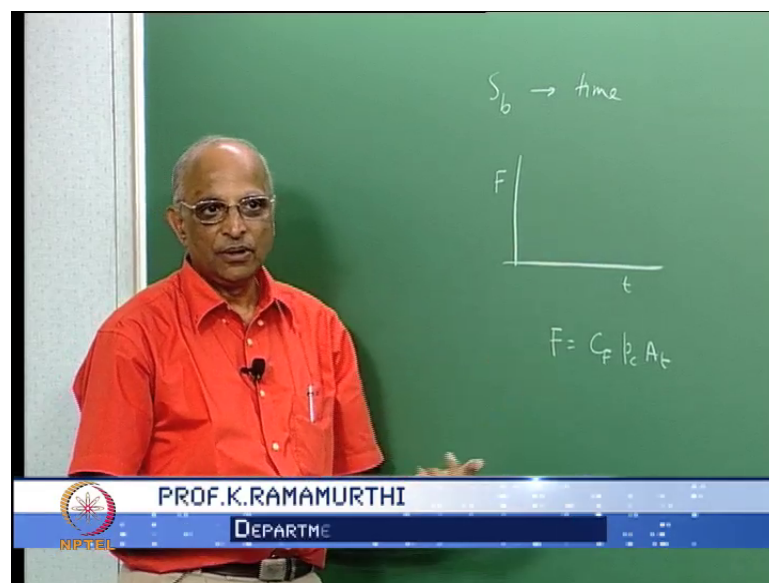


Rocket Propulsion
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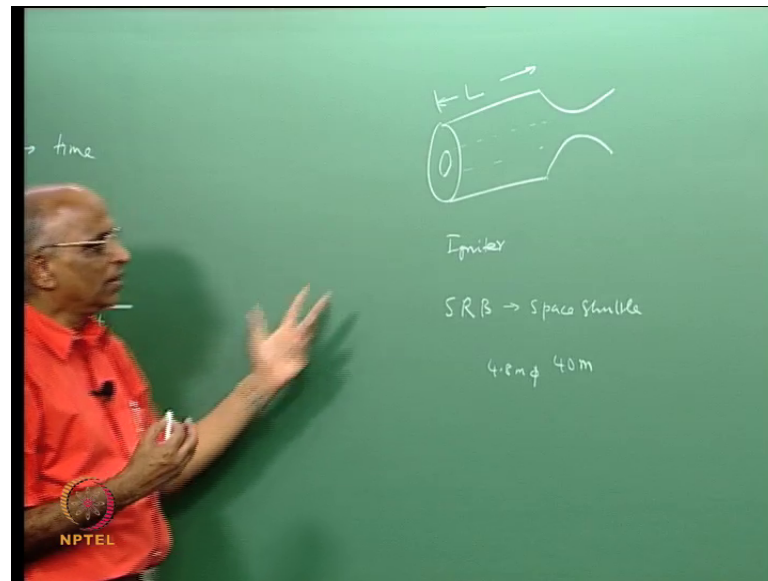
Lecture No. # 24
Ignition of Solid Propellant Rockets

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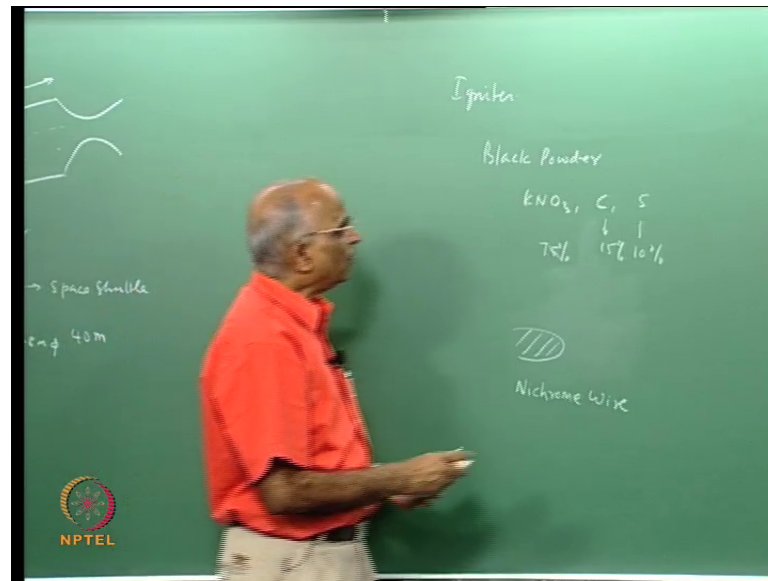
We have seen how the burning surface area can be calculated as a function of time and therefore how the thrust of a solid propellant rocket will change with time. This is because once you know the burning surface area, you can calculate the value of equilibrium pressure in the rocket, and equilibrium pressure \times thrust coefficient \times the throat area is equal to the thrust of the rocket. And how did we calculate the equilibrium pressure; based on the burning surface area? We considered it the last but one class. We have considered propellant grains of different shapes.

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And a nozzle is connected to the case containing the propellant grain. We said the grain could be radial burning or the end burning. Now the question is how do we ignite the propellant grain. How do we start the burning in the grain? We must use a heat source, something like an igniter, which will start the burning process. But then we also know that the grain surface is quite large; may be it could be something like several meters, like for instance if we take an example of the world's largest solid propellant rocket. We call it as solid rocket booster for the space shuttle. It is something like 40 meters long. Therefore, the question is how do we make sure that the grain surface ignites, and that is what we will be dealing with in the first half of the class.

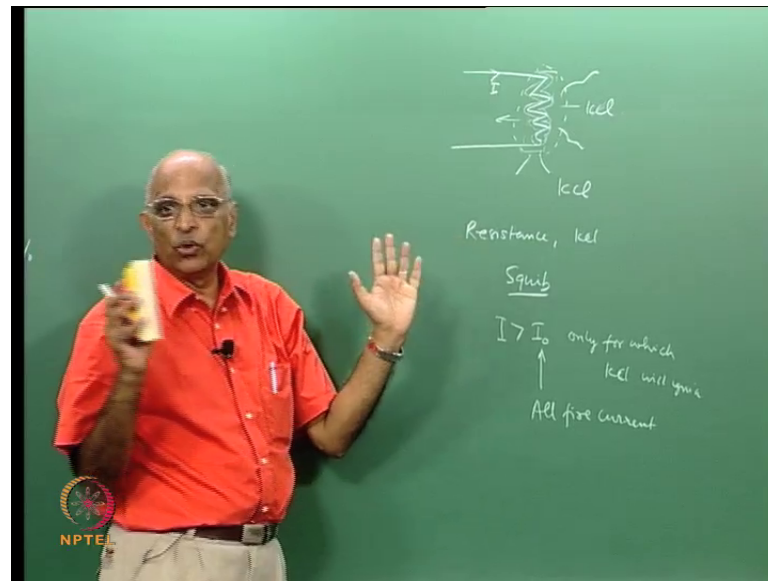
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What should be the attributes of an igniter? Namely how do you make an igniter? Well, we can immediately say igniter must be capable of catching fire easily. And therefore, maybe we will use something like a black powder, which is used for making fire crackers, and this consist of potassium nitrate, some amount of fuel carbon and some amount of fuel sulphur. Typically around 15 percent carbon, 10 percent sulphur and the balance viz., 75 percent of KNO_3 is used. And this composition is easily ignitable. You may recall that we light fire crackers with match sticks and it begins to flare up. We have something like a flower pot type of cracker in which have the black powder; we light it with a match stick and we get sparkles coming out. Therefore, may be this could be one of the contenders for igniters. If we were to use it, how could it be adapted for use it in solid propellant rockets? Maybe I could have a small bag or container and in the bag we have this particular composition of black powder.

But we want to ignite it. How do we ignite it? May be we take a resistance wire. May be an electrical resistance wire like a thin nichrome wire. And why nichrome wire? A thin wire of nichrome has high electrical resistance. If we pass a current through it is gets red hot very soon.

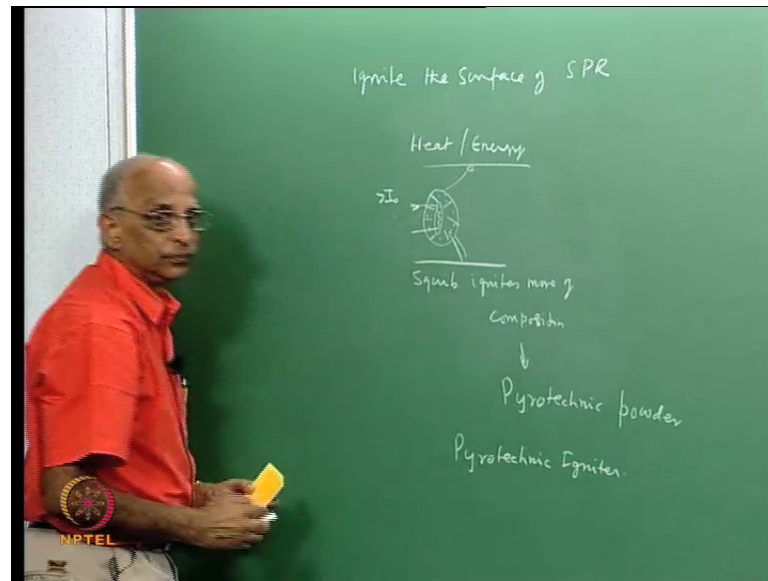
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And therefore, we take this particular resistance wire, coat it or we coat on its surface some easily ignitable composition may be black powder or something like KCl which immediately catches fire. And then may be surrounding it we put more of this composition, black powder or equivalent.

