Advanced Control System Design
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Lecture No. # 04
Classical Control Overview – III

Last few classes, we have seen first and second order systems. The time response as well as some characteristics followed by some (( )) criterion, and steady state error conditions. We also discussed type 1, type 2, type 0 all those kind of systems, and their implication of like steady state errors and other thing.

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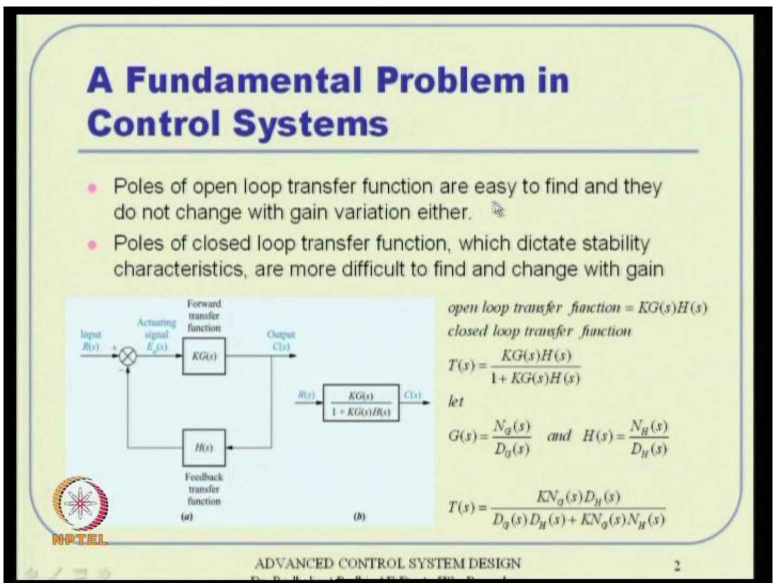

We will continue our review for further and this particular class we will discuss about root locus analysis. Then, we will move into further topics actually. So, if you see the classical control system this we can summarize it in this block diagram sense, this is a plant here, and there is a feedback transfer function which comes into picture, the error signal gets amplified by gain factor K, which is sincerely serves as a controller here. And if you see the closed loop system, the effective transfer function is dictated by this. So, this is a reference input to output, the closed loop transfer function dictated by there. So, that essentially K times G, G of s into H of s divided by 1 plus K times G, G of s, H of s provided this negative feedback; positive feedback then you will have minus sign over here actually.

There is a fundamental problem here as for as poles 0 consideration we have primarily interested in poles and zeros of transfer functions, because we seen that the stability characteristics are largely dictated by pole locations. However there is a fundamental problem here, because poles of open loop transfer function are rather easy to find and open loop means we mean this K into G of s H of s that is the open loop transfer function. It is rather easy, because G of s and H of s once you know and typically they are given in fractional decomposition sort of a form s minus a into s; minus b sort of sort of things, once you do that it is rather easy to find that the poles and 0 location.

However, if we talk about close loop poles which are of our primary concern then we have to do this further algebra, and that is dictated by the poles of the roots of this particular polynomial which will appear here in the denominator. So, I mean if possible we want to avoid this complicated algebra and quickly want to infer some stability characteristics of this closed loop behavior from the open loop pole location the question is seen is it possible is it visible? And the answer turns out to be yes it is visible, and that leads us to this idea of root locus analysis actually. And one more thing to notice this pole locations of the closed loop transfer function are also functions of the gain.

However, the open loop pole locations are typically not functions of the gain I mean, they are functions of the gain I mean, if we equate this equation to 0, then it takes whatever solutions to pops up is actually function of gain K. However here, if you make it equal to 0 then the K cancels out. So, typically it is not a function of gain basically. So, that is another advantage actually.

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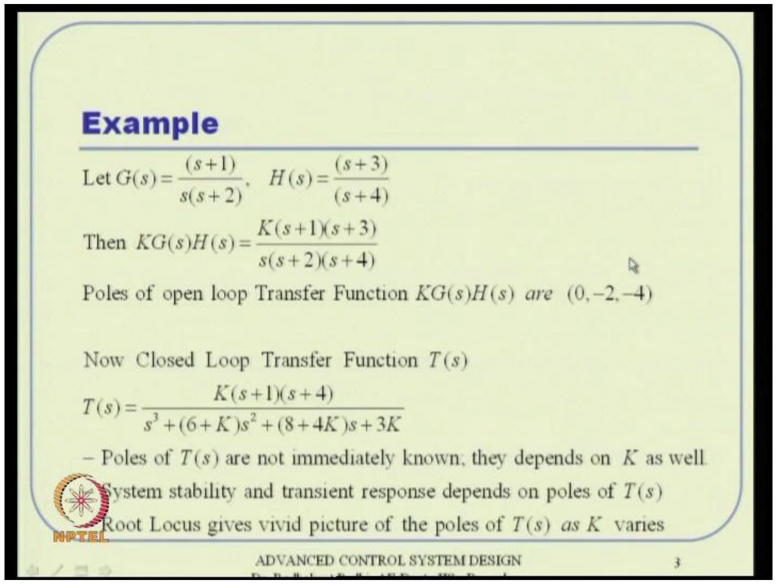

So example, sense we will see let us take G of s like that H of s like that and then K times G of s H of s becomes the simply like that just multiplication of these two transfer function. And hence the open loop transfer function poles will be at 0 s equal to 0 s equal to minus 2 and s equal to minus 4. So, these are open loop transfer function poles actually. However the closed loop transfer function is dictated by something like this.

So, what happens I mean we can clearly infer that poles of for these closed loop transfer functions are not immediately known I mean, once you know the value of K then only we can solve this and find out that particular location for that particular gain K? And then system stability and transient response however depends on pole of this one not pole of that open loop thing. So, root locus essentially gives a vivid picture of the poles of T of s, has K varies, so one particular value of K we will not be interested, we will take all possible values of K and typically when we talk about negative feedback system we take the gain is positive transfer system varies from 0 to infinity. So, as gain varies then how do the, where do they, this pole go where do they start things like that where that is our concern.

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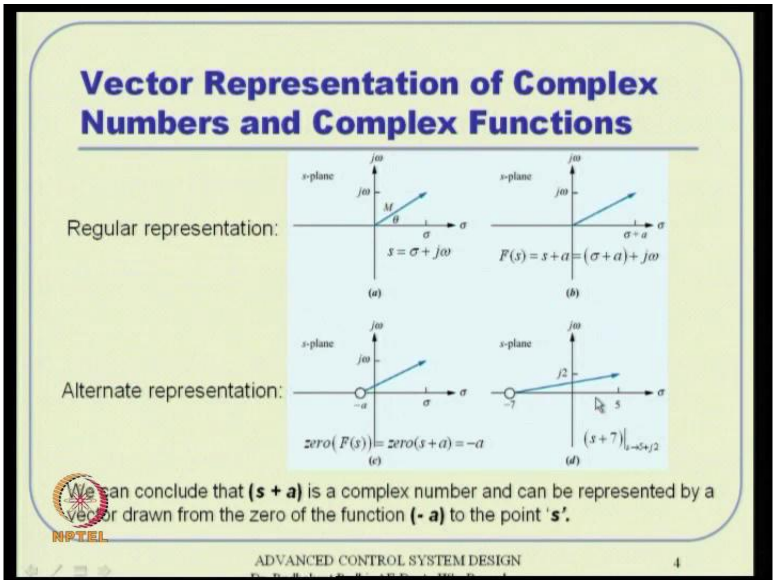

So before, we before proceeding further, we can see this, vector representation of complex numbers and complex functions. So, if we talk about a simple complex number which is s equal to sigma plus j omega, then in a s plane we can we can have the real component sigma and j omega is the imaginary component, etcetera. And if you simply take F of s which is a function s plus a then a being a real number, then essentially the real component gets change it is sigma plus a becomes imaginary part remains same.

So essentially, you can also give it another vector representation with real part sigma plus an imaginary part just omega. As an alternative representation however, you can also you can change this origin to the 0 location. So let, I mean let us not start this vector from origin of this particular complex plane we will start this vector with minus a. Then it will terminate as a omega only. So, either we represent it by this vector or you represent by that vector. So, as an example if you take s plus 7 as s plus equal to like 5 plus a 2 then it starts with minus 7 and then ends at 5 and j 2. So, it is essentially how you represent how you visualize this vector diagram essentially. So, we can conclude the s plus a is a complex number and it can it can be represented by a vector drawn from the 0 of the function which is minus a to the point s that is alternate representation essentially.

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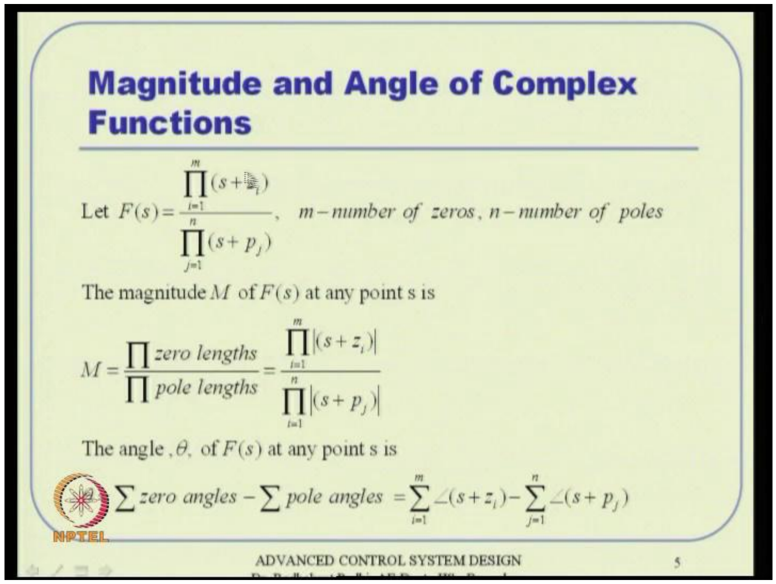

So, how does it help? Now if you in general we just took s I mean, here you take s very simple function. Now in general we are interested in transfer functions, which is a numerator polynomial by a denominator polynomial so in general, we will be able to represent it something like this. So, it is a multiplication of terms in the numerator s plus z i that is s plus z minus plus z 2 like that up to m; s plus z m, and then denominator polynomial s plus p 1 s plus p 2 and things like that z transfers 0 and p transfers pole that is why these constants are easier.

