Advanced Control System Design
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Lecture No. # 01
Introduction and Motivation for Advanced Control Design

Welcome you all to this advanced control system design for aerospace applications. So this particular lecture will be about giving an overview of the entire course followed by the motivation, and I mean the objectives of this course, motivation for this course and all that.

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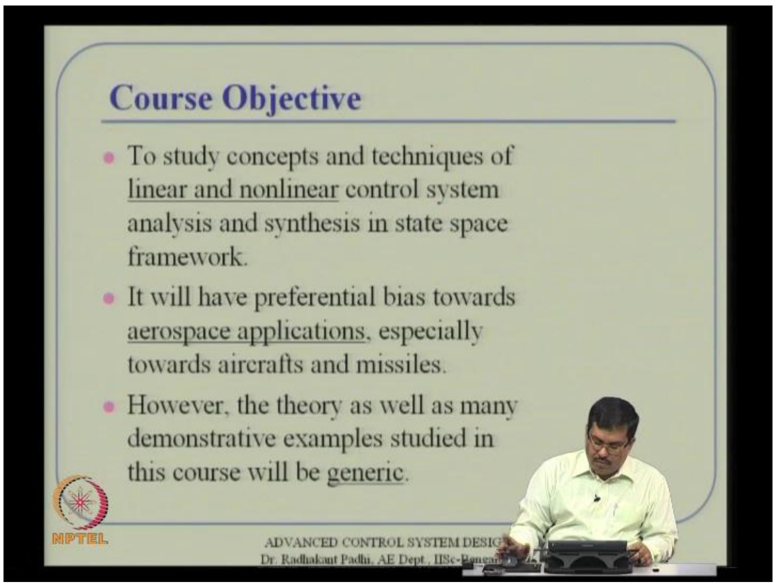

Once I ready with the proper motivation and all our learning becomes easier. so, let us see first what is our objective here. First objective is to study the concept and technique of linear and non-linear control systems, both linear and non-linear will be covered here in this particular course. And then it will have some preferential emphasis towards aerospace applications; that does not mean that it is completely aerospace, we will have lot of examples, which are also generic. That means, the concepts and the mathematical formulations and all that we talk here will be in the generic formulations. So, even though there is some emphasis towards aerospace applications, all other applications can also be very well within the course material for say so.

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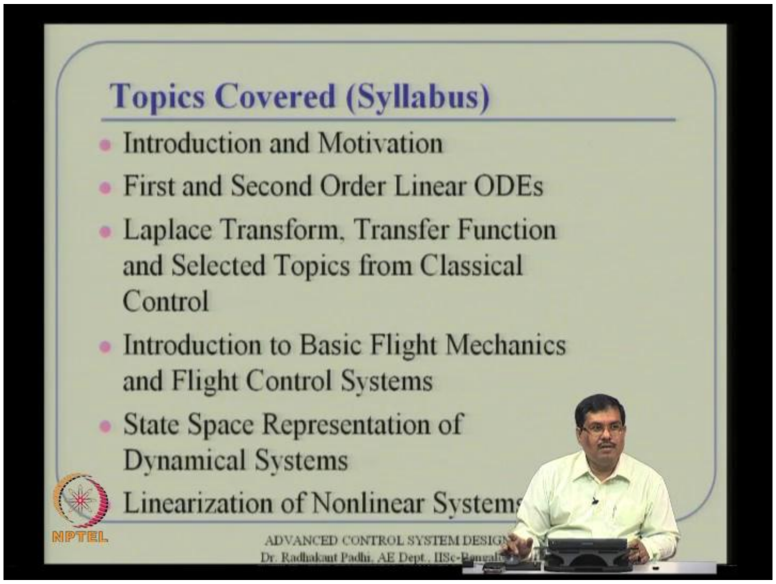

The topics covered are roughly the syllabus part is something like that; we will first talk about introduction motivation, which is roughly the course material today. And in other words, this particular class will talk about some sort of giving an overview along with proper motivations. Then we will talk about some overviews of first and second order linear ordinary differential equations, typically what are covered in undergraduate courses. And some of those will be like that, the entire course is geared up towards like that. So the next next set of topics will be overview of laplace transform, then transfer functions what we normally study in undergraduate courses.

Then we will talk some overview of classical control techniques, in other words it will not be entirely exhaustive, but some sort of overview of the classical control technique just to get ourselves ready for modern control, we will try to appreciate more once you have some fundamental knowledge or basic overview of classical control systems also. So, again once again it is not going to be a full overview of entire classical control, but just some sort of important topics for classical control.

Then on the way we will also introduce this flight mechanics and flight control system overview. So, remember this is a aerospace related course, in other words we have some preferential emphasis for aerospace control system, so understanding the basic flight control systems and flight mechanics is also part of this course. So, we will not devote too much of time, but specially one or two lectures we will leave out on the way.

Then we will go little more serious, then talk about how about state space representation of dynamical systems in general. So, more or less it is again now a days become undergraduate material however, we will we will review that and then will move further on that, we will talk little more depth compare to what you really learn in undergraduate subjects. Next we will talk about linearization of non-linear systems, because we there are… I mean, as I told in the beginning this entire course is geared up both linear as well as non-linear systems.

So, we will see how we talk about linear… Before we talk about linearize methods or linear control designs technique, we need to know what is linearization process. So, we will probably talk I mean, devote one lecture for linearization of this non-linear systems. Especially, since most of the real life systems are non-linear anyway, so it makes sense to study that in a little more depth, in other words given a non-linear system how do we find out a linear approximation of that system around an operating point, and all we will be… that topic will be covered in that lecture.

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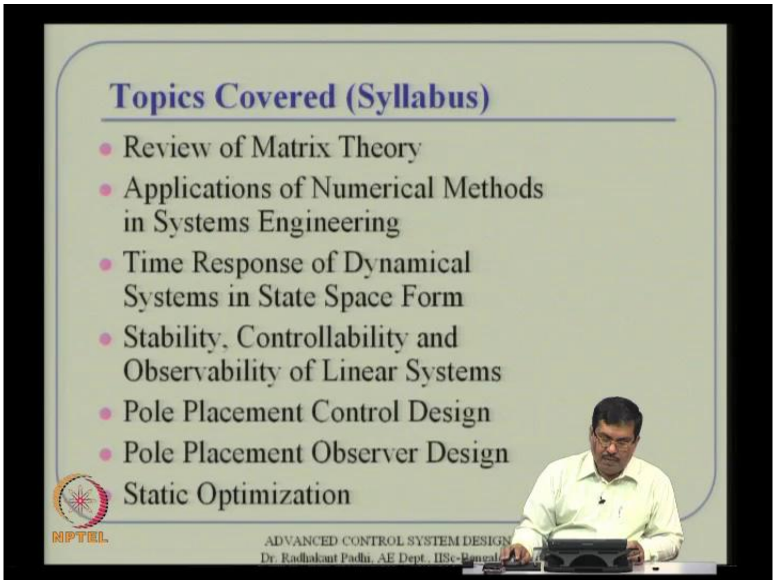

Next, we will talk about some review of matrix theory, which is a key component of modern control systems. It will be fairly exhaustive, so I will not talk about all sort of theorems, proves and all, but the at least important concepts along with some degree of important results, and and some degree of little bit examples and all we will we will study on the way, which will help us understanding the linear system control in a much better way.

It is also useful for non-linear control system design anyway, so that is how it is put there. Then we will have quick review of numerical methods in system engineering, as we do research or do more on non-linear method, linear method all sort of things, especially non-linear control design techniques. We also needs some numerical methods as our tools to talk about linear I mean especially, non-linear control design techniques.

So, it make sense to have a quick review of numerical methods what is typically useful in in systems engineering. Next, we will talk about dynamical systems time response, and this time at we will be talking in terms of state space form, so if you given a state space form of the system dynamic, how do I really come up with this this time domain solutions from form a set of initial conditions? So, that is what we are interested in this particular topic.

Next, we will discuss about stability, controllability and observability, which are very critical for linear systems analysis. Some equivalent concepts are also there for non-linear systems in a limited way, but we are not going to talk too much on those aspects. But for linear systems the clear cut results are available, so we will have to we will be able to discuss in a very precise and clear way for linear systems, that is what the motivation here.

Next, we will discuss about a specific control design technique called pole placement, this is a specially useful for linear system design. The same technique can also be used for designing observers, so that will also be a part of that part of the course. Next we will go to some quick overview of optimization and optimal control, so one lecture will be for optimization only, so parameter optimization static optimization essentially.

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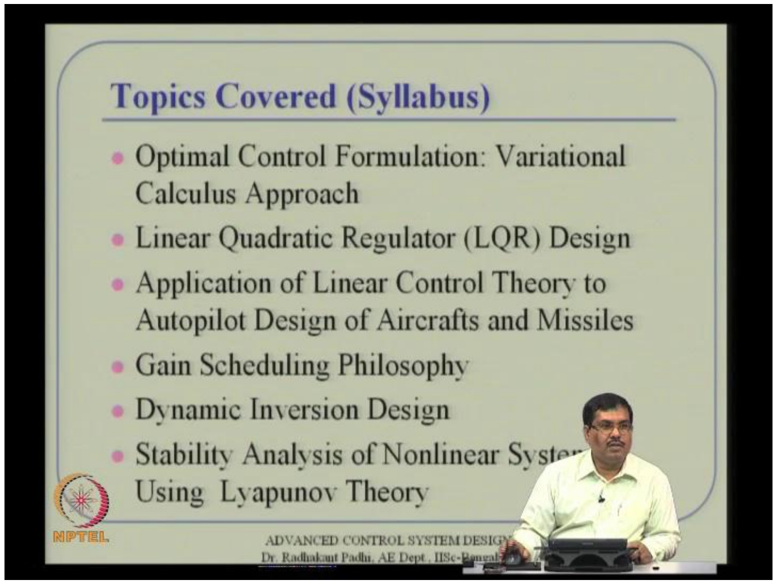

Next, we will talk about how do we extend that for, or how do we kind of formulate control design techniques from optimal control theory point of view. Here we will need some sort of variational calculus approach, so that will be part of one lecture probably, we will see as you go along. But mostly in one lecture I will be able to summarize the important concepts there for optimal control, which will subsequently be used for what is known as linear quadratic regulator design, and there we will spend about one or two lectures to kind of give a good overview of what is LQR design really.

Next, we will see applications of all these linear control design techniques the to autopilot design of aircraft and missiles, which is again we will kind of tie it up together for the concepts that you read to the applications that we have in mind, which is essentially aerospace engineering. Next, we will talk about non-linear control design philosophies, and there the first thing comes to mind is what is called as gain scheduling, in other words you design gains for several operating points where you have a different… I mean, gains for different you select a different operating point, surround which we we found out some sort of a linear approximation, we design specific gains for all those linear I mean, all those operating points.

