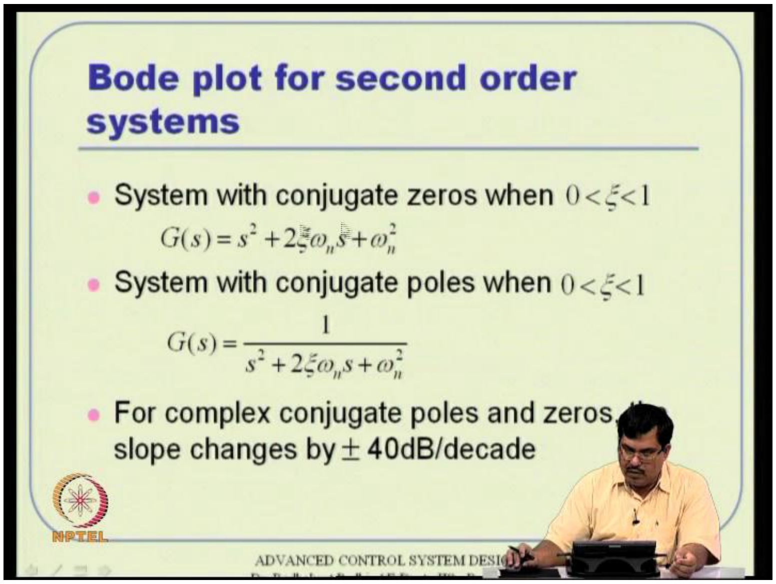


Now, just remember that what we did here, these are all asymptotic approximation that means we are just interpreting at one corner frequency everything below then everything below that is one range one one sort of behavior; everything after that is one sort of behavior. So, that is not true, actually it is a it is a fairly continuous manner.

And if you see that, the actual response actually turns out to be something like that. If you really plot this transfer function and all, it is not going to be 0 0 0 up to 1 and then suddenly start the I mean, there is a slope there actually. But there the slope is also 20 d B per decade actually. So, that is that is also there actually. The slope is 20 log a right, so this 20 log a will give you this 20 degree for decade sort of things actually. If you have a second order transformation, transfer function it becomes 40 d B just double, because omega square will take you 2 times of that actually, we will see that in a while. So, this is a difference actually. So, what you are what you are doing in a graph sheet is this to asymptotic thing what whatever is the reality something like that.

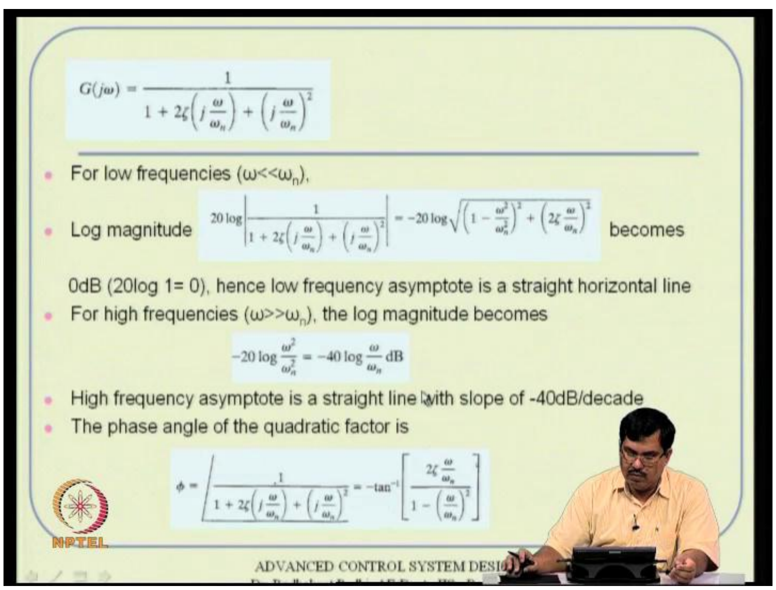
Remember, the maximum difference happens somewhere around the corner frequency, and this difference happens to be of the exactly 3 d B rather; this difference of this what you are interpreting as 0, but actually there is there is a 3 d B magnitude here. So, that is that is how it is happening. And the phase angle also happens to be something similar actually.

(Refer Slide Time: 36:22)



And like now it coming to the second order sort of behavior, we have a transfer function something like this, either in the numerator or in the denominator.

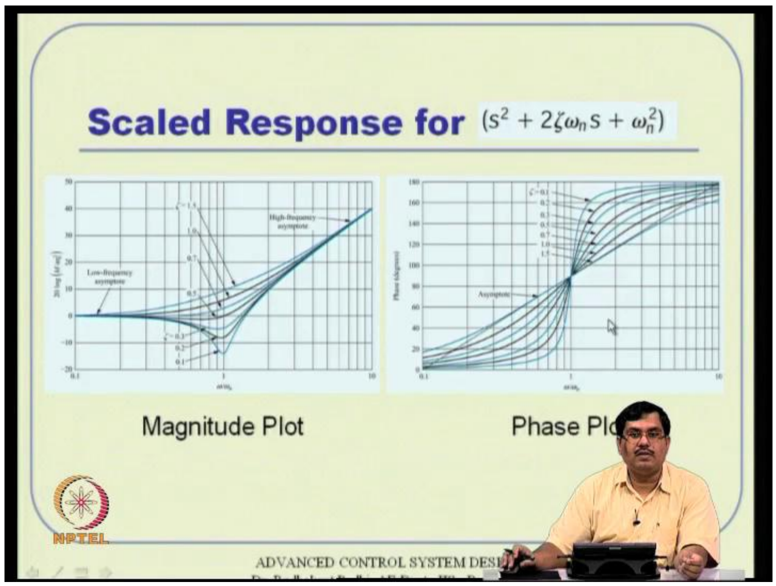
(Refer Slide Time: 36:30)



So, how does it happen? So, you again substitute j omega instead of (( )) and then carry out the analysis. Now, it happens to be a function of this omega by omega n whole square and that whole square will come, so that will be like 40 d B actually.

