Advanced Control System Design
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Module No. # 14
Lecture No # 36
Neuro-Adaptive Design – I

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Hello everyone, towards this end of the course, this couple of lectures will see applications of lyapunov theory. And one of the good applications of lyapunov theory as the I mean as, it appears in recently literature is something called neuro-adaptive design, which I have going to study in this one or two lectures, here the various I mean approaches under this title that means, lot of people will propose different ideas within this frame work. But will take you through, I will take you through this one or two approaches, which are very promising.

So, this particular lecture will study one approach, which is there in the literature of about 10 15 years. And it is it has been applied successfully, it verity of problems in different engineering fields including aerospace actually. So, that you what I will take you through this particular lecture, next lecture will follow of with another approach that we use in our researches will actually.

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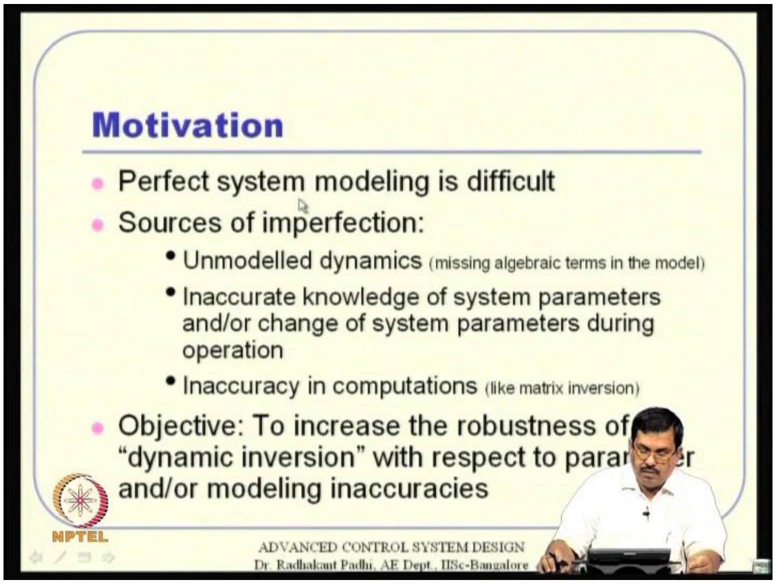

So, will study this, when motivation for this neuro-adaptive is perfect system modelling is difficult, we know that see first up all you may like sources of imperfection. So, if you see where this imperfection comes the first thing is like on model dynamics means, part of the system dynamics. You can you may not be able to model or intentionally, do not want to model. Because the two have a simplify model. And one example is probably like you have number of states are probably, same that is state space dimensional other thing remain same, but you intentionally one to like some miss some algebraic terms in the model, to our simplified model or I mean is it is not possible for us to have this algebraic terms, actually another example.

Let say even if the model all algebraic terms are contain, then that the accurate knowledge of system parameters, may be say actually like exactly. How do you know what the system parameter that goes through that for example, like mechanical system was point of inner say all that, you can how can estimate, you can up to a certain level of accuracy. And not only that the system parameters changed during operation as well for example, like if you are aircropt is flying is continuously was you feel is burning out anyway. So, that particular thing exactly predict how much feel is gone and all that is is typical will no payable to that.

And it say passenger moments inside the (()). So, even if the mass remains constraints distribution of mass is different enhance the moment of inner say allocation is an actually. So, these are issues that comes in to picture. Then another issues probably in accuracy, in computations for example, like if you talk about matrix inversion and all you may have I mean there are some finite truncation may happen and then, this is this issue you comes, because of numerical problems and all that. So, will the objective here is to increase the robustness of dynamic inversion design especially.

And in this particular design, where studied in detail from our last preview, I mean sometime before, when the look I think probably lecture number 30 around that. And we can I hope everything is clear there in the n within this dynamic inversion framework, where going to study how to increase the robustness does a dynamic inversion as that one, the term we realize that, it is sensitive to modelling in accuracy. And that one time also told that, will study in approach to make it robust with respect to this particular issue, that is what the primary objective in this particular lecture.

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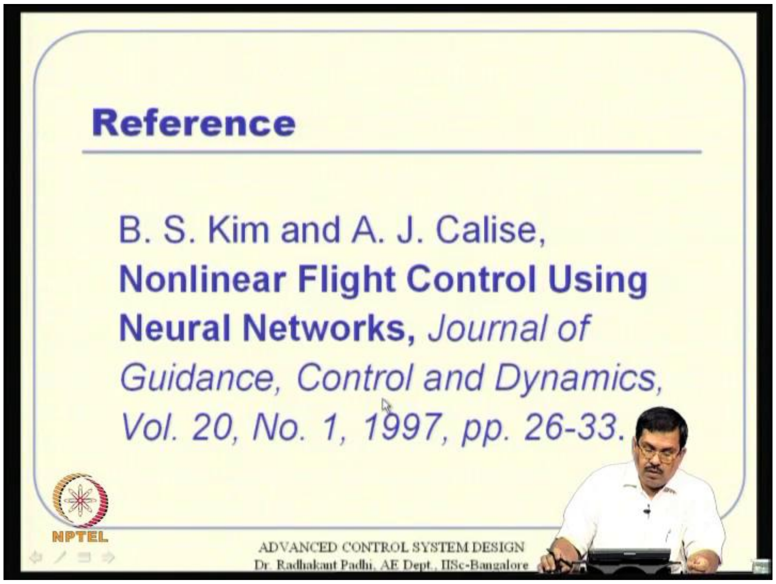

So, want increase the robustness of dynamic inversion design with respect to parameter, in accuracy and or modelling inaccuracies. So, modelling does not mean only parameter it can also algebraic terms alright. So, with respect that we want to increase the robustness of the d

I design. This particular lecture is taken from this in vive, it was one of the seminal papers actually, the conform was one of it probably appeared around 94, the general version his what is here is more complete. And the complete detail analysis and all, I think I have given in the appendix of this paper. And this particular paper talks, about flight control design especially, I am not going to take through details of flight control design for say.

And I will take through detail details of the method and then what just one or two results, that are reported in the paper, I will just likely through it demonstrate, how this method is powerful actually. But also in another verity of versions this authors propose especially, anthalica listen jerji taken all that as several co-authors, with that in the (( )) appear around this theme actually. So, once you understand the basic part of it probably the understanding all branches out of that as soon as been typical actually, will stop about what is see here alright.

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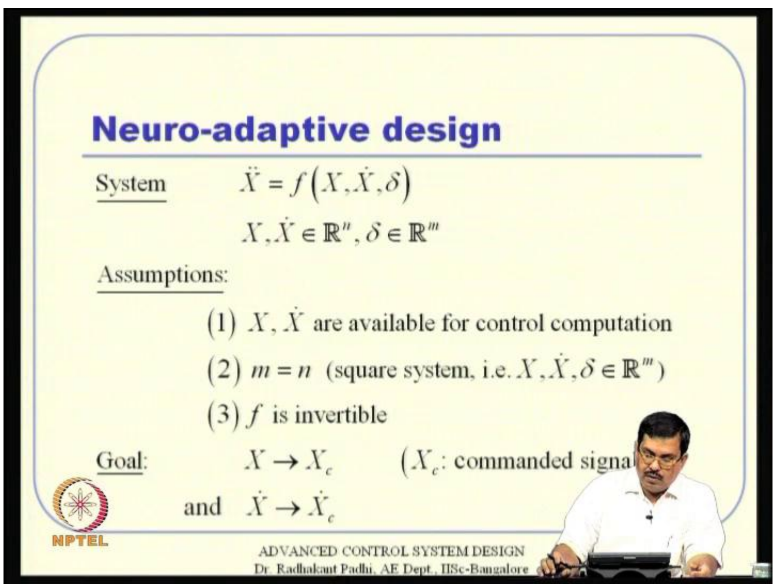

So, system dynamics that you are I mean considering here is X double dot is f of X f dot and delta delta as appears to be likely control variable and intentionally it is taken, because delta e delta a delta R all that are flight control variables. Actually, like control surface reflections and all. And why we take double dot in all because of this is one of the forms that appears naturally in system dynamic. And especially, if you say neutron second low it is a something like relates two double dot m X double dot is f actually and there is how start doing that.

So, if you really want to control it say I mean, if you want design a control based on the attitude dynamic actually. So, for example, that if I theta if you remember then find or theta root side out or functions of e q R, but they are not functions of control actually, but if you take double derivatives then period are it appears and then the deltas will short appearing. So, a never wards the flight control problem that, we typically interested in is of related degrade to actually. So, that is what we were considering here, we in general will take this particular system dynamics e X double dot is if f of X x dot delta X is typical not a state variable you cannot call it that way, but X and X dot taken together only define the takes place system dynamics, that you are considering is in this form where X and X dot R of n dimensional there accurate in a n dimensional space and delta accurates in m dimensional space.

So, what are the assumption, assumption is first of all X and X dot are available for control computation increase of there available with us to use. Second regressions n is m equal to n that means, it is a square system in the since that X n dimension of X is equal to dimension of delta here remember X is just half of the state space. In or the complete state space actually in the case X and X x and X dot both are like n dimensional thing, but n is equal to m, that means, X and delta R of same dimension here means typically, happens in implied control is an, if you see five theta is I are three and delta a delta e delta R also three physically.

So, that it typically happen that way also there is an assumption that, f we invertible for all time and that is what we need for calculating the control delta. If you now what is the like ideal value of f actually, if you know what is what is ideal value of a as in the later on it will be defined as pseudo control actually, but z if you know the ideal value of what it should be then this is what it is. So, from extractive delta out of that what it should be is something like in inverse function. So, in yield. So, fusion veritable actually. So, what is the objective or goal here that X should go to X e and X dot should also go to exide dot actually, where X is the common dot signal that, if you remember dynamic inversion it is something like star actually star, but here I will be I mean, I will try to just comfortable to the notations as reported in the vapour. So, that reading that vapour will be also easier for actually. So, X x goes to X e and X dot goes X e dot.