When we pass a current through the wire, it gets heated and KCl or black powder begins to burn. It generates heat, a flame, and this flame could be used for ignition of solid propellants. This particular arrangement of a resistance wire heated by electricity or electrical energy and using some easily burning composition such as black powder is what we call as squib. But whenever we use electrical current for heating, it is also possible that such electric current could be accidentally generated when we have some electrostatic or electromagnetic disturbances. We could then have a current and even when we do not want to ignite, we could have a small current which could heat the wire. Therefore, it is necessary to ensure we have current greater than some threshold value only for which the composition like KCl or black powder will ignite. And this threshold value of current is known as all fire current. And if the current is less than the threshold value formed accidentally by stray electrostatic discharge, it will not catch fire and the rocket cannot be ignited.

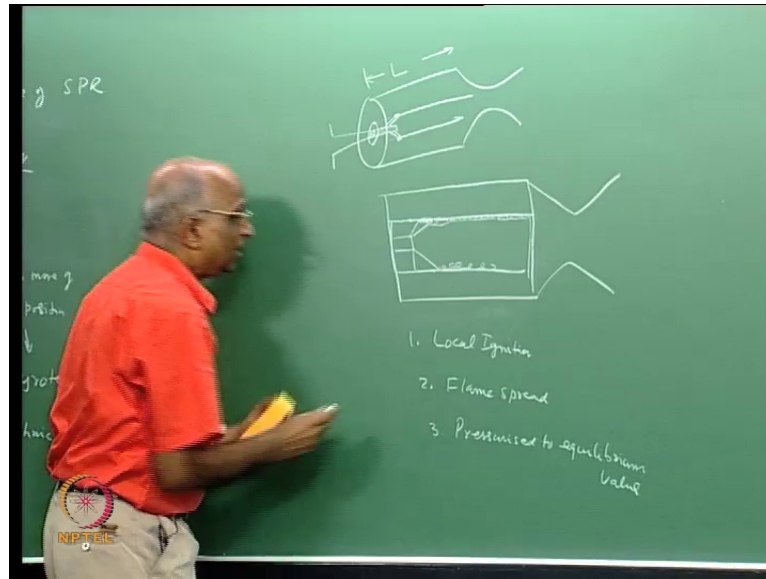
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We want to ignite or burn the surface of a solid propellant rocket. And to be able to ignite the solid propellant surface, we need to add some heat or some energy to the propellant surface. And therefore, all what we do is to release the energy in the propellant cavity. We cannot go and put a fire inside it. Therefore we have a composition which is easily ignitable with a nichrome wire; start the ignition process by passing current through the wire, this generates heat. But then a squib has only a small quantity of charge like KCl or black powder. We add some more charge surrounding the initial charge and when we pass a current greater than some threshold value of current. It starts a chemical reaction of the black powder which is easily ignitable. The black powder or some composition around the squib ignites, and a fire is formed and this fire impinges on the propellant surface and makes it catch fire. That means, we have a squib surrounded by some of these easily ignitable powders. We call the easily ignitable composition as pyrotechnic composition or pyrotechnic powder.

Such igniters, which make use of a squib with pyrotechnic composition around it to generate sufficient energy and ignite the solid propellant rockets grains, are known as pyrotechnic igniters. The igniter seems to be a simple device.

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Since we need electrical current, we have a battery or some other source of electric current. We put something an igniter over here in the cavity of the propellant grain. And then we pass a current through the squib and ignite the squib. The squib ignites the powder, which is around it, and it sprays the flame that is a plume which is formed ignites the propellant surface. The volume of the cavity gets pressurized and the flames spreads over the surface and in this way the propellant surface ignites.

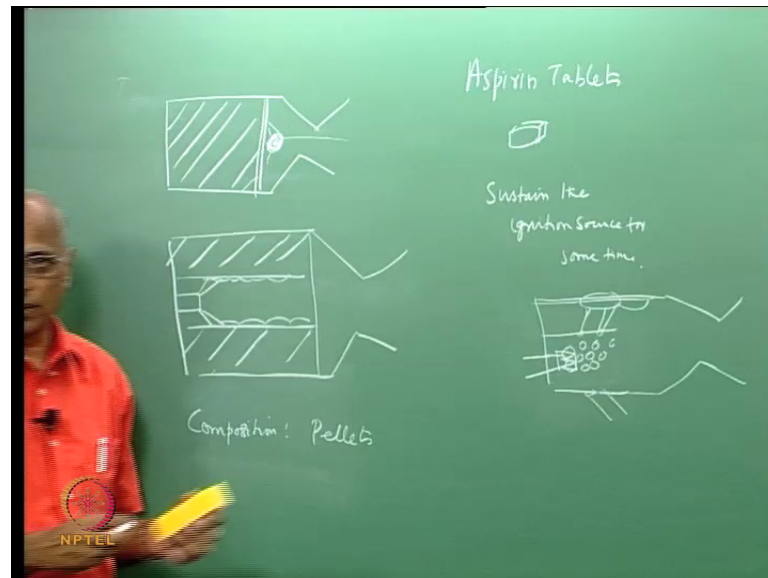
We had discussed the internal surface of the grain, the outer surface of the grain; and just for the sake of simplicity taking a radial grain, this is the nozzle; we put an igniter over here, as shown. We pass a current and a flame or plume originates from the igniter, it impinges over the propellant surface. It ignites the surface over which it impinges and when this surface ignites the next or the adjacent surface gets ignited and so on till the entire surface ignites.

In other words, we have first something like local ignition, where in the sparklers or the plume or incendiary impinges on it. Then we have a flame, which is spreading over the surface, and once the flame spreads the pressure may still not be the equilibrium value corresponding to the burning surface area. Thereafter, as the last part chamber fills up and gets pressurized to equilibrium value.

These are the 3 events, which would happen: local ignition, flame spread and chamber filling to equilibrium. If we have a bag igniter like this and we have a small rocket like

an end burning charge or an end burning grain let us look at the sequence of events during the ignition process.

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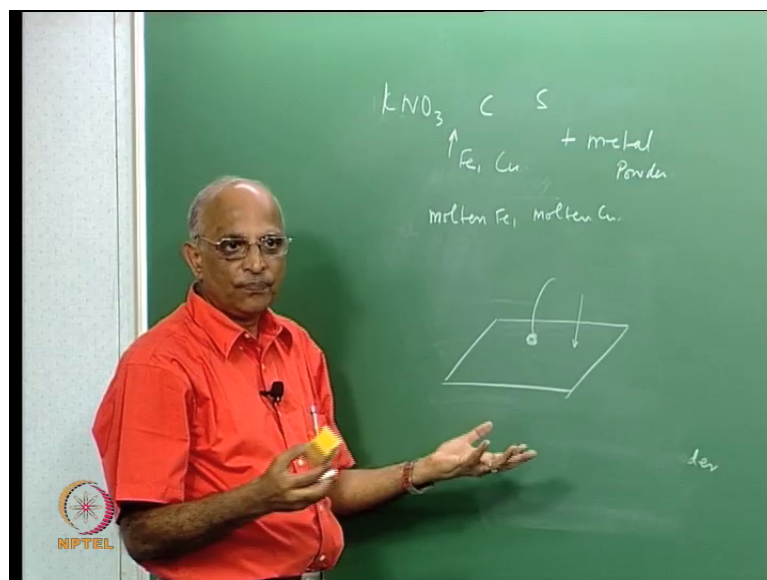
We have a solid propellant in an end burning configuration; we have the nozzle over here downstream of the grain. We want to ignite it. From the nozzle side we introduce a bag of pyrotechnic charge, inside the bag we have a squib; we ignite the pyrotechnic charge over here, a flame is formed and it impinges over the surface of the propellant and the surface catch fire.

Whereas if we something like a radial burning grain or a grain like a star grain which burns from inside to the outside. We put the igniter in the cavity volume; maybe we would like a part of the exposed propellant surface to catch fire. And then this fire spreads over here till the entire propellant surface catches fire. This is how we ignite the total propellant surface. If we have a bag of pyrotechnic powder, we cannot have controlled burning and therefore very often the pyrotechnic composition is compressed and made in the form of pellets. What do you mean by pellets? You know we take this Anacin or Aspro for mild headache and the tablet is in the form of some pellets. We say Aspirin tablets. Instead of having a powder charge; you form pellets like this small Aspirin pellets like this. And what is the advantage of having solid pellets like this. The burning surface area that burns can be controlled; it is not like a powder, which immediately burns. It takes some time for a pellet to burn and therefore, it can give better

ignition. That means, we can sustain the ignition source for some time. Instead of having a bag containing powder, we could have something like a firm bag or a let us say a cylindrical tube containing the pellets as shown.

