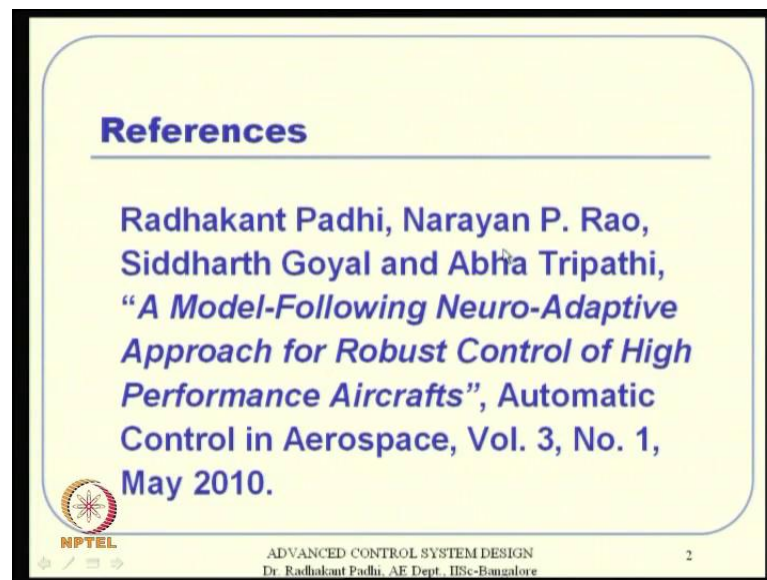


Advanced Control System Design
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Lecture No # 38
Neuro-Adaptive Design for Flight Control

So, this particular lecture I will demonstrate that through a typical aircraft control example, and tell all the associated, I mean developments around that actually.

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Anyway, so this is this particular lecture is taken from this reference which is our own paper developed very **recently**, I mean published very recently rather. So, let us take a take you through, so that you will understand how these techniques do work for a real life problem actually.

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Problem Statement

- **Pilot Commands:**
 - Case -1: Longitudinal
(Roll Rate = 0, Normal Acceleration, Lateral Acceleration = 0, Total Velocity)
 - Case -2: Lateral
(Roll Rate, Height, Lateral Acceleration = 0, Total Velocity)
- **Turn Coordination:** *Lateral acceleration command is zero*
- **Goal:**
 - Airplane responds to the pilot commands "quickly" & "nicely"
 - The control design should have sufficient robustness for parametric inaccuracies (mass, MI and aerodynamic coefficients)

Note: Essentially it is a **Robust Tracking** problem

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So, the problem statement is something like this, we are **we are** dealing with an aircraft autopilot development or aircraft like pilot command implementation sort of thing. Pilot give certain command, and then and the flight control is supposed to take that command and executed through flight control reflections and all that. And previously, we have also studied flight dynamics. So, you understand, what is this I mean, control surface reflection that I am talking about.

Anyway, so this pilot commands that you are assuming here is primarily of two modes, one is probably will give longitudinal command or will give lateral command. There is also a development, where you can actually give combined longitudinal and lateral command, and which is typically called velocity vector all and thing like that. So, anyway we will just confine ourselves primarily to do these two. And then in longitudinal command, we will typically I mean, assume that the pilot gives some sort of a normal acceleration command, that is the primary thing.

And then associated with that we can give, I mean this total velocity command it can also give, like what the speed of the aircraft and all that. But then primarily what we will bother about is that. However while executing these we will also assure that roll rate remains 0, X goes to 0 is possible all the time. And it will also lateral acceleration also become I mean

remains 0, I mean as much as possible actually. So, roll rate is 0 primarily because you are not interested in lateral commands. And lateral acceleration is 0 because even if the dynamics are coupled and think like that it should not develop lateral acceleration, because it reduces, I mean the moment you have lateral acceleration it actually introduces induce drag and think like that, lot lot of drag it will come. So, that we do not want to kind of induce that.

In the lateral mode, however we will assume that pilot gives roll rate command, as well as an altitude command, height command actually. But in the same lateral mode as well, we also have to make sure that lateral acceleration remain 0. And total velocity pilot either can give what you want, so or we can have some default value, like the initial velocity have before starting the maneuver should remain same and all that. So, this lateral acceleration command is built in; that means, the lateral acceleration command is 0 for both longitudinal and lateral and the meaning of that is we want to assure turn coordination.

So, if you, I mean no matter what mode you are executing, the turn coordination has to be maintained actually. Now, the objective, the goal is the airplane should respond to the pilot command both quickly and nicely, and then what is quickly and nicely we have, we will see that. Quickly, we will we understand, but nicely is also like we want to assure what is the minimum there actually. And second objective is control design should have sufficient robustness for parameter inaccuracies, like we can never predict, what is the total mass of aircraft, moment of inertia, aerodynamic coefficient precisely and think like that. So, essentially even if you have some uncertainty; that means, the numbers that are we are dealing with here are inaccurate, but still the control design should have sufficient robustness. So, essentially it is kind of robust tracking problem.

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Airplane Dynamics: Six Degree-of-Freedom Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, 1995

$$U = VR - WQ - g \sin \Theta + \frac{1}{m}(F_{Ax} + F_{Tx})$$

$$V = WP - UR + g \sin \Phi \cos \Theta + \frac{1}{m}(F_{Ay} + F_{Ty})$$

$$W = UQ - VP + g \cos \Phi \cos \Theta + \frac{1}{m}(F_{Az} + F_{Tz})$$

$$P = c_1 QR + c_2 PQ + c_3(L_A + L_T) + c_4(N_A + N_T)$$

$$Q = c_5 PR - c_6(P^2 - R^2) + c_7(M_A + M_T)$$


$$R = c_8 PQ - c_9 QR + c_4(L_A + L_T) + c_5(N_A + N_T)$$

$$\dot{\Phi} = P + Q \sin \Phi \tan \Theta + R \cos \Phi \tan \Theta$$


$$\dot{\Theta} = Q \cos \Phi - R \sin \Phi$$

$$\dot{\Psi} = (Q \sin \Phi + R \cos \Phi) \sec \Theta$$

$$\dot{h} = U \sin \Theta - V \cos \Theta \sin \Phi - W \cos \Theta \cos \Phi$$



$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$



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So, let see how do we do that actually? So, this is that, I mean this is standard six top equation that you discussed before. And the, I mean the, so far as long as we talk symbolically this is same for any aircraft anyway. But we will typically take you through I mean the dynamic, the coefficients numbers and all are typically. I will take you through this high performance or flight aircraft which is like f 16 aircraft; I think everybody knows what is f 16 aircraft anyway. So, this f 16 aircraft model, I mean this is very generic, this is this model is I mean whatever equations you will see in this phase we discussed that before, we derived that that is same for any aircraft. But the difference comes when you do, when you will have this parameter things coming into picture, the mass of aircraft, moment of inertia of the aircraft, then the aerodynamics coefficient all sorts of thing when you do that, then it become different.

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Aerodynamic Model (F-16)

Ref: Eugene A. Morelli, American Control Conference Philadelphia, Pennsylvania June 1998

$$C_x = C_x(\alpha, \delta_e) + C_{x_q}(\alpha)\dot{q}$$

$$C_y = C_y(\beta, \delta_a, \delta_r) + C_{y_p}(\alpha)\dot{p} + C_{y_r}(\alpha)\dot{r}$$


$$C_z = C_z(\alpha, \beta, \delta_e) + C_{z_q}(\alpha)\dot{q}$$


$$C_l = C_l(\alpha, \beta) + C_{l_p}(\alpha)\dot{p} + C_{l_r}(\alpha)\dot{r} + C_{l_{\delta_a}}(\alpha, \beta)\delta_a + C_{l_{\delta_r}}(\alpha, \beta)\delta_r$$

$$C_m = C_m(\alpha, \delta_e) + C_{m_q}(\alpha)\dot{q} + C_{m_{x_{cg}} - x_{cg}}$$


$$C_n = C_n(\alpha, \beta) + C_{n_p}(\alpha)\dot{p} + C_{n_r}(\alpha)\dot{r} + C_{n_{\delta_a}}(\alpha, \beta)\delta_a + C_{n_{\delta_r}}(\alpha, \beta)\delta_r + C_{n_{x_{cg}} - x_{cg}} \left(\frac{\dot{c}}{b} \right)$$

$$\dot{p} = pb / 2I_T \quad \dot{q} = q\dot{c} / 2I_T \quad \dot{r} = rb / 2I_T$$





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And that part of it has been taken from this particular reference and interestingly this f 16 model, what is called as a low fidelity model; that means, subsonic model and think like that, which is available in the open literature now, means anybody can pick it out and do some experiments and think like that. And also I mean remember those are typically available for experimental purpose actually. So, that way, I mean there is nothing classified about this particular exercise basically anyway. So, this reference somebody can grab it and then see all the details, how this details are given? This is just given a very compact notations sense, but each of this let say C is for example, C X T then it is C X of these and C X q of that and then each of that coefficient will expanded in a polynomial way. So, every very detailed modeling has been done and then to a sufficient degree of accuracy and all that actually. So, that is all details can be found in this particular paper.

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Equations in Desired Form

$$\dot{X}_V = f_V(X) + [g_V(X)] U_c$$

$$\dot{X}_R = f_R(X) + [g_R(X)] U_c$$

where

$$X \triangleq [V_T \quad \alpha \quad \beta \quad P \quad Q \quad R \quad \Phi \quad \Theta \quad \Psi \quad X_B \quad Y_B \quad h]^T$$

$$X_V \triangleq [U \quad V \quad W]^T$$

$$X_R \triangleq [P \quad Q \quad R]^T$$

$$X_A \triangleq [\Phi \quad \Theta \quad \Psi]^T$$

$$U_c \triangleq [U_{AER} \quad U_T]^T \triangleq [\delta_a \quad \delta_e \quad \delta_r \quad T]^T$$

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So, with this entire model being available to us, we want to experiment our technique, like you want to apply it aerodynamics inversion followed by Neuro adaptive technique actually. So, how do you do that? Now, I go back to this equation and then I try to club this, the similar quantities together sort of thing. So, the dynamic level equations that we have seen before are U V W and P Q R anyway, so this half of the equation. Other things are dynamic level equation. So, the dynamic level equations and all, I will club them like this. That means X V, I will define it as like U V W, X R I will define it as P Q R and X A is something like altitude variables and all that, phi, theta, psi. And U C is the control variable which typically U is not taken here, because U is also like forward velocity actually.