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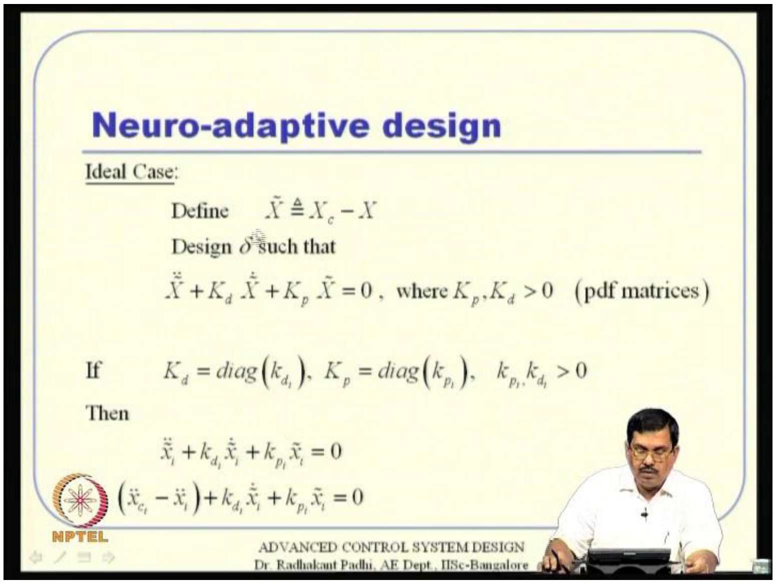

So, but X is the commanded signal we more all this time varying signals actually, they are not constant values and all that. So, what do you that let say this function is exactly, known to us that means, there is known uncertainty and all that. So, that that falls in the case of regular dynamic inversion and, if you really. Now, want to have this objective meant then, what do you I mean we define as something like an error, error is let say X tilde, which is X e minus X either we can define X minus X you are X e minus does not materially.

So, we define X tilde is X e minus X and what to design a control delta such that this error dynamic satisfied, I mean that is what we discussed in the dynamic inversion class right. If it is like e double dot plus k d dot plus k p equal to 0 certain thing, but instead of e we are talking as X tilde, X tilde is error between X and X actually. So, if you design delta such that this equation is satisfied, were k d and k p are posture define matrices then. Now, objective was might we know that does in that situation both X x tilde and X tilde dot will go to 0, if this equation is satisfied exactly then X tilde an X tilde dot both will go to 0 enhance this equation both objectives will be mat actually all it is.

So, one more selecting this K p K d being positive definite matrixes is you select K d and K p both diagonal matrixes and, if typically select. Let say I mean K p I K d, I they are ith element of the diagonal matrix to be positive. And typically, way to select it as is also like b I is something like to zeta I omega I omega n I and K K p I is omega n I square, it will follows typical it was standard form that, we know X x double dot plus 2 zeta omega n write 2 zeta omega n omega 2 zeta omega n I X tilde I dot plus omega n n square and all that actually. So, essentially we have discussed all that in the d I dynamic inversion class. And the way to select this zeta omega n I zeta I omega n I or to select plus this performance objectives like, settling time percentage also like that actually, other anyway (( )).

So, if you select this error dynamics and if you know that, this system matrix this function is known exactly, then model two we want to design in delta. So, this error equation is satisfied. And, if this error equation is satisfied let say this is the ith, ith element equation actually and this ith element equation, I can exponent that way. So, because remember X is nothing, but X e minus x. So, X i tilde is nothing but X e i minus X i actually. So, based double dot remains actually. So, I will return this as it is this 2 terms, but this one I will put it into in the definition art of it. So, X i tilde double dot is nothing but X e i double dot minus X i double dot. So, that is that is (( )) size of X one that is particular term.

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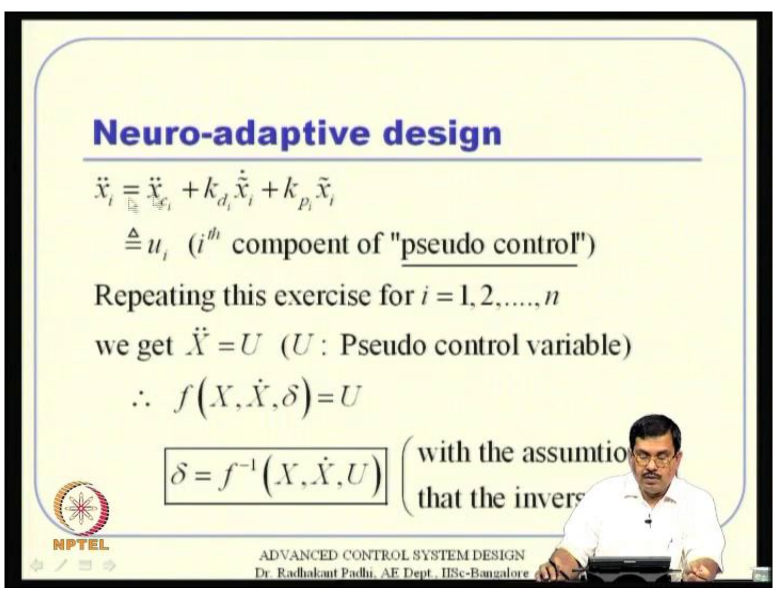

So, now want to solve for a X i double dot. So, this particular term phase e I solve it for X i double dot and X i double dot answer to be all this term, this one plus that plus that one right. So, and this particular term is something that define as u i which is nothing but ith component of what the curl I mean lot of people also call it is pseudo control, actually why is not really the control, but what you really want, you want this X i double dot to bigger something like this where, this K d and K p are design gains anyway. So, the something that we are enforcing, this dynamic something, we are enforcing this nothing do with the system dynamics at.

So, this particular thing, is something that we are enforcing and whatever, we are enforcing in a nothing but like a control variable actually, that is why it called pseudo control actually. So, if you say if you see X i double dot is u I nothing but ith component of the pseudo control. So, if you repeat this X size for I equal to I mean one to n pseudo thing. And are you can directly start with this definition and indirectly, that big capital X double dot in all that that is also possible write, if I instead of going through ith component, I will start directly from here and then put it this is expression like here that X e double dot minus X double dot is equal to all that in the exactly like what is going on here. And then I arrival it something like X double dot is equal to u this u is nothing but pseudo control vector actually, pseudo control variable in general basically.

So, remember so what do you what you are doing, I mean in this in the dynamic inversion design is they one way to interpret this region, in is also like let me design is look control first, because that is hat you what is meant by pseudo control here. So, if I region, it then this equation is something that you know this X double dot is f of this the that is system dynamic and this X double dot is something equal to pseudo control, that I want to design it. So, let me equate the equate that rule is among the same thing, what about wave we have to interpreted before is one of the same thing, just talking in a little bit on about actually, but lower people in literature will talk about pseudo control. So, it is better that we with know what mean actually anyway. So, X double dot is equal to u and then X double dot is equal to f of X x dot a delta also.

So, this two function let me quite now and this local two will contain the gains remember that. So, all this error error dot gains. And all are all hidden in pseudo control actually. Now, if this function is exact then I can compute delta is inverse of that remember this, number of equation, I send because number of a X by assumption what you started is m equal to n. So, number of states I i mean number X variables are equal to number of X dot variables, number of is equal to number of delta variables. So, what you see here is the dimension compatibility is their actually, the number of variables are equal to number of equations. So, then will level will want to solve delta, how do you solve delta symbol equalities represented as f inverse this.

So, that is way we need the third assumption and we tell look this for this function, what we have is invertible, because in veritable I can talk about inverting this actually. And if it is really like control f I hat sort of thing the matter is all already therefore, is no actually like is a suppose, it is a f of X dot plus g of X dot times delta then this delta is the directly solvable. That is what we part of an before, while symbolic speaking delta is nothing but f inverse this function.

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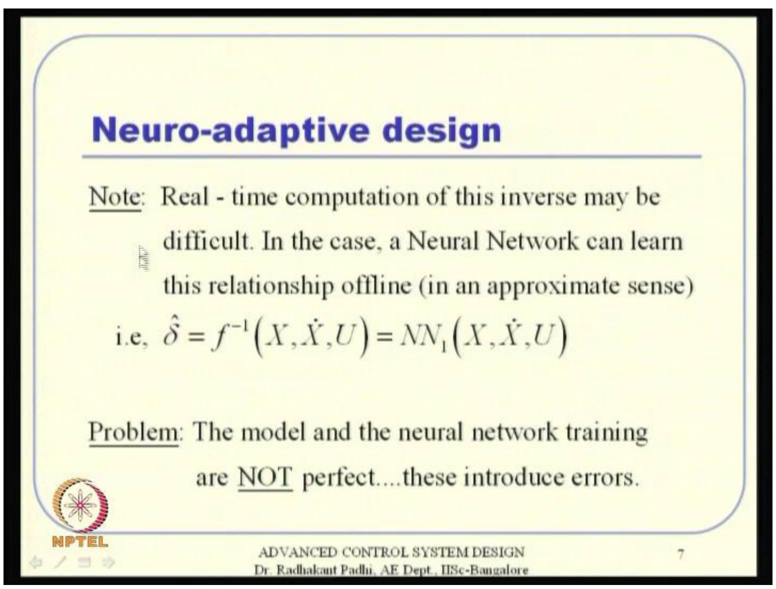

So, as long as we know X and X dot and u, u contains all that actually remeber that is commanded variable and it is in command variable X e. And it is first derivative as well as second derivative, if you know we can complete all that then you put it here actually. So, with the information of commanded variable I mean commanded signal whatever, we have and the gains that, you are selecting will be able to compute this is like regular dynamic inversion actually. Now, where the problem comes on case, problem comes because of several receives and one issue is how do you talk about inverse function actually. In general I mean if it is a problem function is simple on all that, I can talk about doing inverse symbolically what. Let say that is possible in general.

So, what so, proposed in here let us use some sort of neural network to train this been inverse function of like. So, I know where my flight dynamic (( )) where, my flight envelope is within that flight envelope, I will train an a neural network and those of do not know, what is neural network, we can think of it is like something like a function approximation to actually. So, this is actually a function delta equal to some function of X h dot u. And you can train this one and one, network a priory with a spectral values of the this X x dot in all that with in the domain that, we are interested. And then this neural function is directly is captured using in on neural network. This spot may are may not be necessary, suppose you we can directly compute delta. And your let say your modelled model of the flight vehicle is in control affine from then, directly will be able to solve it actually.

So, the no need of doing this neural network training a priory, but in general it is should I mean it is possible actually. Now, it is the problem is the model that, we started with this model there are imperfection, because of this is issues that model itself is not good. And then, where the out a define about training a network any amount of training that, we do this training is never go affect any way, any function approximation can never picture a function exactly, there will be some function approximation errors actually. So, this two issues will introduce errors there. And because of dynamic inversion will not work actually, in a very good way.

So, how do you improve that actually, that is the problem actually. So, what you real what you real say here is this there is an error. So, can we really cancel that error in adaptive way. So, will try to learn that error some of some where. And then try to kind of cancel that error, that is the whole philosophy of this neuro adoptive design. And that function of approximation or something that, that learning part will be done in adoptive adaptive theory base sort of thing adaptive. And inverse instant inverse learn be able to cancel, but using the information an sequential way, then should be able to adoptive a function in such a way, that the imperfection will get cancelled out actually.