And then we tie them out or we stitch them out through some sort of interpolation routine I means, so that interpolation will kind of give us some sort of a non-linear control design called as gain scheduling, so we will talk about that in a quick overview sense. Next, we will move on to what is called as dynamic inversion design, where it essentially attempts to eliminate the requirement of gain scheduling, because gain scheduling becomes a tedious process, it become problem specific, suppose the problem changes next time you have to repeat the exercise again, all sort of problem been there.

There are attempts in current literature to get away from there, and that one way of doing that is going into this dynamic inversion design. We will also talk about this particular design in detail in couple of lectures. Next, we will move on to this stability analysis of non-linear systems using lyapunov theory, as we kind of appreciate that lyapunov theory is a very handy technique for non-linear systems, both analysis and design. So, it makes sense to study that in somewhat fair detail of what is this lyapunov theory, and how do you make use of that in both analysis and design techniques.

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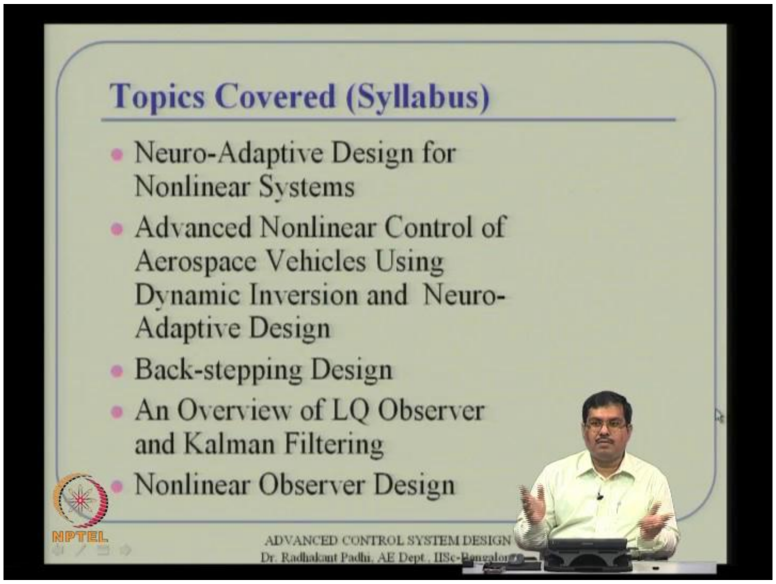

Next, we will we will discuss a very important recent concept which is called neuro-adaptive design for non-linear systems, that will be followed by some sort of application lecture, which will discuss about how do we use all this advanced non-linear control design techniques for aerospace applications, essentially this aircrafts and missiles again. And in this particular lecture we will focus on dynamic inversion and neuro-adaptive design, so these are the two focus areas here, this dynamic inversion and a neuro-adaptive design.

Next, we will discuss back-stepping design, just in 1 lecture will be sufficient to do that, and that is also a powerful robust non-linear control design, so it repeatedly uses this I mean, this lyapunov theory in a loop structure which will give us some sort of a robust design, and that is what is back-stepping design. Then we will next we will discuss about an overview of what is LQ linear quadratic observer, and a very quick review of what is very commonly known as kalman filtering. And then probably, we will also give you some sort of, I will try to give some sort of very quick overview of what is called as external kalman filtering which is very useful in practice.

So, that is how it is structured, but towards the very end we will discuss also one quick overview of what is called as non-linear observer design also, not necessarily the external kalman filtering. But using lyapunov theory and nonlinearization concepts like that, how do you discuss how do you come up with some sort of a non-linear observer design. So, that is how the entire course is kind of structured, you will see all sort of starting from classical concept overview to review of matrix theory laplace transform, to transfer function, to linear design analysis stability theory controllability observability, to polar placement using both as control design and observer design.

Next, followed by this optimal control LQ o design sort of thing, along with that some flight dynamics and applications in aerospace engineering. And next, we will move on to non-linear control design, where you discuss dynamic inversion, back-stepping design, lyapunov theory and neuro-adaptive design, and again towards the end we will discuss about something like linear quadratic observer, and then extending on to that we kalman filter concept followed by some sort of non-linear observer design.

So, its bit… So this entire course give use of this is some sort of overview of the spectromob topics that we discuss in non-linear and and linear designs together.

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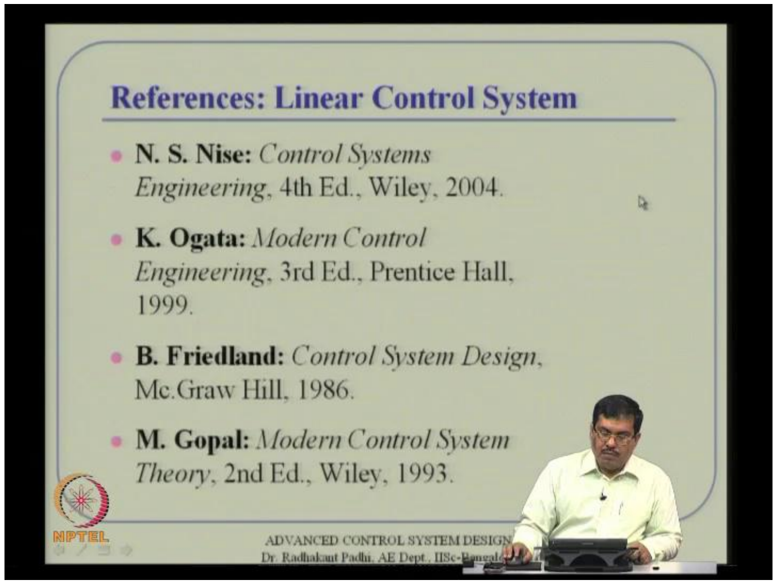

So, that is how this syllabus is structured for this this course, about references you can see there are nice references available for linear control, and the first book is my first reference all the time, that is the Norman Nise very cleanly, very neatly written for good understanding.

This is also a nice book written by Ogata sometime back, and then some concepts of especially controllability, observability LQ I mean, LQR design and all we will take it from Friedland. And then this is also a nice book by Madan Gopal, who is a professor at IIT Delhi about modern control systems theory. So, some of you can sincerely buy these books to get more depth knowledge here.

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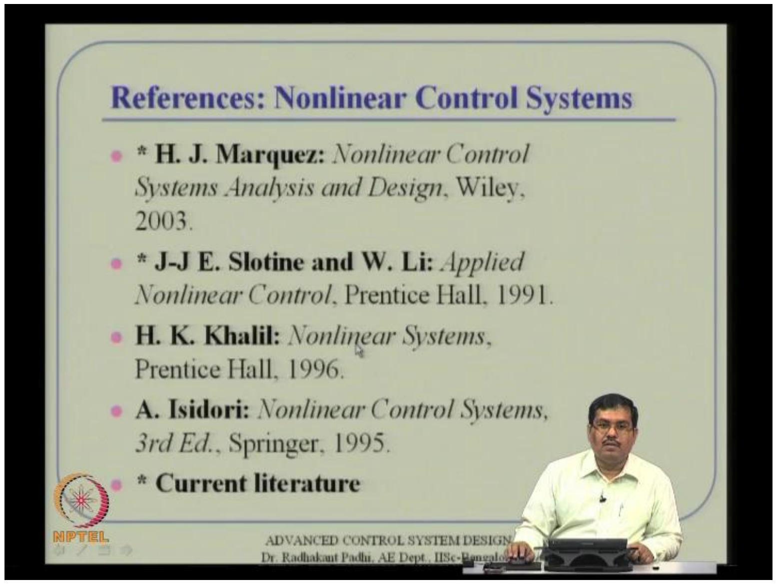

About non-linear control my favorites of first two, the first is Marquez which is non-linear control system analysis and design, just published not too long back in 2003, and I also like this Slotine and Li book very much, because it also a engineering perspective, it does not give you too much of math details and all but gives you fair amount of math, but it has also engineering insights into that, so these two are my favorite books and then there are support books which are written by Khalil and Isidori also, and especially non-linear control systems should also include some sort of a current literature which are not available in in text books for say.

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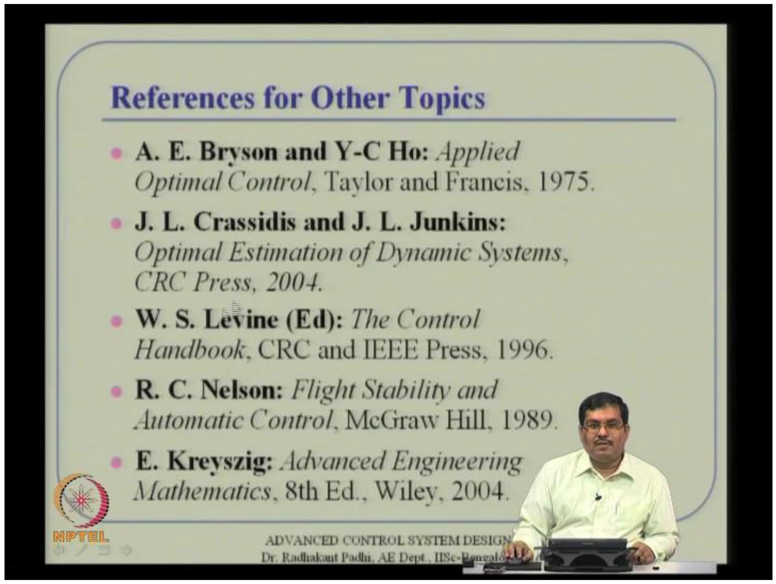

Reference about some other topics for example, optimal control will be taken from Bryson and Ho. There are some filtering like Kalman filter radius and all, will be taken from Crassidis and Junkins. There is a very neat Bible sort of book available here which is control system hand book, just about any topic you can have a quick overview by having this book. And then flight mechanics concepts and some review of flight control we will be taking from Nelson, and some math concepts will be taken from other books as well, but primarily from Kreyszig. That is how it is there, it is not an exhaustive list there may be concepts from other books as well. So, that is how this topics are selected from various places and put together actually.

Now, let us move on to lecture one that is an overview of the entire course. So, the lecture one is introduction and motivation for advanced control design, that is the first thing that I like in any particular course before I study those material, what is the utility, what is the motivation, why should I read that and that kind of idea. So, let us have a quick overview of why do I need to study this concept?