So, if this ratio, remember these are all base 10 actually log all logarithmic are of based 10. So, if I have 10 here then that log 10 10 will become one sort of thing. So, this will be 40 d B per decade sort of thing, the slope that you are looking at is 40 d B for decade that is how it happens actually. And the angle is also of given something like that, magnitude is given something like this actually. We can still do the corner frequency sort of idea analysis and all that, and that is possible, remember that these are not as neat as first order term actually, these are now functions of zeta basically.

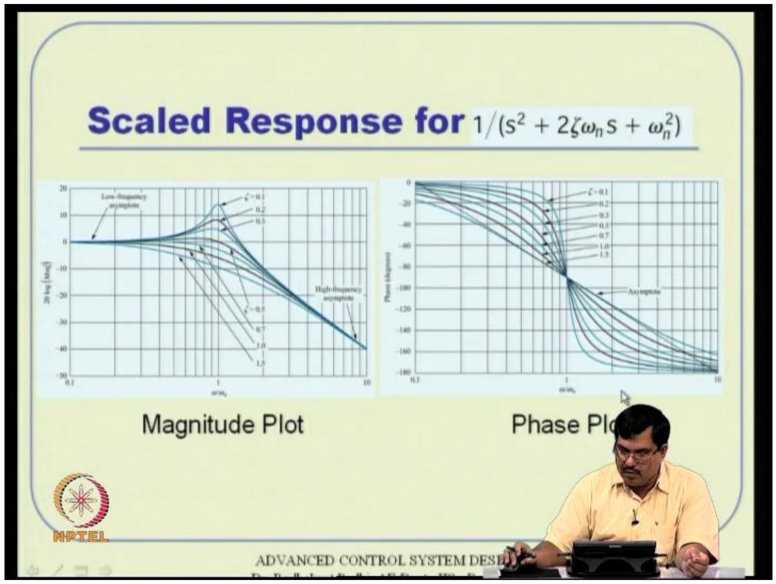
(Refer Slide Time: 37:18)



So, we can still have some asymptotic behavior sort of thing, but remember just remember that the the departure from that approximation is not small, it they and they can be significant depending on what value of zeta you have. For example, if you have very low damp system that is zeta is 0.1, then you have a departure correction or something that requires actually around that frequency is quite high actually. And say, the phase angle also actually same thing happen.

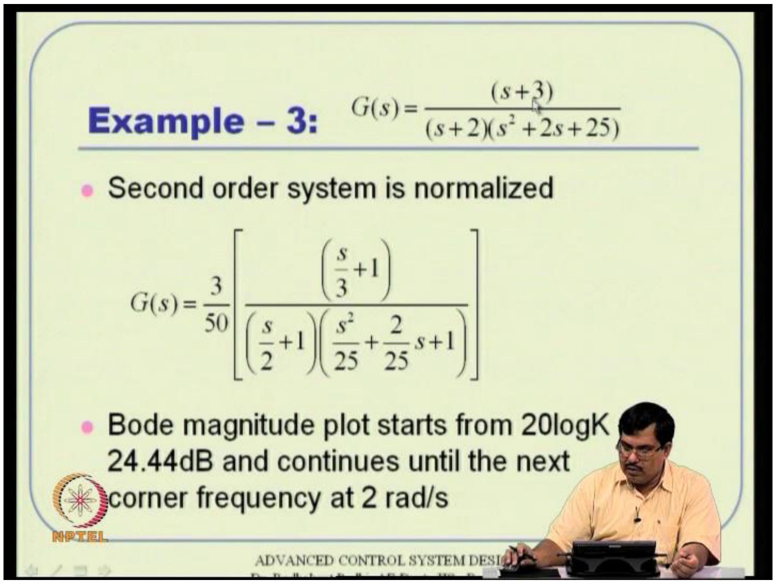
And it is very close to each other like asymptotic behavior and other things are very close to each other around the operating zone; remember most of the second order system, either by nature or by design will will require zeta to be 0.7 that has some certain beautiful characteristics actually. Now, if you see 0.7, 0.707 or 0.7 something like that, then it this particular thing what you looking at, is very close to the asymptotes that we will talk actually; and same thing happens here also.

(Refer Slide Time: 38:19)



And there are and further analysis which you see at the end actually, why these 0.7 have different number for zeta actually. Anyway, so this is what happens in the if it is happens in the in the denominator instead of numerator, what you have it actually it becomes a mirror image actually; instead of going up again it will come down actually, and instead of that way varying this phase plot it will vary this way actually. So, that is how it is.

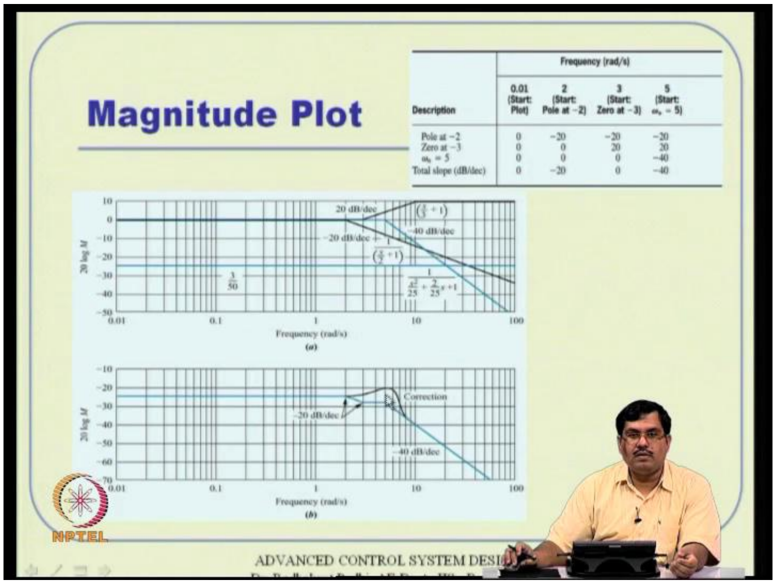
(Refer Slide Time: 38:43)