So, we define U C which is control variable contains aerodynamic control as well as thrust control actually. So, these, but we also remember the aerodynamic controls are club together primarily because they, I mean they can be activated in a faster way; that means, there as, they are primarily responsible for altering P Q R dynamics which are faster dynamics also. And they can be reflected in faster manner as well actually. However, the thrust control is typically a slow control variable, it is a control variable nevertheless or still it is a slow control variable actually. So, we will not be able to treat that in a same category as aerodynamic control. So, that is another thing that we have to remember all the time.

However, we are talking about the full system dynamics anyway. So, this is we are not linearizing anywhere and think like that. So, the entire state vector will consist of all these things. Alternatively either we will use $U V W$ as state equation or sometimes we may use $V T \dot{\alpha} \dot{\beta} \dot{\gamma}$ in terms of, instead of $U \dot{V} \dot{W} \dot{\alpha} \dot{\beta} \dot{\gamma}$. This $V T \dot{\alpha} \dot{\beta} \dot{\gamma}$ equation in place of $U V$, I mean $U \dot{V} \dot{W} \dot{\alpha} \dot{\beta} \dot{\gamma}$ if you do that, then that is something called, I mean this event frame equation all that, if we use $U V W$ it is body frame equation. So anyway, so that is interchangeably these are available, so we will be able to exploit whatever we want actually.

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Definitions and Goal

- Total Velocity: V_T
- Roll Rate (about x-axis): P
- Normal Acceleration: $n_z \triangleq -(F_z/m) = -(1/m)(F_{A_z})$
- Lateral Acceleration: $n_y \triangleq (F_y/m) = (1/m)(F_{A_y})$
- Goal: $P \rightarrow P^*, n_z \rightarrow n_z^*, n_y \rightarrow n_y^* = 0, V_T \rightarrow V_T^*$
 where P^*, n_z^*, V_T^* are pilot commands

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Now, coming back to the problem that we have we are discussing actually, this we want this, like total velocity V_T and then roll rate is actually like P and then normal acceleration n_z which is defined as negative of F_z by m . So, that is nothing but $1/m$, I mean 1 over m into F_{A_z} sort of thing, so that way. And also remember the thrust is primarily acting in the X direction. So, Y and Z direction there is no thrust action that is available to us, that is another assumption and all that which is very very close to realistic. So, this is, so n_z and n_y are defined as this external force component whatever is coming here, so the divided by mass control. So, the things that you see here, this part only, remember these are typically zero anyway. So, that is 0 and this is this is 0 .

So, whatever value remains here with a, you know whatever value remains here, these are the kind of normal acceleration quantities and all that. So, negative of this quantity 1 by n, so F A Z is typically normal acceleration. And I mean and positive of this F A Y divided by m is lateral acceleration. The primarily because normal acceleration we want to, if you give positive normal acceleration we want to visualize that the aircraft should go to up actually. Whereas, you remember the body frame z axis is typically downwards, that is why this negative sign is coming here that way. Anyway, the objective in mathematical terms are something like this or if the pilot gives a P star command; that means, the roll rate command then my actual P of the aircraft should go to P star as soon as possible. n z should go to like n z star if the pilot gives a normal acceleration command then n z should go to n z star very quickly.

Similarly, n y should go to n y star which is typically 0, I mean no matter whether it lateral longitude or whatever, n y star is maintained at 0. And V T the total velocity should go to some result velocity vector, which is I mean nothing is given by the problem default values typically the velocity hold sort of thing. That means whatever initial velocity is there, you just maintain that basically. So, this is the, this is in mathematically speaking, how do you put your objectives into the, I mean control design part of it actually.

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Control Synthesis Procedure

- Define new variables: $a_x \triangleq n_x + W^i$, $a_x^* \triangleq n_x^* + W^i$
 $a_y \triangleq n_y - V^i$, $a_y^* \triangleq n_y^* - V^i$
- Key observation: $\dot{V}^i = \dot{W}^i = 0$

$$([n_x \ n_y]^T \rightarrow [n_x^* \ n_y^*]^T) \Leftrightarrow ([a_x \ a_y]^T \rightarrow [a_x^* \ a_y^*]^T)$$
- Known:

$n_x = f_{n_x} + g_{n_x} U_c$	$\dot{P} = f_P + g_P U_c$
$n_y = f_{n_y} + g_{n_y} U_c$	$\dot{V}_T = f_{V_T} + g_{V_T} U_c$

In Wind Axis Frame: $n_{wz} = f_{n_{wz}} + g_{n_{wz}} U_c$



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Now, once you understand these are the objectives, now how do you go ahead and synthesize the control basically. So, here we I mean the typical way of doing that, also remember that n_z and n_y will typically get converted to Q_d equivalent Q^* and R^* . And then your P^* is available anyway, so $P^* R^*$ will be available. So, in an inner loop we will go back to that this part of it, that $P \dot{Q} \dot{R}$ and then control surface deflections are computed to execute those $P^* P^* R^*$. But here we will not follow that approach; we will follow a little bit alternate approach.

So, let us try to understand that approach. And later on also remember we will compare our results with both of the approach, this approach as well as the, our approach that I just discussed. That means, n_z will be converted to equivalent Q^* and n_y will be converted to equivalent R^* in an outer loop setting. Then, $P^* Q^* R^*$ will be available from which we will be able to compute air control surface deflection. And $V T$ we are tracking to V^* is, $V T^*$ is anyway independent that is primarily done through this thrust control mechanics. So, that is independent anyway. So, anyway coming back to this, let see let see how do we make sure primarily these two quantity; that means, n_z goes to n_z^* and n_y goes to n_y^* which is 0, how do we do that?

Now, for doing that let's define some new variables. Directly we cannot do that, in other words somebody can think, let me take this part of the y vector like desired output that we discussed in the dynamic inversion and then try to put some sort of a aerodynamics to that actually. But if you want to put that, then remember n_z is directly direct function of δ_a and n_y is direct function of δ_R , I mean δ_a δ_R both actually. So, that way, what happens is like if you take derivative set of n_z and n_y , they are functions of derivatives of control and we do not want to solve your control variable some sort of a dynamic equation, we do not want dynamic controllers.

So, we want to still solve it as a control variable, I mean we want to still solve control variable, a static variables only. And because of that we want to avoid these direct derivatives of n_z and n_y . How do we do that? I mean if you define this variables a_z and a_y something like $n_z + \dot{w}$ and a_y something like $n_y - \dot{V}$ and a_z^* and a_y^* also like that, then you can see that if you go back to this equation this six top equation.

Remember, if the moment I take this part to the left hand side and this part to the left hand side as well, somehow and then whatever left out here they are simply functions of state variable, they are not functions of control variable.

So, even if I take derivatives of those quantities, then it will only control variable appear not its derivatives actually, that is the key point to observe. So, we do that and then on the, in the process we also have to assume that \dot{V} and \dot{W} always kind of remain 0, this is quasi state assumption for \dot{V} and \dot{W} . So, that means, they are not really 0 in all time, but I mean these are anyway slow varying quantities V and W . And then if \dot{V} and \dot{W} will be still further slow varying \ddot{V} and \ddot{W} will be still like they will vary, but it will vary at a very slow rate, so that we will just assume that they are maintained at 0 for all time.

Now, wanted those assumptions, what you can see is if I want to assure this anyway n_z and n_y should go to their corresponding desired values. Now, that is equivalent to assuring that this a_y and a_z they will go to the desired values actually. Because if I make sure that a_y go to like a_y^* , then $\dot{V}^* - \dot{V}^*$ will cancel out here and then we will left out with n_y should go to n_y^* , that is the only way it will happen. Similarly, if a_z goes to a_z^* then the only way should happen is n_z should go to n_z^* because \dot{W} is common to both actually. So, that is the key observation.

Now, also remember that n_z and n_y can be written as something by control affine form and this is not dynamic equation again. Because n_z is static variable, n_y is the static variable they are just algebraic expressions. So, that is also that will also be required. And then \dot{P} can be written as some control affine form that way and \dot{V}_T can also be written as written in some sort of a control, that is a small print mistake probably this is U_T thrust control. Anyway, so if you really want to write it in wind axis frame work, then you can write $n_w z$ is something like that. So, that is, sometimes it may be quite to do that; anyway, going back to our system dynamics.

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**Control Synthesis Procedure:
Longitudinal**

- Dynamics:
$$a_z = UQ - VP + g \cos \Phi \cos \Theta$$
$$a_y = UR - WP - g \sin \Phi \cos \Theta$$
- Differentiate:
$$\dot{a}_z = f_{a_z} + g_{a_z} U_c$$
$$\dot{a}_y = f_{a_y} + g_{a_y} U_c$$
- **Case-1: (Longitudinal) Pilot commands:**
 - Roll Rate: $p^* = 0$
 - Normal Acceleration: n_z^*
 - Lateral Acceleration: $n_y^* = 0$
 - Total Velocity: V_T^*

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So, a z now with definition of a z a y and all that, we will be able to write a z and a y is those terms that I told you sometime by this form. Only this coefficient and this term which is like correlation gravity component of the system equation, this part is correlation component and this is gravity component, if you remember the six top equation derivation and all. So, that part will already appear here and then a z and a y will be like that. So, if you take derivatives of a z and a y; that means, a z dot and a y dot and all that, you carry out with the algebra and all, so ultimately will be able to write in a control affine problem. That means, a z dot and a y dot will be written in some sort of a f plus g times U C sort of thing actually, which is nice to see.

So, what you really want, let us try to see longitudinal command like if the pilot gives longitudinal command, how is that control actually getting computed in the flight control computer. I mean however, you propose into do that actually. And in this mode you are assuming that let us pilot does not give any roll rate command, so that is 0 and the normal acceleration command is anyway there that is n z star, this is n y star is zero and V T star can be there. Or, if it is not there then total velocity is the star is the command actually. Now, how do you do that? So, we will again, now at this point of time what we will, what you are doing here is, n z and n y star we are not interested in direct tracking. This one we are not interested, we are interested in tracking this through a z and a y tracking actually.