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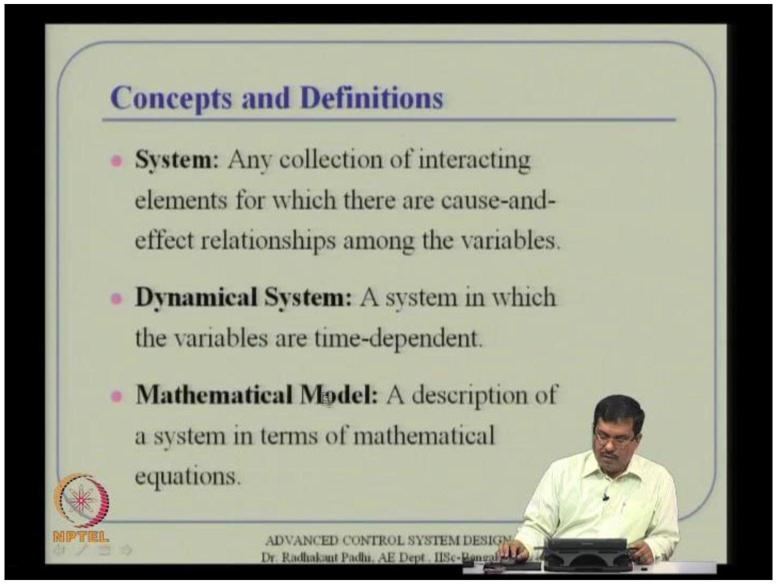

So, concepts and definition first, so first we call… First we need to know what is a system, and any collection of interacting elements for which there are cause-and-effect relationships among the variables, so that is the formal definition of a system. So, that means you have a collection of interesting elements, and there should be cause-and-effect relationship among those variables as well, then only talk about that as a system in general. And this particular subject we are more interested about dynamical systems, and that means that is a system in which variables are really time-dependent, that means the variables really evolve with time they do not remain constant for say, so that is a dynamical system in philosophy sense.

To study anything in those direction we really rely on this mathematical model, and this mathematical model is a description of a system in terms of mathematical equations. So, if you have a static system then you really talk about algebraic equations, if you have a dynamical system we take the help of differential equations or in a limited transfer function either ways, but transfer function representation is also an equivalent representation of differential equation form. So, that is how we really need a mathematical model before we analyze the system dynamics, whether you analyze or try to design a control or synthesize something, for both we need mathematical model actually.

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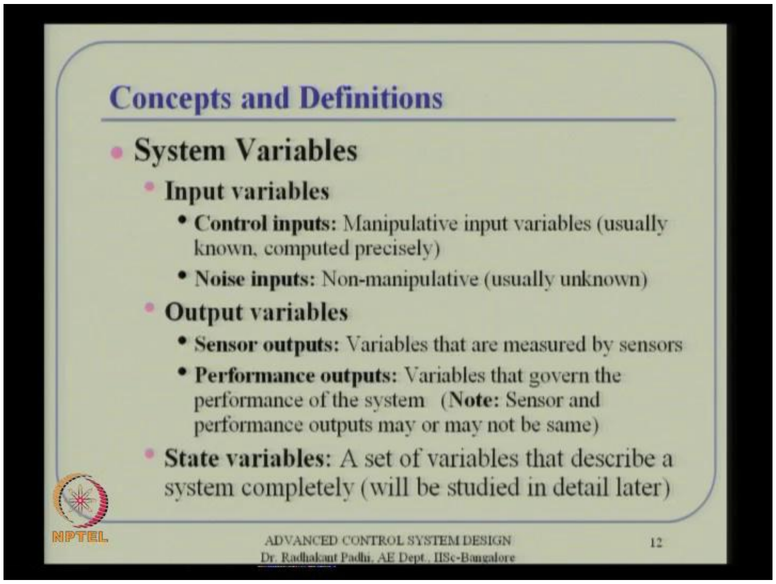

Then system variables, there are various types of variables and first type of first variable is what in call is input variables, and this input variables can be of two type, one can be of control input and the other one can be noise input, so noise is also an input I mean whether you like it or not, so control input is is essentially is a set of manipulative variables, and they are usually known because they are computed precisely, we are the one who is computing that.

So, we kind of know it what we are computing anyway, so control variables are essentially the manipulative input variable that you really want to manipulate so that your system behaves nicely. However, the noise inputs are actually inputs, but they are usually non-manipulatives, they do not obeys. And they are usually unknown that means, you cannot talk about something that is known to me and that is a noise. I mean, if you really know to that if you really know that then that is no more a noise.

If you know any input really, and you exactly know what is the value of plot and things like that, then really that cannot be considered as a noise for say, so that is how we kind of visualize that. So, we like to mainly deal with control inputs however, we want to reject these noise inputs in general, I mean that that is that is our motivation actually. So, the next set of variables are output variables, and these output variables are again of two type, one is sensor output and the next one is performance output.

And again sensor outputs are something that are usually measured by sensors, we have a set of sensors which will keep on measuring these variables, so those are sensor outputs. However, performance outputs are variables that govern the performance of the system, and usually these sensor outputs and performance outputs need not be same, they can be same, then they can be different. For example, if you really want to land an aircraft, so the height becomes a performance variable however, you may not have a direct measurement of the height of the vehicle.

So, you may be can still measure your position of the vehicle, that is a different answer anyway. So, you want to land that means height is your performance variable, but height is not something that you are directly measuring, you can measure also there are height sensors available anyway, that means the variables may or may not be same I mean, that is a typical thing that we really need to remember that. Then the next set of important variables are state variables and these are the set of variables that describe the system completely. That means, if you know the system states really, that means you know almost everything about the system including what is going on internally, even though your sensors does not give you that, sensor input can remain silent about that, but when we talk about states for say that means you really know everything about the systems, because that is how the states are either come from let say physical relationships for example, Newton's law or Kitchers law like that, or they can come from system identification also.

So, there are techniques available to know that, but the whole idea is once you know the state variables, we know almost the entire thing about the system, that is how is those variables they mean that.

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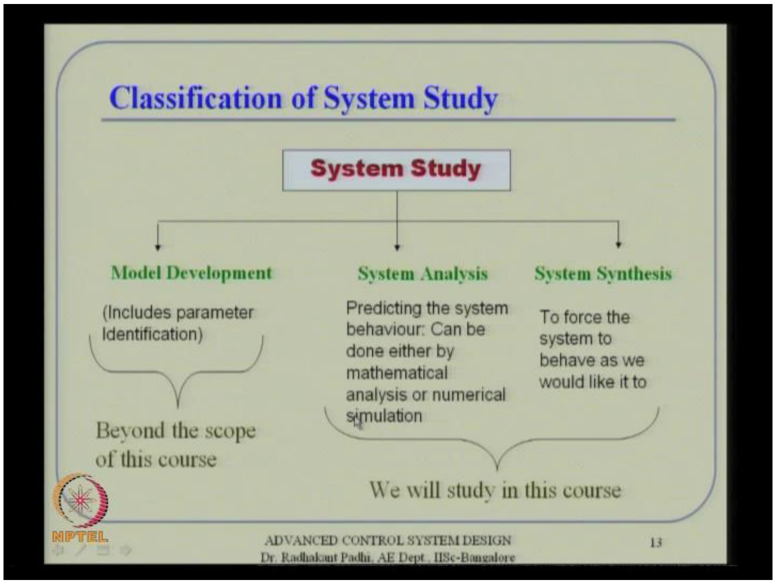

Next, about system study, so when you talk about studying a dynamical system especially, then it can be classified into three parts, one is the model development, and next is once you have the model that means a mathematical model which actually includes parameter identifications as well. So, essentially the laws of nature what we know like equal to n a, that is a universal law. However, we really do not know that particular value of n for a particular system, so that is the type of problem that you discuss about suppose you do not know the mass of the system, then can you really identify that from a sequence of measurements actually. So, that is the that is the type of study which falls under parameter identification, it essentially falls under the broad spectrum of model development.

But, this particular system I mean, this particular study is beyond the scope of this course, so in this particular course we will not discuss about that. However, we will assume a model that is known to us, and then we will go into that next what is what is known as system analysis, so once you have a good description of a model, and then next is to study that system, whether the system is really behaving nicely or not before you do anything anything on that.

So, that is that is the concept that we study about let say stability of the system, controllability of the system, observability like that. Those are the analysis tools that we kind of take the help of, and then study the system behavior first. Next, once you study the system behavior if it is doing well, if the system is really performing well then there is no need to do anything, we can just rely relax actually, and in that sense there is nothing much to do either. However, unfortunately most of the systems are not like that, so there will be some issue or other.

And then we really want to do some system synthesis, that means we really want to do some sort of a control design, essentially we want to force the system to behave as we as we would like it to behave, that is that is our job to do that. So, in this particular course we will study both system analysis, as well as system synthesis. In one particular lecture for that is like neuro-adaptive design, we will partially address this issue. In other words, we will know the model to a good extent, but there will be some degree of unknown quantity in the model itself, then can we address that is this. So, there is very loosely, we will talk in one one or two lectures about that little thing in this course also, but largely this particular course will be about system analysis and system synthesis.

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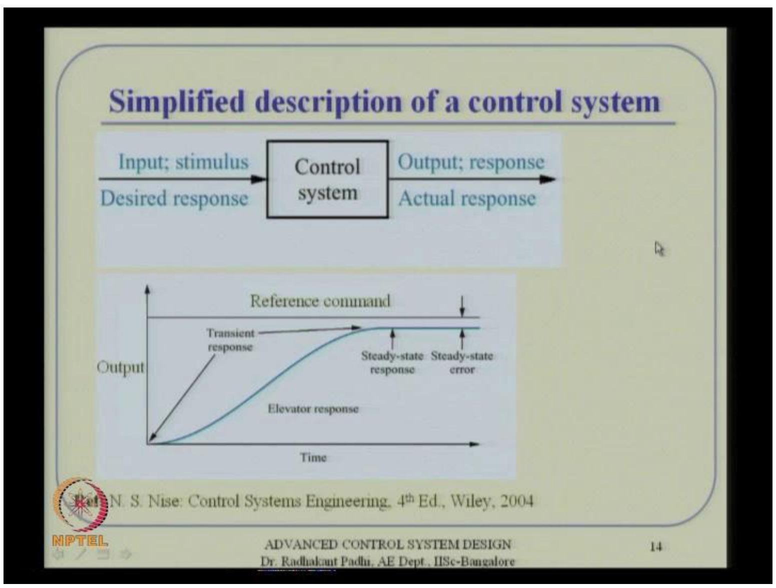

Next, we will have a simplified description of a control system, then all of us probably know this that we can visualize this is a block diagram, and then in this block there are input which are essentially stimulus, and input again can be both control input as well as noise input.

And then there are desired response that is the performance output variables how do you want them to behave, that is also you have to give that as a input to the system, then only the system will know what you really want to do.