Now, how do you do this this third particular example let us say, you have two first order terms and one second order term. So, first thing first process is normalization. So, I will know below I mean the corner frequency is and all that actually. So, here I will take out 3 here, I will take out 2 and here remember it is a 5 square actually. So, I will divide everywhere 25 and then take it out.

So all these thing become 3 by 2 into 25 that mean 3 by 50 here, and numerator corner frequency is 3, this fellow is 2 and this fellow is 5 actually. Remember this is what we discussed here, this omega by omega n, this this place a roll actually. So, at omega equal to omega n that is log 1 sort of thing that is 0 actually. So, that is what will look at at actually.

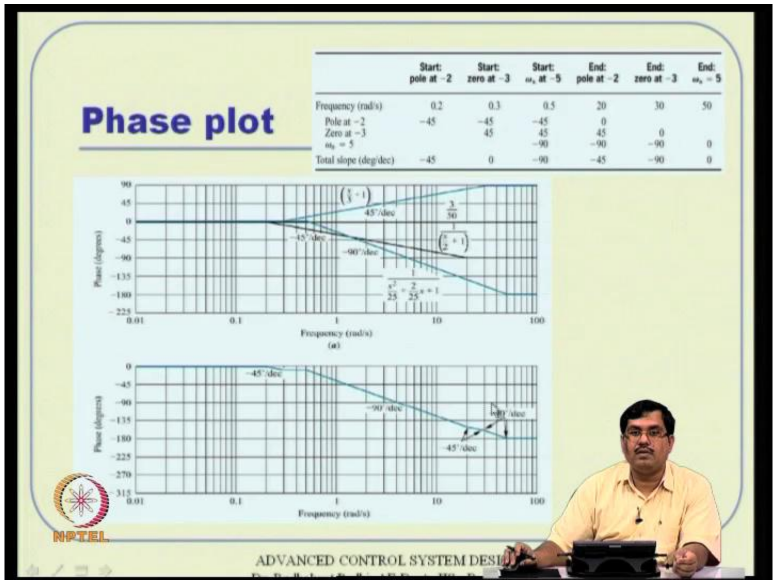
(Refer Slide Time: 39:30)



So, this this is something that we plot. And then this individually will plot that and the remember this is the second order part, second order part compare to first order part will be higher slope actually; first order part will vary at 20 d B per decade, but second order will vary sharply the I I mean double the slope actually minus 40 d B for decade sort of thing here.

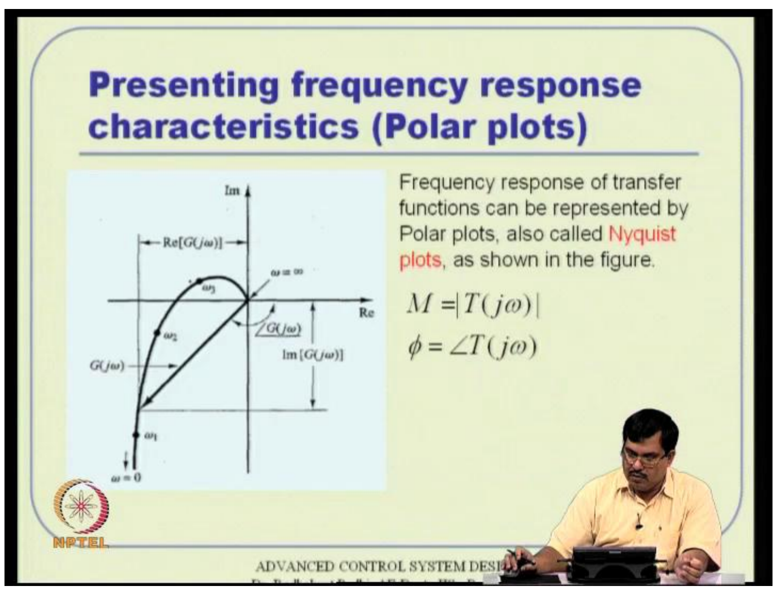
And then, after you do this addition subtraction corrections I mean the final asymptotic plot then, there there will be a requirement of correction term also actually. So, this correction terms are also calculate I mean roughly you can calculate that around the corner points and you can put that some some details are there in books like (( )) and (( )) all that actually let me see that.

(Refer Slide Time: 40:10)



Similar thing you can do for the phase as well actually, phase phase plots you draw individually then sum it up actually.

