Advanced Control System Design
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Lecture No. # 02
Classical Control Overview – I

Good morning everyone. And, as promised before planned before, we will cover this classical control topics in a very quick way; some sort of an overview before we move on to the modern control topics. Next couple of lectures will be kind of some sort of an overview of classical control system; may not be in very detail, but I assume that this particular audience have some knowledge of classical control systems already. So, that will kind of help us moving these topics rather in a fast way. First thing before we talk about any classical control system is review of Laplace transforms.

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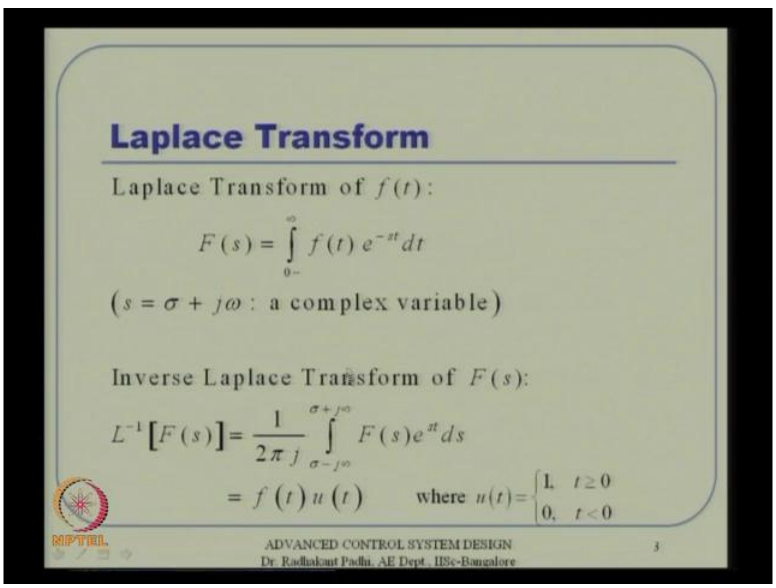

Let us see what does that transform do. Laplace transform of any function f of t can be defined as something like this – is an integral from 0 to infinity; strictly speaking, 0 minus. Just about little bit before 0, this integral f of t e to the power minus s t d t; and, s in general can be a complex variable, which has some sigma as the real part and omega as a complex variable (( )) part of it. When you have this Laplace transform, once you take Laplace transform of the time-varying function, essentially it gives us some sort of a frequency-dependent function; where, s can be treated as some sort of a complex frequency.

Then, suppose we also have F of s already known to us, then we can talk about inverse of Laplace transform. So, essentially, we are attempting to get back this f of t (Refer Slide Time: 01:53). And, the very fact that these two functions are uniquely defined for t greater than 0, makes our life much easier especially in linear control systems, especially classical control system. This little u of t, what you see here, is something like a unity step input defined from t equal to 0 onwards; for 0, it is 0. So, we are mainly interested in positive time sort of response in this inverse of Laplace transform. And, that is what mainly we are bothered about in control systems for t greater than equal to 0 anyway.

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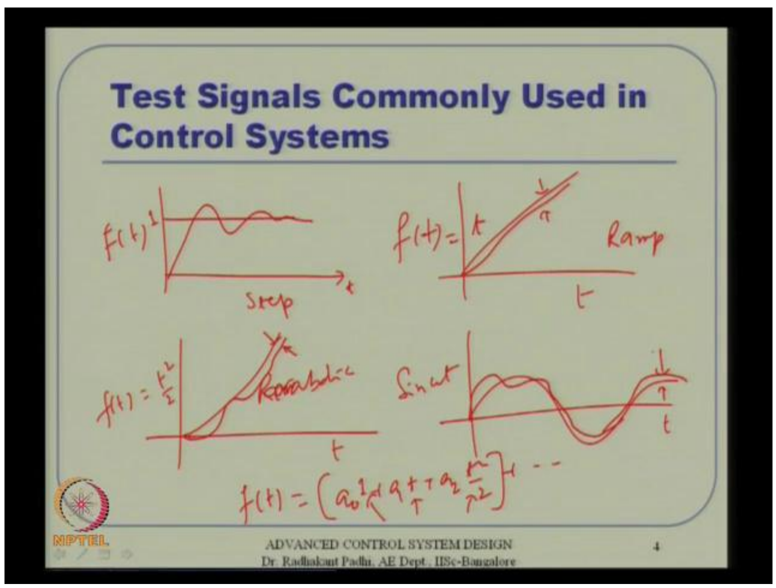

Common test signals used in control systems as we know are… Roughly, this one thing can be treated as something like a step input; that is a step with t and something like F of t; just a constant of number. And typically, this is unity step input. Then, we have this ramp input. So, this is t and this is f of t; this is ramp. And then, there can be let us say parabolic; f of t is equal to typically t square or t square by 2. So, this is parabolic. And, we also can use something like a sinusoidal function – sine omega t sort of thing in many applications.

And, the reasons for choosing these functions are like this. Suppose you have any continuous function; then, we can have something like a Taylor series expansion; then, if this constant function pops up first and then there is a ramp input. So, that is like t sort of thing. So, you have this Taylor series (Refer Slide Time: 03:50) f of t is something like a 0 plus a 1 t plus a 2 t square by 2 like that. So, you have this at least one sitting here. So, you if you see these functions 1, t, t square by 2 and things like that; and, if you really test some function with respect to these three signals, then at least up to first three terms, we have confidence that things are OK. So, primarily, when you see this step input, you have this transient response as well as steady state. Ramp inputs are primarily used for like steady state errors and parabolic as well sort of thing. So, there will be something like that and then we… There are various reasons for that – why do you need to test for all these signals.

In sinusoidal, again similar way; it is used both for transient response and mainly for steady state response. If you see this, this (Refer Slide Time: 04:40) will continue like this. And then, it is also used for modeling like system identification for example; if you give sinusoidal input, then you can talk about what frequencies can respond and things like that. So, there are various utilities not necessarily only in transient response sort of things, transient and steady state response, whether the other applications as well. Sinusoidal functions – similar way, the same function; you can decompose that in Fourier series for example; then, sine and cos terms will pop up. So, if you are interested in decomposing a function in Fourier series way and try to interpret whether my system is behaving well or not, then sinusoidal test signals are various useful.

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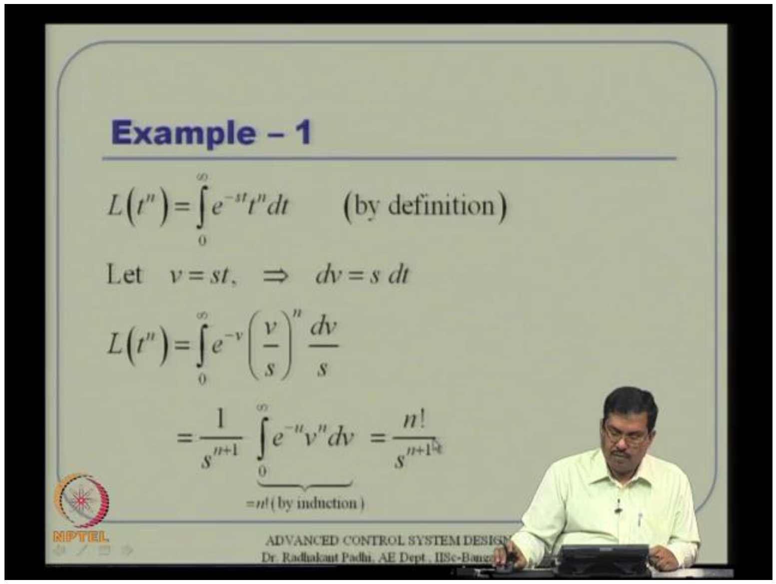

Now, let us see how do you take Laplace transform. First thing is one example. Let us talk about… This is t to the power n. You have this standard definition – e to the power minus s t into t to the power n dt. So, you introduce change of variable – v equal to st. So, dv equal to s dt sort of thing. And then, you go back to this integral and see what all… e to the power minus st will become minus v; t becomes v by s, like that. Then, 1 over s to the power n and then there is one more, where n plus 1 will come out here. And then, you have this… This particular integral what you see here, can be shown as n factorial by induction. So, then, it essentially turns out n factorial by s to the power n plus 1.

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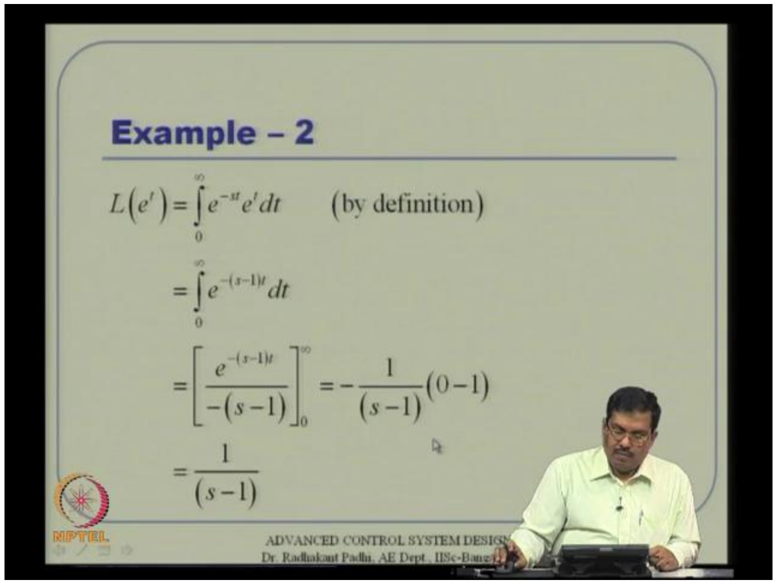

Similarly, example 2 if you see, e to the power t; then again, you plug it back in the definition and then you carry out the algebra. This is rather fairly straightforward; you have s minus 1. So, it is s minus 1 t – e to the power minus of s minus 1 t divided by minus of s minus 1 evaluated from 0 to infinity. And then, you are left out with this one; then, that infinity; this is 0; e to the power minus infinity sort of thing. At 0, this becomes 1. So, 0 minus 1. Essentially, you are left out with 1 by s minus 1. So, that is the way to kind of see these functions – how these functions are popping up from time domain function to frequency domain thing. We cannot keep on doing every time like this; we cannot keep on plugging to the formula and get some results.

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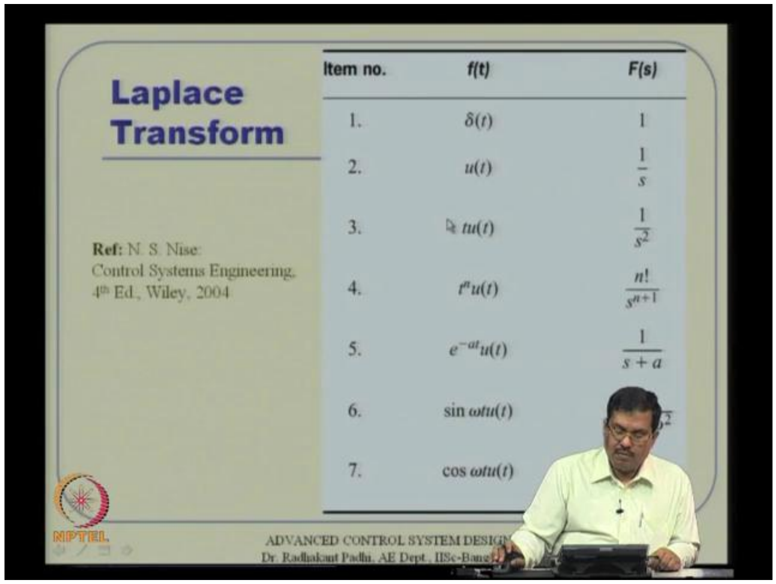

So, there are standard tables available for various standard functions. For example, if you consider this simple function, then the f of s has already been evaluated as 1. And, this is 1. Say 1 and 2 is u t. So, you have essentially 1 over s. This u t is the same u t that we have discussed sometime back. This is this kind of a function (Refer Slide Time: 07:29). If t is greater than 0, then it is 1. And, if t is less than 0 (( )) So, that helps us in the algebra part of it. This one (Refer Slide Time: 07:41) we have already shown; some of these you are going to show it also. So, these tables are available. This is rather a limited table. But, extensive tables are also there in different functions and things like that. But, these 6 or 7 functions can be like some sort of bases function sort of an idea that… Using these functions, we can do many algebra.