So, both of these are actually going as input to the control system, and its output you can talk about actual output that is the that what is your system sensors give you, and this can be really performance output, that means how is your system is is actually responding, so that sense these are all sort of a overview that you can conceptually talk about. Then mostly we will discuss some I mean throughout the course we will we will need this response behavior sort of thing, and when I discuss or when we talk about dynamical system response there are two parts essentially, one first is transient response and next is steady-state response.

The steady-state response is something that happens after sometime, and transient response is something that happens initially, so this part of the system response is transient response, and that part of the system response is actually steady-state response, and in steady-state there may be some error also like wherever you want the system to go you may not be exactly doing a 100 percent perfect job, so that means there may be some steady-state error. And in the transient response there may be I mean, this a typical transient response does not mean every response is like that, there may be over shootings and then it may stabilize and things like that, we will see little more as you go along actually.

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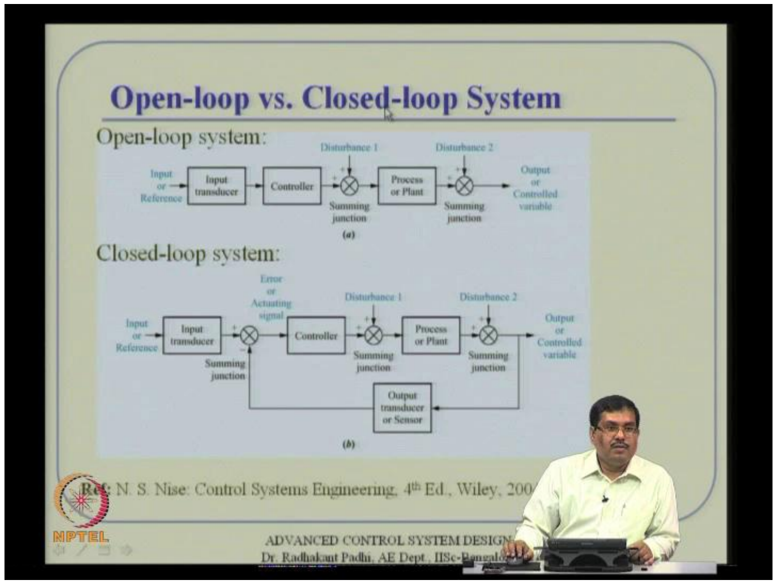

Then there are concepts call open-loop versus closed-loop, and open-loop system means you just give by input to or a reference command to the system, and then just observer system what it is doing, you are not really doing anything else more than that. So, you are just asking the system to do anything, do something, and then you have designed your I mean your controller where you I mean for an open-loop system open-loop controller means there is also a controller by the way, but that controller is designed apriori, that means you visualize all sort of scenarios that you are talking about.

And considering those scenarios you actually, design the control input apriori, but online there is no correction, so that is the difference actually. So, you have some some controller which is available to you, and then there will be disturbance coming here which is like a process noise, and then there will be sensor noise here, that is how going to the system anyway. So, there are process noise there are sensor noise, all sort of things will be there with you, but we will not be able to discuss that online, you will everything offline.

Then you take that controller and then just apply that I mean, that is whatever happens is not on our job anyway, so that is an open-loop control system. But on contrary there closed-loop systems, closed-loop system will have a feedback loop in between, which will take these sensor outputs well they are noisy, but we still have some information there.

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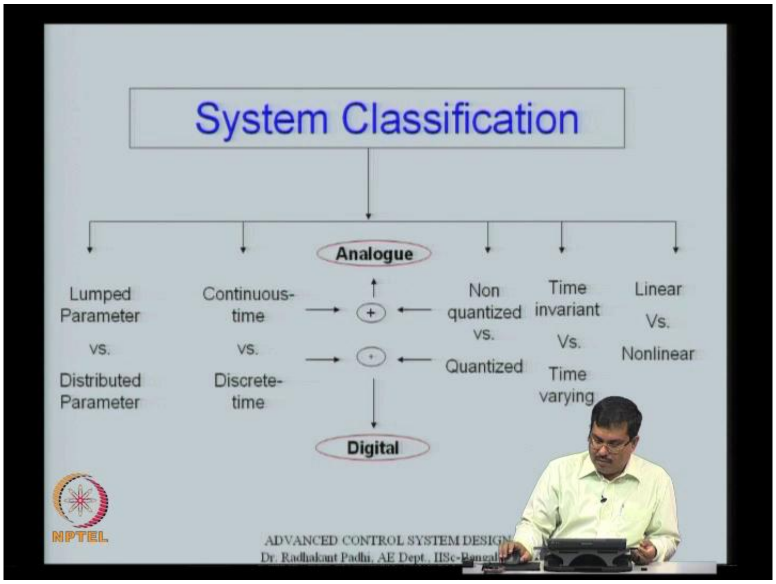

So, we will take that noisy output or whatever, noise free output whatever is available to us, and then we will use that that information, and then we will have some sort of a error signal which will help us actually, there is a there is a input or reference signal, and then the what is really happening is going there as s as a feedback information.

And that will give us some sort of a error signal based on which we will be able to correct the system behavior even further. So, that is the difference between open-loop and closed-loop system. Next, we will discuss about system classification process a different classification for example, I mean it is a different sense we can discuss about that. And first thing that comes to mind is whether, you talk about system as a lumped parameter system or a distributor parameter system.

So, lumped parameter system is what we visualize is a single unity, the single entity, the entire system dynamics is considered… entire system means visualize as a single unit actually. In other words, there is no relative dynamics between different molecules of the system, that is the precise statement probably. That means, if some some I means molecule x moves, then everything every other molecule also moves by the same amount, same direction all sort of thing that is a lumped parameter system.

Typical examples, are like automobiles or aircrafts and missiles, all sort of things what we discuss normally will visualize is as lumped parameter systems, and there will really need a set of ordinary differential equations to describe the system system dynamics completely. However, if you if you if you go a little more deep into that there are systems which we cannot describe as lumped parameter system for example, temperature control, or flow control, or vibration control like that.

There has to be some dynamics between different molecules of the system as time evolves, so that means you really need some sort of a partial differential equation to describe this distributed parameter systems. So, there is a remarkable difference thing, because here we talk independent variable as single variable, single independent variable particularly time, but here you can have time as independent variable, as well as some special variable as independent variable like time as well.

So, here we will in addition we also need to discuss about boundary conditions whereas, here we will need only initial condition, in distributor parameter system you need both initial condition and and boundary conditions as well. Next, we can visualize systems from continuous time versus discrete time, so whether the time variable which is independent variable, that evolves in a very very continuous manner or it really evolves in a discrete sense. That means, the time signal just jumps from one value to other value to other value actually.

So, that is the type of system that we visualize as discrete time system otherwise, if the time is very continuous smoothen all that, then we visualize that is continuous time system. Next, there is a similar visualization you can do from the dependent variable sense also, this is about independent variable. When you discuss about discretization of time, time is an independent variable anyway. Now, here you can discretize the dependent variable also or not, I mean if you do not discretize the different variable it is non quantized, and if you really discetize the variable then it is quantized, and good example is probably about digital clock, digital watch.

So, the time itself is discrete, but the the position of the watch hand or whatever that pointers and in the watch are fairly discrete, it jumps from one point to one point to one point actually. So, that is kind of a quantized description of the system. Next, we can classify the system from time invariant point of view, as well as time varying point of view. That means, if the parameters of the system or the nature of the system dynamics do not change with time, that is actually time invariant if it changes with time that is time variant. Next, we can visualize that from linear versus non-linear point of view, so we can have linear system dynamics or you can describe that as non-linear system as well, that we will discuss little more on that as we go along actually.

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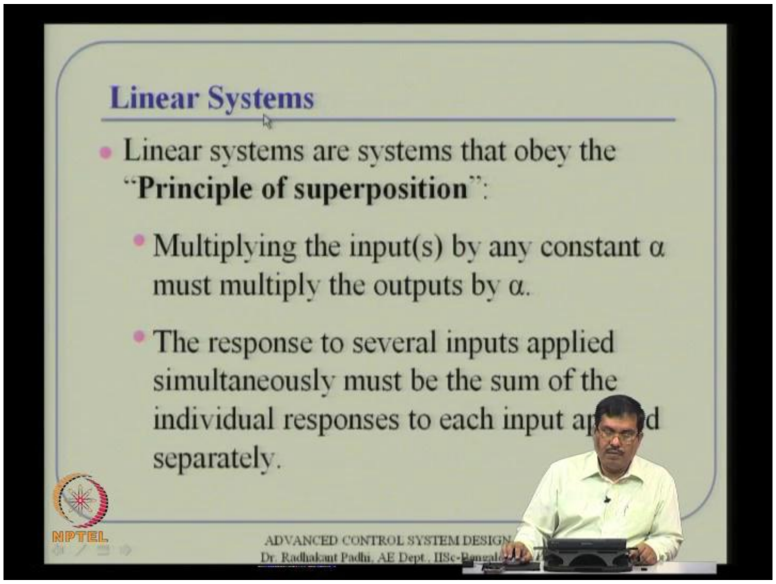

So, here it is, so we describe the linear system as systems that obey the principle of superposition essentially, and what is principle of superposition? There are two criterions; one is multiplying the inputs by any constant alpha must multiply the output by the same constant alpha. And next, the response of several inputs applied simultaneously must be the sum of the individual responses to each input applied separately.

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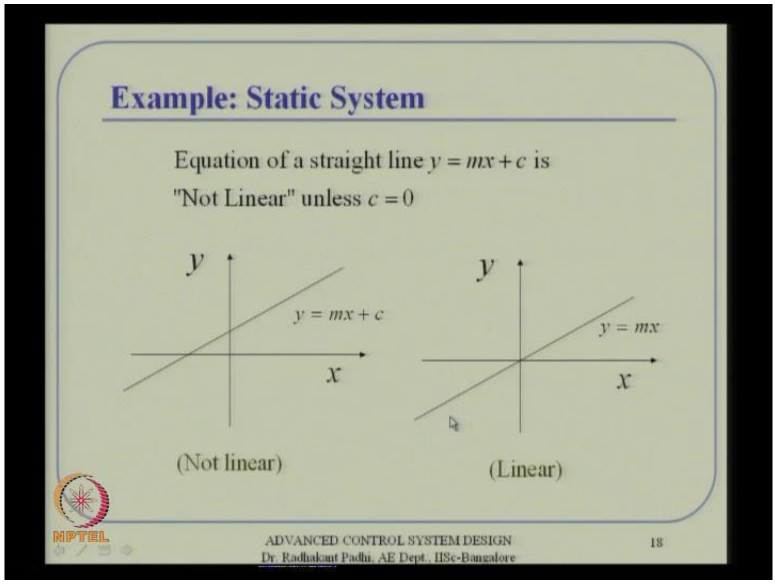

So, these are the two criterions that is that we really require here. Examples, well there is a static system, there is no dynamical system but there is a static system in mind, then we can very well take this common example, what y equal m x plus c we all know that represents equation for a straight line, and that equation for a straight line in general does not mean that represents a linear system. So, you can very clearly see probably we will see that in next example also, that y equal to m x plus c is a linear system provided c is 0, c is non 0 it does not obey the principle of superposition actually.