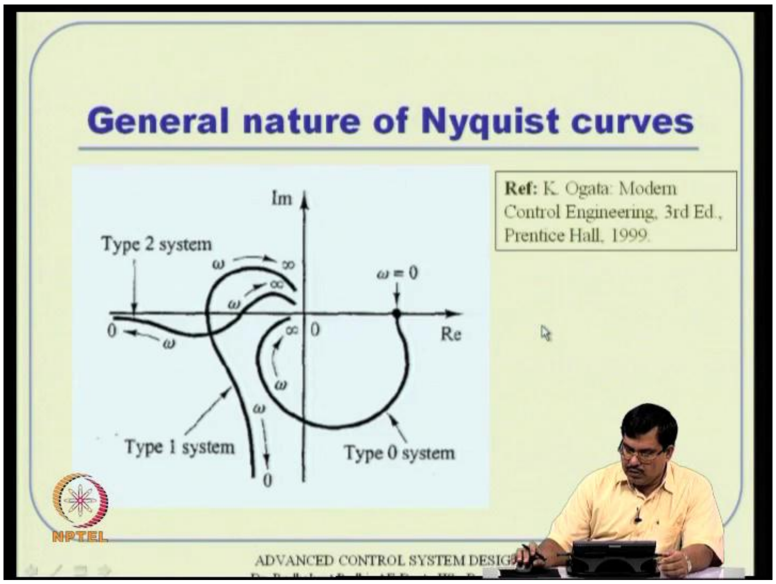
(Refer Slide Time: 40:24)



So, that is summary about Bode plot analysis. Now, very quickly we will see Nyquist plot analysis also. And essentially, I told Nyquist plot analysis happened probably before Bode plot, where Bode plot happens to be quite easy to draw analyze and all that. But lot of nice things also happen in in frequency response from Nyquist conditions actually.

So, what is it? In this particular thing we are looking at simply the polar plot actually, whatever this frequency response that we are talking this M and phi will simply plot it in a polar coordinate actually. So, this angle is something that we are interpreting as angle of G j omega; and this magnitude what you are looking at is something like magnitude of this particular vector for that particular omega is nothing but the magnitude of this particular transfer form. Actually this T is also G, but it is depicted here, we are interpreting T as G actually here.

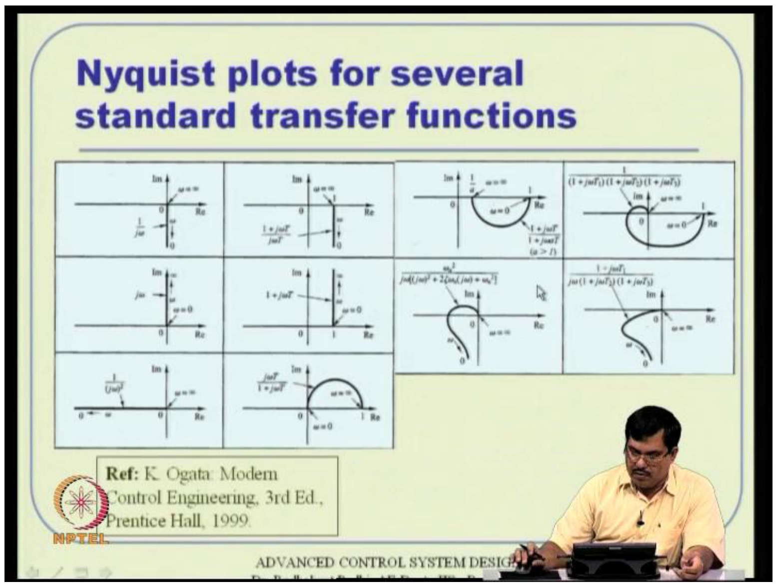
(Refer Slide Time: 41:14)



Now, this particular thing is actually a quite intuitive thing is that means there are, if you have type 1 system, type 0 system or type 2 system, a very neat property that you can see actually. If you have a type 0 system for omega equal to 0 it will start from real axis and it will just travel that way; for type 1 system, the the plot turns out to be of this nature; and type 2 it will turn out to be that nature.

So, knowing this this typical building block will help us in constructing the Nyquist plot in an intuitive way. Some of these things, if we know already then, it will it will be easier actually. Now remember, if you multiply by this gain K this plot is going to be just amplifying that means it is like a balloon actually, it will just (( )) shrink actually, this is a function of gain K, because radius R is directly reflected in gain K actually right. So, if if you multiply by gain K, this is going to just (( )) shrink actually.

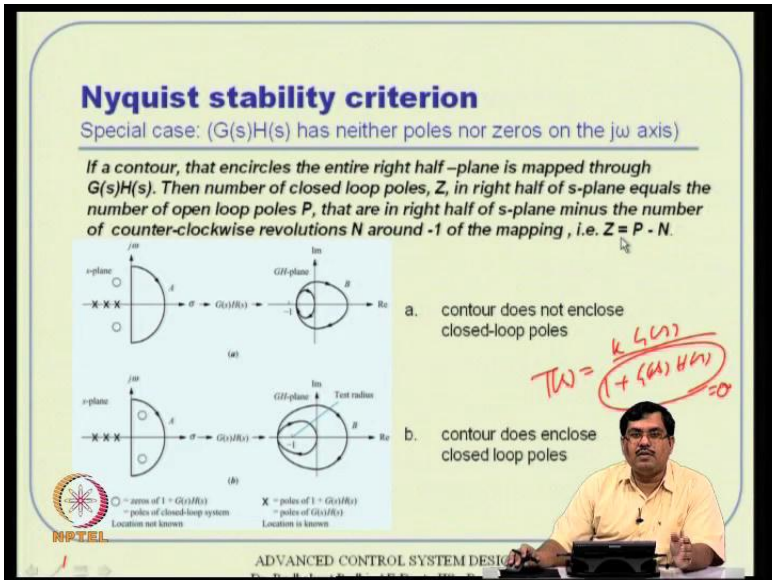
(Refer Slide Time: 42:15)



And this is this things are very neatly given in in Ogata book actually there is you can see interested you can see. But standard transfer function you can have the frequency response from Nyquist diagram something like this, you can Nyquist plot turns out to be like that. It is nice to remember some of that because, it will it these are nothing but building blocks actually.

