Advanced Control System Design
Prof. Radhakant Padhi
Department of Aerospace Engineering
Indian Institute of Science Bangalore

Lecture No. # 34
Dynamic Inversion-1

Hello everybody; we will continue with our series on control design and previously we have seen some linear control design techniques and then, applications of that for aerospace application I mean, autopilot design and things like that. So, towards end of this course we will also want to see some of the non-linear control design techniques and to from beginning of this class to rest of the course, we will study mainly on some of the non-linear control design aspect actually.

So, this particular lecture I will take you through what is called gain scheduling and followed by dynamic inversion and then, we will continue further from there. So, let us see this non-linear control design techniques are sometimes I mean, most of the time non-linear control design becomes a necessity and even if you do linear control design then you make it operate like a non-linear control design by invoking the philosophy of gain scheduling. Otherwise, you directly go for some non-linear control design technique either based on Lyapunov theory or based on feedback linearization or dynamic inversion think like that. So, we will see some of those concepts as we go along actually.

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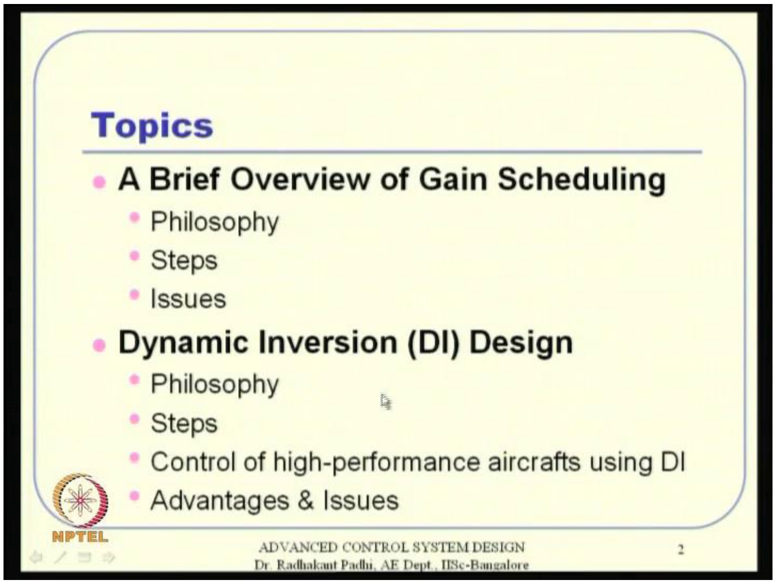

So, let us see first gain scheduling and the topics that we need to discuss today is something like that primarily we will be looking at this philosophy of gain scheduling and some of the steps and issues followed by dynamic inversion design; that is also philosophy, steps like issues and all that and that probably one of the classes later we will also see that some of this like high performance aircraft. That is, like fighter aircraft control design using dynamic inversion so that we will also see not necessarily this class but, somewhat later on actually and then we will see advantages and issues, how this design actually.

So, let us talk; I mean, study first this gain scheduling and it is not a design approach for say but, it is a philosophy actually and that is for making any linear control design work for any non-linear systems that is probably a necessity. So, we see this philosophy and that is a very much accepted practice in industry as well including aerospace industry. So, what is the idea there? Idea is kind of simple; what you have here is let us say you define some sort of a operating point.

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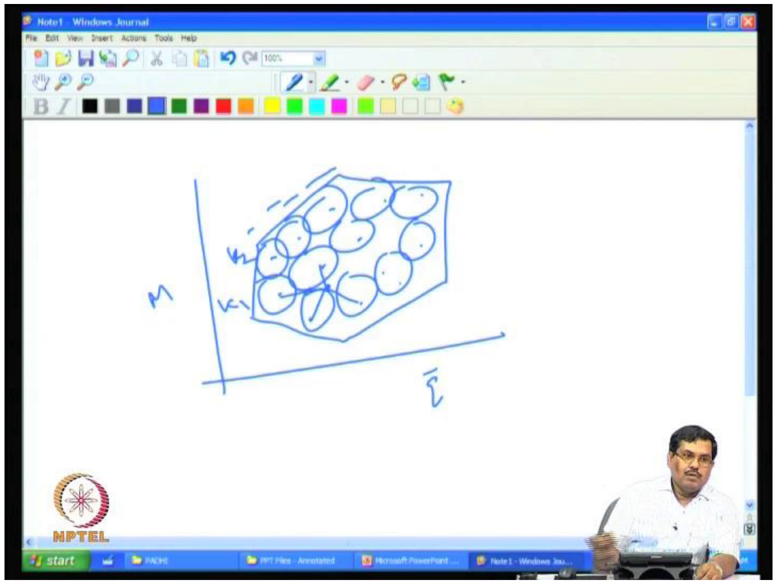

Let us say for example, you there are some flight variable let us say something like dynamic pressure as well as well as Mach number.

So, your aircraft or flying object has to follow certain let us say, some object some domain of operation basically. So, in this domain it should fly; what you do here is like various operating point, you take various operating point throughout the region depending on your design approach and all that and because, this like about each of the operating point you linearize and linearization is locally valid. We know that in the local neighborhood you design different gains. In other words, let us say for this one this will be k 1 this is for k 2 and think like that. So, everywhere you design local linear controls based on local linear plans actually.

Then, you stretch them up using some sort of a interpolation routine. Now, if you have this kind of a interpolation that means you are invoking this Mach number end and you prefer as free variables means. That means, it becomes like a two dimensional interpolation actually. So, anywhere to anywhere you invoke suppose your operating somewhere, here it should see this neighboring gains and based on the neighboring gains it should try to interpolate. Actually, what is the gain should be at that point of time? That way you will we will be able to cover the entire domain actually. So, that is the philosophy of a gain scheduling and that is what is written here.

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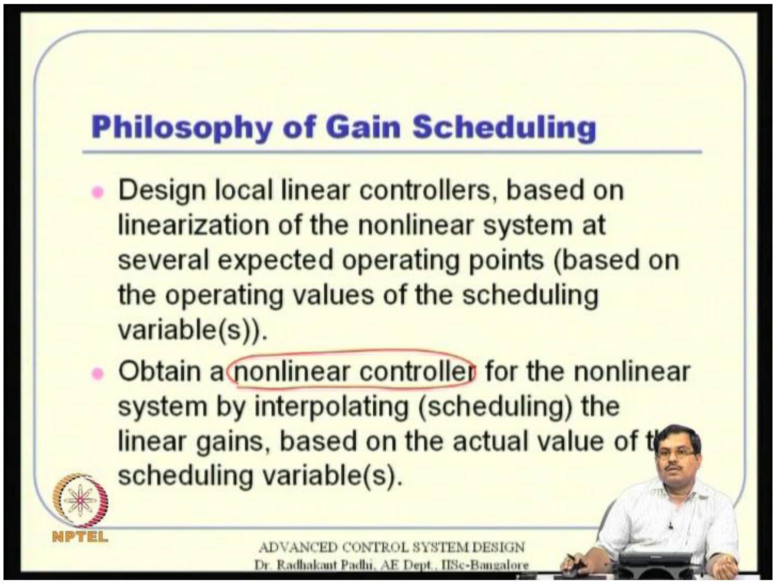

So, first what you do is design a local plants and then design local linear controllers based on those linearization of non-linear system at several expected operating point based on the operating values of this scheduling variable and remember this scheduling variable or the interpolating variable need not be gained they need not be straight vector at all actually because state vector is high dimensional and we do not want to interpolate the gain based on a very high dimensional I mean, interpolation.

So, what you do you normally? Select some sort of meaningful output variable and in this case Mach number in dynamic pressure are very much meaningful aerodynamic parameters. Based on that we try to interpolate and we want to take this these interpolating or in other words meaningful output variables as much low dimensional as possible. Actually, so we if possible just one variable or may be two variable maximum three variable like that which is tightly coupled with a system dynamics number one and number two which is which has to be meaningful I mean, you cannot talk about something which is the which is independent of the physics actually.

One important condition that you need to see is the scheduling variable has be slow varying. Actually, it should not be very fast varying because, you do not want to excite this interpolation based on a rapid change of variables and all that actually. So, it should be physically meaningful as well as the quantities should preferably be slow varying and we all know that Mach number and dynamic inversion are kind of slow varying quantities actually.

Even for a fighter aircraft these are like the vehicle can maneuver that is a different issue but, as long as the Mach number and dynamic pressure these are functions of altitude and vehicle velocity for say so for those things are kind of slow varying quantities actually so that is how it is shaken there so coming back this is what we need to do design local linear controllers based on linearization of non-linear system at several expected operating points and then, obtain a non-linear controller remember this what you are really looking at is actually a non-linear controller design with ultimately when you interpolate again becomes only I mean, the controller becomes non-linear actually. So, we obtain a non-linear controller for the non-linear system by interpolation or that interpolation interpolating algorithm is nothing but, this scheduling algorithm actually. So, at any point of any operating point we should have actual value of those scheduling variable and the grid point data is already available with us. So, based on the grid point data we try to interpolate and kind of get the control gain actually.