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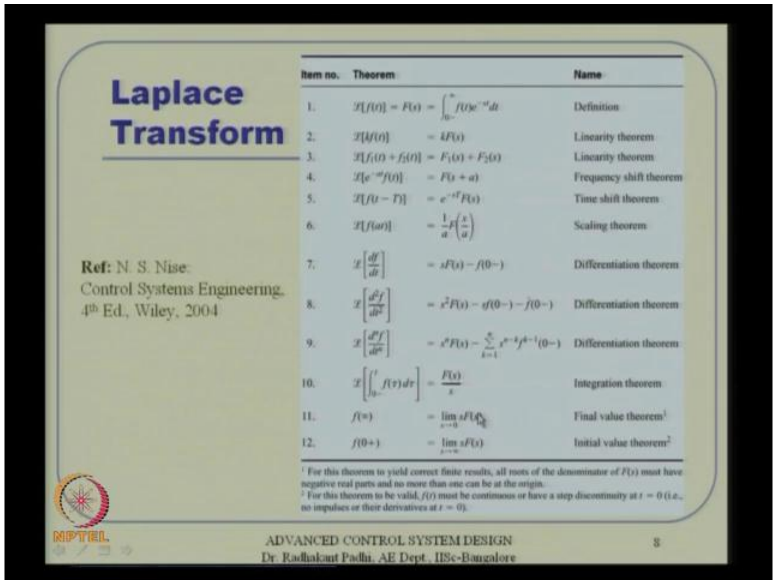

When there are a further ramifications; that means suppose you… This is the definition anyway. Then, suppose you multiply by k just a constant; then, just k pops up. Simply from this definition, you can see that it will come out of the integral. And similarly, F 1 plus F 2 will satisfy this. So, these two together will tell us that this Laplace transform is a linear operator. Then, if you have e to the power minus at into f of t, then it turns out to be that F of s plus a. This is something called as frequency shift theorem. Then, there are time shift theorems; there are scaling theorems available and things like that. What is very useful in our control system analysis is this part of it. When you have df by dt, then it is s F of s minus f of 0 sort of thing. And then, d square f dt square up to n-th order you can do; no problem. Whatever is the order, it will satisfy the same thing. But, remember, this side of the story – they are all time domain initial conditions. So, we will see that as we go along.

And, there is integration theorem also that if you take integral of a function, then talk about Laplace transform; then, it turns out to be F (s) divided by s (Refer Slide Time: 09:11). What this Laplace transform essentially do? If you see this derivative, derivative operator essentially turns out to be a multiplication; integral operator turns out to be division; that means this differentiation and integration, which are rather little complex operators in time domain, they turn out to be simply algebraic manipulations in the Laplace domain; that is the great simplicity that it brings in. Further analysis and all – it becomes easier.

Then, there are theorems like final value theorem and initial value theorem; that means you really do not have to do… Once you know the (Refer Slide Time: 09:45) F of s part of it and you want to find out what is this… Like when t goes to infinity, what is the value of function; then, essentially it is the steady state value. Then, you do not have to do like inverse of Laplace transform and then see what is going on at t equal infinity as a limiting case; instead of that, you can just use this limit rules and all that. So, you just take s tends to 0; s into F of s; that is it.

And similarly, when you talk about initial value theorems, then you do the other way; like s tends to infinity – s into F of s. And, these theorems are not universally true rather; there are some conditions available. For example, this final value theorem – all roots of this denominator of F of s must have negative real parts and no more than one at the origin. That essentially tells us that this function what you are looking at, has to be a stable function. We will see the stability theorems slightly later. And similarly, there is a condition for theorem 2 also. Now, let us go back and try to justify some of these theorems. For example, if you want to see how this one is coming out, because 2 and 3 are very easy to see that from 1.

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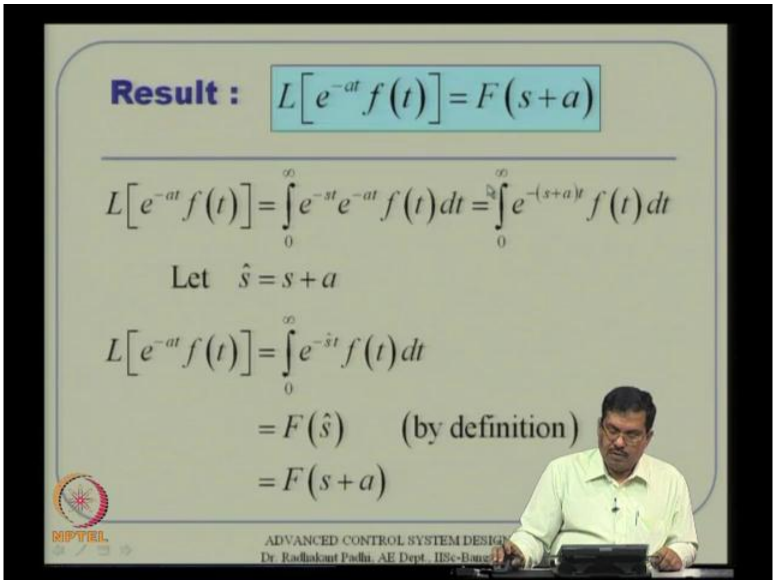

Now, for 4, if you just go back to this, then this is by definition – e to the power a t and things like that. So, this simply from definition; this is Laplace theorem integral definition.

Then, you combine these two and then interpret some s hat as s plus a; some sort of a change of variable. Then, Laplace transform of e to the power minus at into f of t – that is what you are considering here. It turns out to be simply e to the power minus s hat into f of t dt. And, that is again by definition, F of s hat. That is simply by definition. But, F of s plus a – it essentially tells us that this is nothing but this F of s plus a. This is as simple as to kind of show that. It is not very difficult to show that at all.

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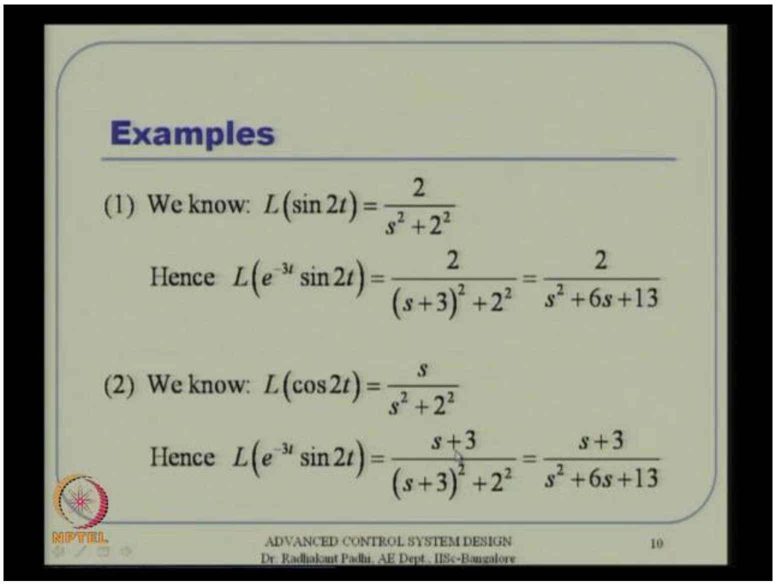

Examples – we already know that sine of 2 t is this one. Using this (Refer Slide Time: 11:48) table you can see that probably the sine of omega t is omega by s square plus omega square. That also can be derived by the way. When you know that (Refer Slide Time: 11:57) sine of 2 t is this like this, you really do not have to do one more time the moment e to the power minus 3t pops up. So, simply use that result. So, it is 2. And, wherever s is there, you just replace by s plus 3 essentially. So, you carry out with… And, this transfer function results in that way. Similarly, if you have cos 2t instead, then this transfer function is like that. And then, if you have e to the power minus 3t, then wherever s is there, you simply replace it by s plus 3. That is the theorem. So, this results in that kind of a Laplace transform.

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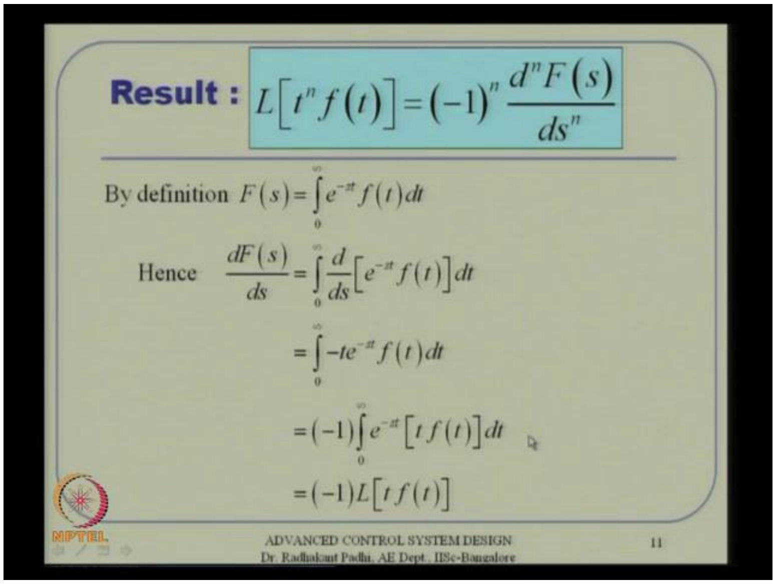

Now, there is another result, which tells us that t to the power n f of t is this one. It is also not that difficult to show. We will start in a rather reverse way. This is F of s. This is again by definition. Then, if you take derivative, then this derivative goes inside, because this integral is with respect to time; remember that. So, s is actually a independent variable of the integral. So, it goes inside. And then, this f of t is not a function of s. So, you can take out. And then, take derivative only with respect to s sort of thing. So, t is some sort of a coefficient here. It is minus t e to the power minus st because of that; and then, this one. So, minus 1 is taken outside the integral. Then, you are left out with that kind of a thing. And, this by definition is Laplace transform of t f of t.

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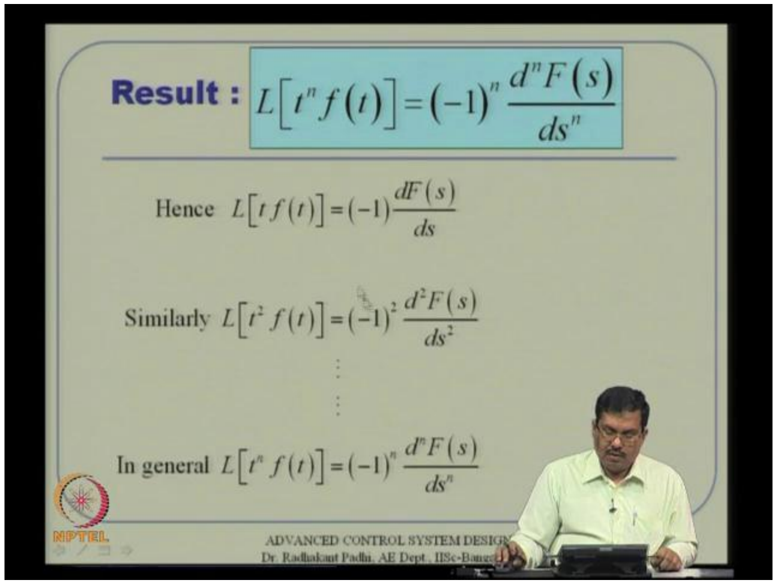

If you rearrange the things and tell Laplace transform t f of t is nothing but this, then I multiply one more time; then, consider that is t into that entire function. But, that entire function is already available to me. So, I keep on repeating this and then try getting these results. So, if I do that n times, then I will have minus 1 to the power n and then this n-th order derivative also. These are little manipulations in algebra here and there, which gives us some sort of important results.