We put a lot of pellets in it of the pyrotechnic powder. We put the squib and we ignite the pellets. We make some holes here, through which the flame or the plume goes out and impinges on the propellant surface and ignites it. This is the local ignition and is followed by flame spread. This is how the igniter functions. And what is the requirement of an igniter? To transfer heat or energy to the propellant surface for which the black powder could be used

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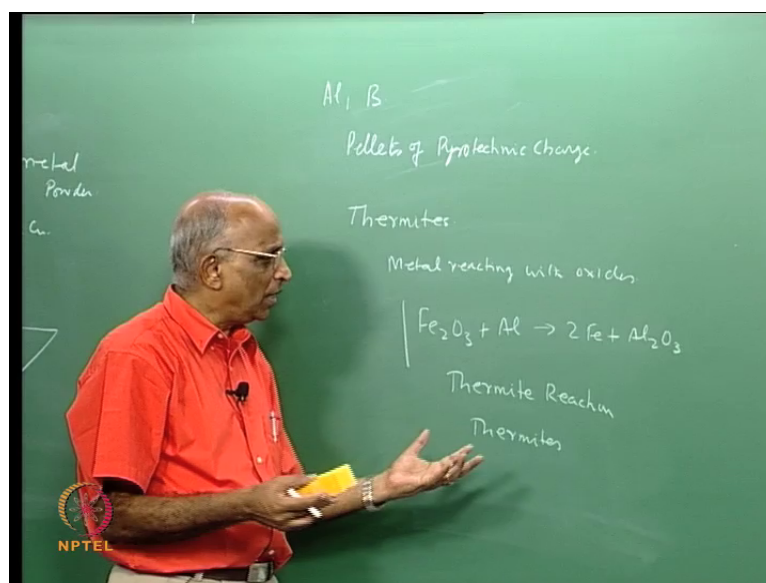


And black powder consists of KNO_3 , carbon and sulphur. You know the products of burning are essentially gaseous except for little bit of carbon in solid phase. But all of us know that if we can put some metal into it like let us say I put iron powder or I put something like copper filings into it; when we heat iron or copper to high temperatures, we get molten iron or be molten copper. And what is the advantage of using this metal powder or filing in the igniter composition? A gas cools down when it expands whereas a solid retains heat for some time. And also if I have a surface and on the surface may be a molten iron falls it will transfer the heat of the molten iron into the surface more effectively than a gas will transfer it.

And a molten iron or a molten hot substance is in better contact with the surface, and it is able to able to conduct the heat to the surface much more effectively than a gas. Therefore most of the igniter compositions also have metal powder. And why do we add metal powder? The reason is a hot liquid metal conducts heat much better onto a surface than a gas.

The simple experiment, which we saw when we lit a sparkler; the composition that did not have metal it was not that violent, but when we had metal filings in it, and when the sparkles fell on my hand it got burnt. The reason is that the hot metal is able to conduct heat much better. Therefore, the pyrotechnic composition will consist of may be some metal powders and metal powders which are used include aluminum and boron. These are also used in the composition of pellets of the pyrotechnic powder.

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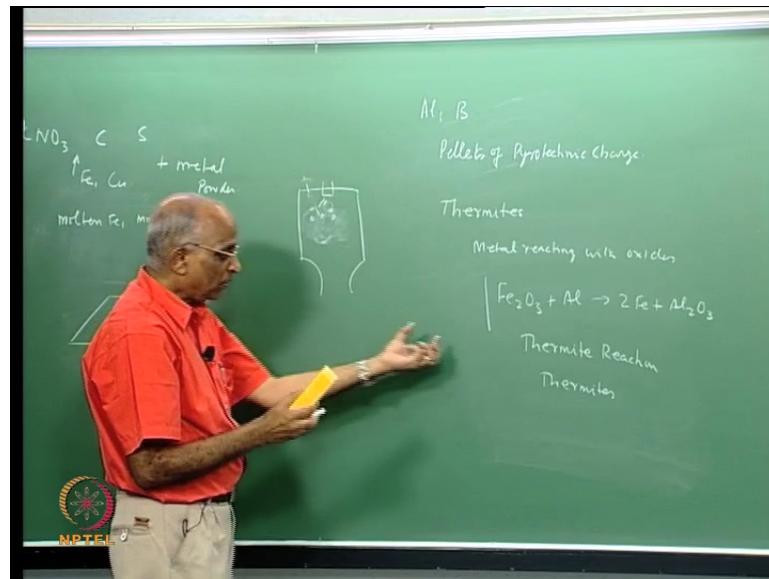


We therefore also include some metal powder in the igniter charge. However, there are some basic issues and let us let us try to resolve some of these issues. You know instead of having metals in the pyrotechnic charge, there are certain substances known as thermites. Thermite reactions are those in which we have metals reacting, metals reacting with oxides. If we take rust Fe_2O_3 and react it with aluminum Al what we get is $2\text{Fe} + \text{Al}_2\text{O}_3$. This reaction is very exothermic and what we form is molten iron and aluminum oxide. If we can use this as an igniter well it has a metal constituent in it in molten form

that will touch the propellant surface. It will ignite much easier. Such reactions are known as thermite reaction and these substances are known as thermites.

You know this is the time we should be looking at thermites as we find research work going on in the area of nano thermites, which are more effective in producing heat.

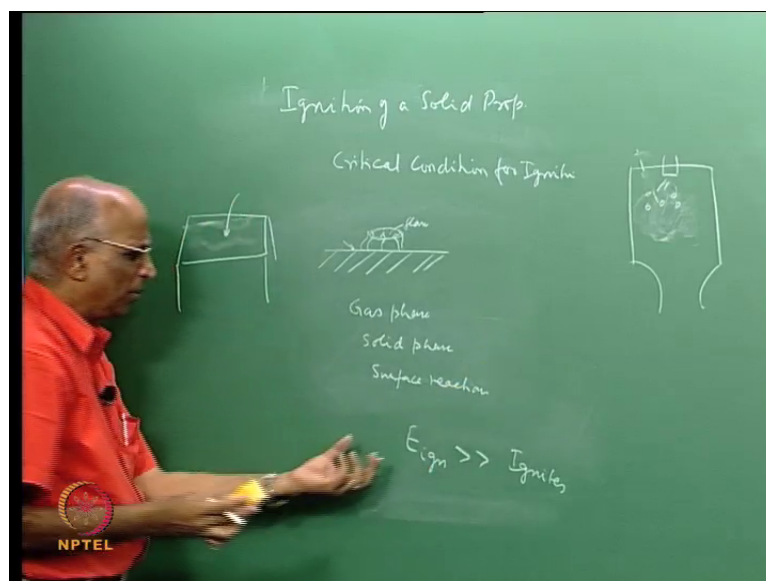
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If, instead of a solid propellant rocket, we want to ignite let us say a liquid hydrogen liquid oxygen rocket. I have hydrogen oxygen which must be ignited. And one of the contenders for this you just use the thermite mixture as an igniter. We have an igniter over here. We spray Fe_2O_3 plus molten aluminium and ignite the mixture of liquid hydrogen and liquid oxygen. We get molten substances and molten substances retain heat for a long time and it will ensure that the mixture gets ignited. Therefore, such of the igniters using thermite mixtures are called as a thermite igniters.

Therefore what is it we have considered so far? We said an igniter could consist of a composition which generates something like a plume or a hot gas jet or if we were to put metal in it will also create some metal or some hot molten metal which will transfer heat to the surface better and give rise to ignition of a solid propellant surface.

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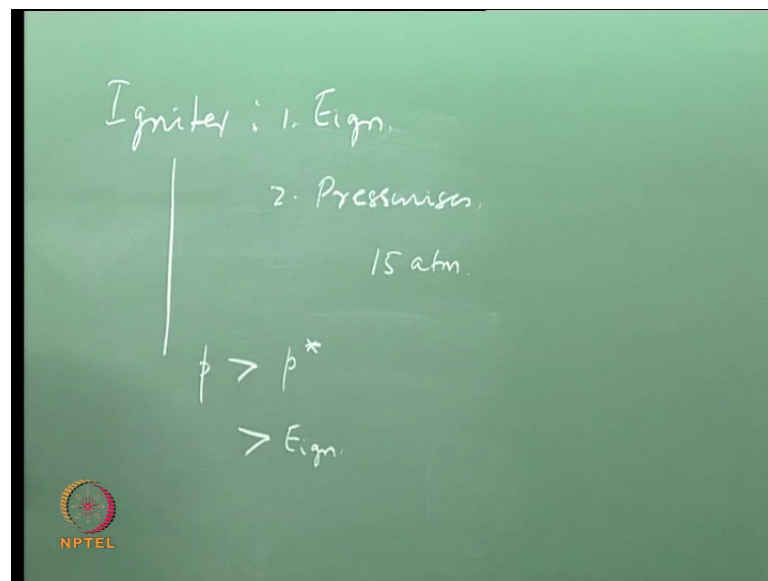
When we say ignition of a solid propellant, what is the mechanism by which a solid propellant ignites? When heat is transferred to a solid propellant, some vapors of the hydrocarbon are generated from the fuel and AP dissociates into a mono propellant flame. The products so formed could mix together and form a flame. And therefore, the process of ignition could happen in the gas phase, wherein vapors ignite. It could happen at this at the solid propellant surfaces, wherein the products of dissociation formed in the gas phase could readily react at the surface, giving rise to surface reactions. Reactions could also occur in the solid phase. All these three reactions viz., gas, surface and solid phase are possible, but it is difficult to say which one dominates and under what conditions. And we will assume that all the three reactions take place viz., gas phase, heterogeneous at the surface and in solid phase, which lead to ignition of a solid propellant.

That means a solid propellant, if we have a slab of solid propellant, this is the surface and we consider transfer some energy to it, in the solid part of it some reactions take place, in the gas phase above the surface some reactions takes place, some surface reactions also take place. I could model it using any of these three theories or combination of two or three theories and I could find out what is the critical condition for ignition. I will not get into details other than say that we should supply some ignition energy greater than some threshold limit so that a propellant ignites. This is a subject by itself. But we know that

if the energy is sufficient, the plume ignites the surface of propellant over which it is incident and ignition is achieved.

However, if the pressure in the cavity is small, the ignition energy must be large; this is because if pressure is higher the flame surface will be nearer the propellant surface. We therefore should ensure that a minimum pressure is formed in the cavity by the igniter.