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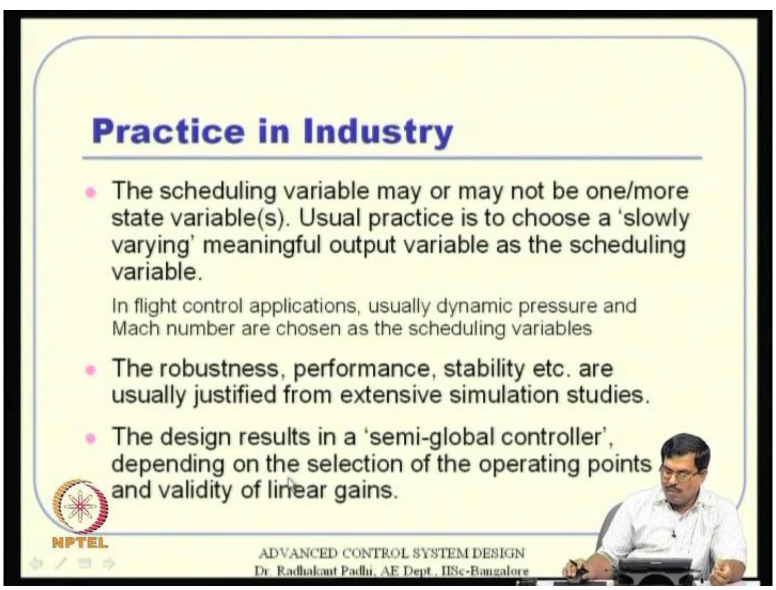

So, that is the philosophy of gain scheduling. This is something what I have already told what you what is the usual practice in industry the scheduling variable may or may not be one or more state variables. So, we do not really go for kind of state variable interpolation actually. The usual practice is to choose slowly varying meaningful output variables as the scheduling variable and as I told in the flight control applications, dynamic pressure and Mach number are typically the accepted thing. But, they are very tightly related to physics and they are also slowly varying actually.

Now, the problem is like the gain values that you are selecting the individual gain values that you are selecting those are I mean, they can be selected based on like robust control phenomena or whatever you want to do actually. But, there the robustness is with respect to the linear plant or rather linearize plant; so the moment you interpolate these gains the interpolated gain need not have all those robustness duties actually. So, what you do even the performance condition may not be met simply because of linear I mean, interpolation.

So, because of this difficulty and what is typically done is the robustness, performance, stability, etcetera, are usually justified from extensive simulation studies. So, this is what is practiced in industry actually but, any way having said that after you do all this what it really results is a semi global controller because, nobody restricts us from enlarging the domain as and I mean, if you can build up your data base in a big way so you can keep on expanding your domain actually.

That way it is what is like semi global control it is not really a global controller for say may not be everywhere but, you can keep on expanding your domain. If you have a good amount of memory and good amount of I mean, computational efficient interpolation scheme actually because interpolation need not be computational efficient either the moment you have large amount of data table to locate where you are in the data table itself takes lot of time actually. So, even the… I mean, even though philosophically it is possible to expand this domain infinite sense you may not be able to do that in practice actually.

So, essentially what it results in is a semi global controller depending on the selection of the operating points and validity of linear gains. Also, if really and the problem here is a we cannot be able to predict a priory the sub domain in which the usual gains will operate actually because sometimes the domain can be small sometimes can be large. The individual balls around those operating points, the size of the balls all kind of not known to us; a priory that is a difficult actually; so anyway that is how it is done in industry anyway.

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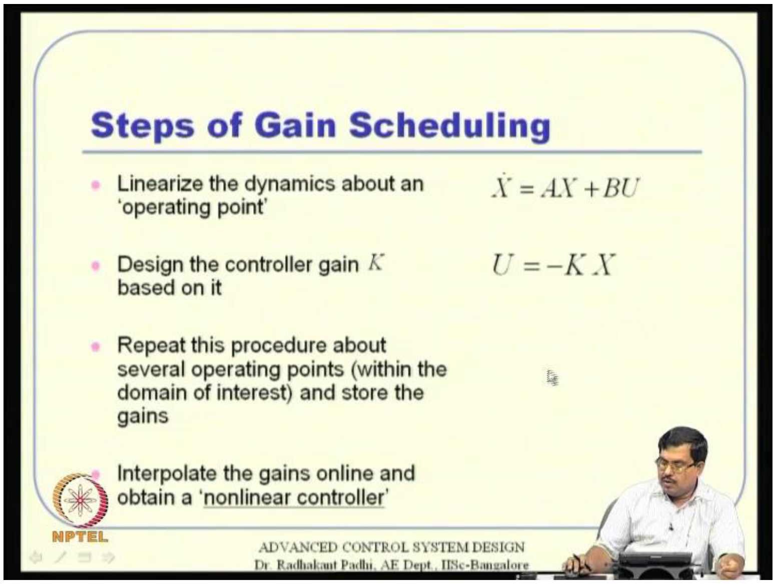

So, steps of gain scheduling we have already told you. We have to linearize the system dynamics about some of operating point and then design the control gain based on that linearize plant actually you can use formulation you can use LQR you can use gain expression you can use I mean, you can use robust controller theory anything actually whatever you want to do so based on this linearize plant you design a controller gain actually.

Then, you have repeat this procedure about several operating point as I describe those the diagram actually and you have to obviously store those gains and memory so online what you have do is you interpolate the gains store gains whatever you have in the memory and then essentially you get an a non-linear controller that is what it is one actually so where as the issues I mean, the advantage is a very clear I mean, we do not need whole lot of non-linear control design technique theory we just relay on our knowledge on linear system design, linear, non linear control design theory and then, validity of this performance robustness and all we do it from simulation studies and that is the advantage actually. So, the

I mean, this too much of a theoretical knowledge is not required and not too much of problems that are associated with non-linear control design or also not non-linear itself gives us lot of advantage. But, is also throw some sort of difficulties say so those are also not of a primary concern to the designer actually because, designer does not have to do all that actually that way. What are the issues? There are several issues actually in this design that is why we need to study non-linear control design as a non-linear control system design.

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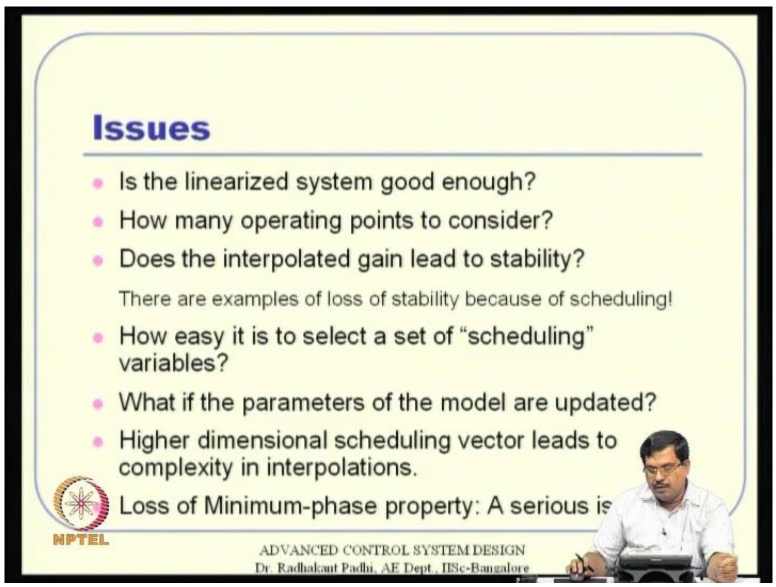

So, what is this? What are the issues and this rather simplistic approach first thing is a linearized system good enough I mean, that means what I told you before about each individual point. What is the size of the ball, we do not know actually? Sometimes, it can be big, sometime small whether we really cover; we are really covering the entire domain or not, we do not know actually. So, there may be some sort of wholes on the way that means singularity on the way but, the gains are not sufficient it may not stable plan think like that actually so that is a big concern. What is the second one? Obviously, how many operating points to consider anybody tell us?

So, even at the given this particular plant you may select best number of points you give another plant you may select high number of points think like hat actually. How many operating points to consider I mean, that become certainly working mode kind of working proposition actually. So, that is the interpolating gain lead to stability may not be so and there are clear examples of loss of stability because, of stabling where the individual gains where good but, once you study it loss stability actually; so that there are clear examples for that actually.

That is a major concern for because of the because of its we need to do several loss of simulation different random simulation what is properly call a multi control simulation actually we need to do lot of simulation like that you have confidence on the design basically and that is as in that may or may not be necessary in the non-linear control design technique actually next issue how is you select a scheduling variable so that means in flight control application especially in stable aircraft design and all people have done so much of study if and come up with Mach number and dynamic pressure in scheduling variable about that is related to physics and all in general. That means, very difficult to do for other system actually; somebody else do lot of study to find good scheduling variables. That will work actually because, you are not the interpolating the gains based on the state variable there interpolating based on some input output variable which is much lesser dimensions actually that may not be able to capture the physics of the point actually; that is the problem. So, the next big issue is, what is the parameters of the model are updated? And, this typically happens when you go for a design change actually like if you go for different design of the system all together; for example, like wind configuration. You want to modify little bit like bigger sixe smaller size of the that angle and think like that. Whatever we talked about then the system is different; so with respect to different we will have different parameters.

