Rocket Propulsion

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Lecture No. 06

Rocket Equation and Staging of Rockets

In today's class we continue with the theory of rocket propulsion. In order to focus ourselves and just make sure we are on the right track, I will go through a few slides which will illustrate what we did in the earlier class. We will also find out that not only rockets eject momentum to propel; but in nature itself we have some creatures which make use of the same principle.

Let us briefly go through some of the slides. Rockets are used to launch satellites or may be objects in space. We talked in terms of circular orbits, we talked in terms of geosynchronous orbits, polar orbits, sun synchronous orbits and retrograde orbits; different orbits.

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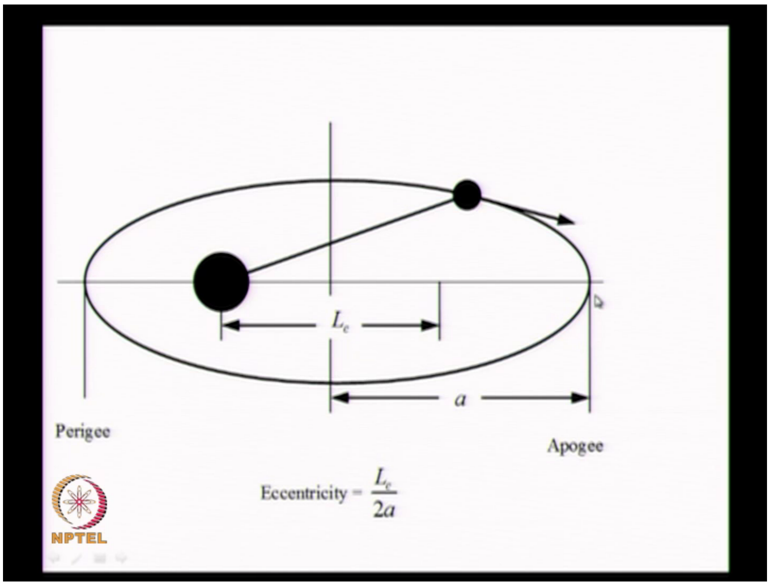

We also talked in terms of elliptical orbits and we said elliptical orbit is one wherein you have two foci or focal points. This is shown here; the Earth is at the focal point and this spacecraft going round in an elliptic orbit. We defined something known as eccentricity which was the distance between the two foci and the major axis which is 2a over here.

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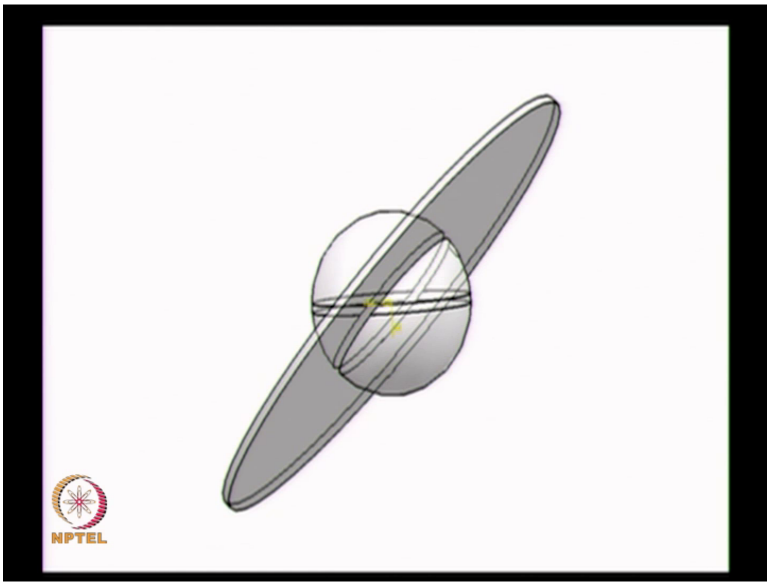

We also said for an elliptical orbit you could have an inclination, even a circular orbit could have an inclination and the inclination is between the equatorial plane and the plane of the orbit. I want to make it clear because we defined one orbit known as a Molniya orbit and we told that in a country like Russia in the northern hemisphere wherein the satellite has to stay in the orbit above the northern hemisphere for a longer time; we have highly elliptical orbits wherein the apogee is at a distance of something like almost like 60000 to 70000 kilometers and the perigee is quite small of the order of 6000 kilometers.

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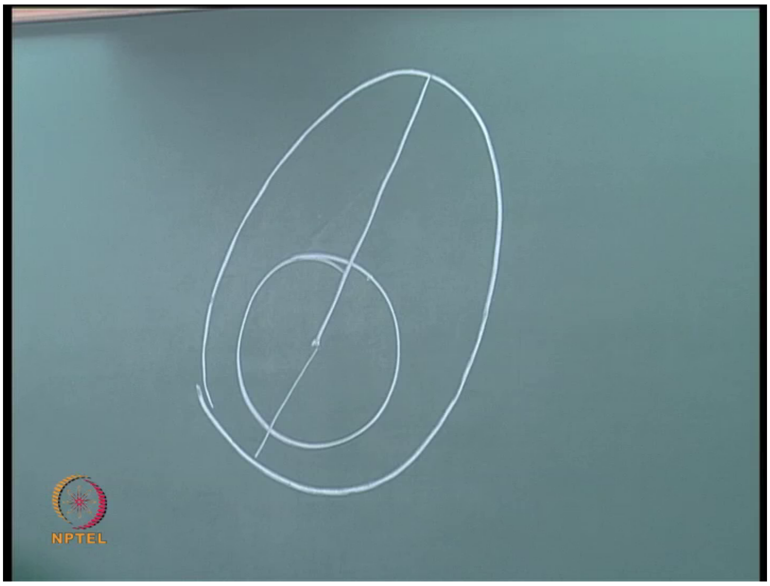

This Molniya orbit is illustrated with the northern hemisphere in the upper portion, the satellite is in the northern hemisphere for a much longer time. Something like 11 hours it is over this northern part and only one hour in the southern region. The countries in the upper north can view the satellite for a longer time. This Molniya orbit is particularly important and was developed by Russia. The apogee is of the order of 70,000 kilometers. The inclination of this orbit was something like 63.4 degrees.

We also talked in terms of the rocket principle and in the example of the sled we reviewed the velocities achieved when stones were thrown out simultaneously and one after the other.

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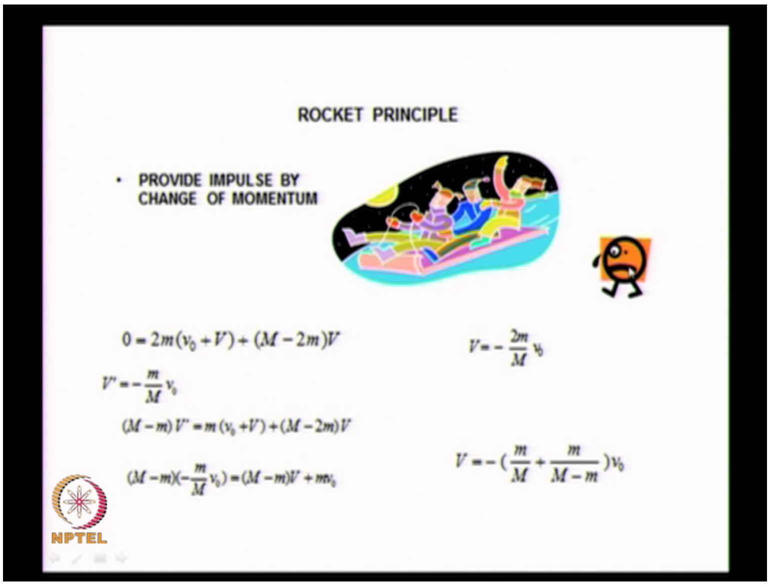

We got the net velocity of the sled was something like the mass of the stones thrown divided by the total mass into the velocity with which the stones were thrown. This was when the stones were thrown simultaneously. The important thing here is when we are looking at the sled which is moving, we are talking with the inertial frame in mind. I look at the sled. I am standing outside as shown by the observer.

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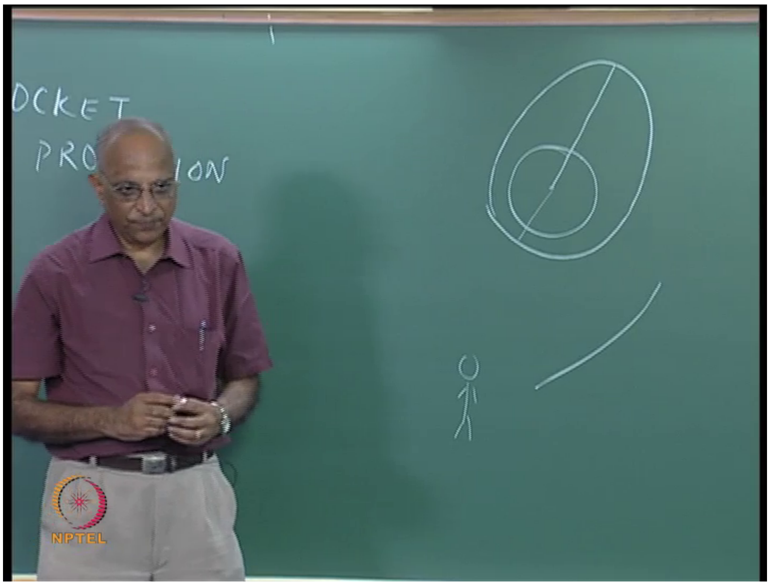

You know, I am observing it from the inertial frame of reference and writing the momentum conservation equation.