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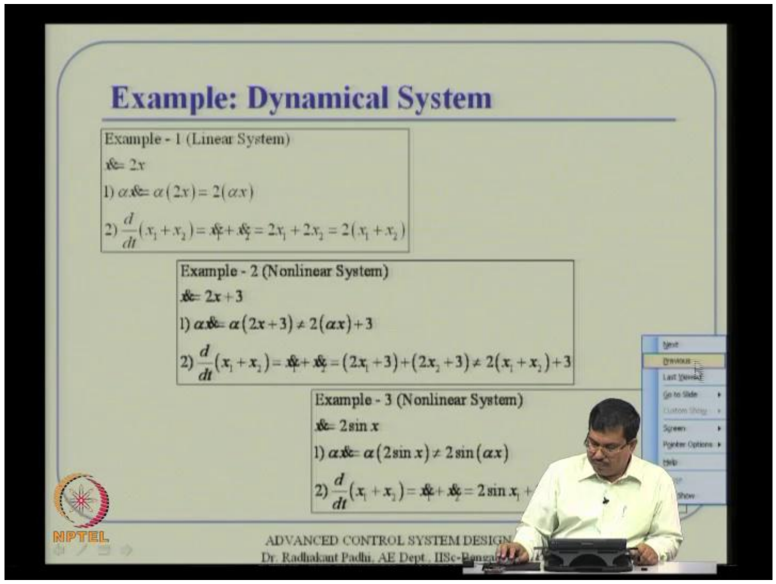

And this particular thing I will demonstrate that through example, so let us first discuss about a linear system, so x dot equal 2 x in a dynamical sense actually. So, if you talk about dynamical system lets talk about example one, which is like x dot equal to 2 x, then you can very clearly see that if I multiply like let say alpha time x dot, then alpha times 2 x I can substitute that.

Then this particular thing this particular, so this particular alpha times 2 x I can manipulate that like two times alpha x. So, essentially if I multiply the input by alpha then the output is also getting multiplied by the same quantity, so that is actually a linear system from first condition. Now, second condition also you can very easily verify, that if you start from here and then go to this next step, then you can substitute that a 2 x 1 plus 2 x 2, then you can take two common, then you can see that if I multiply that if I add that essentially, x 1 plus x 2 then output is also similar actually.

So, essentially the x dot equal to 2 x is a linear system in that sense, it satisfies both the principle system. Now, a non-linear system lets take the very similar example, but we just add the quantity three here then what happens, if you just carry out the same algebra here, then here you will see that two dozen multiply the entire thing, so two multiplies only this particular component, and hence it is really not satisfying the multiplicative criterion.

Similarly, the addition addition criterion is also not getting satisfied that you can very clearly see here.

I start from here d by d t of x 1 plus x 2, then I go to the next one x 1 dot plus x 2 dot I substitute it x dot equal to this 2 x plus 3, that means x 1 dot is 2 x 1 plus 3 x 2 dot is 2 x 2 plus 3. However, it is not two whole into that x 1 plus x 2 plus 3, it is not going to be like that, so that is where we follow (( )) Anyway so now, next example also you see that x dot equal to 2 sin x, again it will not satisfy the conditions that we are looking for, it will not satisfy the condition of multiplicative property, and it'll not satisfy the addition property either. So, that is how we can see that the first one is a linear system, it satisfy the principle of superposition whereas, the next two are really not linear systems actually.

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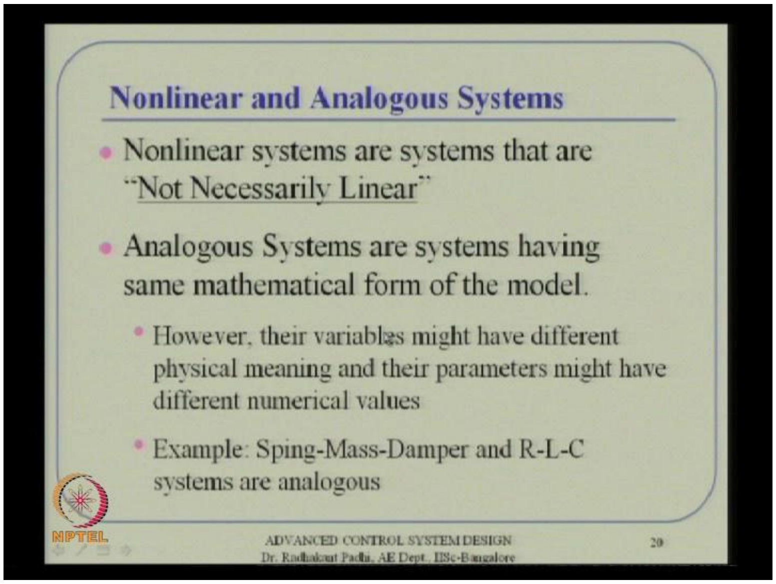

So, then that is how it is, then how about this non-linear system. So, by definition what it means non-linear systems are essentially systems that are not necessarily linear, that is a precise definition. So, why non-linear system we really does not I mean, we really do not mean that this actually a not linear system, it actually a not necessarily linear system.

That means, all linear systems are part of the non-linear systems also but not necessarily vise versa, that is the definition actually. Now, how about analogous systems, analogous systems are systems having the same mathematical form of the model, that means if you really discuss about two analogous system, you take the system dynamics for system one, and system dynamics for system two, mathematical they are identical they not very different either. But the variables for system one may mean something, and variable for system two may mean something like essentially.

So, that means suppose for example, sping-mass-damper system and r l c system are really analogous, one is mechanical system another is electrical system really, but they are two… I mean they are the meanings are different, but the mathematical description of the system dynamics will remain same, and that actually this kind of ideas essentially gives up the flexibility to discuss concepts in a generic way, you do not have to discuss about very specific particular discipline for say.

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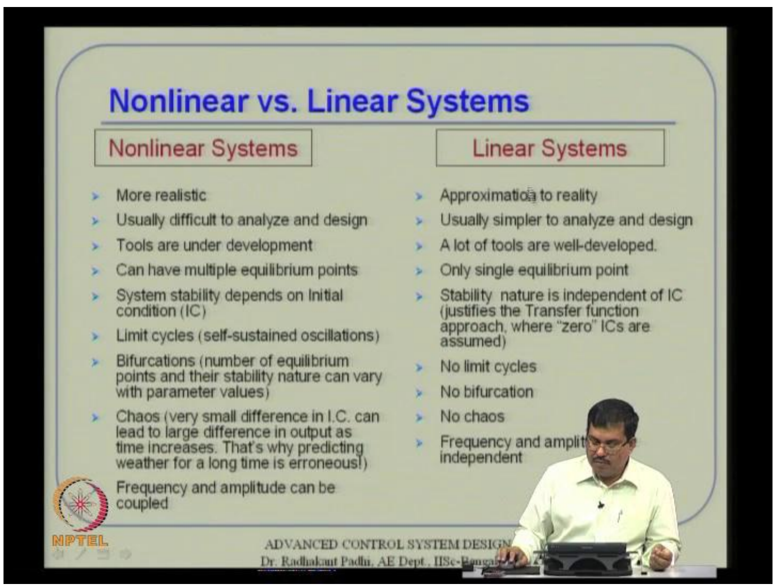

Next, will the lets know little more about non-linear versus linear systems, and first of all we all know probably that non-linear systems are more realistic whereas, linear systems are only approximation to reality, we know that. And second, the non-linear systems are usually difficult to analyze and design however, there are linear systems design technique, analysis techniques are many tools available, so they usually become simpler to analyze and design. There are many tools which are well developed for linear systems whereas, it is not very much well developed for non-linear system, there are many tools that are available, but still there are many issues that people are doing research on.

So, tools are still under development for non-linear systems. These first three are philosophical differences, now let us discuss more about very precisely, quantitatively what are the differences that the common actually. About linear systems, the very great thing about linear system is, it can have only single equilibrium point, that means for linear system we cannot have really multiple equilibrium point, Whereas, for non-linear systems the very fundamental difference is, it can have really different I mean multiple equilibrium points.

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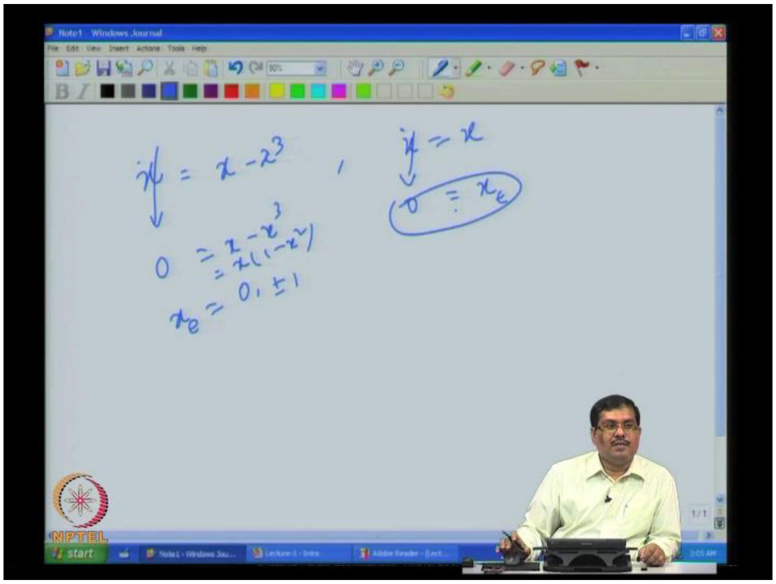

Let us well you can see that in a example sense, if you if you really… Lets discuss about something like x dot equal to, let us say x minus, let us say x cube is very standard example anyway. So, if you really talk about equilibrium point, or linearization of the system will be something like x dot equal to x about x equal to 0. Now, here if you really discuss about equilibrium I put it 0 then the only solution is x equal to 0, that is the only equilibrium point I can have here.

And this is actually, not a very different system than the original system what we started with. However, if you really talk about non-linear system equilibrium then this will be 0 if I put that, that gives me a non-linear equation to solve, and then I gets a solution is both 0, because this is actually x into 1 minus x square so which is 0, as well as plus or minus 1. So, here I have only one equilibrium point, and here I have really three equilibrium points.