For example, this is a typical second order response turns out to be like this actually, omega equal to 0 it starts with somewhere here and then travels when omega infinity it it it converse this here actually like this. And similar things are available for for many of these things for example, 1 over S if you have then, it is it is just travels in the negative real axis; if you have a simple S and a numerator it goes in the imaginary x axis actually; j omega, so j omega is is get in to travel that way; and 1 by j omega is also like minus j omega, so it will travel that way actually that way. So, like that some of these things are handy actually.

(Refer Slide Time: 43:05)



Now, what is the Nyquist stability criterion instead of going into detail, I will directly tell the result. And result is essentially contained in in imagining something I mean it comes from something like contour mapping theorem in the complex number actually and that is the beauty that Nyquist kind of thought about actually.

So, contour mapping theorem the details are there in (( )) book some I mean you can read that. But essentially out of all that analysis what it turns out is this beautiful theorem actually I just read out probably. If a contour, that encircles the entire right half plane is mapped through this transfer function G s H s; remember this G s H s is nothing but the open loop transfer function and all the time we are interested in analyzing the open loop transfer function to conclude about the close loop transfer function 0's I mean poles actually.

So, if you consider the characteristic equation that is 1 plus remember that this transfer function is nothing but T of S equal to K times G of S divided by 1 plus G of S into H of S; and this if I just take this denominator and make it equal to 0 that is characteristic equation.

So, if I have a if I have some 0's of this characteristic equation they are nothing but the poles of this transfer function. But what happens is fundamental series like if I, I can study this G s H s very easily, because G of S is known to me, H of S is known to me. G of H s, G of H s what is called is open loop transfer function the poles and 0's are kind of intuitive obvious actually. So, knowing those those loop I mean those details can I conclude what is happening for for the closed loop system, that is the problem that is how the root locus also same thing actually.

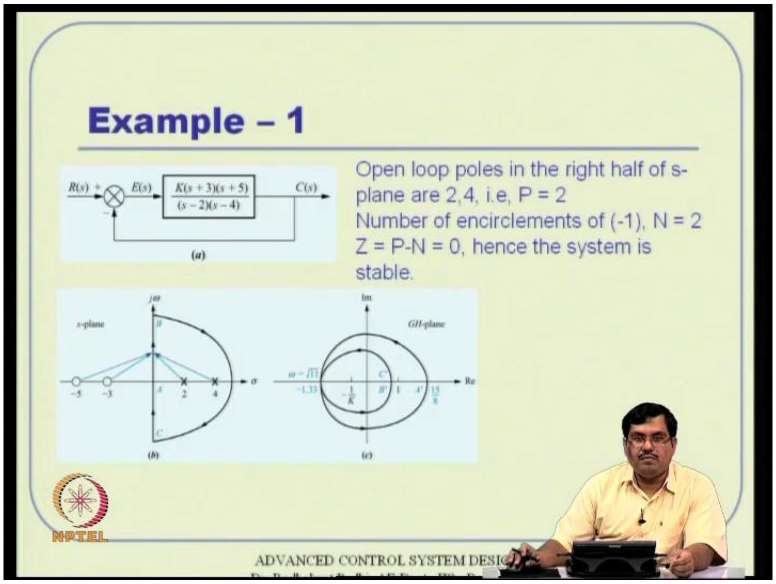
So, here we talk about some theorem this this is like this, we consider the entire right half plane and then consider how many poles are there in that right half plane; and then kind of imagine a contour around that contains the entire right half plane; and then, this the this number of close loop poles in the right half plane is given by this actually, this expression. That is z transports 0, 0's of the characteristic equation remember that, that is nothing but poles of the closed loop transfer function. So, what is it actually number of poles in the right half plane minus the number of counter clock wise rotations, this revolution that actually encircles this minus 1 point and minus 1 comes from this equation again, this 1 plus G s H s is equal to 0. So, G s H s is equal to minus 1 actually. So, that is minus 1 plays a critical role here.

So, if I just imagine some some contour like that, and then then draw this this one this particular contour I will map it to G s H s plane, this is just simply S plane. In the S plane I will I will imagine a contour which contains the entire right half plane, then I will map that to G s H s plane; on the G s H s plane I will observe two thing I mean here I will observe one thing that how many poles are there open loop poles are there in the right half plane.

Here I will observe how many counter clock wise rotation are there around the minus 1 axis actually. Then I will use this formula and tell this Z a nothing but P minus N. And this Z is nothing but 0's but 0's of this characteristic equation and hence, it is poles of the close loop system that is how it is tied up actually.

So, if I if this Z happens to be 0 then obviously there are no right half poles of the closed loop and hence this closed loop system is stable actually. If 0 if this Z happens to be positive that means there is let us say if it is 1 then there is some 1 pole in the right half plane somewhere for the closed loop transfer function; and hence the system is unstable like that actually.

(Refer Slide Time: 46:47)



Example again if you see that, this is a contour that I am imagining first and this for this is a open loop transfer function, remember H of S is 1; so that means G s H s is only this what you are looking at. And this contour contains two poles in the right half plane actually, so that count is actually 2, P is 2 here; and if I if I do this, this encirclement actually you can dump this K in the transfer function and talk about encirclement about minus 1; or you keep this K I mean take out this K and then, encirclement about minus 1 over K both are same thing actually.