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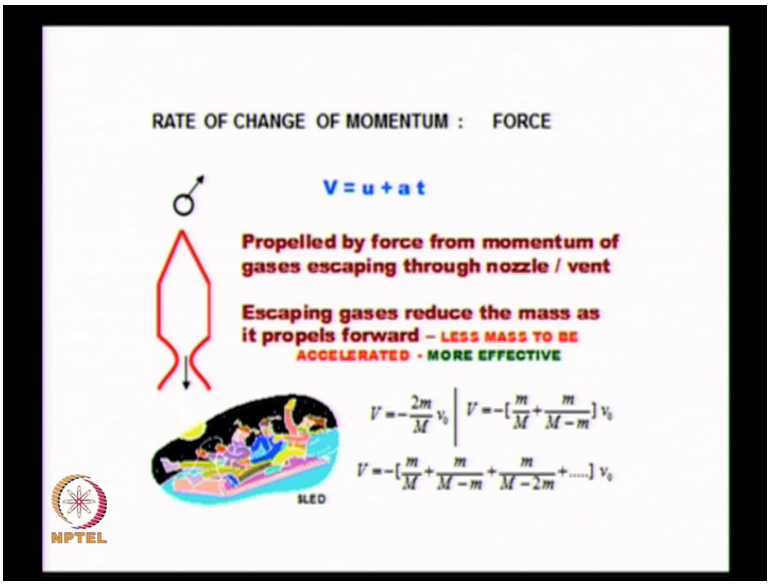

When the two stones are thrown not together but one after the other; we got a higher value of velocity because instead of 2m/M when the two stones were thrown simultaneously, we got m/M + m/(M−m) when the two stones were thrown one after the other. Since M−m is less than small M, we got an increased velocity for the sled. And so if I have a series of stones thrown one after the other; well I can get a higher velocity than when the stones thrown together. The question is why is this so?

Because when the net mass is decreasing I accelerate a lesser mass and I get a higher value of velocity. Therefore, we find that when I throw one stone after the other, I do not have something like stones being thrown once at the initial time in which case I have the net velocity is equal to u + at where a is the acceleration; but, since I am accelerating gradually in view of the reduced mass, I get much higher velocity.

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Let us look at some examples. I chose the above example from the national geographical magazine. What is shown here is something known as a squid. Squid is like a large fish something like 10 meters long and is found off the coast of Japan and off New Zealand. It periodically visits these coastal areas and it is an endangered specious and it propels using the rocket principle. Let us look at the parts of this particular squid. You have something like a mantle or a funnel here through which it sucks in water.

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The above slide shows a squid which was caught off the coast of new Zealand and you see the length compared to this man; it is something like 10 meters long.

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Let us go to the principle of is motiont. You know there is a funnel here at the rear of the squid in this slide. To move itself it opens its funnel or mantle and gulps in water. As it gulps in water it gulps in some sand may be eggs may be small fish or whatever it is available around it in the water. Then it closes the funnel i.e., it closes its gate to the funnel or mantle or the mouth as it were and then it through it contracts the muscles such that it builds some pressure of water which it has gulped.

And then when it wants to move, it just opens the gate again and spouts out the water in this direction and when it spouts out the water in this direction it moves in the opposite direction. It keeps on squirting the water and it propels itself. Therefore, when it squirts out the water, it moves. Whenever it wants to move; it expels water, waste eggs and all that and that is how it moves. The principle is very similar to a rocket. It collects water, pressurizes the water, releases the water gradually and it moves.

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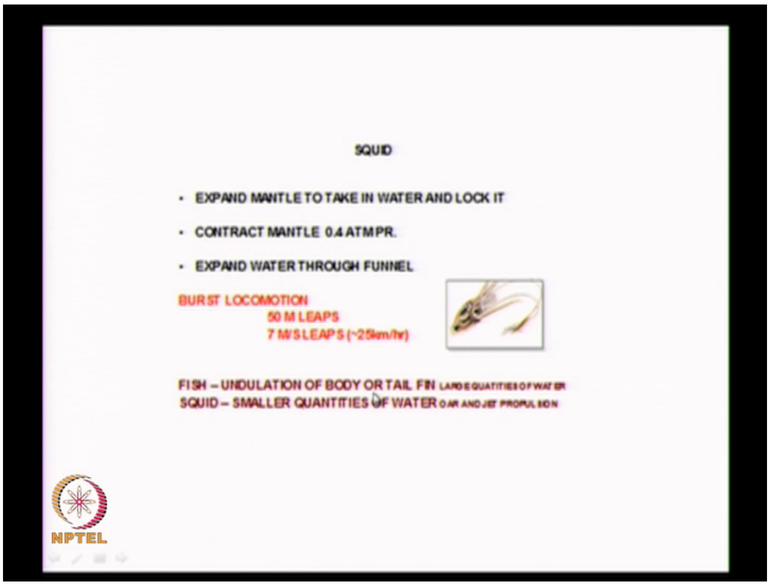

And the type of velocity what it gets is quite substantial. It pressurizes the water to around 0.4 times the atmosphere and once when it pushes it out, it is able to leap something like a 50 meters and gets a velocity of something like 2.5 kilometers per hour. This is quite phenomenal when you consider that this is in water that is a viscous liquid. It is able to go at that speed because of the gradual release of pressurized water from its mouth i.e., funnel or mantle

Now I show the funnel again. It opens its mouth, water enters, it contracts itself and then it opens it, closes the gate, compresses it, releases the pressure, then the water squirts out and whatever is available along with the water is squirted out and it moves.

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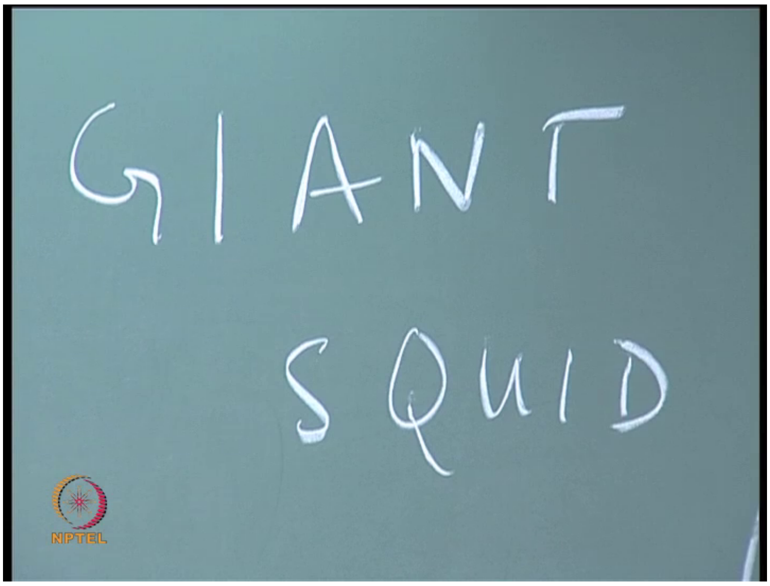

And this is the principle of what we call as the GIANT SQUID. In fact in US we have had rocket projects named as Project Squid. In what way do you think is the motion of a Squid different from a fish?

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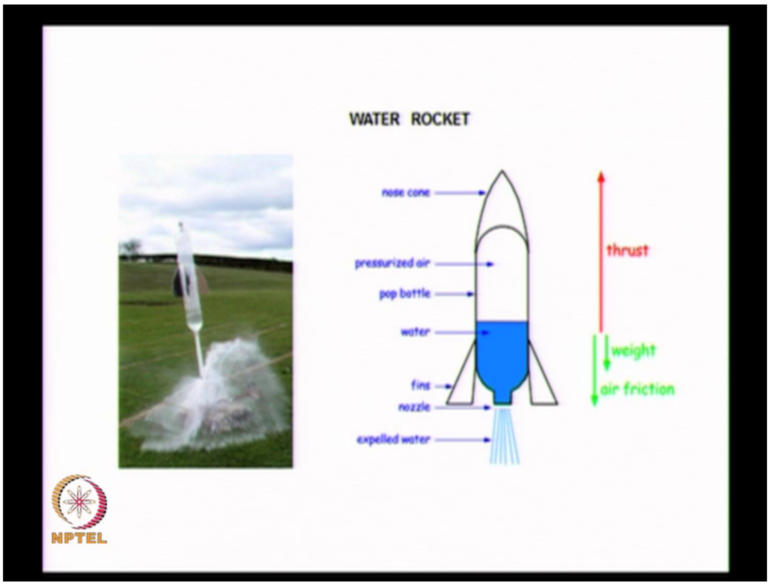

A fish has fins by which it displaces the water. That means it slowly displaces the water like I go on a boat let us say. I am sitting in the boat I have an oar. With the oar, I displace water. I displace some water i.e., provide velocity to the water but, the mass of water displaced is large though the velocity during the displacement is small.

In the case of a squid: It takes in a small amount of water, it pushes it out at high velocity and its able to do a much better job. This same principle is used in a water rocket. I show a water rocket; if you want to make one, all what you do is take one of these bottles, partially fill in with water and then invert it and pressurize the water. I remove the cap and when I do so water squirts out as a jet and the water rocket moves up.

The principle of any rocket is quite identical. Only major difference is that you need higher velocities. Therefore, you put more enthalpy into the working medium of the rocket, which is then expanded out. The medium, which is expanded would have a higher value of velocity if its enthalpy is higher. Therefore, we get higher velocity with a gas heated in chemical or nuclear rockets.