And then the magnitude of this particular value that F of s, actually if we just compute this m magnitude then it turns out to be multiplication of magnitudes of this divided by magnitudes of that by complex number theory. So, magnitude is rather easy the angles when you try to find angle theta F of s at any point is given by angles animated from this like angle that comes out from this, minus the angles that come out comes out from there that is the simple complex number theory actually. So, what you tell the magnitude of this particular function is dictated by this formula whereas, the angle that that comes along with this F of s is dictated by this subtraction formula.

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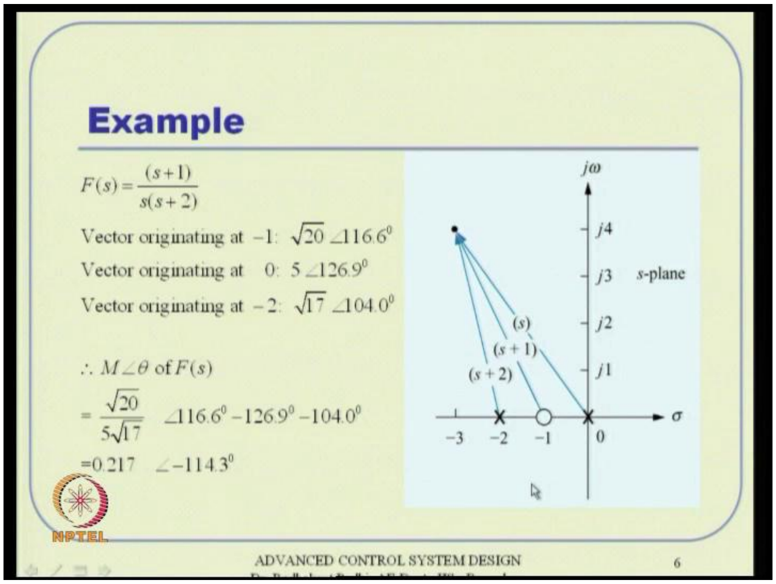

Example, again if we take a function s plus 1 by s into s plus 2, s this particular I mean, if you see this vector is originating at minus 1 there is a 0 here, at minus 1 there is a pole at 0 and there is a pole at minus 2. So, if you take let us say one particular point you consider and probably like, minus 3 plus a 4 that is the point that we are considering. And then what you are telling here is I mean this various vectors and there magnitude as well as angles. So, the magnitudes for this particular vector we start from minus 1 goes all the way is given like this, square root of 20 angle, angle means it makes angle from positive to this particular angle in anticlockwise sense, and then vector originating at 0, that that is given by that 5 angle 126.9 vector is originating at minus 2 again given by something like that.

So, this information we know basically, as far as this particular point is concerned. Now the question is like what is the angle of I mean what is the magnitude and angle associated with this particular function evaluated at this point? This a function s plus 1 divided by s into s plus 2 that needs to be evaluated at this particular point s equal to minus 3 plus a 4. So, then this will go to a different point and that point is dictated by what magnitude and what angle actually that is what our concern. So, then using this previous formula what you have here the magnitude turns out to be like square root of 20, because that is a that is a 0 divided by this magnitude 5 and to magnitude square root of 17. So, that is the magnitude part of it and then angle which angle of 0 go first positive.

You have to you have to sum it up all zeros angles you have to sum it up, so there is only one 0 out here so that is the part minus the angles going from poles actually. So, these are the 2 poles and the minus 126.9 minus 104.0 and if you simply do this algebra it turns out to be this is the vector that you are talking here, so if you just take this point pass it through this functions, it will turn out somewhere like a 0.217, so that means somewhere here. So, it will turn out to be something like 0.217 that is the magnitude going from here, in whatever direction starting from this region that is the magnitude with 1 0 minus 114.3 that what it is. So, it will turn out to be in the somewhere in the in this direction actually probably you can verify that so that, with this concept we are kind of ready for Root Locus Analysis just to see root over what it talks about actually.

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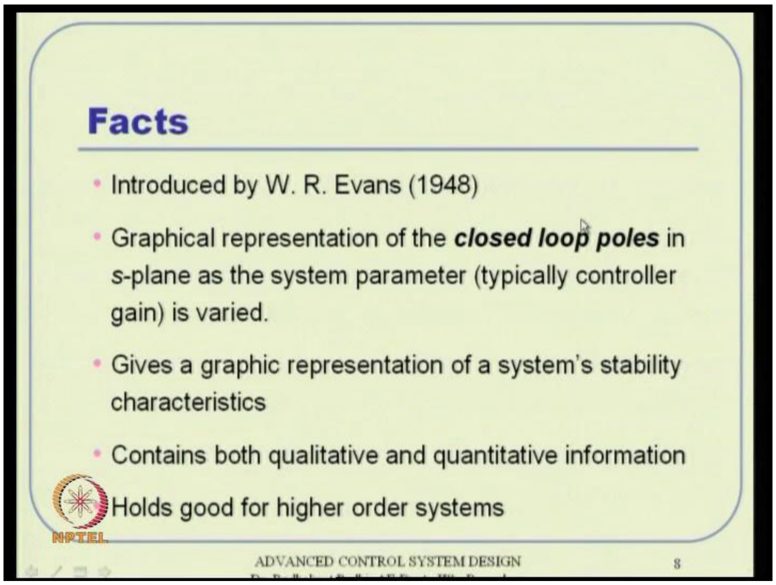

Some of the facts last it was introduced in 1948 almost after the second world war, so the region that is gone into this first, and second world war is probably not took help of root locus analysis they were sincerely, based on frequency domain analysis which we will see may be next class some of that. And then it is introduced around 1948 and then there is a graphical representation root locus is sincerely, talks about some sort of a graph of the closed loop poles in this plane has the system parameter varies that means, the system parameter essentially what you mean is the controller gain actually, so as you as you vary the controller gain from 0 to infinity in that particular set of what I discussed.

So, this particular thing I told that if it is a negative feedback K varies from 0 to infinity, then as the as K varies from 0 to infinity where do this poles go actually. So, that is that that will give us some sort of a locus that means some sort of a graph which we are interested in actually.

So, and then essentially it also contains qualitative as well as quantitative information. So, it is just not only qualitative it also talks about besides quantitative information as well actually. That means, if your particular if we just freeze your I mean tension to one particular one point on the root locus it actually gives you that particular pole location for that particular value of gain K. That also has a qualitative I mean, quantitative information and the beauty of this particular method is also it actually holds good for higher order systems. See most of the analysis what we know in classical control system or typically limited to second order approximation things like that and this is not limited to that it can it still talks about I mean, this higher order polynomials to an arbitrary degree basically take it to very high order.

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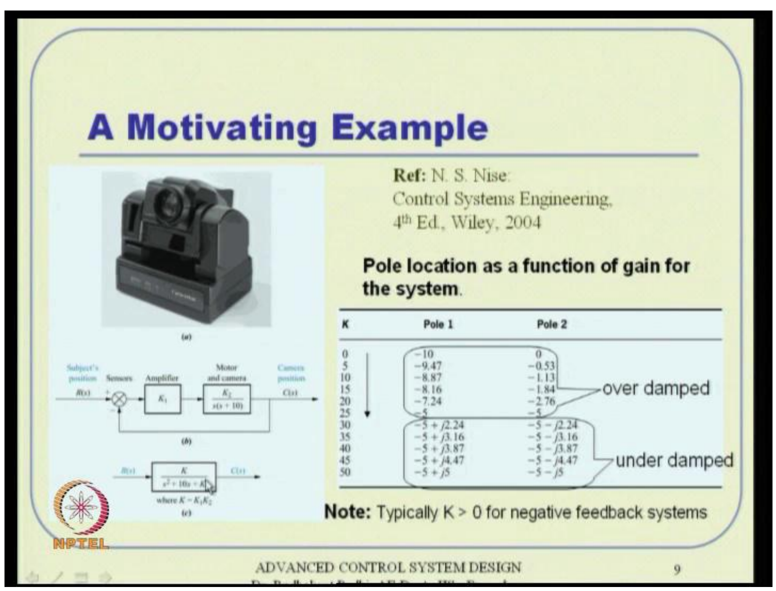

Anyway, here we will discuss a small example again this is a practical example taken from again Norman Nise. So, what it talks about is a some sort of a camera control system and this camera is capable of tracking an object the moment you point out a object, then it will keep on tracking that under its limitations actually. And so, this obviously has a motor, and then that motor is suppose to tilt this camera up and down sort of things in that particular plane which is capable of tracking so this essentially, these systems input and output is given by this kind of a transfer function which is a motor camera transfer function K 2 typically, we do not have a control on the value of K 2 for say, but sometimes it may it may allow us also that not our primary concern here, it also has an amplifier which serves as a controller gain K 1 which essentially we can manipulate. So, then if you see this closed loop transfer function unity feedback gain sort of thing I mean, feedback system, essentially it turns out to be something like that, where K is nothing but K 1 into K 2; that means, if you manipulate K 1 then the gain of this particular system closed loop pole are also effected through this gain K.

So now, what we do I mean we go back and we will vary this K and gain K now onwards we will not talk in terms of K 1 and K 2. We will dump everything in terms of K basically. So, we will vary this gain from K equal to 0 to very high value let say in this particular case 0 5 10 of step 5 basically, so take 0 then 5 then 10, 15 like that, and then compute these roots of this polynomial.

