Rocket Propulsion

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Lecture No: 07

Review of Rocket Principles: Propulsion Efficiency

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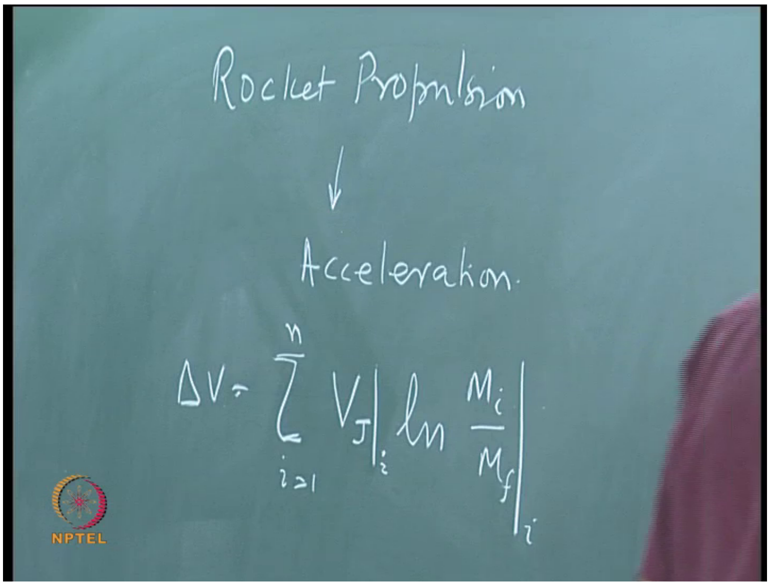

Good morning! In today's class, we will look at the following. We will look at the theory of rocket propulsion again. We derived the rocket equation in the last class. We also looked at staging; we looked at clustering of rockets and also the strap on rockets and what function they do. But, we did not really calculate what is the type of acceleration what we can get from a rocket at takeoff; We will illustrate this in a better way; why we need additional straps or additional clustering for a rocket.

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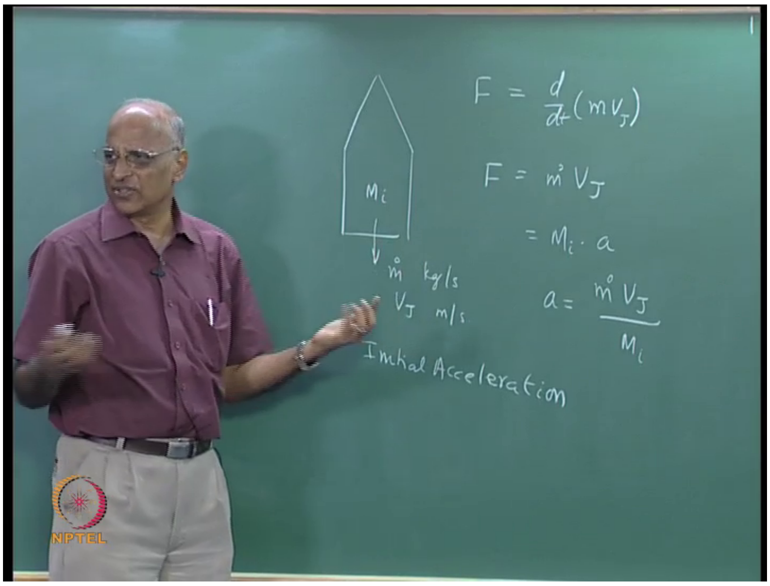

Let us consider this example. Let us say I have a rocket, whose initial mass is Mi. And, let us say it allows mass efflux at the rate m° kg/s. Let us also assume that the rate at which the efflux leaves the rocket (nozzle) is at velocity V_J m/s. Let us put the units together; m°, so much kilogram per second; V_J, so much meter per second. I want to be able to calculate, what is the initial acceleration of this rocket. How do we do it?

In the last class, we told that the force or the thrust with which a rocket is pushed up is equal to rate of change of momentum or d/dt of mv. And, here it is the momentum mV_J. And, therefore the force or thrust is equal to $m^\circ V_J$.

Now, can you tell me what will be the initial acceleration of this rocket? Therefore, the initial acceleration should be equal to the initial mass of the rocket into the acceleration a. That means, force is equal to mass into acceleration or rather acceleration is equal to $m^\circ V_J$ divided by the value of the initial mass. And, what did we tell in the last class? As I keep on adding more and more mass to the rocket, the initial mass increases and it becomes impossible for the rocket to accelerate.

We will work out an example, a numerical example to be able to figure out how we calculate the acceleration and how we decide what must be the level of acceleration; this is because acceleration is also important. Suppose some human beings are sitting in a rocket; it cannot take off at a very high acceleration. The human beings will be adversely influenced by the large acceleration. Therefore, there has to be some control on acceleration.

The ΔV or incremental velocity is equal to summation of; we now use simplified nomenclature. V_J depends on the particular stage and we have natural logarithm ln of the ratio of the initial to the final mass of the corresponding stage. That is i=1,2,3…as i goes from first stage to the second stage and so on. And, this is how we calculate.

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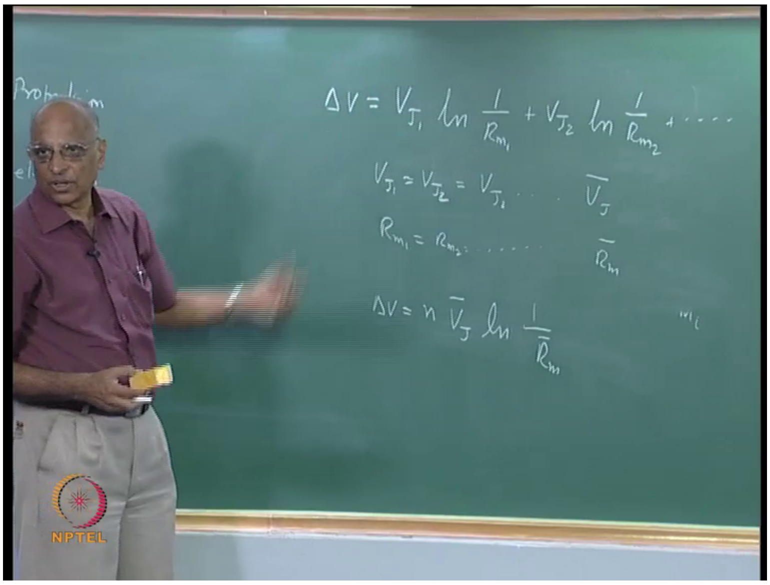

We can try to simplify and get the terms together. And to simplify, we write the summation as: ΔV is equal to V_{J1} 1n of (1 over the mass ratio of stage one). Does it make sense? We said mass ratio of a rocket Rm is equal to the final mass divided by the initial mass. Therefore it is Rm_1. For the second stage we get ΔV_2 is V_{J2} ln of the second stage into 1 over the mass ratio of the second plus and so on. Supposing we have rockets in which the jet velocities are the same for all the stages, V_{J1} is equal to V_{J2} is equal to V_{J3} and so on.

And, further we could have the mass ratios of each stage to be also the same for all the rocket stages. Supposing, if each stage has a same mass ratio and we say Rm_1 is equal to Rm_2 is equal to Rm_3 and so on. Then, what do we get for ΔV. And this expression would be true only when the jet velocity and mass ratio are all the same. Then, we get ΔV is equal to n times the value of V_J ln of 1 over the mass ratio viz., $\Delta V = n\ V_J \ln (1/Rm)$.

That means, by increasing the number of stages we are able to increase the delta V corresponding to the number of stages used. But, this is not possible in practice because the mass ratios of the individual stages may not be the same and the jet velocity of all these stages may also not be the same. You could relax these things by considering V_J to be different, you could consider Rm to be different, and we can keep on getting different ideal velocities. May be we will do a homework problem a little later and try to find out how to calculate the jet velocity taking into consideration a number of stages of rocket together. Well, this is all about the rocket equation and the number of stages, clustering of rockets, and adding a strap-on in a rocket.

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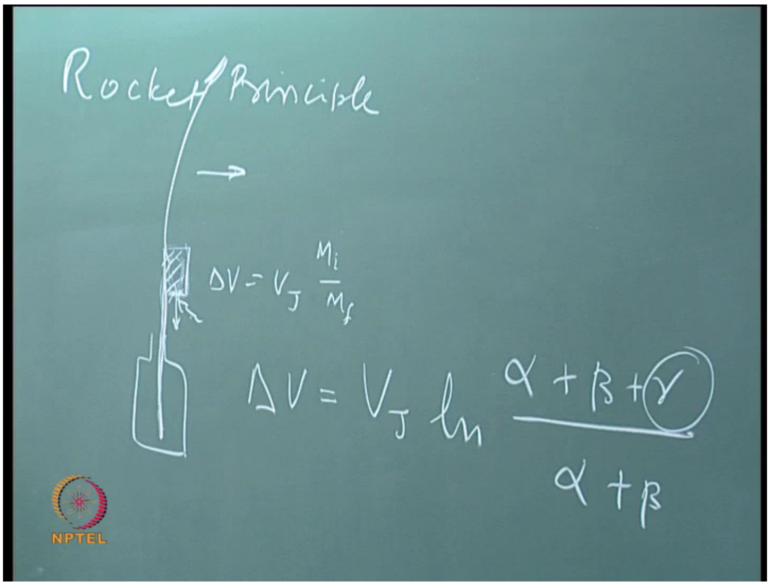

We will look at some examples. But, before looking at examples, I thought can we extend the rocket principle to say a toy rocket used as fire crackers during Deepavali festival. How do we launch this toy rocket? We have a bottle which is used as for launching rockets; we put that stabilizer which is the wooden stick to the fire cracker which is filled with some black powder. We will look at its composition later on. And, you have a small squib over here and you light it and zoom it goes up in a particular direction, if not vertically up. Supposing I want to write the equation for this fire cracker rocket. It should basically be same as the rocket equation?