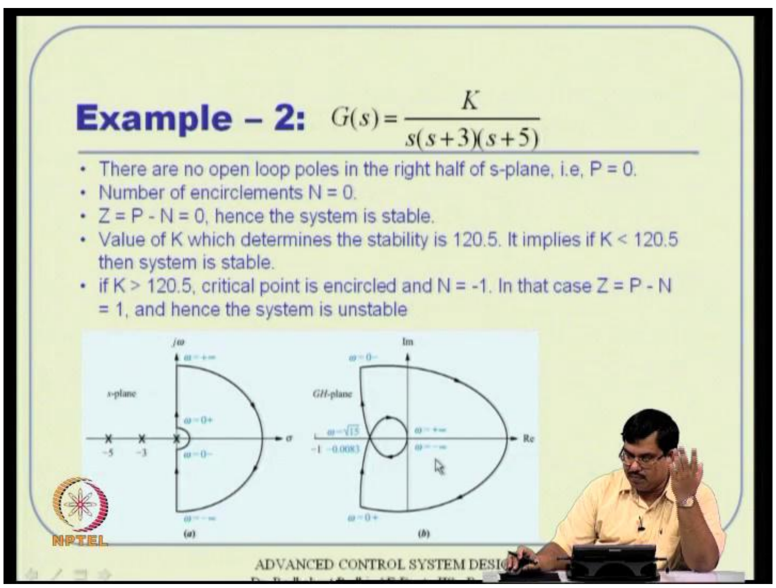
So, what we are looking at, this particular thing it happens to be like if I if I look at this G s H s plane contour, it encircles this this minus 1 over K point twice actually. So, that N also becomes twice actually. So, P is 2 N is 2, so Z equal to P minus N equal to 0 actually; and that tells me that, the system is stable because, the close loop system is stable, because there is no close loop poles in the right half plane. That is how that is how the analysis goes actually.

It is a beauty actually I mean if you just contour I mean if you just draw this contour plot and just do it slightly carefully; then it will do if it just looking at the number of counter clock wise rations remember that, this is one rotation this is second rotation, the counter clockwise rotation will give me the information about stability of the closed loop system. So, it is beautiful in that sense.

Also may remember this is a function of K, if K is high then this point is going to travel more and more close to origin actually that mean I am more likely took encircle that. So, high gain, if I have a very high gain that it may leads to instability also. If I have a low gain that is a small gain theorem and all that it will I mean there is in a non-linear system also, it is roughly true in a way.

So, if I have a small gain thing then what happen this fellow goes outside of this this encirclement and hence, the system remains I mean kind of stable actually; well that we will see in a different example also, this example is just shows that it encircles twice and hence, we have the stability actually. So, this is that another example.

(Refer Slide Time: 48:54)



The one difficulty comes is if you have this this contour then, on this axis you may encounter singular points also. On the contour, you should have not had singular points right. So, infinite poles and all will not be so much worried, but these are these are the poles some poles on the imaginary axis, these are points of singularity actually.

So, you should be able to avoid that actually you should try to avoid that; one way of doing that is put a small circle, small half circle around that and then, interpret that Nyquist plot actually. So, that is done in a details things and all you will find that in I mean in (( )) and and more details in Ogata also. But let us seen this example what happens actually, this is a type 1 system S is there. So, if you see the type 1 system behavior something like that should happen; and that is what it is happening for one part of the plane. Remember, this Nyquist plots are always symmetric about (( )) axis in the G H plane also, about the real axis it will turn out to be symmetric.

So, I will I am able to map that from from 0 to infinity sort of thing, if I infinity I am starting from somewhere and if I am travelling something like this or let me travel (( )) something like that, I start from 0 and go to infinity. So, if I start from 0 I will start from 0 here, then go to infinity something that way; and then, their mirror reflection of that I mean mirror image will happen that way, it is symmetric about real axis anyway.

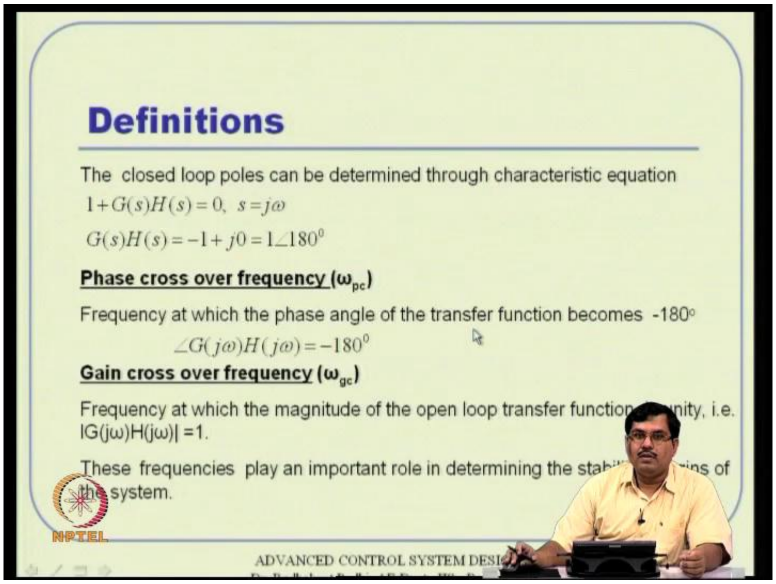
So, the plot happens to be somewhat like this and to know this point where it cuts actually you can see this G of j omega and that you represent in real part and complex part; and make the complex that imaginary part I mean real part and imaginary part divide it to two parts; and make the imaginary part equal to 0, find an omega where it depends to be 0 and substitute that in the real part to get the value of that. So, that is how you get a value.

Now, this particular value happens to be I mean this particular value happens to be this minus 0.0083 or something like that there actually, which is lot lesser than minus 1 point is minus 1 is somewhere here and this point happens to be like that actually. So, what it tells me, like if I if this analysis tells me that, there is no right half poles here open loop poles that is 0; and the number of encirclement is also 0 actually.

So, 0 minus 0 is also 0. So, in further analysis tells me that, if I if I work with some gain K which is less than this particular value, then the system will then the nature will always turn actually turn to be stable. Now, remember if I keep on increasing, increasing, increasing then this this entire plot is actually ballooning (( )) that means, it will it is going to be more and more like a inflation of the balloon actually.