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Control Synthesis Procedure: Longitudinal


- **Define (Errors):**
Fast Variables $\hat{X}_T \triangleq [\hat{P} \quad \hat{a}_z \quad \hat{a}_y]^T \triangleq [(P - P^*) \quad (a_z - a_z^*) \quad (a_y - a_y^*)]^T$
- Slow Variables** $\hat{V}_T \triangleq [(V_T^* - V_T^*)]$
- **Design a controller such that:**


$$\dot{\hat{X}}_T + K \hat{X}_T = 0,$$

$$\dot{\hat{V}}_T + K_{V_T} \hat{V}_T = 0,$$

$$K \triangleq \text{diag} \left(\frac{1}{\tau_p}, \frac{1}{\tau_{a_z}}, \frac{1}{\tau_{a_y}} \right) \quad K_{V_T} \triangleq \text{diag} \left(\frac{1}{\tau_{V_T}} \right)$$







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So, that is that I mean, with that in mind we will propose an error vector for the fast variable like this. That means, for fast variable I will consider this P hat like error P hat is P minus P star error in a z is a z minus a z star error in a y is a y minus a y star. And slow variables it is V T minus V T star, which is velocity control sort of thing. So, let us see, do this velocity control design first I mean sorry the attitude control part tracking control fast and all that. How do we do that? You put, remember these are all like if I take first order of derivatives then it appears like P dot a z dot a y dot P dot control appears a z dot control appears and a y dot also control appears.

So, having that in mind I will propose this first order aerodynamics, where gain value is selected as a diagonal matrix sort of thing. And then I will assure, I mean I will also make sure that V T dot plus k V T times V sorry V T hat dot plus k V T V T hat and V T is the total velocity error and all that, that also remain 0. So, ideally speaking they have to be simultaneously assured, but we will have to update this this this equations and all. I mean the control update that is coming from these two equations and at different time scale actually. That means, we have to execute this aerodynamic control in a little faster delta T sense and then velocity control will execute in a slower delta T sense.

Similarly, the gain selections and also remember that $k_V T$ cannot be $(())$ that because that is also we have to make sure that the time constraint for the velocity control is typically large actually; that means, gain has to be small anyway. So, this is all that you have to keep in mind and then try applying this controller. So, that is what we are doing here, I mean if you put it there and then put this variables and all that I mean whatever the definition and then the dot expressions you put it back, remember this these dots are available, P dot is also available.

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Control Synthesis Procedure: Longitudinal

- Calculate $[U_{AER}]$ for every Δt time

$$U_{AER} = [A_U]^{-1} b_U$$

$$A_U \triangleq \begin{bmatrix} g_P \\ g_\alpha \\ g_\omega \end{bmatrix} + [K] \begin{bmatrix} 0 & g_\omega & g_\alpha \end{bmatrix}^T, \quad b_U \triangleq [K] \begin{bmatrix} P - P^* \\ n_z - n_z^* \\ n_y - n_y^* \end{bmatrix} - \begin{bmatrix} f_P \\ f_{\alpha_1} \\ f_{\alpha_2} \end{bmatrix}$$

- Calculate U_T for every $5 \cdot \Delta t$ time

$$U_T = -[d_{V_T}]^{-1} c_{V_T}$$

$$c_{V_T} = \begin{bmatrix} (f_{V_T} + g_{V_T} U_{AER}) - \dot{V}_T^* + K_{V_T} (V_T - V_T^*) \end{bmatrix}$$

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So, we try to put them together and then try to solve for the aerodynamic control variable, ultimately it will happen in the way that way where A_U and B_U will be appropriately defined actually. I am not giving the entire detail because somebody can always see that, that the reference for detail. And also remember that these are like we know the theory already basically. So, this is just an application problem that someone has to take care of all the details in a good way basically. So, what we are proposing here is, for every ΔT time that aerodynamic control gets updated, we will update the thrust control every $5 \Delta T$ times. So, that is what you are proposing.


So, but nevertheless once you have this aerodynamics is again same thing we propose and that V_T dot is available anyway here, so I will put it back there. And try to solve for

something like U T basically, that is the thrust control basically. So, this d V T and C V T and all that are appropriately defined, for example, C V T can be defined that way basically. And anyway so this is how we are synthesizing control. Now, I mean these two control actions are typically again I repeat. So, this is to updated in a different time scale manner sort of thing, to make sure that the thrust control is updated in a little slow rate and all that.

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Control Synthesis Procedure: Lateral


- **Case-2: (Lateral) Pilot commands:**
 - Roll Rate: P^*
 - Height: h^*
 - Lateral Acceleration: $a_y^* = 0$
 - Total Velocity: V_T^*



- **Generation of ω^* command:**
 - Define $\hat{h} \triangleq (h - h^*)$ and aim for $\dot{\hat{h}} + (1/\tau_h)\hat{h} = 0$

$$\left[U \sin \Theta - V \cos \Theta \sin \Phi - W \cos \Theta \cos \Phi \right] \dot{\hat{h}} + (1/\tau_h)(h - h^*) = 0$$

Solve for $\omega \triangleq \omega^*$


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Now, what about lateral I mean lateral command sort of thing it and here one has to be slightly more careful. Because we are talking about here roll rate command, altitude command and the lateral acceleration command is anyway 0 and total velocity command is 0, I mean some V T star value. But the problem here is this altitude command actually. Once we have altitude into picture, because if I take h dot, h dot is not typically the control will not appear actually. And if I take h double dot the, I mean then things may be different, but we want to kind of avoid that h double dot, I mean even if I take h double dot only q will appear by the way.

So, if I take h triple dot probably control will appear, but we do not want to avoid these complexities of third order aerodynamics and thinks like that. So, what we are proposing here is take you through this command transfer loop in a standard way. So, what we are doing here? The altitude command once I know, I will compute an equivalent pitch angle

command, theta star command and that is done by enforcing this first order aerodynamics of altitude. So, I will make sure that $h - h^*$ is my error that by definition and make sure that, this error equation is satisfied. And then put it back if the $\dot{h} - \dot{h}^*$ all sort of thing. Now, \dot{h} if you see \dot{h} equation is available to us anyway. So, this equation is there with us. So, this equation I will put it back.

So, these altitude \dot{h} is available and then rest of the things are like that. So, I can solve for theta from this equation. And once I solve for theta from this equation, I will get something like a theta star command, whatever solution is there that is interpreted as a command for the inner loop.

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Control Synthesis Procedure: Lateral

- Generation of Q^* command:
 - Define $\hat{\theta} = (\theta - \theta^*)$ and aim for $\dot{\hat{\theta}} + (1/\tau_\theta)\hat{\theta} = 0$
 $(Q \cos \Phi - R \sin \Phi) - \dot{\theta}^* + (1/\tau_\theta)(\theta - \theta^*) = 0$
 - Solve for $Q \hat{=} Q^*$
- Control Computation:
 - Define (Errors):
 - Fast Variables $\dot{X}_T \hat{=} [\dot{P} \quad \dot{Q} \quad \dot{a}_y]^T \hat{=} [(P - P^*) \quad (Q - Q^*) \quad (a_y - a_y^*)]^T$
 - Slow Variable $\dot{V}_T \hat{=} [(V_T - V_T^*)]$

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So, once theta stars command is available, so that, then I will go for generation of a Q star command from theta star because theta star theta dot does not contain control like that. So, I will go with the same idea, but I will define an error quantity theta hat equal to theta minus theta star and then aim for something like aerodynamics like that first order aerodynamics; theta dot is again available to us that is that theta dot expression. So, once I put it back, then I solve for Q assuming that all other variables are available and whatever solution comes out that becomes my Q star variable, Q star value actually. So, that becomes command variable for the innermost loop.

So, what happens here? Now, q star command is anyway available, now an equivalent q star command is also available. Then, a y should go to a y star this is 0 anyway, so that part is kept here because a y dot already contains control. So, then now I am ready to apply my innermost control execution. So, I will define this X T tracking states and all that, something like P Q and a y and then error in that P hat Q hat and a y hat. So, these are by definition like that slow variable like that.

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Control Synthesis Procedure: Lateral

- Design a controller such that


$$\dot{\hat{X}}_T + K\hat{X}_T = 0 \quad \dot{\hat{V}}_T + K_{V_T}\hat{V}_T = 0$$

$$K \triangleq \text{diag} \left(\frac{1}{\tau_p}, \frac{1}{\tau_q}, \frac{1}{\tau_r} \right) \quad K_{V_T} \triangleq \text{diag} \left(\frac{1}{\tau_{V_T}} \right)$$
- After some algebra, Finally:

$$U_c = [I A_U^{-1} b_U]^T T^T$$

$$A_U \triangleq \begin{bmatrix} g_p^T & g_q^T & (g_{a_y} + (1/\tau_{n_y})g_{n_y}^T) \end{bmatrix}^T$$

$$b_U \triangleq -[f_p \quad f_q \quad f_{a_y}]^T - K[(P - P^*) (Q - Q^*) (f_{n_y} - n_y^*)]^T$$



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So, now I will apply the aerodynamic control principle. So, I will define this X T hat as this variable anyway and then I will apply this first order aerodynamics for this X T hat. Now, simultaneously I will also try to assure that velocity control is also there like the longitudinal way. So, we will do that and then come up to this control, I mean this control computation that way, but first part is actually aerodynamic control, second part is thrust control. And again I will repeat the thrust control will be updated in a slow rate, in a slow manner where the aerodynamic control will be updated in a first manner.

