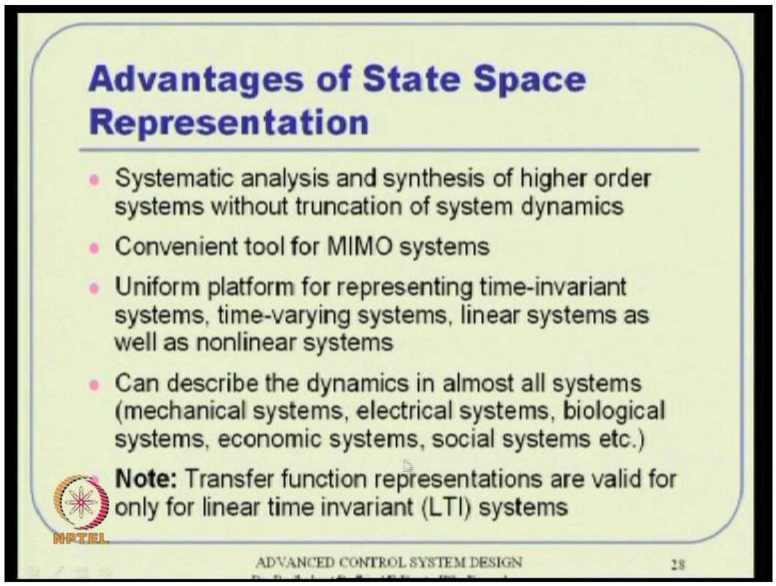
Advanced Control System Design
Prof. Radhakant Padhi
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture No. # 10
Representation of Dynamic Systems - II

(Refer Slide Time: 00:21)



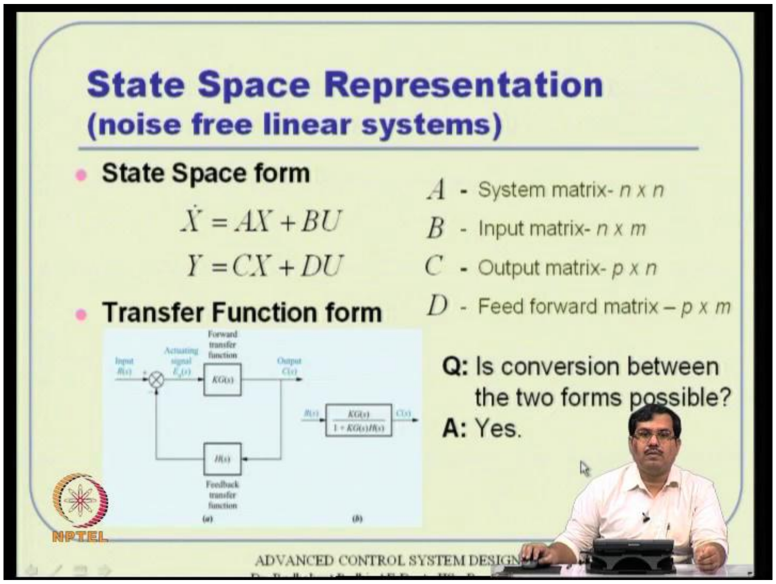
So, recapitulate these advantages of linear systems state space representation one more time. So, this is first I mean, we go for state space representation, because it gives us a systematic analysis and synthesis tool for higher order systems. And we have seen couple of examples in the last class to get kind of getting ourselves comfortable with this idea actually. And we do not want to do any truncation of the system dynamics here, that the state space representation gives us that framework. Second thing it is convenient tool for MIMO systems. Then it is a uniform platform for representing a many system like time invariant; time varying linear system; non-linear system whatever it is. Then it gives us some sought of a it can describe system dynamics for all almost systems mechanical, electrical, biological, economics, social all sought of systems. We primarily confine ourselves for mechanical systems because that is where aerospace engineering falls actually.

So, then also remember that transfer function representations are only good for linear time invariant systems. So, with that in mind we will proceed further to the next class actually. The topic here what we discussed is something like this.

Now that we know that state space representation is possible for linear systems and transfer function representations are there in linear systems already. Is it possible to have conversion between the two? That means, if I know state space representation can I transfer it to transfer function and vice verse. There are certain good things out of that actually if I if I represent the system dynamics in transfer functions domain, there are certain nice analysis I can do certain inferences, I can do in a big way good way rather and say similarly, in the state space representation as well actually.

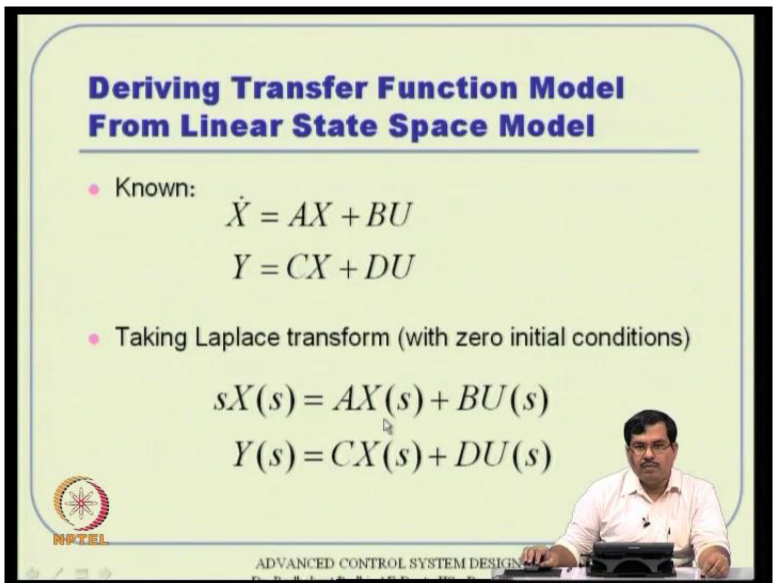
So, the question turns out for example. I mean, if you really want examples, for example, transfer function representation gives us some sought of like this phase margin, gain margin, margin concept. That is a robustness concept we want to see it from that point of view, let say and state space representation gives us the idea of controllability observability that is another big thing actually. So, we want to have some sought of a conversion if it is available or not actually.

(Refer Slide Time: 02:26)



So, the question is like this we know the state space representation form is something like this. In the transfer function representation form for single input single output is something like this, right KG s divided by 1 plus KG s actually. K (g) H g (s) into H (s). Now, as I told that the question is, is conversion between the two forms possible? and it turns out that the answer is yes its possible.

(Refer Slide Time: 02:56)



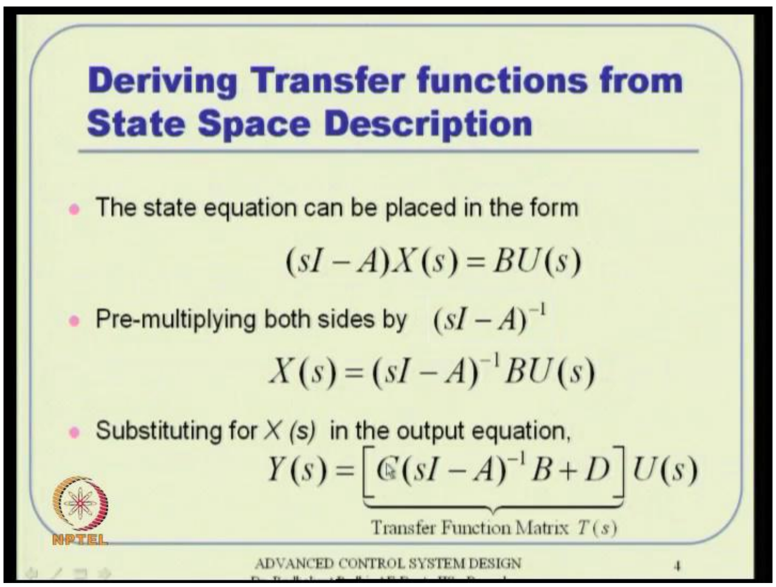
So, let see, how it is possible. Now first we see that how to derive transfer function model from linear state space model actually that means, this state space model is known to us somebody has given us already and what you are interested is in deriving some sought of a transfer function representation. How do we do that? Now let us start with this two equations because it is known to us, we will take Laplace transform and assume standard 0 initial conditions everywhere the linear systems response is not they do not depend on initial conditions I mean, the properties of the linear system are independent of initial conditions that is why 0 initial conditions are there.

So, it take Laplace transform both side, both of the equations and we derive it that. Remember there is X dot, X dot will give us s into X (s) you know vector you will sense component by component if I take I will be able to this actually. And A is a constant matrix; B is a constant matrix that will fall out to your similar, things and D will also fall outside.

So, this is where we will get this expression is coming from the first equation; second expression is coming from the second equation.

Now, we will do little more further analysis and see that X (s) and X (s) are together here. So, let us try to solve for X (s) that is what we need here. What ultimately want to we want to need is a relationship between input and output. So, output is here and input is here. So, we want a relationship between y and x actually. And then this is coming somewhere in between X (s) we want to eliminate that actually. How do we do that? We will be able to do that taking help of the first equation here.