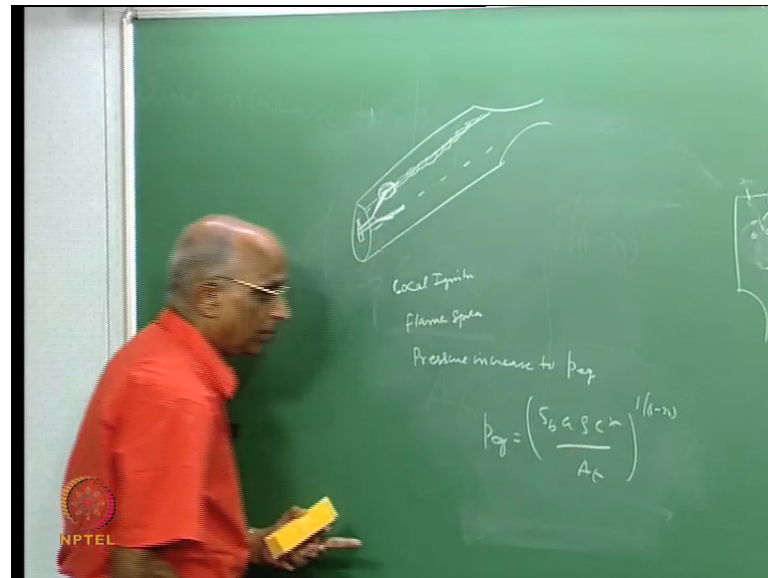
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This brings us to the point on the role of the igniter. It supplies the necessary ignition energy to the propellant surface and also pressurizes the chamber cavity to some threshold value of pressure. The pressure is about 15 atmospheres. What happens is when the pressure is low? We noted that the flame standoff is higher at lower pressures while when pressure is higher the standoff is lower. I make sure that the standoff distance is small such that the propellant surface ignition is sustained much better and this we will see when we study the combustion instability of solid propellant rockets. If the pressure is greater than some threshold value, the solid propellant combustion is more stable and therefore the role of an igniter is to ensure that it pressurizes the chamber to a value greater than some threshold limit. It also supplies some energy greater than some threshold value for ignition to occur. This is all what is required from an igniter.

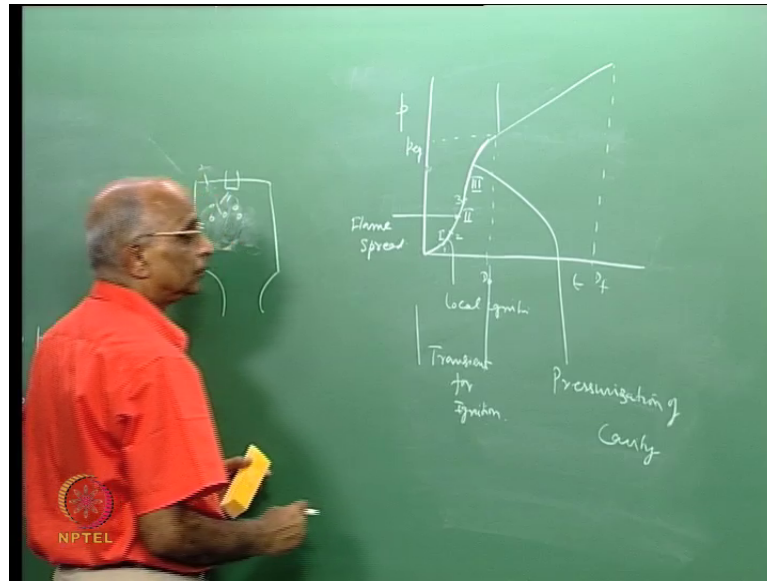
And what does the igniter therefore do. We have a propellant grain and we have an igniter here; it generates a plume may be from the pellets within it burn and the plume impinges over part of the propellant surface.

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If the energy transferred to this surface is greater than the energy required for ignition and if the pressure in this cavity is greater than some limit, the propellant surface ignites. We have something like local ignition of a small part of the propellant surface over which the igniter plume impinges. And when this part ignites, heat which is generated in this zone plus the heat which is generated by the igniter helps to supply the necessary energy to ignite the adjacent surface and therefore, the flame keeps on spreading. And we call this spreading of the flame as flame spread. That means flame spreads from the local ignition area over the entire surface of the propellant. But the pressure is still not equal to the equilibrium value and therefore thereafter the pressure increases to the equilibrium value. And we calculated the equilibrium value as equal to we said = $(S_b \times a \times \rho_p \times C^* / A_t)^{1/(1-n)}$.

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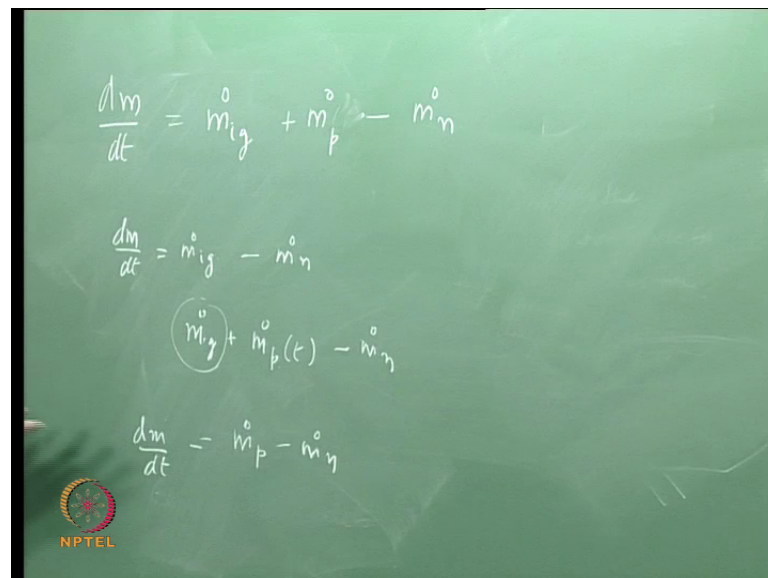
Therefore let us put this down on a figure such that we clearly understand it. What is it we were discussing in the earlier classes? We said that the pressure with respect to time would be the equilibrium value if we consider a radial burning grain? This is the pressure corresponds to t_0 when the surface is ignited and at burn out the pressure corresponds to time t_f . The pressure increases monotonically between t_0 and t_f because of the progressive burning.

Now when we ignite the motor, we start with ambient value of pressure and the pressure increases to the value at t_0 .

We have the igniter composition increasing the pressure to some threshold value. Let us say from 1 when it increases the value to a threshold value and heat transfer takes place, we move from 1 to 2, which is local ignition of a small part of the surface. That means, only a part of the propellant surface gets ignited initially, because the plume is impinging on it, the hot metals impinge on it and ignites it. The energy is further released from here and flow takes place and ignites the balance surface of the propellant. Therefore, we have something like a flame spread from 2 to 3 and flame spreads over the entire surface over the propellant. We call this zone as flame spread and when once flame spreads over the surface, we have reached this pressure, which is still less than the equilibrium pressure. And then the pressure in the chamber increases or there is something like pressurization in the cavity.

These are the 3 processes namely local ignition followed by flame spread followed by cavity pressurization to the equilibrium value. And this is the equilibrium value to get started with and thereafter progressive burning of the propellant grain takes place. We would like to write equations for these 3 phases so that we can find out the time required for ignition and the transient. How do we do it? How do we determine this rate of pressure evolution and this portion wherein ignition takes place? Rather we would like to determine the transient during ignition? Let us try to write an equation for some of these processes. It is quite simple if we really get into the details. Nothing complicated and we have done much more difficult problems trying to determine how the pressure should evolve in a complicated grain shape.

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The image shows a green chalkboard with handwritten equations in white chalk. The equations represent mass balance for a propellant grain. The first equation is $\frac{dm}{dt} = \dot{m}_{ig} + \dot{m}_p - \dot{m}_n$. The second equation is $\frac{dm}{dt} = \dot{m}_{ig} - \dot{m}_n$. The third equation is $\dot{m}_{ig} + \dot{m}_p(t) - \dot{m}_n$, with \dot{m}_{ig} circled. The fourth equation is $\frac{dm}{dt} = \dot{m}_p - \dot{m}_n$. In the bottom left corner, there is a small circular logo with a gear and a star, and the text 'NPTEL' below it.

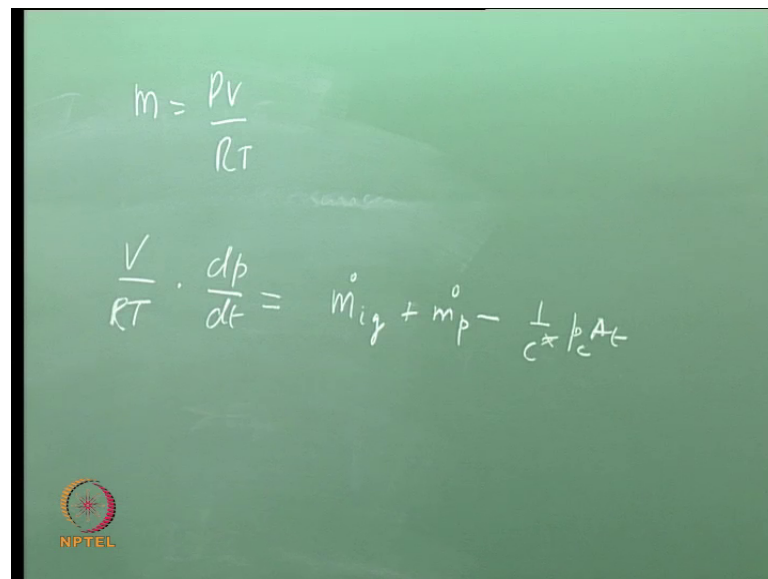
We would like to solve the mass balance equation. We say the rate at which mass is added by the igniter; during the process of flame spread, mass is not only added by the igniter and also the ignited burning surface area is also adding mass as the flame spreads. We are adding more and more mass as the more of surface of the propellant is getting ignited.