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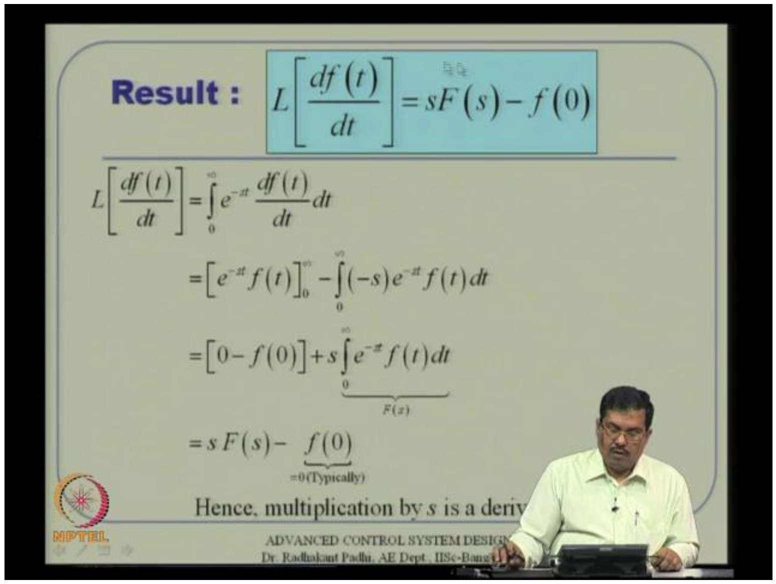

Now, this Laplace transform of the derivative of a function; how does this one happen? Again, you go back to the definition part of it and then this time, you integrate it by parts. So, this is exponential; this is just another function. I take this one times integral of that, which is f of t minus differentiation of that, which is this one; and then, integral of that, which is that one. So, then, this infinity again – e to the power minus infinity is 0; then, e to the power 0 is 1; then, f of 0. So, essentially, this s comes out of the integral minus this. So, essentially, we are left out with s into F of s from this part of it minus f of 0 from that part of it. And, most of the situations in classical control system or linear control theory especially, we assume that f of 0 is 0. And, that is primarily because of the thing that we discussed in the first class that, the linear system behavior or stability nature is independent of initial conditions. So, that is the fact that I will keep on repeating. So, we will not hopefully forget that. So, what it essentially tells us is multiplication by s of F of s essentially leads to this some sort of an interpretation of derivative; that means the multiplication of s is nothing but a time domain derivative property.

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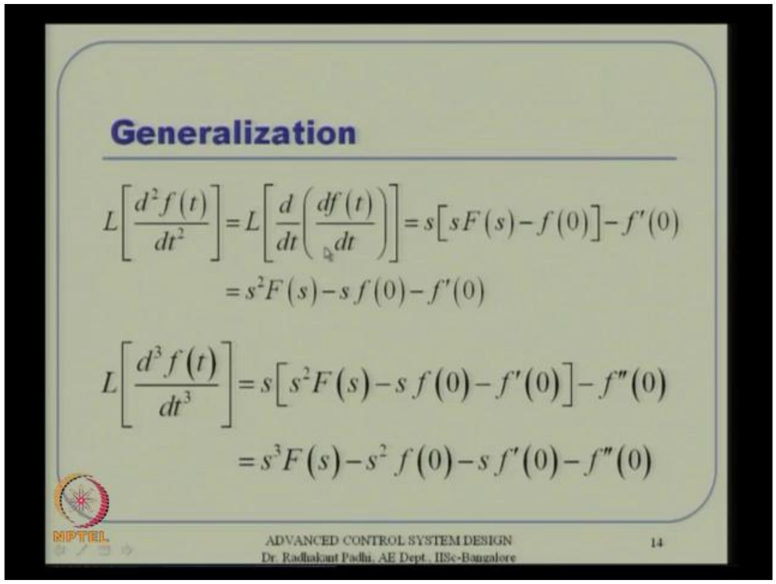

Generalization – you take double derivative and then repeatedly apply just one more derivative sort of thing. This result is already there with us. So, we just repeatedly use s into this entire thing. This result is already available; minus f dash of 0, because this is f dash here. Then, you multiply everything; then, it turns out to be s square into F of s minus s into f of 0 here minus f dash of 0. So, that is how it pops out here. If you do one more time, again, this result is now available. So, you apply one more time to that – minus f double dash of 0. So, essentially, it leads to some sort of a polynomial like this. So, first derivative leads to a first order polynomial – first order s multiplication of F of s; second derivative leads to s square into F of s; third derivative leads to s cube into F of s. And then, if you see the trend, then this s will start diminishing – s cube, s square and s; and then f to the power 0 sort of a thing. And then, this time domain initial conditions will start from 0-th derivative – f of 0; then, f dash 0; then, double dash; like that. It is rather easy to remember that way.

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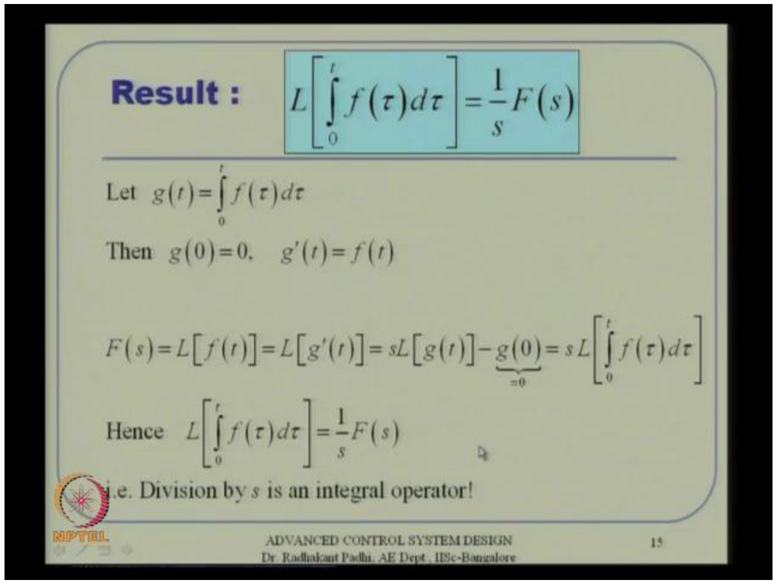

Then, integral operator – we know this 1 by s. How do you go about doing that? We again go back to this and then g of 0 is 0; g dash of t is f of t sort of thing. What we are really doing here is we are just redefining this g of t as integral of that what we are interested in. Then, g of 0 is 0 to 0; that is essentially 0. And, g dash of t – when you take derivative here, that is nothing but f of t. So, this F of s, which is nothing but Laplace transform of f of t by definition; this part is just definition. Then, I substitute f of t as g prime of t here. And then, g prime of t – I will use the result, which is already known to me anyway; and, g of 0 is typically 0; that is what we will assume. Not necessarily assume; g of 0 is 0 here essentially, because of this integral property 0 to 0. This is equal to 0.

And then, we are left out with this (Refer Slide Time: 17:29) term – s into Laplace transform of g of t; but, g of t is (( )) So, if you just see this Laplace transform of this integral, is nothing but 1 by s into F of s; that is what we have done here. So, division by s is nothing but an integral operator; that is what it tells. All these results are available to us. So, it becomes handy for us to do further analysis of this single input single output systems, which are of primary input (( )) classical control theory.

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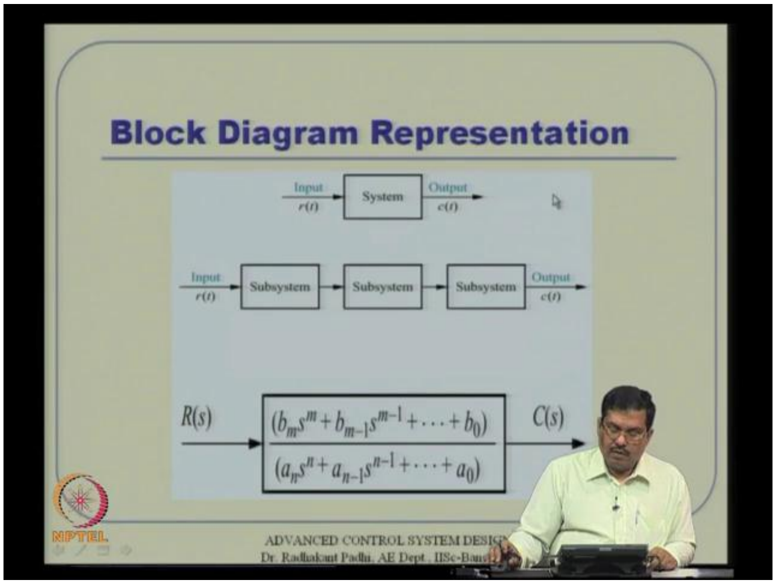

Before we move on, we will need to know what is the transfer function of a system. So, let us see that. In block diagram, any system can be represented as something like this. Many of the text books will represent this way. This is like reference input is r of t; and, some sort of output is c of t. And, some books will follow that as (( )) u of t and y of t depending on what kind of input you talk and what kind of output you discuss here. Probably, these are commanded outputs; c stands for commanded output and r stands for reference input.

And, this (Refer Slide Time: 18:39) system need not be one single unit; a connected subsystems can be there inside this system. So, there will be a system subsystem 1, 2, 3; and, many subsystems are not necessarily connected in a serial way; they can also be connected in a parallel way and things like that. So, essentially what you are interested in is a relationship between input and output. What goes on inside the system – we are not too much keen in this kind of analysis. But, we are really interested in whatever happens there – can it be represented in terms of c of t; or, in other words, this relationship between c to r basically. So, this c to r is typically represented in some sort of a polynomial in a numerator divided by polynomial in the denominator. We will be able to see that in a second. How do we get that?

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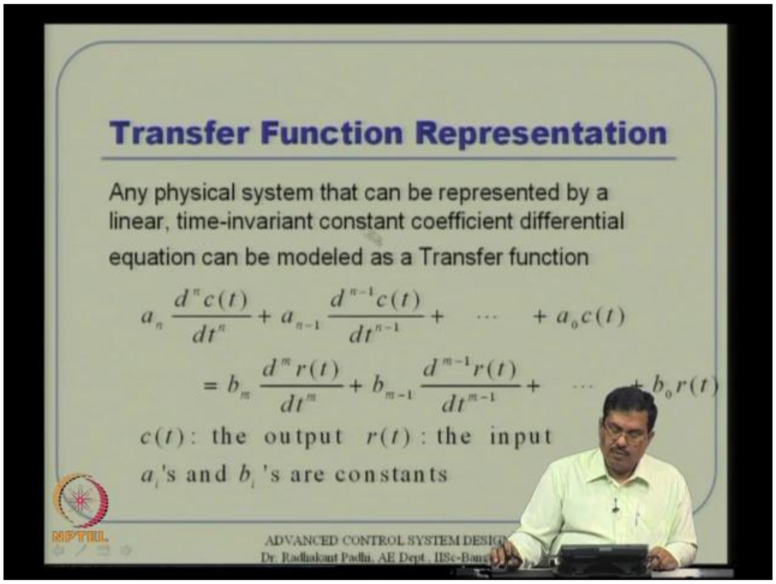

Any physical system we all know that, it can be represented by some sort of a differential equation. However, we are assuming that the system is linear, time-invariant constant coefficient differential equation; that means these coefficients what we have, are all time–invariant; they do not say anytime. That is why when you see this transfer function, Laplace transform and all that, essentially, implicitly, we mean that. We mean single input single output time-invariant system. The order can be high or low, whatever it is; but, essentially, it is a linear system and it is a time-invariant system; it is a single input single output as well.

This is the (Refer Slide Time: 20:08) physical relationship, which will give us either from Newton's law or Kirchhoff's law or black-box modelling, whatever it is. And then, what is this? We are talking about this c of t is nothing but the output and r of t is nothing but the input. So, n-th order differential equation for the output is connected by n-th order differential equation in the input in general. This can be zero-th order also depending on whatever you have. So, what do you do? You start with this differential equation and then simply take Laplace transform on both sides. Then, what happens? This is n-th order polynomial. So, this a constant approach. So, a n into s to the power n.

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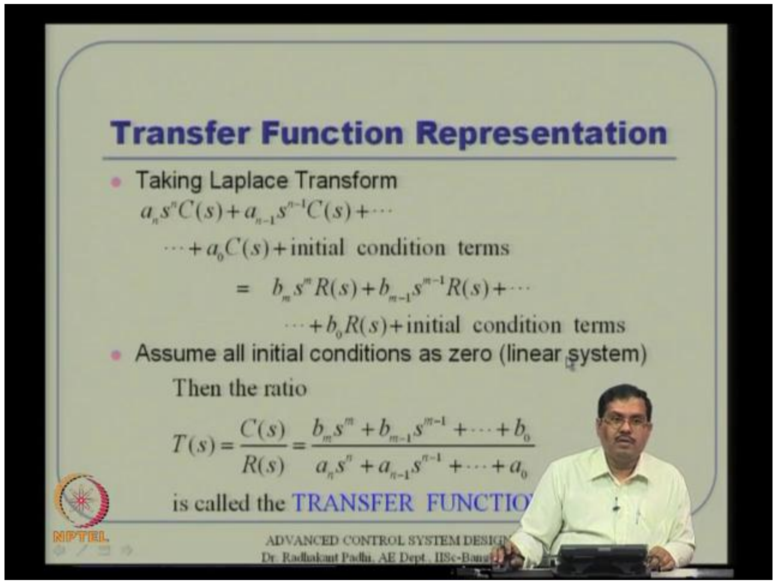

Then, Laplace transform of this C of s. That is what it pops up here. And, next one is like that; next one is like that up to this term plus all sort of initial condition terms. And, soon (( )) on the right-hand side, it is all sort of this polynomial of n-th order and then all initial conditions. And again, we will all assume that initial conditions are 0. Again, this is because of linear system property that is independent of the exact value of the initial condition. So, the ratio term… Then, once you take out this one – this initial conditions as zero, then you can take C of s is common from left-hand side and R of s is common from the right-hand side. So, then, C of s and R of s… Once you take common from left side and right side, you can talk about C of s by R of s now. So, it will pop up now this kind of a polynomial. And, that is what we saw over here (Refer Slide Time: 21:38). Then, this is what is called as transfer function in general.