(Refer Slide Time: 04:38)



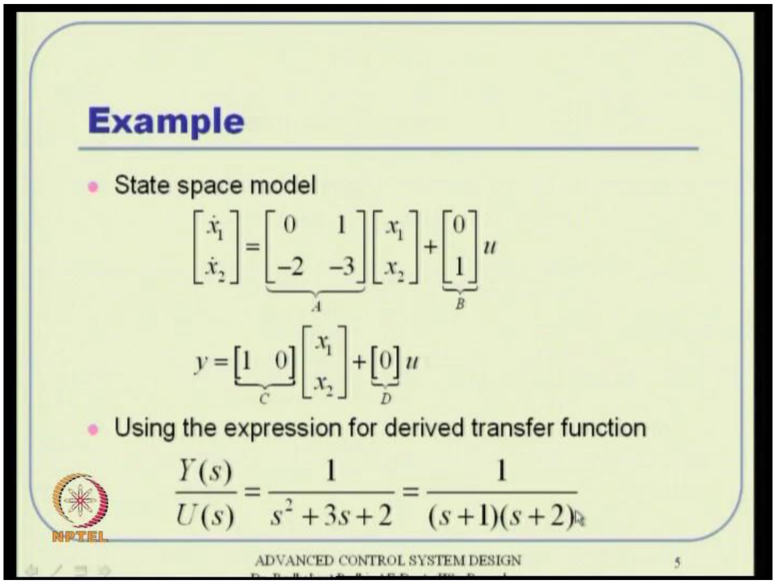
So, let us do that. So, if I take the first equation and try to see this is if I if I want to bring this one to the left hand side and try X (s) common. Then I have to do this algebra s I s into I minus A remember that, because this has to be matrix actually what you see here and in a vector sense if I write it s into x in a component by component multiplication. In a vector matrix sense I have to do s I minus A here. So, this is s I minus A into X (s) is equal to B times U (s) actually. So, what you see here this same equation s I minus A into X (s) is equal to B times U (s) that is what is written here.

So, if I pre multiply both sides by s I minus A inverse, then the first 1 is identity that is that is gone, but this will come to the right hand side. So, you see s I minus A inverse into B times e that is what will happen here. So, now I have got a solution for X of s. So, I will go back and try to plug in here whatever s of X (s) I got and then I am done, because this is what will give me Y (s) is C (s) into s I minus A inverse into B plus D that is, because that D comes from this D actually. So, that is what will happen C into s I minus A inverse plus into B plus D into U (s) actually.

So, this expression what you see here, is nothing but you can represent this as some sought of a t (s) that is what you have seen before, but t (s) is not possible to write Y s by U s in general, because Y s by U s is by writing that you are assuming single input single output again. This is multiple inputs and multiple outputs that means, this is a vector and U (s) is a vector, some vector by some vector is not defined actually. So, we will most likely in I mean, in all possible cases will just stop here, and tell this a transfer function matrix t (s) whatever matrix is coming up here, in the sense that Y (s) is equal to t (s) into u of s. Some books will write Y (s) by U (s) but that is just a notation that is not an algebra Y (s) by U, U (s) is not defined in terms of algebra.

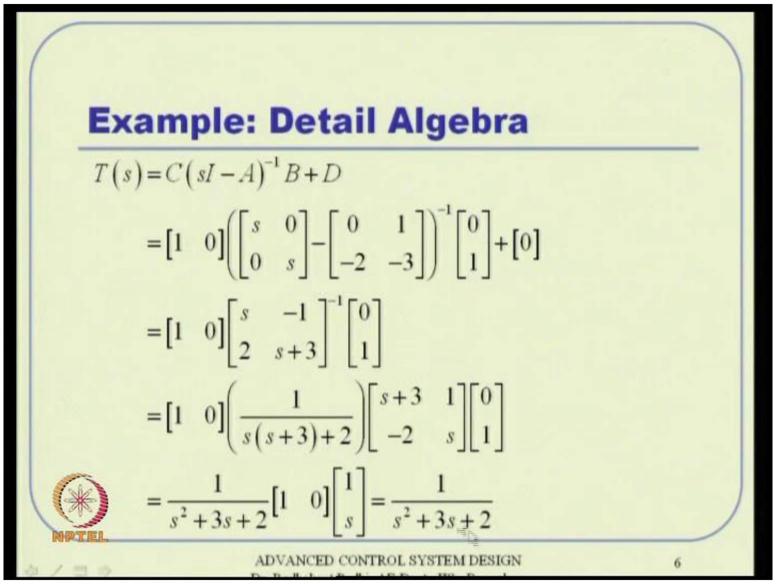
In algebra they mean, this one only Y (s) is equal to t (s) into U (s) and t (s) expression is given by this C into s I minus A inverse into B plus D. And remember this is actually unique way of doing this the moment you know A, B, C, D this is the t (s) there is no there is no second expression available actually. So, that means converting from state space to transfer function representation is always unique and this relationship is given by that.

(Refer Slide Time: 07:21)



Now example, sense if I talk about a simple example that we discussed last class probably this is a 0 1 and minus 2 minus 3, and then 0 1 actually. So, we knew this A, B, C, D matrices. So, can we talk about a transfer function, remember this is a scalar input this is a scalar output. So, it is actually a SISO - single input single output systems. So, I can still write Y (S) by U (S). So, I will not carry out the all the steps again I will simply apply this formula remember D is 0. So, I will simply apply this formula what I have C into s I minus A inverse times B D 0 anyway. So, if I do this algebra as will turn out to be like this

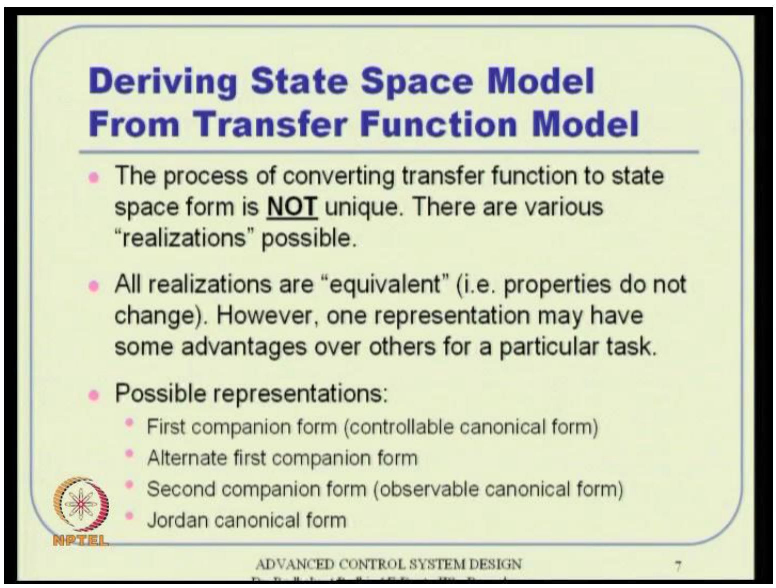
(Refer Slide Time: 08:00)



How is that let see the algebra C into s I minus A inverse plus into v plus d. so, C is this s I turns out to be a 2 by 2 matrix. So this is s and s here, 0 0 here and this is s I minus A a is a is what you have here. So, I will just pick up a from there so this is s I minus A the whole inverse I have to take into multiply by this B matrix what I have then plus 0 actually. So, if you simplify this actually this two subtractions first it will turn out to be s here and 0 minus 1. So, it is minus 1 here 0 minus of minus 2 that means, plus 2 here and then s minus of minus 3 that means s plus 3 here. So, you have C you have s I minus A and I have to still take inverse of that then, I have to multiply with d, then d 0 anyway actually. Now C is here then this matrix inversion for a 2 by 2 thing it is it is very easy you have to do 1 by determinant of that determinant is this 1 s into s plus 3 minus o minus o that means, plus 2 here, this 1 by determinant of that and as far as ad joint matrix is concerned for a 2 by 2 it happens that you can exchange these 2 elements and you can change the sign of these 2 elements.

So, you can exchange these two elements to s plus 3 will go here and s will come here. You can see their form the ad joint matrix formula it is very clear actually why it happens. So, one way determinant into ad joint matrix and the ad joint matrix for a 2 by 2 is diagonal elements will exchange s an off diagonal elements will change their sign actually. So, it take minus 2 and plus 1 here and then you put it there, and then we are ready with a some sought of a I mean, matrix here non matrix inverse anymore. So, this is a scalar. So, I will take it out and this is s square plus 3 s plus 2 which will come out here then this is C matrix then this 2 matrix if I multiply 0 into s plus 3 0 plus 1 1 into 1 that is why 1 is there and minus 2 into 0 is 0 plus s into 1 that is s actually. Again if I multiply this 2 quantity this is 1 plus 0. So, ultimately I am left out with one actually. So, this entire algebra t (s) is nothing. But that that is what you get it here actually. So, that is that is conversion from transfer I mean, state space to transfer function actually.