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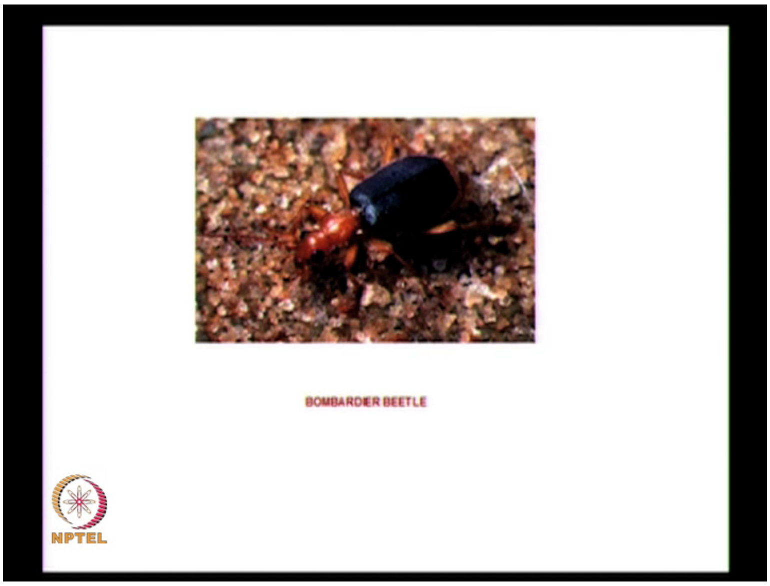

Having observed about giant squid, I discuss another example. There is a creature by name bombardier beetle. You know what a beetle is: in Tamil we call as “vandu” and in Malayalam it is known as “nandu”. It is a harmless creature. It cannot fly well as it is bulky and it goes round either by flight or by walk. Whenever it comes into our house, all what we do is push it out by placing it on a piece of cardboard or thick paper and throwing it out.

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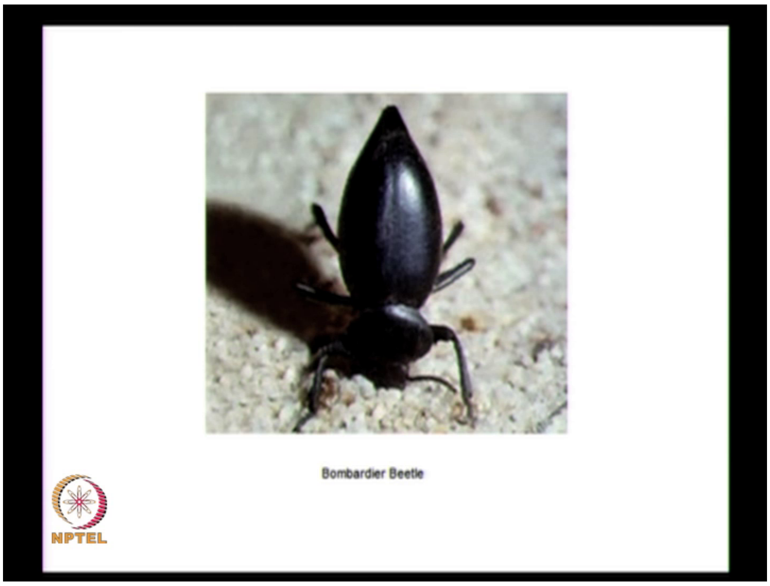

A particular form of beetle known as bombardier beetle has a different means of locomotion. Beetle we had said is a harmless creature; ants sting it and nature has given its some means to protect itself.

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It has something like two stomachs, and after the two stomach there is another receptacle which is like a third stomach. In one of these stomachs, it secretes hydrogen peroxide which is an oxidizer. In the second stomach it secretes hydroquinone which is a fuel. Hydrogen peroxide being an oxidizer and hydroquinone being a fuel can react to form hot gases. When the bombardier beetle is chased by the ants or when it is bitten by the ants it immediately squirts the hydrogen peroxide and hydroquinone into its third stomach which is coated with enzymes. The fuel and oxidizer react to generate hot gases in the presence of the enzymes and these hot gases are forced out in the form a jet which kills the ants or else they are chased out as the bombardier beetle moves forward.

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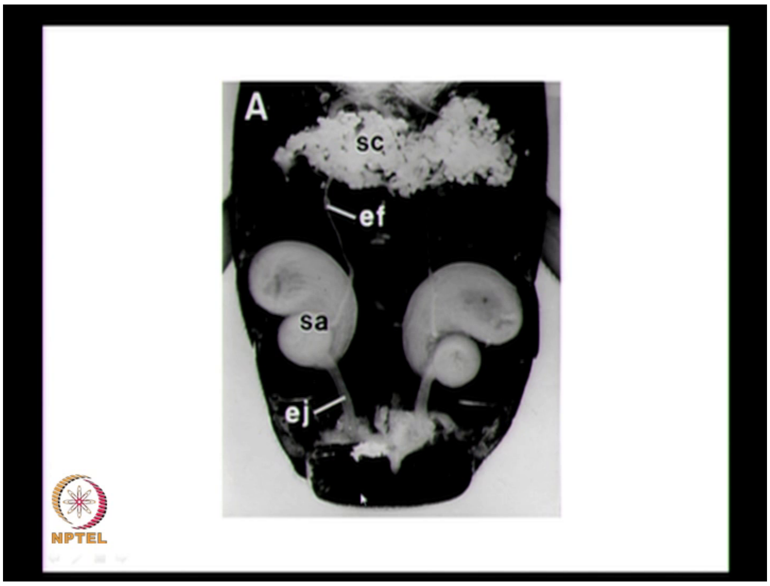

Let us examine the processes again. In the above slide, the two stomachs are shown which form hydrogen peroxide and hydroquinone. They are secreted here. In the third stomach below is a lining of mucus (an enzyme) which acts as a catalyst. The catalyst promotes the reaction and whenever it is attacked it just squirts out the hot gases. This is similar to the processes of combustion and expansion in a liquid propellant rocket.

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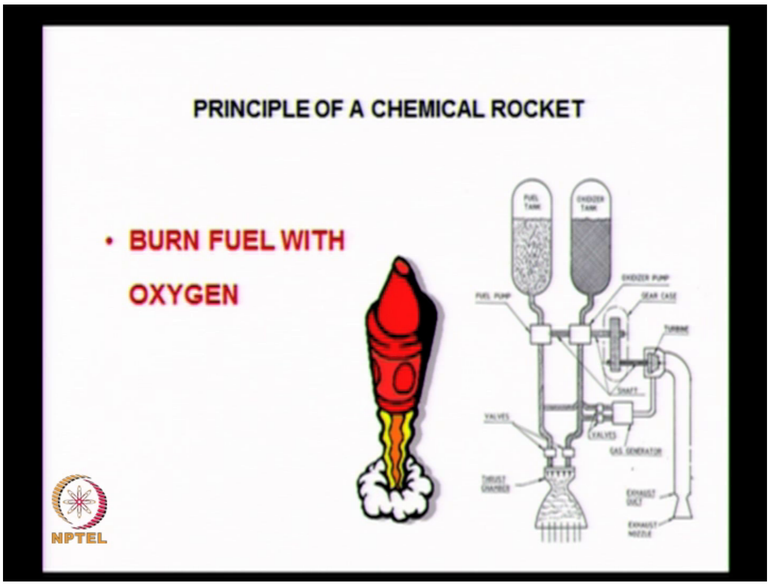

What does a liquid propellant rocket consist of? We are yet to study it. You have a fuel tank, you have an oxidizer tank, you pump the fuel and oxidizer into it, you ignite it and you push the gases out. So, this small insect which is available in nature works on the principle of a liquid propellant rocket or rather the rocket works on the principle of the bombardier beetle.

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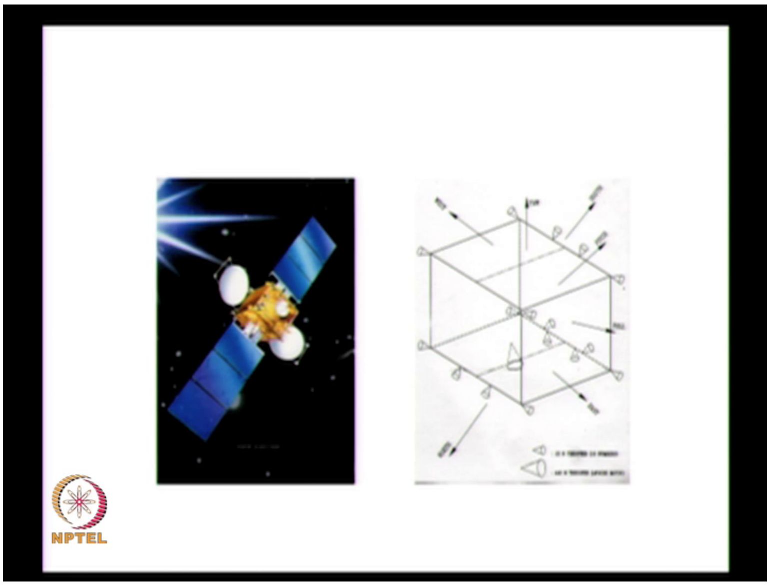

I think we should look at nature to understand many of the things what we are studying. Everything is there in nature and we have to be more observant. Having said that let us get into some more details. Let us illustrate the rockets used in satellites because we told that whenever a satellite is there in space in geostationary orbit and its life is nearing completion, we have to push it out. A satellite has something like 16 rockets which are there at the edges of this satellite and these are used for correcting the attitude. May be for station keeping of the satellite and whenever the life time of the rocket is near to being over, we fire some of these rockets such that we remove it from the geostationary orbit and push it into deep space. That means we make it escape to deep space.