Therefore, the rate at which mass is added to the propellant dm/dt = the rate at which the igniter adds mass + the rate at which the burning propellant adds mass; let us call it as $\dot{m}_{ig} + \dot{m}_p$ and this minus the mass flow rate which nozzle leaves through the nozzle \dot{m}_n . When we talk of local ignition the only the \dot{m}_{ig} is from igniter adding mass, because

we are still in the first phase wherein the propellant surface is not yet ignited. It is only dm/dt the mass added by the igniter is equal to \dot{m}°_{ig} . When we talk of the second phase, we have $\dot{m}^{\circ}_{ig} + \dot{m}^{\circ}_p$, which is changing with time as the flame is spreading. We should have \dot{m}° , which is leaving through the nozzle as $-\dot{m}^{\circ}_n$ which is leaving. And during the pressurization to equilibrium time, the igniter function is over we just have \dot{m}°_p when the whole surface of the propellant grain is burning $-\dot{m}^{\circ}_n$, which is leaving which is equal to dm/dt . If the igniter mass is present during this last phase, it would be very much smaller than the mass generated by the surface of the propellant.

During the local ignition phase followed by the flame spread phase, the igniter is still supplying energy while part of the propellant surface which is burning is also supplying energy or supplying mass to the hot gases. And some hot gases are leaving through the nozzle. We have in the final phase, when the entire propellant surface catches fire, the pressurization of the cavity. These are the equations that we could solve to determine the variation of pressure with time during the process of ignition.

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$$m = \frac{PV}{RT}$$

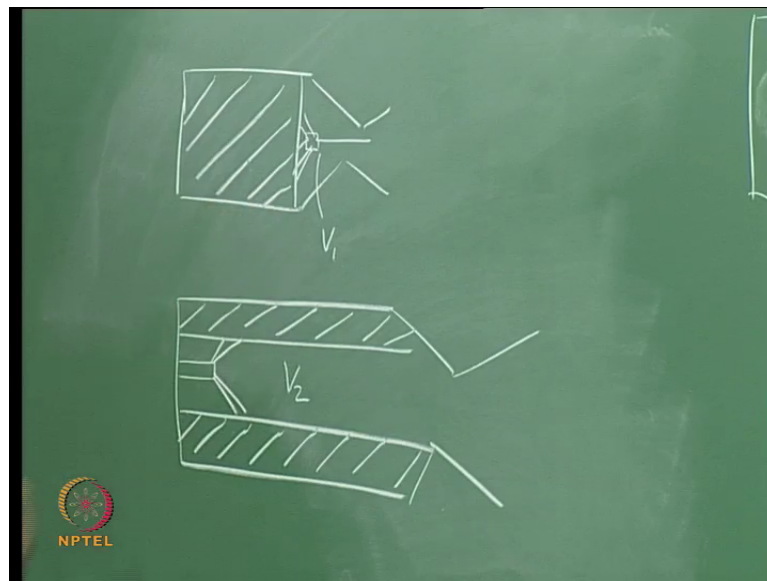
$$\frac{V}{RT} \cdot \frac{dp}{dt} = \dot{m}_{ig} + \dot{m}_p - \frac{L}{c^*} \frac{p_c}{A_t}$$

From ideal gas equation, we get $m = PV/RT$, where V is the volume of the cavity or the cavity volume and therefore we can write the equation for dm/dt . The volume V during ignition is about a constant since there is hardly any significant regression of the surface. We can take volume V as a constant and also temperature of the products T as a

constant. We therefore write $dp/dt \times V/RT = \dot{m}_i g + \dot{m}_p - \dot{m}_n$. But $\dot{m}_n = 1/C^* \times p \times A_t$ where the chamber pressure is denoted by p .

And now if we are given the rate at which the igniter is supplying mass, we know \dot{m}_p is equal to the mass from the burning surface area, we can find the value of dp/dt . The burn rate is a_p^n . We can solve for dp/dt and we can determine the pressure and also the thrust. Now, let us take a look at this figure once again, and try to draw some inferences and see if some rational approximations may be introduced.

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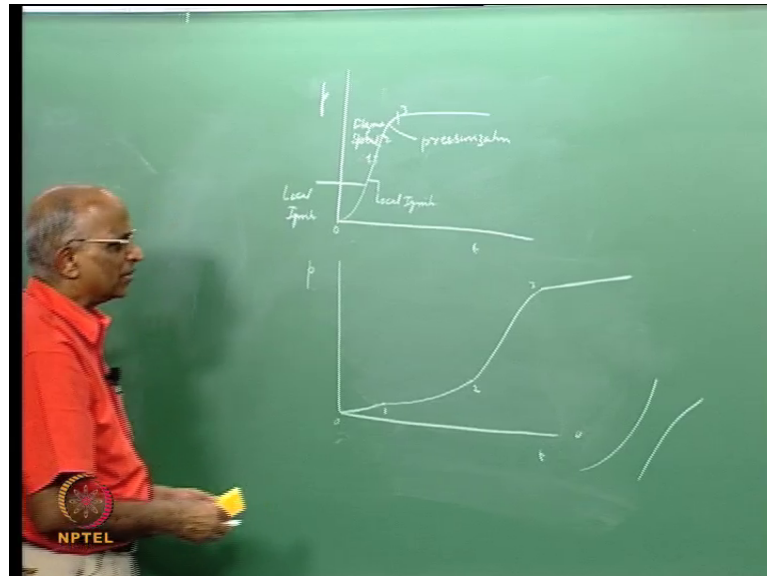


Supposing as a first case we consider a small solid propellant rocket; and let us consider for ease an end burning grain. This is the propellant grain, the cavity volume is quite small. In the second case, we consider is a huge solid propellant rocket, which is very much larger. And when we say the initial volume is large it means that we have something like with a radially burning propellant grain. We want a large burning surface area. Correspondingly, we have a larger area for the cavity or port volume V . We know the volume case 2 is very much larger than the volume for case 1.

We put an igniter; a controlled igniter with pellets in the case of the end burning small rocket. We make sure that the pellets squirts fire and firebrands on the end - burning surface of the propellant grain. Now what is going to be the change in the transient for pressure for the small solid propellant rocket compared to a large solid propellant rocket? Apparently in the small end burning grain, the surface will locally ignite. The entire

surface of the propellant grain is directly ignited by the igniter in the case of the small end - burning rocket. This is what we would expect.

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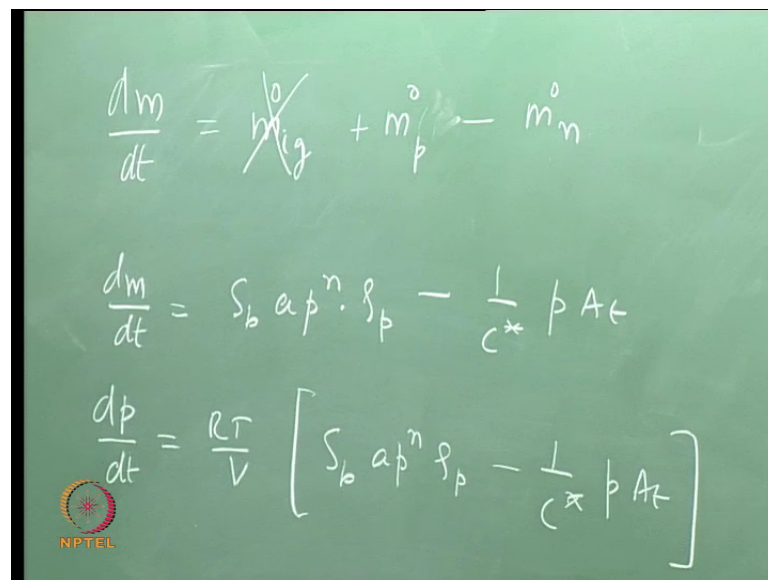
The sequence of events may be seen in the pressure versus time figure. At the start of ignition, the pressure immediately goes up and reaches the equilibrium value for neutral burning. The local ignition mainly constitutes the process of ignition. In other words, the plumes from the igniter directly impinge and ignite the propellant surface. There is hardly any flame spread over the propellant surface. We can say that 0 to 1 will be now the local ignition, 1 to 2 is the flame spread which is very small region compared to 0 to 1, then we have the equilibration to final pressure taking place which is again very small. This last part is negligible because the volume V is small.

In other words for a small solid propellant rocket the ignition process is governed by local ignition. If we have a large cavity volume and a large rocket, well local ignition first takes place. Since we have a large volume, it takes time to pressurize it therefore, the pressure build up starts very slowly. We need a minimum pressure and minimum ignition energy to initiate the burning. Therefore, from 0 to 1 which now we call as local ignition slowly takes place. Then the flame spreads over the surface. The surface area being large, the flame spread continues to spread at low pressure till flame spreads over the total surface of the propellant and we reach point 2. After this we have to the pressure

from pressure at 2 changing to the equilibrium value and we have something like pressurization of the cavity between 2 and 3. At 3 equilibrium pressure is reached.

Therefore the sequence is consisting of local ignition followed by a significant portion of flame spread compared to a small rocket. And this portion of chamber filling to the equilibrium value also tends to be a large fraction. We would like to predict what must be the shape of this curve viz., how the pressure increases. Let us address this particular curve. We write a simple mass balance equation.

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The image shows three equations written on a green chalkboard. The first equation is $\frac{dm}{dt} = \cancel{m_{ig}^0} + m_p^0 - m_n^0$. The second equation is $\frac{dm}{dt} = S_b a p^n \rho_p - \frac{1}{C^*} p A_t$. The third equation is $\frac{dp}{dt} = \frac{RT}{V} \left[S_b a p^n \rho_p - \frac{1}{C^*} p A_t \right]$. An NPTEL logo is visible in the bottom left corner of the chalkboard image.