Therefore, here also we note that the initial mass of the rocket will include the stick, the paper which binds the black powder or gun powder together, and the mass of the gun powder or black powder used. This is initial mass. And, when the rocket is all consumed, we are left with this wood and left with paper which has still not got burnt. This will be the final mass. And therefore, what is the value of V_J? A small hole is provided here through which the gases are escaping and the velocity of gases is the value of V_J.

But the mass of powder used as a propellant is very small. And rather, if I were to go back and write the equation what I wrote in the last class namely $\Delta V = V_J \ln [(\alpha+\beta+\gamma)/(\alpha+\beta)]$ where α was the payload mass fraction, β was structural mass fraction and γ the propellant mass fraction. The amount of gun powder or the amount of black powder which I keep is very small compared to the weight of the stick and the cracker assembly. And therefore, gamma tends to be negligibly small. And, in fact it is not only in the Deepavali rocket that it is small but also in a booster or first stage of a multistage rocket.

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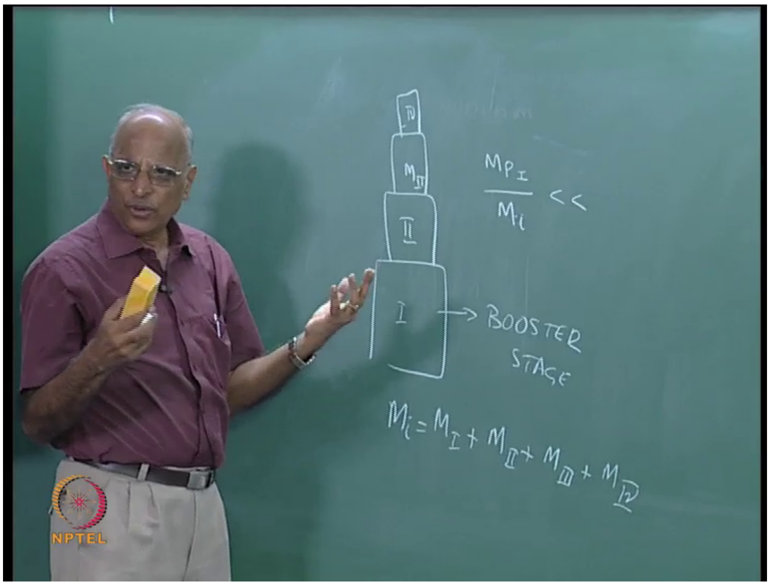

But, if I have to look at another example say that of the multi-stage rocket. We have one rocket stage after the other. I say let us have a four stage rocket. The initial mass of this rocket is going to be Mi summed over i = 1 to 4. This would be mass of first stage M_I + the mass of the second stage M_{II} + the mass of the third stage which is M_{III} + the mass of the fourth stage which is M_{IV}. Now, we have some propellant in the first stage and propellant in the successive stages. Whatever be the propellant we have for the first stage, since the total mass of the rocket is large, the mass ratio of the propellant of the first stage divided by the initial mass of the rocket Mi would be small.

So, also in these cases of multi stage rockets, the first stage, which we sometimes call as "Booster"; what is the "Booster"? It boosts the rocket; it allows the rocket to takeoff from the ground. It boosts the acceleration of the rocket. Therefore, it is known as "Booster stage". And in a Booster stage, the mass of the propellant divided by the initial mass is the small number. If it is a small number can we find out whether I can simplify the rocket equation.

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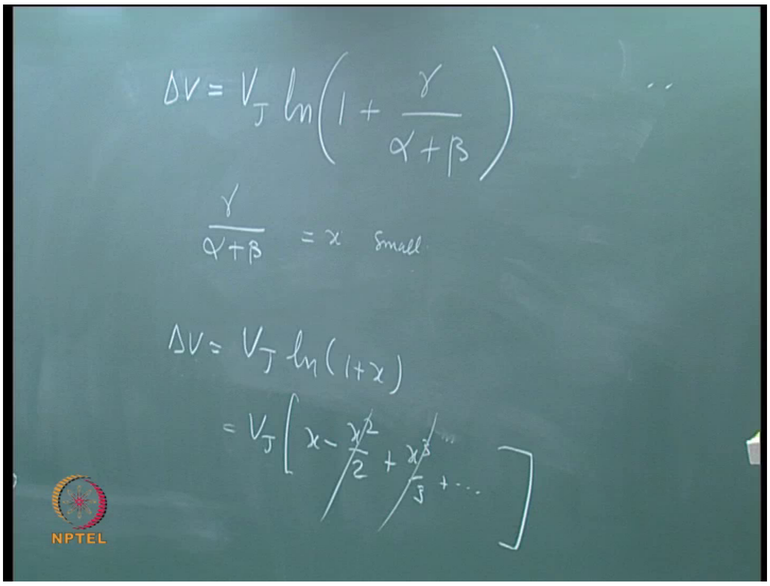

Let us go back and see what happens? Let us write that equation again. $\Delta V = V_J \ln [1 + \gamma/(\alpha+\beta)]$. This was the general rocket equation, which holds good for each of the stages.

Now, I come back to the Booster stage or to the case of a Deepavali rocket. For that, I say the propellant mass fraction γ divided by $\alpha+\beta$ should be small. It is small because there is so much of mass above it. The mass of the propellant is going to be small over here. Therefore, let us say this $\gamma/(\alpha+\beta)$ is equal to x, which is small. Therefore, now I get the equation; $\Delta V = V_J \ln (1 + x)$, where x is a small number. And, what is ln of (1 + x)? $x - x^2/2 + x^3/3 + \dots$ .But, x is a small number. Therefore, I can as well forget about square, cube etc., of x and write ΔV for a rocket, like a booster rocket or the bottle rocket which we used for Deepavali time as $V_J \times x$ or $V_J \times \gamma / (\alpha+\beta)$.

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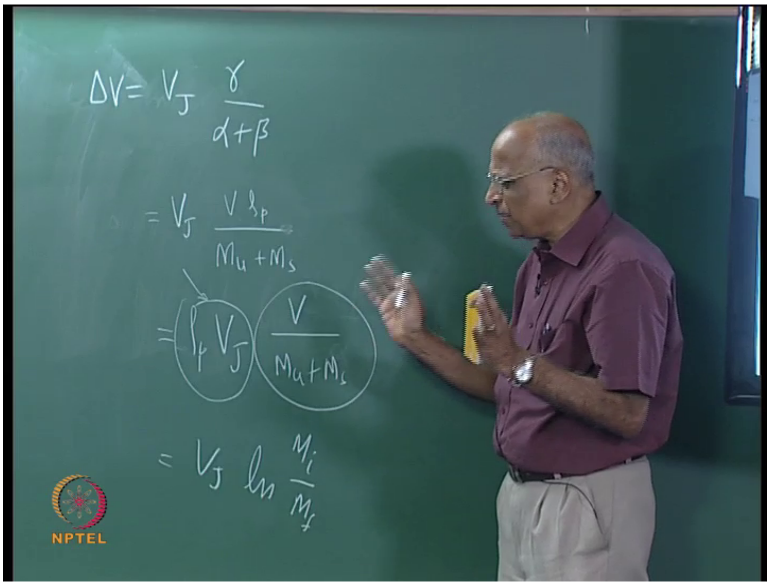

This is something like $\Delta V = V_J \times$ mass of the propellant divided by mass of the useful part plus mass of the structure. I am just writing these terms here. And, what is the mass of the propellant? Mass of the propellant is equal to the volume of the propellant into density of the propellant. And therefore, this becomes equal to density of the propellant into jet velocity into the volume of the first stage rocket divided by Mu plus Ms. In other words, we find that in comparison with the rocket equation wherein ΔV was $V_J \times \ln$ (Mi/Mf), we now get it as density of propellant into V_J into volume divided by the masses. Rather, instead of V_J, density of propellant into V_J becomes influential.

Therefore, for a Deepavali cracker or for a booster stage rocket, instead of V_J being a figure of merit, it works out that the density of propellant into V_J becomes a figure of merit and that is the difference. And, of course this becomes the volume proportional to mass of the propellant and Mu by Ms. This must be kept in mind when we design the boosters. I will come back to this point when I show some slides. But, this is something which is important, which comes very simply by looking at the expansion of this particular expression containing the terms within logarithm.