So, somewhere at at this particular value, K equal to 120.5 it will exactly cross through minus 1 actually and after that it will become unstable, that is how it happens actually. So, that is that is the beauty of this this Nyquist diagram, and more details are there in the in classical control books actually. Now, all these frequency response is well respected even now in industry, because of its beautiful concept of robustness actually. So, let us study that in a slightly in careful manner.

(Refer Slide Time: 52:08)

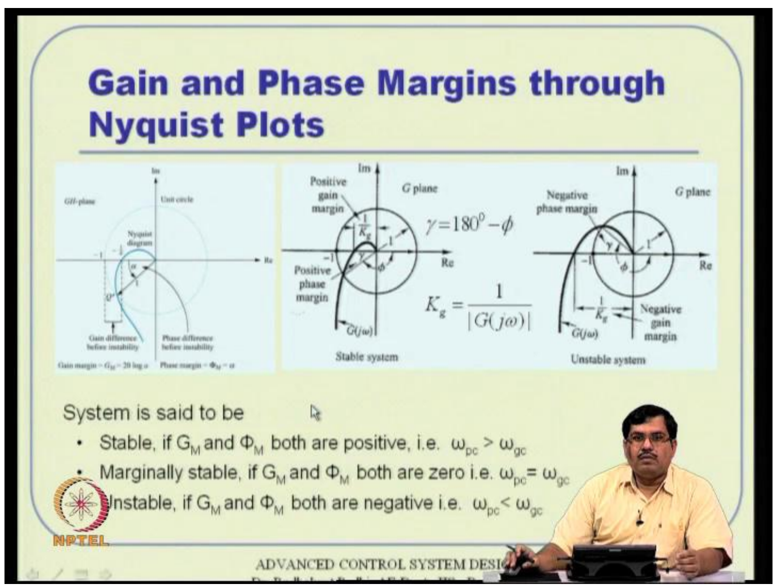


So, what it is before you go there, we just talk about two things here; one is called phase cross over frequency omega, essentially these are corner frequency sort of thing actually. Phase cross over frequencies omega p c and gain cross over frequencies omega g c these are not really corner frequency. Because, remember that this phase cross over frequencies are somewhat related to this characteristic equation. We will study that in a little (( )) here, they are related to that, but they are not same actually.

So, this is the characteristic equation, if I plot S equal to j omega this is the relationship that I am getting actually right G of H s or G j omega H j omega has to happened I mean it should be equal to minus 1 actually; and this minus 1 is nothing but magnitude 1 and angle 180 actually. So, phase cross over frequency is defined something like this actually. So, it is a frequency at which the phase angle of the transfer function becomes minus 180 degree right if if the frequency where the angle becomes minus 180. And the gain cross over frequency it is the frequency at which the magnitude of the open loop transfer function becomes (( )) actually. Remember, this G j omega G j omega H j omega that is what we are studying here actually.

So, this at this particular thing contains angle and magnitude. So, phase cross over frequency is a frequency where this angle becomes minus 180 degree; and gain cross over frequency, the frequency where the magnitude becomes 1; and both are troublesome by the way. Because, if you have this then the Nyquist plot will pass through that minus 1 point actually right what we are looking at this minus 1 point actually here.

(Refer Slide Time: 53:48)

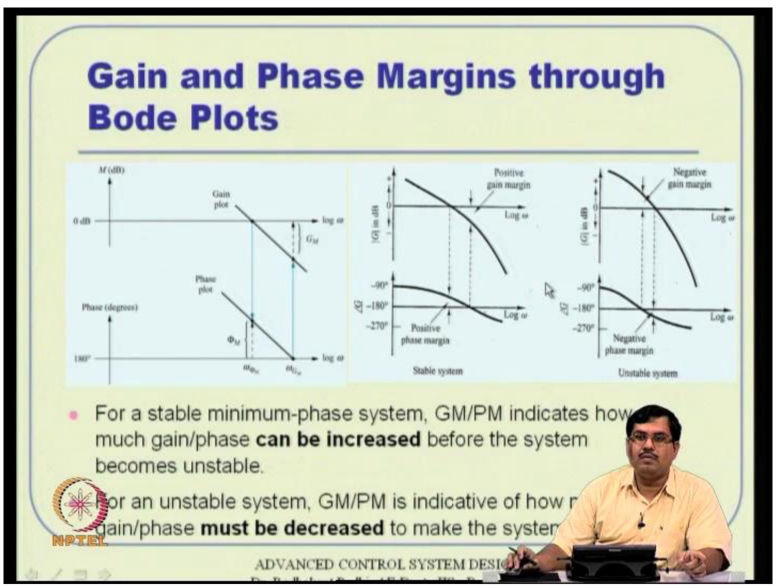


So, this is let us study that in a detail. So, let us say this this particular typical Nyquist diagram let us say we study. Now, we are happy with that because, this particular point what why it crosses the real axis is away from that. So, that the difference what we are having here is actually is indication of robustness, how much robustness we have, the moment this point travels here we have a robustness you see there actually.

So, what I am looking at in I am looking at this looking at this plot, it turns out that if I multiply this gain by some quantity, it will pass through (( )); or if I add this phase angle by some quantity, it will cross that actually. So, as long as I am somewhat like this minus 1 is outside I am fine. All that I want to see is by how much quantity I can multiply this gain, so that this curve will pass through that; or by how much angle I will add this quantity, so that this plot this plot will still go through minus 1.

So, those those two values are something like stability margins actually. So, the one is called gain margin, one is called phase margin. So, these the magnitude whatever happens to be, you have to multiply by 1 over K G right. Suppose, this this this this magnitude is K G then 1 by K G if you multiply then, magnitude becomes 1. So, this particular quantity becomes gain margin; and if this angle what you are looking at, that angle becomes phase margin actually.