What happens during the chamber pressurization in the case of the large rocket? Well igniter has finished its job; it has ignited the surface and flame spread has happened. Therefore, we can write dm/dt as corresponding to the entire surface of the propellant catching fire. Therefore, the burning surface area $\times a p^n \times$ propellant density is the rate of mass generation. Even if the igniter continues to supply mass, it will be negligibly small compared to this large value of mass flow rate from the propellant surface. The mass which is leaving is $-1/C^* \times p \times A_t$ where p is the chamber pressure. We can call the chamber pressure as p_c or p . Now dm/dt has already been written as equal to $dp/dt \times V/RT$. Therefore, we have $RT/V \times S_b \times a p^n \times \rho_p - 1/C^* \times p \times A_t$. This is what the mass balance equation gives. Let us solve this equation. But since we have so many variables can we take the variables out and put it in terms of some non-dimensional numbers?

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$$\frac{dp}{dt} = \frac{\Gamma^2 C^{*2}}{V C^*} \left[\frac{S_b a C^* \rho_p}{A_t} p^n - p \right]$$

$$L^* = \frac{V}{A_t}$$

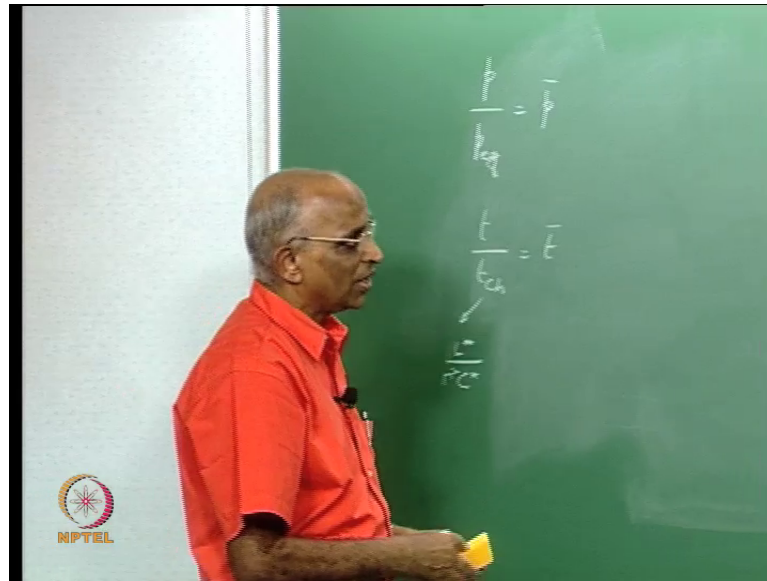
$$\frac{dp}{dt} = \frac{\Gamma^2 C^{*2}}{L^*} \left[p^n (p_{eq})^{1-n} - p \right]$$

We already know that $(C^* \times \rho_p \times a \times S_b / A_t)^{1/(1-n)} = \text{equilibrium pressure } p_{eq}$. We also know that the value of $C^* = \sqrt{RT/\Gamma}$ where $\Gamma = \sqrt{\gamma \times (2/(\gamma+1))}^{(\gamma-1)/2(\gamma+1)}$. And we can write the non dimensional pressure in the chamber at any time as the ratio of p by p_{eq} instead of p .

We write the value as dp/dt is equal to: instead of RT we write $\Gamma^2 \times C^{*2}$ and simplifying by taking C^* outside $\Gamma^2 C^*$ we $S_b a C^* \rho_p$; we also take A_t outside. Therefore we are left with the pressure p left over. We find C^* and C^* gets cancelled. We have volume of the cavity divided by the throat area. This gives us the length called as L^* . We define it as equal to initial volume of cavity or port volume divided by throat area (L^*) or the initial value of L^* . And therefore, we can write this equation as equal to $dp/dt = \Gamma^2 C^*/L^*$ and C^*/L^* represents a velocity divided by length which corresponds to one over a reference time. We have $S_b \times a \times C^* \times \rho_p / A_t$ is equal to equilibrium pressure p_{eq}^{1-n} . This is because the above term to the power $1/(1-n)$ is p_{eq} .

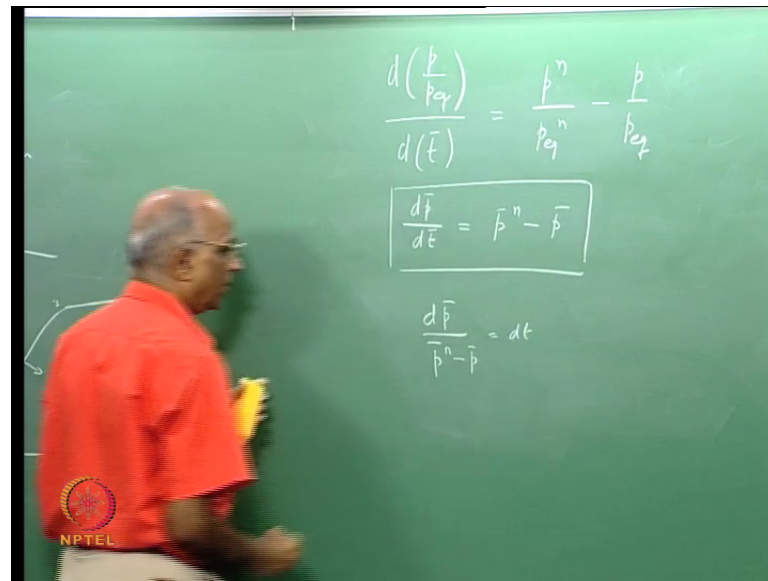
The burn rate of the propellant is given by $a p^n$. Therefore we have p^n over here. We have $S_b a \rho_p C^*/A_t$ as p_{eq}^{1-n} . Simplifying, we have: $dp/dt = \Gamma^2 C^*/L^* \{ p^n (p_{eq})^{1-n} - p \}$ as shown in the slide.

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We non dimensionalize p by the equilibrium pressure p_{eq} and call it as \bar{p} . And now we have time t in dp/dt . We non dimensionalize t with respect to a characteristic time. We have L^* , Γ is anyway a constant and is divided by the characteristic velocity C^* . This represents a time which is again a characteristic time. Therefore, now we say t by t characteristic is equal to a non-dimensional time \bar{t} . The characteristic time is equal to $L^*/C^*\Gamma^2$. Length divided by velocity gives unit of time and we call this as characteristic time. We will develop on this further when we study combustion instability in rockets. And now if were to introduce these two non-dimensional terms in the equation, what is the final equation that we get?

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The image shows a professor in a red shirt standing next to a green chalkboard. The chalkboard contains the following equations:

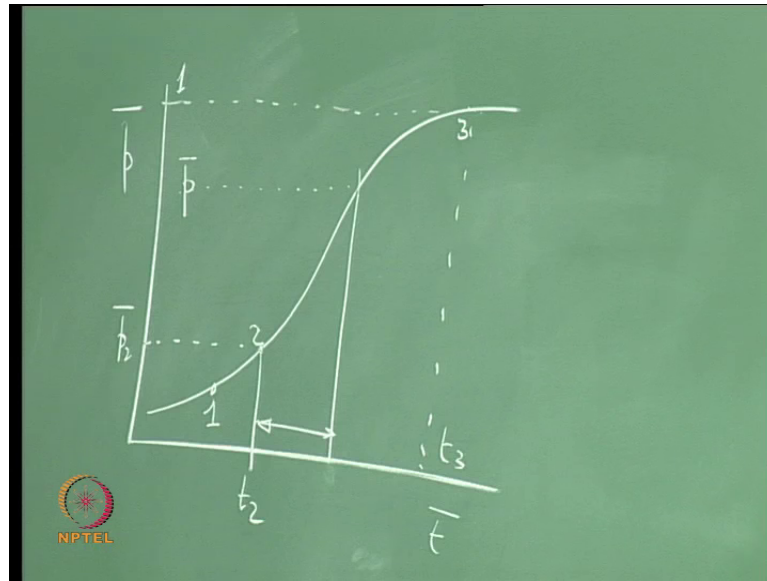
$$\frac{d\left(\frac{p}{p_{eq}}\right)}{d(\bar{t})} = \frac{p^n}{p_{eq}^n} - \frac{p}{p_{eq}}$$

$$\boxed{\frac{d\bar{p}}{d\bar{t}} = \bar{p}^n - \bar{p}}$$

$$\frac{d\bar{p}}{\bar{p}^n - \bar{p}} = d\bar{t}$$

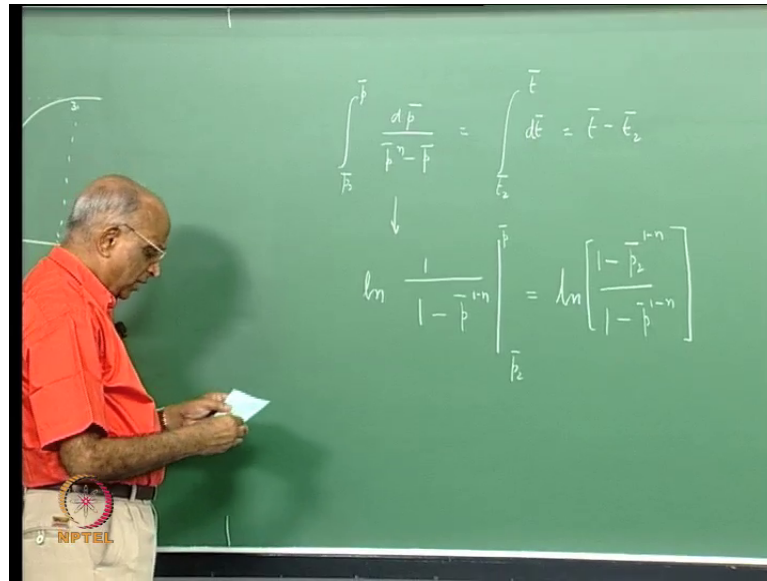
We get $d(p/p_{eq})$ on the left. And since we have brought 1 over p_{eq} on the left therefore, it should also divide the terms on the right. Dividing t by t characteristic gives dt in the denominator on the left side. Since we divided the right side by p_{eq} , this is equal to p^n / p_{eq}^n and the next term is p by p_{eq} . Or rather this equation is now telling us that d of non-dimensional pressure divided by d of non-dimensional time $= p^n - p$. This is a final equation that get. And now we can solve this equation by writing as dp bar by dt bar as equal to p bar to power n minus p bar. Let us integrate this equation from a value of p bar at time of flame spread to the equilibrium value at time when the equilibrium pressure is reached.