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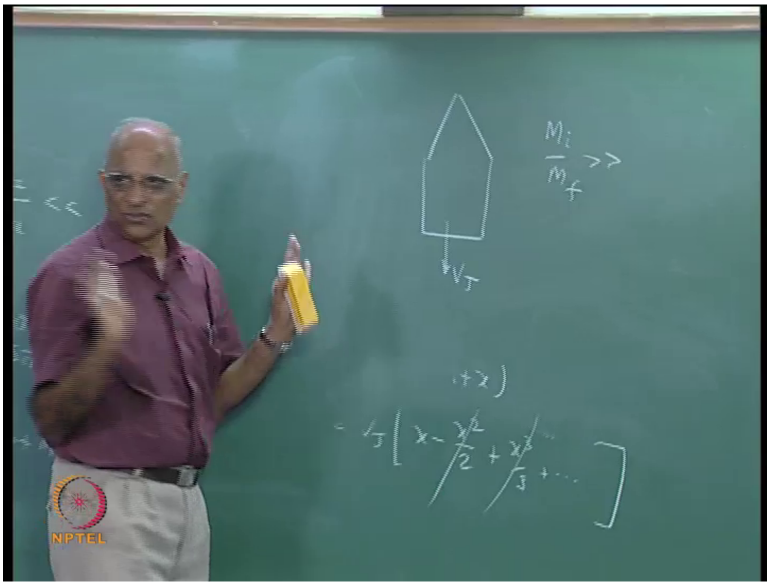

When we want to make a rocket, there are two important parameters that we must consider as we have discussed. One is V_J. It must be very high. The ratio of Mi by Mf must also be large. But, when we make a booster rocket or when we are making a fire cracker rocket, then in that case what is going to be different? Instead of V_J, I would like the density of the propellant into the value of V_J to be large. And in fact, as we go along we will see, since the density of hydrogen is small, the use of hydrogen and oxygen as in cryogenic rockets is not that advantageous for boosters. Whereas if we use solid propellant, which is a dense material, may be it is better for the booster stages. Let us keep this in our minds.

Well, this is all about the theory of rocket propulsion, rocket equations, staging, etc. Let us go back and refresh ourselves on what we have learnt through some slides. And then, we will come back and see what we mean by propulsion efficiency and then we will solve one or two problems.

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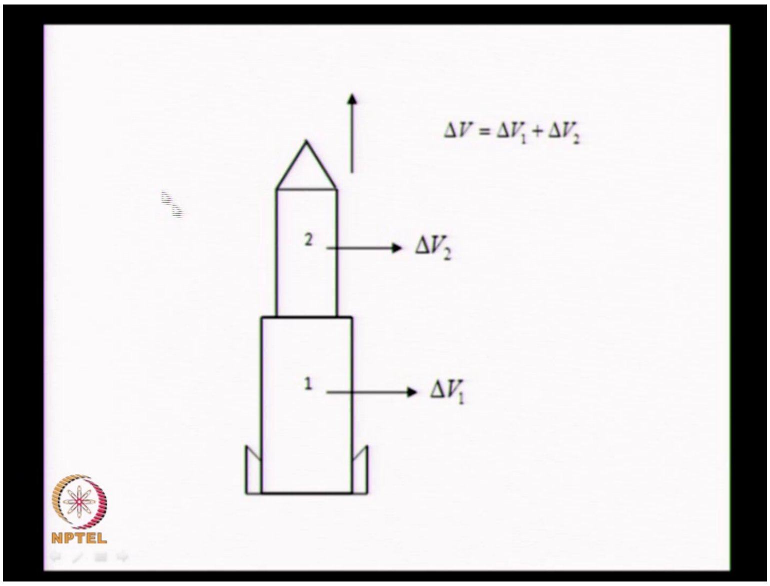

We go through a few slides. You know, here we see a two stage rocket. May be the first stage gives you a velocity ΔV_1, the second stage gives you a velocity ΔV_2. The total velocity of this combination of the first and second stage is $\Delta V = \Delta V_1 + \Delta V_2$. This is a two stage rocket.

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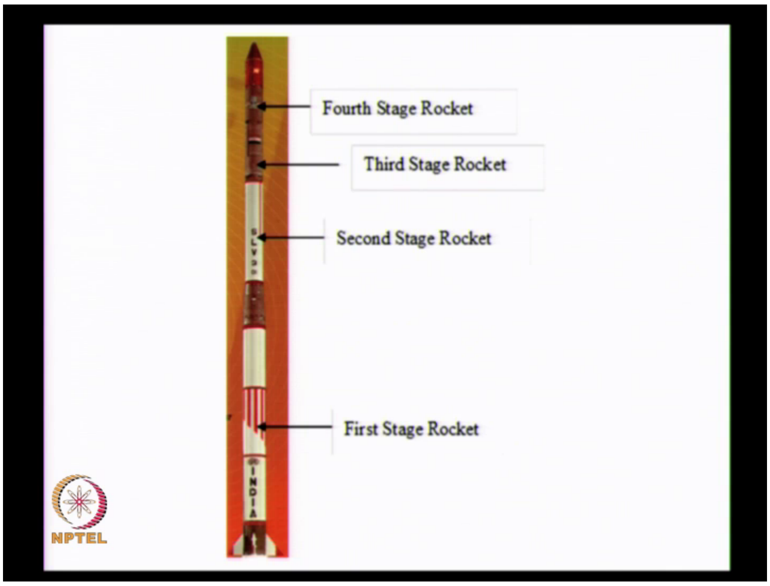

Let us go to the next one. This is the first rocket which was designed in ISRO at Trivandrum. This was in the period, may be nineteen seventy to seventy eight. And, the first successful launch was in 1980. It was a simple rocket, you know. It consists of four stages. And, the all the stages used solid propellants. I will give you a problem involving this rocket, and we will do it today. This is the second stage, this is the third stage, this is the fourth stage. The total velocity, which the rocket gives is ΔV_1 for the first one; ΔV_2 for the second one; ΔV_3 for the third one; ΔV4 for this fourth stage.

We had four stages here. You know it is very deceptive to think that you can make a good rocket to give the desired incremental velocity by increasing the number of its stages. You know sometimes we feel we can keep on increasing number of stages, looks very straight-forward. The more stages we have, the more commands we have to give to the rocket. We have to ignite this second stage and the subsequent stages, we have to separate it out; it becomes more complicated. The reliability of the rocket comes down. And therefore, the trend today is to go only for two stage to orbit (TSTO) or three stage to orbit. People are still working on a single stage to orbit.

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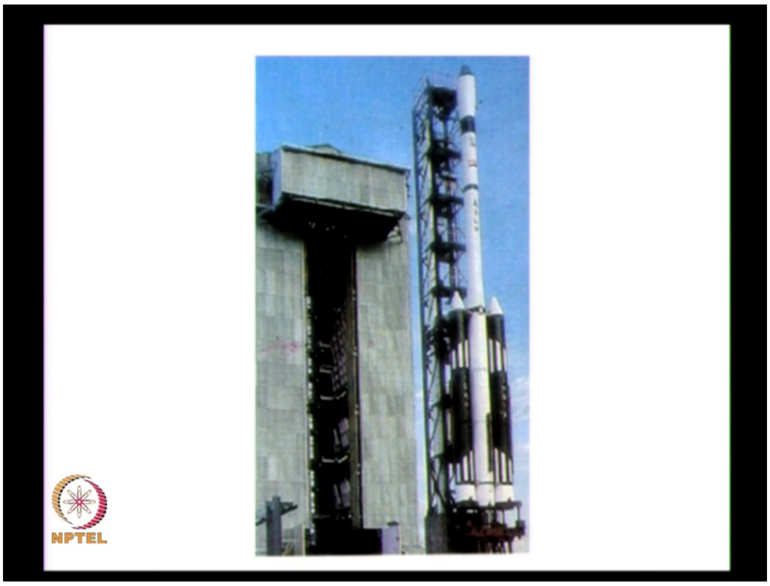

We go on to the next slide. The SLV 3 rocket, what I showed you here, it can only take a payload of something like 40 kilogram. It is very small. Therefore, it was necessary to go to higher payloads. With higher payload, you cannot have the vehicle to accelerate adequately and provide the incremental velocity required; you need to increase the propellant weight. It was necessary to put two straps. Therefore, we have a strap on either side. The straps are same as the first stage in this vehicle(you have two strap on) and then the second stage, the third stage, fourth stage. This is known as Augmented SLV or “ASLV”.

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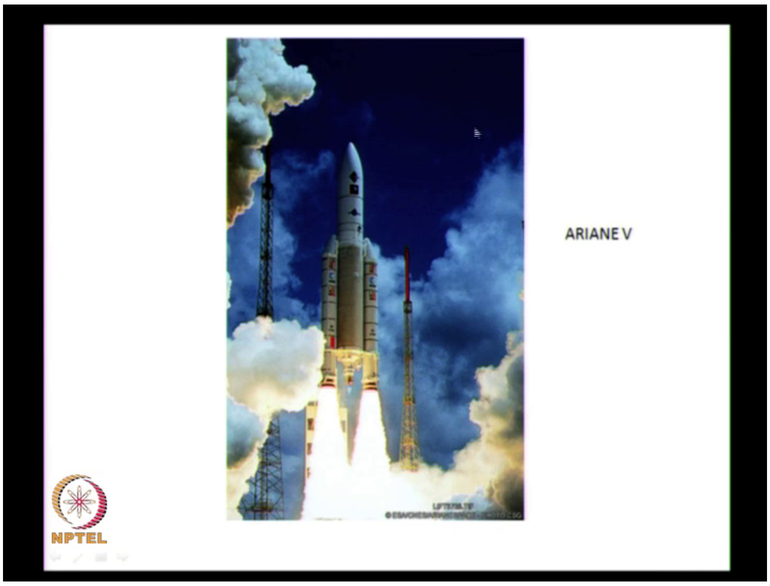

And, now I show some more examples. These are the current rockets which fly; “ARIANE V”, by which we launched several INSAT satellites. We have two straps; first stage and the second stage

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This is the “SPACE SHUTTLE”. You know, it has been work a workhorse for US Space program though it has been decommissioned now. The last flight of “SPACE

SHUTTLE" is over. In this we have the space plane, which comes back. It has something like three liquid engines, all clustered together so that, you get the high thrust. And, you also have two straps. One strap here, the other strap over here; that means two straps for giving the initial acceleration. It starts off with the three liquid engines and straps burning and it pushes itself up. The brown thing what you see is the hydrogen tank, which stores hydrogen which is required for the liquid propellant rockets in the space plane.