Now once we fix a value for this then the roots are available anyway. For example, if you put K equal to 0, then this is like s square plus 10 s equal to 0 here. So, essentially it gives us s equal to 0 or s equal to minus 10, so that s equal to 0 or s equal to minus 10 like that the moment you put some value 5 then you again go back to this quadratic formula that minus b plus minus square root of b square minus 4 divided by 2 a, that will give you 2 4, 6, 7 that way you will compute keep on computing actually.

So, then we list this generate this table and then this gain K varies from 0 to 50 let us say, then pole 1 will be of this order and this way it popes up, then pole 2 pops up this way. And what you observe at first part that this particular set of values, if you see the first part of it, up to K equal to 25 the poles were all real there is no imaginary component that means this is actually up to K equal to 25; 0 25, this system behaves as if like a over damped system remember this is all second order system here. And by design it is also like a stable system sort of thing. So, we are interested in for observing some properties here and then that is when first observation is when until K equal to 25 the system behaves as if it is a over damped system after that it starts behaving as if it is a under damped system.

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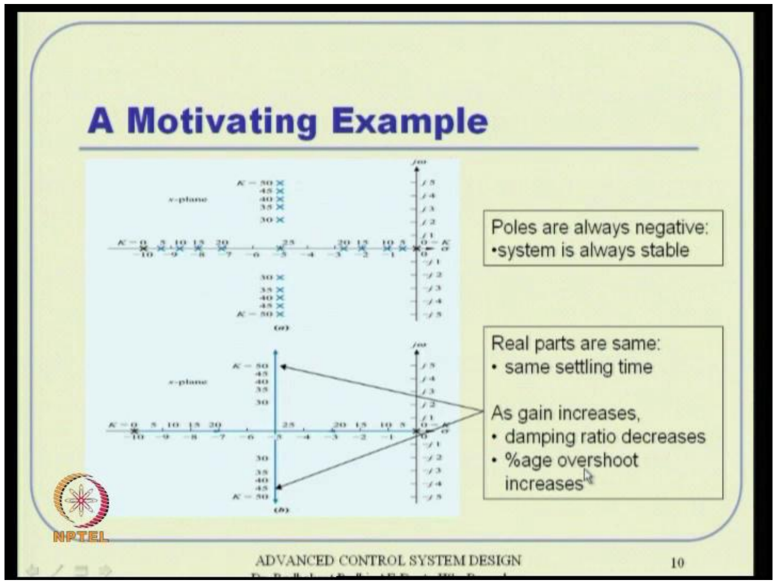

Now further things, so poles as you vary this gains from 0 to 50 the poles what you see here one pole is at origin, but after that they are all negative sign all negative real parts actually what you see here, so that is what it pops up here that I just located this locations of the poles under complex plane, and then as K varies the poles are always in the negative side that means the system is always stable. And then further observations the poles will start from like that. If you see this table, it is 0 minus 10 then minus 9.470 minus 0.53 like that. So, the one pole starts here, one pole stats here then it starts moving this way this starts moving that way and then at this point of time one goes that way one goes this way actually, so what happens actually?

So once, it starts going this way vertically once it really becomes a complex number it is actually becomes a complex conjugate phase that is the property we know for real physical systems the coefficient said to be real and the coefficients are real means the roots have to be complex conjugate actually. So, once you starts doing that way what does it tell actually, because this real part remains same you want the real part remain same, because settling time is roughly 4 by zeta term again and this component is kind of zeta omega and component etcetera we know that.

So, as long as this component remains same the settling time remains same no matter what gain you select after that whatever gain you select after that, the settling time will remains same actually. Now as the gain increases; that is where the poles will travel going one going up and up one going down and down what happens actually. We observed that this angle keeps on going up more and more high actually, one the angle this theta component what means if I draw a any particular location, if I draw some sort of a line here sorry this straight line suppose to be this a straight line and then the next one suppose to be one more straight line. So, this traveling is through this actually like it this happens this way. So, as a as the angle keeps on increasing we know that zeta that the damping ratio is a factor of this Cos theta component this the theta part of it. Then it happens to like when theta increases more and more than cos theta decreases, so that means the damping ratio start decreasing actually.

The damping ratio starts decreasing actually and for the same time the percentage is starts increasing also, because the pole locations. So, essentially what we have done we started varying this gain K then we started plotting this values and then we joined this points that came across and then it gave us lot of qualitative information actually. And if I really freeze one particular value of gain then I have a particular value of this poles and hence it will give me exact values of setting time exact values of damping ratio and things like that, so it also give us the quantitative information as well actually. So, this is the motivating example. And let us study further formalizing these concepts actually.

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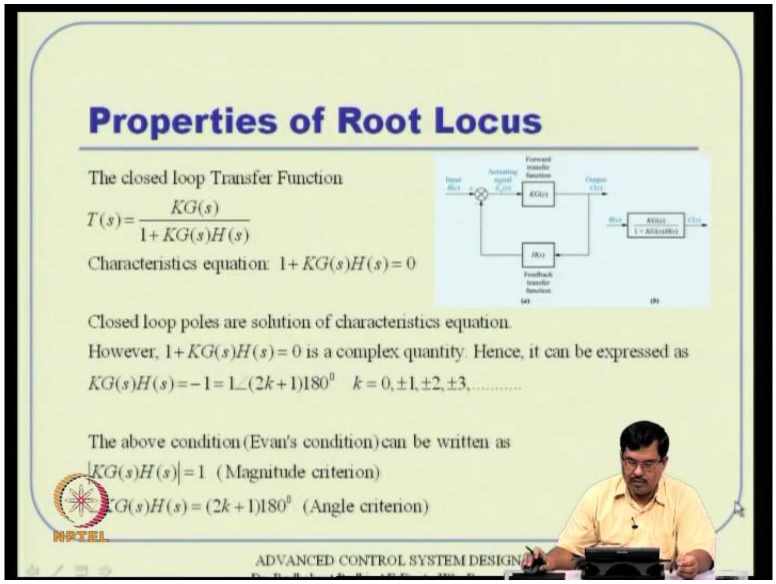

So, now we are ready for discussing some of these properties of root locus in a generic sense. So, we start with this transfer, I mean this close loop system, this is a close loop system dynamics and this is the close loop transfer function and hence, the characteristics equation when you see this is given by this particular equation is equal to 0.that way. So, if I really all that I have to do is analyze this equations slightly carefully actually. This is the characteristic equation what it comes up for close loop system I have to analyze it little more carefully actually, but in general this particular equation I can represent this I can interpret this rather as a complex equation sort of things, because this G of s and H of s where s is actually s varies in a complex plane. So, this entire equation is a complex equation sort of thing and hence this equation has to be satisfied. What you see here this is a primary equation that we are talking actually. K into G of s into H of s should be equal to minus 1 in complex sense actually. So, what that mean? This minus 1 2 information that the magnitude has to be 1 that the magnitude has to be 1 and the angle of minus 1 is an odd integer multiple of 180 degrees actually.

So, if my s that particular value of s is actually part of the root locus, then that particular value if I just draw the vector diagram then the magnitude has to be 1 and then the angle has to be this odd multiple of this 180 degree actually. So, this above conditions are called Evan's conditions, and you can formally write it as 1 first is a magnitude condition and the second is angle condition angle criterion actually; these 2 essentially dictates the root locus behavior, and root locus plotting the root locus how do you this all these things hover around these 2 conditions essentially.

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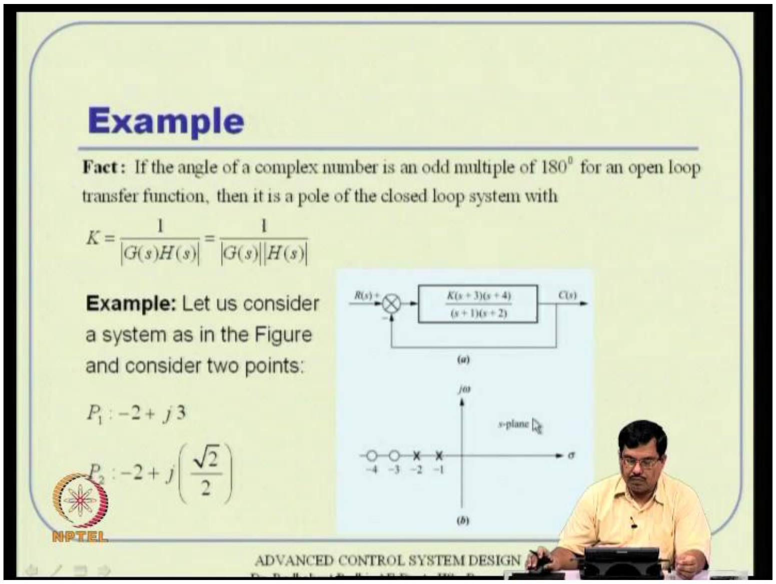

Again before proceeding further let us see a small example here. Now we know that this angle condition actually place a little pivotal role or primary role and deciding whether this is part of the root locus or not, actually one particular value of s. So, let us see that and then if it is a part of the value, if it is really part of the root locus then the gain is actually dictated by this formula. Because then we are interpreting all through we are interpreting them is positive value, so this positive value comes out of here and they are left out with 1 by this formula actually first thing the angle conditions helps us to know whether it is really part of the locus. And if it is part of the locus, then the magnitude is given, magnitude of that gain value the gain value that corresponds to that particular value of s is given by this formula actually.

So, let us consider this transfer function, remember this is open loop part of it H of s is equal to 1 here. So, this is all that open loop transfer function what you have here. So, we are considering 2 points - P 1 and P 2, and remember these are the open loop zeros and poles here, and later we will also utilize this particular example to draw root locus formally.