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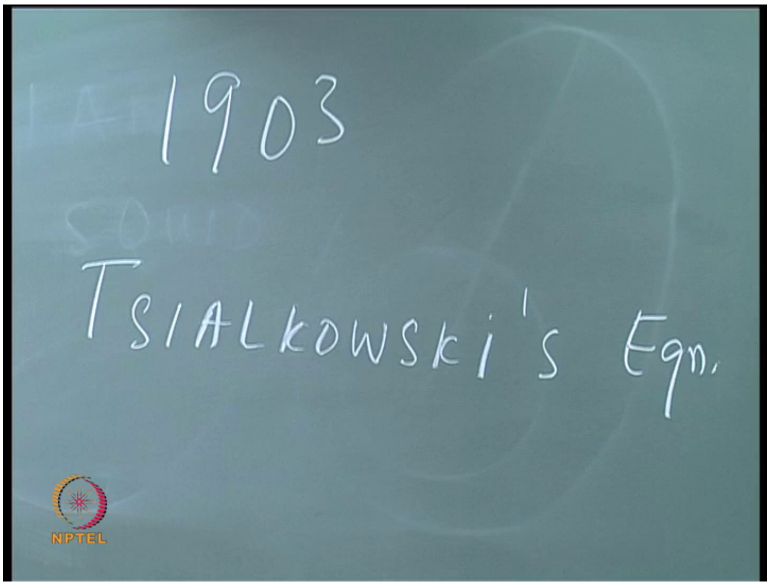

Based on the above background how to develop the theory of rocket propulsion which leads to the rocket equation? The rocket equation was developed not very early, only in the year 1903 and that also by a Russian school teacher by name Tsialkowski. Now, let us see how this is done and what is this rocket equation also referred to as Tsialkowski equation.

Let us derive it first. We will follow the same procedure what we adopted while finding out the velocity gain by the sled, these two boys standing on it, throwing one stone after the other. We will make some simplifying assumptions.

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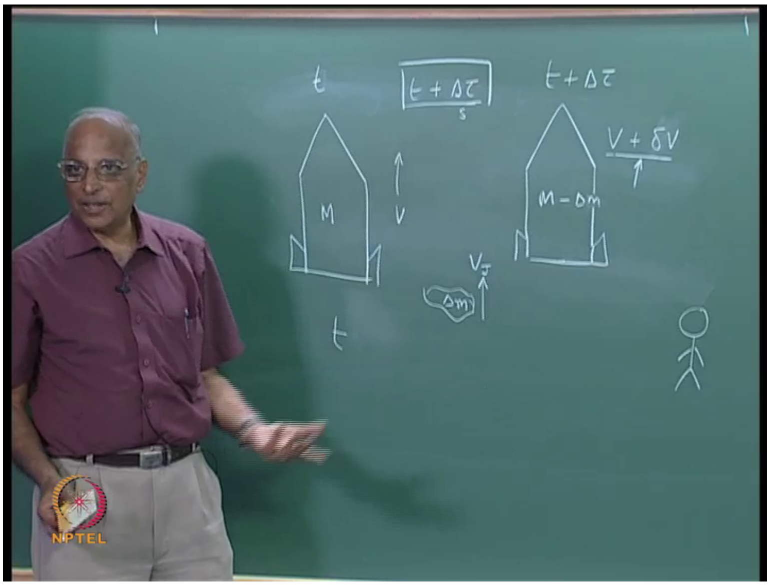

Let us assume I have a rocket as shown in this figure. We will assume rocket has a shape something like this. It need not really be the only shape and may be you will be come out with better configurations of rockets. Let say at time t it is moving with a velocity let us say V. Let its mass at time t be M. After a small time $\Delta\tau$ after time t i.e., time t $+\Delta\tau$ let it gains a small velocity ΔV.

Now how does it gain this velocity? It is moving forward and then during this small time, a small mass Δm is being ejected out. The rocket is ejecting matter and in a small time $\Delta\tau$ a small mass Δm is ejected out with a velocity V_J. I do not know the direction in which the mass is ejected out. We will presume it to be in the direction of motion of the rocket.

And therefore, the final mass of this rocket at time t $+\Delta\tau$ is going to be $M-\Delta m$. Let us presume that the velocity of the rocket at time t $+\Delta\tau$ is V $+\Delta V$. Since the rocket has lost a mass delta m in this small time, the final mass of the rocket is M $-\Delta m$.

Now we talk in terms of inertial frame of reference and therefore we watch the rocket from the ground. I watch rocket go up with a velocity V at time t and then after a time $\Delta\tau$ I am looking at it going with a velocity V $+\Delta V$. Now, I do the momentum balance in the inertial frame of reference, and what is it I get?

The initial momentum of the rocket is equal to MV. It is mass into velocity at time t.

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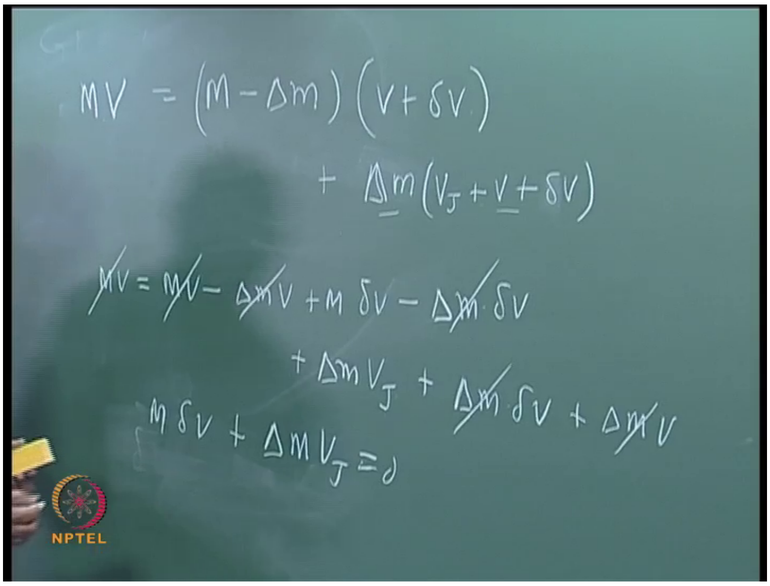

Now what is the momentum of this rocket at time t +Δτ as a I am seeing from the inertial frame of reference? I have (M−m) × (V+ΔV) plus you also find that Δm has been pushed out of the rocket with velocity $V_{J.}$. The velocity V_J is with respect to the rocket and from the inertial frame of reference when the rocket is moving with velocity V+ΔV, the mass Δm will appear to leave with a velocity V+V+V_J. This is what is shown here. That is the velocity of this parcel of mass Δm and is the relative velocity V_J + V +Δ V. This is how we wrote the equation for the sled. We ignore the gravitational field and the resistance to motion of the rocket by the air and we get the momentum to be conserved exactly in the same way as in the sled problem. In the case of the sled which was initially stationary, the initial momentum was 0. In this case, the initial momentum is MV. This equals the momentum after t +Δτ seconds.

If we simplify, what is it that get. MV is equal to MV−ΔmV. Then we get M ΔV − ΔmV. And Δm × (V_J +V +ΔV). I should have written here small δV instead of ΔV because I want to reserve the capital delta for something else.

And now I find that this Δm δV and Δm δV cancels. So also MV and MV cancels. Therefore, I am left with the term M into δV plus delta mV_J is equal to zero. This gives $\delta V = -\Delta m\, V_J / M$.

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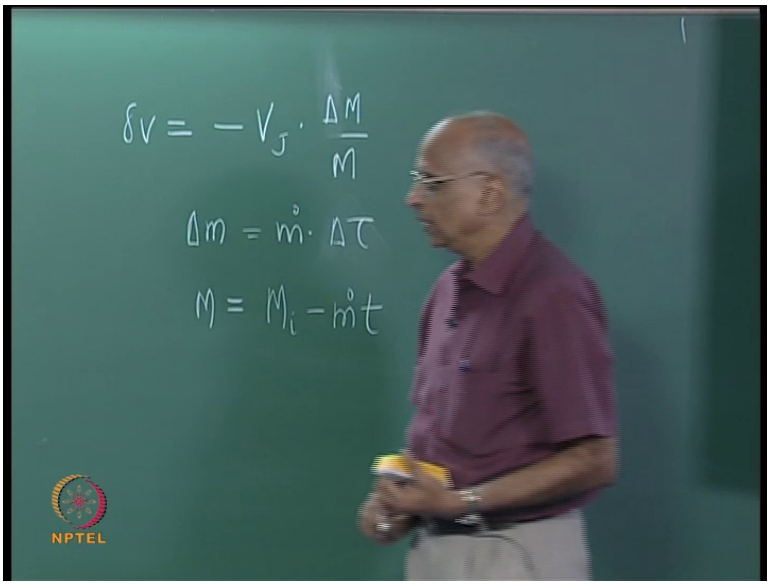

Therefore, we get $\delta V = -(\Delta m/M)\, V_J$.

We need to solve the above equation. What is the value of Δm? Let us assume that the mass which gets exhausted from the nozzle is something you are constantly pushing out. Mass at the rate let us say mdot; m°. Therefore, the value of Δm should be equal to m° into the small time Δτ. You know the rate at which mass is leaving the nozzle is m° over a small time Δτ; it is equal to m°Δτ and what should be the value of M? Capital M must be equal to the initial mass of the rocket at time t which is equal to its mass at time zero minus m° into t. This is the mass of the rocket at time t.

In other words at time t is equal to 0; the rocket had a mass equal to the initial mass; it continues to eject mass at the constant rate m° and therefore, at time t its value M = initial mass Mi − m° t.