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Let us put the value pressure and time. At time t_2 the non-dimensional pressure is p_2 bar while at time t_3 it is 1. As a function of the non dimensional time starting from a value t_2 bar, it goes the equilibrium value. Equilibrium value is p bar is defined as a pressure by equilibrium value which is 1. We have t_2 as the time at which the flame spread is completed. And between 2 to 3 wherein we get the equilibrium condition we are interested in finding out the time taken namely from the value of t_2 to the value of t_3 . The pressure value is to be integrated from p_2 bar to one. Let us find out what is the time required for the pressure to go from the end of p_2 bar to one. That is the entire propellant grain has ignited to the condition when there is some pressure p bar in the system. Now that means, I am interested in this particular time t bar to reach the equilibrium value from the initial time t_2 bar.

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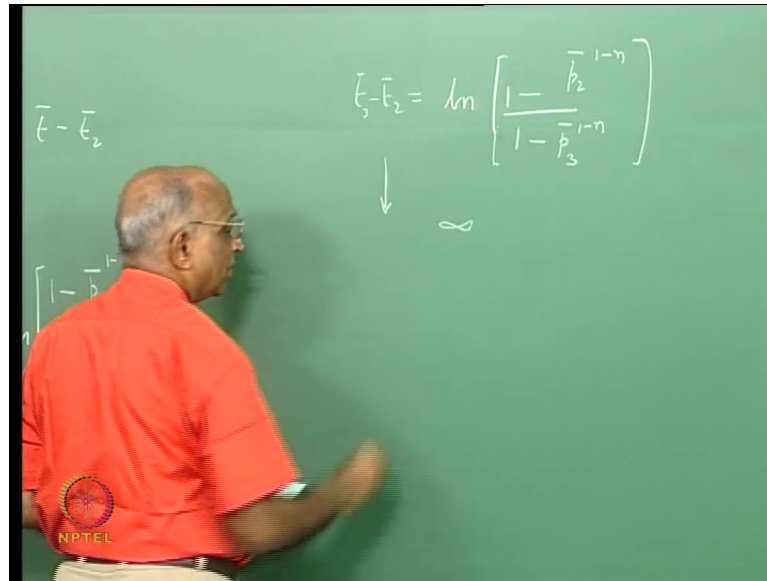


$$\int_{\bar{p}_2}^{\bar{p}} \frac{d\bar{p}}{\bar{t}^n - \bar{p}} = \int_{\bar{t}_2}^{\bar{t}} d\bar{t} = \bar{t} - \bar{t}_2$$

$$\ln \left[\frac{1}{1 - \bar{p}^{1-n}} \right]_{\bar{p}_2}^{\bar{p}} = \ln \left[\frac{1 - \bar{p}_2^{1-n}}{1 - \bar{p}^{1-n}} \right]$$

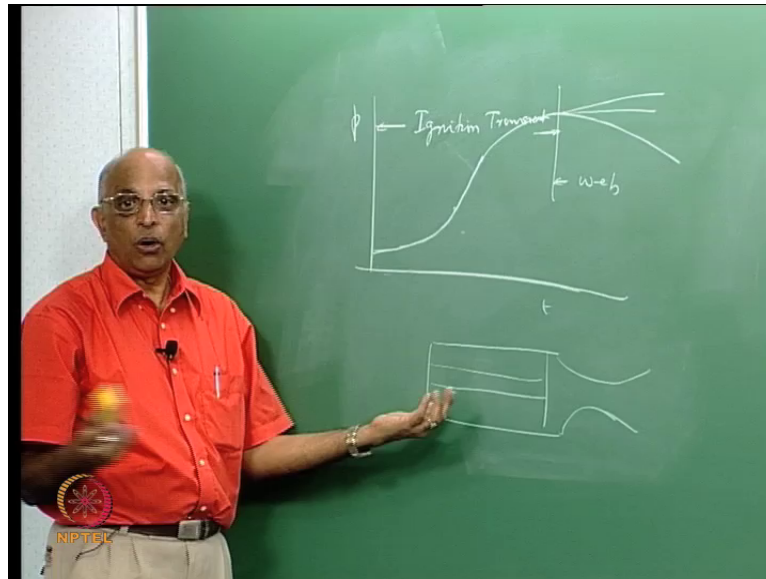
We integrate the equation between \bar{p}_2 and a value of \bar{p} corresponding to time between \bar{t}_2 and \bar{t} . The time changes from \bar{t}_2 over here to a value \bar{t} and if the time is also non dimensional \bar{t}_2 to \bar{t} . Therefore if we want to integrate we have on the left side $d\bar{p}$ by \bar{p} to the power n minus \bar{p} . And this is a standard integral which can be integrated by parts. And we get the value as natural logarithm of 1 divided by 1 minus \bar{p} to the power $1 - n$. And this changes from \bar{p}_2 to a value of \bar{p} . And the net value will therefore, be equal to \ln of 1 minus \bar{p}_2 to the power $1 - n$, divided by 1 minus \bar{p} to the power $1 - n$. When we look at the right hand side it are quite simple I get the value between \bar{t} minus \bar{t}_2 over here. The above expression is shown in the slide

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Therefore, the final expression what we get is $\bar{t}_3 - \bar{t}_2$ (non dimensional times) is equal to natural logarithm of 1 minus \bar{p}_2 to the power 1 minus n divided by 1 minus \bar{p}_3 to the power 1 minus n . If we are interested in finding out the time to reach equilibrium pressure, well I substitute the value as 3 over here; I substitute the value as equilibrium or \bar{p}_3 here which in non-dimensional form is 1. Therefore, the denominator becomes 0 and therefore, the time taken to reach the equilibrium state is infinite. Therefore the trend of variation of pressure is such that initially there is progressive increase thereafter it droops and it takes infinite time or a long time to reach the equilibrium pressure.

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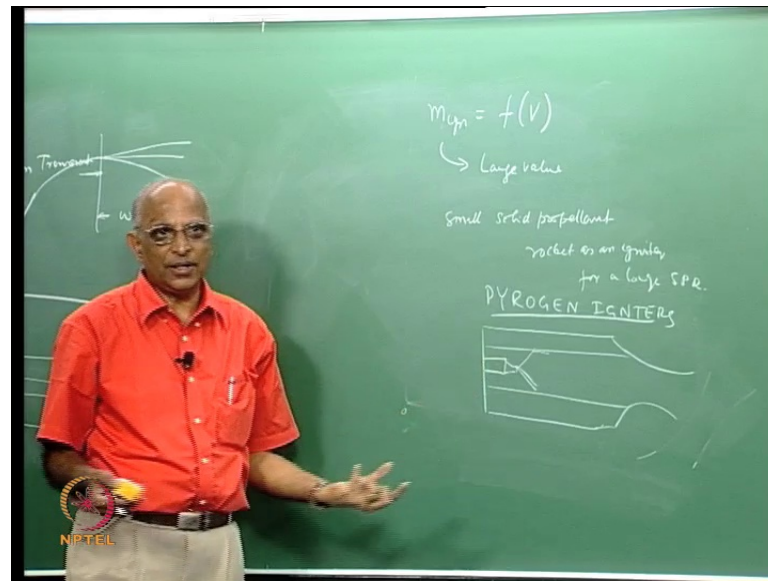


The ignition transient is such that the pressure starts off and increases but it droops later and approaches the equilibrium value. Using this trend we will try to define some characteristic times for burning and ignition subsequently. The ignition transient is seen to have a characteristic pattern, which starts increasing initially but then droops and reaches the equilibrium value. And thereafter if it is neutral burning it goes like this, if it is a progressive burning it increases thereafter while if we have regressive burning, the pressure drops further. And this is how we predict the ignition transient.

Therefore we talked in terms of pyrotechnic charge; we talked in terms of metal powders, we talked in terms of thermite igniter. Initially for a large rocket, local ignition takes place followed by flame spread and then transition to equilibrium like the rise followed by the droop. If we have a very large solid propellant rocket and most of the solid propellant rockets whether it is for missiles or whether it is a launch vehicle, the initial port volume is quite large.

The amount of charge also increases with the volume. Would we be able to calculate the mass of charge required in the igniter?