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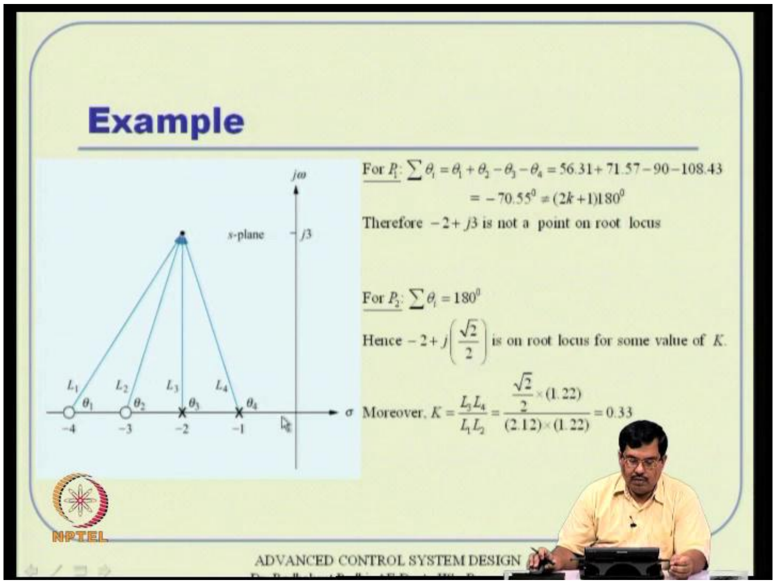

So, this P 1 and P 2 let us analyze a little more and then just talk about P 1 for say then P 1 going back to the same formula what we talked about angles? And all the 2 angles are coming from zero's this theta 1 and theta 2, and 2 angles are coming from poles theta 3 and theta 4. So, if you compute these angles theta 1, theta 2, theta 3, theta 4 and carry out the algebra, then it turns out this is the angle which is not an integer multiple of 180. So obviously, this point p 1, what you are talking here is suddenly not part of the root locus actually. However, if we talk about a point P 2, this point is like this minus 2 plus j 1 by root 2 essentially that that particular point. If you do the same algebra again it will turn out that it is the summation of these angles is actually 180 degree that means point P 2 is actually part of the locus. And when it is part of the locus, what is the gain value for that particular value which corresponds to this particular point is given by this formula. So, that is a L 3 L 4 divided by L 1 L 2 and carry out this magnitude condition, this is given by this that is what you are using here, and it turns out that the magnitude is something like that 0.2 and 3.

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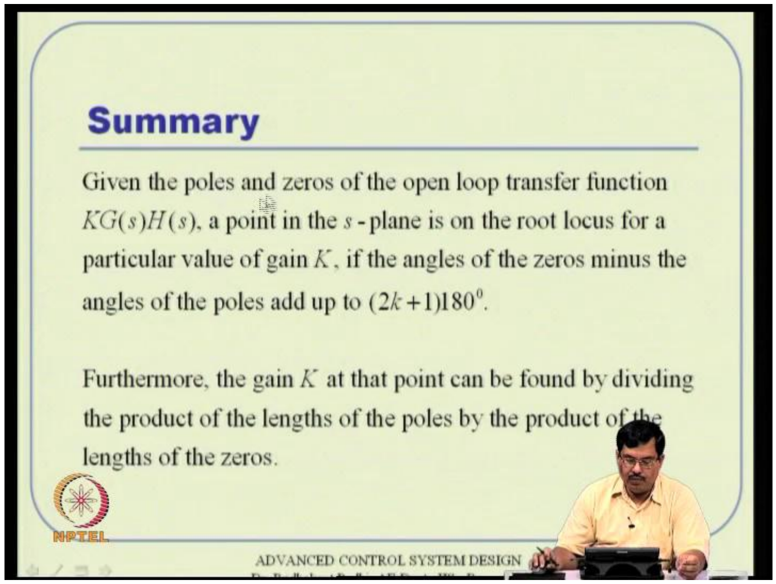

So, what is the summary? The summary of this observation is given the poles and zeros of the open loop transfer function. A point in the s plane is on the root locus for a particular value of gain K, if the angles of the zeros minus the angles of the poles add up to an odd multiple of 180 degree again the repetition of what you have already studied just now. And further most the gain K at that particular point can be found by dividing the product of the lengths of the poles divided by the products of the lengths of the zeros. Remember it is not length of the zeros by lengths of the poles its lengths of the poles divided by length of the zeros; the gain because of the 1 over 1 by this factor comes in actually that is why.

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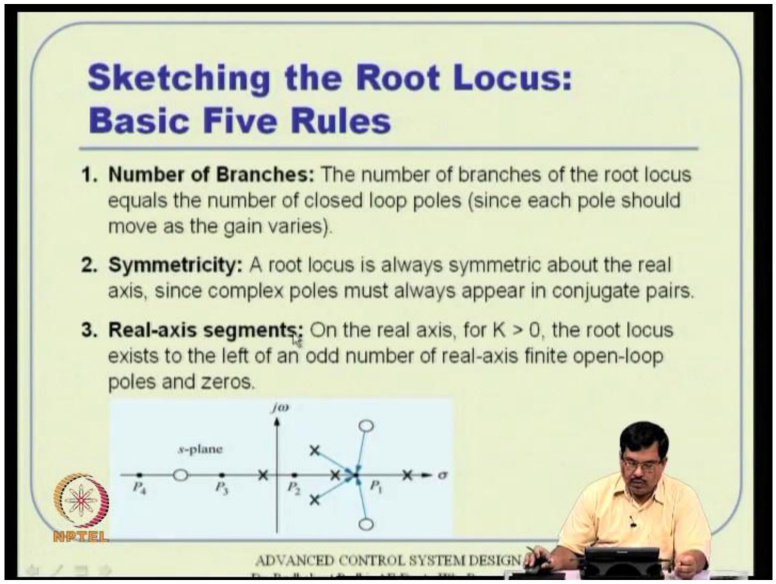

Now, formally it is telling how do you sketch a root locus and remember there are some basic 5 rules and further rules are also available for thing and all that. Essentially these rules were I mean the root locus were developed when the computer where not there, computers were computer power were not there. And that is actually the whole idea of classical control system in a way classical control system talks about transfer function, and the transfer functions was a necessity to avoid differential equation as it is actually.

The large handicap at that point of time was unavailability of computers which are very I mean, obvious now a day’s actually everywhere. So, one value was without solving this all the values of gain K we cannot this particular example that we studied for the second order system and we know the root location I mean, how to solve this second order polynomial, because the closed loop formulas are available. So, rather we can do it not in a I mean it is not that difficult to carry out this algebra that we discussed.

The moment it becomes higher order things, higher order polynomial I mean, the getting the closed loop solutions that way is not so and that is not a very I mean straight forward manner we cannot get the values calculations however we want to sketch the root location we have to we need to have an idea of how that locus I mean, appears in the complex plane. So, there are some beautiful results there and some 5 rules, let us study first thing is number of branches, how many branches a root locus can have and this very obvious I mean, because root locus by definition is locus of the roots of the root means locus of the poles. So, the number of poles is number of the branches, so if you have the number of poles dictate how many branches of the root locus are there that is that is rather easy to see.

Now symmetricity rule a root locus is always symmetric about the real axis, because the complex pole must always appear in conjugate pairs actually, if there is a complex part then it should appear in the conjugate pair otherwise it is not good the transfer function is not good for a physical systems, so that is that is the problem actually. If it is valid for a physical system, then the coefficients of the polynomials are always real and for that and those of polynomials the roots must be conjugate pair, and either it travels on the real on the real axis or it appears conjugate pairs. So obviously, root locus is always symmetric about the real axis that is that is also now difficult to see.

Now, here is the critical observation, and this observation tells us that the real axis segment I mean, if you just consider the real axis of the complex plane in which segment the root locus appears on the real axis that is the question actually. Now it turns out that if I just analyze any point P 1 let say. So, these are all open loop poles all zero locations let us say. Now let us assume that these 2 of zeros are appearing here, and 2 poles are appearing here. Now, if I simply consider the angle between these gains into these angles that is coming to this actually the contribution of the angles from these 2 poles will cancel out, and the contribution of the angles of the 2 zeros will also cancel out actually.

So, what happens essentially, so they really do not contribute any as far as the angles are concerned actually? We know that if it is part of the root locus then this must satisfy the angle conditions somewhere actually. So far the angle condition these poles and zeros do not play any role actually. So, what does it what is it left out we are left out with only poles and zeros that are appearing on the real axis basically. Now we know that if there is a pole I mean, if there are 2 poles just say I consider a poles somewhere here then there are 2 poles here, then these 2 poles will contribute to even multiple of 180 degree. If I consider these angles as 180 degree then it will turn out to be even multiple and hence they are not part of the root locus.

We always want odd multiples actually. So What? It essentially tells us that on the real axis for gain K positive I mean greater than 0, the root locus always exists to the left of an odd number, because any pole and 0 that appears to the left of this particular point p 1 will always contribute an angle 0 basically. So, that is also not a not a candidate for odd multiples of 180 this always 0 if I consider this particular pole then angle to that is this kind of 0 and this angle to that is also 0 like that it actually. So, I will not consider that part of thing actually so, all that I am left out is poles and zeros to that particular point. So, essentially what it means the root locus exists to the left of an odd number of real axis finite open loop poles and zeros, so if I just consider this poles and zeros on the real axis and I start counting from the right there is 1 pole here, so that is a odd multiple of that is an odd integer. So there should be some root locus here, but once I come this segment there are 2 poles here, so there should not be any root locus here, once I come here again there are 2,3 poles, so there be root locus in the segment like that actually you can continue do that. That will give us some idea of which segment of the real axis my root locus lies actually.