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This shows the Space Shuttle taking off. These are the two boosters. Solid rocket straps as I said and you have three engines which generate the thrust.

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Another work horse of US is the DELTA vehicle. Again you have a number of straps over here or cluster of engines.

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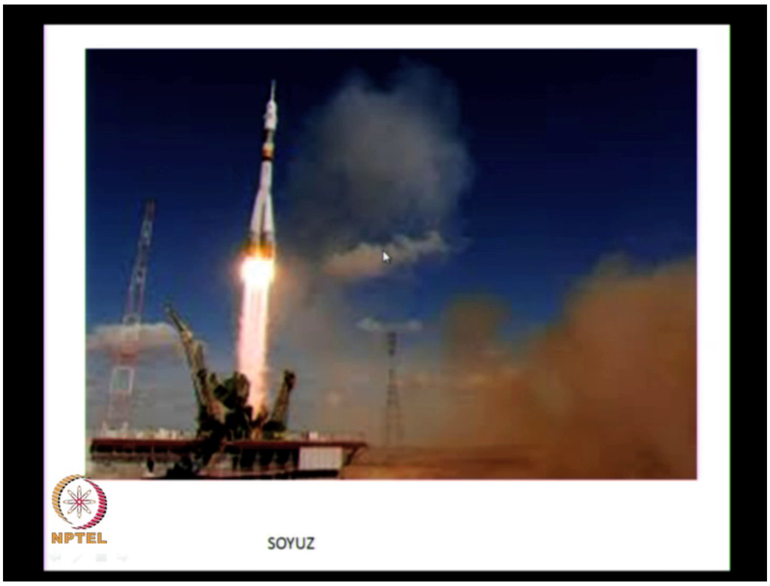

This slide shows "SOYUZ", a Russian rocket. Here you have straps here or a cluster of engines here and the main engine is firing. The exhaust from the several rockets is interacting to give the shape of something like a ball. And, "SOYUZ" was used for launching our first experimental satellite namely "Aryabhatta". This was in nineteen seventy five to seventy six time period. The slide below shows a powerful Russian rocket by name “Proton”. The multiple stages and cluster of stages are seen clearly.

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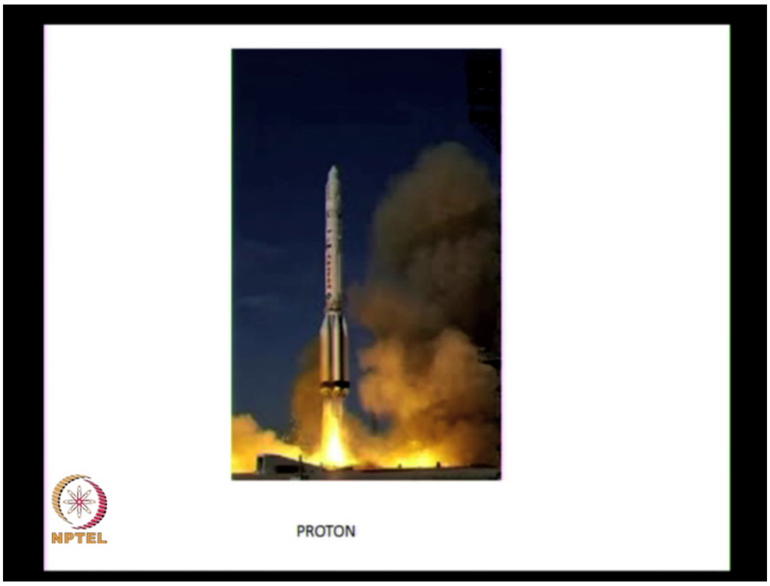

All what I want to communicate is that, most of the vehicles have a number of rockets which are clustered along with a number of stages. This is why I showed these examples.

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We told had said that a rocket can be launched from the sea viz., from a submarine it takes off and you see the water droplets splashing over here.

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The slide above shows Saturn rocket for which I will give a problem which we will try to solve. This was one of the very powerful rockets; which was used to take men to the moon. It was known as Saturn V. And, here again and you have a number of stages. I think it is almost like a five stage vehicle; the ground having straps, then one after the other. Then, you have the spaceship module on top which carry the astronauts and which comes back. Maybe we will take a re-look when we are solving the problem.

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This is our PSLV; Polar Satellite Launch Vehicle of India again. And here, again you find straps around the first stage - you have straps. Six straps or four straps could be put. Then, you have the second stage, third stage.

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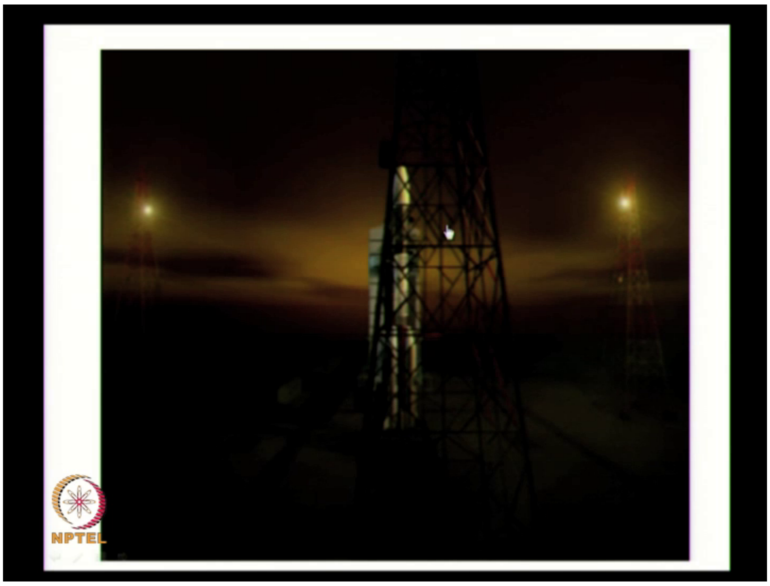

And, of course this is the GSLV. The movie shows GSLV going up. Let us watch it closely. First see the configuration. The configuration consists of, as you see it has four straps or cluster of four engines; one, two, three, four. Then, there is a central engine. First, the four straps fire. It generates thrust, the vehicle takes off. Then, the core also fires and then you have a huge thrust that keeps accelerating it. And, once the four stages clustered to the first stage have finished their operation, they are separated and falls down to the ground. So also the first booster stage. Then, the second stage fires. This is the inter stage. Then, it keeps going further. Then the third stage fires and the rocket keeps accelerating. After all stages have fired the payload which is a spacecraft gets separated and proceeds forward. And, once it reaches the particular orbit, it sort of deploys. This is how we get the rocket goes up. Therefore, staging and clustering are very important in the configuring a rocket.

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And, this is the GSLV. As we just now saw, these were the four straps. This is the first stage, second stage and the third stage. First, these four stages are ignited. Four straps are ignited, gives you the thrust to take off. Immediately after takeoff, the core is also burning. Therefore, you have the huge thrust which pushes it. Then the second stage fires, then the third stage fires. And, this is how a rocket functions.

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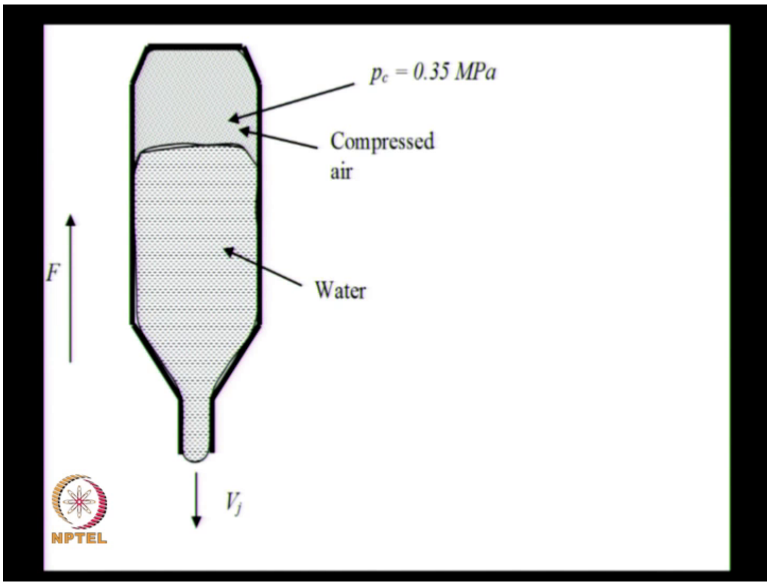

Well, we also talked in terms of a water rocket; wherein we could have water and I could pressurize it and launch it. Maybe we will solve this problem in class a little later.