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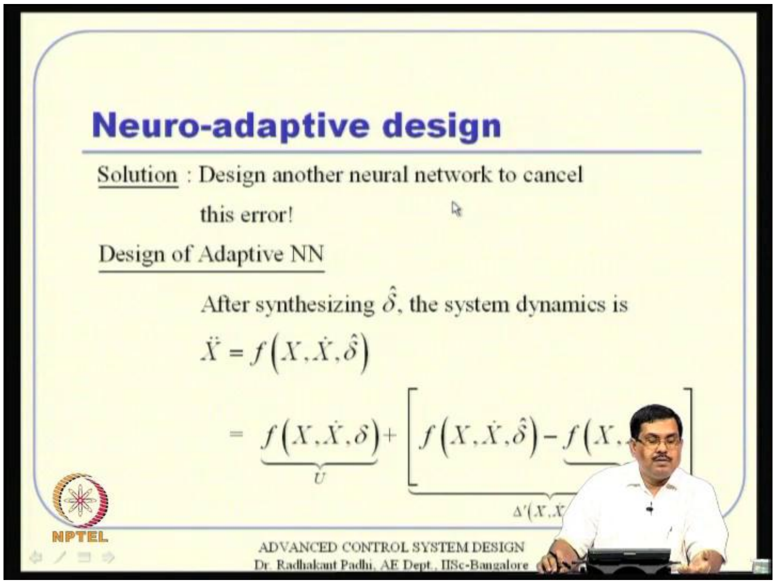

So, let us see than how do you that. So, what the form was idea here we want to design in another neural network. So, this neural network one. So, that is just to that inversion function radian all that. So, non best of another neural network to let say to cancel this error actually, now how do you do that. So, let say synthesize delta delta hat actually, using neural network. So, that is our actual whatever, neural network predicts whatever, this function inverse is giving us that is nothing but delta hat equal to actually. So, what is my system dynamics, system dynamics if I go back and look at this is my system dynamics actually, right original system dynamics, if I look at it then this is my system dynamics actually.

Now, this delta hat is now perfect by the way, in the whatever is exactly need for the system dynamic is not perfect. So, what I have do I will kind let me add and subtract this two terms actually, this instead of del I mean this f of X h dot delta I will keep it. And I will aid this term and subtract that term again. So, that you have this is same as X double dot actually. Now, what is this now, let me interpret that this particular thing is nothing but my ideal pseudo control actually, my delta is modified in such a way. Now, delta prime, I mean delta hat is something that I am computing with this function, but delta is something X actually. Now, delta is computed in such a way that, a nothing but exactly my pseudo control actually.

So, that is the already other than what is the error, error is nothing but this minus that what whatever, I have here this minus that introduces some error is actually. So, that error I will call that is delta prime, because if I not delta for whatever, if I some our notice that delta for my h l system, then that is what I will need for inverting my function actually, but I do not know that.

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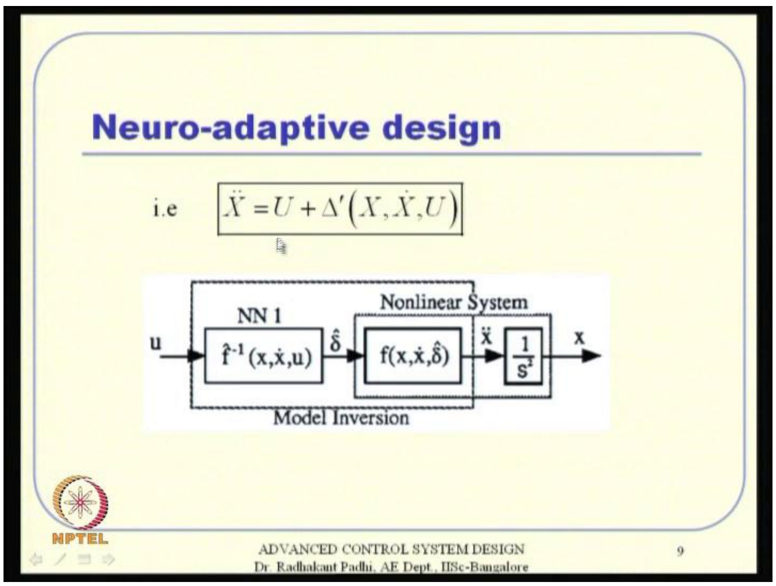

So, let us how to how to synthesize that know all that actually, but is essentially this is what, if I know that then I will do this hidden subtract operation. And you see that this is my U plus this delta prime. Now, if I so what is my X, my real X double dot is my pseudo control. The ideal pseudo control whatever, I should have plus there are actually enhance this entire error dynamics that, we started with this error dynamics is not satisfied actually right, I mean in this error dynamics that is started with is not satisfied. Because this is if it is equal to I mean that X x dot is equal to U sorry X double dot is equal to U than, it is it will satisfied exactly, but it is not U any more.

So, it is U plus delta prime. So, and then enhance, if you go back and substitute that this error dynamics will not satisfied actually. So, the error may not go to 0 that is that is the whole id there. So, how this how this approximation is doing on here I like, if you see here the if I some or now this exact U and all that. Let me go through this neural network, one which is the inverse of that function I will get, it enter prime. And would that the delta prime I will put it through, that this function actually. And then X double dot appear then double integrate take X an all that actually, I can do partially integration like 1 by S one prime then X dot will be available 1 by S one more is one more time it is will be available, I can take signals whatever, I want actually this just a block diagrammatic representation is nothing more actually.

So, mathematically so, we can still here I mean still at the receive that X double dot is not equal to the ideal pseudo control, but it is ideal pseudo control plus some error term actually. So, how do you deal with that so, whole idea is can I design control properly. So, that this delta prime will go to 0 actually and then it when we are actually.

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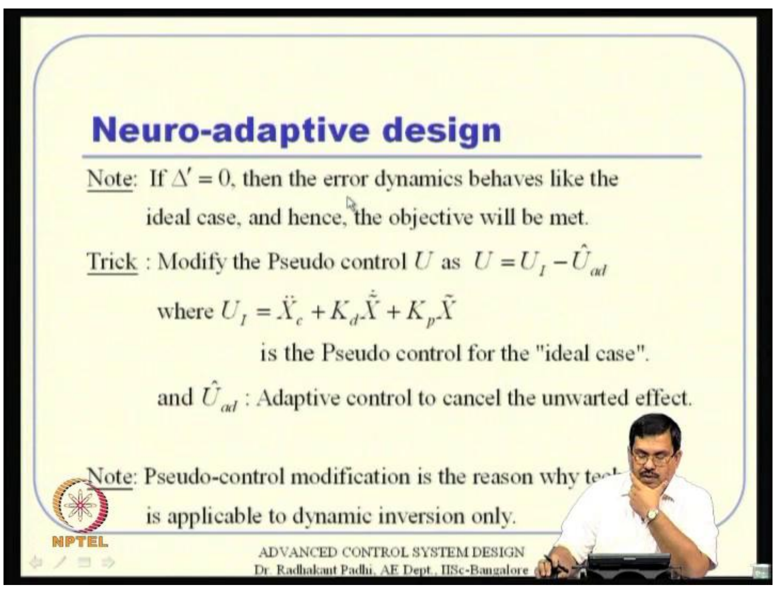

So, just observe some in delta prime is 0 then my X double dot is equal to (( )) fine actually. So, how do you do that so let us talk about this pseudo control value is something like, U I minus U adoptive hat actually that is U I transform the ideal pseudo control ideal case, I like if the system dynamics is perfect and all that I mean those condition. What about one pseudo control is available to us that is that is denoted as U I that is all actually, then this actual pseudo control what I am talking here, is nothing but U I minus another time, that I have been here this is nothing but adoptive part it part of the pseudo control actually.

So, this will become computed adoptively. So, U I is nothing but the in the original case, pseudo thing that ideal pseudo control part, of it gradual case and U a b hat is adoptive control to cancel, the unwanted effect actually. And because this enter operation here in this, if you see that this pseudo control modification is doing on here. And pseudo control is concerned which is a applicable to (( )) inversion only.

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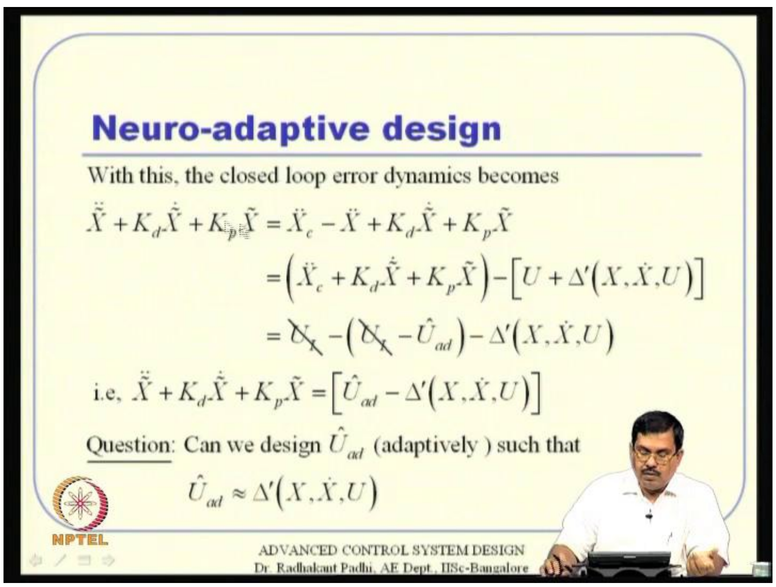

So, the entire philosophy that you talking here in this class is applicable to dynamic inversion only actually, it does not probably contain this sufficient general it is to apply to apply to any other design actually. So, the method that will talk in the next class will have that that general property basically, any way conduct to this is what you want to. So, U is U I minus U at hat and U I ideal pseudo control U at hat is adoptively, what you can what you want to I mean, compute physically. So, with that, but how is system dynamics where in. So, if I go back to this error dynamics, that is what I want to say it is 0 actually ultimately.

So, error dynamics is some the first term is X e minus I mean the first term is expanded here X e double dot minus X double dot other terms are kept as it is. And this one I will put it here this particular thing, when then this two last to terms I will keep it together. And that is nothing but my ideal pseudo control, that is the by definition that is the my ideal pseudo control. Now, what about rest of the part this particular remember X double dot a nothing but a U plus delta prime that is what you as computed. So, X double dot is something I mean what about I U plus delta prime. So, this is nothing but X double dot. So, this X double dot and substituting is U plus delta prime. And this U is nothing but U I minus U I d hat. So, this U and substitute is U I minus U d hat and this minus will come here actually anyway.