(Refer Slide Time: 10:23)



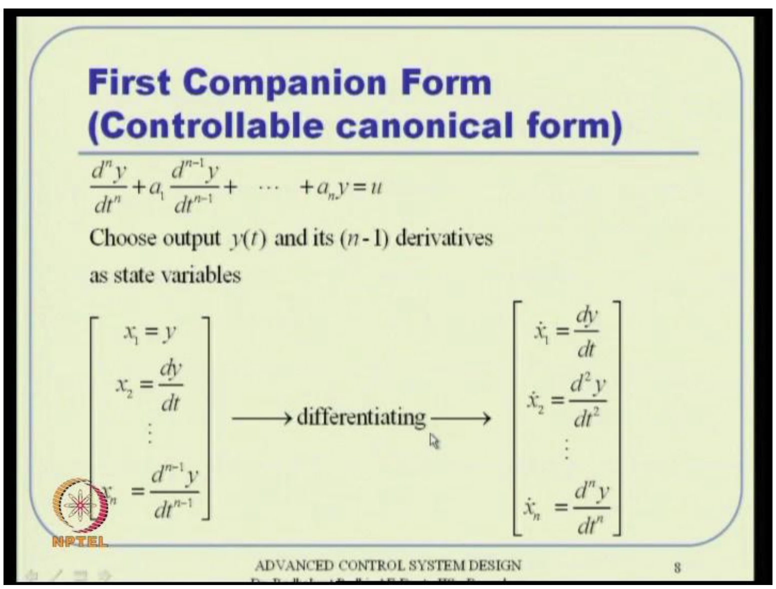
Now can it be possible is it possible to do the reverse actually. That means, we want to derive the state space model from transfer function this time. And unfortunately this happens to be non unique that means, this conversion process what you see here is certainly not unique you can do it one way I can do it other way actually.

So, each of that getting that A, B, C, D matrix from a transfer function representation is something called realization. So, obviously, because this process is not unique there are various realizations possible, but it turns out that all realizations are equivalent that means, the properties do not change actually. The properties of stability especially or control ability if you see though they would not change.

However a some representation may have some advantage over the other for a particular task for example, if you go for control design, then probably we most likely we opt for control level canonical form, but if you talk about to some sought of a observer design or filter design probably the natural choice will be going for a second companion or observable canonical form. Now certain advantages especially this numerical computation algebra and all sought of things there are advantages by doing one form of realization over the other actually. So, but nevertheless also remember that all realizations are equivalent that the properties normally do not change so much actually.

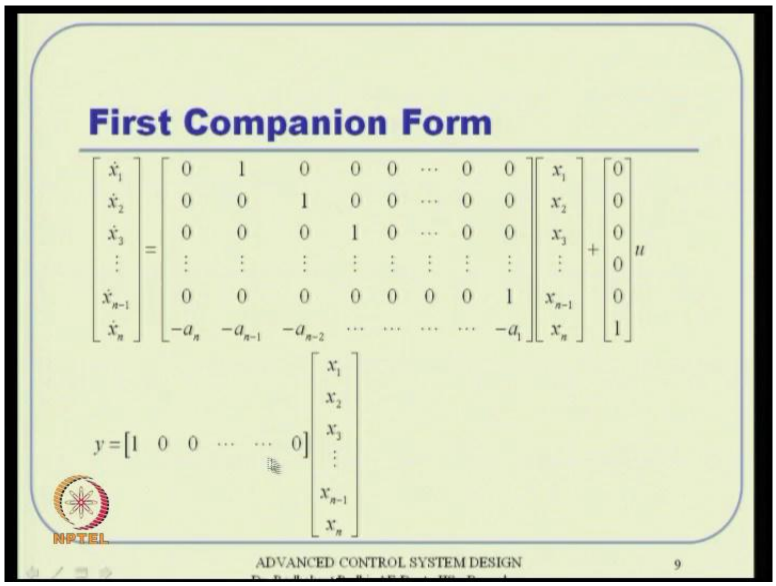
Now, possible representation we have already seen, what is called as first companion form, First companion form which is also known as controllable canonical form. These alternate first companion form possible, the second companion form possible, the Jordan canonical form possible some of these form we will see. And this is not an adjustably throughout the way there are various realizations people have done extensive course and a linear system theory probably will tell you some of these more detail in a rigorous way, but here we are interested in getting the ideas in a proper way.

(Refer Slide Time: 12:22)



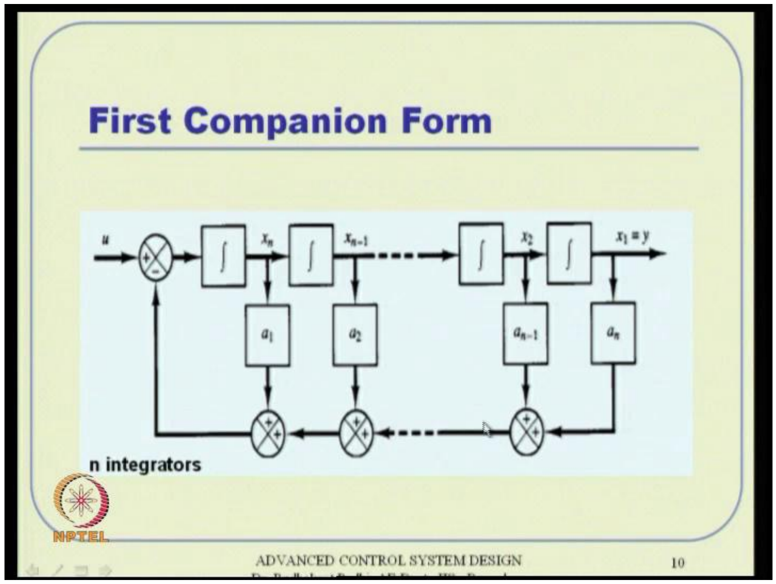
Let us see one at a time. The first companion form is something that we already discussed in the last class just to recapitulate. This is the form that you are this is the differential equations of course, we start with and we now started with a with a transfer function remember that what will soon tie up with this actually. Let us start with a differential equation then we have seen that these are the ways the to define these state variables and this is the derivative I mean, x 1 dot, x 2 dot all will fall in that way.

(Refer Slide Time: 12:52)



And then taking the help of the last equation it is possible for me to write it this way. So, we discussed that all the details in last class actually.

(Refer Slide Time: 12:58)

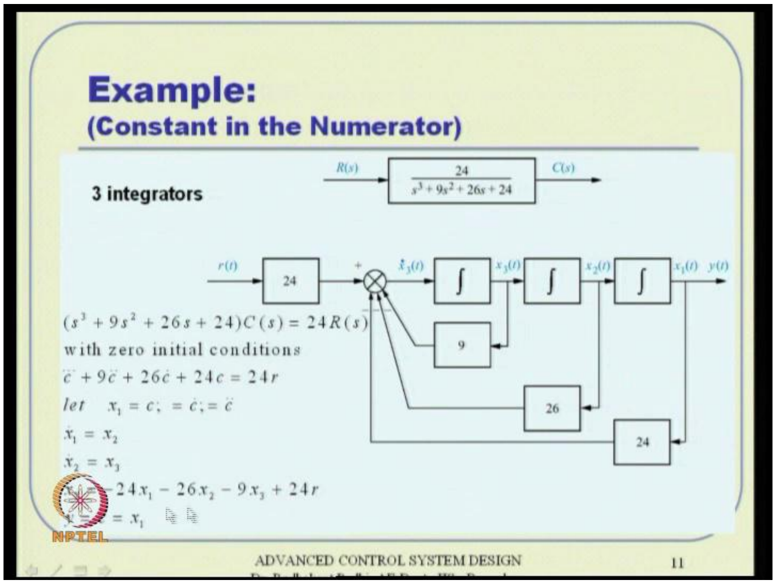


Now, if you I mean, if you see this realization why you are studying this primarily is, because suppose you are given a transfer function is possible to write this system equation in this form actually. Once it is written in that form differential equation form, then we will be able to proceed further this way we will see that in a example also. Now if you see this representation. What you see here now really want to realize this system when I let say hardware design and things like that. How do you go there? How this signal float express? remember this a definition sense what I am telling is x 1 dot is x 2 x 2 dot is x 3 things like that actually.

So I will put n integrator first let say on the line first row actually and what is happening x 1 dot is x 2. That means x 2 integral is x 1 and similarly, if you see x n dot is integral of that is x n. So, there is x n dot here integral dot x n integral dot integral of that is x n minus 1 it will all the way continue to x 1, so this n integrators I will put it first actually this way and now I have x n dot to take care here and this x n dot is nothing, but this expression actually what I see here.

So, this is something like minus a 1 times x n minus a 2 times x n minus 1 like that actually. So, if I see this diagram what I see here, is this is x 1 I mean a 1time x n a 2 times x n minus 1 all taken together all the way sum a sum together with a negative sign here ultimately plus u and u is coming directly here that way. So, x n dot is constructed through all these integrators with multiplication of a 1, a 2 and all that, sum together then finally, summed with a negative sign actually. So, that is way to kind of realize this integrators actually.

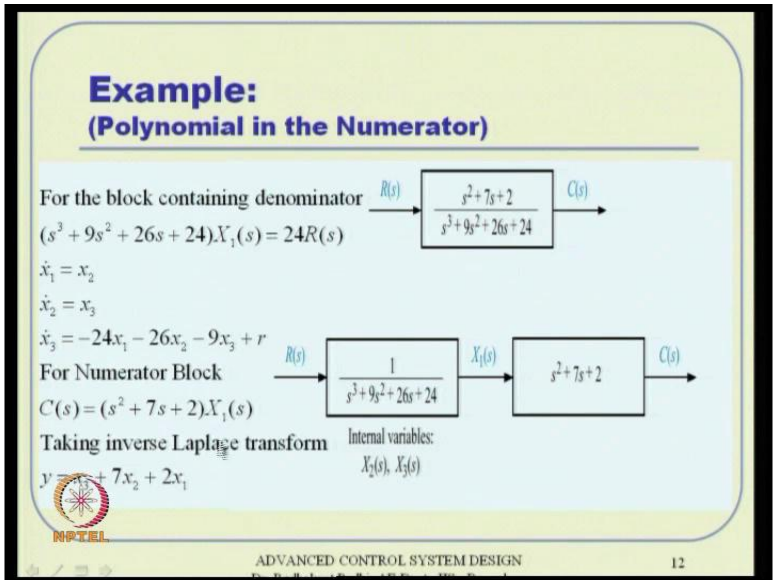
(Refer Slide Time: 14:54)



Now, an example we will see, this lets say start with this example actually remember this starts with this is a transfer function representation. Now starting with this transfer function if I what is the meaning of this transfer function C s by r s is this transfer function right. So, if I start with that then this into C s whatever this denominator into C s is equal to numerator into r s C s by r s is this expression.