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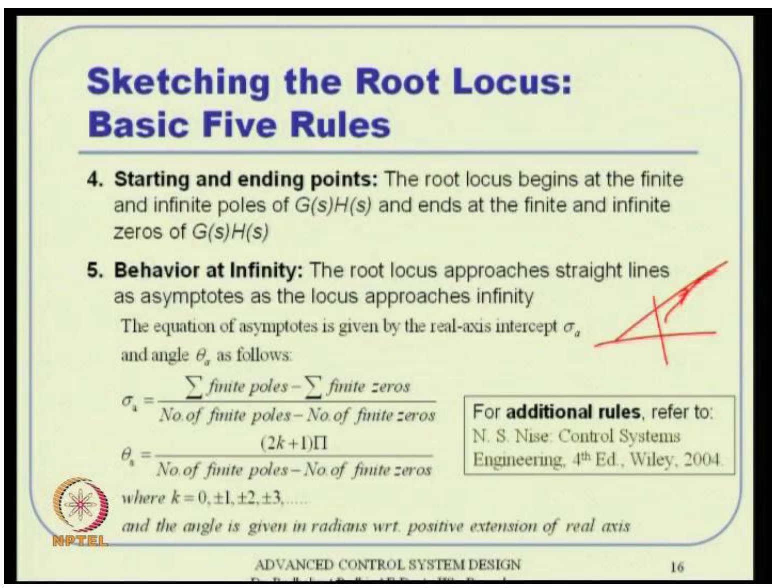

Now, there is number 4 let us see what is it starting and ending points, where does the root locus starts and where does it end actually. And obviously, the root locus begins at finite and infinite poles of this, and ends at finite and infinite zeros of open loop; I mean, open loop 0 actually. The root locus essentially starts at the poles of the open looped transfer function, and it ends at zeros of the open looped transfer functions, and it is easy to analyze also. Like if you just take the closed loop transfer function and then carry out the algebra in the limiting sense that what happens? When K tends to 0 and what happens when K tends to infinity, then this is also observed actually details are in the books actually you can see that. And then suppose there is a pole equal number of poles and equal number of zeros, then the matter is because if it will start from a pole and end at 0 that is there.

Most of the time what happens is number of zeros are typically lesser than a number of poles, and in those sense one of the ideas in classical control is for any pole 0 deficiency those can be assumed to lay at infinity formally it can be shown also that there can lie at infinity actually. So, any root locus that does not end to finite 0, it actually ends at infinite 0. So, that is there actually, now if it really an infinite 0 and s plane in which direction it will go actually the s plane is all over any angle it can go to infinity. So, then the question is then this rule number 5 comes into picture that tells us the behavior at infinity. So, the root locus approaches to straight line as asymptotes as the root locus approaches to infinity.

So, let us conceptually let us see this suppose this is something like that and the root locus where it will go to infinity I mean, in all direction it tends to be I mean which direction it will go. So, now it turns out that, if you if you use this formula what is given here with this particular value of sigma a, and this particular value of theta a I can draw some sort of a asymptotes actually and any pole that starts to somewhere here. And there is now here to go then it will actually do something, but eventually it will go towards the asymptotes actually it will merge in the asymptotic (( )).that is the all idea basically.

Remember this is the sigma a, that we are talking here is not the y axis asymptotic it is x axis asymptotic. So, this is the part that we are talking here and the angle is that particular angle. So, these are the basic 5 rules another other rules do exists other rules something like the give us ideas of the y intercept like the imaginary axis intercept where it will intercept on the imaginary axis that will come from the root locus criterion. Then it will also tell us there is one more rule which he tells us like on the real axis at which point it will start going to the complex domain that is the point of departure actually. So, that also is a is available then there I mean some of these rules are actually, there in this book you can this particular book you can you can study that actually. While this in a review class I will not talk too many details, but let us use in this 5 rules lets study that particular small example that we discussed here.

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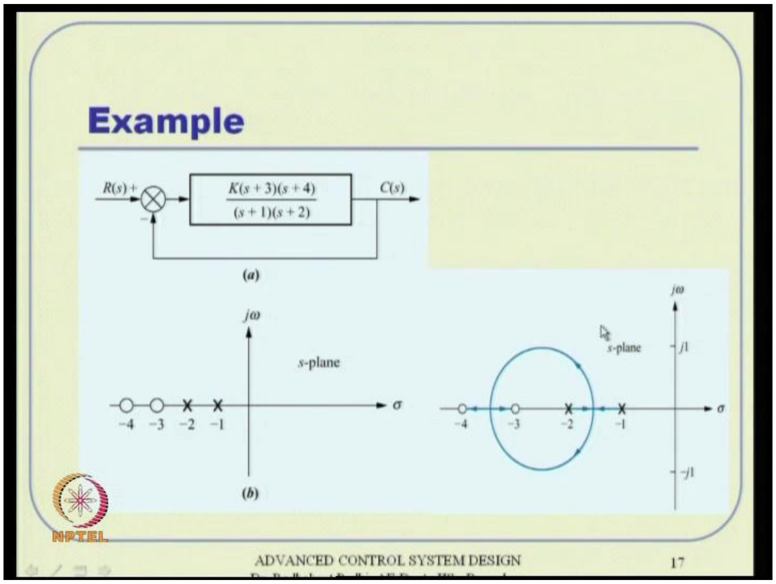

So, this is a this the properly, this is the system that I am talking here, and these are 2 poles, these are the 2 zeros of this open looped transfer functions. So, let us go back to rule number 1 number of branches obviously equal to the number of poles which is 2, so that must be 2 branches it has to be symmetric about the real axis, so these are there. Now, let us talk about rule number 3; real axis segment which segment it will appear obviously this one pole here, so this odd multiple, so the root locus should appear in this segment and then 1 2 3. So, this root locus will appear in this segment as well on the real axis as far as real axis is concerned.

So, it is one part to start from here one part to start from here, and both the things one should end here one should end here and then this part of the real line I should have a root locus and this part of the real line I should also have a root locus that the rule number 3 says and starting and ending point we already covered that we use to start from a pole and end at zero. And using this under further conditions were as I told that the point of departure and point of arrival can also be I mean, can also be computed from some derivative conditions actually.

So essentially, we know that once thing to start here one thing to start here this points of departure, and arrivals are known and where will where it will go ultimately that is also known, and it has to be symmetric about the real axis. So, all this condition gives us an idea that the root locus should start should behave something like that one part I do not know which part goes where? That part is kind of that information is silent, but you can assume that this sigma into this part probably starts from here it goes, and then goes here and goes there. And this part starts from here and it goes through that it goes there so that is the kind of representation of root locus. Further details are there in the book you can see actually.

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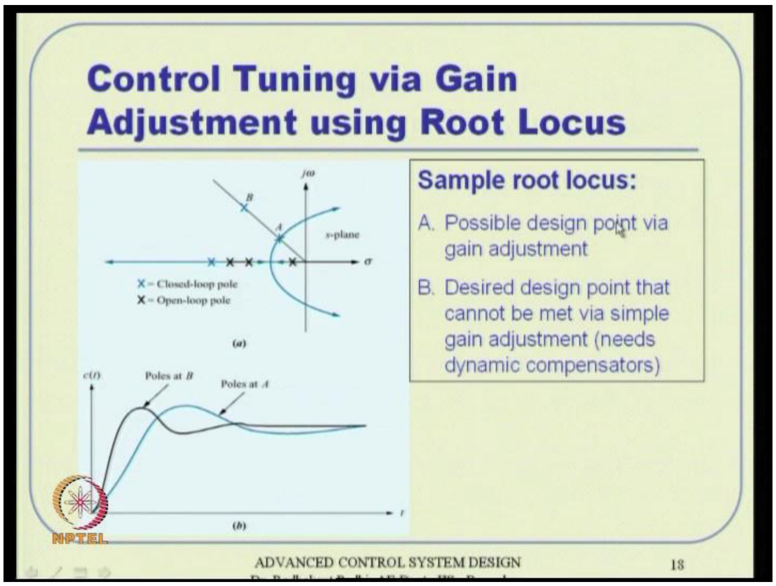

Now, after we plot the root locus we should take advantage of that to the best extent possible actually and how do you really it actually helps us in tuning the gains in a I mean in a good way basically. Now there are some ideas that in classical control that these higher order systems you can roughly interpret that as a dominant second order poles I mean, dominant second order systems and you can keep only the dominant poles then you know the settling time formulas, and all that actually assuming that there is no zeros in the loop and then assuming that the only 2 dominant poles actually. Further, particular approximation you know this settling time characteristics and then percentage overshoot things like that actually.

So, you can use those things and tell if my system is largely like a second order system, then that should be my that is should be my damping ratio let say, then this will give you some angle that is the that cos theta is that, so theta will be you will know that. So, you draw a line and wherever it intersects this line with the root locus. Let us say you pick up that particular pole actually, so that particular locations on the root locus will corresponding will correspond to certain value of gain K you know that. As each of the each of the location corresponds to a particular value of gain, so I will select that particular value of gain. So, that then I will go back and assimilate the original system actually, so to some extent it will give some sort of a design procedure of select in the controller gain K actually.

Now that mere may not be sufficient though, because if it is sufficient I am done, but if it is not sufficient that means, remember that gain variation can only allow me to select poles on the root locus anything other than that is not visible. What really if I want some other location let say location b here for different this angle remaining same means damping ratio will remain same, but that is not everything let us say let me see that for example, these 2 points will correspond to different setting time. And let us say these setting time is really good, looking at these plot also it is very clear let this pole will give large settling time which I probably.

So, I will go for a for this particular pole location which is your same damping basically, that means it will use it will help same percentage overshoot also, but it will help a better settling time characteristics also. Then only root locus plotting only the root locus will not help us actually we need dynamic compensators that means, we really need some sort of a integral loop or derivative loop which essentially leads us to see what you what we have so far is only proportional gain K constant, so that that p loop is already there in this part of. but in addition we should also have some sort of an integral loop I or derivative loop b so that essentially leads us to PID control design actually. So, if any pole on the root locus is then the simple gain adjustment will do otherwise, will go for PID design. So let us see an overview of the PID design and we will see probably little more detail next class also actually.