So, that is how this non-linear systems are really different from this actually, from the linear systems. Anyway, next is the system's stability nature what we discuss here, not only there multiple equilibrium points, but the system's stability nature can depend on the initial condition as well, where you start with, whether your system trajectory will remain stable after the starting point or not actually, that is also a concern for non-linear systems. Well, that really does not happen for linear system, essentially that is a very great behavior of linear systems, which essentially tells us that stability nature of the stability nature of the linear system is independent of initial condition.

And that probably justifies the transfer function approach, where we typically assume 0 initial conditions everywhere which we start with a differential equation, and then you proceed next step for laplace transform and all. Very typical we will assume 0 initial condition for all system variable and the derivatives. So, this is the this is the reason why we really do that, because the system stability nature is actually independent of initial conditions, so we put 0 to make our life simpler and then then proceed further.

For same thing we cannot do for non-linear system, because non-linear system stability behavior depends on the initial condition, so that is the just the difference actually. Next, non-linear system we have some concepts call limit cycle, linear systems we really do not have either limit cycle, there is no bifurcation and no chaos. So, these are all simplicity for linear systems whereas, we have limit cycle bifurcation and chaos for non-linear systems actually, so let us discuss one by one. So, what you what you mean by limit cycle is something like self sustained oscillation, that means if you go to that the example, if you discuss about this limit cycle behavior, it is a self sustained oscillation.

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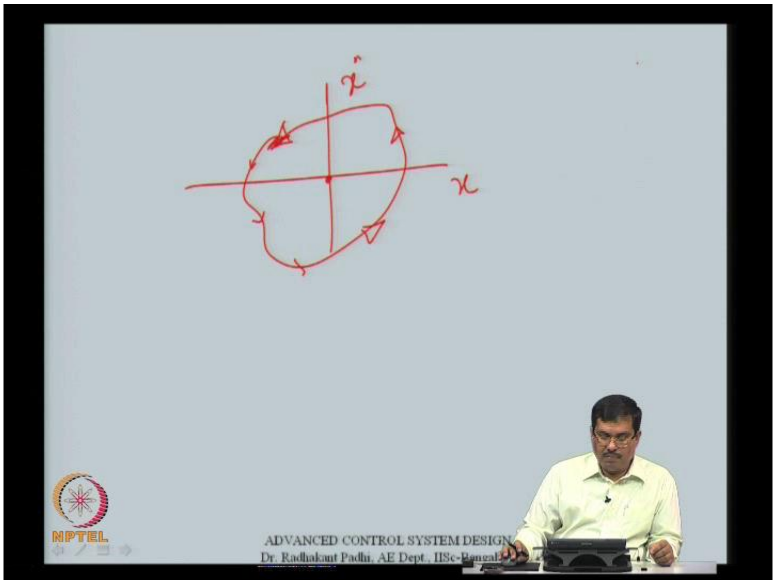

So, let us discuss about what is that, if you draw something like a phase plot, that means x versus x dot this is x dot, then essentially it will have some sort of a closed plot actually, that means the trajectory can keep on moving around that. That means it can start from some point, but it will never go to 0, this is let us say 0 0, but it never go out or it never come back to 0 either, so that means it will keep on the trajectory if you see, it will exchange energy between x and x dot however, it will keep on moving along the same path, need not be very circular it can be some sort of a close path actually close, and typical example is van der pol oscillator. So, van der pol oscillator will typically give us this kind of a trajectory, and there are fatal problems in aerospace engineering where the wing of the aircrafts, they derive energy from the out stream, then they keep on vibrating in while they are flying, that is a typical problem of non-linear systems.

Now, next bifurcation, and what is bifurcation is essentially the number of equilibrium… So, the number of equilibrium point, and the stability nature they can vary with the parameter values. That means, initially you can have just initially you can have just let us say one equilibrium point, but as the parameters vary for the system then the number of equilibrium points can really vary as well, lets go to that same example where I will be able to demonstrate a little more.

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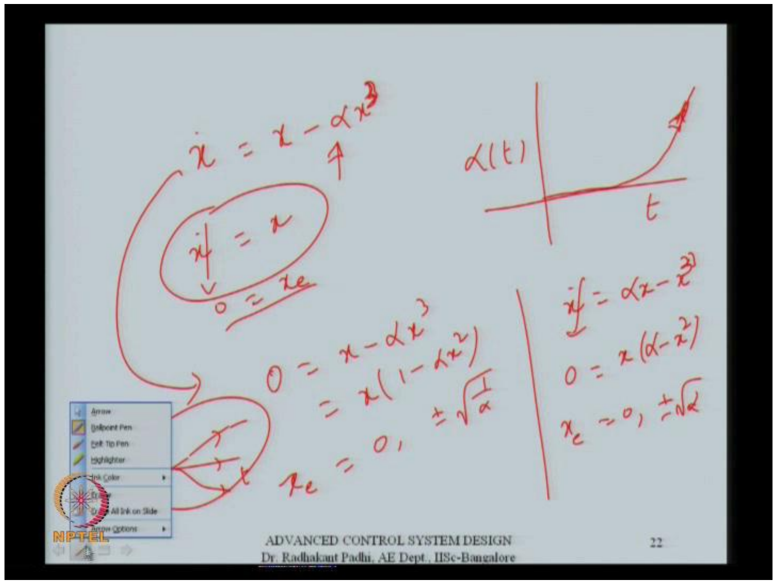

We are discussing about a system dynamics, let us talk about x dot equal to x minus let us say alpha x cube again, and alpha is really small. Let us say alpha if you discuss like that lets say time versus alpha of t, then it starts with a very small value stays like that, and then suddenly it starts going up actually that is very possibility anyway. So, when alpha is small typically, we neglect that and then it tell that is our approximate system here, so it roughly behaves like a linear system.

But, when alpha starts moving up, alpha is more and more we cannot do that, so we will go back to that and tell initially, my equilibrium point was was x equal to 0 anyway, so if I put this is 0 that is equal x my one equilibrium point, but when I go back to that and talk about 0 is equal to x minus alpha x cube, then essentially it is giving me again depending on alpha it is really the solution is will be like that 1 minus alpha x square, so that is the equilibrium points are both 0, as well as something like plus or minus square root of 1 by alpha, you can very clearly see from this equation.

So, as when I cannot really neglect this alpha then the equilibrium points start showing up, initially it was only one later it will start showing up actually. So, in other words suppose that is not the system, lets say your system was something like x dot equal to like let us say alpha x minus x cube, this something the parameter being there then the the equation will be something like 0 into like x into alpha minus x square sort of thing, and then your equilibrium point will be 0 and plus or minus square root of alpha really, not one over alpha.

Then what happens when alpha goes on increasing then your achieve if you plot x x e versus time initially, it was 0 0 0 suddenly it will be itwill going up and all that actually, so there are the three equilibrium point later. This kind of bifurcation behavior is something call pitchfork bifurcation. It is like some sort of a fork kind of area which is coming (( )) So, these are the concepts that are useful for non-linear systems that is typically not the case for linear system. Next, is chaos, chaos is like for very small differential initial condition can leads to large difference in output as time increases.

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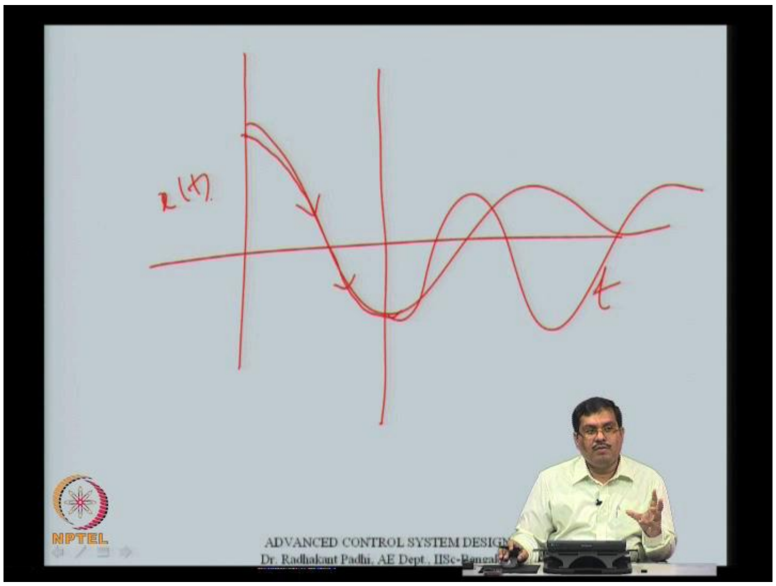

So, what is that, suppose you have something like a system response and suppose it is like this, then the initial condition is very close then you expect the system trajectory to follow that very close, but normally I mean for chaotic system it remains close for little time and then it will be very different up to that, and typical examples are weather forecasting system, weather models are typically chaos models. So, initially your prediction will be good, so the weather prediction for a day or half a day is normally, but weather prediction for a week is typically erroneous, that is that is primarily because they are the truncation computation what the…. they use the model to forecast and all that, they will dominate after some time very quickly rather, and then we will have problem about wrong prediction later.

So, that is that is the behavior of this chaotic system, so we we will be able to do that in a very little time we are correct, but then in a long run it we are not the (( )) They are typically chaotic systems. Next, last one the frequency in amplitude can be coupled here, and in this non-linear system it can be coupled, typically what we visualize in linear system they are decoupled, the frequency is one aspect and amplitude is another aspect in a signal. But here they can become really dependent, and there are some analysis tools which will really analyze that in a coupled manner, describing function analysis especially. Let us keep that

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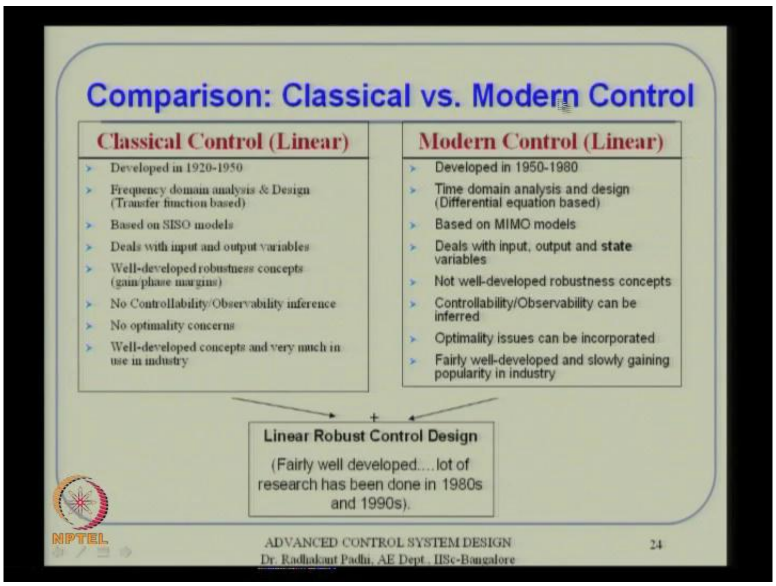

Next lets quickly compare some sort of classical versus modern control, classical control was developed in 20s to 50s, primarily between 1st world war and 2nd world war, and modern control was developed mainly, and here what you mean by modern control is linear control essentially, so they are developed between 50 to 80, they are still under developments many concepts and all, still not completely over for say, but majority part of the development happened during 50s to 80s.