So, c s into numerator denominator is equal to r s into numerator that is what is happening here. Now if you go back to a Laplace transform ideas then x q means, standard derivative s square means, second derivative like that actually. So, you land up with some equation in the differential equation. Form the third order differential equation and again we will I will go back to this system variable representation and thing like that. So, we will discuss x 1 is equal to see I think there is small print mistake here, x 1 is Cc x 2 is c dot and x 3 is c double dot. So, then x 1 dot is x 2, and x 2 dot is x 3 and, x 3 dot is actually all these things that you see here and that is something like x 3 dot will give us this expression actually. So, y equal to c if I take then that is x 1.

(Refer Slide Time: 16:17)

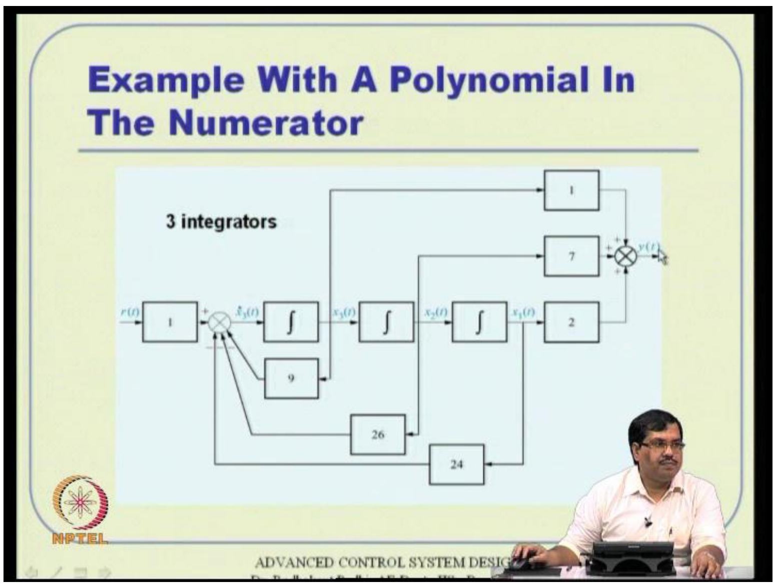


So, I will be able to proceed further and then write it in the form of state space representation that is all if I have a constant term in the numerator. Now what happens if there is a polynomial in the numerator that is also very much possible right, in the transfer function may not be a constant term actually? Now the same trick applies what you do is this transfer function what you see here, I can represent that as multiplication of two blocks actually. One will be the one by this term then the next block will be multiplication that way. And we are all talking about proper transfer functions remember that that means, the order of the numerator polynomial is less than equal to the order of the denominator polynomial and strictly proper means, it is strictly less than that actually.

So, here is a strictly proper transfer function where do decomposing that into two blocks one in this bock one in other block. And this particular block we have just studied actually right other than that 24 constant which really comes here, I will if it is one this one will come here actually. That part we already know and also remember this the way to way to realize also there 3 integrators here, x 3 dot, x 2 I mean, x 3 dot integration will gives us x 3 x 3 integration will give x 2, x 2 integration will gives us x 1 then multiplying with that appropriate constant what I have and then sum it altogether with a negative sign actually. Now coming back to this example, this is one here. So, we will have one there that means, this part of the equation that is already known to us. Now what happens to the output actually output happens to be as this polynomial into x 1. So, this polynomial gives me something like x 1 double dot plus 7x 1 dot plus 2 x 1 and x 1 double dot is nothing but x 3 x 1 dot is nothing, but x 2 and x 1 is nothing, but x 1.

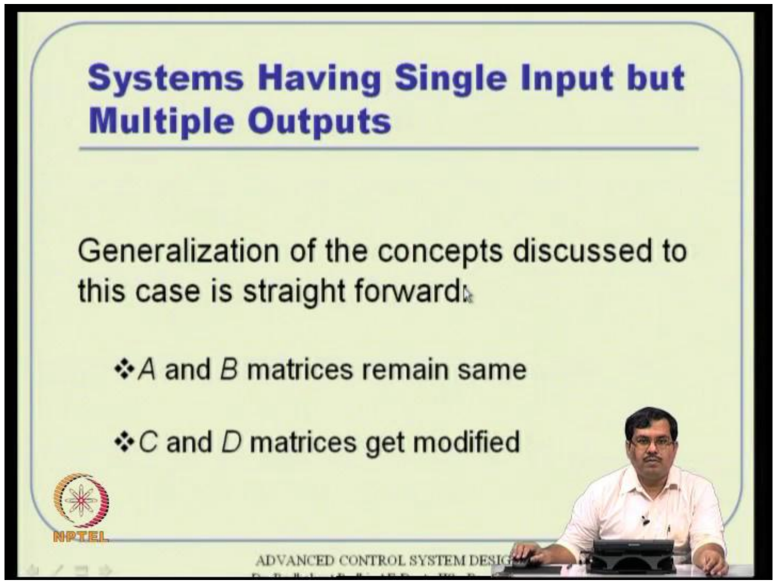
So, the output equation is something like this x 3 plus 7x 1 plus 2x 1 x 3 plus 7x 2 plus 2 x 1. So, the state equation remains same in the dynamic equations remains same, but the output equation takes this form actually earlier remember it was just 24 just x 1 sorry now it is not only x 1 but it is like this actually.

(Refer Slide Time: 18:30)



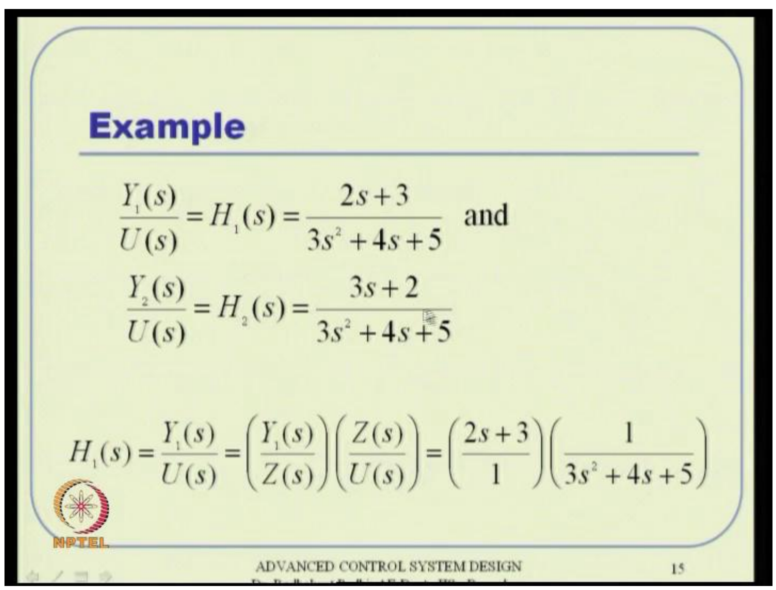
How to realize this? again that this bottom side of the story remain same, because the state space realization remains same. And then, it talk about the top side you take this branches out here remember this is x 3 plus 7 x 2 plus 2 x 1. So, this x 3 component I'll take through one this x 2 component I will take it multiply through a 7 this x x 1 component multiplied by 2 then sometime together I will get y. So, that is how we represent actually.

(Refer Slide Time: 19:02)



So, system saving what about generalization further generalization will talk about single input but multiple outputs they are all single outputs. What you saw that y right single outputs actually how you generalize for multiple output. A and B, remember and B matrices will remain same but C and D matrices will get modified that same example, for that.

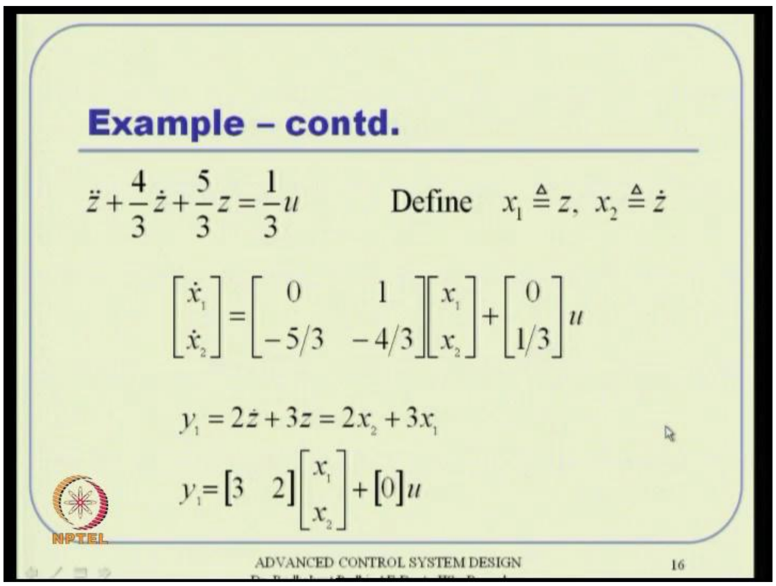
(Refer Slide Time: 19:23)



Let us, talk about two equations Y 1 of U (s) Y 2 by U (s) H 1 and H 2 given by this polynomial. And mostly the denominator polynomial will remain same, that is how the system transfer function representation will pope up actually. Once the denominator remains same the numerators are different actually. This is 2 s plus 3, this is 3 s plus 2 rather.