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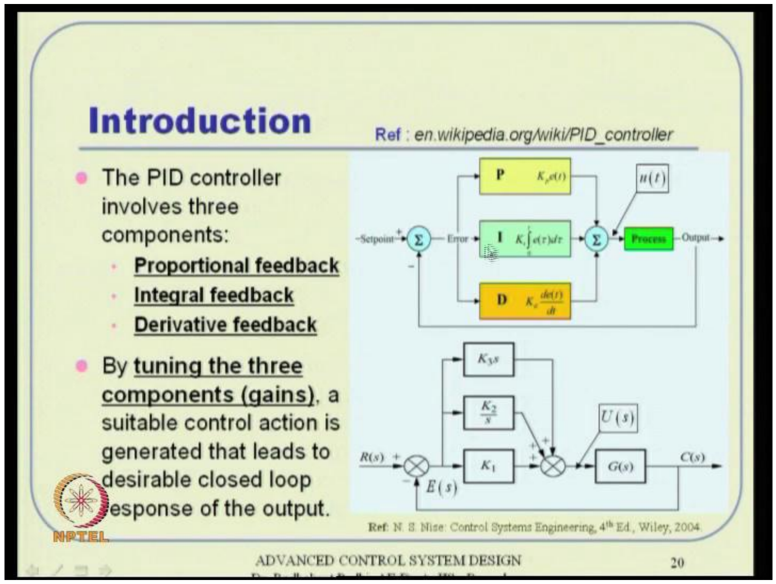

Essentially what is PID design PID controller essentially has 3 box in the loop and we already have this P loop so far in the in the discussion actually in a recent way we are talking about a I loop which is integral loop integral feedback and there is a D loop which is a derivative feedback actually. And it essentially turns out the different manipulate this error signal which keeps on coming to me in a time distributions then probably I will have a better characteristics of the closed loop process actually. Instead of simply instantaneously adjusting the I mean, error signal I will also take care of the history of the signal all I will also try to predict the error signal little later actually, that means the derivative information gives us an idea what the error signal should be in the future.

So, if I manipulate these characteristics then I will have a better property actually there. So, what essentially it turns out this will have 3 gains one is the proportional gain, one is integral gain, one is derivative gain and by manipulating these 3 gains I will essentially synthesize a controller which is summation of all these 3 signals actually. So, by tuning these 3 components or gains a suitable control action is generated that leads to desirable closed loop properties actually. So, we are not handicapped by only this P part we are having a maximum powerful tool by being bringing in the d and i component actually and in transfer functions sense we can represent this way. Earlier we had only this part of it K 1 part of it.

Now we are talking about both K 3 as well as K 2 remember this is an integral part and this is derivative part all right.

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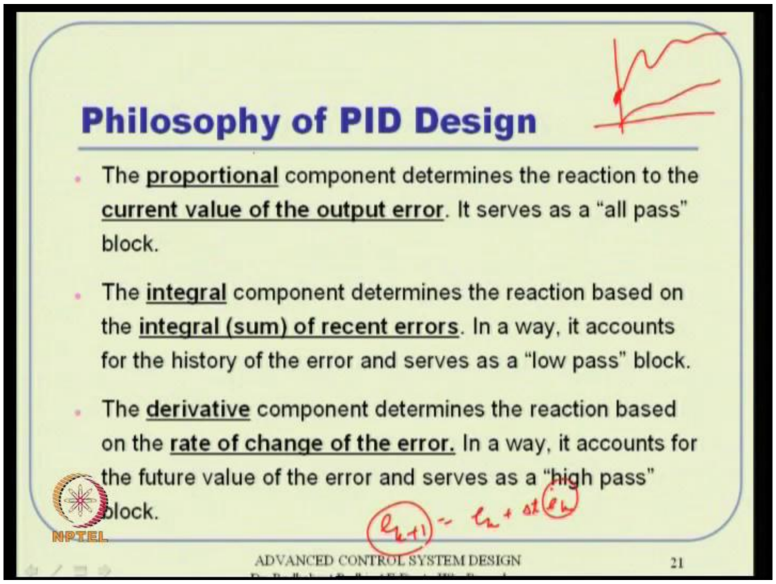

So, what is a philosophy? Let say what these different components do actually what are the capable of doing? The proportional component determines the reaction to the current event of the output error. So, it is ignorant about the history neither history nor creature it just talks about the current value of error and tries to kind of minimize that actually. So, it essentially serves as some sort of a all pass block it does not contain any frequency properties basically, just whatever error comes it just manipulate that error and then gives you control actually signal. Now integral component determine the reaction based on the integral or some of the recent errors integral K what we are talking here is finite time integral. So, we will have some finite length of integration, so that particular value is multiplied by gain K, I then it is use as a control signal actually.

So, in a way it accounts for the history of the error and essentially serves a some sort of a low pass block actually, because high frequency component we know that if there is a signal the integral value will have will very slowly actually. Even if the let us say if we talk about some signal which is like that, if you talk about integral value integral starts at 0 obviously at time 0, then it will not of that variation actually it will have low frequency variation. So, we will I mean, we will essentially can interpret this is some sort of a low pass block actually for integral component part.

The derivative however is actually fast rating and it actually takes care of that, I mean it manipulates the rate of change of error actually. So in a way it accounts for some future value of the error, but if you use this other integration formula then essentially it tells us that e K plus 1 is actually e K plus delta T into e dot K and this e dot K is a derivative component. So, by knowing this current value and this derivative component, I can actually predict what will happen to my K plus 1 in a rough way. So, I am using this property (( )) of time actually. So, essentially it tell it accounts for some sort of a future value of error and serves as a high pass block actually, so proportional is a all pass block integral is a some sort of a low pass bock and derivatives some sort of a high pass block actually.

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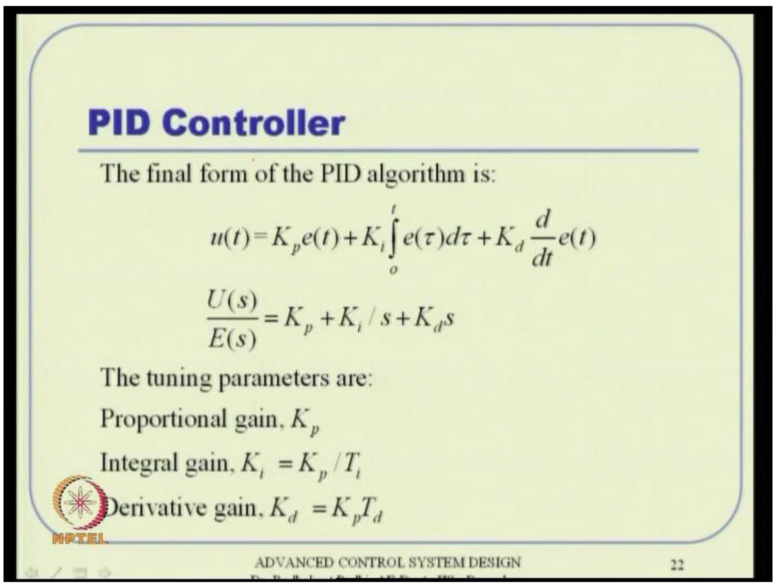

So, in time domain we can we interpret this way this is the errors signal what it comes. So, this control signal is partly from the proportional part and I mean then the integral part and then the derivative part any combination of that we are free to use by the way. We do not I mean we not necessarily have to use all the gains all the time in some part of the gain you can take it as 0 and then may not use it also that depends on the particular application what we want to do here actually.

So, in Laplace variable sense, it I mean the same thing can be represented something like this actually. So, the tuning parameters are something like of a personal gain and then the integral gain, and there is a derivative gain and these I mean. these integral and derivative gains are always compared to proportional gain, whatever proportional gain what you have how much scaling you are doing actually, that that essentially place a role and hence this integral gain I can I can write it something like this divided by some constant T i and the proportional derivative gain are something like I multiply by some constant T D actually.

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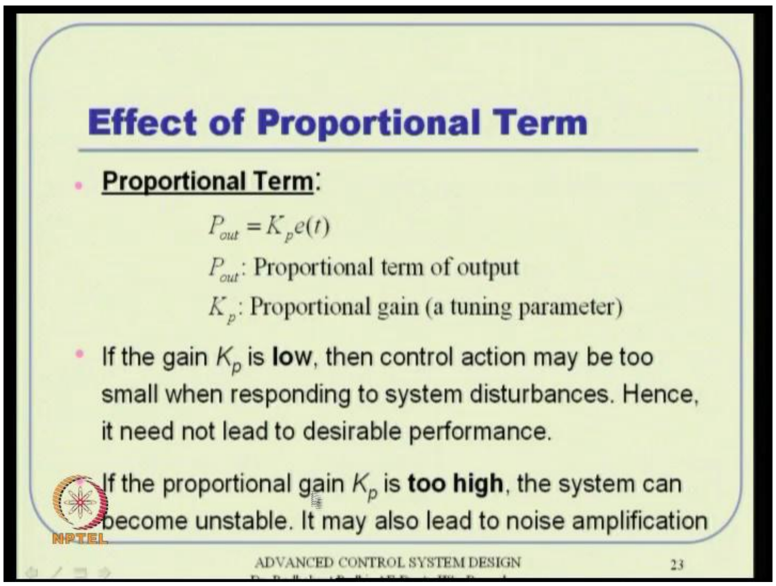

Then effects let us talk about a little bit elaborate sense the proportional term is essentially coming out to be something like this K p into e(t) that is the proportional term. So essentially, in this proportional term what you are having is K p is the proportional gain actually. So, if the gain K p is low then this component turns out to be very low and it may not be sufficient for our need actually. It may I mean, it may not give us the desirable settling term properties actually. It will have a large settling and things like that which you may not require. but on the other hand, if it is very high then essentially it has a property of making the overall system unstable that the high gain draw batch actually.

So, we really do not want this K p to be very high actually and the moment K p is very high remember this all parts I mean, all parts blocks certain things. So, any noise component will also get amplified by this y value of K p and it will affect the system dynamics so that is also not desirable actually.