Then here the necessary tools are frequency domain analysis, so essentially transfer function based, here we necessarily talk about differential equations based directly in the time domain. Here we discuss about single input, single output models primarily, even though its MIMO system we discuss that in a single input to single output somewhere, and then we take that relationship into account and then we visualize that as a single input single, output system at a time.

But, in modern control there is m i m o models and we manipulate those models directly, and this one deals with input and output variables only, the classical control systems whereas, the modern control systems, it will deal with both input output and state variables as well, it does not stop at input and output only.

Here about the goodness or the good thing about linear classical control system is some sort of well developed robustness concepts about gain and phase margin like that, these are all robustness concepts nicely well developed, its not fairly well developed here, but there are lot of effort going on now a days to develop equivalent concepts of robustness from modern control point of view, and even people now discuss about adaptive control point of view, how do you come up with criterions for robustness and all that.

So, there are many research topics, there are many people who are doing research on these lines and all that. Then another drawback of classical control is essentially, there is no controllability, observability inference directly from the system model, because it fairly remain silent about the state model, but if you state variable it does not talk anything about that, so that information remains hidden there actually. But if you have a model that is completely… they have a transfer function that completely describe the system, then you can create a kind some sort of a m I m o model out of that, there are equivalent transformations available from transfer function to time domain and all that, then the inference turns out to be like this that there should not be any poll 0 cancelation for this, for both observability and controllability.

So, these are inferences there available when I lose sense and all that, but it is not very tight sense they are not available here. Another drawback is no optimality concerns, it all talks about stability only whether the closed-loop system will remain stable or not, that is the only concern that you have. But in modern control we really talk about optimality issues can be directly incorporated, that is how we discuss about optimal control design techniques. And these are well developed and very much uses industry whereas, these are fairly well developed, and slowly gaining popularity in industry as well actually. There are ideas of merging these two, which is actually linear robust control design ideas there.

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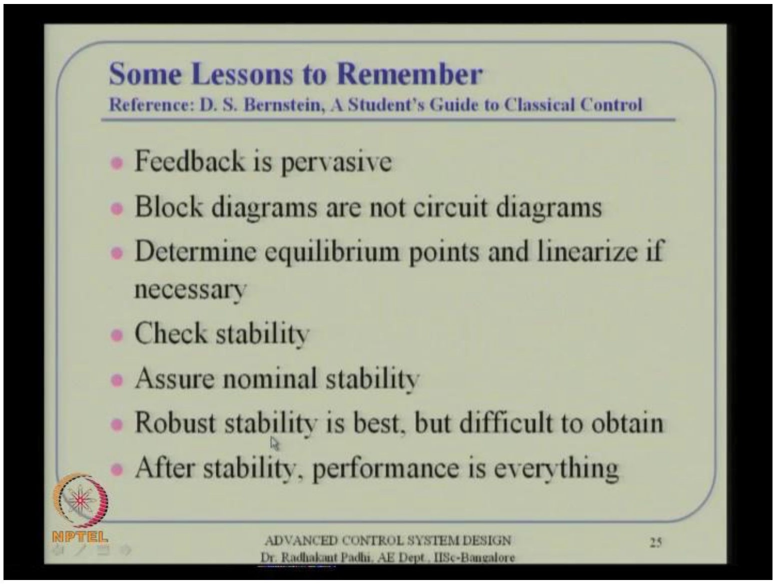

And some lessons to remember before we proceed further that the feedback is always there whether you like it or not, and we have to make use of that that, there are some lessons to remember this is a nice article by Bernstein, and you can see that the feedback is pervasive actually. Then also we need to remember the block diagrams are not necessarily circuit diagrams, that means the block diagrams that we discuss in control system or system analysis tools and all that are essentially, conceptual ideas put together in a structured way, it is a absolutely no connection with circuit diagrams, there can be connections if you discuss about electrical system for say, but in general block diagrams are actually conceptual ideas put together.

Then next thing what you do given a system, you find out the equilibrium points and linearize it necessary about that, next thing check your check stability then assure nominal stability, that means if you assume the transfer function is very correct there is no error or there is no parameter inaccuracy like that, then only concern that we have there at that point of time is assume stability for the nominal system, that is the first condition.

Next, you have to kind of make sure that there is robust stability, that means after stability you have to discuss about robustness issues actually, normally those are difficult to obtain, but robustness is a must for any practical control system for any practical use. So, essentially after stability, that means after both nominal and robust stability, then you we have to worry about what is the performance behavior. So, without compromising too much on this aspect, especially like nominal stability there should be no compromise at all, robust stability you may like to compromises a little bit because these two are really counteracting with each other anyway. But without compromising too much you cannot discuss about performance of the system also, so that is a another system.

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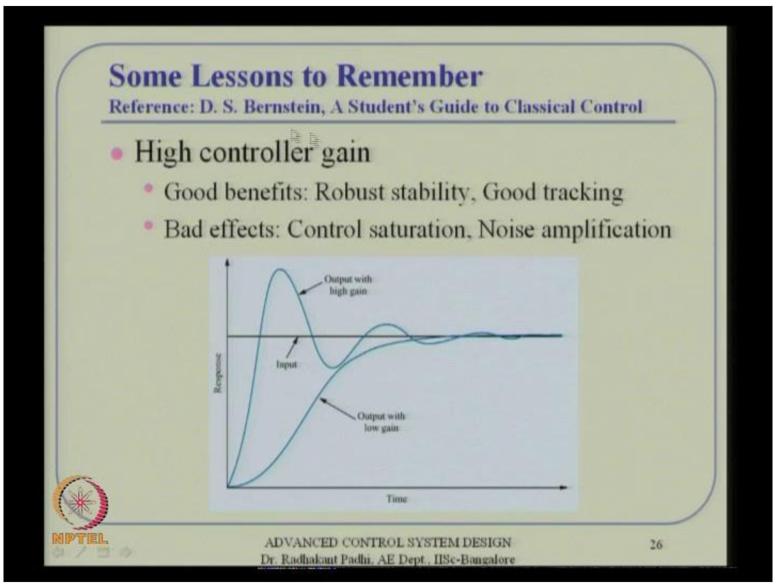

Now typically we like to use high control, but high control gains has good benefits as well as bad benefits, I mean bad effects and all that. Good benefits is it leads to some sort of robustness behavior, it gives robustness actually, so it assures good tracking also. However, the bad effect it leads to control saturation, it leads to noise amplification like that actually, so you have to be careful about selecting a gain properly.

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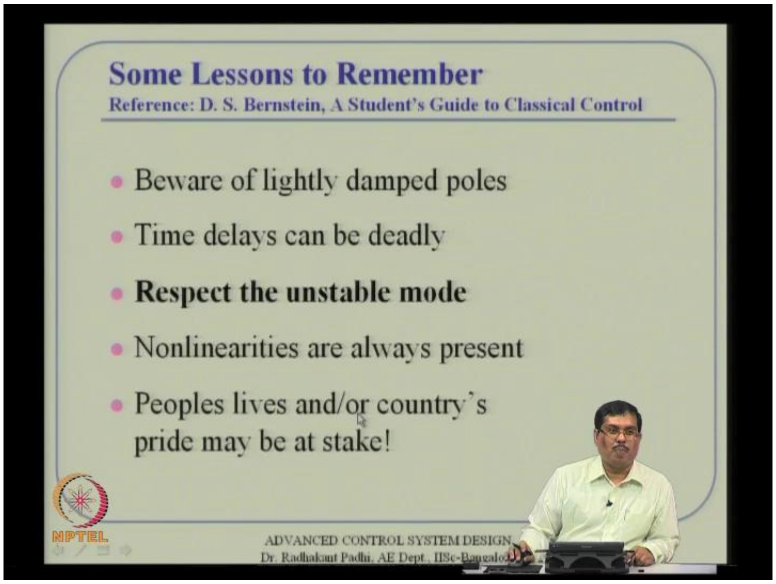

Now, you have to also see that we have to be worried about this lightly damped poles, because they cannot be rejected so easily, and the behavior of those lightly damped poles or long term effects actually, so these are… And they are also remember they are like close to imaginary axis, that means what happens is there is a danger that these poles may go to the height plane and the system becomes unstable after that, so you have to be very careful about these lightly damped poles which are close to the imaginary axis actually.

Now, time delays can be deadly so we are be about that, the very good very good part of it in this article I found is this part, what we have to do is respect the unstable mode, if there is one unstable mode somewhere is going to couple with others and pull everybody into instability, so neglecting whatever amount of unstable mode whatever it is is not a good idea so, little down the line it will devastating actually.

Nonlinearities are always present, that means we we have to live with nonlinearities, and how do I handle that? Whether in a approximate sense or true sense that is our way of looking at the system. And last you can also remember that people's lives, and or countries pride may be at stake, if you talk about good missile system for example, strategic missiles, mostly I hope there will not be any use in the future for that, but that is a country's pride, sending somebody to moon is country's pride, you may or may not gain in a monetary sense in a long run, but that is a country's pride, you demonstrate capability actually. But you if really develop a 500 sitter aircraft, and then you put it into danger there are people's lives involve involve in that. So, both are kind of important to us.