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Control Synthesis Procedure: Combined Longitudinal & Lateral

- **Case-3: (Combined Longitudinal and Lateral)**


Pilot commands:

- Roll Rate (about velocity vector): P^*
- Normal Acceleration: n_z^*
- Lateral Acceleration: $n_y^* = 0$
- Total Velocity: $V^* \hat{d}$

- Generation of P^* command:
 - Define P^* in the desired form as: $P^* = f_{P^*} + g_{P^*} U_A$

where $f_{P^*} = (1 / \cos \alpha (\cos \beta + \tan \beta \sin \beta)) (P_{\beta}^* - R \sin \alpha (\sin \beta \tan \beta + \cos \beta) - \tan \beta (f_{n_{zm}} / V_T) + (g / V_T) \cos \gamma \cos \Phi$

and $g_{P^*} = (-\tan \beta / \cos \alpha (\cos \beta + \tan \beta \sin \beta)) (g_{n_{zm}} / V_T)$



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And, as I told you in the beginning there is some idea of combined longitudinal and lateral maneuver also, in that sense what you are doing is many times the roll rate command given to high performance aircraft, that is why aircraft and all typically not body roll rate, they are given in terms of velocity vector roll rate. And velocity vector, as you know can be different from body X y z, I mean they need not it need not be airline with any of the X y z of the body X plane. Why it is done? Because rolling maneuver becomes very faster around velocity vector also, we can take quick maneuvers of that.

So, if, but if you do that it exerts both longitudinal and lateral maneuver together sort of thing. So, the how do we do that? And again that is not very, very different because if you see this something like this P W dot sort of thing, that is a function of P anyway. So, what you do is, you assure that P W expression whatever P W expression is there, you solve for P and then get some sort of a expression for equivalent P and all that. So, that is the, I mean, I will not take you through the very detailed sort of thing and then it is available to you in the vapor. Exactly, similar way you assure that first things like, once you have a equivalent P star then it is, this as good as a lateral I mean maneuver sort of idea there. So, you go through the same thing and then synthesize the control.

Some of that point and time then see this lateral maneuver is also like a little bit funny situation, many times sometimes P star command is directly given to get a quicker roll sort of things. So, that you get out the way and then our combined superior T and think like that. But sometimes when you are doing nice flight for long duration and think like that, it is not a good idea to control through P star. So, what you really want to do is straighten level flight sort of things where wings are typically available. And if you want to do that then P star command is not a good command.


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Alternative way to compute P* Command

- Goal: $\Phi \rightarrow \Phi^*$ (Not $\Phi^* = 0$ for longitudinal case)
- Define $\tilde{\Phi} \triangleq (\Phi - \Phi^*)$
- Desired error dynamics:

$$\dot{\tilde{\Phi}} + (1/\tau_\Phi)\tilde{\Phi} = 0, \quad \tau_\Phi > 0$$
- Substitute for Φ and solve for $P \triangleq P^*$

$$P^* = \Phi^* - \left[Q \sin \Phi \tan \Theta + R \cos \Phi \tan \Theta + \frac{1}{\tau_\Phi} (\Phi - \Phi^*) \right]$$



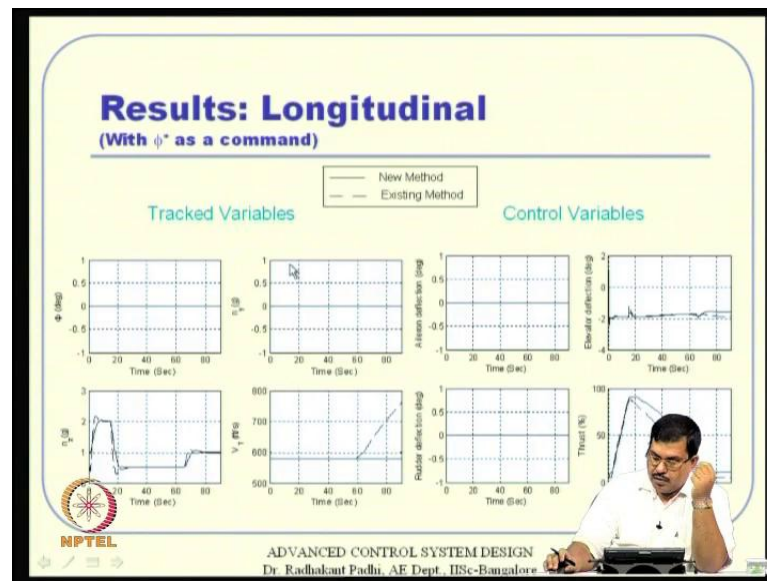
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So, what you are really need to do is phi star command, and phi star is typically 0 for longitudinal case, for lateral case also like if you know what is your desired phi and all that you can give that it is phi star. So, in those situation and typically this is what is required for commercial aircrafts also, because commercial aircrafts are never control through direct P star command sort of thing, there is only control through a nice altitude command and all that. But if you want to do that, then phi, this phi goes to phi star then what happens is you define error between that. So, phi minus phi star is error between them and again enforces some sort of a aerodynamics, but the problem is, this is not directly function of control variable.

But also remember phi dot is primarily function of p, so we have to all noting the in the and relative remember those, like which is which I mean rate of change of some variable is a primary function of some other variable and that is what you should solve for in the outer loop. So, phi dot is a primary function of p. So, I want to solve for P from this equation and what about P comes out, I will interpret that is P star command. So that then, I my P star command is available So, with all that then I will proceed further with whatever P star I already have.

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So, with all that we saw experiment some of these results and all. So, we can see that the results also been compared with that earlier rather existing idea that are available in the literature, where this n_z and n_y are converted to equivalent Q and R then experimented. So, we try to maintain as much similarity as possible and when you compare two methods you have to slightly careful also, that you do not gave emphasis to one over the other either I mean (()) by the way.

So, we try to (()) atmosphere is possible and then I am trying to do some sort of a experiments here actually. So, if you see that the adjusting method what it turns out is that sometimes around like 60 seconds and think like that. So, the command part of it, first of all it is given something like positive 2 G plus 2 G command for some time, then it is negative

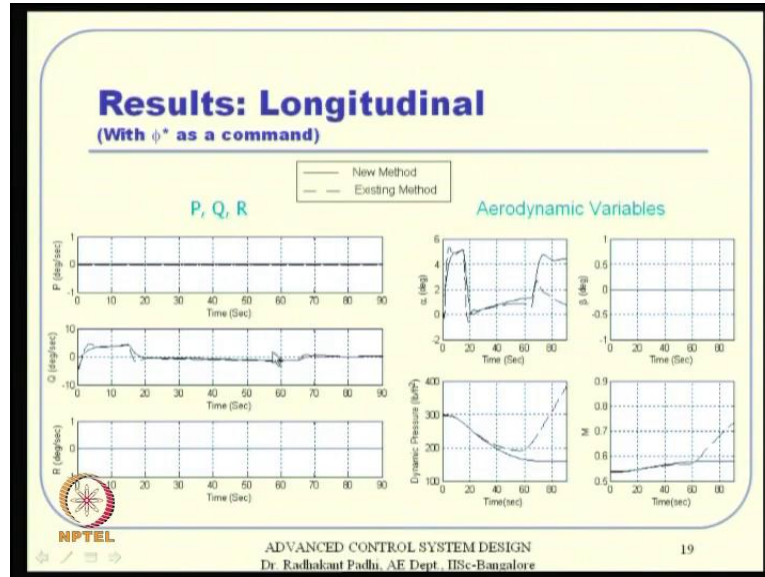
like command for some time, I mean and then again it is like 1 G command for some time. So, these are all some of things that you can experiment plus minus slow sort of thing. So, to make sure that tracking happens every time.

So, that if you see closely then what it turns out here is, first of all this thrust control is getting saturated, so very quick this is the saturation level. You have put something like 5 percent of maximum thrust and below that we do not want to go; because once you start of the engine it may be difficult to restart the engine during flight. So, you want to maintain some sort of thrust level all the time. So, that 5 percent of the available thrust is the minimum limit sort of thing, but you see that this existing approach goes towards saturation value and after it reach saturation, the velocity tracking is not good anyway. So, that will depart from what you want.

And, all these experiment, also remember it has to be done some sort of, I mean these are numerical studies where things partly some information available partly not and all that. So, you have to make sure that you start with something like a trim condition straightens level flight actually. So, getting that trim condition itself is an exercise and that is there in the vapor, but I am not telling with all that. So, you have to make sure that the aircraft keeps on flying at some straighten level trim condition. After that you may set these experiments actually. So, that is the first observation.

Second being observation is like if you see this plots slightly close, I mean closely I do not think this is very clear here, but if you see that very clearly, then what happens is I mean very closely then the new approach there is nothing like if the command is given to positive side, then the development also happens towards positive side only, there is. But the other one first it dips actually; that means, it goes in the reverse direction and then it comes out. And that is typically expected, I mean whenever we are talking about the stain control aircraft and all that. We all know that it is it is supposed to be non-minimum phase behavior manner. And mathematically speaking, very strictly speaking the second method that we are proposing also is that, but it is so less that, it is just almost as if it is not there actually that way.

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So, the non-minimum phase behavior is quite minimum over here. And same thing you can also see in the Q response basically that is one method goes one side and the other one first goes to other side and then comes out. And similar thing you see in alpha also basically there and because of the saturation problem that we discussed here, all other things will go very bad actually, after one control goes to saturation which is no more control variable it just becomes a some sort of a parameter. So, that control action will not be activated at all actually. Similarly, if you see control variable and all that these are well within the dimension all that, because so we have to also see almost control action is necessary to generate those 2 G values, 3 G values like that actually. And so you can see that the control deflections are all within limits like 2 degree, 5 degree like that actually. So, that is not a very big problem for say.

So, all these plots are available in the vapor also, that you can see what variable like P Q R and then the alpha beta, these are also important quantities how they behave and think like that in both the approaches by the way. This beta remains very close to 0 anyway and that is one of the requirements for turn coordination actually. And the new approach beta will be remained even more try to up to 0, I mean close to 0 basically; that is a point to observe what is not a very major point to your claiming because the other approach also does not give a very high value of beta basically.

So, you can see some of this combined longitudinal lateral maneuver that is something that we cannot compare. So, this is actually, I forgot to tell these values what I told here is for the longitudinal maneuver, these are the associated control with that you know all that. So, what we are talking here is lateral maneuver, where P^* is given as a command value. So, command value is directly given in terms of P first minus 10 degree per second, then plus 10 degree per second, then 0 initially it was 0 anyway actually. So, with that your primary X_1 is aileron and n rudder and elevator deflection is minimum. And here it is primary X_1 is I mean longitudinal maneuver, primary X_1 is elevator where the aileron and n rudder is close to 0, anyway.