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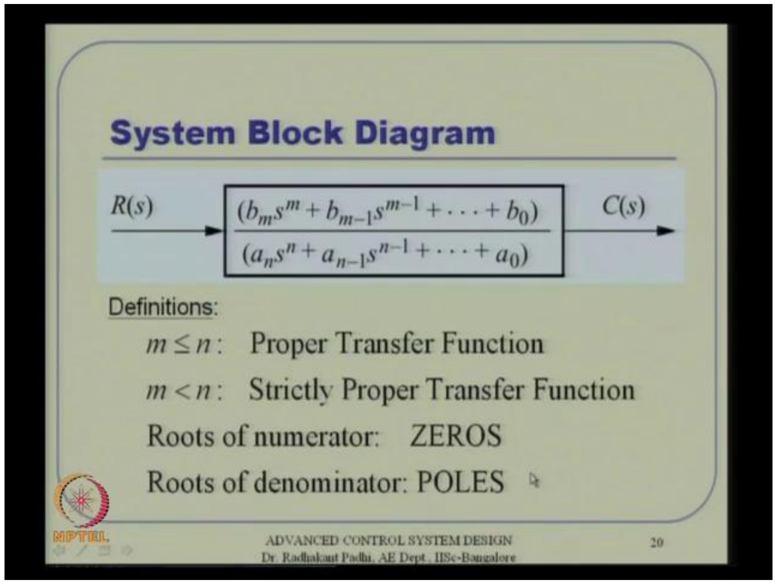

That is why we get that (( )). When you talk about system block diagram representation sort of thing and we are interested in output-input, then in general, this kind of a thing will pop up here. And, one thing to note here is, if m is less than equal to n – this typically happens in many systems, then it is called a proper transfer function. And, if m is strictly less than n, then it is also called strictly proper transfer function. And, these kind of notations are useful in modern control theory as well especially in robust control theory and all that. And then, by definition, the roots of the numerator; that means if you make it equal to 0 – then numerator part of it, that will give us zeroes. And, if we make equal to 0 in the denominator part of it, that will gives us poles; that is by definition.

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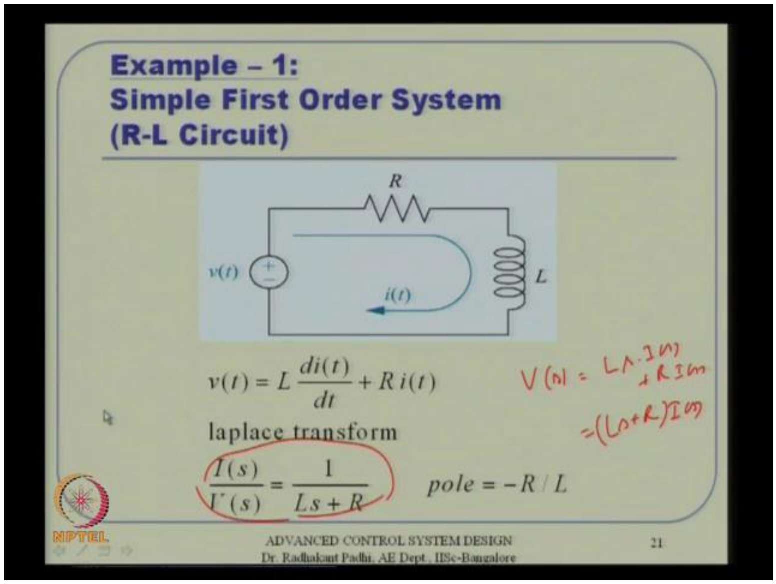

Now, move on to the example part of it. We will start with a small example. These examples are primarily taken from Norman Nise. So, you can see these examples there also. This is kind of R-L circuit; that means you have a register and just this inductor in the loop. And, v of t is nothing but the input to the system. So, we know that this voltage drop across R is R times I and voltage drop across an inductor is L d i by d t; that is from physics part of it. And, there is a current circulation, which goes on there; and, that is nothing but i of t. So, the relationship that governs this system is nothing but the voltage drop across this, which is R times I plus this voltage drop across that, which is L into d i by d t, is nothing but the voltage applied; law of conservation of energy will give us that. So, this relationship we already have in time domain.

Now, we go to Laplace domain, because we really do not want to deal these (Refer Slide Time: 23:39) differential equations directly as it is in classical control. We want essentially this (Refer Slide Time: 23:43) algebraic representation, so that we can manipulate them easily and things like that. So, we go back to like playing this Laplace transform both sides; then, this is v of s, is nothing but s into L of s and plus R times that I of s. So, if you carry out algebra yourself… Let me just quickly do that maybe. So, this is something like V of s in the left-hand side, is equal to L into s into I of s plus this R into I of s. So, this is L s plus R into I of s. So, then, this I s by V s; that is what… I s is the output here. So, that is how we get it here – I s by V s here. So, essentially, if you see the pole location, then the denominator part – you make it equal to 0. And, hence, you get some sort of a pole location at minus R by L; that is what we do. That is the first order system.

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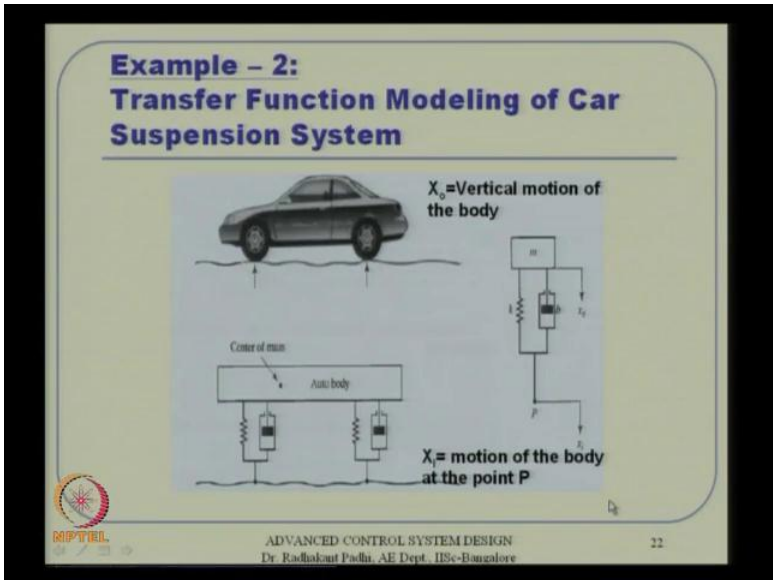

Little more complex system – we all ride on these roads with a car and things like that. So, those can also be modeled with respect to this input that is coming to the vehicle as some sort of a displacement, that is, X I sort of thing; that is nothing but the input to the system. And, X 0 is the vertical motion of the body; that is what we are interested, because there is a suspension system in between. So, whatever motion comes to the tyres, it does not go directly to the vehicle. That is how the we have some sort of a smooth ride on the road all the time; otherwise, the ride will be much more bumpier like some sort of a bullock cart for example; there is nothing there. So, directly it gets transmitted. So, compared to a bullock cart, relatively well-designed car runs much more smoother on the same road by the way. This is because this X I that goes into the system is not the x 0; that is vertical motion of the entire body. So, how do you take care of that? Because this entire auto body, this center of mass under that – there are different levels of modeling.

You can consider the auto as kind of a single point mass. That is what you are doing here (Refer Slide Time: 25:56). But, next level of modeling – we will discuss about how many inputs you have; whether you want to model it in a pitch plane only; that means only in one plane; or, you want to take care of all these four wheels independently; and then, discuss something like a 6 dof model sort of thing. It will be like rolling motion, pitching motion, all sorts of thing. So, there are different levels of modeling here and depending on what your motivation and what you want to do with that. What you are doing here is this auto is kind of a single entity – just a one point mass sort of thing. There is X I to the body – input to the body; and then, we will consider that as some sort of a lumped mass with a lumped spring and lumped damper. Essentially, it is spring mass damper systems doing mass and damper.

And, we are interested in the vertical motion of the body is x 0 or output of the system; and, input of the system is X I. So, the entire thing what you see here (Refer Slide Time: 26:50) in reality, you can really represent it based on this lumped behavior sort of thing provided we know these values accurately – k, b and m all that. But, here we assume that the values are kind of known to us, because somebody has done this parameter identification already.

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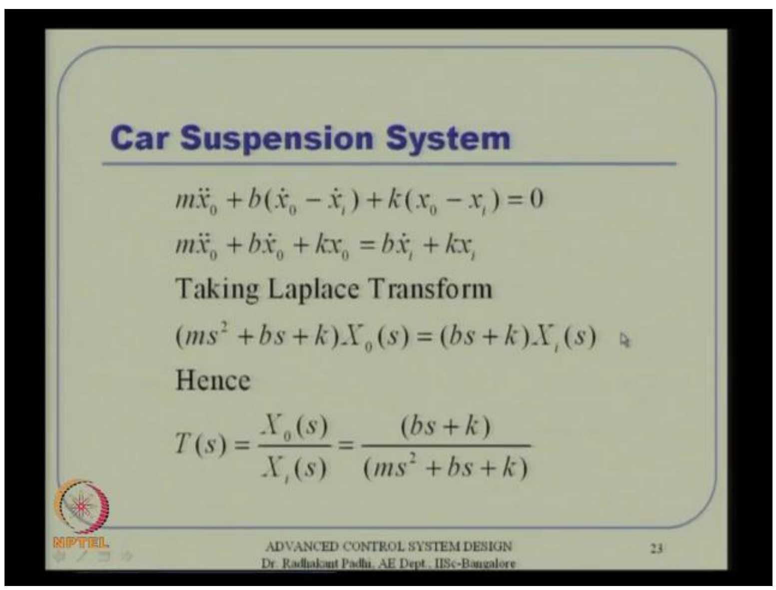

What is this? If we go back, then this loss of motion, which is a essentially Newton's law – we can draw the free body diagram and things like that, which will give us this kind of an equation. And then, this equation I can separate it out, put the input to the right-hand side and output to the left-hand side. Then, I take the Laplace transform of this left-hand side, which will give me something like this; and, the right-hand side will give me something like that. And hence, the transfer function turns out to be b s plus k divided by all these. So, that is how you see that these transform functions are derived based on physics of the system from the (( ))

Now, this (Refer Slide Time: 27:51) is all about utility. If you see this… We go back to this. This (Refer Slide Time: 27:55) is what you are looking for. And, there are variety of ways to get that. And, one primary way to get that is using laws of nature physics part of it. Starting from this differential equation, starting from these (Refer Slide Time: 28:07) laws of nature, whatever we visualize here, we will be able to derive this transfer function. That couple of examples I have given there. Next, once we know the system transfer function, essentially, we want to know what kind of inferences or what kind of conclusion we can get from those transfer functions. And, first thing to analyze there is the time domain response – first and second order system especially with respect to the step input let us say, what you get there.