So, if I see this is nothing but my U I this is m I U I minus U hat. And this is n minus delta prime actually. So, what you what is getting operated here. So, this error entire error dynamics is not 0, but this is U a d prime a sorry U a d prime hat minus delta prime. So, ideally what it what it should be the objective what this U I d hat design, that is adoptive control design is to mica equal to delta prime is must possible. So, if it might it equal then I it is 0 the entire remix will be perfect actually. That is the whole K physically here.

So, can we design this adoptive control and such a way, that U hat a hat will be at least approximately equal to delta prime will learn be able to be perfect job, Because any function approximation is also in our perfect in way, because of that will long will be do, but at least convey might it approximately equal to that. So, the entire error dynamics will operate like ideal case, even with the presence of system dynamics doing imperfect inverse is being in perfect in all that actually.

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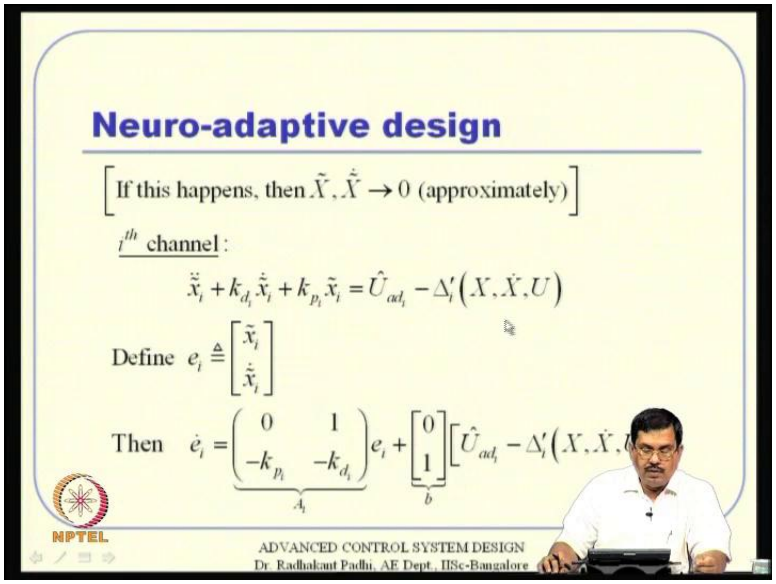

So, that is the objective here now, if this apex if it is approximately equal to that then we know about just will be 0 approximately, can I actual approach something like this right if at apex to be like this, then this will be approximately equal to 0 at least. Now, under the in anyway convey to this, let me again analyse the ith general of this error dynamics, this is the vector. Since and all that let me consider only ith channel and ith channel will appraised something like that, way of this particular term. Now, a scalar component (( )) in this is I mean, again to want to put the small notation an all, because of this follow actually, I mean the paper follows anyway. This is whenever you see I substitute I that means, it is component of that vector.

So, it is a scalar variables so anyway so this is ith channel have a error dynamics, happening in this is all. Now, ith general this is the full vector what that is no ith general of that double actually. Now, this error dynamics I want to represent that in terms of states basic reason let us on. So, then I will define e something like X tilde and X tilde dot standard definition of phase by of full form in all that. And then this e I dot, which is vector equation. Now, two by one vector actually this standard vector form, why you do need that, because we know the Lyapunov theory is at sort of, we want to apply here is demands that system dynamic should been in states form. So, the double derivative forms and all that not applicable with respect to Lyapunov theory that we know actually.

So, we want to convert this into states phase form actually. So, this is the two by one states, for every channel of this error will have a 2 by 1 vector in a states a space actually anyway. So, that is all it is also get remember that because this K d I K p I selected to positive to satisfy, that if it is this is equal to 0 then, this is actually like in ideal error dynamics right. so that means, the K d I and K p I are selected an such a way, that the roots of this characteristic equation are always in the left hand side. An if that happens then this a e I matrix is certainly then is actually. And because that a I comes from that is there in any way, I mean does not come in error, just comes from the way U write this is just expression alright.

So, if you see that does not value of Eigen value look at some sort of a I matrix nothing but the wholes of this characteristic equation anyway here. And that will always this is the error dynamic, that we are selecting and will always select an error dynamics, anyway between's the follows are always in the left hand side actually. So, a I size always 1 is actually, that is that is necessary down the line actually are again talk about that on this entire thing. What you see the writing side is interpreted as some something like, a control input this is the system dynamics here.

So, if I see this X I double dot and the that is what I am writing here, everything here and this entire term is like my single input control. And what you have is 0 1, here that is standard phase state variable forms sort of thing phase variable form.

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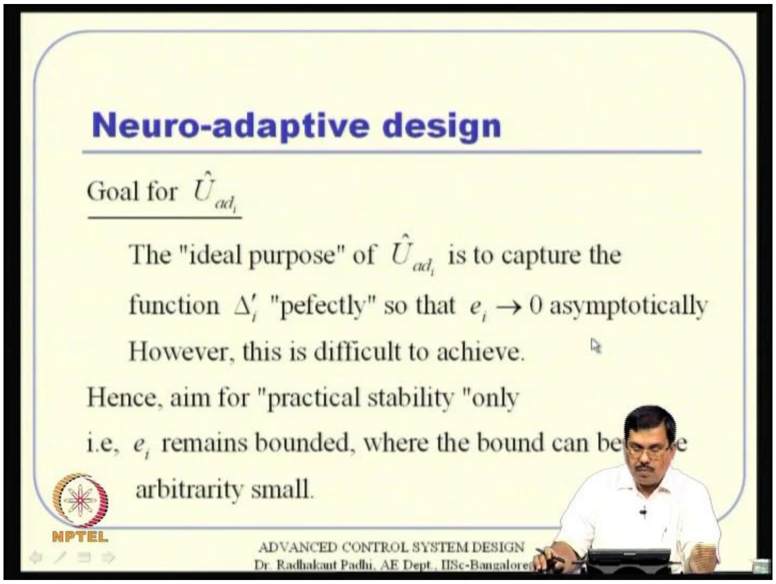

So, ideal purpose go along this U I d I hat a I mean the ideal purpose of that capture the function delta hat perfectly, almost perfectly let say. So, that e I goes to 0 synthetically what this is difficult as. And hence we what is called is practical stability, what do you mean by vertical stability means, e I will remain bounded, what the bound convey made literally small once e I. First of all a should remain small, but we are belonging to left to with the occasion early bounded. And the bound I mean I it leave it what about bound pox of actually, that is not a practical stability the bound can be made small as, we want will now, we going to make a 0, but we will by appropriates selection of design by variables. And all that you can make it small actually and not only that the moment this trajectory, if the state the system dynamics both outside the bound. Then it is granted to come inside the bound it is not whenever, it go it is outside it will come inside I mean that, we write is in is always there. And that non convenient a hat literally small actually, that is what particular stability means actually, that what to lot of engineering problems we are satisfied with that actually alright.

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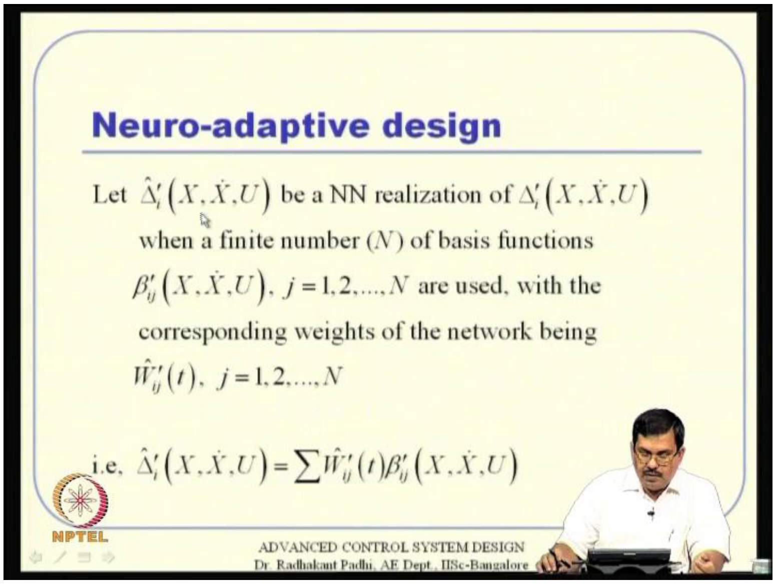

So, if you see this like now (( )) to this will let us consider, delta I prime what you have talking about. Now, the let us talk about a delta I prime hat now, which will nothing but a neural network realization of this function. So, the this different notations are necessary, because we have talking about different things any way. So, the so delta I prime is the ideal error that is there in the system. So, dynamics and delta I prime hat is neural network realize an out that obviously, that been error between the two. They will be like neural network cannot approximate a functional, I mean with 0 error and any function of approximation cannot approximate a function, which is error any way basically.

So, because of that there is a necessary to choose different notations and then talk about a null in that actually. And why is that because will talk about finite number of basis function, if you talk infinite number of basis function, probably you can literally small then error can be 0 and also thing. So, but then will not be able do that and all practical realization, particle implementation will always talk about finite number is because each of basic function, will I have a weight associated with that. And that weight vector also it is a in (( )) see that is U go along go along actually. So, we certain need a finite number of basis vector and because of that this two an error equal, enhance various and of notation actually.

And with respect to this particular thing this neural network, we talk about this basic functions actually, this basic functions that you are going to use is notation wise will define that way, with the corresponding weights of the network will like that. What the meaning of that, meaning is this approximation of this for this function that going talking about can be represented something like that, something like in the ith general it is something like with y get write in that this like j equal to 1. To some finite number on that we are talking about this.

So; that means, a W i prime W i 1 time times beta 1 beta i 1. Let say plus w I two times beta i 2 I would like that it will continue that finite (( ))or thing. And beta's are something that is design lecture, that is basic function or something chose a net priory. And typical one way to select that is Gaussian network here, because the weight set we are linearly, networking about linear in the weight network actually. So, if select this as goes will basis function then weights of where interpreting that is linear in the weight actually.

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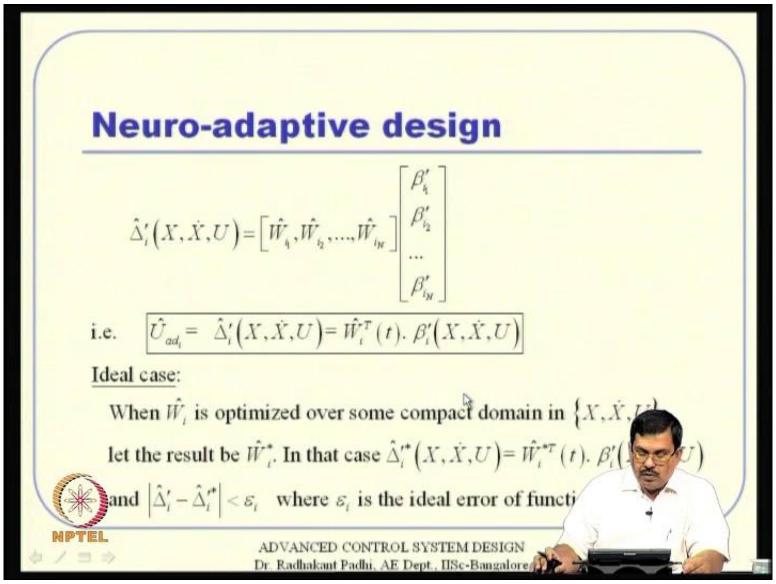

So, the this is, this is how we will interpret. So, in other words you can write it something, what I told this just now, I mean this summation sign can be return in vector matrix way that actually. So, ultimately what you can write here is U a d by I at that particular ith component of that optic control, can be return as W i transpose terms beta i prime with beta prime is the vector at a vector actually. So, ideal case, when W i it is optimized some compact domain neural network, it want typically want compact domain.