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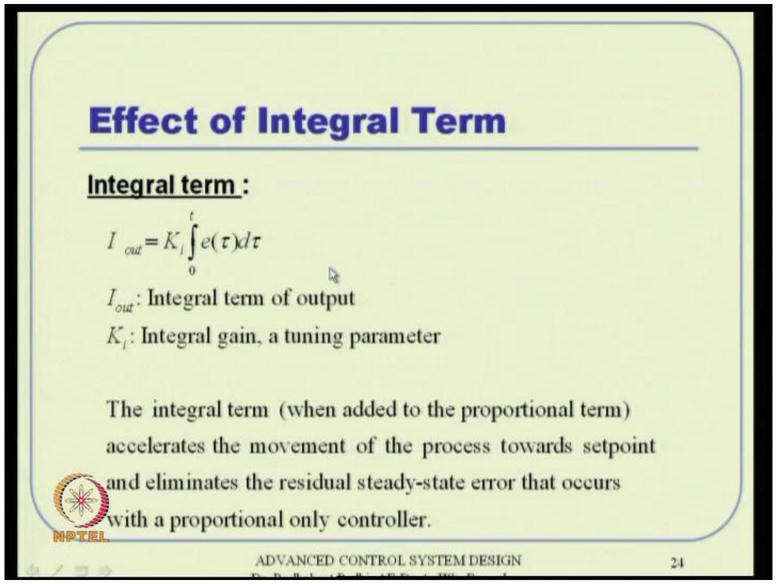

So, just select this K p judiciously sort of thing, now coming to the integral term, so this loop so this is the component that we are talking here. And here the K i is essentially the tuning parameter. So, the integral term when added to the proportional term actually it helps us in accelerating the moment of the process towards its end points and, because there is an extra enforcement you can think of that way. Even though K even though K i value is constant, this integral term is suppose to build up actually as long as suppose even, if you talk about some sort of a bias value let say steady state error sort of thing. So, let say this is there and we know that this is second order system and there is a steady state value, and if you really start your integral block somewhere here, then you know this integral area under the curve keeps on building actually.

So, even though this gain value K I is a constant value this area term keeps on building and hence this component keeps on building up actually. So, that essentially helps us in eliminating the error in a good way and you have seen in the steady state analysis also that if you have a integral in the loop, then essentially it gives us some sort of a additional I mean it makes the type 0 system type 1 and type 1 system type 2 like that because of this divided by s property.

So, any finite value of error that popes up in a type 0 system let say then only the type becomes a type 1 system and hence the steady state error becomes 0 actually it helps in eliminating the steady state error basically. They are the good things, but what the bad things the integral loop should always be used in a judicious manner, because of there are certain bad things actually first thing is there is some sort of a destabilizing effect of the integral term, and probably we will understand little more when we know about gain margin phase margin things like that in a frequency domain analysis.

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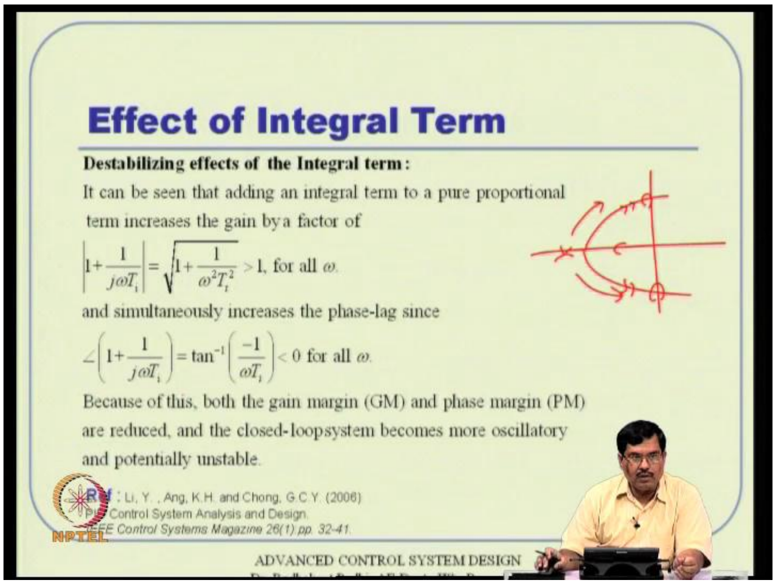

Now, if you assume for a second that you know that then it turns out that the magnitude part is given by something like this which is 1 plus there is some component here that. So, essentially what it does if you will talk about this sort of a tuning here something like that and essentially comes about comes up with something like this and hence this is an invariably a number which is greater than 1. So, this gain the overall gain of this proportional controller you can interpret that that there is a gain multiplication factor actually which is greater than 1. That means, the gain become more and more that means the gain margin becomes less and less actually.

So, essentially you can see that in a little elaborate sense something like this let say root locus is something it goes like that sorry it comes like that and it goes like that way, so there is a gain value after which the system goes and stable actually. Now, as gain increases the root locus travel like that that means, if you go more and closer towards this values we are more approaching more and more in stability region actually. They are becoming closer and close closer towards the unstable domain. So, if the so essentially this pushes you towards that kind of towards that side basically.

So, the gain margin becomes lesser and lesser margin is the value that is left out before the system become unstable actually. So, if the gain value keeps on increasing more and more the margin becomes less and less actually and for the phase angle characteristics also essentially gives us something like this and we know that tan inverse of a negative quantities also a negative quantity. So, essentially it actually an increases the phase lag also basically this is large vector any negative value is a large is a large vector s essentially, it increases a large vector also actually. So, that means what happens it essentially leads us to the reduction of gain margin and phase margin both that means, you are essentially compromising one characteristics actually.

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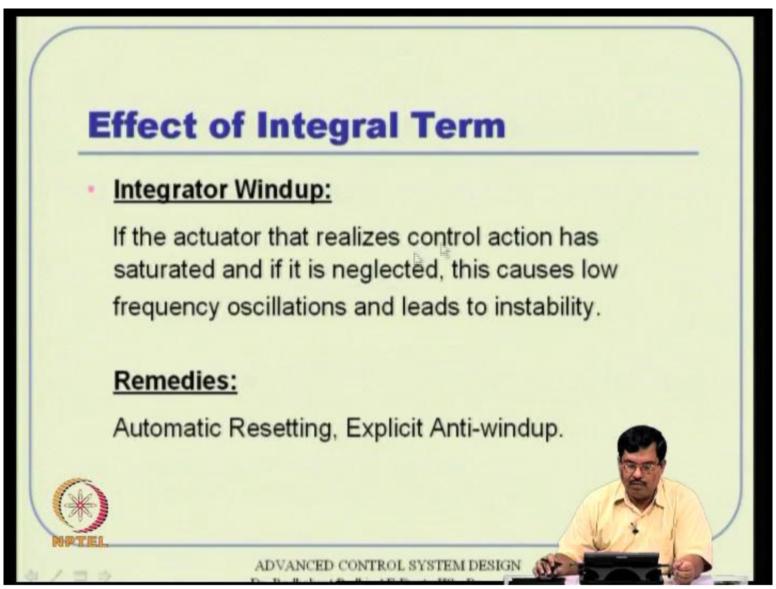

So, that is here to be slightly careful about that there is a larger problem of what is called as integrator windup that means, if there is something like the control action is saturated and the saturation even after it saturates, we keep on evaluating this integral term then the evaluated term becomes more and more and more and more large and large, because the error never dies the error never goes to zero. The integral term actually stabilizes once the error becomes zero after that there is no integral component actually, but if you does not stabilize the integral component keeps on building actually.

So, then it essentially keeps on predicting more and more and more and more, so you will never be able to come out of this actually. That is a larger problem which is integrator windup the remedies for that is the periodic resetting of the integral value, because see this particular term the zero starts from zero you start from T 0 rather and keep on resetting the T 0, so this integral value starts again from T 0 to T 0 that means again if it becomes zero actually. The integral component periodically said to zero basically. Then it is again that is one remedy. And there are explicit anti windup logics also people are devoted lot of attention to this particular problem even system that is a problem so there are anti windup logics available in an explicit manner actually.

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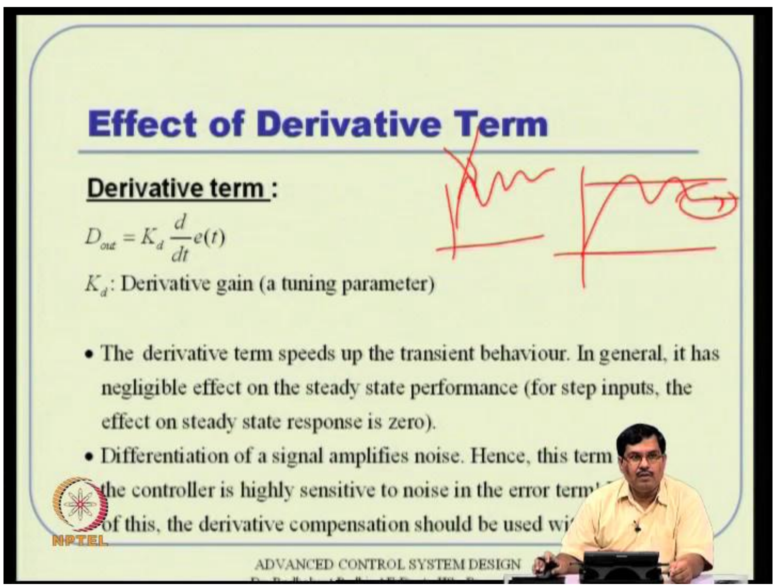

Now coming to the last term derivative term what does it do the derivative term actually helps us speeding up the tangent behavior, because we talking about accounting for feature values of the error in a way as well and it also accounts for high pass terms sort of things actually. So, in general it is negligible effect on steady state performance though, steady performance though steady state since it does not matter so much actually, because ultimately that the derivative itself becomes zero if you take about a if you talk about a some sort of a I mean, the step response reference then ultimately it happens to be derivative here zero.