Now, if you linearize linear system design then everywhere the heavy matrices are changed because the original non-linear system is different now I have to go back to that I mean, almost like around 0 and then you talk about designing the gains again and then interpolating the gains and think like that so that is a very commercial process in general actually so we do not need to do that if possible I mean, we want to kind of 3 4 2 avoid that actually so this parameter update in the model or the some plant dynamics to get in change becomes a big issue as per design cycle is concerned actually it takes lot of time to retain at again when as I told high dimensional scheduling with needs to complexity in interpolation.

So, do not think that is also we deal of interpolation lets go for like state variable interpolation. That is all not very good idea at all actually. As you multi dimensional interpolation goes exponentially as you as the dimensionality increases actually. So, one dimensional interpolation you just need that two neighbors interpolate two dimensional interpolation that means 4 variables interpolate three dimensional interpolation in day variables interpolating like that actually then never inward points will be it I mean, the two power and sort of thing.

So, we do not need to keep on expanding the number of three variables like that way so if you really like for example, if you have like system of equation or where 12 threads there and think like that way when if you are very let us do it 12 dimension interpolation directly then the number of grid neighboring grid points that will see for interpolation is 2 the power 12 actually that is a very big number to work with an realistic system actually so that is may be big issue there actually and there is a very serious issue what is call loss of non minimum phase property and loss of minimum phase property. That means, it lead to this non minimum phase system for which the so called the other dynamics you will go unstable or this is the, if the system is of non minimum phase then 0 dynamics or the internal dynamics will go unstable and all that actually; so that concept we will see in the dynamic inversion anyway.

So if you are designing a tracking controller for example, pilot comma and implementation is nothing but, tracking controller pilot this comma and they will control system so track that whatever it wants implement. So, in this situation may be tracking itself will be was suppose you want to claim a height claim is probably but, the later dynamics may go unstable actually so those kind of situation are not very nice and even if I tracking is good that minimum phase property and non minimum phase property think like that will have reverse response in the beginning and this reverse response in the beginning is a very great for the pilot actually because it loss handling quality properties actually so pilot will feel that is the system is a if is not waving the weight it wants to be have actually anyway so those are many issues there are many many issues because of this an this gain scheduling is not a universal design we certainly need better approaches than these to kind of give some better design tools for the and then nears actually.

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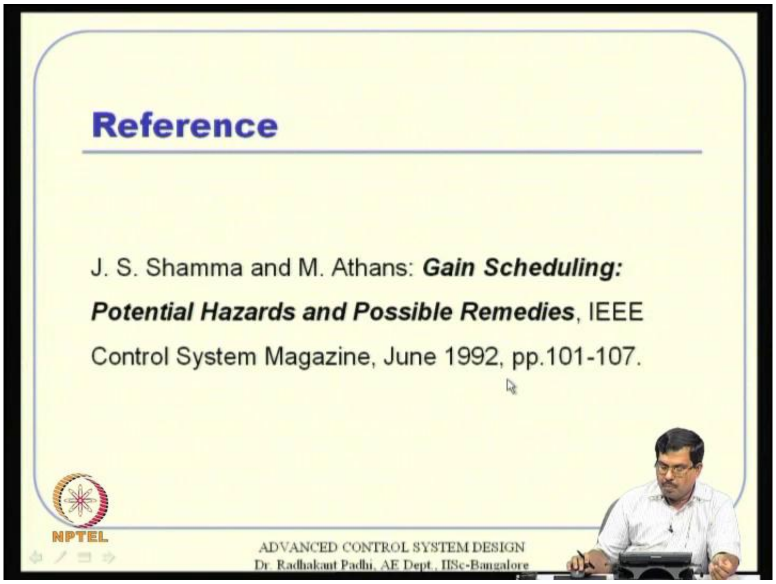

So more on this gain scheduling probably you can read from this is actually high typically control system otherwise it will taken so it is not very difficult to read the but, was very violent for anybody kind of follow it and there are examples to show demonstrate while gain scheduling is a effective in some cases not effective in other cases like that actually so my suggestion is to head of read this and then have comfort feeling so because there the all these issues and all what we really wanted is some sort of substitute design and that is what we proceed further and they when can we design one such can you proposed one such design approach and there alternatives can be discuss later actually and this is where we talked about dynamic inversion where it is a actually a promising substitute for gain scheduling if you want to prevent this.

Gain schedule may not be necessary actually it but, when scheduling is still an option within it so it will may not be able I mean, may not need to invoke it for a large class of systems but, if you really want to do that of finish lower level within this so that is how its operate actually so let us proceed with this already we notation and think like that what it what does it do actually.

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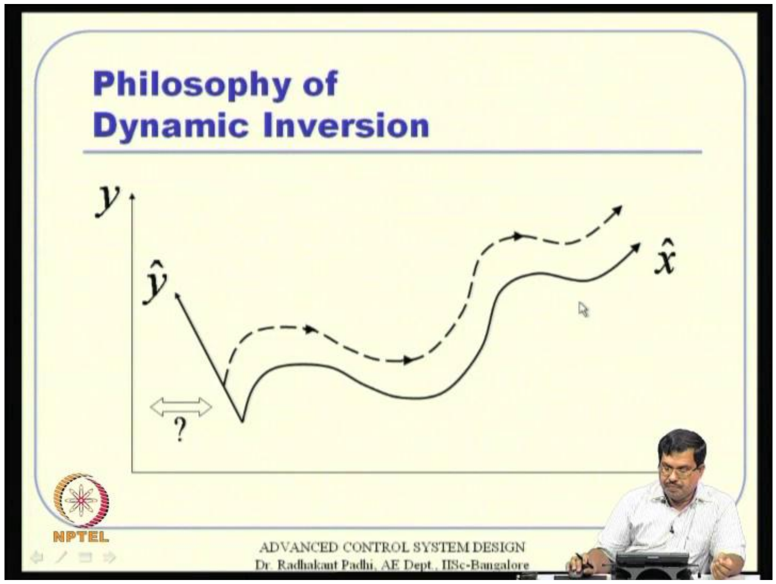

So, let us talk about some sort of curve like this I mean, you can you can visualize from a number of application for example, like a FM your flying at different locations and then going somewhere and think like that it that is for example, this can be like way 0.12 way 0.3 think like that can it as to fly and then proceed further let simplify about our ideas little bit and tell let us consider this as a some sort of two d trajectory.

So if I ask the question that whether this trajectory looks linear or non-linear I am very said in the first third many of you will think it is a non-linear trajectory and which is not very non intuitive whether actually is very intuitive because without even knowing any other additional information the mind tells us that this is the coordinate system actually this is the coordinate system certainly this curve looks like non-linear curve actually but, without saying the problem that mean without entering the trajectory itself let us say lets visualize the problem in a very different way than how about designing coordinate frame that way that mean I will not take this x y coordinate system but, I will interpolate this problem from the coordinate system etcetera y actually.

So if I design this coordinate frame if I am able to interpret this problem from this x y coordinate frame then the distance remains constant actually distance from x I two y I and find out time when point of x remains constant actually so instead of this frame if I look at in this frame suddenly this frame this a looks much more different problem actually now within this modified coordinate frame I can design a control system as if it is tracking a step input actually this is nothing but, step input so I can design control system from this a in this coordinate frame x and y so that there are the reference signal is like a step input I need to do struck it there.

Now, once I do observe in that coordinate frame I have to kind of a go back to the original coordinate frame and then interpret the results in my original coordinate frame so obviously the question here is can you come up with some procedure for the for designing a coordinate frame first depending on the reference trajectory basically depending on the problem whatever is given to us can we design can we design a coordinate frame which should be invertible actually by the way for example, if I solve this problem in this coordinate frame as we able to come back to this and y source actually it is so if the if the transformation is invertible certainly I will be do able to do that is how actually here much more simplified.

Actually, sort of otherwise what you have to do I mean, I just talked to odi originally if I look at it that way then I have to design let us say many linear localized I mean, many local linear controllers and then have to interpolate and think like that actually now I do not have to do that in this coordinate frame because every where the command is step input anyway.

I just design it and go basically so that is the whole idea there but, without going through too much of mathematics and all I mean, that is what you will find out in what is called feedback linearization sub states and all in a non-linear control books I will try to simplify the kind of this approach and then try my best to give you as a direct approach for controls and thesis actually alright so will amt will not digress this too much into well when it is possible to transform what is the transformation and then it will if you really want to do it then it will demand this concepts of lie algebra lie derivative lie bracket then the concept of diffeomorphism invertibilty lot of concepts from differential geometry actually.