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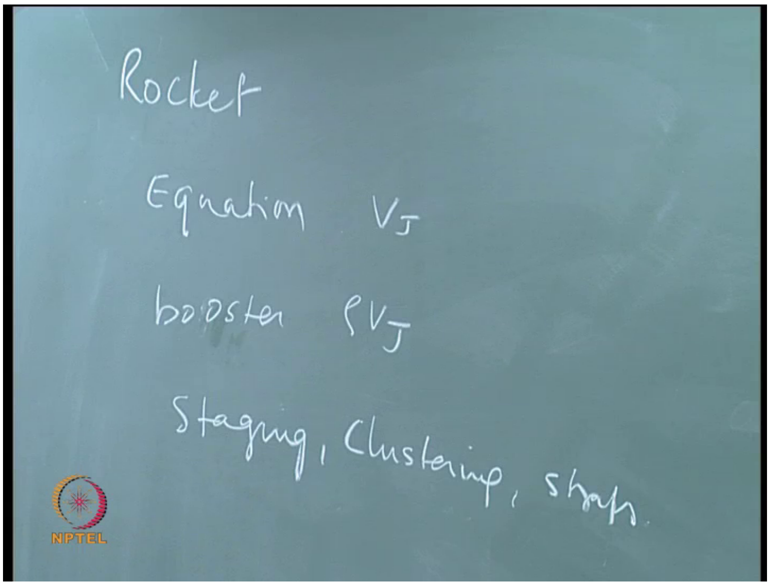

Well, I think by now we should be very clear about what is the principle of rocket, what is the rocket equation as it were, what will be the rocket equation modified for a booster, in which case rho V_J becomes more important than V_J itself. And then, we talked in terms of staging, clustering or and also straps.

If the above is clear, let us go to the next part. We will address efficiencies.

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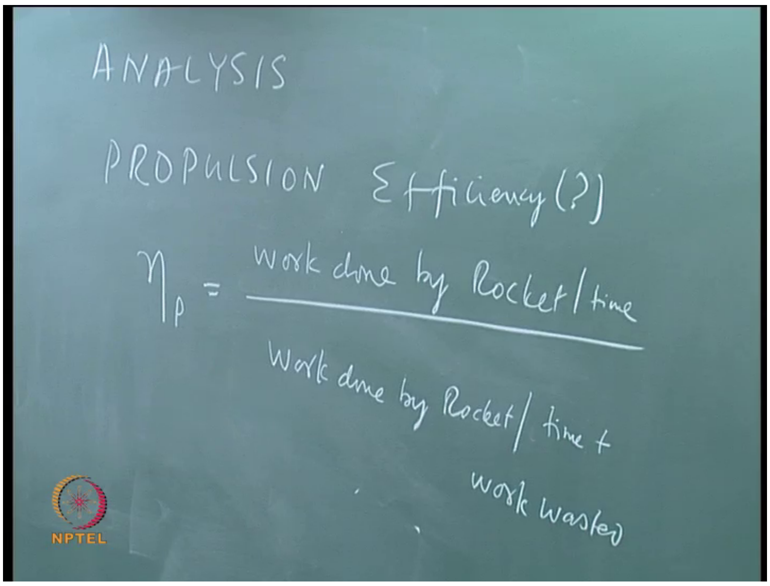

We would like to know what is the efficiency of a rocket. What do you understand by efficiency? We are looking at the rocket flying up; therefore I want to find out how effectively or efficiently it is being propelled or pushed. Therefore, we talk in terms of propulsive efficiency. How do I define it? It has to do something about the forces, how it is moving up or the power which you are giving, the power which is being used. Therefore, I say, well, propulsion efficiency is something like what part of the power generated by a rocket is converted to useful work done by the rocket per unit time. What would it be? What is your guess? Useful work that a rocket does while it goes up. And, what should it be? The actual work done by rocket. Work done by rocket per unit time plus the work wasted by rocket is the power generated. That is the total work which is done by the rocket per unit time. How to put these aspects together as an efficiency?

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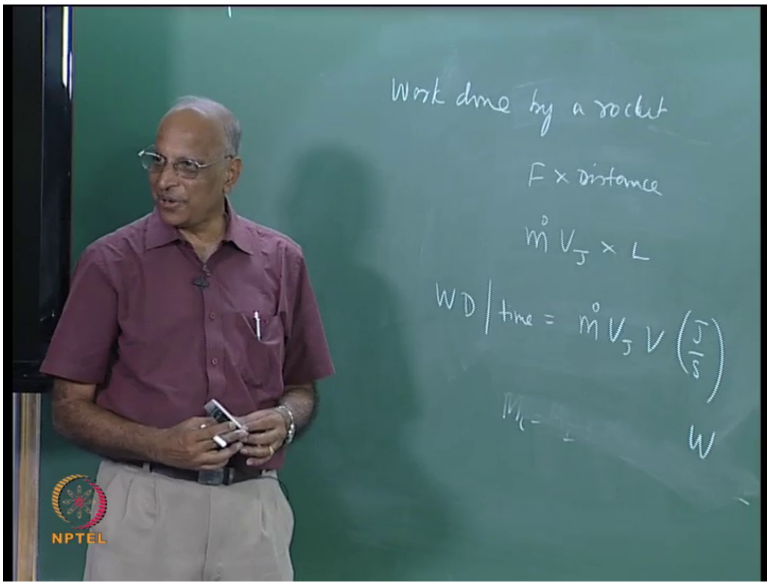

The work done by the rocket is equal to force into distance. The force of the rocket, we found is equal to $m^\circ \times V_J \times$ distance, let say L. Therefore, I say work done by the rocket per unit time is equal to $m^\circ \times V_J \times$ the velocity of the rocket. And, therefore we say so much joules per second or so much watts is the useful work done by the rocket. Is it all right? Useful work; or the useful power.

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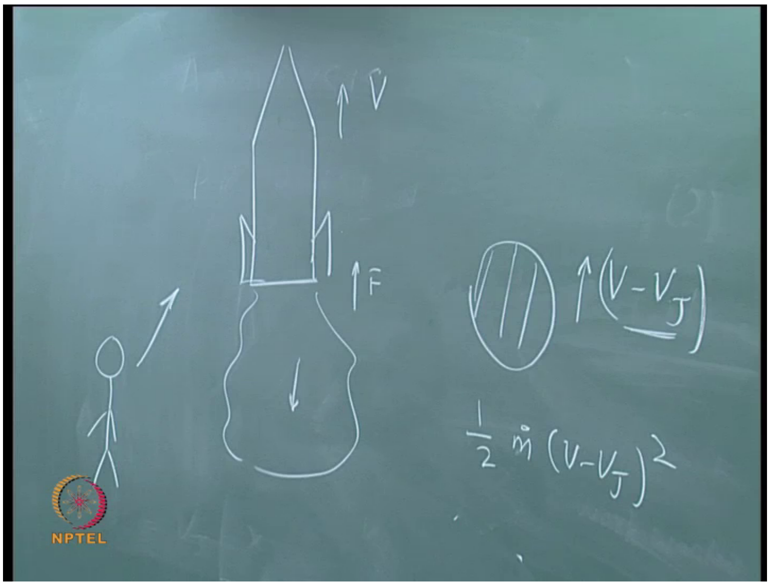

Let us try to sketch the factors involved. We have a rocket going up. It goes with the velocity V. It is pushed up by a force. Therefore, the useful work which is done by the rocket is the force into the velocity per unit time. In the process is anything getting wasted? What is the waste? How do we get the waste energy or work?

As the rocket is getting pushed, the plume from the rocket is going down. Again, we picture the rocket going up. I am in the inertial frame of reference. I am standing here, watching the fun of the rocket going up. What do I see? I see that this plume is now going down with the velocity V_J with respect to the rocket or rather if V is the velocity of the rocket is going up, the plume is going up with a velocity V minus V_J as I see it from the inertial frame of reference.

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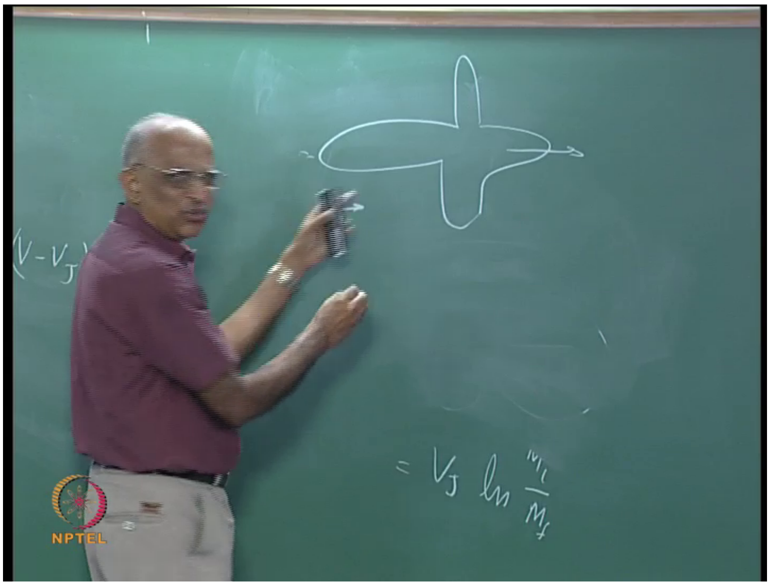

I will give you an example to illustrate. See, on some days, maybe early morning say 5: 30 or 6: 00AM you watch a jet aircraft up in skies. May be some of these jets skip Chennai and you will see the trail behind it as the aircraft is moving. You see the aircraft is going and the trail follows it.