So, what will happen this H 1? What I see here, Y 1 by s then this Y 1 by s I will represent this polynomial what I see here. As I multiply and divide by Z (s) actually rather. And what this Z (s) I will take this Z (s) nothing, but one actually. Let me represent it that way. So, this is 2 s plus 3 I am remember, I am still analyzing only H 1 H 2 I am not considering here. I know H 1 I will represent it that way again this multiplication up to blocks right is not first block and I can multiply both ways I mean, this one I can always putting back right hand side actually.

(Refer Slide Time: 20:36)

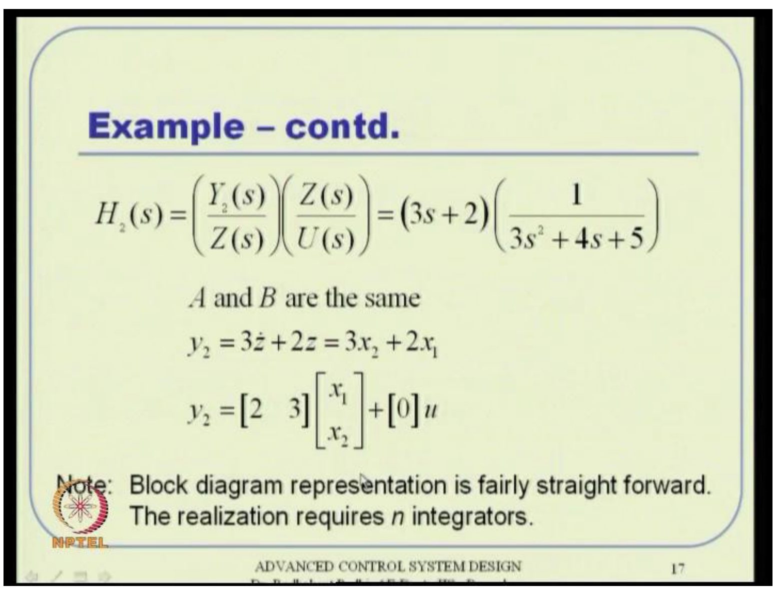


So, then I will get again similar things what we discussed before, I will define let say x 1 is z and x 2 is z dot. And hence, I will I will from this polynomial what I see here the denominator part this right hand side of the story. This part is corresponds to right hand side from there I will talk about I will be able to extract this equation x 1 dot and x 2 dot actually right. And then, when this 3 what you see here will actually divide by everywhere right 4 by 3, 5 by 3 that kind of expression will come you can do the algebra little actually.

So, the equation will turn out to be something like that 4 by 3 plus 5 by 3 equal to 1 by 3 u. So, then you would define x 1 and x 2 that way and then this if you do that algebra that is x 1 dot is z dot, z dot is x 2 again same thing will pope up here 0 1 and 0. And x 2 dot is nothing but z double dot and z double dot is all these equation in the right hand side. So, if you put them together it is 1 by 3 into u so, that is 1 by 3 comes here minus 4 by 3 into z dot that z dot is x 2 so that is where it is minus 5 by 3 into z.

So, this expression is given like this x 1 dot, x 2 dot together like that then output equation is given as 2 of 2 times z z dot plus 3 times z right. 2 of z dot 2 time z dot plus 3 times z actually. So, that is where you go back and sees what is z and z dot? they are simply definitions actually. z is x 1, z dot is x 2. So, y 1 is nothing, but that so in a in a vector matrix representation it turns out to be 3, 2 which is my C matrix and plus 0 times u that is my d matrix actually. So, this is my this is my B; this is my C; this is my D; in this way of realization basically.

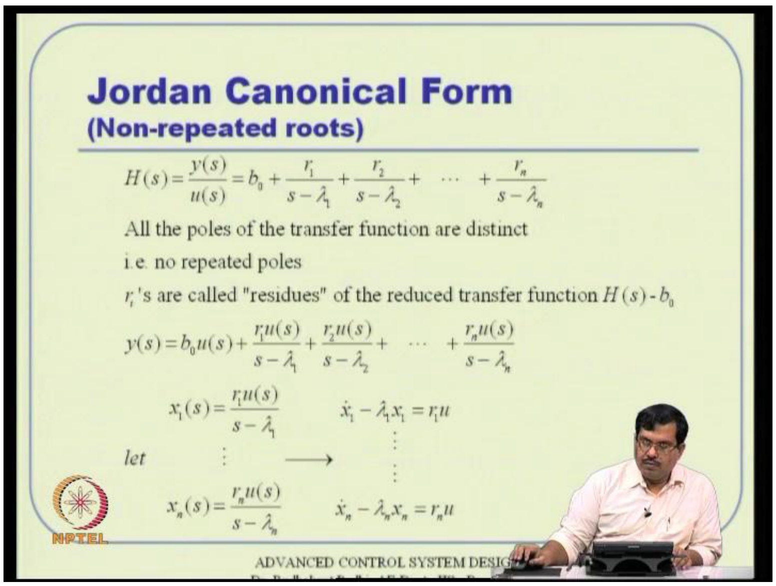
(Refer Slide Time: 22:36)



Now, H 2 will see what is H 2 here, and H 2 is the second one and second one remember that that comes from this actually. This expression so this 1 by term will remain same 1 by 3 s square plus 4 s plus 5 that will remain same, that is that is what remains intact and then instead of 2 s plus 3 we will write 3 s plus 2. So, obviously A and B matrices are same we discussed that before. Whereas this y 2 if you if you discuss about that then this is a 3 of z dot plus 2 of z is equal to again going back to the definition and all this is 3 of x 2 plus 2 of x 1 actually. So, this is 2 time 2 into I mean, 2 3 which will come here times x 1 x 2 plus 0 into I mean u basically.

So, the block diagram representation is again fairly straight forward and also remember that no matter whether you have multiple input and I mean, multiple output, but some input with same polynomial here. Then the state equation remains same that means, the number of integrators that you need to represent this system dynamics or the number of state variable that remains same actually. So, it is the system is still realizable with an integrators here actually. The realization requires integrators only.

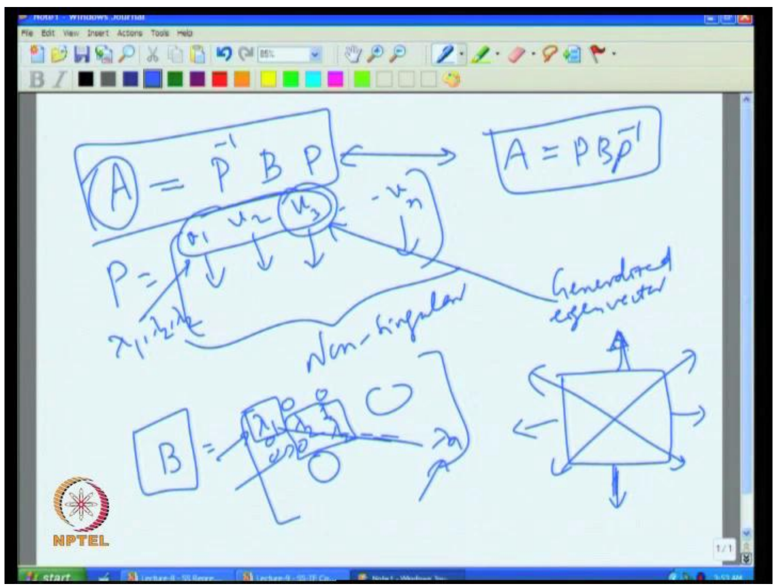
(Refer Slide Time: 24:07)



Let us go to a very different form of representation see that is what we discussed about first companion form all the time actually. Now there is a very different notation which comes from what is called as Jordan canonical form. And this is its own beauty actually like if you see to see go back to the matrix representation and things like that we will discuss the matrix algebra in couple of classes later in fact, details actually. Whatever is necessary for this particular course actually.

So, it turns out that there is a matrix transformations there and then if you given a matrix whatever n by n matrix can you reduce it to some form or some simple forms or all that actually. And those simple forms will lead us to something like one reduction what is called as simulated transformation and all that. The simulated transformation will reduce a matrix a to matrix b to certain invertible transformations actually. We will see that little down the line actually. Now what happens is in those simplified form the meaning turns out to be little more appropriate actually that means, if you if you really have some sought of a matrix form let me try to demonstrate that little bit further probably.

(Refer Slide Time: 25:29)



Now, let us see you have something like a matrix A. Now the idea is, can I can I get a equivalent representation in some form or rather tying of that with matrix B basically. And it turns out that I can do this thing by p inverse multiplying this and p here where p is given by certain Eigen vectors and all that.

Let us say I compute an Eigen vectors for these v 1, v 2 like that I put this in the form of matrix I mean, this v n and whatever is that v 1, v 2, v 3 these are nothing, but Eigen vectors of this a matrix actually. Now this turns out to be something like a non singular matrix. And now if I carry out this algebra what I have here, then this B matrix interestingly turns out to be a very nice form actually, and sometimes if the Eigen vectors are linearly independent that is a primary requirement actually. Then what happens is this B matrix that you see is nothing, but a diagonal matrix that is all. It is simplest form that is possible and knowing B and p I can always recover A, because A is nothing, but p times B into p inverse. If I do that some backward algebra basically then this turns out to be from this expression and these 2 expressions are equivalent. As long as the p matrix is non singular actually, and this p matrix turns out to be a non singular if all these Eigen vectors are linearly independent actually that means, what I am doing here.