Before we if really go to the topic actually so with without doing that much details so we will we will not know that much detail we will hear some sort of engineering prospective to directly formulate a problem and then solve it without compromising the philosophy actually philosophy remains something similar but, you will not be able to answer your question directly that what is the coordinate transformation you are looking at and what is the error the I mean, what is the reverse transformation inverse transformation all that actually those if you want to know obviously non-linear control books are available and probably solution and is kind of a good book to study this one chapter which is given there actually.

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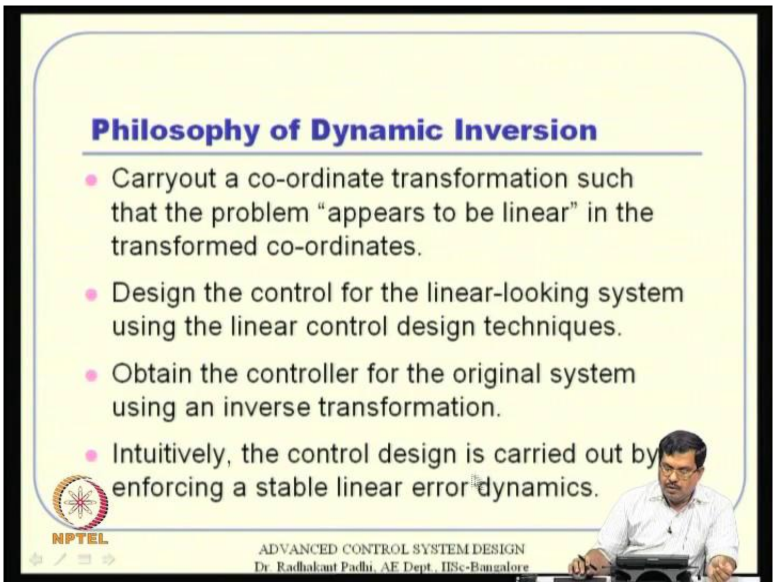

In a coming back to the problem the problem remains like this we want to talk about a some sort of a design approach for non-linear systems directly and philosophy of dynamic inversion is something like this we first carry out a coordinate transformation such that the problem appears to be linear in the transform coordinate is really not a linear problem we are not linearizing the problem at all but, in the coordinate transform the transform coordinate system the problem appears to be linear and then design the control for the linear looking system it is not really linearize system but, it is a linear looking system using linear control design techniques then obtain then non-linear obtain the final control.

Obtain the control of for the original system using an inverse transformation that is what that is what the entire philosophy of dynamic inversion ash relays on so intuitively the control design is carried out to enforcing some sort of a stable linear error dynamics so that is what we do and that is the key point that will assume to actually now so we want to design a controller by enforcing a stable linear error dynamics so whatever is the error that error philosophy should continue to see some sort of a linear stable linear dynamics where soluble need to do so control design has to be done in such a way that whatever error between your actually output to the desired output. So to keep on we keep on seeing some sort of a stable linear dynamics actually that is what we will enlarge on to actually.

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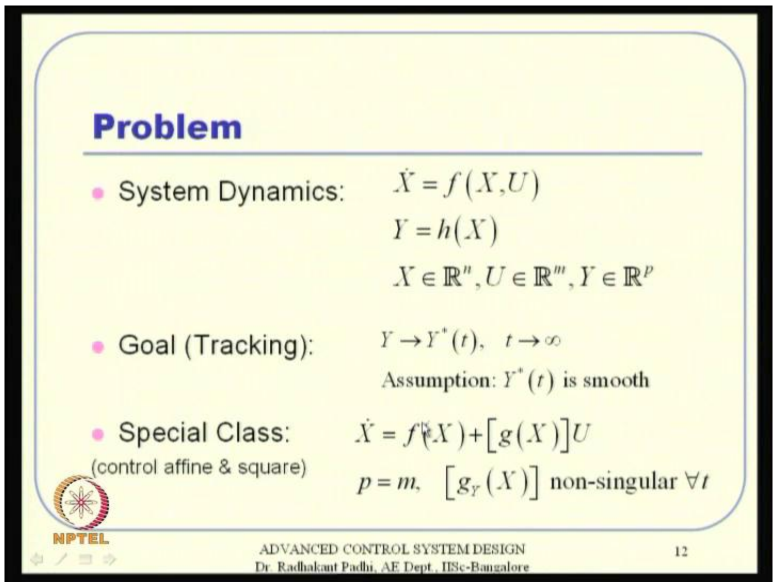

So, let see this problem formulation and mathematical details a little bit what we are doing here is let us say we have a system dynamics of this form X dot equal to f of X U Y equal to X of X and Y is the desired output remember this need not be sensor output sensor output is something different and control design output is something different control design output is the kind of a objective output of the design goal what you want to take it actually for example, if you want to land an aircraft then height becomes some sort of a performance output.

But, you may or may not measure the height directly you may have you may be able to measure the range and the bearing angles sort of thing and that height information is indirectly there in that actually that way so you may not the measure the directly the height as such so this one may or may not be sensor output actually. So, that is what we need to clearly remember in control and filter design actually there are one some output which are performance outputs for which control design is primarily responsible and there are once of outputs which normally the sensors keep on sensing these two may or may not be the same they may be some overlapping they may not be overlapping think like that actually.

Anyway certainly it is a good idea to directly measure it I mean, because the if you have a direct information nothing kike that actually anyway so come back to this is the system dynamics that we are worried about and this is the performance output that we are worried about in other words this output has to track some desired output actually and that is the goal or goal of tracking is like this why should track or why should go to Y star of t as t goes to infinity and then Y star of t is suppose be the desired output or commanded output think like that and assumption here is Y star is smooth but, that means if I plot Y star t versus t.

Then I should not see any comma points and think like that at least it should be smooth to go first to the derivative sense actually the first order derivative should be kind of continuous actually and I mean, if the problem is clear then we will I mean, looking at this problem if they looks a little bit too much in the sense that this is a non-linear system directly as it is we do not know how this control I mean, variable operates with that and think like that so to make our life a little bit simplistic we will further confine our discussions to a special class of problem where the problem needs to affine in square actually affine means in sort of control appearing that way directly as a non-linear function.

Let the control variable at I mean, operate I mean, act under plant in a linear manner is in other words the plant is non-linear but, it is non-linear only in state only in X variable it is linear in the control variable actually now the second one is this p equal to m that means if you see this dimensional things and all and in general X is a n dimensional vector U is n dimensional vector where is a R is a I mean, Y is a p dimensional vector and then what you are assuming here is m equal to p that means number of controls are equal to number of performance output so in the input output sense the problem is really square actually and then in addition we are assuming g Y of X is non singular for all time.

And g Y f is not g X g of X g Y of X is something you will define in the next slide but, is certainly not g of X actually but, this particular matrix should remain non singular for all time and this is we see certainly a time varying matrix because it is a function of X actually so we will see that in the next slide what is this g Y of X anyway that is the problem objective is Y should go to Y star and then it should we are the confining ourselves to control affine system which are square in addition to that there is no singularity in this particular matrix actually that is all we are interested and before you proceed further we remember this t goes to infinity and all does not really mean t really goes to infinity here this is a like I mean, notationally we also do that.

The what it means is asymptotical tracking actually so and then what time it will closing and all that it will depend on the settling time that we choose I mean, ideally speaking it will never close in I mean, in the 0 error sense basically but, I mean, it will approach to almost 0 error in some sort of settling time that we consider and this problem actually alright now proceed further lets go back and try to see what control value should take in other words what is the control solution for this so that this objectives is matrix actually what we do that so before doing that let us I mean, let us see what is our output dynamics as what we have is state dynamics actually.

So we need to consider what is our output dynamics first actually so that we can talk about output error dynamics and that error dynamics needs to be some sort of a stable linear dynamics actually so first we need what is our output dynamics.

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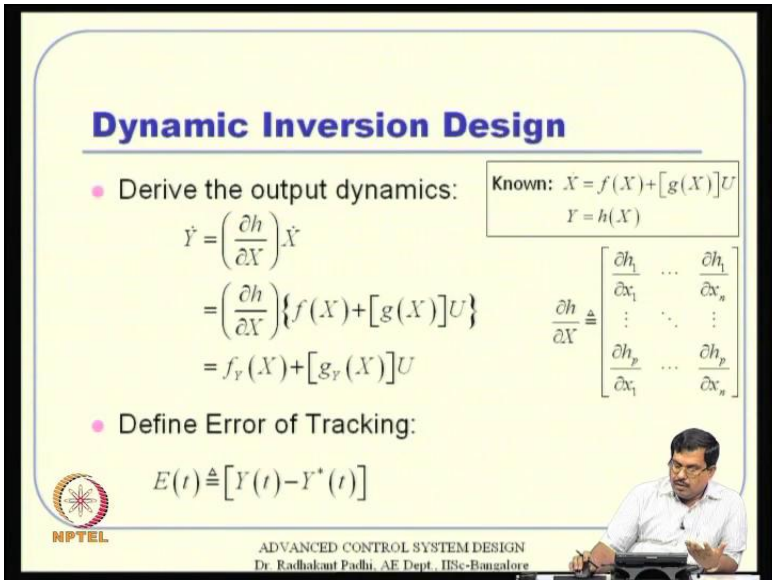

So remember Y equal to h of X that is where you will start with because y equal to x of x then if I take derivative both sides Y dot equal to this del h by del X to X dot because Y equal to X of X actually that way and X dot we know that is f of X plus E if x times u so we will substitute that is why use of substitute and then try to simplify this matrix multiplied by f of X is Y of X and this matrix this del h by del X remember this in general is a matrix by the way.