So, longitudinal variable, control variables also are there and then we also experimented all with respect to combined longitudinal lateral maneuver case. So, that detail are also available in the paper actually. So, the point here is all control actions will be activated. So, that it actually, the objective is met this P_w is the velocity vector; however, I can see the how it, I mean how good it goes there and then tracks the commanded variable actually, so that is nice to see that.

And n_y also if you see its very close to 0; that means, you have a no matter even if you have combine longitudinal lateral maneuver, your turn coordination is always assured which is again a nice thing to see. And in velocity, initial velocity vector velocity value is trim velocity is 580 feet per second that is what we want to maintain and it is maintained actually, that is another point that I want to tell here. This is the model that is available for f 16 that is openly available is all in the in terms of f V s unit actually. So, somebody has to be careful to either deal with that f V s unit throughout or you have to convert S I unit and then overcome that. So, then everything needs to be converted to f 1 unified framework fast.

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Summary: Nominal Controller

<u>Existing Method:</u>	<u>New Method:</u>
<ul style="list-style-type: none">Assumption: $\dot{V} = \dot{W} = 0$ $\ddot{\Phi}^* = \ddot{\Theta}^* = \ddot{\Psi}^* = 0$	<ul style="list-style-type: none">Assumption: $\ddot{V} = \ddot{W} = 0$
<ul style="list-style-type: none">More number of design parameters (11 & 12)	<ul style="list-style-type: none">Less number of design parameters (5 & 7)
<ul style="list-style-type: none">Works	<ul style="list-style-type: none">Works better...!<ul style="list-style-type: none">Lesser control magnitudeSmoother transient responseBetter turn co-ordination

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So, these are the summary of this control design, which I call nominal control design because we are still not talking about any adaptive control, inaccuracy in the model and think like that. So, first thing is that existing method, again that is I am not talking anything only any details on that, somebody can find that literature as well. So, this is a 1994, if I remember it was published as A W conference paper. That developers details can be found from our paper that is there anyway, summary of that method is also available is a appendix of one of our papers anyway. So, that list of the paper I will put it at the end of this lecture.

Now, existing method if you see assumptions that they have done, what is required is that V dot W dot has to be 0 and this after you converted to this altitude values and all that double derivatives there, the commanded I mean double derivatives of the commanded values of this phi theta psi should also go to 0, I mean remain 0, that is an assumption. And if you see the new method the assumption is only that we required is V double dot W double dot become 0. So, this is actually one order improvement over V dot W dot being 0. So, this is, I mean if you see this assumption to this assumption, this is one order close to reality basically and this is, that is how it is this assumption is also not required here.

And, here we talk about more number of design parameters. So, if you talk longitudinal it requires something like 11 parameters (()), lateral you have required 12 parameter (()). But

here it is less number of design parameter longitudinal 5 and lateral 7. Remember, the lesser number of design parameter is also good thing for control designer because he does not have to keep on wasting his time tuning this values to get a good performance.

And, the point here is this method works, I mean certainly it works it not claim anything beyond that. But what is your claiming in the new method is it certainly works better; that means, if you do this, we propose your new ideas and think like that and compare it in a fair manner. you can is very good to see some of those that it leads to something like less control magnitude, it leads smoother transient response, we know non-linear phase behavior thing sort of thing, leads to better term coordination, that means beta remain very closed to 0. And probably that like design parameters being less and all that is also available.

So, this with these new methods seems to be very promising. But whenever this I mean after getting all those results, so also we were curious to see the robustness part of it. So, then we want to experiment this Neuro adaptive control design for increasing the robustness part of it.

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Objective:

To increase the robustness of the
“Nominal Controller” with respect to
parameter and/or modeling
uncertainties.

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So, objective here is to increase the robustness of the nominal control with respect to the parameter and other model modeling inaccuracies, primarily we are interested in parameter inaccuracy here.

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Robustness Study

- Parametric uncertainties was introduced in aero-coefficients and inertia parameters using Gaussian distribution around the nominal parameter values
- Since no analytical method for analyzing robustness behavior is available, a stochastic approach has been followed in this paper as an alternative.

Legend: ● Perturbed Coefficient, ● Unperturbed Coefficient

Diagram: Shows a Gaussian distribution curve with a mean value. A vertical bar represents the nominal parameter value. A blue dot (Perturbed Coefficient) is shown to the right of the mean, and a pink dot (Unperturbed Coefficient) is shown to the left of the mean. A dashed line connects the two dots, and a solid line connects the pink dot to the mean. A mouse cursor is pointing at the diagram.

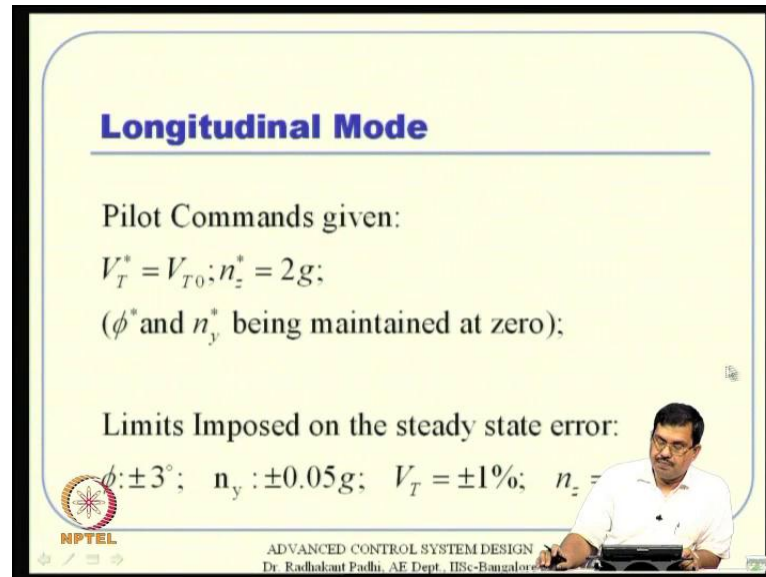
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So, what is the issue here? Issue here is that the numbers that are typically given to us, I mean the coefficient of the polynomial that is fitted and all that are typically coming from internal experiment and think like that.

So, with the numbers that comes out from C o T study or internally something like this lets say. And then here actual number may be somewhere like, that this value is part of around that and this value is part of that. So, whatever slope that your interpreting here this point that point is actually not a good slope, what should be is that slope actually. And that is a quite a bit our difference I mean difference and that may lead to lot of problems there actually. Because that slopes are computing the derivative actually like let say $\frac{\partial C_m}{\partial \delta}$; that means, $C_m \delta$ probably, if that is inaccurate then the δ value that will compute is a anyway inaccurate. So, that will lead to difficulties and all that.

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Longitudinal Mode

Pilot Commands given:

$$V_T^* = V_{T0}; n_z^* = 2g;$$

(ϕ^* and n_y^* being maintained at zero);

Limits Imposed on the steady state error:

$$\phi: \pm 3^\circ; \quad n_y: \pm 0.05g; \quad V_T = \pm 1\%; \quad n_z =$$

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So, what I mean what is going on here, we just gave some pilot commands like that and then we impose some sort of limits on this steady state error as performance evaluation. That means, if you give this, I mean we remember this what exercise is, you go back to this model let say see these models are there with us. So, if you see these models and then pick up one of that and all that and then we lot of coefficients see that. And then we assume that the coefficient values are wrong; that means, 5 10 percent error whatever your putting there, as far as control design is concerned you take the nominal values, but as far as the state simulation concerned you say you picked a different value which is (()) actually.

And, with respect to that you see the performance and without performance you put this, I mean performance bounds and all that to tell that if this performance bounds are met, then I will assume that it is a it is a success, if it is not met in the steady state then I will assume that it is a failure actually.


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Lateral Mode

Pilot Commands Given:

$$\phi^* = -40^\circ; \quad V_T^* = V_{T0}; \quad h^* = h_0; \quad n_y^* = 0$$

Limits Imposed on the steady state error:

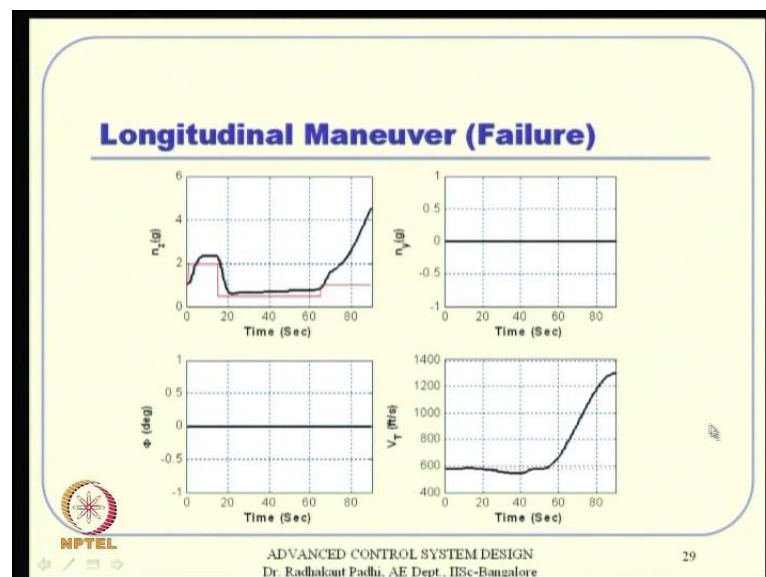
$$h: \pm 1\%; \quad n_y: \pm 0.05g; \quad V_T = \pm 1\%; \quad \phi = \pm 10\%$$


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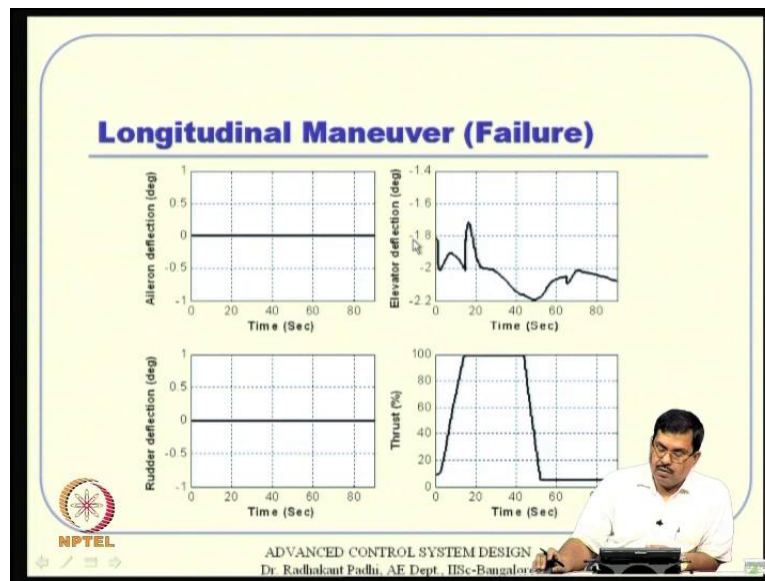
And, pilot commands are also put something like that and limit imposed on the steady state error in the lateral mode are something like this, altitude should be plus or minus 1 percent and n_y should be plus or minus 0.05 g, V_T is plus or minus 1 percent ϕ is plus or minus 10 percent actually.