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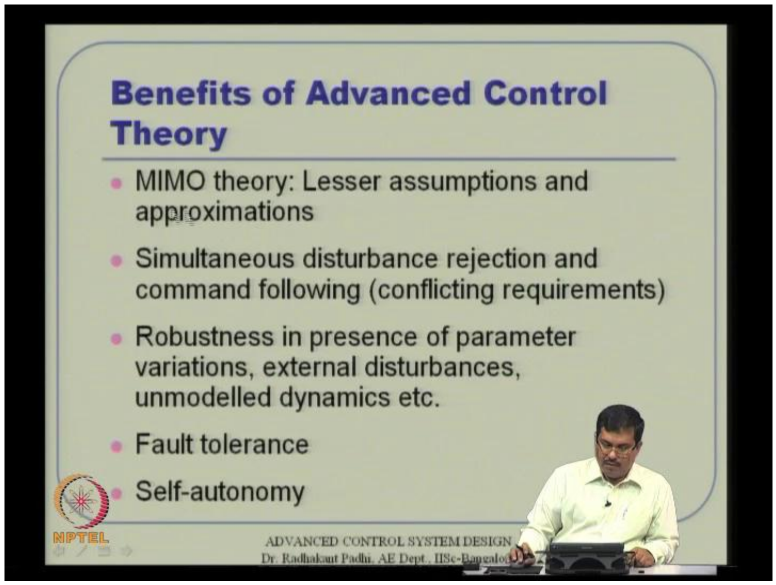

So, in summery benefits of advanced control theory, we discuss about MIMO concept, multiple input multiple output, then there are lesser assumption involved and lesser approximations. And next we will be able to do simultaneous disturbance rejection and command following both together. We will be able to do robustness in presence of parameter variations, external disturbance all sort of on model dynamic etcetera, there concepts of fault tolerance, self-autonomy thing like that.

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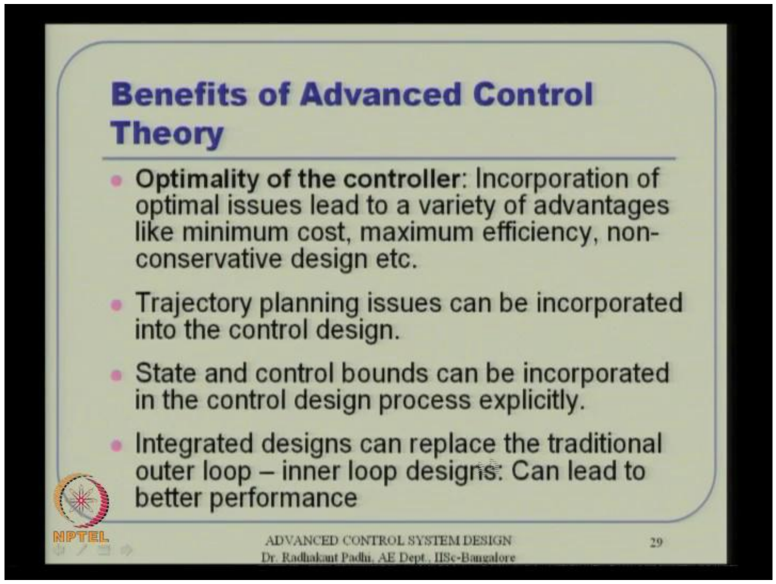

Next, we will be able to discuss about optimality of the controller, that means we will be able to incorporate optimal issues away which will lead to variety of advantages, like minimum cost, maximum efficiency, non-conservative design etcetera, like that there many things we will be able to discuss here. And then we will also be able to use this trajectory planning issues, which are typically concerns of outer loops that can be incorporated into the control design also, that those are concepts for integrated guidance and control typically in aerospace. So, we or that the even before that we will be able to discuss what is my my optimal path to follow, that means the trajectory optimization which is typically carried offline normally.

That also can be discuss directly in the frame of a control system theory, so that is a great advantage of advance control. Then state and control bounds can be incorporated you do not have to live with whatever is coming you know the bounds apriori, then accounting for those bounds will be able to design a control system based on those bounds, because we know that information apriori, so we will be able to use that.

And next integrated designs, what I just discuss couple of minutes back, that generally in traditional sense we will we will help outer loop then followed by inner loop and things like that but you can eliminate that in some sort of integrated design, which will lead to some sort of better performance altogether for overall system .

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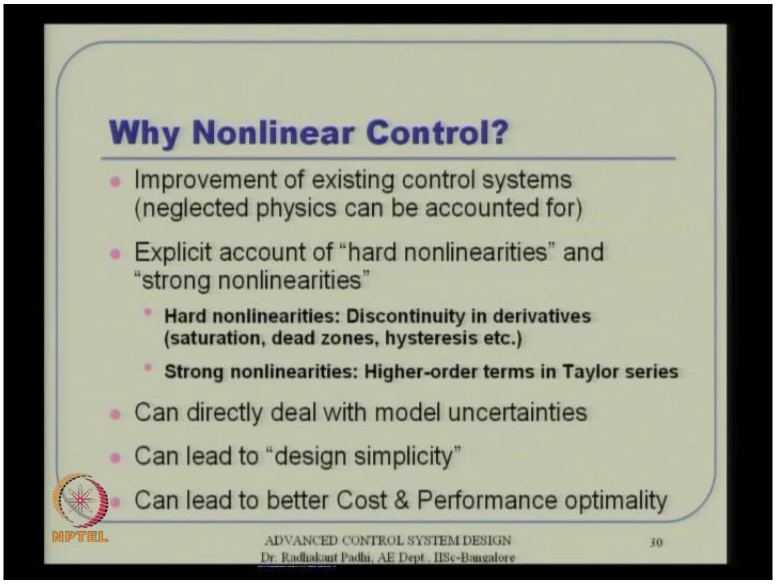

Why non-linear control for particular not necessarily linear system theory, first of all it improvements of existing control systems, so neglected physics can be accounted for. A non-linear system we are not really linearizing the system dynamics around anything, so any operating point, so you can discuss about improvement of the existing linear controller there is a linear controller you want to improve the performance. So, you now account for nonlinearities that you neglected before and you try to improve on that, that is one aspect of non-linear control design. Next, explicit account of hard nonlinearities and strong nonlinearities can be there, so what is hard nonlinearities? these are discontinuity in in derivatives, that means saturation, dead zones, hysteresis all sort of things those are hard nonlinearities.

Strong nonlinearities are essentially, you cannot neglect this higher-order terms in taylor series, they are small signals but there are strong, I mean there are large component from higher-order terms as well, so you cannot neglect that at all but in both sense we will be able to discuss for for non-linear control and in this particular course, we will give importance towards emphasis towards strong nonlinearities, all nonlinearities we will not discuss too much in this course.

Next, we can directly deal with model uncertainties and there are something unknown about the model, we will be able to incorporate that. Next there is a big advantage of discussing about non-linear control and in some cases it will actually, leads to design simplicity over the linear system, which is slightly counterintuitive but as you go along we will see, some non-linear design things are actually, simpler than linear design techniques and one example, is dynamic inversion for say.

Well, comes to mind is essentially the feedback linearization or dynamic inversion essentially eliminates the gain scheduling philosophy and hence, it is simpler than the linear design technique. Next is, it can also lead to cost and performance optimality, if you really talk about optimal control design for the non-linear system, it is not optimal control of for the approximated approximate plan, it is actually optimal for the for the true plan. So, that is that is how it can lead better cost and performance optimality for the true plan.

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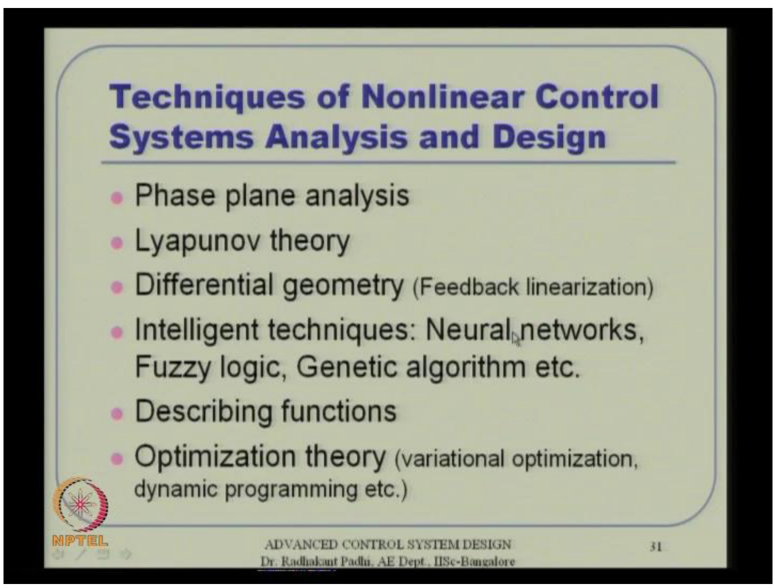

Then, techniques of non-linear control design, there are many techniques available, one is the phase plan analysis next is lyapunov theory, then is differential geometry, which is essentially feedback linearization or dynamic inversion, then there are intelligent control techniques which are based on neural networks, fuzzy logic, genetic algorithms, like that and there are earlier ideas, which are like describing functions analysis, which is like amplitude and frequency getting coupled, that kind of ideas there.

Whereas, optimization theory based, I mean ideas which is variational optimization dynamic programming like that, and about all that we will not discuss everything in this course, we will be discussing some sort of overview of phase plan analysis, lyapunov theory, differential geometry, for intelligent techniques we limit ourselves to little bit on neural networks, and then optimization theories we will also be able to talk a little.

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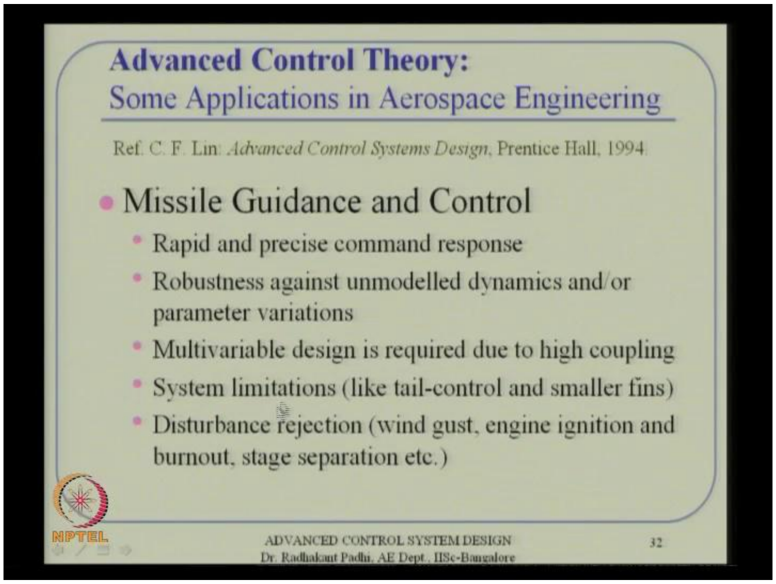

This is some sort of an overview, and then there there are some applications of aerospace engineering that is how we discuss all that. Essentially, it is like some variety applications for missile guidance and control, there are aircraft flight control, there are guidance and control of UAVs essentially. So, all this actually I have listed out here many different things like various command following, you can discuss about reconfigurable control, you can discuss about eliminating gain scheduling and all sort of things. So these are all applications of this. So, that is all about this particular lecture, thanks a lot for the attention.