So this del h by del X into g of X is what is what is defined as g Y of X and that is what g Y of X is all about so g Y of X is nothing but, del h by del x into g or g of x that is g Y of X so that is our output dynamics now we have to talk about an error dynamics actually so what is error is Y minus Y star that is what that is our error of output tracking so in that what is error dynamics then so let us talk about E dot and then what you really want the error dynamics needs to be some sort of a stable first order dynamic let say by the way if this del h by del X is defined that way so this del h obviously it is a jacobian matrix so that is defined that way in partial derivative sense and all that actually.

Now, this the now coming back this you defined this error like that Y minus Y star sort of thing and do some control design U in such a way that the E e f t so it is always see some sort of a stable first order I mean, an like time invariant linear dynamics actually.

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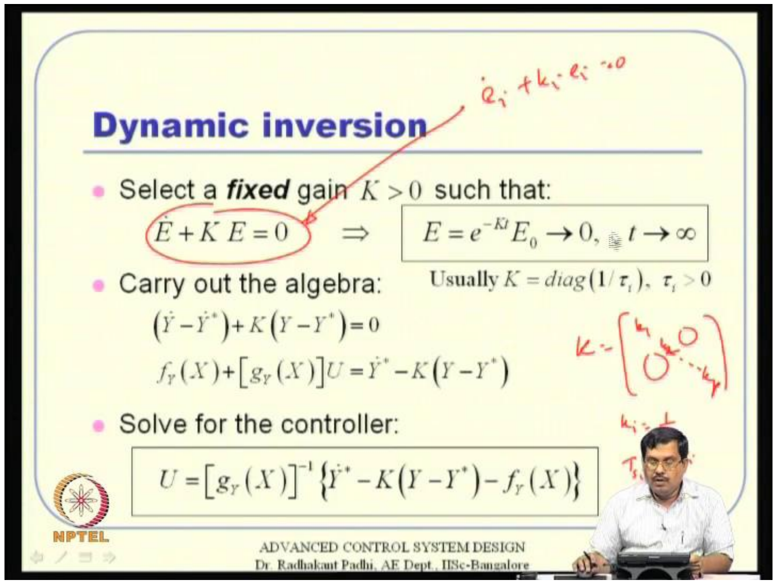

So how do it how do we need to enforce this dynamics by some gain selection K which will come and operate here actually so when mostly we will see some sort of a fixed gain K that is what to most of the time will be able to operate that means just one gain we will able to operate E dot plus K equal to 0 and then that scheduling and all we will see if we if necessary you can always do some limited amount of scheduling in other words you if say keep on changing this dynamics as we go along actually.

But theoretically speaking if you really want to operate this dynamics as some sort of a linear time invariant first order error dynamics then K has to be fixed actually it should be varying at all then only it will take it will be some sort of a LTI system linear time invariant system anyway so coming back to this will a E dot plus K equal to 0. So, that means that is what we want to do we design a control in such a way that this dynamics should always keep v like this we say so that means so this if this dynamics is continuously satisfied that means they are the solution is like that equal to E to the power minus K t e times is 0 if this is dynamics where K is the control gain fixed gain let us say and K is positive definite in a scalar sense it is positive.

But in matrix sense is positive definite then what you got is the is that a is solution of a is nothing but, w to the power minus K t times e 0 and certainly this small e to the power minus K t into e 0 goes to 0 because K is a positive definite matrix and hence minus K is a negative definite matrix so this a this e certainly goes to 0 as t goes to infinity so our controller should do such good job that it should keep on satisfying this error dynamics know whatever it actually whatever is operating condition it whatever point outing it should continuously try to enforce this system dynamics actually so that is the that is what the entire philosophy all about actually.

Now, the question here is how do you select a gain matrix K first of all because a symmetries after all when it needs to be positive definite so how do we go and select that and one easy way to do that is to select a diagonal gain matrix K and because the diagonal if it is a diagonal then the Eigen values are the diagonal elements actually so if you simply select a gain matrix as diagonal matrix with positive elements in the diagonal entries basically all the diagonal entries needs to be positive numbers and then obviously this gain matrix will be positive definite and how do you select those positive numbers we will select in a reverse way telling that I will select the gain matrix K some sort K equal to some sort of a k 1 k 2 like that I will select k p or k k m whatever have then this k I that we are talking about will be k I is one about of I and this tau I is selected based on the philosophy that the settling time is nothing but, four tau i.

In any design if you have a settling time specification then you can go you can in select a appropriate tau for that because four tau is settling time so we can complete the tau and you can compute the gain like one over tau and then this gain and this gains will go as diagonal entries in this gain matrix K so my strong suggestion is always to operate from time domain specifications like this instead of looking at your behavior later and all that once you if you want implement this control is an any problem start with some specifications like that then reverse computer gain that should go into the diagonal elements actually so that this error dynamics will continuously satisfy this kind of I mean, I mean, this kind of settling time property actually.

Now second thing to notice if you have diagonal elements actually in the for the gain matrix then if you go back to this dynamics this without plus K whatever we are talking about here this e dot plus K e this one if I have this gain size diagonal element then a individually I can think the u i dot plus k i e i equal to 0 that means the any e i is not a function of any e i minus 1 i plus 1 nothing actually it becomes an independent dynamics by itself so that is how you get d coupling behavior in this dynamics directly basically for example, linear dynamics I mean, this longitudinal and lateral dynamics you want to decouple then this will automatically give decoupling in each of the general actually so that is another nice behavior of this control design basically anyway.

So coming back to this is what we want to kind of enforce or satisfy if for our show plant so how do you design a controller to satisfy this dynamic that becomes some sort of a very straight forward approach because we know the definition of e now y equal one minus of star so that substitute equal to y minus Y star and then e dot is nothing but, y dot minus Y star dot where Y dot is already was something that you compute it this is what we computed Y dot so Y dot you substitute Y dot is of f Y of X plus e Y of X times U and then take everything as to the Eigen sight and ultimately you try to solve for u and remember this assumption that you have done that p equal to m that means U Y of X is a certainly a square matrix. And, e Y f X is non singular for l time that is what that is the that is the reason that is why we assume that g Y of X we have assumed that is non singular it is square and non singular for all time that is the assumption actually so we should be able to do and inverse I mean, it should be able to solve control for here and from this equation if you want to solve the control variable this takes this form actually so ultimately the control a formula is like this and remember this matrix has an inversion and this is a dynamic matrix that means it keeps on changing with time and that is one of the reason why it is called dynamic inversion actually the matrix one matrix inversion is to be taken all the time and that matrix itself keeps on changing in a dynamic manner so that is why it is a dynamic inversion actually so we get a solution for the control of this way.

So if everything is clear by that by now I hope so then we will proceed further and see some of this discussion around that the basic philosophy like this we have a output to output tracking problem and we have the output dynamics on the way we assume that this problem is clear and this e Y of X is non singular g Y f X is clearly defined that if the del h by del X into g of X del h by del X is like that and g of X is already known to us so we will be able to do all these computations in a in an nice way f Y we know g Y we know and if you plug in all these error dynamics in this formula then we will able to compute u as a function of these a known information actually.

We know Y star we know Y star dot these are coming from the command reference side actually then Y is already available with us [n] f Y is known to us and g Y is also known to us so we will be able to compute our controller in a direct manner now the we have started from here then gone here then gone here and then gone here actually but, you remember that none of the steps are only one sided that means you can start from here and go there and go there and go there in a reverse these are all implies and implied by relationships as long as to like the control can be in computed in a continuous manner like this actually if have most of the time implementation sense we compute it in some sort of 0 order whole sense.

Then we will not be able to do that reverse way in a very precise mathematical manner but, as long as delta t as small we can continue consider that is as continuous variable and this all this relationship are implies and implied by that means if the control look control is computed like this and there is absolutely no problem in implementing this control magnitude whatever it is I mean, there practically problems as we all known for example, this control that you compute can go outer bound if there is outer bound and just cannot implemented this formula is no more valid so the obviously at that kind of a dynamics no more valid so if it is a if it violets similarly, if its violets like red constant or something then also this dynamics is not valid.

So like that there are several issues that will pope up here and then we will try to kind of see what are the issues are there actually where as long as this issues are not then none of the issues are relevant and in other words this control you are able to implement in reality in a continuous manner then this is aerodynamics that you are enforcing and hence he has to go to 0 and e goes to 0 and in exponential manner remember that is not necessary only asymptotic manner but, this exponential manner actually.