Let us try to picture it out. Let us picture the trail. Aircraft is moving. You see the aircraft going, and then you will find the whitish trail behind. The trail is also following at a slower speed. Why does it have to happen? Maybe because this fellow is leaving the aircraft with the velocity V_J; the aircraft is moving with the velocity and therefore you see this particular jet or plume as it were following it a velocity $V - V_J$.

Therefore, what is being wasted in the rocket? The energy content of this is getting wasted because it is getting lost. And, what do I see from the inertial frame of reference? In the inertial frame of reference, we look at the work done by the rocket per unit time. But, we also see that this work of the plume is getting wasted. And, what is my waste? That kinetic energy is getting wasted or ½ mass of this into $(V - V_J)^2$ squared or what is the rate at which I am seeing is: ½ $m° \times (V - V_J)^2$. That is the waste. That means the rocket is going up; this plume is still following it up like this. And therefore, this is waste. It need not have got wasted.

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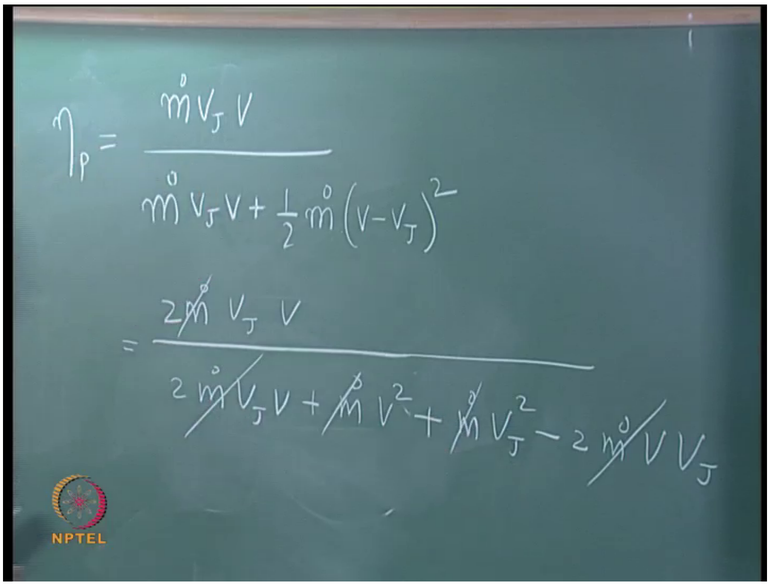

And therefore, how will I put the expression for propulsive efficiency together? I will now write the efficiency as η_p (propulsive efficiency) is equal to the useful work done equal to $m° \times V_J \times V$. This is divided by useful work $m° \times V_J \times V$ + the kinetic energy per unit time viz., $\frac{1}{2} m° \times (V - V_J)^2$.

Please let us be very clear. You know, we will have to define the efficiency of the scram jet. We will have to define the propulsive efficiency of an airplane. We will find that there are some optimum values of efficiencies. And, I find some research work going on. I will refer you to a paper in today's class itself. The way people tend to think; can we improve the rocket by looking the propulsive efficiency?

Let us first simplify this equation. This is equal to $m° V_J V$ divided by the term. We bring 2 on top. In the denominator, we get $2 m° V_J V + m°V^2 + m° V_J^2 - 2 m° V V_J$. Is it all right? $V^2 - 2 V V_J + V_J^2$. You find that this $2 m°VV_J$ gets cancelled; m dot gets cancelled in the numerator and denominator. And, what is the propulsive efficiency therefore equal to?

Propulsive efficiency is therefore equal to $2V V_J / (V^2 + V_J^2)$. Is it alright? $V^2 + V_J^2$ in the denominator. Let us simplify it. Let us divide the numerator and denominator by V_J^2 square and we get $\eta_p = 2 V/V_J \div 1 + (V/V_J)^2$.

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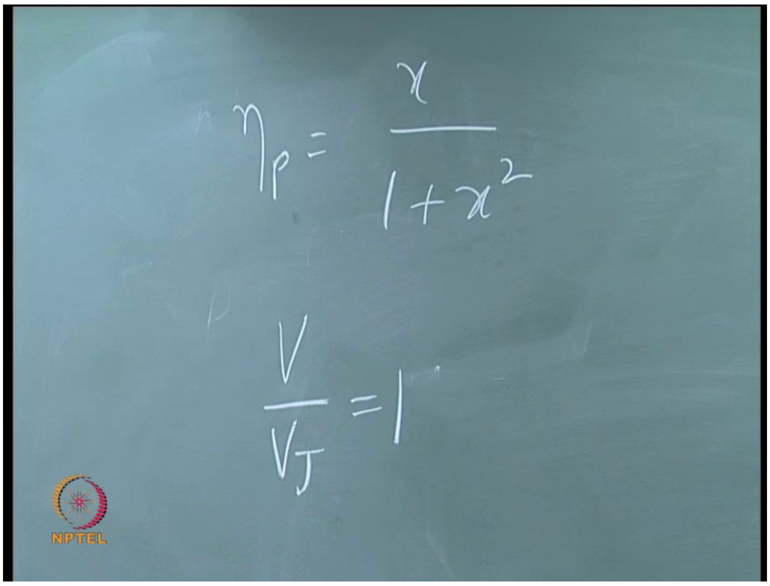

Now I want to ask you, when will the propulsive efficiency be a maximum. Just look at this expression. Just be unbiased and tell me whether I can identify a condition for the propulsive efficiency to be a maximum. We will anyway solve for the maximum. We will find out the maxima and get the condition. But, by looking at this expression can you tell me when should the efficiency become maxima?

Let us substitute V/V_J by x. All what we are saying is $\eta_p = 2x/(1 + x^2)$. What should be the value of x which for which eta p is the maximum? Efficiency cannot be greater than 1. It has to be 1. And, we find that the moment x is 1 or V by V_J is 1. It becomes 2 by 1 plus 1; 2. Therefore, by inspection itself I can say when V by V_J is equal to 1, then the propulsive efficiency will be a maximum of 1. And, how do we do it? Normally, we find the maxima.

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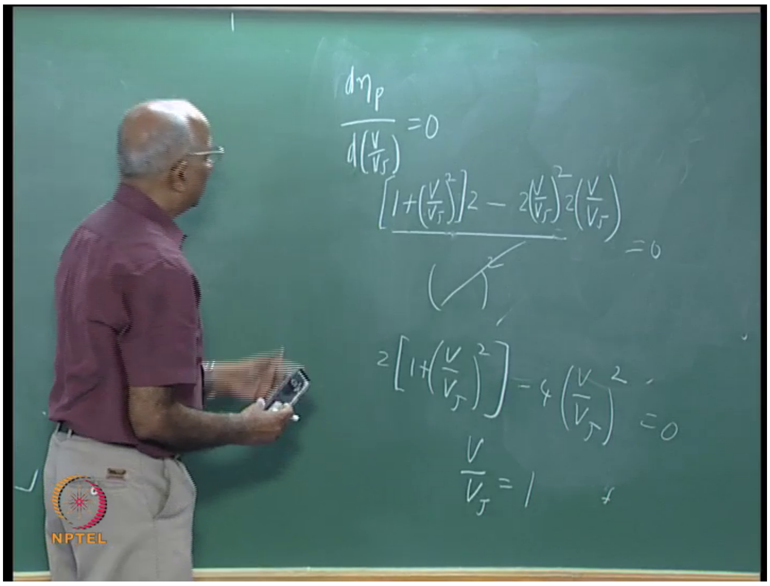

Let us differentiate it. Let us determine the condition for $d\eta_p$ by $d(V/V_J)$ must be equal to 0 to give the maxima. And for that, you would say: denominator into differential of numerator minus numerator into differential of denominator divided by denominator square must be zero. Therefore $1 + (V/V_J)^2 \times$ differential of numerator which is $2 -$ numerator which is $2\ V/V_J \times$ differential of denominator which is $2\ V/V_J$. And, this must be equal to zero. Therefore, I am not really bothered about the denominator of the differential and I need write $1 + (V/V_J)^2$ over here. And therefore, what does it give me? It gives me $2 \times (1 + (V/V_J)^2) - 4\ (V/V_J)^2 = 0$. What does this give you? $1 + 2\ V/V_J + (V/V_J)^2 - 2\ (V/V_J)^2$ or rather it gives $1 - (V/V_J)^2 = 0$. This means V/V_J must be equal to 1 to get the maximum.

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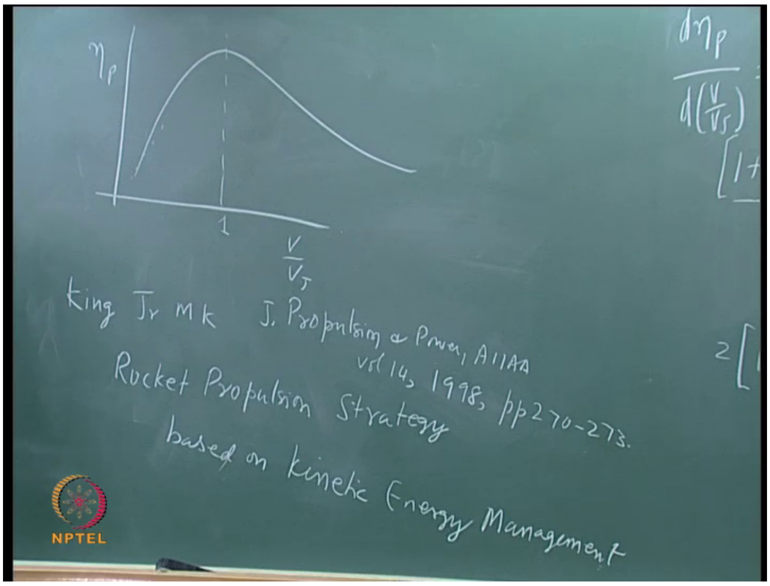

Therefore, if we were to plot the propulsive efficiency of a rocket η_p as a function of its flying velocity divided by the jet velocity, the efficiency becomes a maximum when the velocity ratio is 1. It increases initially till the ratio is one and thereafter it begins to decrease. Therefore, this gives us some suggestion viz., that if I can have the exhaust velocity equal to the velocity of the rocket which is high then the rocket flight will have maximum efficiency when the rocket speed is also at this high value. Or rather, as the velocity of the rocket changes, if I can somehow keep on changing my exhaustive velocity, I operate the rocket at its maximum efficiency.