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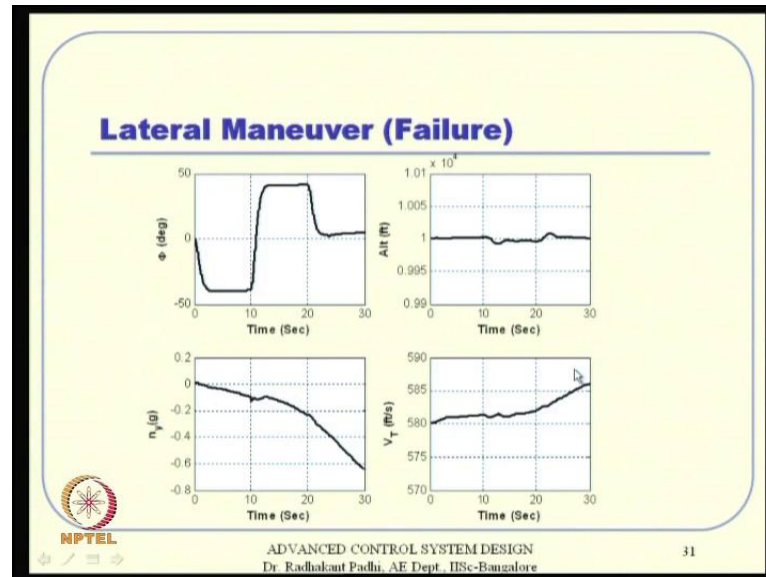
So, with that you see this some of this, I mean surprising order the well you can tell it is expected also sometime. So, you see that this is typically responds where if you take nominal parameter values it appears everything remains good. But if you, the moment you put this inaccurate parameter values and then simulate it obviously the control that you are computing is not able to enforce the aerodynamics that you want to enforce. And hence nothing is going good then that means, the tracking is there and after sometime it actually goes unstable.

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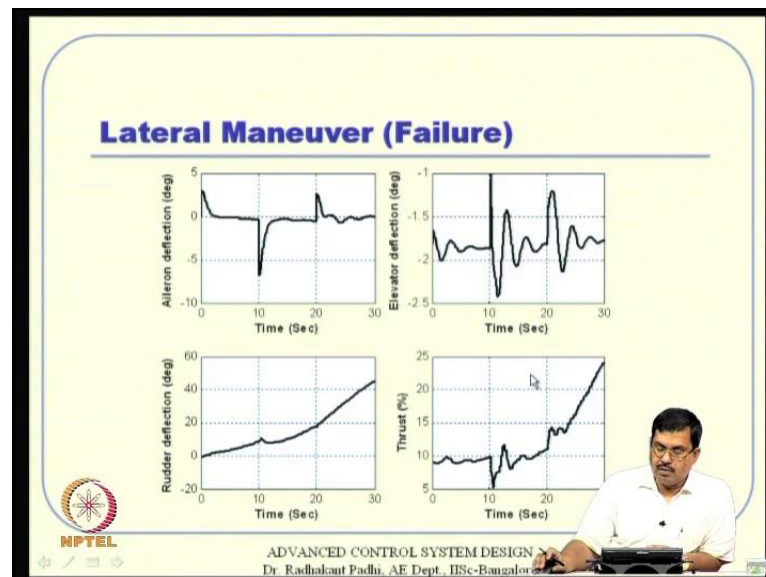
So that means, it actually certainly a failure case actually. And if you see that why it happens then actually thrust goes on increasing, it goes to the 100 percent of that and that is actually upper bound of the saturation ,then it comes back and it saturate in the lower bound anyway.

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So, these are all reasons that why it actually happens that way. And similarly if you see the lateral maneuver case, if this is also like not good because n_y which is should remain 0, it starts developing I mean non-zero and starts going diverse, I mean its diverging. Similarly, V_T also start diverging all those issues are not good to see.

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So, this is the failure case, why it happens and think like that, remember there is nothing like a saturation value here, they are still within that. And then what the deflection demands are so high here, they may be very unrealistic you can think like that. That issue a part you can see that this values, I mean the, if these happens then we certainly consider that as a failure case.

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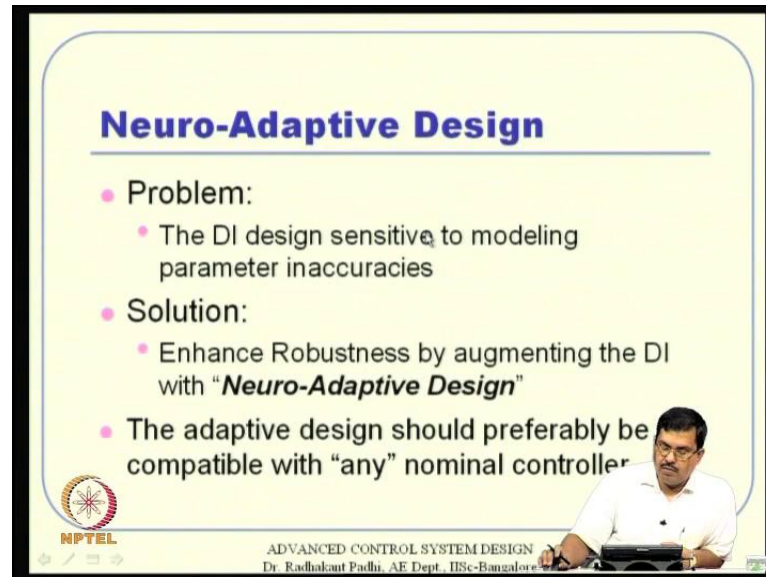
Robustness in Longitudinal Mode

<i>Aerodynamic coefficient</i>	1%	1%	2%	2%	5%	5%
<i>Mass and Inertia coefficient</i>	5%	10%	5%	10%	5%	10%
<i>% Success</i>	100%	100%	98%	96%	78%	76%


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So, with that we are lot of rigorous, I mean lot of simulation studies that you would like you say aerodynamic coefficient and mass and inertia coefficient, you combine them in a different percentations you (()) of them and then 1 percent 5 percent 1 percent 10 percent 2 percent 5 percent like that. And each of those combinations you are on something like 50 to 100 cases. And then count for how many number of cases you have success, how many failures and all that. And it turns out that if it is 1 percent 5 percent 1 percent 10 percent nothing to worry so much actually. But the moment these aerodynamics coefficient start becoming more and more inaccurate there is lots of robustness actually. And if it is 5 percent and 10 percent that combination which is not unrealistic by the way, the success is almost like, let say you can three-fourth actually. So, one-fourth cases you certainly get failure sort of thing. So, that is not a good thing to observe.

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Neuro-Adaptive Design

- **Problem:**
 - The DI design sensitive to modeling parameter inaccuracies
- **Solution:**
 - Enhance Robustness by augmenting the DI with "**Neuro-Adaptive Design**"
 - The adaptive design should preferably be compatible with "any" nominal controller

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Similar thing happens for lateral mode also. So, then we thought we will go back and have experiment this Neuro adaptive design to increase this robustness and can be really address this issue. So, then in that setting as I told you in the previous class we have experimented in the output setting only. That means what about this $P \dot{Q} \dot{R}$ those equations, that equation needs to be robustness actually. That is the innermost loop needs to be, that is all we are enforcing not the total straight tracking and all that actually. So, then Y should go to Y_d as soon as possible, then Y_a should be Y_a dot is defined as some like a auxiliary dynamics and all that. Then, Y should go to Y_d what will take through y should got to Y_a then Y_a should go to Y_d all that actually, so that kind of two loop sort of situation. And not (()) loop it is just like interpreting both the things in a different setting both the things are happening together anyway.

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Steps for assuring $Y_a \rightarrow Y_d$

- Enforce the error dynamics
$$\dot{E}_d + KE_d = 0 \quad E_d \triangleq (Y_a - Y_d)$$
- After carrying out the necessary algebra
$$f(X, U) = h(X, X_a, X_d, U_d)$$
- In case of control affine system
$$f(X) + [g(X)]U = h(X, X_d, X_a, U_d)$$
- The control is given by
$$U = [g(X)]^{-1} \{h(X, X_d, X_a, U_d) - f(X)\}$$

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So, for steps for assuring Y_a to Y_d we have studied that it is done in a dynamic inversion setting. So, you solve it and then because it is control affine and the number of input outputs are equal to number of input the square system, I will be able to solve for the control variable directly actually, that is the point here. And then if you see this Y to should go to Y_a then you define an error for that and then neural network is for every channel; that means you have three channels anyway P Q R. So, well it is not really P Q R here, it is actually the Y that you are talking actually in terms of dynamic equation for lateral and longitudinal, I will show you that actually. And you have four variables, in fact, not three and four control variables also. So, that way this is still square system anyway, you do that actually.