And compact domain as we discussion lyapunov theory some time, where it is something like the set is the domain least is to be closed and bounded both actually. Then neural network are approximations are valued actually, the universal function approximation theory. That is there in the neural network is valid only in a compact domain actually. So, that is why it is needed in most of engineering problems, they are anyway compact actually. So, not able to this is actually anyway. So, one W dot is optimized over some compact domain in this, I mean in this at pseudo thing after this optimize in over. Let the result be W i hat star actually.

So, that is the very optimal weight after the training is done, in that sense, what will happen this delta i prime hat star that is notation, that only is this star is nothing but the ideal function, cut of function that is captured more than that you can do. That is the best thing that you can also actually, that is something that goes with W i star that is the optimum weight actually. So, that is how will, I will write it actually then obviously, W i star W i hat star is something that we do not know. And, if we know i m in directly, will selected and that problem is over, we do not need optical lecture actually.

So, the problem I mean a you know that, there exist some weight W i hat star, which is ideal, but the value is something that we do not know. So, obviously, in an adoptive is we want to go there want to approves, that only hat star is not proceeding actually, see in that case, if this delta i hat prime star is a is given like that. And when this error between this two will remain something very small actually.

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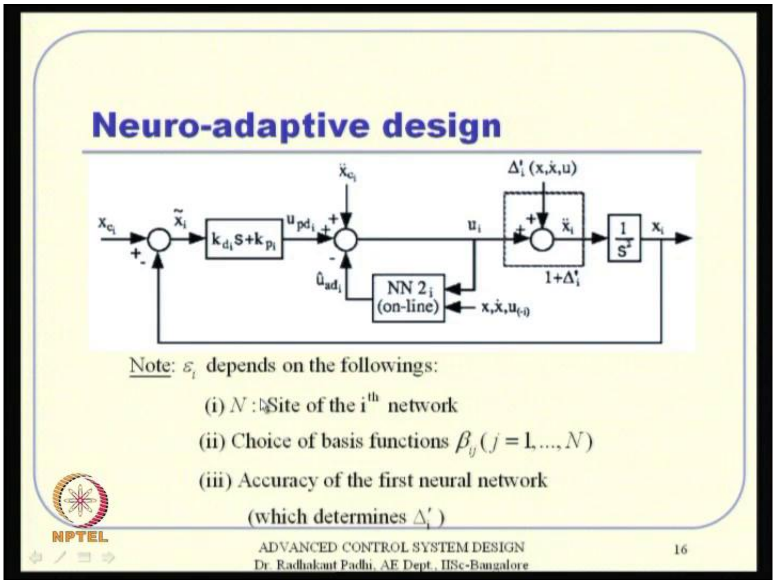

So, that is the ideal error function approximation, that I certain what I talking actually. So, I mean the whole idea the actually, U i d hat is given the something like W i hat transposed transmit i. And ultimately will be able to at the maximum, we can go to this W i hat star. And in that sense will have some function, which is a delta i prime star right. And this two windows will have some errors, in between and with error is the that is the ideal function is approximation error, which are the denoting this epsilon i.

So, will this algorithm some sort of picture, will diagram is something given like this. And I will not discuss to some, but I mean that give this command it thing than, you formulated error then you pass it through that. That function you get a this p d part of it and then you get adaptive control, then will get in subtract both of that X double dot also this one will go through neural network through, which is online this one will go through the adapt on this of line thing with is not. So, on here this will go through and once you one this is realize, will take the appropriate signal. And then positive through the offline signal to get the real control actually.

So, that the part is so on here anyway, that is the function let us just a picture will diagram sort of thing, but you cannot that epsilon i depends on the following that means, when this is a spell errors, spelling errors here it is size the network. Now, let us so N is epsilon i the ith the ideal error actually, depends on size of the ith network, the choice of the basis function. And accuracy of the neural network not required are essentially, the third depend one first to anyway. So, that first to a primary.

So, size of the network and source of the basic function increase that. in the that will ultimately detect how much is general network accurate actually. So, be carefully while selecting in the basis function, that is the message actually is a same thing will repeated. So, will not able to actually alright. So, this epsilon i, that we are talking here there will be talking in the two, will depend on size of the ith network and choice of the basis functions actually alright.

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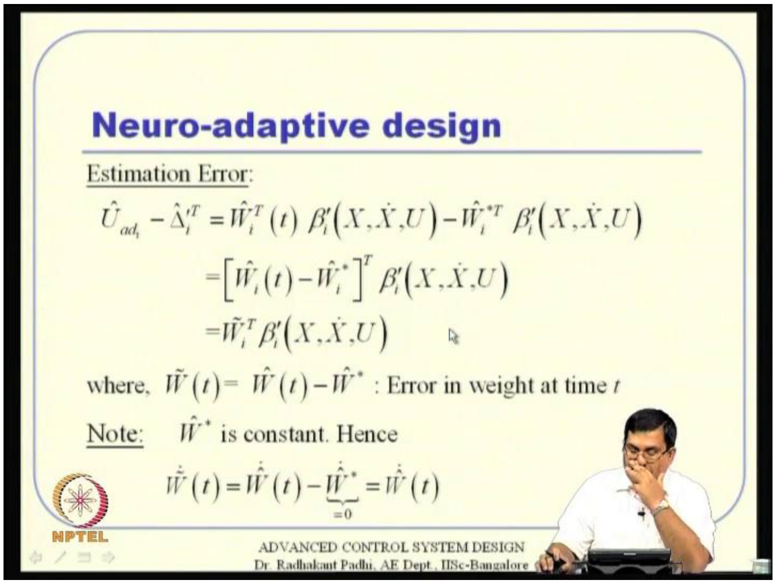

So, now will go back an analyse what is (( )) here. So, what is our estimation error, because this is what ideal in this is what my actually thing, that is aerodynamics not equal to 0, but equal to this quantity now sort of thing. So, the right, but this prime is not there is I mean is this set is not the theirs actually, you have this what here talking size let us analyse this particular term know. Now, if you successive this appear this quantities this one is nothing but the actually bit, this small is nothing but the ideal bit. The basic functions are same any way.

So, I will able to combine the basis in this W i, W i minus W i star here and like this beta i prime, it is quite lecture this is goes of it. So, and tell this error, that I am looking at here in the left hand side is nothing but the error. in the west of the network times basic function actually. And also know that this W tilde, which is error in the weight is nothing but weight minus W is star and this W at star is constant weight actually. So, this does not change with that is actually, because that is the ideal weight in our changes with time. Because the function that, we are talking here is also like the function that we are talking that is static function, the system dynamics system is dynamics that means, X double dot i mean X dot and all that. If the changes will change with test, but the function that, we have talking about in the right hand side, if the equation actually that is the static function in other word the function itself does not change it.

Change a time actually, because of that this function is static and null that is f d here is all static actually. So, that mean double at star is constant way, vector actually. So, if this is constant then that the if I take ten derivative. So, W i tilde that, the that is nothing but the W write in this term, if I see did not before that this particular thing, if I, if you see here this are the this W and beta can be vectors actually, even the this is scalar quantity this will be depends on the size of the I mean network can all that actually. So, that means, there are they are quantity actually. So, if you want you can put i everywhere. So, this is i here and then i everywhere actually.

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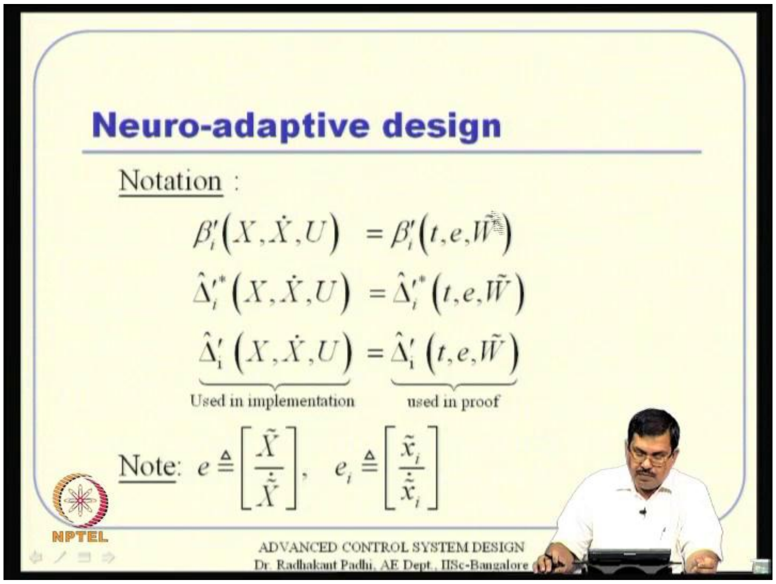

So, here of factor quantities actually for way. So, then when the do not here is well is 0 that is 0. So, that means, W a tilde dot is nothing but that y hat dot that is important will do not important not a actually. So, then this some little bit of change of notation in all the autho here. So, whenever, you appears that quality that whenever, W tilde appears because all something like that actually. So, it is I mean interchangeably that one a one a mean beta I prime it can beta argument of this argument of that. And this is use improve this is implementation, I mean the interchangeably you can use that probably.