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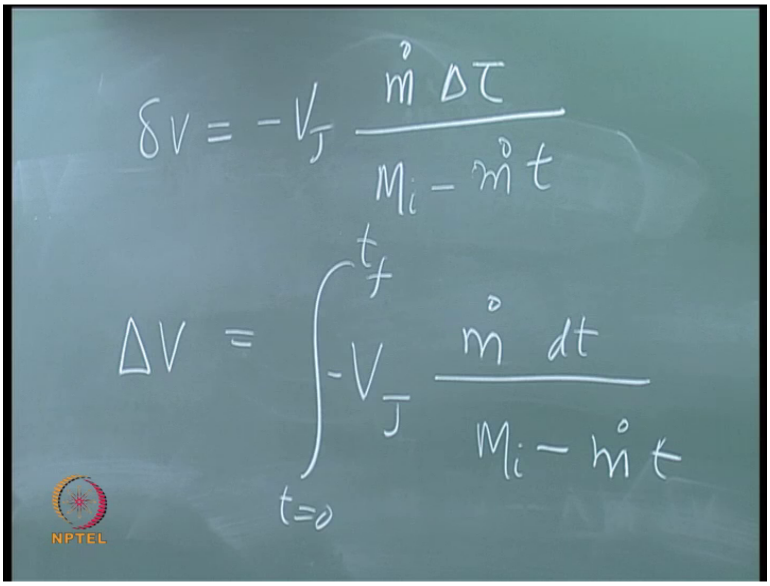
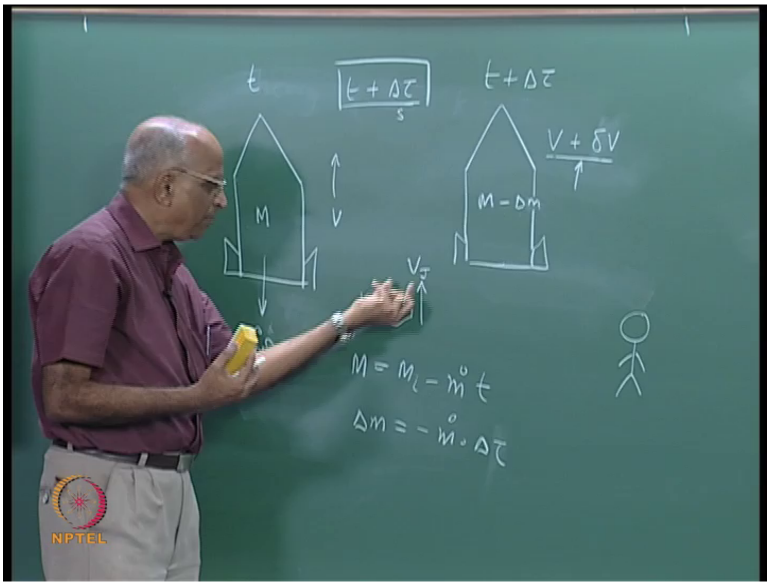


And therefore, now I can erase this part and we can write the value of delta V as equal to minus V_J into Δm which is equal to m°×Δτ ÷ (Mi − m° t).

I want to integrate this equation and if I have to integrate this equation from initial time of 0 to a final time t_f at which I get the total velocity increment of ΔV. I know the value of m°. The mass at time t is the initial mass Mi − m°t. The value of ΔM equals −m°×Δτ. With the two negatives the minus sign will not be there in the expression for delta V. I show the derivation of ΔM as being − mdot × Δτ in the following.(Refer Slide Time: 27:16)



The minus sign shows that the mass has left the system and therefore, when I substitute the value of ΔM, I should have substituted − m° ×Δτ; that means this should have been a negative sign and this negative sign and this negative sign would have given me a positive sign. The same is seen from the expression for ΔM.

To summarize: We solved the momentum equation in the inertial frame of reference and balanced initial momentum MV with final momentum (M−ΔM) ×(V+ δV) + ΔM×(V_J + V +δV) and then we got an expression which gave us δV as equal to − V_J ×ΔM ÷M. We find δM is equal to − m°×δτ and therefore we got it as V_J m°×Δτ ÷ Mi − m°t.

We now integrate it to determine the net velocity gained by the rocket.

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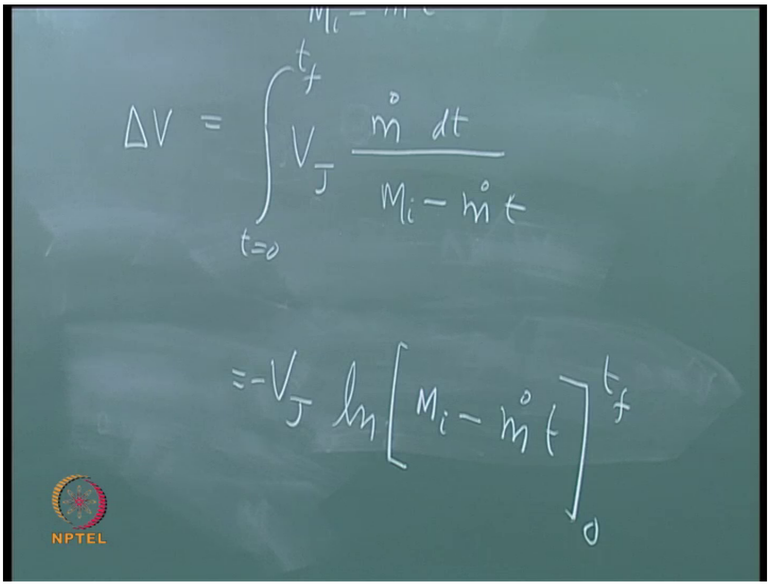

And what is the expression we get now? Let us assume that V_J which is the velocity at which the gases leave the rocket is a constant. The limits of integration are from t = 0 to t= t_f. Integral of m° divided by (M_i − m°t) dt is natural log of M_i − m° t. We also get a negative sign from the − m° t in the denominator and this expression is between the limits t = 0 and t= t_f.

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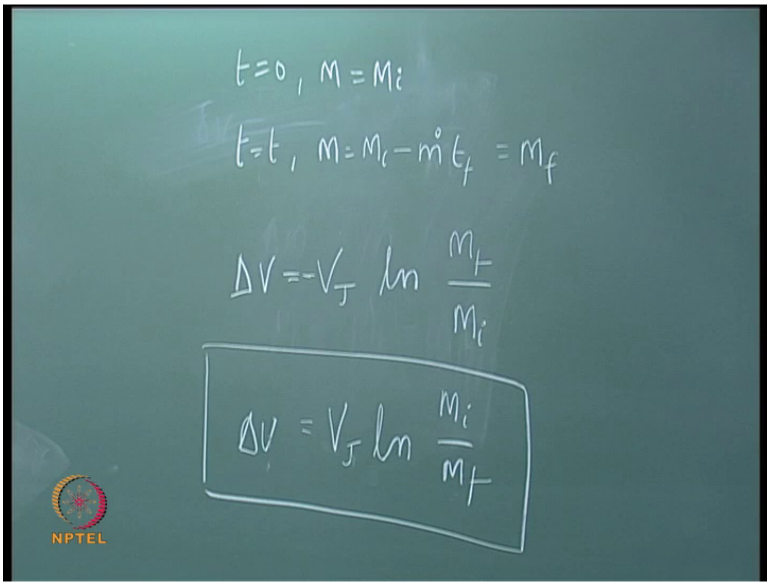

The value of the expression $M_i - m^\circ t$ at $t = 0$ is M_i viz the initial mass of the rocket while $M_i - m^\circ t$ at $t = t_f$ is its final mass M_f. And therefore, the value ΔV is equal to minus V_J into natural logarithm of M_f by M_i. The negative sign can be removed by inversing the term within the logarithm. The velocity change or increment provided by the rocket is therefore V_J into logarithm of the initial mass to the final mass of the rocket. This velocity change is spoken of as incremental velocity and the equation is known as the rocket equation.

All what the rocket equation tells is when I burn a quantity of fuel between the initial value and the final value and I am exhausting it out at a constant velocity V_J, the final velocity of the rocket is equal to the velocity with which I am ejecting matter out into natural logarithm of the initial mass to the final mass. This is what we call as the rocket equation. The Russian school teacher Tsialkowski derived it and postulated that a high value of jet velocity V_J is required and a large value of mass ratio (M_i to M_f) is desirable if we have to go into interplanetary missions because all what we want is we want a jet velocity and the mass should keep getting depleted and this is what is the theory of rocket propulsion states.

Can I repeat it again because this forms the basis and you should know the limitations?