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Function Learning:

Define error $e_{a_i} \triangleq (y_i - y_{a_i})$

Output dynamics

$$\dot{y}_i = f_{\gamma_i}(X) + g_{\gamma_i}(X)U + d_i(X)$$

$$\dot{y}_{a_i} = f_{\gamma_i}(X) + g_{\gamma_i}(X)U + \hat{d}_i(X) + k_{a_i}e_{a_i}$$


From universal function approximation property

$$d_i(X) = W_i^T \varphi_i(X) + \varepsilon_i$$

$$\hat{d}_i(X) = \hat{W}_i^T \varphi_i(X)$$

Error dynamics

$$\dot{e}_{a_i} = d_i(X) - \hat{d}_i(X) - k_{a_i}e_{a_i}$$

$$= \tilde{W}_i^T \Phi_i(X) + \varepsilon_i - k_{a_i}e_{a_i}$$


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So, the ideal network is neural network is given that the error quantities that. So, output dynamics is defining like that, all the details we discussed in the last class, then e I dot the error dynamics is given like that.

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Lyapunov Stability Analysis

Lyapunov Function Candidate:

$$L_i = \frac{1}{2} (e_{a_i} p_i e_{a_i}) + \frac{1}{2} (\tilde{W}_i^T \gamma_i \tilde{W}_i)$$



Derivative of Lyapunov Function:

$$\dot{L}_i = e_{a_i} p_i \dot{e}_{a_i} + \tilde{W}_i^T \gamma_i \dot{\tilde{W}}_i$$

$$= e_{a_i} p_i [\tilde{W}_i^T \Phi_i(X) + \varepsilon_i - k_{a_i} e_{a_i}] - \tilde{W}_i^T \gamma_i \dot{\tilde{W}}_i$$

$$= \tilde{W}_i^T [e_{a_i} p_i \Phi_i(X) - \gamma_i^{-1} \dot{\tilde{W}}_i] + e_{a_i} p_i \varepsilon_i - k_{a_i} e_{a_i}^2 p_i$$

Weight Update Rule:

$$\dot{\tilde{W}}_i = \gamma_i e_{a_i} p_i \Phi_i(X, X_d)$$



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So, take Lyapunov function that way, take stand derivative do the algebra, the coefficient appearing with respect to the W tilde, I mean W tilde transpose we take the coefficient make sure it is zero. So, that gives you some sort of a weight update rule actually.

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Lyapunov Stability Analysis

This condition leads to $\dot{L}_i = e_{a_i} p_i \varepsilon_i - k_{a_i} e_{a_i}^2 p_i$

$\dot{L}_i < 0$ whenever $|p_{a_i}| > |\varepsilon_i| / k_{a_i}$

Using the Lyapunov stability theory, we conclude that the trajectory of e_{a_i} and \tilde{W}_i are pulled towards the origin.

Hence, the output dynamics is "Practically

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The slide features a yellow background with a blue border. It includes a title, a mathematical derivation, a condition for stability, a conclusion based on Lyapunov theory, and a partial sentence. The NPTEL logo is in the bottom left, and the course information is in the bottom center. A small inset image of a lecturer is in the bottom right.

So, these list to the condition that L I dot is something like that, which is negative provided this happens, all the details we discussed again in the last class. So, we are assuming that that actually, what you are claiming is at least to practical stability. And in the process it increases robustness; this is what I was talking here.

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Problem Specific equations

- Output vector in longitudinal mode:

$$Y \triangleq [P \quad a_z \quad a_y \quad V_T]^T$$

- Output vector in lateral mode:

$$Y \triangleq [P \quad Q \quad a_y \quad V_T]^T$$

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The output vector for the longitudinal mode is like that P a z a y V T and the output vector for the lateral mode is P Q a y V T instead of a z we have Q actually. So, this is the innermost loop that you are bothered about. So, that is where you will compute the control action directly after by enforcing first order dynamics.


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Basis Function


$$\varphi_i(X) = \left[e^{-\frac{1}{2} \left(\frac{\|Y - Y_d\|^2}{\sigma_1^2} \right)} \quad e^{-\frac{1}{2} \left(\frac{\|Y - Y_d\|^2}{\sigma_2^2} \right)} \quad e^{-\frac{1}{2} \left(\frac{\|Y - Y_d\|^2}{\sigma_3^2} \right)} \right]^T$$

where mean values chosen were:

$$\sigma_1 = 0.1, \sigma_2 = 1, \sigma_3 = 10$$

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Basis function selection becomes an issue. You select your Gaussian basis function and the linear in the weight network. And for each of the basis function we selected three I mean Gaussian basis functions actually. And then you varied this sigma values like one thing is 0.1,1,10 sort of thing for each of the selection. That means, for each of the channel your actually talking about nine basis functions and then for each of the nine basis functions you have to we should have a weight actually. And so that weight dot, I mean the weight update rule should be integrated in parallel. So, this weight update rule should be integrated in parallel.

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Design parameters selected (Longitudinal mode)

- The learning rates and the scalar values:

$$\gamma_P = \gamma_{n_z} = \gamma_{n_y} = \gamma_{V_T} = 60$$

$$p_p = 0.0001, \quad p_{n_z} = 0.05, \quad p_{n_y} = 0.001, \quad p_{V_T} = 0.005$$

- The constants selected are:

$$K = \text{diag}[5 \quad 5 \quad 5 \quad 4]$$

$$K_a = \text{diag}[0.05 \quad 0.05 \quad 0.05 \quad 1]$$

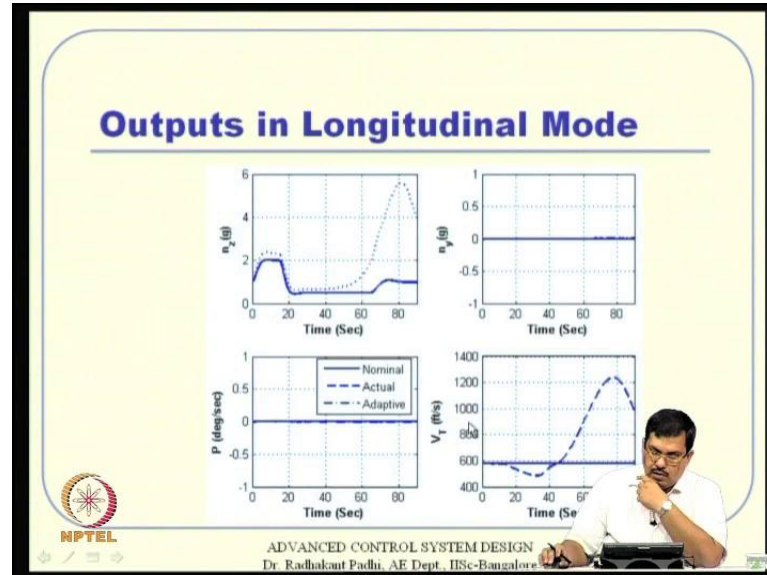
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So, in some of the design tuning parameters have been selected something like this. So, then constraint will you say, these are all like once you start doing some experiment, I mean start with some early one then keep varying until you get that and once you finalize some design after that you do not have to vary. In other words, if you select this variables one bar stain, then from then there onwards it is not varied from case to case actually. So, you think that these are the values that you have to (()) to the best manner sort of thing. So, just keep it as it is.

And, then we take random cases; that means, you select some sort of a like random coefficient number and all that, within the domain that you discuss actually. And then carry

out the simulation studies anyway. Also, again I repeat this P value if you see are smaller, lot smaller than this gamma values. So, that means, the error minimization that we are talking here, this Lyapunov function this component is valid more compared to that component actually, that is the meaning there.

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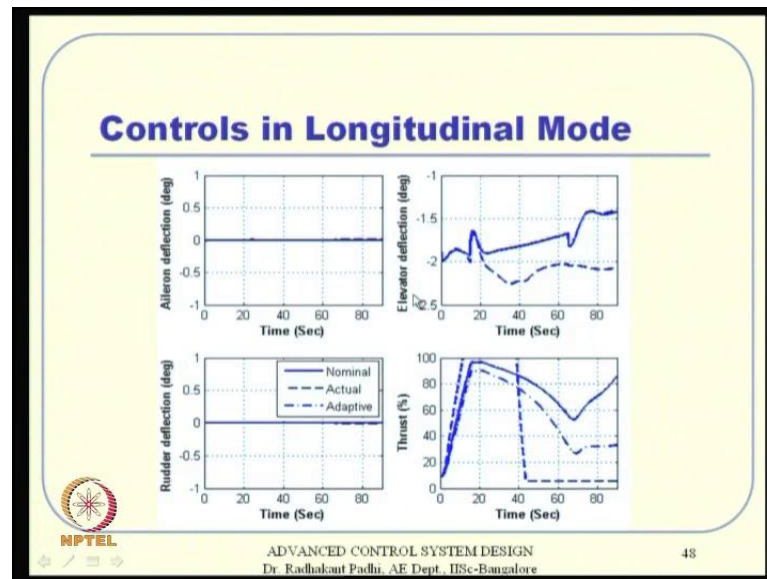
So, our constants values are selected like that and again we can see that the same case, I mean some of the cases that were leading to failure. Now, you can see how good it is to see this response and however, whatever was happening here it was going unstable and going way beyond desired values and all that. That is not happening anymore, we are able to very tightly kind of track these commanded values where is, I mean in other words the response that comes out from here this with adaptive control is as if there is no parameter inaccuracy basically. So, that is how we can interpret that way; that means, we can see that values that are error between the tracking command and the, I mean then actual response are there. But that is as good as the nominal controller actually; nominal control will also from transient and this control will also be transient. And the error between the nominal responses to actual response is quite small, that is sort of that is the point here.

So, you actually like, you say that as if nothing has happened as far as like my actual system is concerned, it is behaving as if it simulation system dynamics. So, something happens here

something for the control, I mean the velocity thing as well and then you can see that whatever it was leading to instability, I mean the saturation and think like that very quickly it is no more saturated, it is always in the bound that we are discussing. And we took it in the, that bound values for the thrust was either it should be bounded below is 5 percent or bounded above by 100 percent, that is what we have taken. And within that, this command value is that given is you can see that is all operating within that. Also, see that nominal controller whatever is here the control values and they have to be different because this control, so that solid line actually is for the actual plant. This control is for the nominal plant, so obviously they have to be different. So, that is the way you are enforcing.