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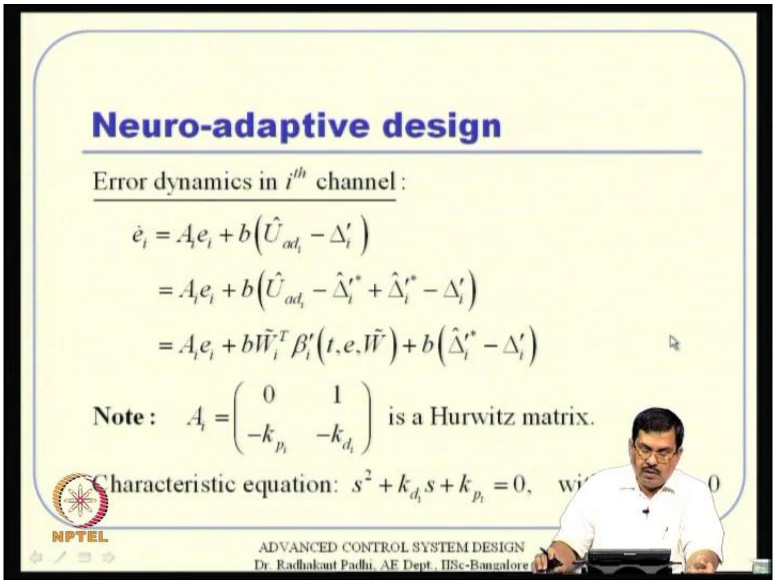

So, the notation wise, if you e can the this is the total vector and then X tilde X tilde dot e i means this is what it tilde. So, their some notation on the way now, in go back to that general error dynamics, that is what critical thing that we stared with. So, error term if you go back this is what the aerodynamics of ith general. So, this where will go back and try to see what is dynamic here actually. So, ith general error dynamics is something like this remember b is as to 0 1 actually. So, that is A i that A i is something that, we define before. Now, I will ideas subtract quantity increase here and then, I will tell look at this is nothing but my system dynamics for the error dynamics part of it actually. And in a more a will as I told that is Ishurwitz matrix, because it the characteristic equation are like that, with that gain thing positive actually.

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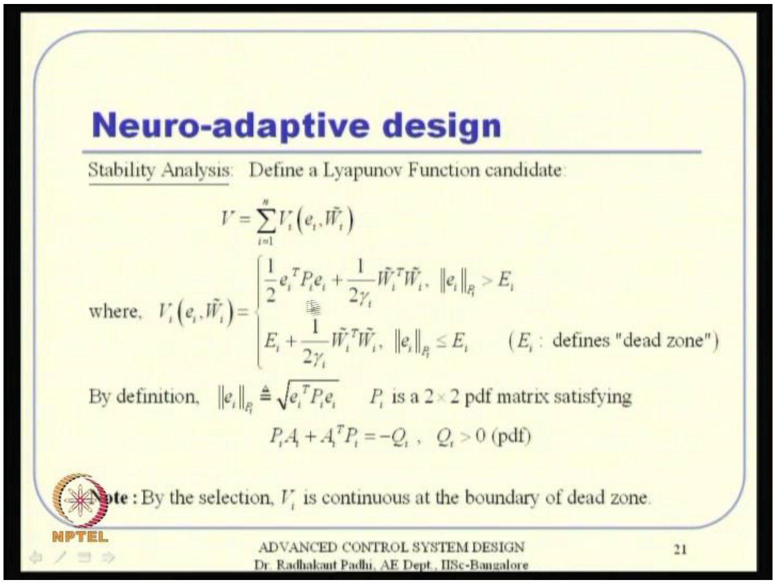

So, certainly going to be a Hurwitz matrix, once will happens and then we tell look a known. Let me try to use this Lyapunov theory and think like that null it is define a lyapunov function, candidate which is summation of over this e i, e i stands for ith generally Lyapunov function V i is lyapunov function for ith error dynamics. So, I will just consider V one plus V two plus V three up to n whatever, I have actually and V i define it something like that, like A if it is define some sort of a dead zone and if there error quantive norm of the error quantity, as defined as pth known, pth known is defined something laser.

So, if the pth known is greater than certain e I then I will take it that way, if it is less than that I will simply consider that part of it I known no more worried about minimize in that error quantity basically, if it is greater than that I will define V i to be like that, if less than that I will simply consider that part of it each. And also remove that by design, this is a actually continues V i is actually a continues concern at the boundary. Actually, if you consider that what happens equal to the this E i suppose, actually suppose e I means that is what it is actually write in. So, if it is the like that is that. So, when where in a if you in a limiting sense, when we have to go to 0 sort of thing then it will 1 that will happen, when V i approaches this boundary of this domain then obviously, it will that. So, that this continuity there is actually by design highlight.

So, now what is this P i that we are talking about this P i is in a positive definite matrix a which is actually a solution of this Lyapunov equation. And this lyapunov equation will admit a positive definite matrix, because A i is of this lets one thing, that we discuss in the Lyapunov theory class right it. In as of Lyapunov theory like e P i is stable matrix sort of thing, I call is matrix then this lyapunov person is given to admit posture definite solution for P for any positive Hurwitz matrix Q.

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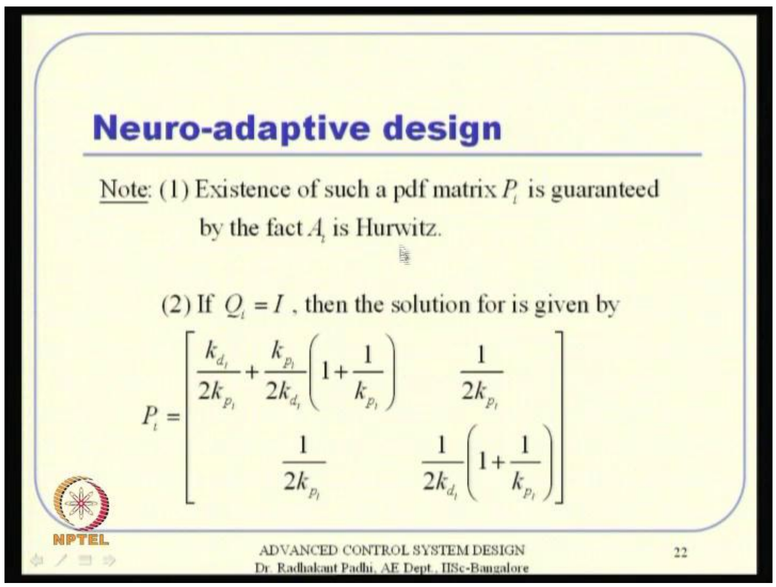

So, that is the theorem that we studied there actually. So, as you told the by the selection we are is continuous are the boundary of the dead zone so, it is fine actually. So, will considered this will this V i something like this where P i is the solution of lyapunov function for positive definite Q I. So, that is why told existence of p d f is granted by the fact that a is Hurwitz and is Q A i equal to identity give just output K i equal to identity A I is something that, you know already right A i something like this. So, I can actually some P i this size symbolically, if I just could identity here and put this this thing here A i is that. So, if you essentially it way to linear equations.

So, it to linear equation, I will a lorenz component by component and light weight to solve for few variable actually, what are peace are there. And know it is actually, if P is it freedom will be three actually, not two in case of I mean this will symmetric matrix with this value are same, in and this two will be different actually. So, two will contain it is (( )) is two by two matrix; that means, four variables, but a taking it is symmetric matrix it will three variable actually. So, 3 equations and all will be available will able to solve it in a cross form get it in way actually. And you can introduce a verify that, if you take any positive numbers for K b K p then this matrix is a certainly guaranteed to be posture definite, we can calculate the Eigen values and tell when there is Eigen values will remain first actually. And select that is not two started with lyapunov function candidate, now let us analyse this is the rate time rate also, that means, V i dot actually.

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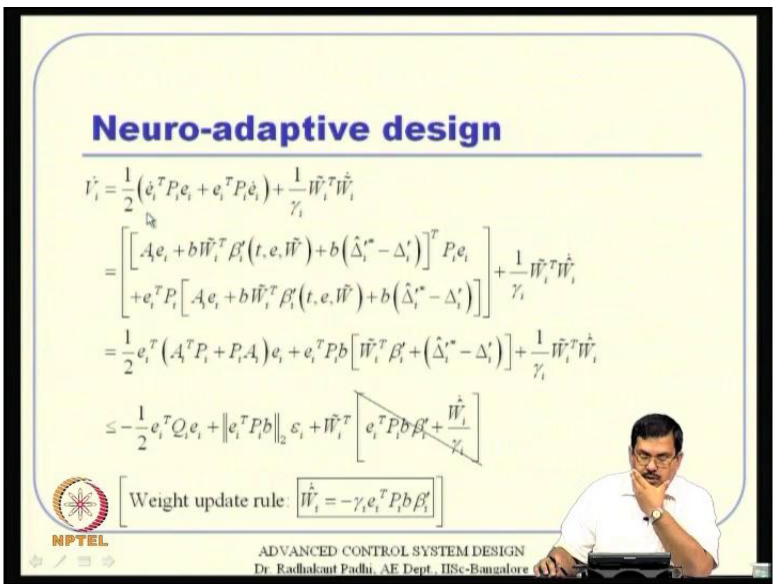

So, if you take V i dot then to this dynamics in general and the started let me the derivative, it is just write it in side in all that. So, that will become V i V i dot transpose P i e i plus (( )) actually goers with an that. And let take it through all the derivation, because it is anyway, there then you substitute is for e i dot here all the term, that you hat for e i dot take transpose of that. And put e i dot I mean this e i expression again may sorry A i dot expression, again in the right hand said of it. This A i dot is something that, we derived here this expression ok.

So, we could it here and then, we analyse is what is going on it try to combine terms it makes is possible. And this past took them here in first here them combine this term box, when this is nothing but I selected it e i in such a way, that the lyapunov equation satisfied that means, this term we nothing but and you get that quantity, basically right left know equation is that. So, instead of that I will be substitute minus Q I. So, I will bale to do that each other. So, whatever terms are remaining is something like this out of each, I will keep something like U i epsilon i actually, epsilon i is a something like an like in error quantity which will confirm this quantity actually, if you see this I think derive it tell. So, that write n is a epsilon i somewhere, this this two term we will will be less than equal to epsilon i.

So, that will being on there this epsilon i is coming here and then this should W i this tilde transfer appears tilde here. And batch of appears to the left here sort of thing, because of this quantity is square quantity. So, this quantity I this taken to be there a just quantity. So, I will be able to proceed to the if I want and I will do when will take this one common W A tilde transpose an then this term here. And this is not do the after this is equal and certain this from here and here is it is less than equal to as, I introduce state norm quantity here ideally. If you do not like than non-part is equal, what I want to make and less than equal to with introduce, where introducing a known quantity.

Now, here is one more problem is something like, turn that will multiplied with W i tilde and W i tilde is consist of both W i hat and W i star. And W i star is something that, we do not know actually. So, until tilde is no something, I will not be able to tell what is my W at this quantity whether, it is positive negative I will not be able to anything actually. So, to aware the difficulty what we do is is just put it equal to 0 actually. So, the coefficient part of it we just make it equal to 0 lets. So, if the coefficient is equal to 0 at gives you a right product gives you some sort something like a learning, I mean near network learning equation actually, that means, W i hat that in a nothing but that. So, that will write that one approach for dating main weights is a nothing but weight update rules sort of thing. So, that is our derive my update rule, but even I, even if I say that this coefficient 0 and stilled after this two term. So, I to analyse that captured.