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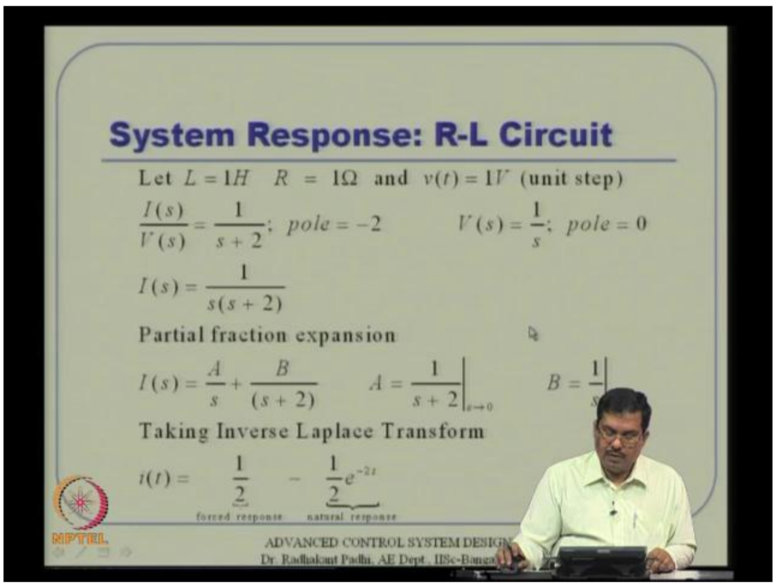

System response – I go back to the R-L circuit. And, R-L circuit was here (Refer Slide Time: 28:41). And, now, I assume some values for sake of simplicity. I assume that this v of t is unity step input; that means that is 1; and, L and I also – I will assume some values here (Refer Slide Time: 28:50). L – I assume (Refer Slide Time: 28:52) that there is 1 Henry sort of thing; R is 1 Ohm and all that just for our simplicity sort of thing. So, if you go back to that transfer function, it will turn out to be something like this – 1 by s plus 2 sort of thing. And then, the pole location turns out to be at minus 2, because the moment you make it equal to 0, s equal to minus 2; that is the pole. And, V of s is actually 1 by s, because v of t is 1. Then, this I of s… If you really plug it (( )) I of s is V of s divided by s plus 2. But, V of s is 1 over s. So, 1 over s by that.

And, how to get time domain solution sort of thing? The first thing to do here (Refer Slide Time: 29:37) is to carry out some proper algebra. And, that is done by partial fraction expansion. I have not discussed too much about that; those of you are interested, you can see Norman Nise book also. There are various ways of doing that depending on what form you have here. And, this particular form gives us independent roots; s equals 0; s equal to minus 2; that is what the roots are. They are independent anyway. They are not repeated. So, they are real and different. So, this form turns out to be A by s plus B by s plus 2. And, A and B – very quickly you can evaluate. A is nothing but s into whatever you have here. So, s into this one; that means s cancels out. And, that is evaluated at s equal to 0. So, s cancels out means you are left out with that. And, that is evaluated at s equal to 0.

And, B is similar. This (Refer Slide Time: 30:26) entire function multiplied by s plus 2, so that this will go from numerator and denominator; they cancel out. And then, you have to evaluate that at s equal to minus 2; that is what you do here. So, A turns out to be 1 by 2 and B turns out to be minus 1 by 2. And then, you plug in back here; that is, I of s is 1 by 2 divided by s. This is nothing but minus half divided by s plus 2. And, we take the inverse Laplace transform using the results that you already have. And, this one will give us 1 by 2 into 1. And, this will give us minus 1 by 2 e to the power minus 2 t; into one is there anyway. So, that is what it is. So, this part is essentially forced response and this force is essentially natural response. And, directly solving in the time domain – we discussed in the last class. You can also solve this using the differential equation – the first order differential equation, time invariant and all that. So, you will essentially arrive at the same solution.

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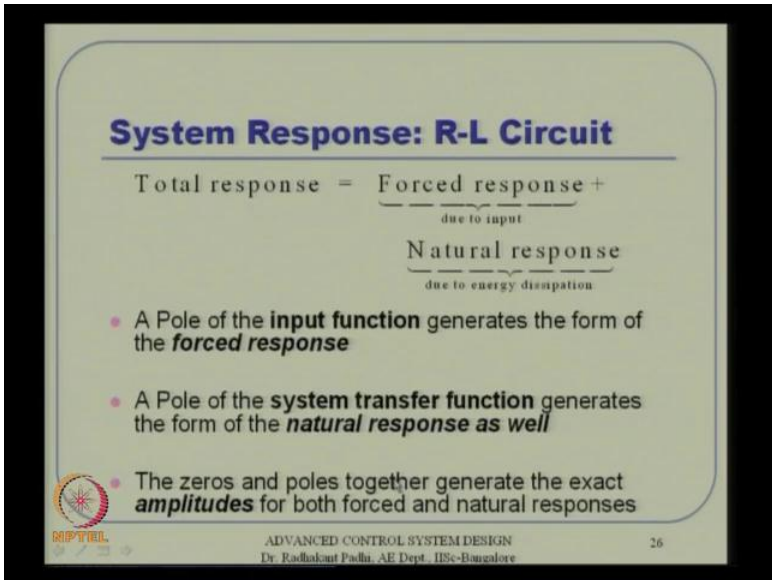

That is what will happen here. If you see the total response, it consists of forced response due to the input primarily. This is because of that. And, there is a natural response, which is due to the energy dissipation and all that. You can interpret that way. And, if you really look at the transfer function the way it is manipulated and all that, you can visualize that the pole of the input function generated the form of the forced response. See here it is very clear. Pole of the input is (Refer Slide Time: 31:59) 1 by s, because input was 1. So, the pole was s equal to 0. And, that is what led us to forced response.

Similarly, system transfer function (Refer Slide Time: 32:14) – it generates some sort of a natural response. So, this part is forced response; that part is kind of natural response. And then, the zeros and poles together – everything that you have to consider together, which will detect the exact nature of the response including the amplitude – how much it will go; where it will go; number wise; all matters on… The zeroes also play some role, which we will see towards end of this class.

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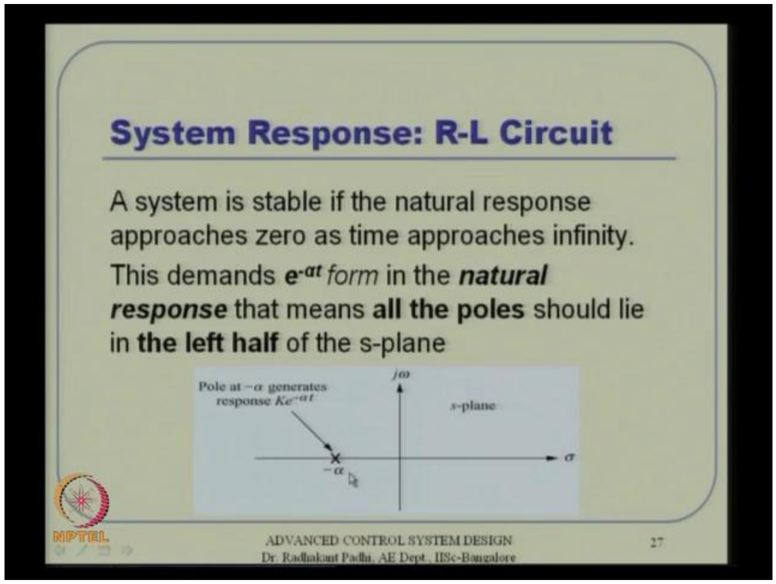

Now, moving on further, we can see that this transfer function what we have here – in s–plane, we can represent something like this, because see this (Refer Slide Time: 32:50) transfer function was something like this – 1 by s plus 2. So, in general, I can put that as alpha – s plus alpha sort of thing. So, the pole location is at minus alpha. So, pole at minus alpha generates the response something like that – K into e to the power minus alpha t. What does it indicate? That all poles should lie in the left half of the s-plane; it essentially leads us to the power minus alpha t some sort of a function in the time domain. And, that is the stabilizing term. Because of that, we kind of guess here that for system stability, all poles should lie in the left-half side. That is why, this point of time, I will consider that as a simple kind of guess we are doing. Later, we will show why it should happen and all that; even for linear M I M O system of course; otherwise, we cannot talk about poles. Then, even in that case, all poles should lie in the left-half plane; that is the result. But, we will see as we go along that.

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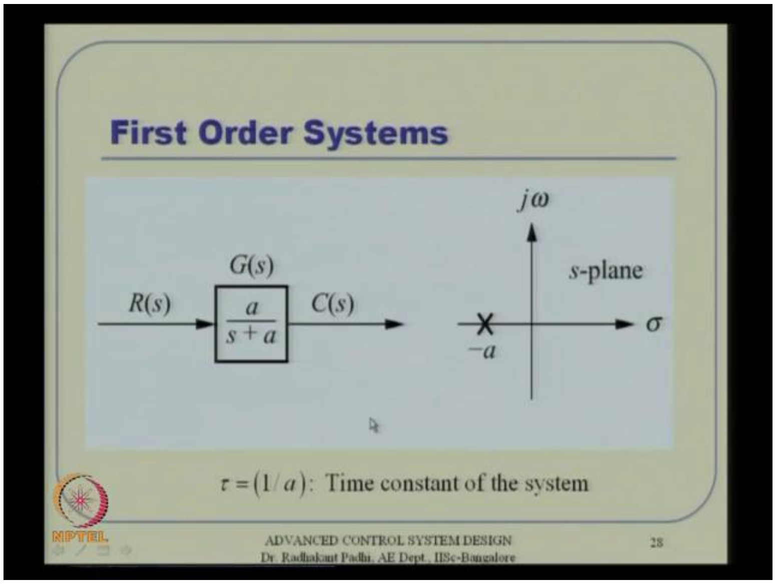

In general, the first order systems – we can discuss like this – a by s plus a. So, this is… And purposefully, we have written a and a here; there is a reason for that. If we have b here, then we write it as b by a into a by s plus a sort of thing. And, this b by a will turn out to be some sort of gain multiplier. So, that gain multiplier aside, we are left out with some standard form – a by s plus a. That is what we want to analyze. The pole location is obviously at minus a. And, 1 over a in this form is nothing but the time constant of the system. Some of that we already saw that in the last class also.

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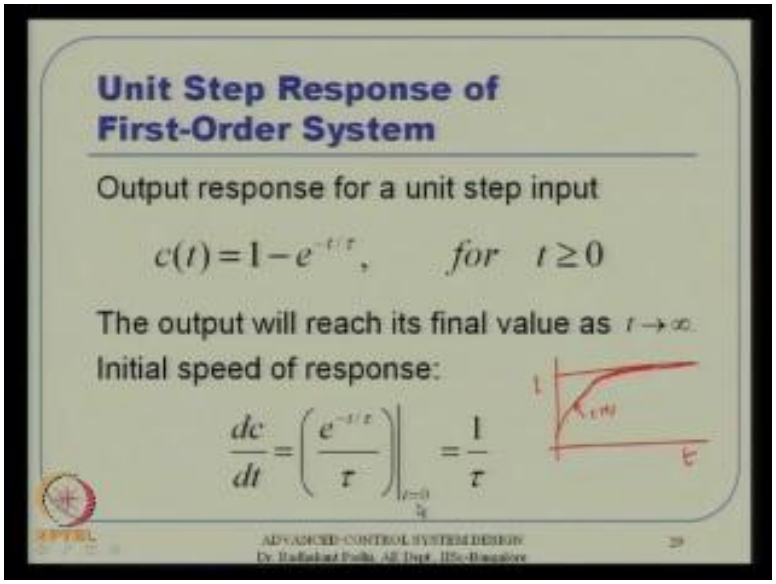

If you really consider a unity step response, that means the input is one, then we have c of t in this form – 1 minus e to the power minus t by tau. That is what is unity step response is all about. Essentially again, if you see the nature of the curve, this will be something like this. We have 1 here. Starting from whatever initial condition, you will go like this – exponential way. And, remember as t goes to infinity, then this part goes to 0; that means you are left out with 1. And, that is primarily because of having a by a here (Refer Slide Time: 35:16). If you have b by a, that may not happen. We saw that last class also. So, that is one of the reasons why we want to consider some sort of a by s plus a. So, this is the time part of it and this is c of t. What you have here is c of t. This is essentially the input part of it. If you already know the response, then you can talk about derivative of that particular function. And, that turns out to be like just 1 by tau at t equal to 0. So, initially, it will start with some sort of 1 by tau slope.