So, the controls values are for control distributes are different. So, the control is modified in such a way that the response is not modified, response is enforced this is similar to nominal system, that is the whole core point.

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So, similarly, you see that the lateral mode commands also, like whatever was not good to see before is now very good to see because you cannot be we will not able to see any different difference at all. And whatever was going away and away it is no more going away, that n y is maintained at 0 in the turn coordination is also assured. So, these are some of the observations that you have to have. Controls in the lateral mode is you concede this

control values do good modified through that the performance values do not get modified, I mean in that is the key point again actually. So, you can see that nominal response whatever is there actual control is actually not doing a good job; that means, nominal control, again I repeat like last class what you mean by actual control or something like that is like this. Either you have a nominal plant and associated with that there is a nominal controller, so those combinations are good anyway.

But where the, in that case where we just want to retain our nominal controller formula, but we want to execute that bus from the actual state information. So, that way the formula whatever the control action is getting into the system is actually operating in a feedback manner. And all these exercises are based on that and if you really make your U_d as a function of X_d ; that means, that nominal control as computed from nominal state dynamics and then you simply apply it here, then it becomes a full control. And the performance behavior is extremely bad if you want to do that. Other way you done it you just seen it for some sort of accuracy actually. So that means, the feedback action itself has a meaning that gives you certain degree of robustness, but once you have this adaptive dynamics action and you exactly enforce what you want to do and that can be substantial amount of robustness increase.

So, the same exercise that you are done before we will go back again and try to do that, repeat that exercise. You have defined your performance condition where it is acceptable or not acceptable and think like that. And again all that you have done this kind of Monte Carlo studies primarily because this robustness concepts are in the norm setting or in the, let us say in the margin setting and think like that are available only for, I mean engineer systems. And because we do not want to extrapolate that too much on that, so we have just want to define this some sort of a Monte Carlo simulation that large number of random studies. And then see what you going on there in the non-linear setting directly.

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Robustness Enhancement: Longitudinal Mode

Aerodynamic Coefficient Perturbation	1 %	1 %	2 %	2 %	5 %	5 %	10 %	10 %
Inertia Parameter Perturbation	5 %	10 %	5 %	10 %	5 %	10 %	5 %	10 %
Nominal Success	100 %	100 %	96 %	92 %	76 %	70 %	48 %	40 %
Adaptive Success	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %

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But one can always go back and try to linearize and then talk about some sort of a, like inversion phase margin for single input single output setting all sort of analysis can always do. And there are lot of exercise, lot of research going on these days to propose good robustness measures for non-linear systems also. But we will, we were not done any of those in this particular exercise. What you are doing here is just like random Monte Carlo simulation, you take this various I mean if you see this various combination of this perturbations and each of the combination you try to outside something like fit a random cases and then see how many success is there, how many failures are there.

So, first is this aerodynamics coefficient perturbations 1 percent, inertia parameter is 5 percent. So, 1 percent 5 percent combination and 1 percent 10 percent combination, 2 percent 5 percent, 2 percent 10 percent, 5 percent 5 percent like that actually you do and you can keep doing actually that way. So, what it happens is you can see that as long as I mean as soon as this aerodynamics perturbations becomes 10 percent, there is actually like a lot of loss of robustness as far as nominal control success is concerned. You can see that it is already, if it is 10 percent 10 percent there, the only success is about 40 percent. And if you take it to 20 percent here, the success rate is almost close to 0, I mean very rarely it will succeed otherwise it very large numbers of failures will happen. And 20 percent inaccuracy

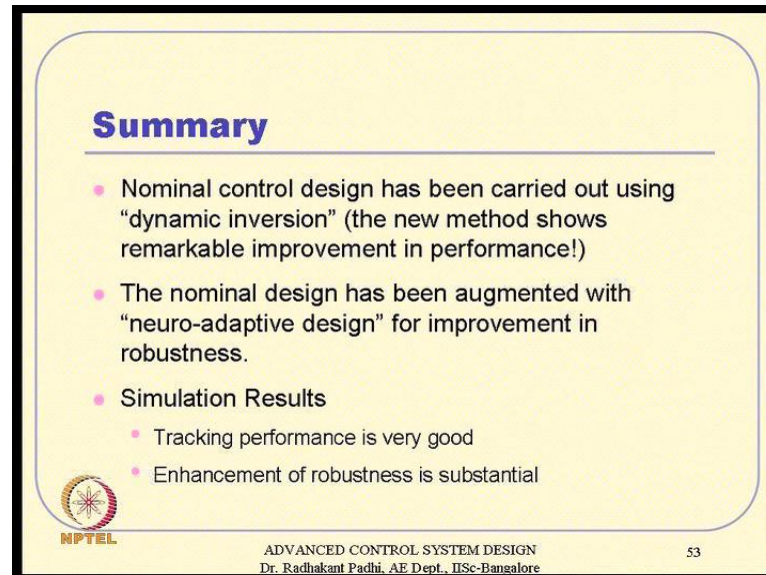
is not a very unrealistic inaccuracy rather; I mean that is there, I mean no matter you want to do.

But if you see this success rate of the adaptive control for the same cases, then you see that it is the last number it is all 100 percent actually. That means, we have never observed one single case with respect to the performance condition that way this specified, we have never seen a single case which will fail actually. So, that is the observation there that is what will happen in longitudinal mode. Again lateral mode is also similar, if you take this lets say towards end of this table if you say 5 percent 10 percent 10 percent 5 percent 10 percent 10 percent, the nominal success quickly degrades actually. And it is actually quite sensitive to aerodynamic coefficient perturbation which is logical also.

So, the moment which 10 percent there, the loss of robustness is quite high, but if you want to put adaptive control back into picture, then the robustness very high actually. So, it I mean again observe no case, where the adaptive control will lead to a failure. I have been said that also remember that we all assumed that ideal state information, ideal error information and think like that. And those are also not realistic in a way. So, we have to also make, I mean if somebody wants to do further experiments and all you can have to, you have to see like some sort of a filter in the loop or something like estimated state in the loop, how does it behave and think like that, that exercise we have not done.

We all assume that the actual state is information is available desired state is available. So, the E I information which is critical for Lyapunov weight training and think like that that is available in a precise manner which may or may not happen in reality just like matter of percent. But nevertheless that issue is that any control design any control design. So, without that issue if you want to see, this it gives you some sort of an idea that if I put adaptive control like into picture, then the success rate can be very very high and that is the point actually.

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Summary

- Nominal control design has been carried out using “dynamic inversion” (the new method shows remarkable improvement in performance!)
- The nominal design has been augmented with “neuro-adaptive design” for improvement in robustness.
- Simulation Results
 - Tracking performance is very good
 - Enhancement of robustness is substantial

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So, summary of this entire lecture is something like this. Nominal control design has been carried out using dynamics inversion, for this flight control problem. And also remembers that throughout this design and simulation we have use the actual non-linear six degree of freedom model that is available. You have never done any; I mean approximations like linearization, so anything like that actually.

So, that this new method source remarkably improvement in performance, as compared to the other dynamics inversion based method that we discussed and we I am not given any details on that. So, what I claiming is nominal control design has been carried out using dynamic inversion, the method that you are compared, we have compared nominal control design are also that method is also based on dynamic inversion. But the approach is different I mean, so that means, even the truly same, we have to also see that what problem we are trying to solve and with respect to that problem what is the good thing to do. I mean that has to be well thought about actually; otherwise just because you are applying the same method does not mean you get the same results actually.

So, first claim is the new method that your proposing for nominal control design is actually at least remarkable improvement in performance over the existing approach, that is available in literature actually. And also remember the same thing that we also experimented with the

very, I mean kind of a what about little information model is available in the Roskam flight dynamics book for (()) 747 aircraft, that is also that was actually our starting point of the experiment of all these things, I have not taken through that anyway, but I will give the reference in that. So that is, so that this same design is also been experimented with same results that you saw with respect to nominal control design. And similar advantages has been obtained for the commercial aircraft as well.

But I mean this entire approach and all makes more meaningful in the flight aircraft on the high performance aircrafts and so, where the command values can be given in terms of n_z and n_y and all that actually anyway. So, second thing what we are claiming is the nominal design has been augmented with Neuro adaptive design, for improvement in robustness. And the results shows that the tracking performance remains very good and the substantial announcement of robustness actually, that is what the claim. Robustness becomes quite high and that is there in evident from this table, the both the tables. So, the last row if you see, everywhere it is 100 percent and nowhere we have observed one case it will go like failure case and all that.

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So, like references **the**, I have told the entire lecture is borrowed from this particular reference, you can see this from more details. So, first one which is again published very

recently like May 2010 and this is also a good thing is, this journal is actually (()) journal and anybody can register there, then download this paper, I mean as of now it is free actually. So, that is an advantage anybody can download, and then you can see more detail. You can see the references wherein in that paper, that you have given some several references, you can see more details on that actually. Prior to that, we have published that in some couple of conference as well, part of the stories and all that, but I everything has been combined to one piece here anyway.

So, the only thing is that nominal control design that the comparison study has been carried out is even a summary is probably not included here. But if you want to see that, then probably either you pick up that reference and see the detail or you see there was other paper, which is appeared here like A I double A conference in 2003, where some summery of that approach is given as an appendix anyway. So, that somebody can see that also actually. And adaptive control approach is we discussed in the previous lecture is borrowed from this literature which is also not very well, this is our own publication this is found 2007 actually.

So, the method part if somebody wants to see more detail can be, I mean can see details on this, the comparison part for the commercial aircraft has been done here, so that also somebody can see. The I mean, the reference from which we have borrowed for comparison sake other approach and all that is also available here as a summary. And the entire lecture for this particular lecture I have taken from this one, which is again it is frees analyze, you have to register and download from there actually. Alright, with that I will stop this lecture. Thank you.