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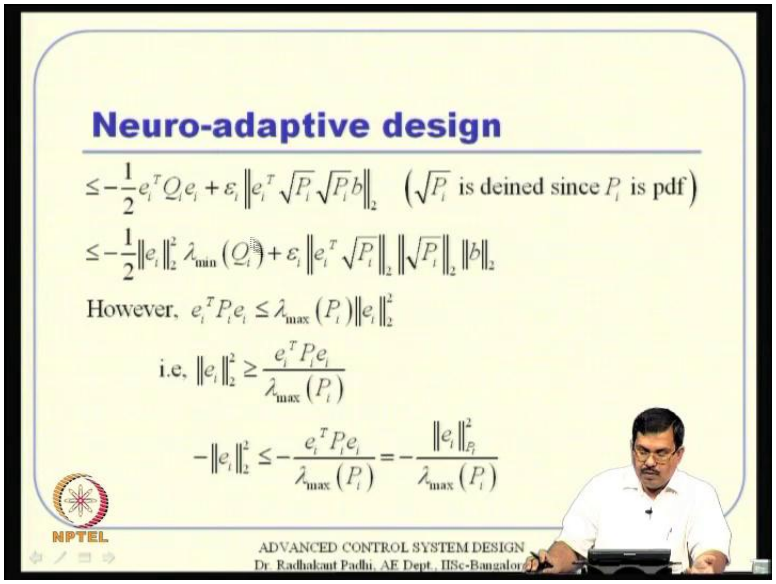

So, this is left of that further analysis will be done that way it more P i and P i is positive definite matrix. So, this is definition of square root and square root can be define, because there has been here and once you do that, when this particular inequality can be define something like, if you tell this particular quantity is less than equal to this quantity, because the any posture definite matrix is bounded between this inequality, we (( )) are inequality matrix theory basically. So, you invite that and then we try to speak of this norms and all that, that will be that way see this is way, this is also less than this particular term. So, we look that turn also at then tell because this learn that is here is that is in equal to that quantity.

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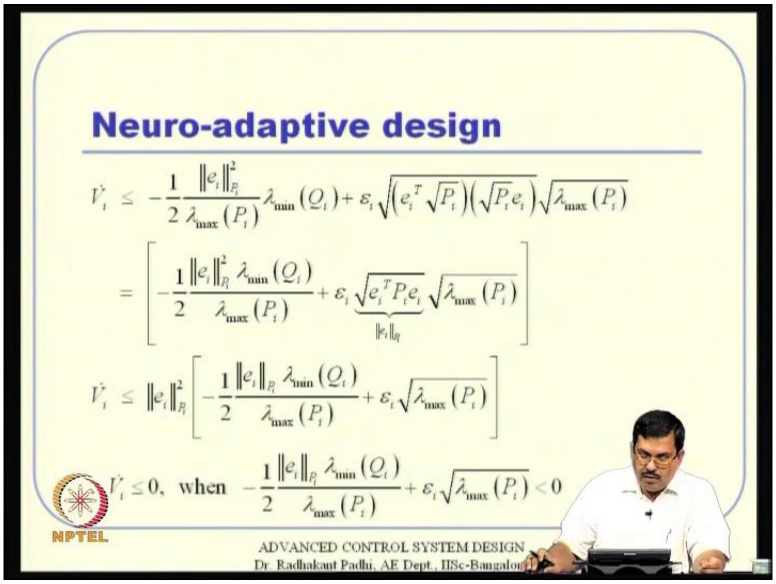

So, carry of this long in algebra it here in that in the sense of this real inequality matrix norm all that. And then we tell V i dot, we less than equal to particular quantity, it I mean at that feels little bit involve, but what it is not you certain with the equations actually, you do it one by one term and I will try to understand one equation is. Turn is long with that difficult. So, in may know all that going on here is this is airline equality applied here in the negative sense, if you tell that and it is replied here in positive sense actually. So, both the terms it is applied an then this square root is split out and then you do that algebra here on do, if you ultimately what it answered this we dot is less than equal to certain quantity like that here actually.

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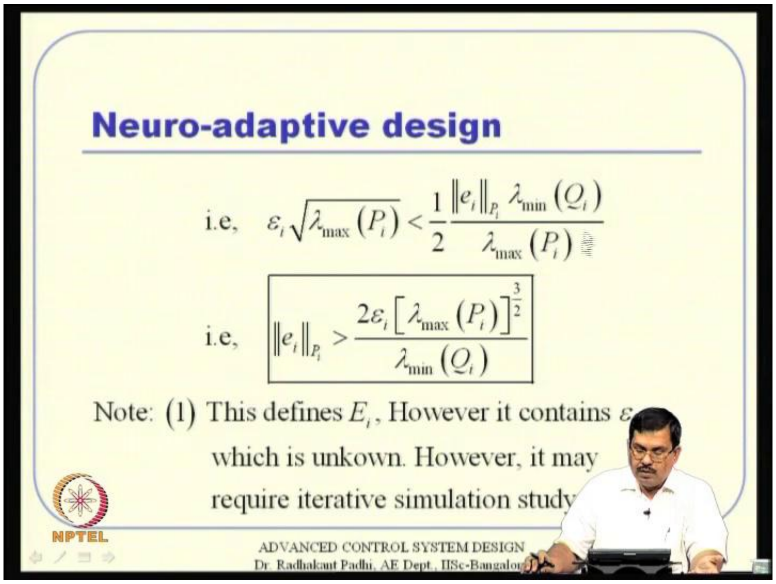

So, with that there is a positive term here and negative term here. So, we are that will be less than equal to 0 whenever, this quantity less than equal to 0 that is the condition that we have to analyse whether this condition is makes meaning full sensor not actually. Universally, it long we definite, if I negative definite than you have done what it long like that the and of this condition. This will be negative less than equal to zero. So, this condition when you analyse this condition gives us a very nice condition, we tells us that norm of you is greater than equal to this quantity then way it is then equal to 0. And this long is a actually, an error when we said quantities sort of think that also been lot of think. So, what it tells as long as this norm of the error amplifies a error grows, we are certain value which certified like this V i dot will be negative it will try to pool it back inside actually.

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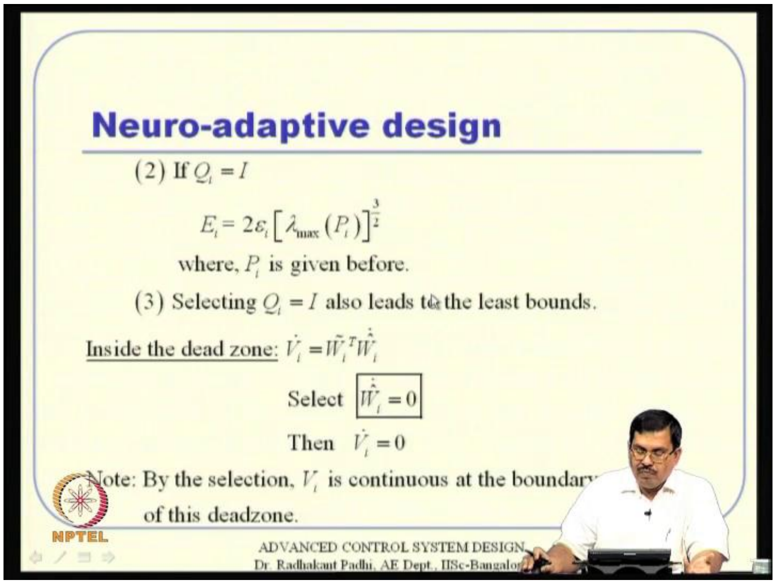

What the problem, where is this a this entire quantity become contains this epsilon item which is unknown actually. If I know then I have, I can compute this thing or because of the I will try to have do it is sunset of iterative way. And it where I it a proper value for this when we write it select one and then if you take it is the done actually, little less than that like that actually highlight. Now, coming it to Q i is equal to I, when this A i quantity this is a basically, this definition reference this quantity is something defined as A i. So, if this way what you see here, if K i is equal to the this A i turn out to be like that, because this Q i lambda a minimum of Q i is 1, when way Q i is identity means, all does not values 1. So, that is that is all of in actually and denominator 1.

So, will let out to be actually where is a solution of that equation is given before actually. So, also then this theorem newspaper as tells us that Q i equal to identity is not just the simplication of algebra it also leads to the least bound. That is what we are interested in actually as the simplification algebra and light, but it also. So, leads to the least bound in the sense of this bound whatever, bound where soon here that will be minimum, when we select Q i is identity this on the powerful result actually than inside the (( )) all that happens, when this one we started here like that the analysing this particular term.

Now, within that what you do would within, if you takes to be insert that in then the it will selecting as 0 actually, that the W i that dot equal to 0 that means, there no change of eta after that. So, that is what we have doing here then, we have dot will be equal to 0 which is so further, in a further will minimization will happen further, we need further error remain minimize not happen for at least it learn grow actually. We have dot will except to be 0 actually.

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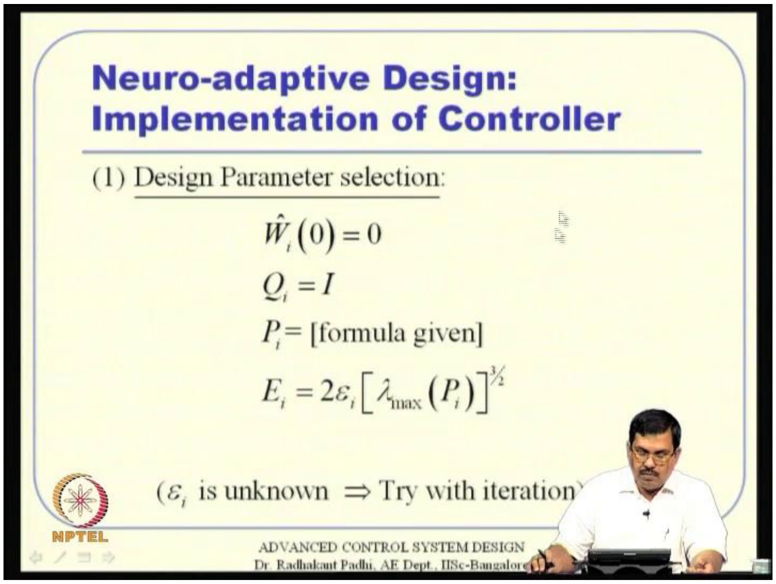

And say what you implementation of this summer is sort of I think in carrier to select this design parameter, these are the design parameters actually, W dot all that expression here derive some square of a W i hat dot actually that means, this is differential equation this is the differential equations. So, you also certainly needs some sort of initial condition to integrate this equation and that initial condition is typically take then a 0. And that actually, is also good in the sense and suppose, they are known imperfection anywhere then we really do not need to exact this control there optic control will not be a excited, if the everything is perfectly known and all that then it will be 0, it will be 0 actually.