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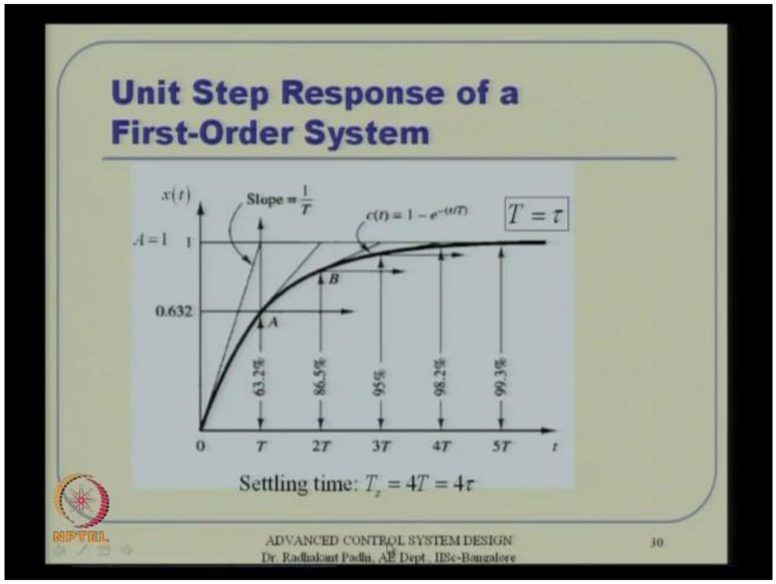

All these analyses are done for further explanations and understanding of the solution nature. So, this turns out to be in general what we saw in the last class also that, if we consider this t as tau and then this first t, then two times of the t, three times of the t, like that, essentially at 4 tau sort of thing or 4 t, we have the response, which is 98.2 percent. And, that is what we normally call that as settling time for first order systems.

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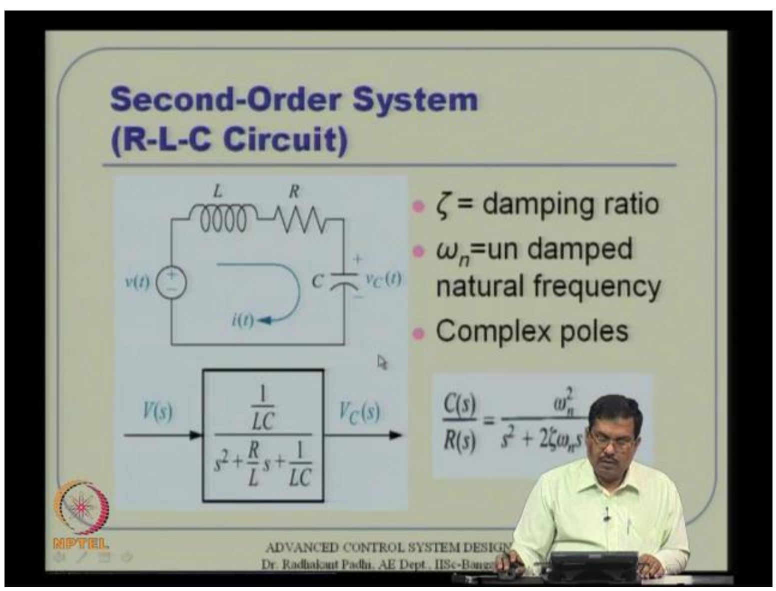

Now, coming to the second order system, again, we go back to the… Suppose we go to that R-L circuit and introduce a C now and consider V C as the output; where, V is still the input. Then, it will turn out that C s by R s is nothing but some sort of a differential equation of this form. And, this is again a standard form, where omega n square and omega n square – both appear in numerator and denominator; that is, purposefully we do that. Something else is there, you divide and multiply, so that this one pops up from the other part of it. So, omega n square appears in the numerator as well.

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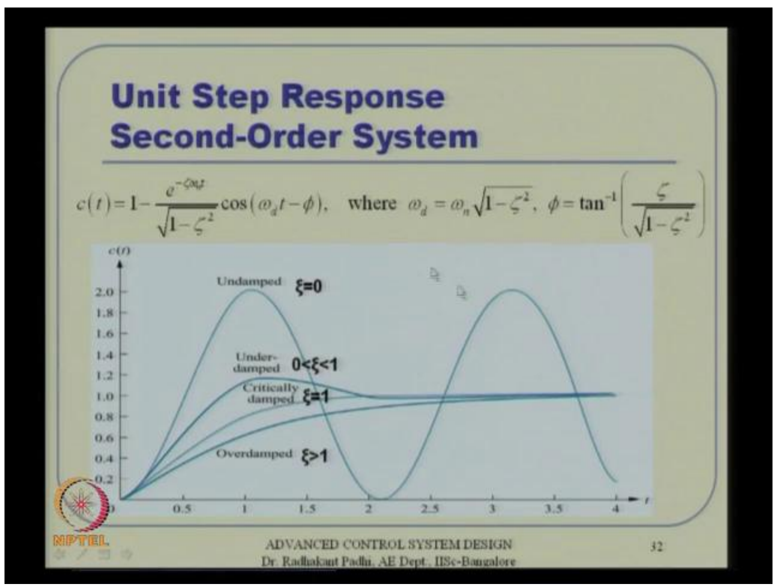

And, this particular form – again, unit step response – the solution can be derived. You apply some sort of 1 by s, because there is this (Refer Slide Time: 37:17) input is 1; R of s is 1 by s. So, you take 1 by s here. Do this partial fraction expansion; it will have multiple solutions. And then, this quadratic equation – assuming that it has these complex poles, it will turn out to be complex conjugate pair. And, you do further algebra and things like that. Then, you will essentially land up with some form like this (Refer Slide Time: 37:37). c of t will give us something like that; where, omega d is the damped frequency given in this form; and, phi is the pair shift given in that form.

And then, depending on whether you have this (Refer Slide Time: 37:49) zeta as 0 or in between 0 and 1 and things like that; and then, you have the response, where… That we already discussed in the last class as well. So, if zeta is 0, it is undamped; it will keep on oscillating that way. If zeta is greater than 1, it will have over-damped sort of behavior. It will take lot of time to go there. But, there will not be any oscillations at all. Zeta equal to 1 is somewhat in between, exactly at the boundary line. But, that rarely happens in practice. Then, zeta is 0 – in between 0 and 1, which is under-damped system. And, that is what the solution we want here.

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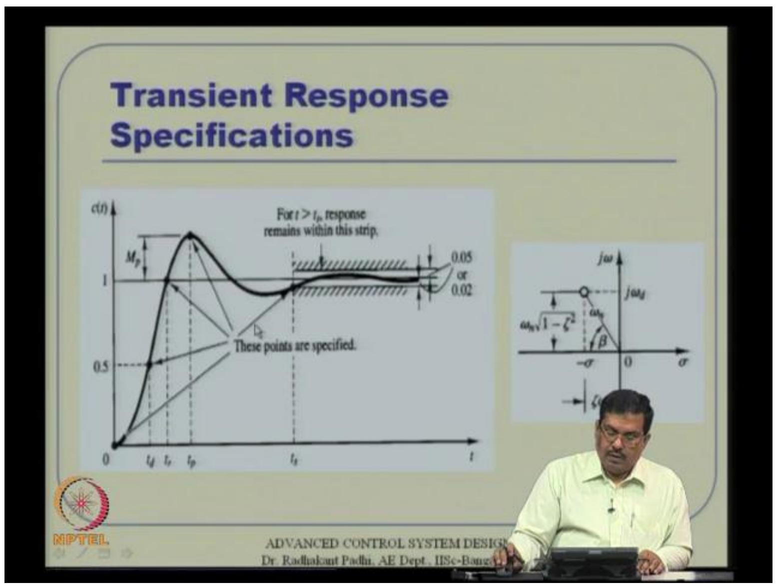

Now, we go on to further things. Then, we talk about what is a kind of peak times, settling time, all that. All that I think we discussed in the last class. For example, settling time is something like if you see the response closely and then once it enters this 2 percent condition or 5 percent condition, whatever you take it as the condition, then it does not come out of that. So, it is stable in that bound. So, the time it takes to go there is nothing but settling time starting from 0. And, this is the over shoot type of things. With the maximum over shoot it has from the reference command, whatever you have given, in this case, it is 1. Then, M p is nothing but percentage… If you convert that in percentage, then it turns out to be percentage over this essentially.

And, in general, the poles will be located something like that (Refer Slide Time: 39:18). Remember it is sigma plus or minus j omega; so, sigma plus or minus j omega d sort of thing. So, the sigma part turns out to be here. And, omega d is nothing but omega n into square root of that part of it. That is the complex part of it. And, remember if there is a pole out here, there is also a pole out here; independently, they cannot occur; they have to occur in some sort of a complex conjugate manner. So, that is what the pole location all about. And then, this radial distance is nothing but omega n. The distance along y-axis is omega d; the distance along the x-axis in the negative direction is sigma. And, this is nothing but zeta into omega n also. Some of these things are very handy in visualizing the response behavior and things like that.

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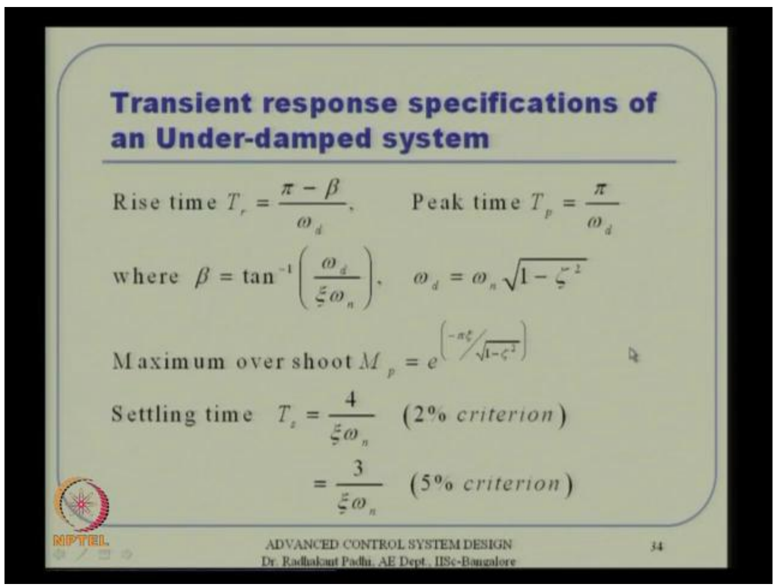

And, there are exact values available for these quantities. For example, rise time can be given exactly like this definition phi minus beta by omega d; where, omega d is already like that; and, beta can be defined that way. Peak time is given like that maximum over shoot is given like that. And, the settling time is given like that. And, these are again… As I told in the last class, these are very handy in design as design tools also including linear as well as non-linear systems. It makes sense to remember some formulas, which can be remembered easily. For example, settling time is 4 by zeta omega n assuming 2 percent criterion comes very handy. But, second order systems – you have to remember at least one more thing. So, probably, percentage over shoot is another thing to kind of remember that quickly. Once you give us how much percentage over shoot we want and how much settling time we have, then we start our design process. And, we can actually design a control system, which will satisfy this time domain conditions.

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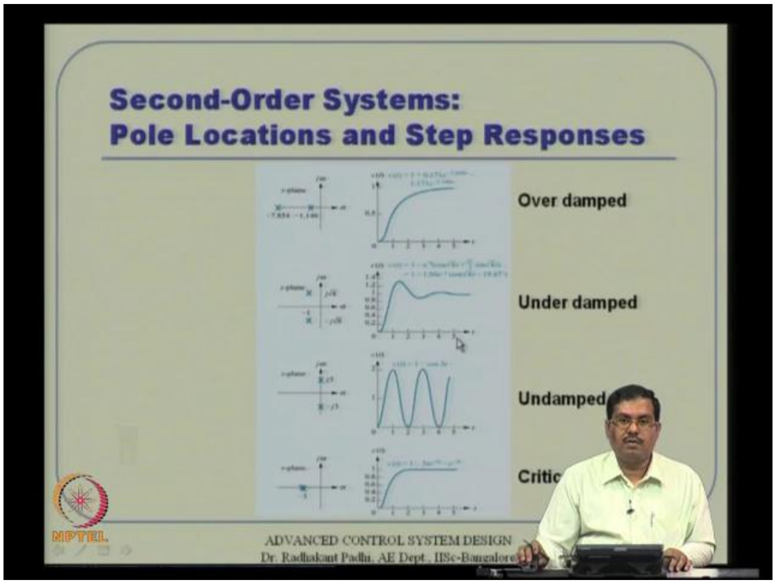

Moving further, now, depending on the pole locations, we will have different response characteristics. And, here is the pole location, which is kind of real, but nonrepeated. Then, there will not be any oscillation. So, it will go towards like that. And, suppose it is like complex conjugate pairs, then it is a classic case of under-damped system. So, it will have this response characteristic. If these two poles sit on the imaginary axis, there is no zeta part of it; zeta is kind of 0. So, it is undamped system. So, it will turn out to be like this. And then, this is very something like critically damped with doubled poles and things like that. So, depending on the nature of where it where it occurs and things like that, you can visualize the response characteristics that way. So, it is closely related to the pole location – the nature of response what you get.