I am getting some sought of a diagonal matrix not only that I will get diagonal elements say something like lambda 1 lambda 2 all the way to lambda n where these are nothing, but Eigen values of a matrix assuming that these are this lambda 1 lambda 2 all these are some sought of I mean, different numbers actually. They are not same numbers then it will happen actually everything will happen nicely. Now if they are same the question turns out to be I mean, what if they are same actually. Now if they are same there are two questions which may rise actually in some cases you may still be able to get a linearly independent Eigen vectors. Then, it still possible to reduce this into a diagonal form actually. And then if it is not possible then it turn to turns out that it little more general sense let us say you have repeated Eigen values, with for some of the repeated Eigen values you really do not have linearly independent Eigen vectors, then what we will do is the corresponding elements.

Let us say this given v 2 v 3 correspond to let say same Eigen value lambda 1 v 1 and v 2 are independent, but v 3 is not independent let say, then in that in that particular case this v 3 is no more an Eigen vectors, but this still consider a something called generalize Eigen vector generalize Eigen vector. If it is generalized Eigen vector then it will turn out to this will no more be very specific forms or what you look at it very nicely then it is actually, what you are having here lambda 1 is independent, but what you have is lambda 2 lambda 2 let say this is repeated actually.

So, in this case what you have is lambda 1 lambda 2 lambda 2 here this is 0 here 0 here and things like that, but here you will have 1 actually not 0. So, these are all like, but it is still much simpler than what you what you really like to have actually, these are nothing but Jordan blocks and all that actually. If you have a real 0 here, instead of 1 then you have a diagonal matrix actually. And instead of that you will have 1 actually, but still it is a much simplified form because all other elements are 0 anyway.

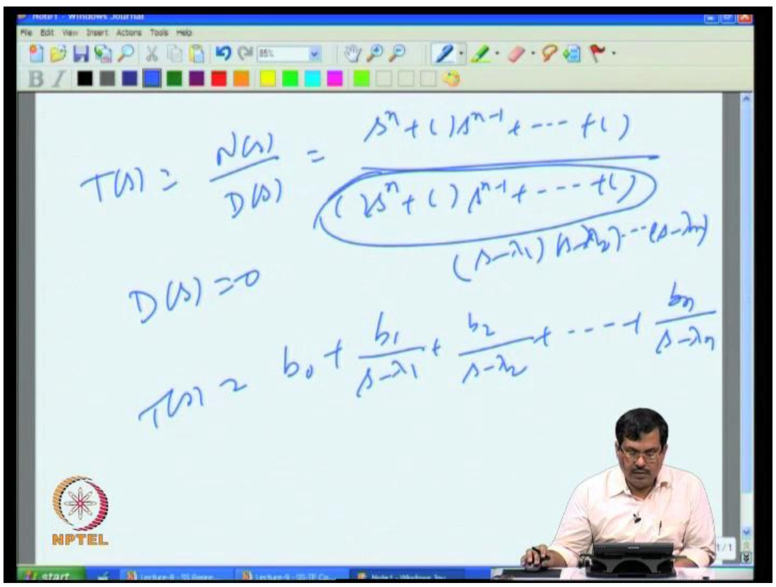
So, that is that is where things are in a easier to analyze they have much more physical meaning as well that means, it is Eigen vectors typically represents some sought of a principal axis in the in the system dynamics. For example, if you if you take some sought something like a piece of paper and try to apply force to tear it off this direction as well as this direction these are all forces applied like let say two people are pulling this actually, in two different directions then the way assuming that homogeneity of the paper and paper properties are homogeneous and all that the tearing thing will not to happen ether of the direction the tearing thing will happen in some other direction.

This direction it may happen this two directions actually, depending on result on forces and things like that where it will lie where the paper is is weak and something like that actually. These are the thing of principal directions actually those are nothing, but Eigen vectors essentially. So, these way to concepts what you study in matrix theory a little bit a later down the line well all tie up nicely with this the system theory basically in a way. So, if you have a transfer function representation somewhere it in some cases, it will be very advantage to us to represent that in terms of a Jordan canonical form and this is I mean come back to this example, these are what you called as Jordan blocks actually and this essentially this representation is nothing, but as Jordan I mean, Jordan transformation what you are talking a equal to p inverse B p simulated transformation we will leave it to a some sought of a block matrix here actually. Block diagonal matrix in a simplified way. So, that is where that name of Jordan canonical form comes and all that actually.

So, easily want to analyze what is happening in these principal axis directions essentially. So, that is that is that is what you are interested in actually. Now let us go back to that and see what is what is going on here actually. So, let us represent this in the form let us assume that there are non repeated roots that means, that the transfer function that you are talking about H (s) has denominator and numerator. So, we are all concerned about the denominator first, that is what will give us roots actually. Eigen values roots and things like that actually. And here we are assuming that there are the roots are non repeated that means, and the roots are nothing, but lambda 1 lambda 2 and all that actually. And remember this is specific reasons for considering them as these lambda 1 lambda 2 symbols they are nothing, but Eigen values actually once you see that realization it will turn out to be Eigen values actually.

So, anyways so these are the roots of the polynomial and what you are assuming here is lambda 1 lambda 2 up to lambda n are non repeated and hence, the Eigen vectors corresponding to those elements of lambda 1 lambda 2 after realization of this a matrix and all that are will turn out to be linearly independent. I mean, just remember that actually. Now what happens here is if it is linearly independent I mean, if it is a non repeated then this polynomial what I have a numerator by denominator I will always be able to factor it out this way. Assuming that, this is the most general way I can have actually right, assuming that these are non repeated that means, the numerator by denominator, what I am talking here let me probably go back to that leave it here to analyze the thing let me clear the board first and then, that let us talk about that.

(Refer Slide Time: 33:08)



So, you have something like transfer function which is nothing, but numerator by denominator polynomial. What I doing here is d of s equal to 0 that is what you are doing and we are assuming that numerator polynomial is of n th order n plus something into s minus n like that actually. Denominator polynomial is also of n-th order basically, and remembers this may not be same coefficient by the way. So, something like that so if there is a if it is say there then I can always divide this polynomial and what I can write I can write this t (s) is something like b 0 which will come from this coefficient analysis and all that I will make this plus or minus multiplication division all sought of thing.

So, I will take out a b 0 component and then, I have some this polynomial what you see here in the denominator is nothing, but s minus lambda 1 into s minus lambda 2, like that it will carry up to s minus lambda n actually. And then, after I take out b 0 the remaining polynomial is strictly proper that means, it is it does not any more contained the power of n actually.

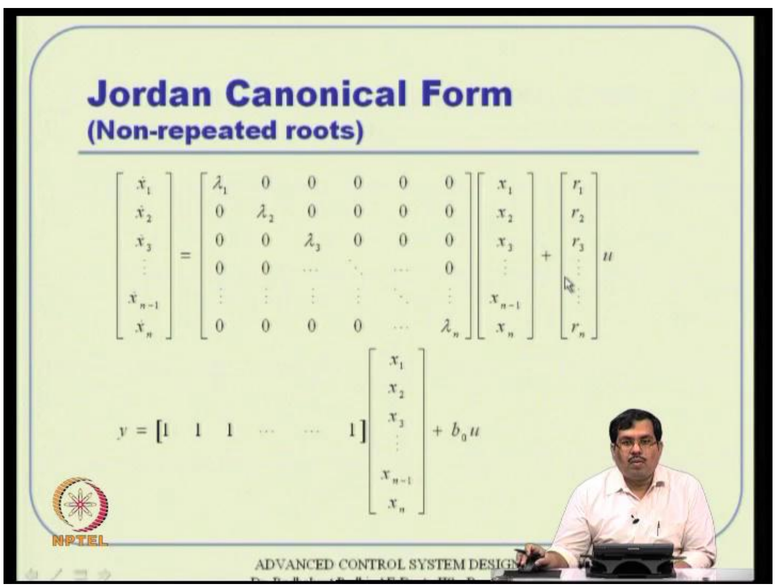
It contains the power of n minus 1 onwards actually, that is why I will be able to write it in the form of b 1 divided by s minus lambda 1 plus B 2 divided by s minus lambda 2 all the way actually plus b n divided by s minus lambda n, and that is what I am doing there actually. So, let me go back to that, and then see what is going on here actually that is a that is part is being done here actually. So, I am starting with this Y (s) divided by U (s) with assuming that I can be able to do that actually. And with the assumption that the all poles of the transfer functions are distinct, that they are not same and no repeated poles actually, very specific case we will try to generalize that also. Now what is the we all know that r 1, r 2, r 3 all though up to r n these are all called residues of the reduced transfer function.

H s minus b 0 if I take that H (s) minus b 0, what I discussed there these are all residues of that transfer functions that is left out actually it is just by definition sought of thing. Now if I have this is what I started with this form actually, then what we what I can do I can multiply this U (s) to the right hand side and tell that is Y (s) is nothing, but b 0 times U (s) plus r 1 divided by s minus lambda 1 whatever I have into u of s, plus r 2 by s minus lambda 2 into U (s) like that actually.