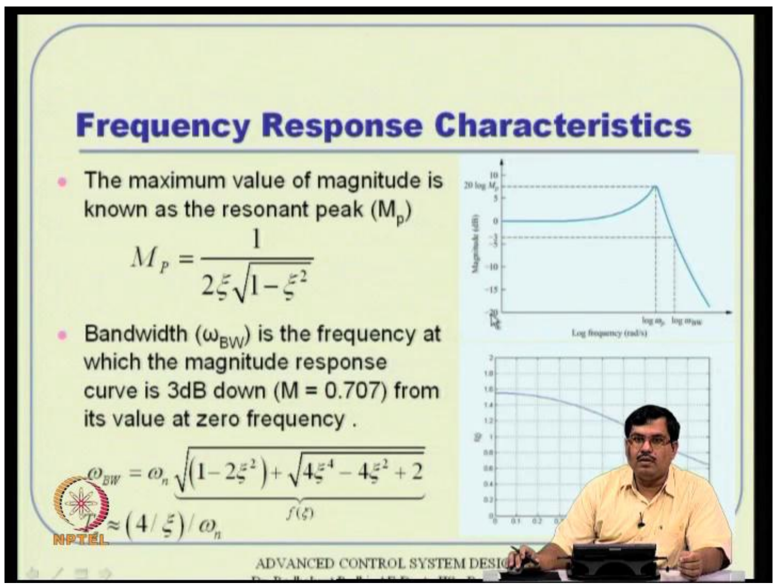
(Refer Slide Time: 55:05)



So, Bode plot looking at Bode plot, this is the gain cross over frequency remember that. Cross over frequency, the frequency where this magnitude magnitude becomes 0 actually right. So, I mean magnitude becomes this log magnitude become 0 what I mean actually, the magnitude becomes 1, so log magnitude becomes 0. So, if you see the 0 line in the in the Bode plot then, this is the plane then you go to the phase part of it, then you see how much is there, that is a phase margin. And if you see the gain plot I mean sorry the phase plot, there is a phase cross over frequency and you look at to, look back to this gain plot and then, tell that is a gain margin that I have.

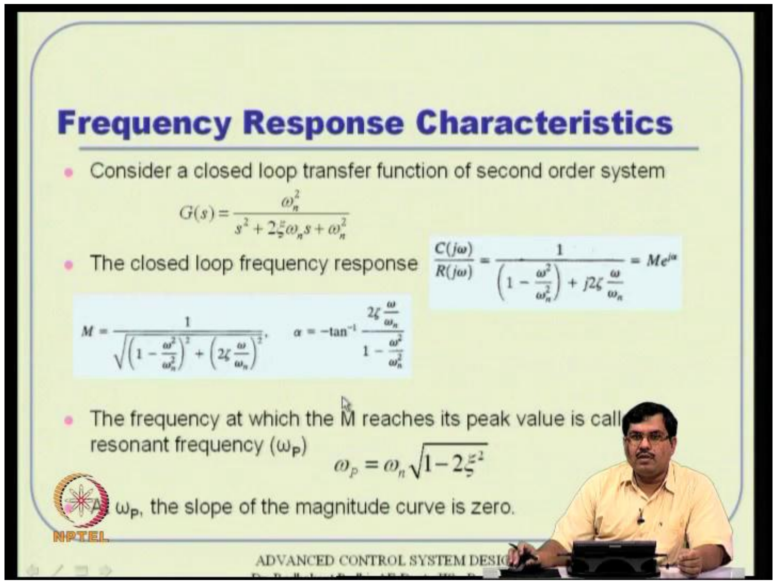
And if this plot happens to be in this way, then the both gain margin and phase margins are positive, so that means my system has some robustness. And if it happens to be other way round that happens to be negative gain margin actually, so that means the system is not not robust. And they will happen to be 0 at exactly the same point actually, that curve should pass through minus 1 point anyway that will happen to be 0. These are the concepts that are much useful there.

(Refer Slide Time: 56:05)



And then the last concept is something called bandwidth actually, which is also important here.

(Refer Slide Time: 56:11)



There are further analyses actually how do you do that, what is called a peak frequency corner I mean this closed loop frequency response, there are studied little more actually. Then there there is a peak frequency, this frequency plot, where this curve actually gets a peak value sort of thing. And there there is a bandwidth definition, which is the actually the frequency range just 3 d B below the 0 line actually, this entire range I will consider a some sort of a filter idea basically that it any signal pass through that and all that.

And then the bandwidth is defined something like this, omega n into this particular function actually. And interestingly this function, if you plot it turns out to be like this (Refer Slide Time: 56:45), that means these are all positive number. And settling time is also like remember in a approximate sense it is 4 by zeta omega n that means 4 by zeta by omega n, I can interpret that that means there is a multiplication term here, and there is a multiplication term here actually that way. So, if you look at this two, settling time is actually inversely proportional to bandwidth. See, if the bandwidth is very high, then the settling time is almost 0 actually. These two you can interpret that a some sort of a scaling factor actually; if I put that together I can I can interpret settling time as an inversely proportional to bandwidth actually, then that happens to be that way actually.

And another interesting thing is that 0.707 that is our zeta that I am talking about, then what happen actually? This zeta value is 0.707 if you see this plot little carefully, this magnitude is 1 actually; that means that particular zeta zeta equal to 0.7 around that, your bandwidth is nothing but the natural frequency. So, that is that is the beauty part of it actually. So, here all concepts coming from frequency response characteristics, and which is these are very useful in classical control system design. So with that, I think I will stop here. There are some points to remember that we have already discussed before probably I will talk that in the next class also actually.