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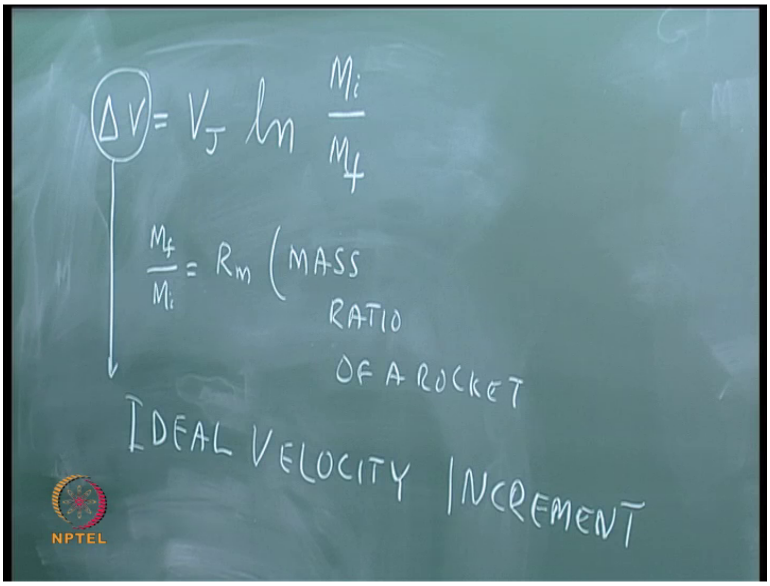
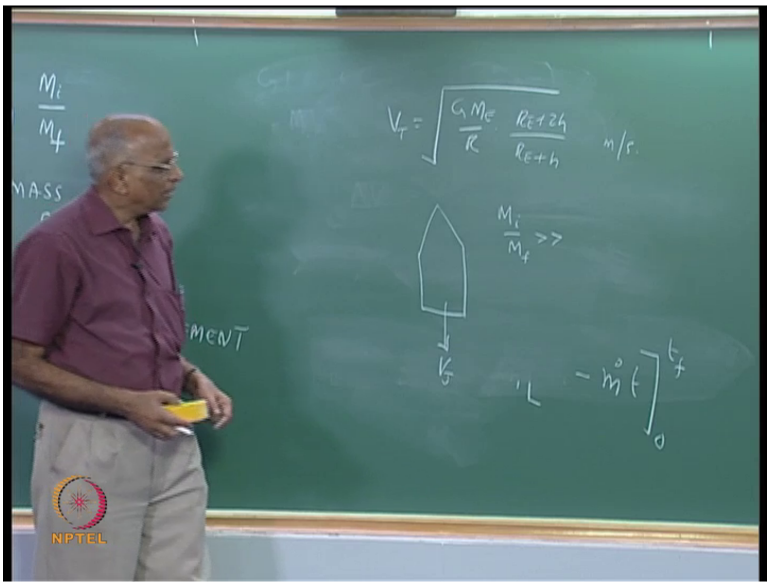


The jet velocity or the velocity which with mass is being removed into natural logarithm of the initial mass divided by the final mass decides the velocity provided by a rocket ($\Delta V = V_J \ln(M_i/M_f)$). The value of final mass of a rocket to the initial mass of the rocket is also called as the mass ratio of a rocket. We did not consider the gravitational forces nor the drag forces in the derivation of ΔV.

Therefore, ΔV is also spoken of as ideal velocity increment. Now, in the earlier classes we saw that the rocket has to supply the necessary orbital velocity and the total velocity for which expressions were derived.(Refer Slide Time: 34:21)



If I can write it, $V_T = \sqrt{(G M_E / R_E) \times (R_E + 2h) \div (R_E + h)}$; so many meters per second is the velocity what was a required.

Now, if I have a rocket and on top of it I put whatever I want to launch; I give it a velocity I can put it in orbit. But we found that we need a velocity of the order of something like 10 to 12 kilometers per second. Therefore, what it is required in order to achieve high velocities? See, you have to have high value of mass ratios which means that the ratio of initial mass to the final mass must be a large. That means that difference between the initial and the final must be large and also the V_J must be a large value. That means the jet velocity is a controlling parameter and higher the jet velocity you can have higher delta V..

Therefore, the figure of merit of a rocket if somebody were to ask us we may say well one is the jet velocity, the other is something related to the masses. Therefore, let us go back and look at this term because this is fairly clear to us. Supposing, I were to exhaust at some jet velocity and I can get as high a value as possible. Apparently you cannot get to the speed of light, you cannot get more than some amount. But, I have some limitations. I will come back to these limitations. In addition I am talking in terms of Mi by Mf should be a large number.

Let us just examine this number before we can design a rocket because as of today you cannot get more than something like 3000 to 5000 meters per second as jet velocity. We will find out where and what the limitation are there but, let us first examine this.

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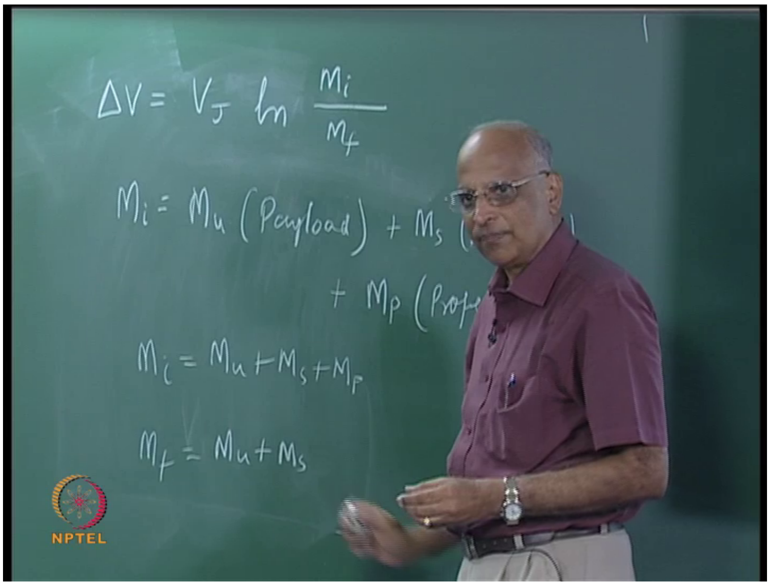

Let us therefore write the ideal velocity increment as equal to jet velocity into natural logarithm (ln) of the initial mass to the final mass of the rocket. What does the initial mass of the rocket consist of? It will anyway have the useful part of a rocket? What is the useful part of a rocket? The object which is going round and round that is the useful part of the rocket which is we call as payload. Then we have the structure of the rocket that means it must have some metal and other structural materials including inert materials to contain it and protect it from heat such as insulation. Thus we would have a structural mass.

Plus it should also have some fuel and we said fuel is used for propelling the rocket and is known as propellant. Therefore, we have a mass corresponding to mass of propellant Mp. That means that the initial mass of a rocket comprises of the payload mass, the mass of the structure plus the mass of the propellant and we say Mi is equal to Mu plus the structural mass plus the fuel or the propellant. This is the initial mass. When the rocket has done its job, what must be the final mass of the rocket? It has done its job that means all the propellant has burned out. Therefore, the final mass will be the mass of the useful component viz., the payload plus the mass of the structure. But, it is also possible that the structure could be removed the payload after the rocket functions and thrown out. But, otherwise the structure will remain as part of the final mass of the rocket. This is generally the case. Therefore, the initial mass Mi = Mu + Ms + Mp while the final mass is Mu plus Ms.

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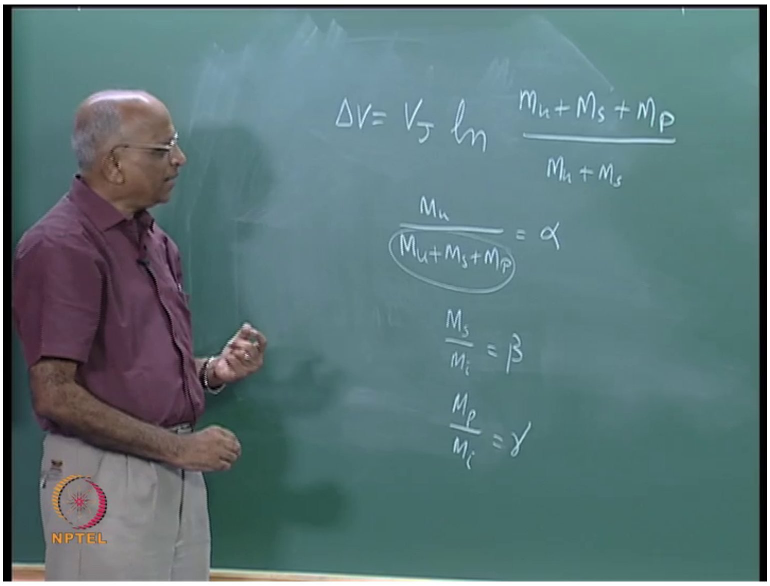

Now, let us analyze the masses. The incremental velocity $\Delta V = V_J \ln [(Mu + Ms + Mp) \div (Mu + Ms)$. Let us express these mass as non-dimensional terms or as mass fractions. Let us call the useful mass of a rocket Mu divided by the total initial mass namely Mu ÷ Mi + Ms + Mp as equal to alpha (α). The value of α is proportional to the initial mass of the rocket and is the non-dimensional payload mass. Similarly, we denote structural mass divided by the initial mass of the rocket as the structural mass fraction and call it as β. The propellant mass fraction is the ratio of the propellant mass divided by the initial mass of the rocket. This is denoted by γ.

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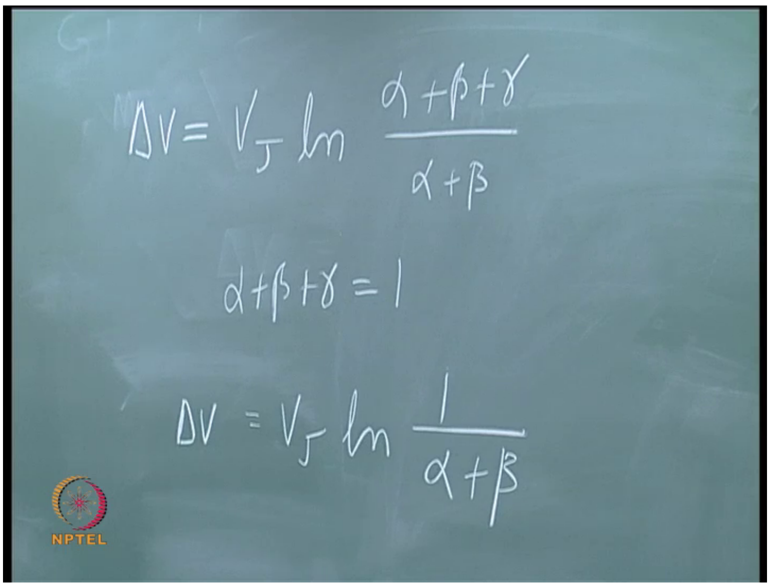

Dividing the mass terms within the logarithmic term by initial mass of the rocket, we get $\Delta V = iV_J \ln [(\alpha+\beta+\gamma)/(\alpha+\beta)]$. However, the sum of the masses is the total mass of the rocket and therefore the sum $\alpha+\beta+\gamma = 1$. And therefore, we get this equation for $\Delta V = V_J \times \ln (1/(\alpha+\beta)$.