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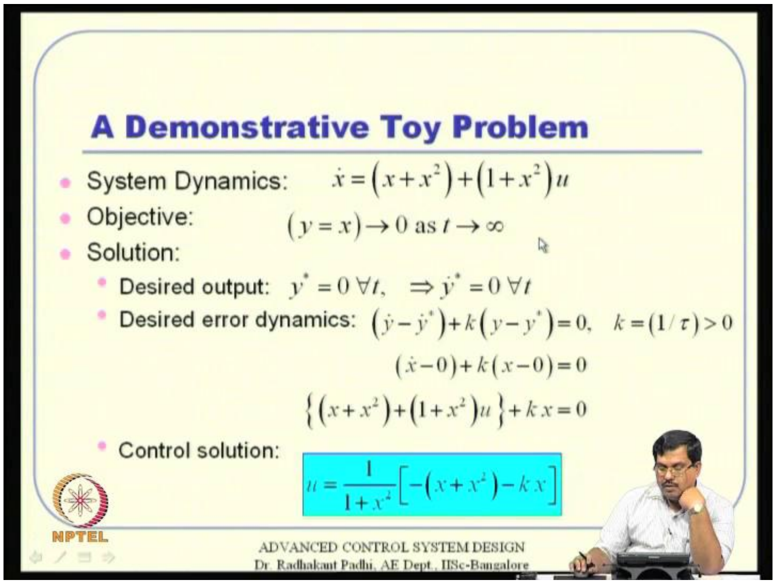

So what are the advantage I mean, at the demo I mean, before if you see further let us see some sort of a toy problem design problem sort of thing.

So, very simple one dimensional problem where you have x dot equal to this dynamics remember this j of x 1 plus x square will never become singular no matter whatever x values are an objective that is just a regulation that means y should go to 0 and y here is nothing but, x so we know x x star or y star is 0 and y star dot is also 0 f of x is this one f g of x is that one and because y equal to x so del s by del x is one so that means f y is this one only as f y is this one f Y of X and g Y of X is also that right del x by del x is one anyway so we do not have this difference actually.

What is the desired aerodynamics let us say e dot plus k equal to 0 where k is 1 by tau which is certain equal to 0 then you substitute what you whatever you know actually y dot is nothing but, X dot so X dot is all that actually whatever you see here that is what you substituted here y star dot is 0 now y star is 0 for all time so y star dot is 0 and y star is also 0 so substitute 0 so we will ultimately what you are doing is this entire expression plus k times x should be 0 so that means u is equal to this all bit in kind of inverse basically.

And then from this equation if you solve for you this is the control formula so that is and this control formula is a is guaranteed to satisfy this kind of aerodynamics that means this controller will certainly lead to stable behavior of the plant you know in other words x will be given to 0 actually know doubt about that actually this is some simply demonstrated toy problem that is all actually.

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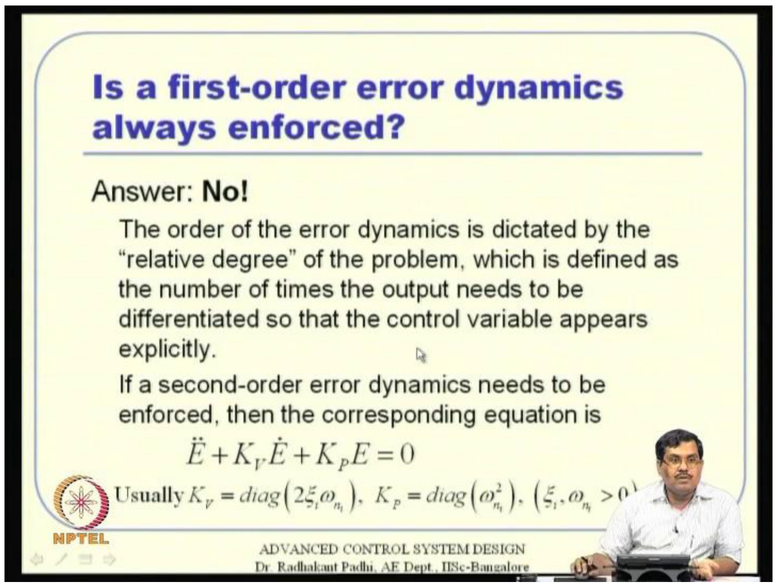

Now, the question is if the faster aerodynamics always enforced will always able to do that and answer turns out to be no and because suppose if the u is not there here but, but that just left one dimensional two dimensional three dimensional problem and for we all know that is if a control can control canonical form and all that x 1 dot is x 2 x 2 dot is x 3 like that actually the control variable is not there in the output variable suppose it is a I mean, why is x 1 and x 1 dot is a x 2 then y dot is x 2 and then no control variable so we will not be able to operate that based on the first order plan actually so what is the answer to that then the first order if the first order dynamics is not always enforce than there is a concept of something called relative degree actually so what is relative degree this is define as the number of times the output needs to be differentiated.

So, that the control variable appears explicitly and based on that dynamics wherever the control variable appears explicitly you enforce in other dynamics of that order and we all know that linear system LTI systems theory linear I will invariant system theory any order linear dynamics can be enforce you select those number of poles or thing like that and you must suppose you have this forth order polynomial you have to select or forth order aerodynamics that you need to select then how do you do that this is rather easy.

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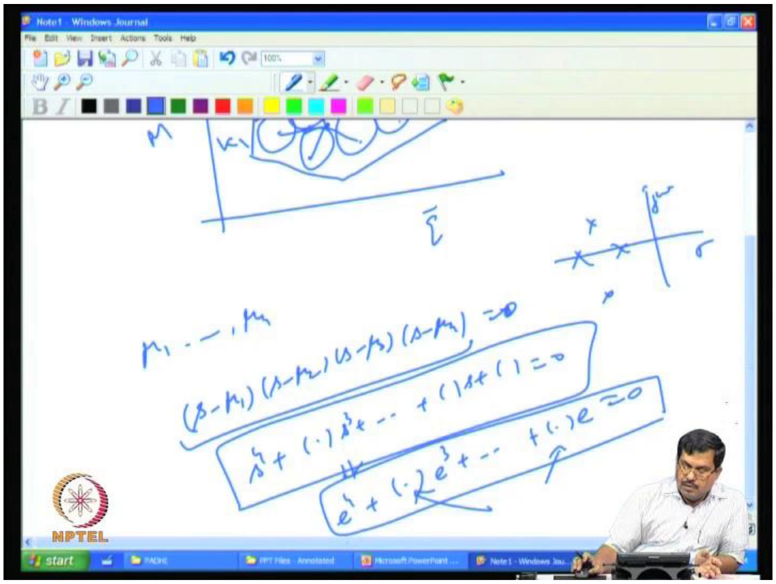

So, you have this you have this let us say mu 1 up to mu up to mu 4 let us say so these are the desired Eigen values like whatever you did in 4 placement control design so what you do here is like s minus mu 1 into r lambda minus 1 either way into s minus mu 2 into s minus mu 3 in into s minus mu 4 equal to 0 if you put that then the that means then the 4 locations also telling mu 1 mu 2 mu 3 mu 4 which are stable actually whatever the you select these are the locations whatever you select.

So, if you expand this expression it will give you some sort of s 4 plus something time x q plus like that actually something terms s plus something equal to 0 so this is the forth order dynamics that means if I take this one and then this is e 4 plus this one whatever is here that will be same as times e q and think like that plus something times e equal to 0 that is that is the stable dynamics that I will select so whatever if the co efficient that goes on here that will help me in enforcing a stable dynamics actually this is not equal to select whatever order actually there so that is not that is not a problem at all alright so coming back to this so the second order suppose you need to fix a second order aerodynamics and then the you have to enforce some sort of aerodynamics like that.

Where K V is turn turns out to be the diagonal matrix of like that and this K P turns out to be diagonal matrix of this one so again this individual elements sense you are operating on kind of in decoupled manner and thing like that actually if you select all this a diagonal matrices now this concept of this relative remember that the relative degree can defer from various asymptotic actually that means y 1 can related degree to y 2 can degree 3 like that actually and then the total relative degree turns out to be addition of all these individual relative degrees what I shown here is possible only all the channels will have second relative degree 2 basically if it is different relative degrees you have to enforce different aerodynamics in different channels actually.

so that is also possible into that way so if the second aerodynamics makes to enforce then the corresponding equation is like that you start from here than simply substitute what is e dot e double dot all that actually and ultimately you will end up with y double dot and y double dot will contain you and then you will be able to solve for you basically that is the whole idea there.