But then, you know that this is just not possible; a rocket is rapidly accelerating and the condition is very difficult to meet this. But, there is some very interesting work on this topic. And, one paper which deals with this and which is very exciting to read is this one. I will give you the reference. May be you should take a look at it. It is by "King Jr MK". The title of the paper is "Rocket Propulsion Strategy based On Kinetic Energy Management". It appeared in "Journal of Propulsion and Power". I just write it down here; "Journal of propulsion and power" of AIAA. The volume number is 14. It is in the year 1998 and the page number is 272-273. I would request each of you to take a look at it.

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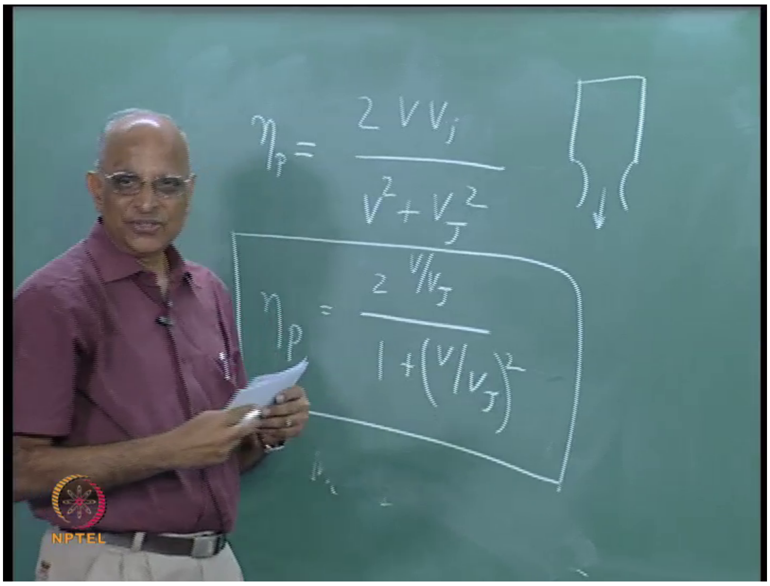

See, we know it is not possible to meet the condition of $V = V_J$. But can you somehow get something to do with V_J and try to see whether you can get a better propulsive efficiency. Such ideas are useful. You know, because later on we will get into electrical propulsion and nuclear propulsion. We will try to see, whether we can somehow make a rocket more efficient by tailoring the jet velocity to be near to its speed of the rocket; because as of today even to go to Jupiter, we saw it takes something like five years. If we have to go to the Kuiper belt it takes something like ten years. Whether for galactic missions - going to different galaxies- we can progressively change the exhaust velocities. Therefore, this article by King is something which is useful. It says it is a blue eyed tutorial.

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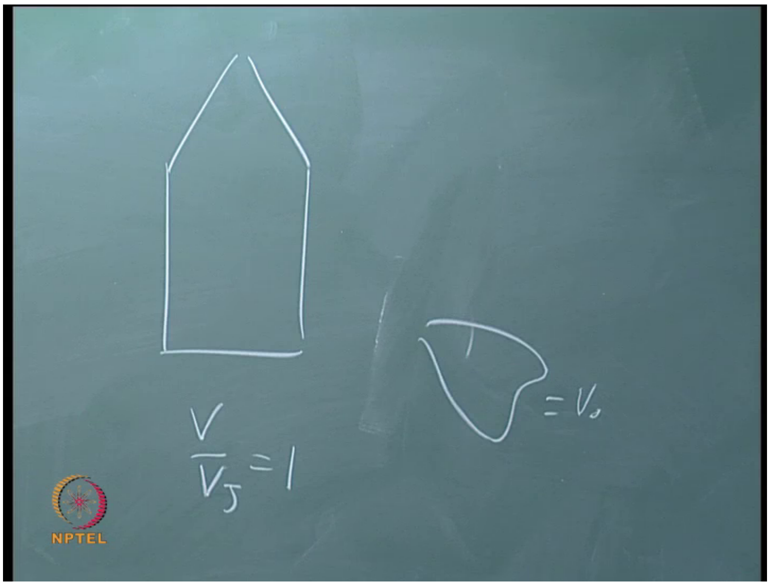

But having said that, I want to ask you one question; Why is it when $V = V_J$ the propulsive efficiency is a maximum? We had said that the rocket is going up and we see the trail following it. What is the condition of the trail or the plume when V by V_J is 1? That means the kinetic energy of the plume is what? Zero. What happens to the trail? I see a rocket going up; the plume is also going up, what will happen to that plume when propulsive efficiency is one or maximum? That means V minus V_J is 0; that means it will be static; that means it has no energy at all.

In other words we are saying is, the plume as seen from the inertial frame of reference does not follow the rocket. It just stays put at the given location. And in other words, we have made use of the total kinetic energy for pushing the rocket up. And this is the reason for the propulsive efficiency to be a maximum.

I shall now go through two or three case studies. I will start with the SLV 3 ~~rocket~~ on how we calculate the masses, payloads, etc., in the first part. And then we shall do a simple problem of the water rocket. But, since this water rocket is here on this particular slide itself, maybe I get started with this water rocket.

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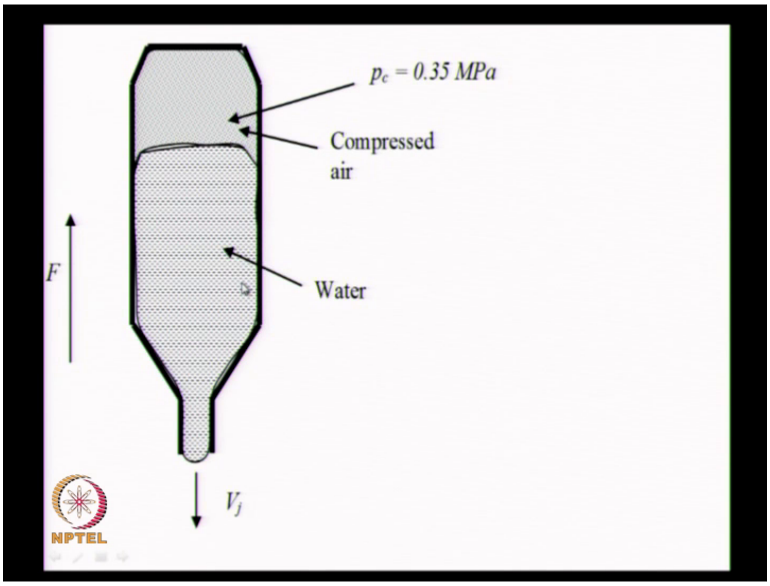

Let us try to solve this problem of trying to figure out the size of the neck or vent through which water should leave the rocket in order to achieve a given value of acceleration. We have the bottle which contains water; we have high pressure gases above it. I want to push out the water out through the neck (vent) using compressed air. The compressed air pressure is told to be something like 0.35 Mega Pascal. That is, 3.5 atmospheres. I want to find out the size of the neck such that the rocket leaves the ground at a given acceleration. The volume of the bottle is given, but I need to know the size of the vent or hole such that the rocket can leave with the given acceleration. Let us do this problem.

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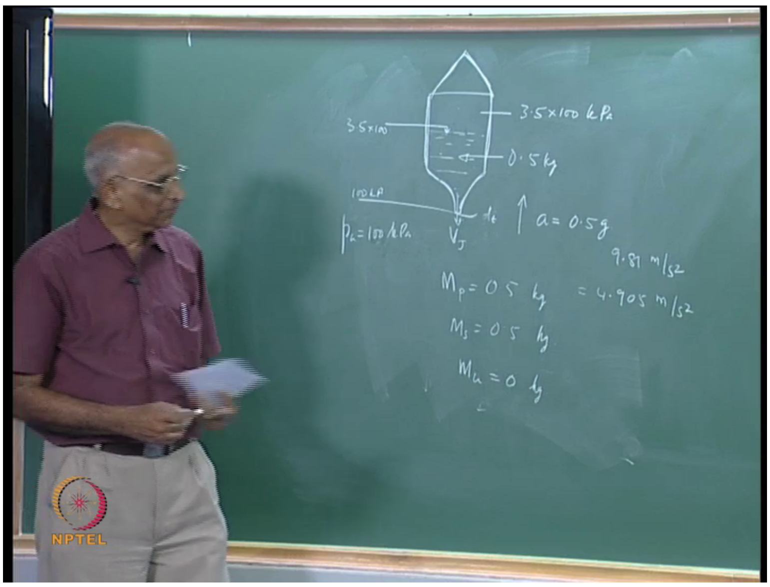

Therefore, let me write this problem on the board. You have in a bottle, 0.5 kilogram of water. The air is relatively massless which is above the water and the pressure of air is 3.5 ×100 kilo Pascal. That is the 0.35 mega Pascal. You know, it is also told that the mass of the bottle that is the structure containing the water is also 0.5 kilogram. And, all what we are interested to know is, we would like this rocket to leave with a given value of acceleration.