Now, what is it that we want to do in a rocket? We want to have as much payload as possible. May be we would like the useful mass to be high. Let us say what is the fraction of the useful mass α.

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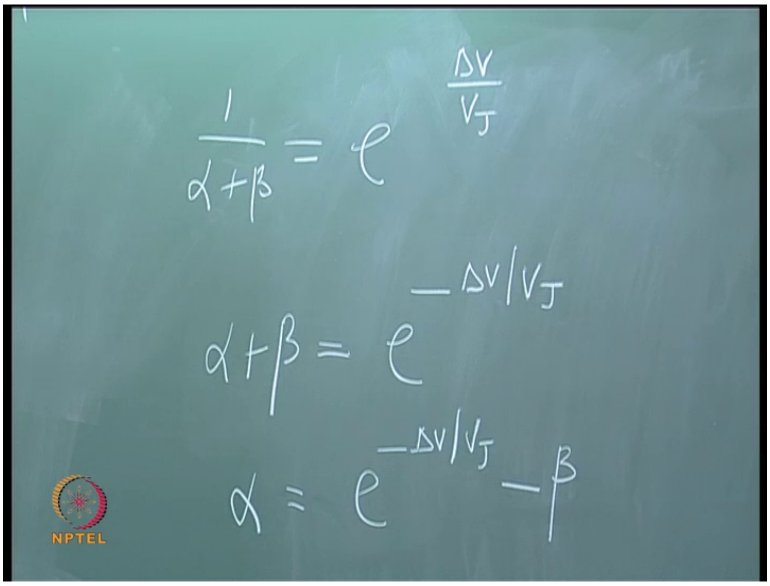

I get therefore, from the above expression one over alpha plus beta is equal to exponential of ΔV by V_J. This is done by taking exponential on both the sides and the exponential of ln becomes unity. Taking the inverse on both sides we get α+β is equal to the exponential $-\Delta V/V_J$. The payload mass fraction α is therefore equal to exponential of the negative of delta V by V_J minus the structural mass fraction β

Now, let us examine under what conditions will we get the value of the useful payload mass fraction α to be high. Let us plot it out for different values of V_J at given values of velocity increments. The velocity increment required could be between 8 to 12 km/s as seen earlier and depends on the mission. There might be some variations in the jet velocity between 3000 and 5000 meters per second. What are the values of payload fraction that we get? Let us just plot it out and see.

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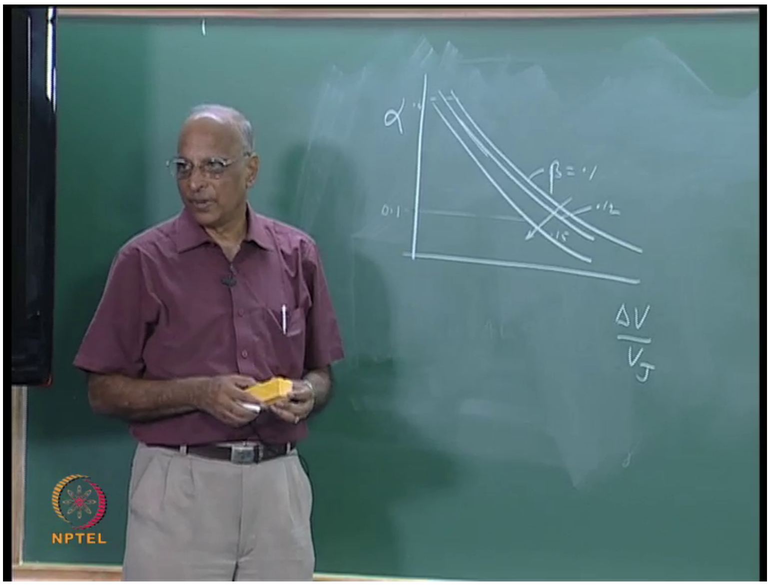

This is shown in the above wherein alpha is plotted as a function of $\Delta V/V_J$. As the incremental velocity increases for a given value of V_J, the value of alpha decreases for a given specified value of β. As the structural mass fraction β increases, the useful payload fraction alpha decreases. If the efflux velocity V_J is higher for a given incremental velocity delta V, the payload fraction α increases. And generally this value of the payload fraction might be around 0.04 and keeps falling. In the operable regions, the fraction could be around 0.1 or even lower depending on the structural mass fraction β.

Further, as β increases α decreases. We have still not put any numbers here because we do not know what it is the range of β. But, generally beta should be around let us say 0.1. As mass of structure increase β increases to around 0.12 to 0.15. That means the payload fraction will keep decreasing as the structural mass increases. If we can have a high value of V_J then we can get a higher value of the payload fraction α. Or if we have a rocket or object, which requires more ideal velocity then I get a lower value α of payload. You know we are just looking at the rocket equation and trying to draw some conclusions from it. The conclusions that we draw are if we want to put payload of higher mass then I need a structure which much be very light. I must have a large value of jet velocity V_J or else if I can have rocket which does not have to go very far away it and the orbit is nearby then I can carry a higher mass. Well this is all about the rocket equation and the conclusions from it.

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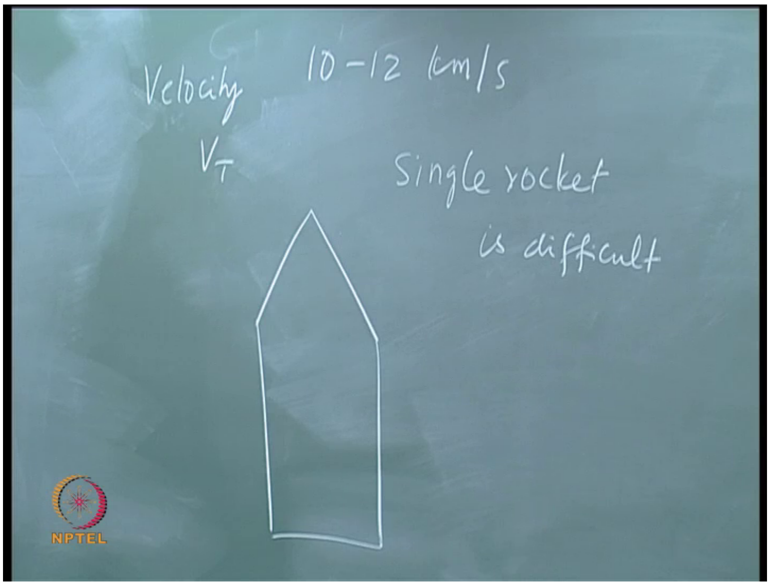

But then the problem is that we need an incremental velocity of about 10 to 12 kilometers per second. We called it as V_T. This is what we said as the total velocity to climb up and orbit. But, then to get a reasonable value for the payload mass fraction for this incremental velocity with the existing jet velocities V_J and structural mass fraction β is very difficult, if not impossible. If I have a single rocket, top of which I have a payload, I may not be able to get a useful mass fraction for the payload, because I have a definite mass of the structure. I have a limitation on V_J and therefore, to be able to launch a payload into orbit using a single rocket is difficult. I use the word difficult since it appears to be impossible at this time. So far it has been impossible but the quest for the rocket engineer is to make a single rocket do the job.

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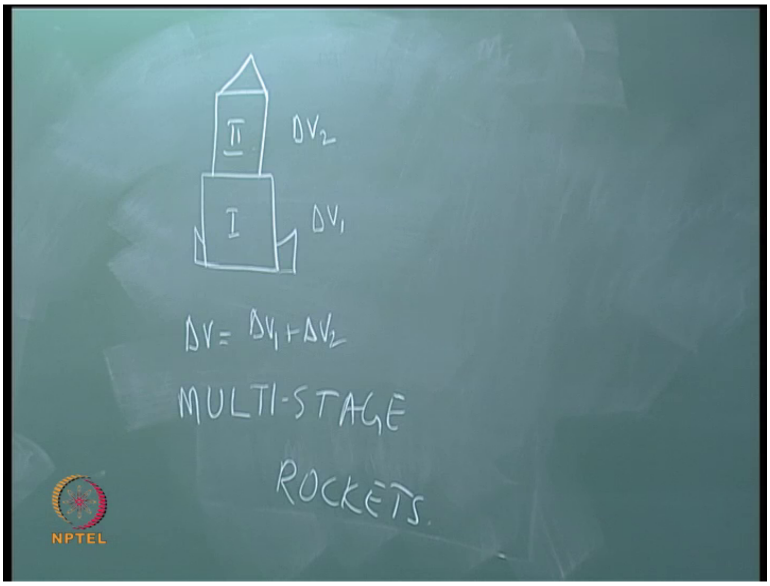

But the job can be done by using multiple rockets. Let us examine this point. Let us say instead of having a single rocket, I make two rockets. I put a rocket on the top of another one and so on. This is my first rocket, this is my second rocket. Now, the first rocket gives a value of the ideal velocity ΔV_1. The second rocket already has a ΔV_1 when its starts functioning. It gives me a value of velocity ΔV_2 and the total velocity of this composite two stage rocket, gives me a $\Delta V = \Delta V_1 + \Delta V_2$. Therefore, by putting one rocket on top of the other, we are able to achieve higher incremental velocities. We call these rockets as multi stage rockets and most of the rockets used today are multi stage rockets. That means you want a velocity increment of something like ten kilometers per second; may be the first one could give you 1 km/s, the second one could give you 3km/s and the third one could be still higher at 6 km/s and therefore, you keep on adding stages of a rocket and this is what we say as multi stage rockets.