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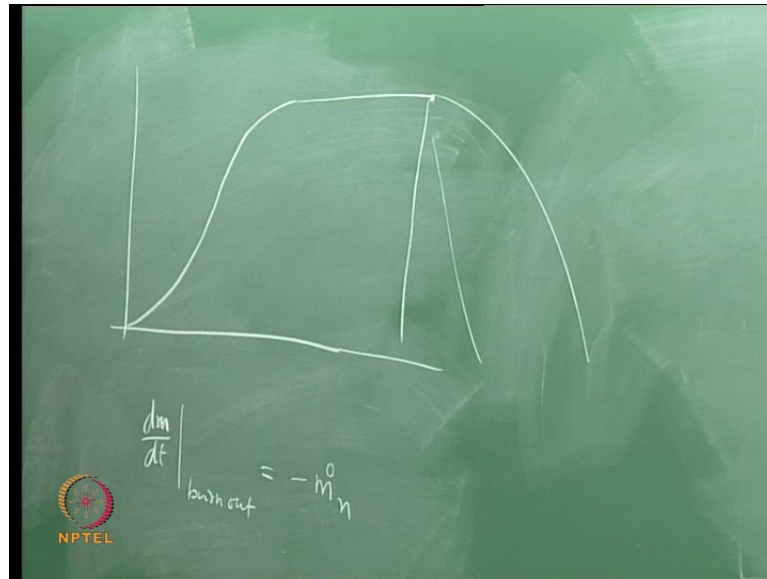


The mass of charge should be capable of increasing the pressure in the cavity to a given stable value of burning pressure and also ensure that sufficient energy is imparted for local ignition of a part of the grain surface. If we have larger cavity, we must have more charge and as the volume keeps on increasing, the mass of charge also increases. And you cannot have large amount of pyrotechnic charge as its burning is not well controlled. If we need mass of igniter as a large mass, we can as well use a small rocket itself for the rocket igniter.

That means, that we use the plume from small solid propellant rocket to ignite the large solid propellant rockets. We get the plume coming over the propellant surface, it ignites the surface of the large solid propellant rockets. The solid propellant rockets, which are used as igniters for large rockets, are known as pyrogen igniters. And almost all the solid propellant rockets, which are developed make use of pyrogen igniters. This is true for both the booster rockets and rockets for upper stages.

Pyrogen igniter is a small solid propellant rocket, which is used for igniting a large rocket, but this rocket should not contain aluminum, because the nozzle will tend to get clogged. And therefore, pyrogen igniters use non-aluminized propellants. It could be HTPB it could be PBAN or CTPB based. Therefore, this is all about igniters.

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When we ignite a solid propellant rocket, we know how to calculate the equilibrium pressure and the transient during ignition. When the rocket ceases to burn, how does the pressure fall? And we use the same theory again.

We say dm/dt after burn out of the rocket motor, is equal to well igniter is not there. All the propellant has ignited it is also not there. And only thing which is there is \dot{m}_n which is leaving the nozzle and therefore, the $dm/dt = -\dot{m}_n$. let us quickly integrate it and examine the result.

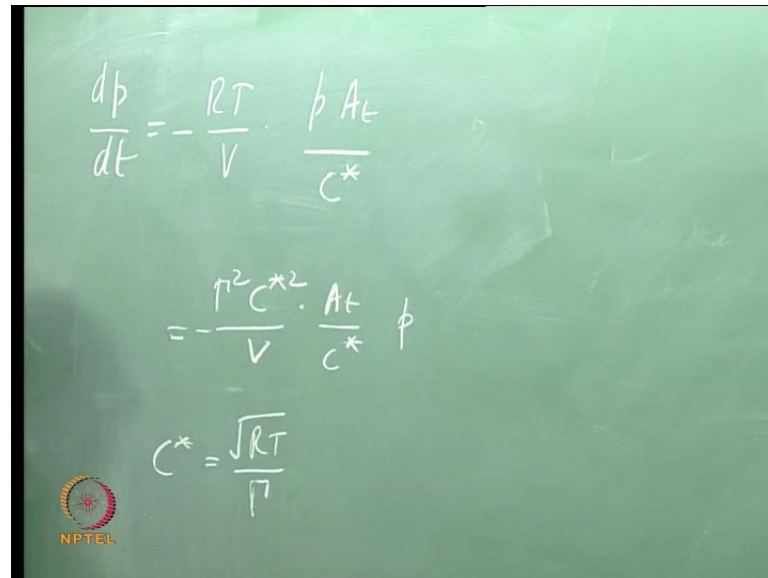
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$$\frac{dm}{dt} = \frac{dp}{dt} \left[\frac{V}{RT} \right] = - \frac{p A_t}{c^*}$$

p_T

We have $dm/dt = V/RT \times dp/dt$. What has happened the entire propellant has burnt out but the chamber pressure is still quite high. The mass leaving through the nozzle is minus chamber pressure into A_t by C^* . The volume in the chamber does not change and V is a constant because all the propellant has burnt. We have the temperature is still the higher value over here and it could be assumed as a constant. Therefore, we can write it as $V/RT \times dp/dt = -\dot{m}/n = -p \times A_t / C^*$.

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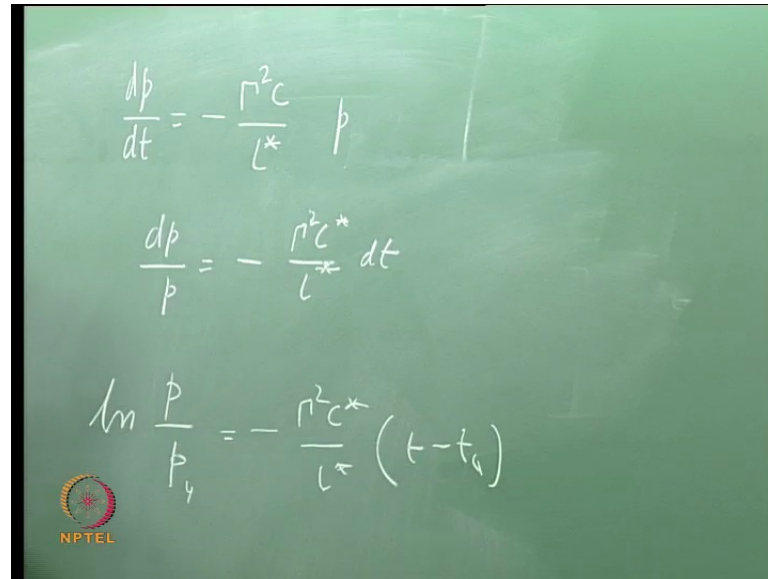
$$\frac{dp}{dt} = - \frac{RT}{V} \cdot \frac{p A_t}{C^*}$$

$$= - \frac{\Gamma^2 C^{*2}}{V} \cdot \frac{A_t}{C^*} p$$

$$C^* = \frac{\sqrt{RT}}{\Gamma}$$

Let's let us solve this equation. We have $dp/dt = - RT/V \times p \times A_t / C^*$. The product of RT can be written in terms of C^* since $C^* = \sqrt{RT/\Gamma}$. Therefore, we substitute $\Gamma^2 C^{*2}$ for RT . This is divided by V and multiplied by A_t and divided by C^* and multiplied by p .

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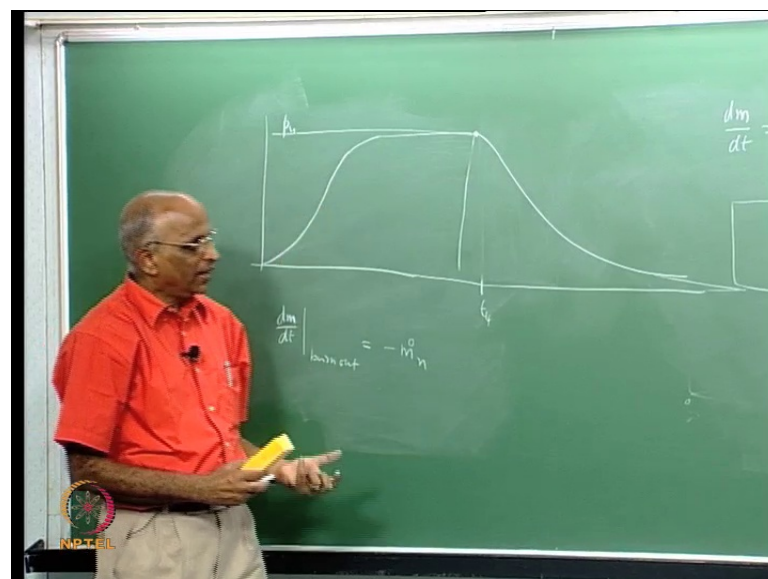
$$\frac{dp}{dt} = - \frac{\Gamma^2 C}{L^*} p$$

$$\frac{dp}{p} = - \frac{\Gamma^2 C^*}{L^*} dt$$

$$\ln \frac{p}{p_4} = - \frac{\Gamma^2 C^*}{L^*} (t - t_4)$$

We substitute V by At as equal to L^* and retain the dimensional form of the equation, we can write it as $dp/dt = - \Gamma^2 C^*/L^* \times p$. We therefore get $dp/p = - \Gamma^2 C^*/L^* \times dt$. And on integration, we get the value of natural logarithm of p divided by the equilibrium value which at this particular point of burnout is equal to p_4 viz., $\ln(p/p_4) = - \Gamma^2 C^*/L^* \times (t - t_4)$, where t_4 corresponds to the time of equilibrium pressure at which the grain got burnt out.

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And therefore, what is it we see? The pressure keeps coming down logarithmically or rather the pressure should come down and to reach ambient pressure it is going to take

infinite time. Rather the slope would decrease with time. This is all about the changing pressure in the rocket chamber once the propellant has burnt out.

We are still left with one or two small things regarding the solid propellant rockets. The characteristic times involved in a rocket and some examples of some big solid propellant rockets. We shall address them in the next class. But in the class today, we looked at ignition and igniters and we also covered briefly about how long it takes to reach the ambient pressure once the propellant grain has burnt out.