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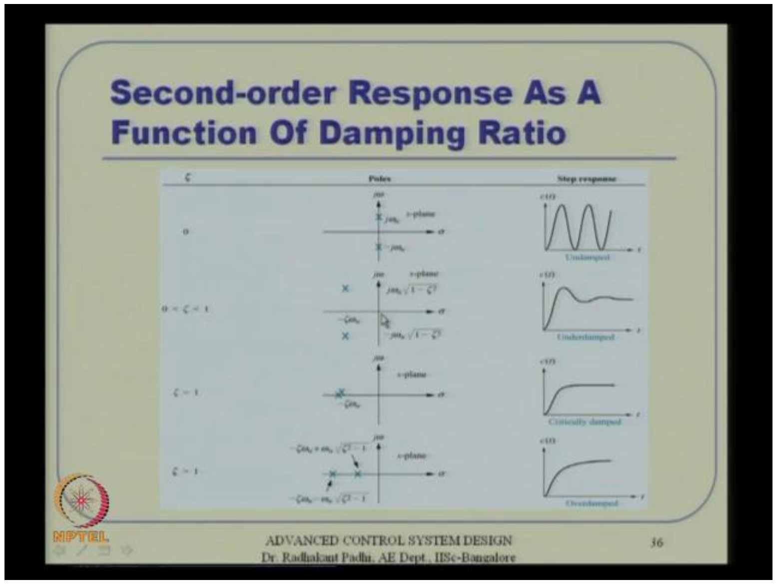

And then, the zeta plays a role here especially for second order systems. If zeta is 0, it turns out to be like this. And, we already saw that this is undamped case. Zeta is in between 0 and 1, this turn out to be like that and this is like that way. So, again, you can visualize these pole locations from zeta conditions also like how much damping you have really. All these are kind of related to each other.

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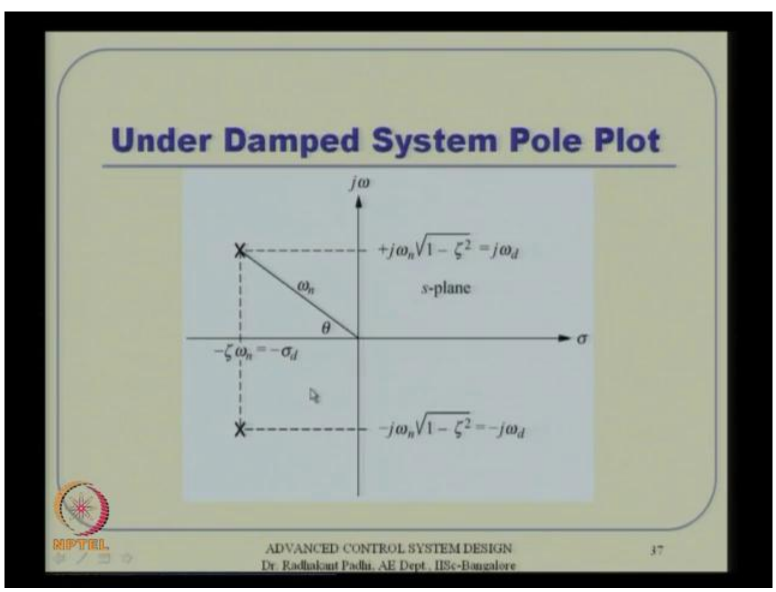

And, this is again the same plot elaborated in a bigger way. So, this is sigma and j omega part of it. This is omega n. And, this is sigma d part of it; that is, zeta times omega n with a negative sign. And, this is the omega d part of it. So, given a pole location, we will really know how much is omega n directly. If you simply consider the distance – just a Euclidean distance between the origin and that pole, then this is nothing but omega n essentially. And, if you know this projection essentially that how were its cartesian and how much is the x-axis; and, once you know omega n, you can find out zeta also.

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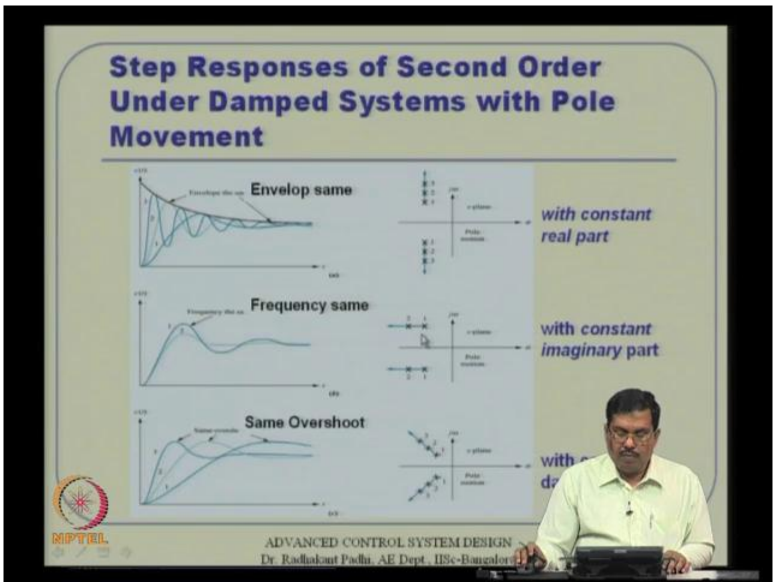

And then, we can see interesting ideas here; that what happens, if the pole starts moving away and moving away in a particular direction let us say; for example, if it moves vertically away… In other words, this j omega part of it keeps on going to plus or minus infinity, where the sigma parts remain same; then, the response characteristic becomes like that. In case 1, it will be something. If it is like that, then case 2 will be like that; and, case 3 will be like that. So, what you observe here, the moment it goes away and away, you are introducing more and more oscillations in the loop. The frequency component keeps on becoming higher and higher.

Similarly, if it goes away parallel to the x-axis or parallel to the sigma-axis rather, then it turns out that if it is 1 corresponds to that, 2 will corresponds to that. In other words, you will have more zeta in the loop, more damping in the loop. So, it will turn out to be kind of more damp sort of a system. And, if it goes radially away, then omega n remains constant. So, we can see this (Refer Slide Time: 44:30) 1, 2, 3 – will correspond to 1, 2, 3 out here. This is with constant real part; and, this is with constant imaginary part; and, this is with some sort of a constant damping ratio, because this (Refer Slide Time: 44:42) cos theta is nothing but zeta also. So, the angle remains same means zeta remains same; the damping ratio remains same.

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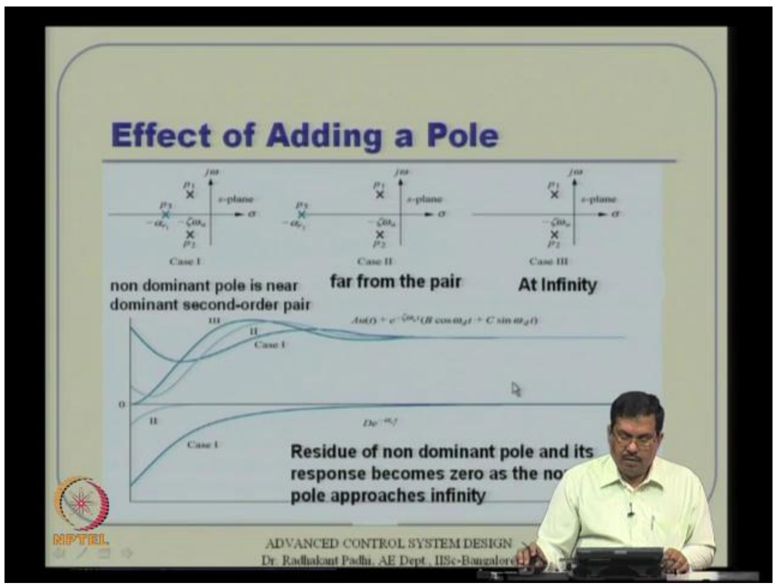

Now, what about adding a pole there? That means we already analyzed the second order system; what if we have a third order system? Then again, depending on where the third pole is, we consider it as some sort of either near to the dominant second-order pair or far from the pair or at infinity, either way. Then, what is the implication? If you see case 1, case 2 and case 3, this response behavior is something like that. And, remember this case 3 for example, this is classic second order system, because there is no third pole is here, is really at infinity sort of thing. So, compared to second, if you consider case 3 as the ideal case, then case 2 and case 1 are little far away from case 3. And, how much response is away from this plot depends on where this pole is located. The moment this pole is located not really at infinity, but still considerably far away, it is close, but not very close enough; that means there is a residual response, which is coming because of the third pole. And, if it is coming real close to that, then these two will go away from that; that means the residual will be larger. This is actually like a third order system. The second order approximation is not good here; whereas, this is kind of OK depending upon the application. And, this is a perfect second order system. So, the message here is, the third pole should not like close to the dominant second-order pole. If it lies close, then the approximate second order system will not hold good.

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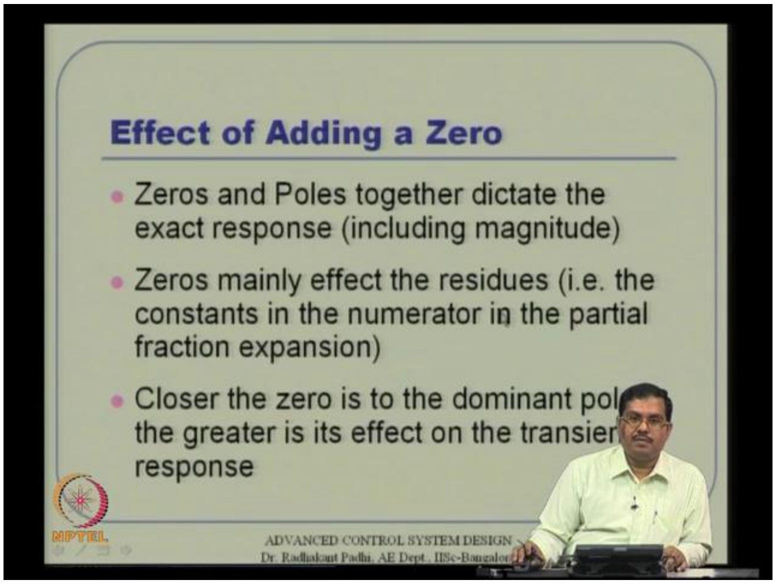

What is this effect of adding a zero in general? These are (Refer Slide Time: 46:38) all adding a pole; that means the third order system, fourth order system, fifth order system, like that. And, when we talk about classical control system; and, specially, this PID design and things like that, which is like… We will see that in a very quick way also –proportional integral derivative sort of control theory, control which is very popular in both in literature as well as practice. They all talk about accounting for the dominant second order poles; that means anything that is away from these poles are kind of neglected in the design process and all. And, from variety of gain tuning and simulation and all that, you assure that the design is OK. The whole idea here is if it is a high order system, I will consider some sort of a dominant pole kind of idea. And, they are still considered as an approximate second order system for the design purpose. So, that kind of ideas are available. But, how much you are away from the reality and all, depends on how much away the third pole is.

Now, how (Refer Slide Time: 47:45) about telling a zero term here? Now, this is (Refer Slide Time: 47:49) all about pole location. We have been discussing about pole movement both in y direction and x direction radially all sort of things; we have been discussing about pole location and things like that. And, we also saw briefly what happens if you add a pole; second to third order, third to fourth order, like that. Then, the next question obviously is what about zero? Does it play a role at all or no? As we saw that previously, the zeroes and poles together dictate the exact response; that means the stability behavior is dictated by poles; that is OK. But, what kind of response you have? Exactly where it goes? Where it starts? And, things like that. Zeros do play a role, because they also effect the residues. And, what are residues? Residues are nothing but the constants in the numerator in the partial fraction expansion. We did this partial fraction expansion sometime back.