So, once you once you compute the derivative in this segment then it is as good as a having nothing actually. So that essentially, now differentiation of the signal see what happens actually differentiating any noisy signal actually amplifies the error quite a big that is a terrible effect actually. Like there is a noisy signal suppose I mean, I have something suppose I have a signal which is something like this, and I am talking about taking derivative of that that means one time I am talking this one time I am talking that. So, there is a slow almost positive infinity to negative infinity So, the noise amplification term becomes quite a lot actually in any derivative part, so if the differentiation is computed numerically for a particular signal based on the accommodation is do not use the derivative loop at all whereas, integral loop is because integral is area under the curve so any noisy behavior essentially there is a soothing characteristics there actually.

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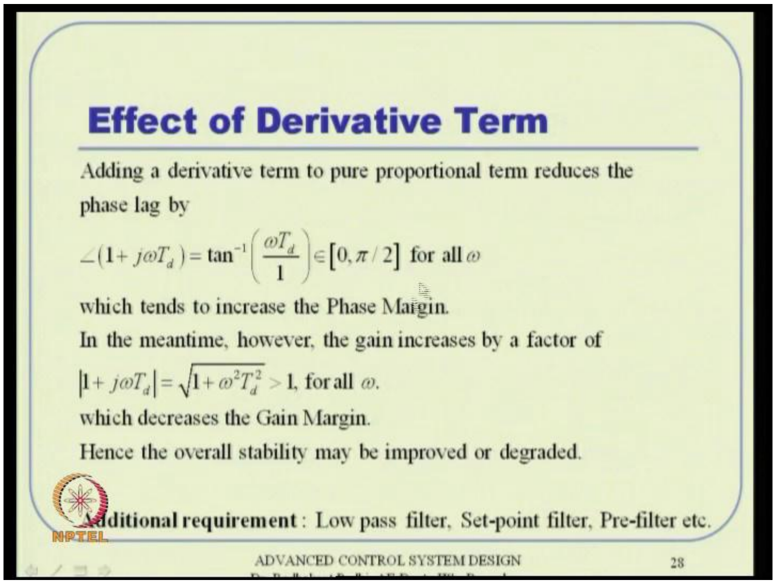

And then if you do the frequency analysis frequency part of it essentially you can do that, but it is, but it is a do the similar analysis, but what it turns out even that even though gain is a gain margin reduces, because of this effect there is nothing can be I mean there is a frame margin increase actually. So, earlier it was both reduction actually both were in the bad side now it is one in the good side one in the I mean, one in the bad side, but one in the good side, so nothing can be said precisely different from the problem actually. So, the derivative term by having a derivative term whether you are losing, you cannot be told that unless you know these values, and you can say that the additional requirements for low pass filter.

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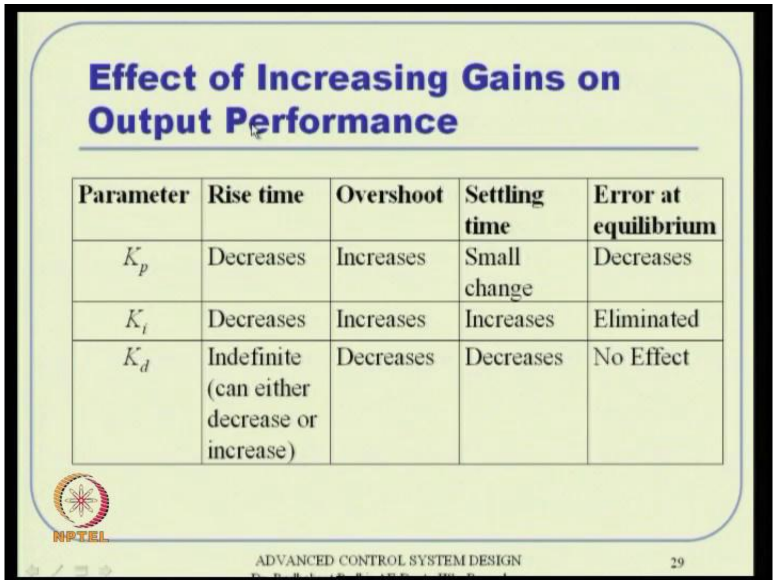

Whenever there is a D block in the PID control you talk about a low pass filters set point filter etcetera actually. Now you can summarize this I mean how do you tune of these values that is the next question actually how do you tune there is a summary of table which you can see that that if you increase this K p increase this K i and increase, this K d what will happen actually. So, these are rise time overshoot settling time and then steady state error sort of thing you can nicely have this table handy ready. For example, if you increase K p then the rise time decreases percentage overshoot increases settling time, I mean kind a very small change there, and the steady state error decreases that helps us in telling you whether what you should do actually and similar things for i and d parts are available actually.

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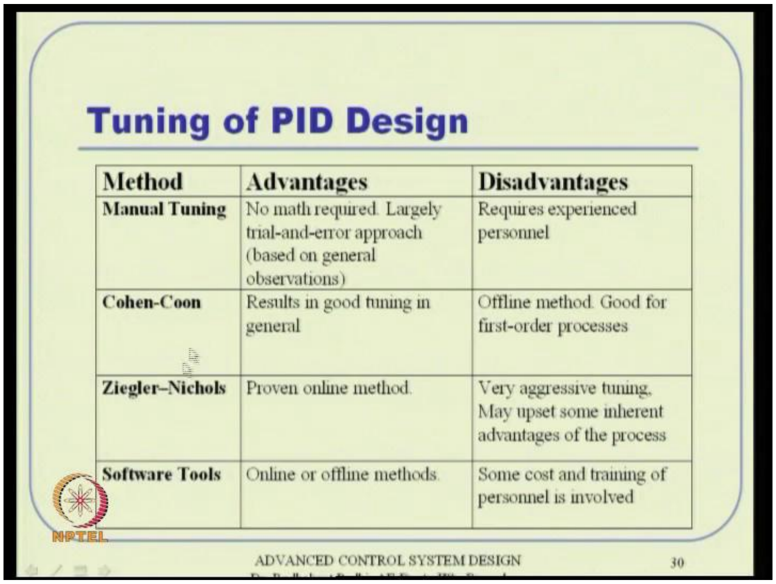

Tuning of PID Design

Method	Advantages	Disadvantages
Manual Tuning	No math required. Largely trial-and-error approach (based on general observations)	Requires experienced personnel
Cohen-Coon	Results in good tuning in general	Offline method. Good for first-order processes
Ziegler–Nichols	Proven online method.	Very aggressive tuning. May upset some inherent advantages of the process
Software Tools	Online or offline methods.	Some cost and training of personnel is involved

Then there are that is a Manual Tuning part so no particular math is required only this qualitative information is kind of you know what it requires some sort of experience a personnel for that particular problem to come up with a very good tuning basically. Then there are formal approaches available there are various things are available in literature and they are some of those one is Cohen Coon, they are the researchers of names and all that which is slightly this particular method is slightly popular in chemical industry actually. It results in some sort of an underline method of automatic tuning sort of thing and however, it is a very aggressive tuning that may that means, it may upset some inherent advantage of the process as well. There are various ways where people even talk about adopted PID designs that means the control structure is same, but the gains are adoptive they change with time based on certain adoptive laws actually that is a concept of simple adoptive control which is also available for non-linear systems actually.

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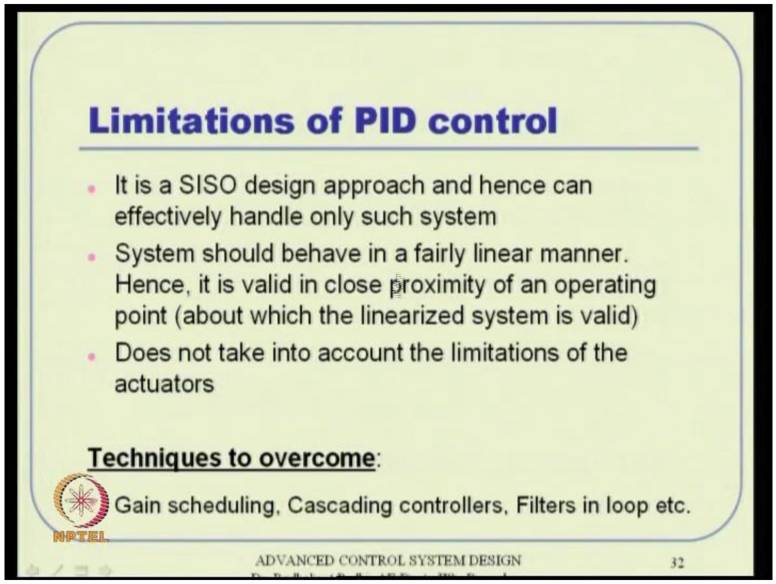

There are many things people interpret fuzzy logics and all other things actually, so that this part I probably skip and then limitations of the PID control we should be aware of. These are that should not skip actually. So, it is a SISO design primarily single input single output design and hence, can effectively handle only such systems it is a m i m o systems so you can have a variety of SISO channels and then it may or may not be sufficient actually. And systems should behave in a fairly linear manner for a non-linear system there is a problem, and then linearity means we should approximately I mean, use it for close proximity of some operating point about which they have linearized system and obviously, it also does not take into account the limitations of the actuators. So, there are various concerns PID control design it is a wonderful tool it is a good tool works in practice there are several limitations of that as well.

Techniques to overcome: one popular technique is obviously, gain scheduling, we will talk about little more detail as you all in this course. Then there are cascading ideas that means you come up with some sort of a loop structure of control design the outer loop come and transfers to intermediate loop then come and transfer to inner loop and things like that and each of the loops you talk about using a PID controller actually. So, that essentially gives us some sort of a cascading thing then filters in the loop and all that actually they are also available actually. So, you use various filters in the loop as well so they are some of the remedies and that is all I will talk in this particular lecture, thank you.