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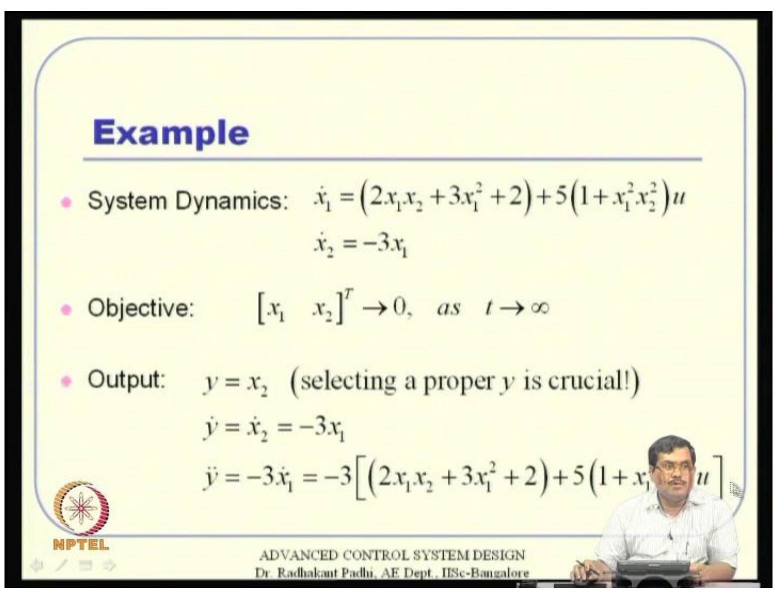

So, example again let us see this is like a second order system x 1 dot is that x 2 dot is that and the objective is both the state should go to 0 as t goes to infinity let us say and in general we one of the able to do that for arbitraries signal tracking there is a theorem that we will see later actually but, for a regulator problem this is possible to do so what we do here is very tempting to see first of all that there is x 1 dot and x 2 is there anyway so what you do here is suppose with x 1 should also move to 0 is very tempting for me to select y as x 1 you I will able to do that because if I select x 1 y dot contains u so if I select y dot plus k e k y equal to 0 sort of thing then I will be able to enforce that x 1 should go to 0 and that is one of the objective anyway but, the problem is once x 1 goes to 0 x 2 dot goes to 0 that means there is no change of x 2 after that so that means even though I will be able to stabilize x 1 two I mean, I will be able to drive x 1 to 0 from I will from any initial condition.

I may not be able to do that same for x 2 actually in the on the way x 2 dot can go to 0 and once x 2 dot goes to 0 there is no change of x 2 basically so that is that is the reason selecting y one I mean, selecting y is x 1 not a good idea actually so let us select by is x to then and that is why I have written that x to selecting proper y is to sell so let us select x 1 y as x 2 then what happens let us say y dot is x 2 dot it does not contain u but, y double is three x 1 dot that contains u.

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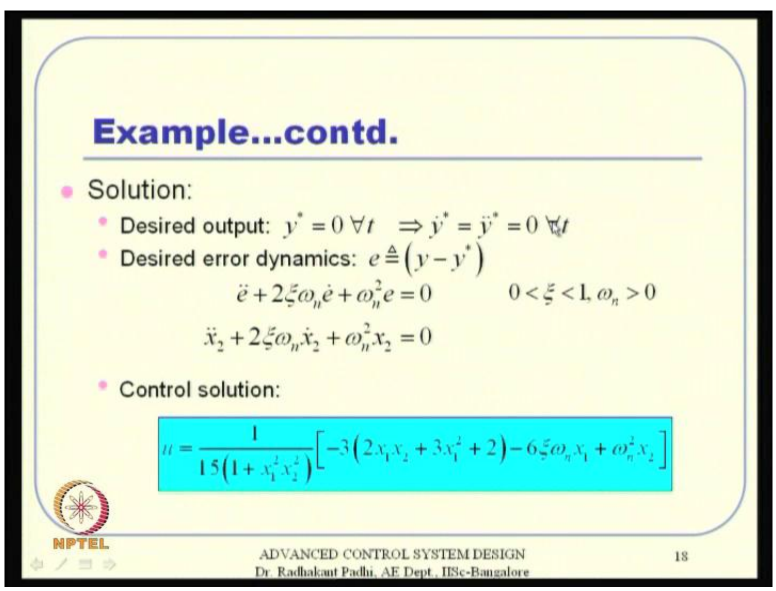

So then what you do we select a desired output and all that to that is y star is 0 so y star dot double dot are also 0 so error dynamics now because this y dot double dot contains control we have to select it double dot I mean, this second order dynamics sort of thing.

And then enforce this dynamics by enforcing this dynamics what we mean is the enforcement of this dynamics actually now what is the beauty of this dynamics the not only e goes to 0 but, e dot also goes to 0 second order dynamics all that u goes to 0 e dot also goes to 0 simultaneously actually as t goes to infinity of course, so if e dot goes to 0 that means x 2 dot goes to 0 and x 2 dot goes to 0 means x 1 goes to 0 and e goes to 0 means x 2 goes to 0 anyway basically so x 2 goes to 0 as u goes to 0 and e dot goes to 0 means x 2 dot goes to 0 that means x 1 goes to 0 also so both the things will goes to 0 actually that way.

So that is how we have to select the I mean, kind of a intelligent manner why should the x 2 basically so we continue further like that we select this aerodynamics and all that then it leads to that that kind of aerodynamic and hence there is a control solution for that because x 2 double dot contains this is x 2 double dot right why is why is x 2 now so y dot is x double dot is there is a u term here so will be able to solve it for that actually and also remember that this term what you see in denominator has no singularity I mean, it does not go to 0 actually this expression has no singularity actually and that is possibly where possibly because we all selected this example artificially is another kind of toy problem actually.

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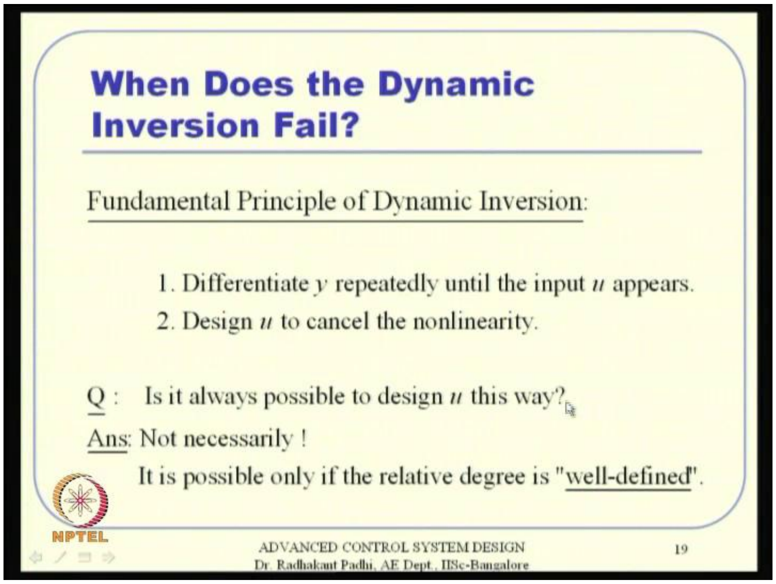

In general it may not have that nice feature actually so when does the dynamic inversion fail that is another condition that we have to worry about so what is the fundamental principle or dynamic inversion it turns out that you differentiate y repeatedly until the input u appears and then design u to conceal the non inertly and enforce some sort of stable I man aerodynamics that what you do that is the fundamental principle so yet possible to design u this way always and that turns out the answer is not necessarily and the concept here is it is possible only if the relative degree is well defined actually now the question is what is well defined related degree obviously so these are all like further questions outer questions it will show us to more and more to have differential the matrix concept actually but, anyway what is well defined relative degree.

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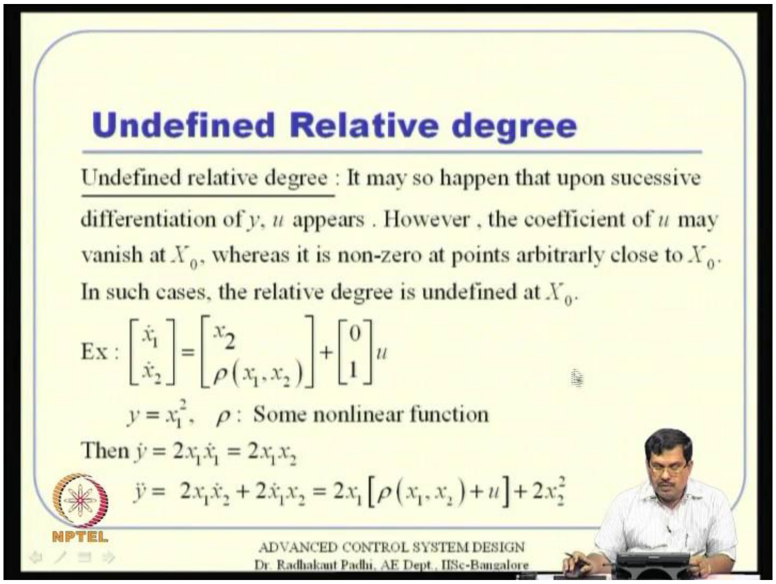

Let me kind of simplify this idea form counter example point of view the question is what is undefined relatively degree whatever is not defined these value for that sense.

So, undefined relative degree is something like this it may so happen that happens successive differentiation of y if appears but, the co efficient of u may vanish at some point x c and the domain actually the co efficient has an expression but, that expression goes to 0 at some point x 0 in that domain then what you have actually you have you almost do not have it that expression you have it everywhere but, some point of time you do not have actually so in that in those situations the relative degree is not defined at x 0 it is defines everywhere but, it is not defined as round I mean, at x 0 because arbitrarily close to 0 it is no similar but, on x 0 it is similar.