This is 0 and that will also turn out to be 0, this equation is a everything equal is ideal then e will be 0 and W i dot will be 0 everywhere increase. So, start will zero will remain a 0 actually, that is another thing any way does not have initially, in the W i hat to be 0. For every channel I remember I is the ith channel of the system dynamic and all Q i, we take is identity P i is a we evaluate within that formula. That the formula that, we discuss actually, this the formula you evaluate that you write it is this is valid the Q i equal to identity. So, will be able to that evaluate that and then this a is given something like that which is unknown.

So, will try with the iteration actually, try with the epsilon some value think that with that is a primarily for after analysis basically anyway. So, the weight update tells like this. So, this is weight update rule will apply along this is this equation, it is specified not pth P i norm something depends norm something like this, like I define this p known is defined this square root e I transpose p I is basically. If that is the that quantity that in the quantity that certain value this E i this E i weight guess remember. So, it is selected that an then you apply the that is that you adopt rule I V i it is 0 actually.

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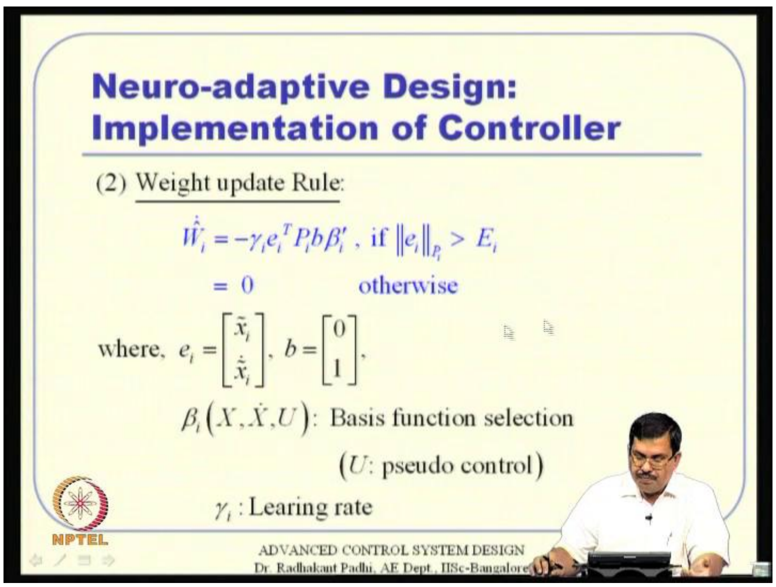

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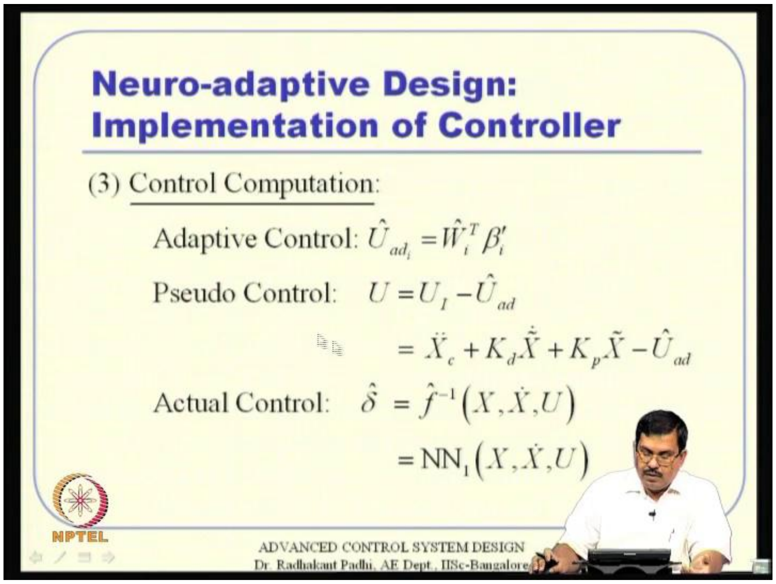

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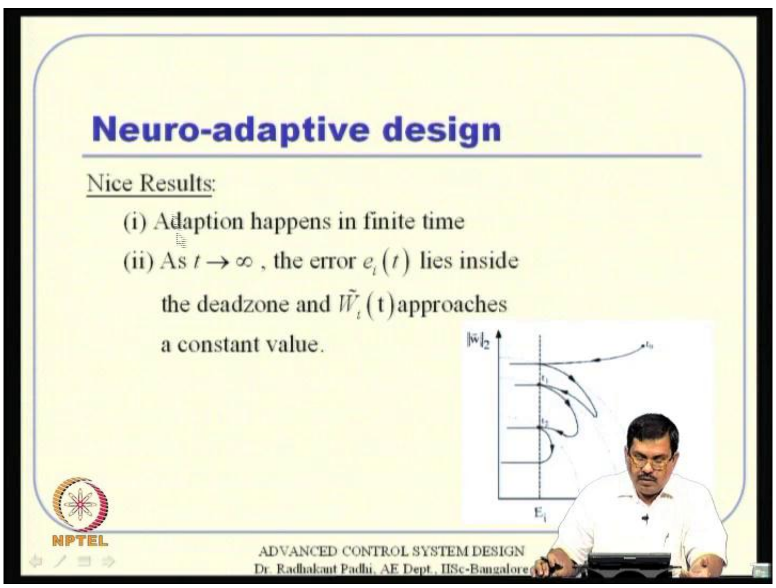

So, this is basis function that, we have to select this is again this I very was prime also, try prime it is a we have to select this beta I this is basic function, this gamma should be learning it is also be selected actually. Then count E i compare the this adoptive control, that way we have to had this adoptive control part of you. And then you control this pseudo control, which you like this we all define on the way discussion. And the actual control is to be like that, each a universal that is given like that is how, we complete actually.

And you nice result also be even in the something like that, the adaption happens in finite time it does not take for a over go, I does not happening take infinite time adapt and also remember that there as to be still by infinity, we lie insert the dead zone this is something lecture on given some sort of dead zone here definition in all that. So, this is form of a E i P i norm this is the so E i put a particular like this dead zone actually pseudo thing. So, it is ultimately weight to be dead zone and it approaches a constant value actually.

So, what do you what mean it is if pictorially speaking that been actually, it is some value initially dose been actually, it is some initially, easily go here then by for some reason of a come side, then it value and if is a some is a beset level it goes a level on that and they also. So, many type that every time it concert of base way it will always apprises to normal value, it level to go back to the no more value actually and the resonal value it will keep, when you go side it will enter to that dead zone that with the smaller value that actually. So, that is another nice part of actually.

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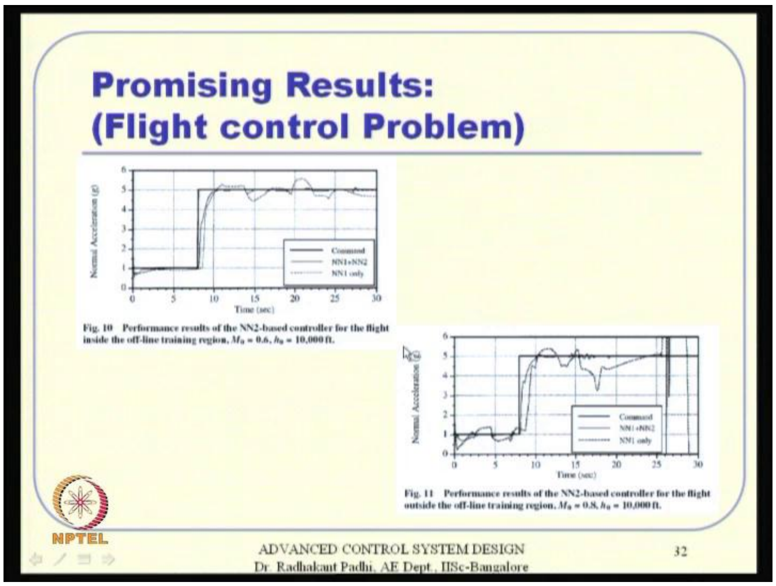

So, certain nice thing that this actually. Now, represented the result sense you see the without detractive control this will have to very well, those this command it is signal sort of thing, but with adaptive control in case, this one (( )) to neural network to that is adaptive control part of it this various, command value it terms any derivation at the remain this plant is something different. And all it try's to adopt it very quickly rather and same thing happens also, here is a then the varied between this (( )) and this particular thing is a small whereas, if you dot of the adoptive control this is going variable pictorially.

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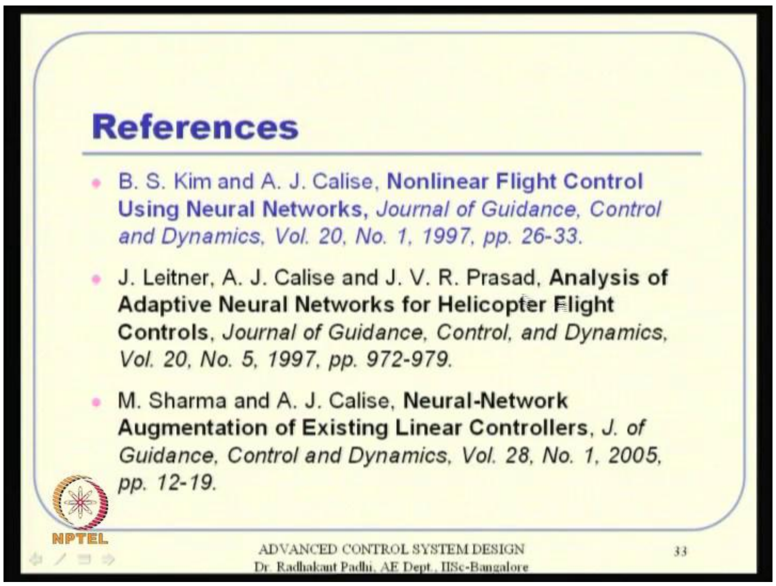

And several results are with the remember of that a told before actually, this is just not this as to represented thing for you to see and what it as you can always, see from the before actually, we are the references primarily whereas, first reference we are told you, but they are also like, when you are which tell you how do you use this theory along with the like linear controllers for example. So, this the vapour that will give you this is another helicopter flight control, in the as a sent brief sort of think or most applying the sent theory sort of thing. So, if you see variety of problem solve again and they we are produce verification program, they are in this reconfigurable control. So, pseudo control dead zone all pseudo control available actually. So, encourage is to it some literature and get comfortable with this, I will stop this class. Thank you.