Right this expression what you see here simply comes from the first equation. And then, what I what I see is my definition of state variables are different here actually. I am defining my x 1 (s) to with this quantity what I have here, defining x n of s remember these are all in Laplace variable actually is to be that that quantity. So, this is my x 1 (s) this is my x 2 of s and this is my x n of s. Now if I have if I know this variable like I mean, this equation like that let me multiply now actually, if I multiply this now to the left hand side and interpret the differential equation then this is s into x 1 (s) that means, x 1 dot minus lambda 1 into x 1 (s) that is lambda 1 into x 1 is equal to r 1 into u. so, this is this is like that actually, this is x 1 dot minus lambda 1 into x 1 is equal to r 1 into r 1 into u.

So, similar thing I can do for all these things and I will represent this differential this all will pope up as 1 1 first order differential equations actually. This is x 1 dot minus lambda 1 x 1 is equal to r 1 u, next 1 will be x 2 dot minus lambda 2 x 2 equal to r 2 u similar, thing will go onto x n dot minus lambda n into x n is equal to r and u. I hope this is clear then what happens is I will take this quantity to the right hand side that is all I will do x 1 dot is equal to lambda 1 into x 1 plus r 1 u like that I write actually.

(Refer Slide Time: 37:43)



So, if I write that and ultimately what it happens x 1 dot is nothing, but lambda 1 times x 1 plus r 1 u. So, this is lambda 1 times x 1 plus r 1 u, all the elements all other things are 0 actually similarly, x 2 dot is a lambda 2 in the diagonal into x 2 plus r, r 20 u. Now see all these things I can I will be able to write it that way right x 1 dot is equal to lambda n into x n plus r n u. So, of you see each of the rows of the a matrix only diagonal elements will be a non 0 all other terms will turn out to be 0.

So, naturally what you get here, this a matrix is actually in a diagonal form already. What the b matrix is non 0 0 0 0 in last 1 element is 1 that is not there actually, all of the elements that you get is r 1, r 2, r 3 up to r n actually that is elementary actually. Now coming to the c and still remember that if, it is I mean, output is given by this expression right Y (s) is all this actually remember that. So, Y (s) is nothing but b 0 u s plus x 1 plus x 1 (s) plus x 2 of s plus x n of s actually all these things are available for me.

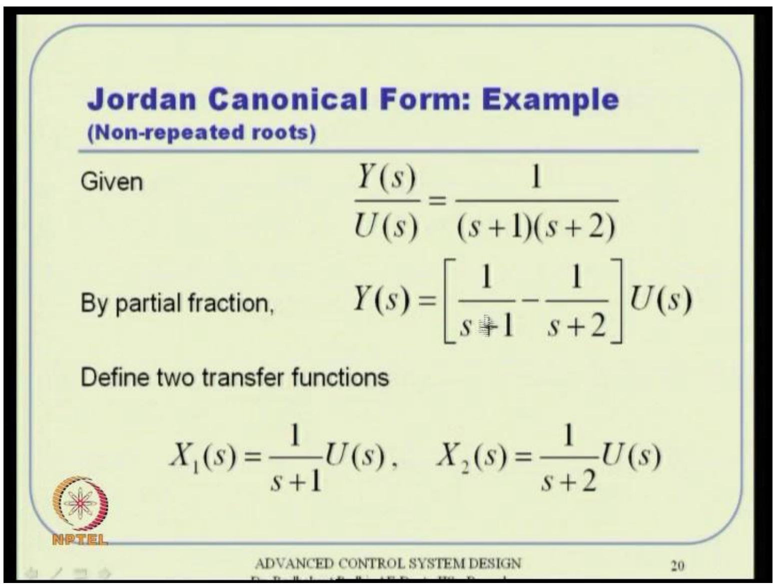
So, as far as y 2 is concerned that is all x 1x 2 plus, x 2 plus, x 3 plus up to all the way up to x n plus b 0 times u s actually. So, that is what I will keep it this way. So, y of t is nothing but x of x 1 of t, plus x 2 of t, x 1 of t plus this 1 into this that is x 2 of t plus x 3 of t like that actually. So, this is why I get C matrix as 1, 1, 1, 1, 1, 1 all the way I mean, that is the C vector, and then this is a b 0 component will also be there. So, that means this b 0 is coming directly from here. So, that is what will happen to be like that b 0 sitting here actually. So, this is your A, A matrix this is b matrix, b vector rather in this case this is your c vector this is a b 0. And if you started with if you start with something like proper strictly proper polynomial, then this component will not be there this b 0 will not be there. So, that b 0 will not come here either actually. So, and most of the systems are that way so, what the difference between this first canonical form to Jordon canonical form you can see clearly, that the matrix turns out to be diagonal that is the big advantage and these are nothing, but now remember the diagonal matrices the Eigen values are diagonal elements directly they are the diagonal elements.

In general if it is a triangular matrix the Eigen values are diagonal elements that you can see from the definition of Eigen values actually. So, these elements what you saw what you saw that is roots of this characteristics I mean, roots of the denominator polynomial equated to 0 is nothing, but the Eigen values of this system matrix actually. And we will see that later that the stability behavior of this system dynamics largely depends on only depends on the Eigen values actually. So, that is what happens actually later. So, this then the way suppose you started with this transfer function what you had and went to it and did, that first canonical form then the a matrix turn out to be in a in a different way I think you can see some of these examples also like in fact, if you go back there the a matrix turns out to be that.

So, certainly the Eigen value information is not as clear as the other 1, you really have to do the Eigen value analysis in a in the characteristic polynomial and things like that to find out what are the Eigen values in this form, but in that form it turns out to be very natural it just it just available for you actually. That is that is already there for you. However does the say the b c and d matrix so we have to be slightly careful actually. So, earlier the B matrix turns out to be 0 0 0 0 the last element was some number here, it is all the elements will have some number actually and why that happen that primarily happens, because of the change of the definition of x 1, x 2, x 3 it is no more your see first companion form the state variables normally carrying direct nice physical meaning.

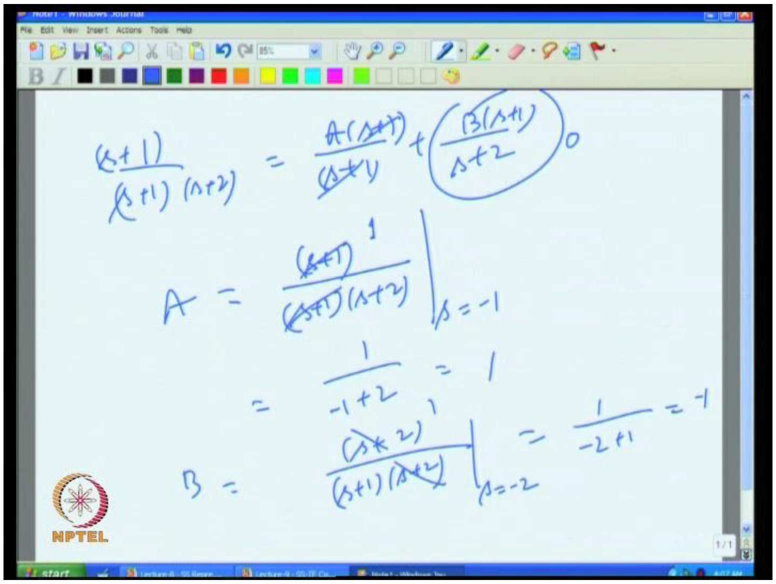
For example, if you see this form let us say this for a system then you what you have z then noting, but position this is velocity. So, that definition is intact in your state variable now if you see this definition what you see here x 1 comes through this one actually this entire polynomial what you have x 2 comes to that actually. So, if the definitions of state variables are no more the direct physical meaning of the system dynamic equations all right.

(Refer Slide Time: 42:36)



Now, let us say a small example here let us say Y (Y s) by U s is given in this form remember this is already factorized and it is already a strictly polynomial strictly I mean, strictly proper polynomial in case of that b 0 term will not be there now if I do this for partial fraction decomposition for this which is actually, very easy rather, let us try to do it here 1 by x plus 1 and into s plus 2. So, what you have here is something like this let me erase this again then try to do something all right. So, what I have here is something like 1 by s plus 1 into s plus 2.

(Refer Slide Time: 43:28)

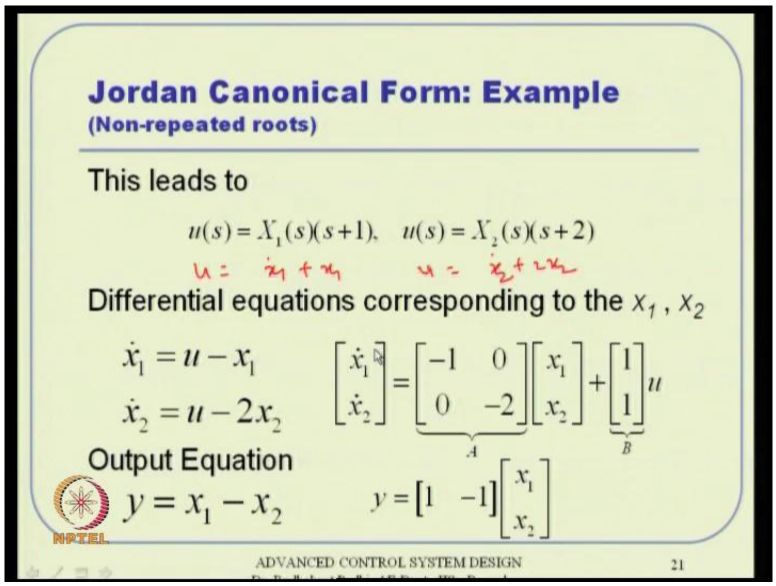


So, what do I carry about with this 1, 1 by s plus 1 into s plus 2 remember this will take a something like a A by s plus 1 plus B by s B by well B by s plus 2. Now what is a A by definition, is I will multiply this side if I multiply that I will get x plus 1 divided by x plus 1 into x plus 2. If I get that if I evaluate that I evaluate that at s equal minus 1 the roots of this actually, and it will turn out to be very nice actually, because if I if I simply multiply x plus 1 both sides here what I get here. If I see that x plus 1, if I multiply x plus 1 and x plus 1 what happens here, this fellow gets cancelled out and if I see s equal to minus 1 then this term happens to be 0. I am left out with only a basically. So, that is that is why this is given like that so what is happening here s equal to like so, what I am left out this is none this is 1 actually. So, 1 by s equals to minus 1 right something like minus 1 plus 2 so this is 1 actually.