In that sense, these are like confusing and these are the and in that sense we tell relative degrees undefined actually so in example sense that is what it is x 1 dot is x 2 let us say x 2 dot is some expression of x 1 and x 2 but, u appear plus u basically and if you select y is equal to x 1 square where some non-linear function and all then you carry out and say that y dot is this expression y dot is to x 1 and to x 1 dot and x 1 dot is x 2 so I will substitute the I will get it to x 1 and x 2 ad y double dot is 2 times x 1 times x 2 dot plus 2 x 1 dot times x 2 so I will have this expression actually.

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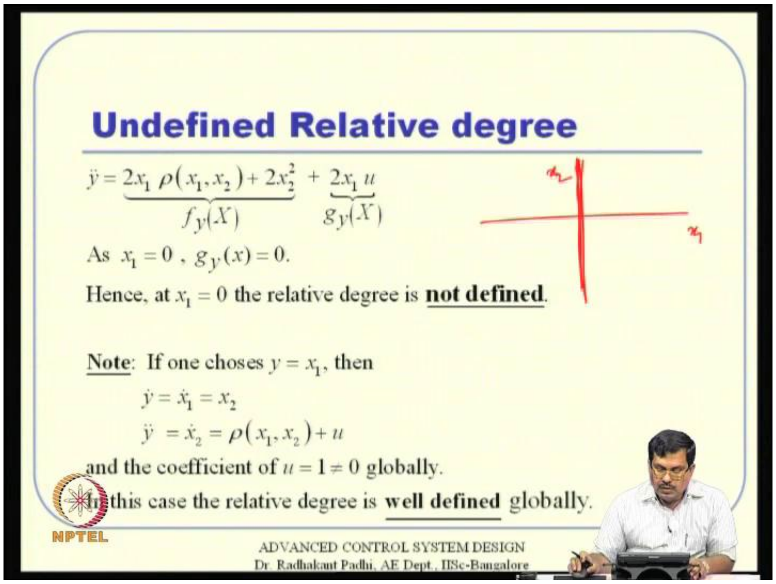

So obviously this y double dot am I control appears so I have to enforce a double kind of double derivative and thing like that so u double dot is like this. Ultimately, u will guarantee whether the relative degree is defined or not so what is relative degree I am this g by f x is like this 2 x plus 1 basically so as x 1 goes to 0 when certainly g Y of X 0 and here is a problem actually so if I draw this phase plot or something we have x 1 and x 2 so x 1 0 means what and the entire imaginary line x 1 is 0 so entire imaginary line I mean, not imaginary line that x 2 x is basically entire x 2 x is x 1 is 0 so on the entire x 2 is x of the of this phase plane I have a problem of non not defined relative degree basically relative degree not well defined actually.

So, if you choose y equal to x 1 then this is this problem happen because we selected x 1 square if you select x 1 then this problem does not occur because y dot is 2 x 1 and all that they the 2 x 1 term comes from this expression because y is x 1 square come on here y is y is x 1 square so y dot is 2 x 1 times x 1 so this 2 x 1 fellow comes from because of that fact actually so y what if you select x 1 instead of x 1 square then obviously this y dot is x 1 dot which is x 2 and y double dot is x 2 dot which is like that so the co efficient of u is one now naught and expression really it is just one and obviously one is not equal to 0 and that is to globally and in hence this relative degree is well defined globally actually so this relative degrees are well defined then obviously you will you can discuss about this like dynamic inversion is possible actually.

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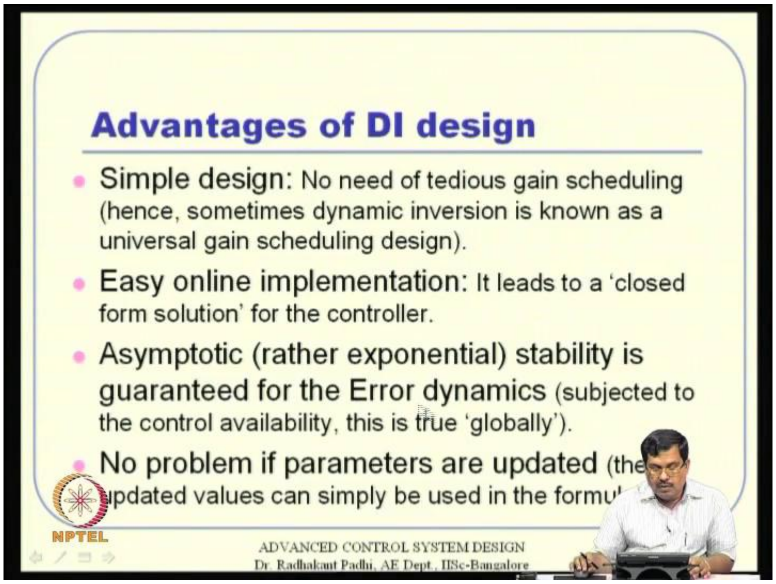

Now, what are the advantages to summarize of DI design? First of all it is a very simple design I mean, as for as understanding of mathematician other things are concern I do not think that is to raise the I mean, it requires too much of a understanding of complicated math and think like that to understand why it works and all that.

So, it is some sort of a very simpler design in my view actually and obviously we no need of t d s gain scheduling because what we the really started with gain scheduling demand lot of operating points and then you design local controllers from linear system and then interpolate also, sort of things are not required so as long as the control is not incisive it can keep on enforcing this aerodynamics and then it keep on getting the stable behavior no matter where while whatever is your Y star basically so wherever you are operating basically. I mean, in some sense scheduling the vein as in different advantage because of other issues for example, you want to avoid the slow rate limit or that control derivative limit and all that actually so probably you can gain tune the gain little bit as an automatic function somewhere.

So, then it may help you actually I mean, that is some sometimes people do that and that is there also they do not make use of like a high were gain and all that actually there is a slow varying parameters something you will introduce to have additional advantage but, in principle you do not need to do that is a select of x and k and then you are done actually so gain scheduling is not required and hence sometimes this dynamic inversion is also known as universal gain scheduling design because if gain scheduling is not required it operates everywhere and thing like that so it is also known as universal gain scheduling design and also known as what I dint write it there is also known as some sort of non-linear PID design but, ultimately if you go back to that expression what it really operates.

E dot equal to minus k sort of thing so if you see that this is actually operating on p gain actually e dot is minus k e so k times e e is nothing but, proportional term actually so it is operating on some sort of a p gain now the nothing wrong in operating based on PI gain so, somebody can always tell k one times e plus k 2 times integral of e equal to 0 so that is also possible so that that will become PI design if you have a second order dynamics and thing like that this is already PD design there is a p term and e e dot term so this already PD design you can always make it some gain times integral of e so it will become PID design basically that way it is also called as non-linear PID design so all the advantages of PID design tuning and all sort of things can be brought in here and that gives additional degree of comfort and power for the control design actually.

Number two it is easy to implementation because it will lead some ultimately leads to close form solutions so control is a control is actually after all a formula so as long as you have the state information you can compute the formula directly just one expression actually so there is no certain known not necessary to have some sort of like things two point boundary value for problem in optimal controls and then robust control design you talk to record equations thing like that do not do all sort of things there is just a formula that you want to evaluate and carry on actually. So, it is very easy for online implementations now asymptotic rather exponential stability is guaranteed for the aerodynamic and of course, I keep on telling is subjected to control level ability basically.

If your control is not saturated it is still within the bound rates are not incisive then it that expression that is derived it can be interpreted in a reverse way start from here then go there go there go there actually but, this expression is not compromise. So, whatever you are computing we are able to give that, ultimately your enforcing this one actually. This dynamics only that is why this asymptotic rather exponential stability guaranteed further for anyway because if you see that the solution we all started with this solution actually and this solution is actually exponential decaying is all based on LTI theory time and invariant theorem is always globally exponentially stable.

Because of this beautiful theory as far as output tracking is concerned, remember, this is all about output error as for as output tracking is concern it satisfies global exponential stability actually. Of course, subject to control ability as a big problem that we discussed in gain scheduling that, what if the parameters are updated. That means you have a design – ( ) design or design update and things like that suddenly is not a major issue because all the parameters values can directly go into the formula actually. Whatever you have a formula for control, if you can directly take it and then make it operate you do not start with basically kind of sequence of non-linear I mean, sequence of linearize plant and then we linearize I mean then, design gain and something like that you do not have to do that you have a control formula you simply forget and operate actually.

These are all the major advantages simpler design easier online implementation asymptotic stability guaranteed for the aerodynamics for the tracking aero sort of things and not a major issue if the system plant is updated or the parameters are updated. So, I think with all these in mind, we will stop here in this class and then continue with the further discussion of aerodynamic inversion in the next class actually. There are several issues and there are several remedies also will discuss some of the concept and discussion will do in the next class actually; thanks a lot.