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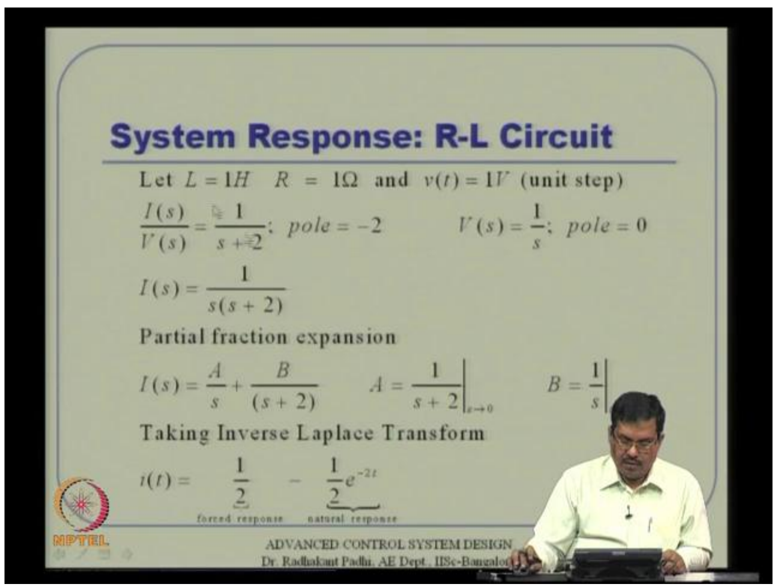

If I can quickly go there… Let me see if I can. Here this kind of a thing. The moment I do this partial fraction expansion, the exact value of this A and B depends on what you have in the numerator also. Suppose instead of 1, I have s plus 1, then things will be different here. A and B will be different, because s plus 1 will pop up here and then it will be… Here it may not matter, because s is 0. But, here it will matter. Probably s plus 5 let us say here; then, everywhere it will matter. It will be s plus 5 divided by s plus 2. This will become 5 by 2; something like that here. So, the exact value of this constant that you are getting here, the zero does play a role here. So, that is what the message here (Refer Slide Time: 49:31). So, zeros mainly effect the residues; then, the constants in the numerator in the partial fraction expansion are effected by zero locations as well. And hence, the exact value of the response and all that. Another observation is, closer the zero to the dominant poles, the greater is its effects on the transient response; that means zero – if it is close to the dominant pole, it will have more effect on the transient response as well. If it is far away, probably you can think as if it is not there. But, if it is close to the dominant pole, it will have an effect for sure.

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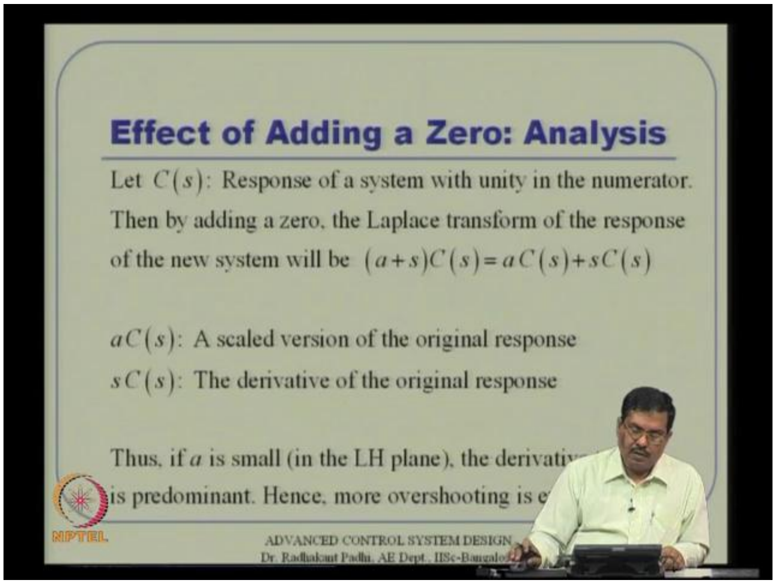

Now, let us see that in a little analysis sense, let us assume that C of s is the response of a system with unity in the numerator; that means there is nothing in the numerator other than just 1. That is what C of s – the response of the system in Laplace domain; that is C of s. Then, by adding a zero, the Laplace transform takes this form; that means this is with zero. That is the response that you are suppose to get; like a plus s into C of s; so, that means this entire… because this Laplace transform also satisfies this linearity rules and all. So, this is essentially nothing but a times C of s plus s times C of s. And, a times C of s is nothing but a scaled version of the original response. And, s times C of s is nothing but derivative of the original response; we have seen that. Multiplication of s is nothing but derivative and a is just gain multiplication sort of thing.

If a is (Refer Slide Time: 51:12) small – that means as this term is kind of not there, then what happens? If a is really small, then this plays a major role; that means the derivative term becomes predominant. And remember, if it is some sort of a let us say step input response that you are talking about, this is the type of response we are dealing with; we have some response like this; that means the derivative is also positive in the beginning. This value what you are expecting is positive anyway; the derivative is also positive in the beginning. So, that is what is going to give us some sort of idea. So, if a is small, the derivative component plays a more important role here, because derivative is also positive in the same direction sort of thing. It will lead to some sort of more overshooting; the overshooting that you expect is more if this a is less compared to this term.

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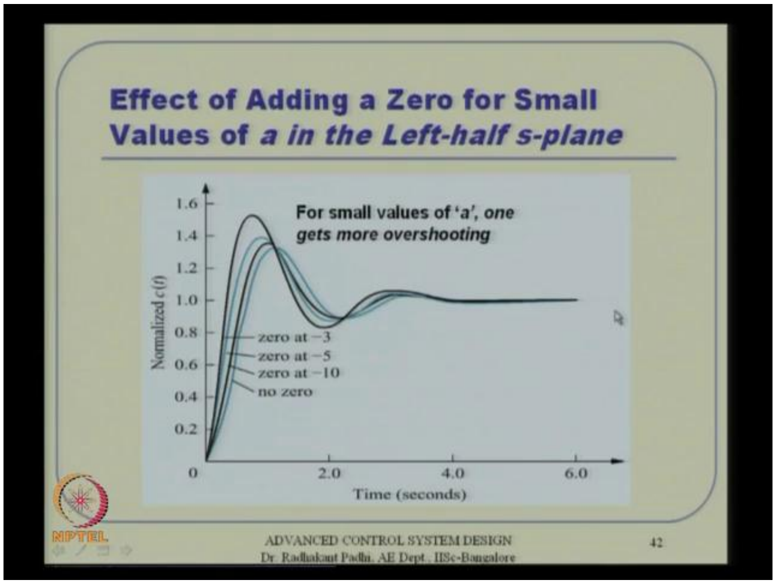

That is what you see here. With no zero, that is the response. And, with zero equal to minus 3, that is the response. But, zero (( )) far away, that is minus 10; that is very close to as if there is no zero; minus 5 – little far away; minus 3 – even further away, like that. So, the more it is closer and closer to the kind of origin, then the response becomes more and more overshooting behavior. That is all about in the left-half s-plane what happens. Now, what happens in the right-half s-plane? If you have a zero location in the right-half s-plane, even more interesting things happen.

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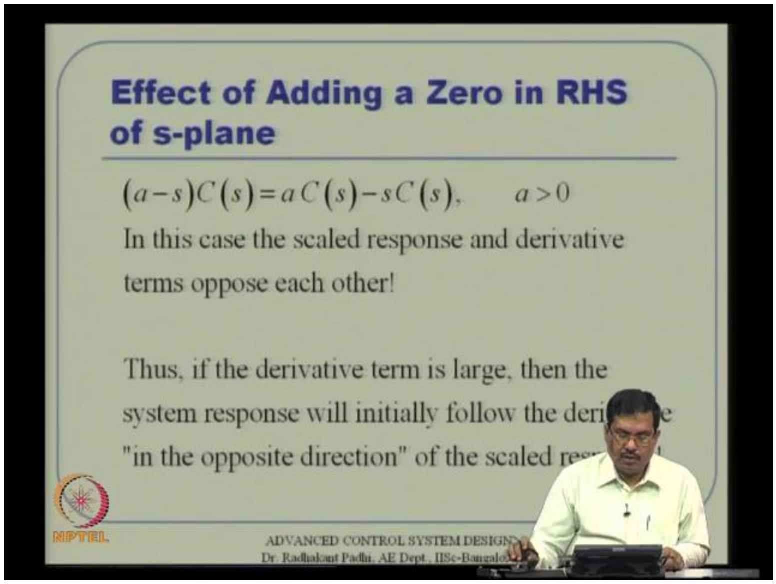

Let us see that part of it. If it is right-half side of the s-plane, then we have something like a minus s into C of s in the numerator, because s equal to a will turn out and a is positive here. Again similar way, it will have a kind of scaled version. But, here this is not plus, but minus – minus of the derivative quantity. So, that means if the derivative term is larger compared to this one, because again, this is (( )) Previously, it was plus here. So, both were kind of adding to each other. Now, here it will be minus to each other. They will kind of counteract to each other. So, if this term is larger compared to this, then what will happen? Then, the system will initially follow the derivative, but in the other direction. This is the direction that you want to go, because this is simply a scaled response. But, this will pull you in the other direction, because derivative is also positive, but it will act in the negative direction.

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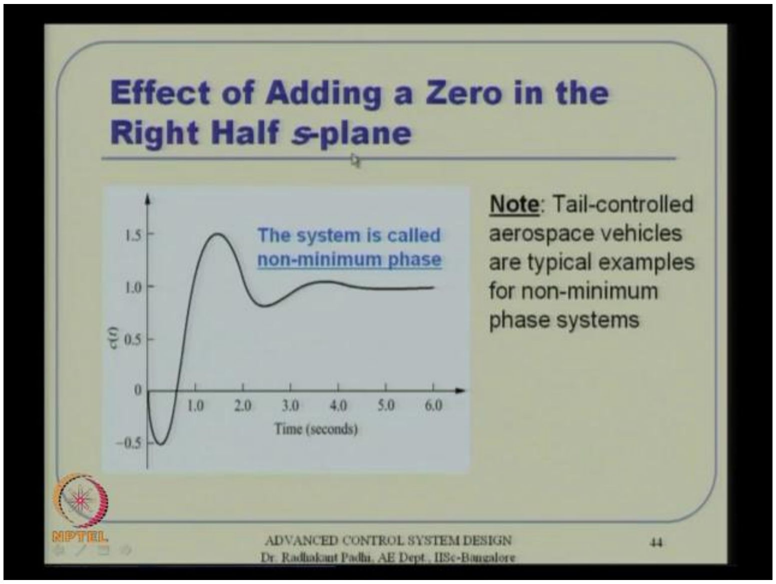

So, essentially, it will have some sort of a negative response in the beginning. And then, it will have… That is what the message tells here (Refer Slide Time: 54:11); negative response in the beginning because of the derivative. But, eventually, this will be powerful. Then, it will turn out to be like that. Remember – as t goes to infinity as time evolves, then this derivative component also becomes 0, because of… See this (Refer Slide Time: 54:30) response what we discussed here – if you have some response like this, then initially, it is very high; but then, gradually the derivative component becomes 0. So, derivative components really do not play a role after sometime. So, as time evolves, that loses significance.

But, initially, it will show its power. That is why initially it (Refer Slide Time: 54:52) will be a negative response and then it will turn out to be a positive response. And typically, these systems are called non-minimum phase systems. And, classic example is like tail-controlled aerospace vehicles. Most of our airplanes are typically tail-controlled because of maintaining nice aerodynamic behavior and things like that. But, the moment you have a tail-control, the mechanism turns out to be like… Suppose you want to go up, then first you apply a force downwards, so that the vehicle will rotate something like that. If you have more force, then it will go up; that means initially, you are applying a force downwards, which is acting on a rigid body; that means the entire body will drop down and then kind of moves up.

The force is like this (Refer Slide Time: 55:34). It will go a little together down. But, simultaneously, this is also rotating and then it will show a large angle of attack. It will have more lift and then it will start going up. So, that is what normally happens. So, most of the tail-controlled aerospace vehicles are typically non-minimum phase system. And, non-minimum phase systems do play a very important role when we discuss about inverse dynamic base control design; that means if we really talk about dynamic inversion or feedback linearization sort of ideas there, then non-minimum phase systems are very critical. In other words, the pole will become zero; zero will become pole; all sort of ideas we will see later.

And then, if you have a… This is (Refer Slide Time: 56:15) actually a zero in the right-half plane. So, for the internal dynamics later, that will turn out to be a pole in the right-half plane. And, that is disastrous; that means the internal dynamics will really behave in an unstable way, because zero will suddenly act like a pole there. So, all these things we will see as we go along. So, these little things what we are studying here are also important for non-linear control design later all the way.

With that message probably, I will stop here for this particular class. This is kind of an overview for time domain response and all. We will see little more further things in the in the next class about some more different concepts of classical control theory including stability behavior, root locus plot and things like that. So, next couple of classes will be on classical controls theory; we will not move away so rapidly. With that, I will stop here.

Thank you.