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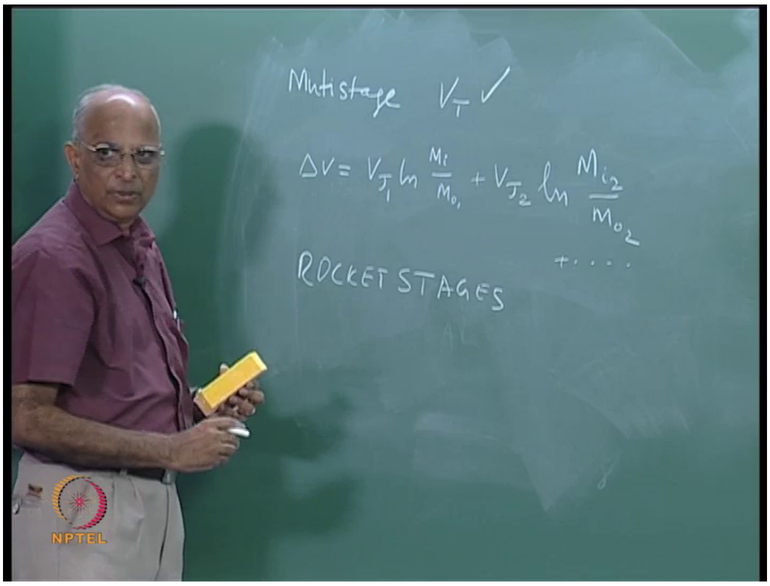

For a multistage rockets, we have ΔV = V_J of the first rocket into logarithm of the initial mass to the burn out mass of the first rocket + V_J of the second stage multiplied by the logarithm of the initial mass to final mass of the second rocket and so on. I get the final ideal velocity. Maybe we should we should try to analyze this in some detail. This is about staging of rockets. We have something as the base or core stage, then on the top of it we have the first stage, then the second stage and so on.

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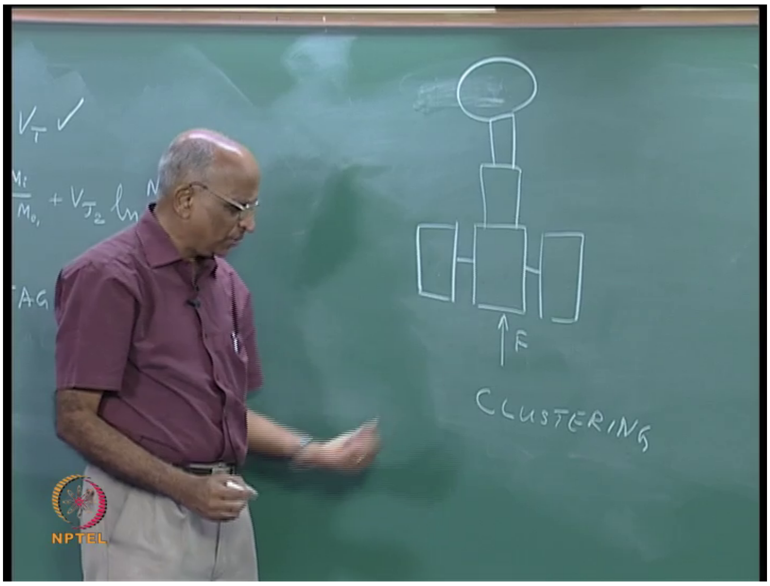

Let us taken an example: Let us examine the construction of India's GSLV rocket, which is very much in the news. We know that this rocket consist of a core rocket, it consist of four rockets attached to the core stage, then it consists of the second stage and the third stage on the top of the third stage sits the satellite.

Therefore, you have the first stage, second stage, third stage. This gives you ΔV_1, ΔV_2, ΔV_3, the sum of which gives you the velocity to put it into orbit. Therefore, we talk in terms of staging. Staging means one after the other but, what about these four rockets attached to the core? Why should they be required?

We have put one stage on the other to get higher incremental velocity; however, in the process we have increased the mass of the total rocket. When you have increased the mass and you want this to be lifted, the core stage should develop sufficient force. However, the single core stage may not be able to generate that level of forces.

Therefore you need additional rockets so that the force or the thrust is able to take off from the ground and that is why we put rockets together and this is known as clustering. Why do we need clustering of rockets? To provide sufficient force for propelling. Even the upper stages may need clustering.

Let us just put things together and summarize what we have learnt so far.

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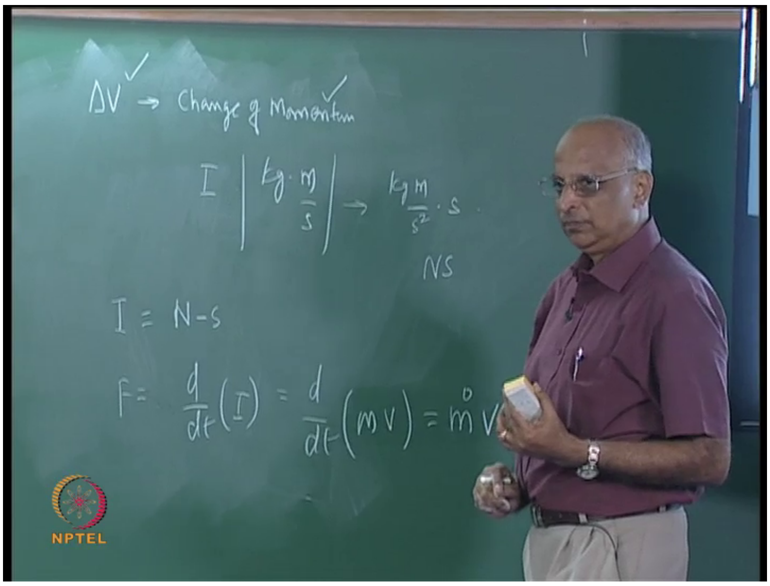

We found that a rocket gives you incremental velocity ΔV. How does it give you ΔV? Because of change of momentum, produced by the efflux of the jet. We wrote the momentum balance equation from an inertial frame of reference and found out the value of ΔV. What is the change of momentum known as? It is known as impulse. What is the unit of impulse? Same as momentum viz., kilogram meter per second. But, kilogram meter per second can also be written as kilogram meter per second square into second which is same as Newton second. Therefore, I can write the impulse as equal to Newton second.

Impulse in Newton second is what gives ΔV; therefore, what is the force with which the rocket is pushed up? Rate of change of momentum means d/dt of mv or d/dt of Impulse I. That is, we get so much force, which is equal to d by dt of momentum; this is equal to mass flow rate m° of the exhaust which is going out with velocity V_J. This equals m° into V_J and therefore, I can also write the force is equal to $m^\circ V_J$ (Newton) or compared to momentum which is equal to mV, I write force is equal to $m^\circ V$ and this is the force pushing the rocket. There is a limit to the mass m° that can be released.

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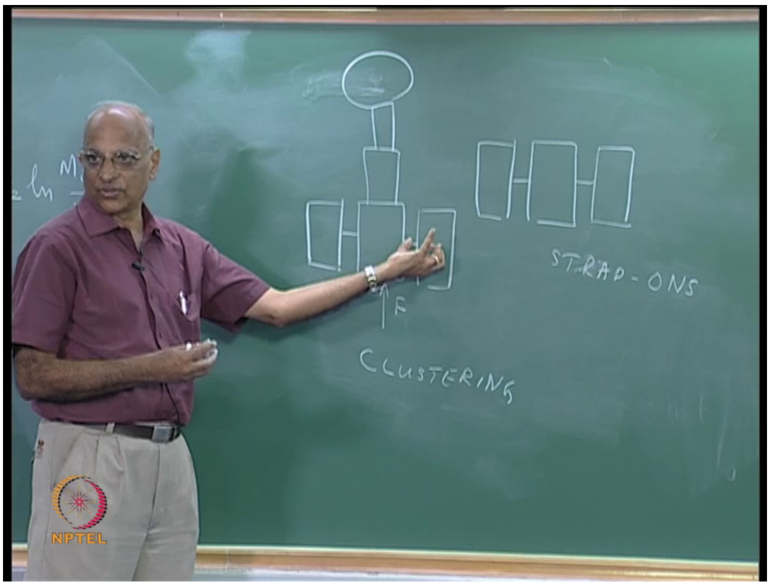

And therefore, we allow more mass to be going through by clustering and thus achieve the desired force. That means that we need a larger force to push and that is why we require clustering. Sometimes, we have a booster rocket to whose sides we attach two rockets. These are like straps. I strap something on to it. The side rockets are also known as strap-on.

Let me take one or two examples. May be I will show it through slides when we meet in the next class. But, to be able to just conclude at this point of time, all what I would like to say is we derive the rocket equation from the change of momentum. We looked at the inertial frame of reference, watched the rocket go up and we found out what is the ideal velocity increment.

We also discussed about some creatures in universe which make use of the rocket principle. Then we found out that the structural mass of the rocket plays an important role just as much as the jet velocity plays a role. Then to get a high value of ΔV, we found the need to operate in stages and to be able to take off with the larger mass of the stages we needed some additional side rockets, which are known as clustering and strap-on.