Similarly, if I do B is nothing, but s plus now I have to multiply the other 1 that s plus 2, I have to multiply. So, s plus 2 divided by s plus 1 into s plus 2 evaluated it s equal to minus

2. So, this is this is gone I am left out with still 1 so 1 by minus 2 plus 1. So, this is minus 1 actually. So, a is 1 and B is minus 1 and that is why you are you are left out with a equal to 1 and B equal to minus 1 therefore. So, that is where you start with actually that is so what we will do I will take U (s) inside then x 1 I am defining that particular term this is my x 1 (s) this is my x 2 of s that is my definition actually.

(Refer Slide Time: 45:31)

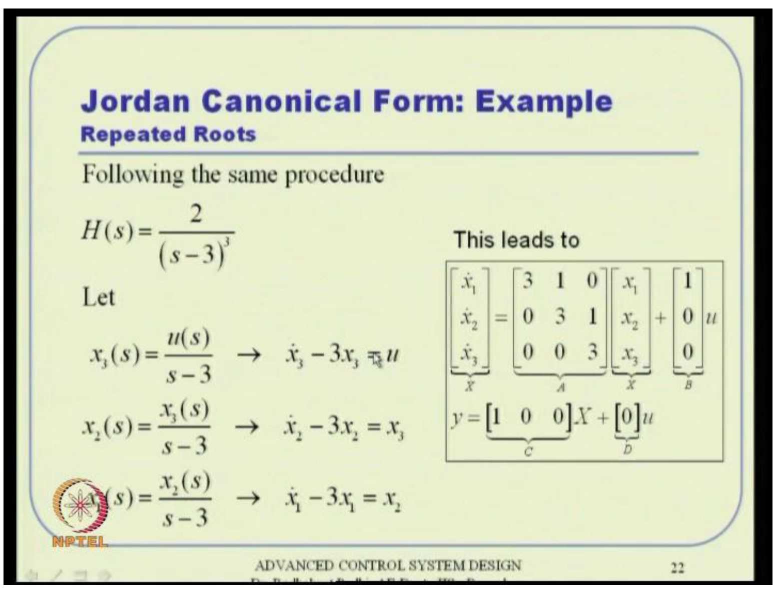


So then I will carry forward and then tell this leads to what does it lead to I multiply both sides s into X 1 (s) plus x 1 (s) like that actually, now multiply that both sides U (s) is nothing, but x 1 (s) into s plus 1 U (s) is nothing, but x 2 X 2 of s into s plus 2 actually. That is simply coming from this expressions actually and then I tell I will formulate the differentia equations here, that x 1 dot x 1 dot plus x 1 is U (s) remember that this expression what you see here, x 1 dot plus x 1 is u and similarly, x 2 dot plus x 2 is u. so, let me probably write it here also.

What I have here is something like this say u is nothing, but x 1 dot plus x 1 here. And u these are all time domain actually u is x 2 dot plus 2 x 2. So, if I if I rearrange these terms what I what I get here, if I if I here in this term x 1 dot is nothing, but u minus x 1 plus y here and x 2 dot is nothing, but u minus 2 x 2 that is what I have here. So, now I will go back and put it in standard form x 1 dot and x 2 dot, I will write it see normally the way to write it is you just leave this matrix blank and simply put x 1 dot x 2 dot here, x 1 x 2 here give a long matrix and put u here. And then, try to see what my x 1 dot is? x 1 dot is minus x 1 plus u. So, minus 1 here minus 1 into x 1. So, that is minus 1 plus u that means, 1 here that is how I will fill up the elements actually for a b matrix, and then x 2 dot is a minus 2 x 2. So, this is 0 into x 1 minus 2 into x 2. So, that is minus 2 here and then plus and plus u actually u is 1 here. So, I get my A matrix here and I get my b matrix here that way. And then output equation remember that what is my output here output is this expression minus that expression right multiply by u by u a.

So, what it is u by s s plus 1 minus u by s plus 2 that means is nothing, but x 1 (s) minus x 2 of s. So, that is that means, y is nothing but x 1 minus x 2 if I really want to put in the matrix vector format. So, again I leave this leave this blank for a second and put x 1 x 2 here then see it is x 1. So, that means plus 1 will come here and minus x 2 that means minus 1 will come here. So, this is my A matrix this is B this is C that is all I have and b 0 obviously and d happen to be 0, because I started with a strictly proper polynomial function strictly proper polynomial otherwise actually.

(Refer Slide Time: 48:31)



Now, these are all nice thing about a non repeated root. What you saw here. Now what about repeated roots? Well again see that in a in a example repeated roots that means, we have some expressions somewhat like this. It can be partly non repeated partly repeated so that means, we will concentrate here only that repeated portion what is going on here actually. Now non repeated portion will go regular way whatever we have discussed all these examples and all actually, now for repeated part let say this some 1 term is something like this then what we do. In that case, let me define my state variables this way. I will start let say 3 terms it is repeated right. So, I will start with x 3 as my regular definition x 2 is not u anymore here, but x 3 x 2 is x 3 divided by x plus 3 and x 1 is x 2 divided by x plus 3. Then, there is a third order polynomial here so I start with x 3 definition x 3 is u by s plus 3 regular way, and then x 2 is x 3 by s plus 3, and x 1 is x 2 by s plus 3 and once this is defined this way then this 2 algebra is fairly simple.

So, you again do that multiplication and then take it to time domain. So, s into x 3 is like x x 3 dot minus 3 into x 3 is like minus 3 x 3. So, that is the differentia equation and similarly, what you have here is x 2 dot minus 3 x 2 is equal to x 3 remember this is not no more u, but x 3 here and s into x 1 that means, x 1 dot minus 3 x 1 is actually x 2 here. So, what I have here is something like x 2 I mean, this sorry x 1 dot minus 3 x 1 is equal to x 2. So, what I have here this is system of equations if I really want to put it in state space form now what I see here is x 3 dot is 3 x 3 plus u so x 3 dot what I have here is 3 x 3 plus u.

So, that is that is that is clear actually probably this a small mistake also here, this is once here actually it is not a 1 0 0, but 0 0 1 probably actually. So, x 3 dot is 3 x 3 plus u so 3 x 3 plus 1 times u this is 0 actually now if you see this 1 x 2 dot. So, what is what is that 3 x 2 plus x 3. So, this is 3 x 2 plus x 3 that means, 1 term here similarly, x 1 dot I mean x 1 dot is 3 x 1 plus x 2 that means 3 x 1 plus x 2. So, what is happening, this 1 and 1 are appearing here actually. So, that is like a Jordan block actually it is not an it is not a diagonal block, but it will happen to be this part of this overall system matrix will turn out to be a Jordan block.

Remember we are considering only one part here, other parts are non repeated, that is what you have assumed actually, then that part will turn out to be diagonal only this part will turn out to be a Jordan block, but if its original system is this no other thing, then this is a system dynamics anyway this is your this is how you will get it get into a matrix this is how will get a b matrix then c and d matrix actually. So, this is how what will do for repeated roots. So, it is possible to get a Jordan form whether you have a non repeated root this is a how we started with non repeated roots and all, that and go and do that and if it is it is also possible to get a Jordan form even if there is a repeated root. And once again these Jordan canonical forms have some nice characteristics already what you see here are nothing, but diagonal elements actually I mean, diagonal say Eigen values even if it is a Jordan block it is still a triangular matrix and a triangular matrix Eigen values are nothing but diagonal elements. So, you already have a diagonal matrix I mean the Eigen value information actually.

So, these are properties of a Jordan canonical form, that is all I will probably discuss in this lecture, and we will see one or two more forms in the next lecture, and then proceed from there actually. So, in general what you saw here is if it is possible it is possible to transfer from a transfer function representation to state space representation and vice versa as well, but what you should remember is transfer function given a state space representation.

The equivalent transfer function representation essentially what you come up with system like a transfer function matrix basically, and that way is unique that is given by a single formula c into c into s I minus A inverse in times B plus d that is t (s). Whereas the reverse procedure that means given a transfer function to realize the state space representation. That is A, B, C, D matrix that is not unique and depending on the particular application that you are talking about. The particular form can be of use actually, some little advantage may come, but largely the properties of the system representation remains fairy same actually, that means Eigen values do not change thing like that actually. So, more on that we will probably try to build on from the next classes, thank you.