The level of acceleration is to be 0.5 g where g is the gravitational field due to the Earth. The g value is 9.81 meter per second square. With respect to 'g', it is half. That is the value of acceleration with which it must get pushed up. Now the question asked is, what must be the size of the diameter of the vent or the hole by which the water should escape from the bottle?

You have the mass of water 0.5 kilogram. Let us say the mass of water is mass of propellant which is used for pushing up 0.5kg of the bottle. The mass of the structure is equal to 0.5 kilogram. There is no payload in this problem. Just the bottle is moving up. Therefore, the useful payload Mu is equal to 0 kilogram. Now, you can find out what must be the rate at which water is getting pushed out and if you determine the velocity at which water is getting pushed out, we can find out the diameter of this hole.

Therefore, how should I do this problem? What are the things that I should do? Let us say I first want to find out what is the velocity with which the water will leave this particular hole. In other words, I am interested in finding out the jet velocity V_J. How do I get V_J? The gas pressure is 350 kPa; the ambient pressure is 100 kilo Pascal. That is, the air pressure pa is equal to 100 kilo Pascal. Water is incompressible. Let us neglect the height of water. May be from Bernoulli's equation, we can say p for the compressed air by rho plus what?

The pressure differential driving the water is p-pa which is 350 minus 100 which equals 250 kPa. From Bernoulli equation velocity square by the 2 is equal to delta p divided by rho. And, that would have given us 3.5 minus 1 i.e., 2.5 into 10 to the power 5 divided by the density of water; 1000 kilogram per meter cube. Then, what is the value coming out to be? If you calculate, you get it to be equal to 22.36 meters per second.

We have neglected the height of water in the bottle because the height is small and because you have the pressure which is so high, then the height will not really matter. But in a real problem, yes, I would like the height of water to be considered.

Now, we would like to make use of this jet velocity and find out what must be the diameter of the hole. But, what is given to me? Something important is given to me. It is told that the rocket should leave with an acceleration of 0.5 g or rather with an acceleration of 4.905 m/s^2. How do I get this? That means I must be able to calculate the force. And, that force I have to convert it to acceleration and make sure I get this acceleration. Let us revise what we have just now done and do this problem.

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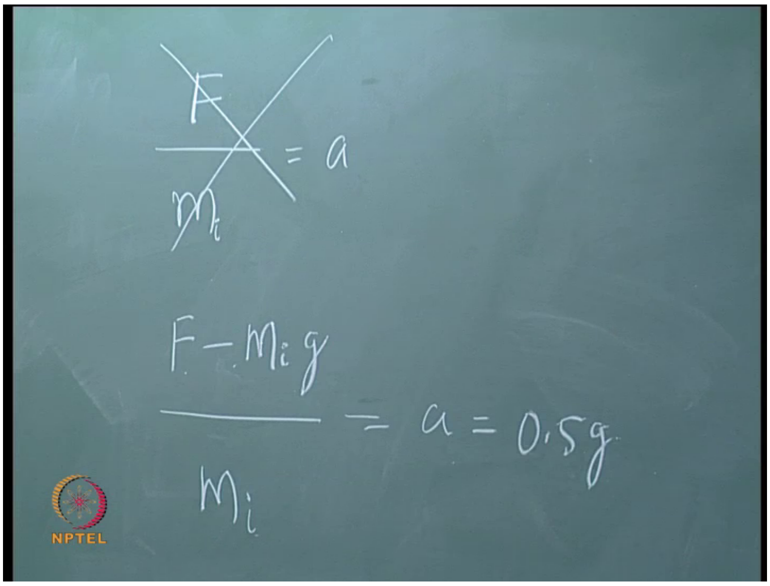

I would like to find out what is the force, which is generated by the particular rocket divided by the initial mass of the rocket, which is the acceleration of the rocket. And, the rocket is going up. Therefore, what is the acceleration with which it is going up? It is equal to F minus the gravitational force of the mass divided by mi (weight of water); is the acceleration with which it is going up. See, this differentiation is important. I say force; force is what it is pushing it up. As it is pushing up; the gravitational field is also exerting a force is equal to mi × g on the mass of the body. Therefore, F minus mi g divided by mi must be acceleration and not Force/mi shown here. This later one is what would be the acceleration when we have no gravitational field.

Therefore, let us get back to the problem. The acceleration a is given to be as 0.5 g. Let me erase this portion and write V_J is equal to 22.36 meter per second.

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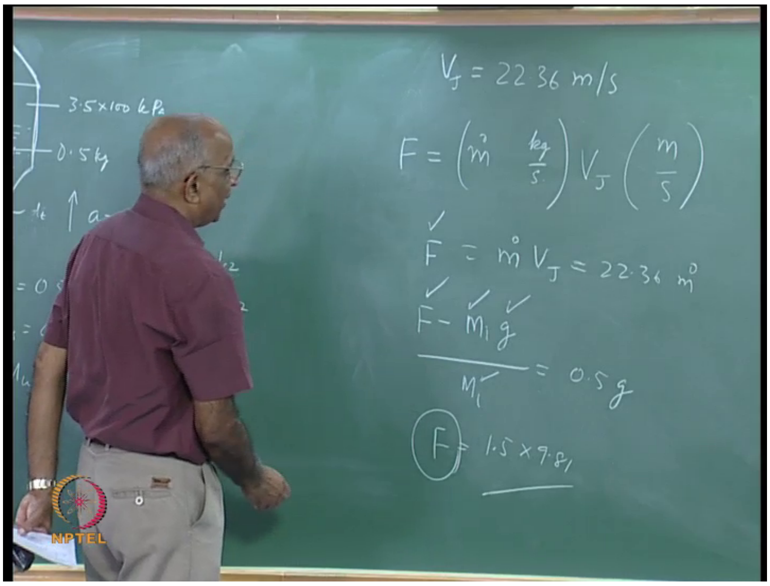

What is the value of F? Let us assume, let water flow at the rate of m° kilogram per second; because if I know the water flow rate, I can calculate this diameter. Therefore, F is equal to m° kilogram per second into what? How do you calculate the force? We have done it. Change of momentum, m V_J is momentum; force is equal to d/d t of m V_J which is equal to m°V_J. And, V_J we have already calculated. It is equal 22.36 into m° is equal to the force.

Now, what is the force? I go back to the equation what I wrote here. I get (F − mi×g) divided by mi is equal to 0.5 g. Here g is 9.81. What is the value of initial mass of the rocket? 0.5 kg plus the structural mass is 0.5 kg, which is one kg. Therefore, F is equal to 0.5 g plus mi into g. mi is 0.5 plus 0.5 is 1. It becomes 1.5 g; that is 9.81 and mi was 1. Therefore, force is equal to 1.5 into 9.81. I put it over here.

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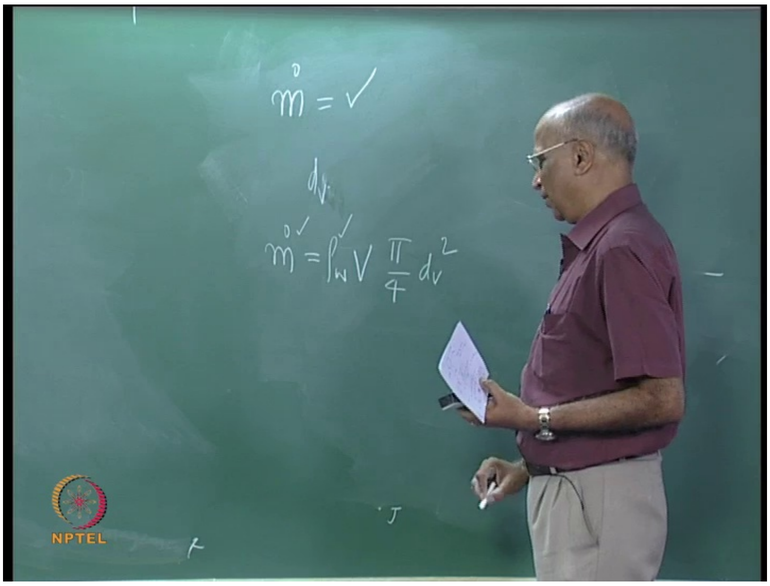

And, what I get is the value of m dot. And, if I know m dot, how do I calculate the hole diameter or diameter of the vent? m dot is equal to; we get the value, the density of water into the velocity of water into the area. Area is equal to pi by 4 into the hole diameter square. You have already calculated the mass flow rate; density of water is 1000 kilogram per meter cube. You have calculated the V_J. This is the velocity with which the liquid is leaving; V_J square pi by 4. Therefore, the only unknown is the diameter of the hole or vent.

I think I leave it as carryover homework for you to complete. All what I want to tell you is that it is possible to calculate the thrust. You need the value of V_J; V_J we find through simple calculation involving Bernoulli's equation. And, once you know this, I can always relate it to the acceleration.

I would like to do a problem on the masses; structural mass, may be the propellant mass and the acceleration for a multi stage rocket vehicle. May be in the next class, I will just go through one or two small examples on it and then we go to the next topic, which is on nozzles.

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